

# Laboratory 01

## Finite Element method for the Poisson equation in 1D

### Exercise 1.

Let  $\Omega = (0, 1)$ . Let us consider the Poisson problem

$$\begin{cases} -(\mu(x) u'(x))' = f(x) & x \in \Omega = (0, 1) \\ u(0) = u(1) = 0 \end{cases} \quad (1a)$$

$$(1b)$$

with  $\mu(x) = 1$  for  $x \in \Omega$ , and

$$f(x) = \begin{cases} 0 & \text{if } x \leq \frac{1}{8} \text{ or } x > \frac{1}{4}, \\ -1 & \text{if } \frac{1}{8} < x \leq \frac{1}{4}. \end{cases}$$

**1.1.** Write the weak formulation of problem (1).

**1.2.** Write the Galerkin formulation of problem (1).

**1.3.** Write the finite element formulation of problem (1), using piecewise polynomials of degree  $r$ . Write the associated linear system, and express the integrals involved by means of numerical quadrature formulas.

**1.4.** Implement in `deal.II` a finite element solver for (1), using piecewise polynomials of degree  $r = 1$  and with a number of mesh elements  $N + 1 = 20$ .