Laboratory 01
Finite Element method for the Poisson equation in 1D

Exercise 1.

Let $\Omega = (0,1)$. Let us consider the Poisson problem

$$\begin{cases} (\mu(x)\, u'(x))' = f(x) & x \in \Omega = (0,1) & \text{(1a)} \\ u(0) = u(1) = 0 & & \text{(1b)} \end{cases}$$

with $\mu(x) = 1$ for $x \in \Omega$, and

$$f(x) = \begin{cases} 0 & \text{if } x \leq \dfrac{1}{8} \text{ or } x > \dfrac{1}{4}, \\ -1 & \text{if } \dfrac{1}{8} < x \leq \dfrac{1}{4}. \end{cases}$$

1.1. Write the weak formulation of problem (1).

1.2. Write the Galerkin formulation of problem (1).

1.3. Write the finite element formulation of problem (1), using piecewise polynomials of degree r. Write the associated linear system, and express the integrals involved by means of numerical quadrature formulas.

1.4. Implement in `deal.II` a finite element solver for (1), using piecewise polynomials of degree $r = 1$ and with a number of mesh elements $N + 1 = 20$.