Linguistic Modality, Expected Utility, and Confirmation

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Overview

An expected value-based semantics:

Must/should/ought ϕ is true iff the expected measured value of ϕ is significantly higher than the expected measured values of its alternatives.

Linguistic evidence:

The semantics is derived from Korean modal expressions

Additional evidence from the psych literature:

The conjunction fallacy, the lawyers and engineers puzzle

Lawyers and engineers (Kahneman and Tversky 1973)

The lawyers and engineers scenario presented in Kahneman and Tversky (1973) has been claimed to support the hypothesis that people rely on a heuristic dubbed representativeness rather than the reliability of given evidence or prior probability in making intuitive predictions.

A panel of psychologists have interviewed and administered personality tests to 30 engineers and 70 lawyers. On the basis of this information, thumbnail descriptions of the 30 engineers and 70 lawyers have been written. Below is the thumbnail description of Jack, one of the interviewees:

(1) Jack is a 45-year-old man. He is married and has four children. He is generally conservative, careful, and ambitious. He shows no interest in political and social issues and spends most of his free time on his many hobbies which include home carpentry, sailing, and mathematical puzzles.

Lawyers and engineers

Kahneman and Tversky report a surprising result that the subjects assigned a higher probability to Jack being an engineer than him being a lawyer.

In our introspection, upon being asked to guess whether Jack is a lawyer or an engineer, it is also reasonable to utter the following:

(2) Jack must/should/ought to be an engineer.

This is just as surprising as the reported result in the original experiment, as modalized statements also seem to ignore the prior probabilities.

The conjunction fallacy (Tversky and Kahneman 1983)

To reinforce the theory of representativeness, Tversky and Kahneman (1983) introduce the famous Linda scenario which manifests the conjunction fallacy:

(3) Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

   a. Linda is a bank teller.
   b. Linda is a bank teller who is active in the feminist movement.

The standard theory (Kratzer 1981) is too rational

According to the the standard theory of modality, ‘must/should/ought ϕ’ is true if and only if ϕ follows from a certain set of assumptions.

(4) $[\![\text{ must } \phi \ ]\!]^{w} = \forall w' \in \textsc{best}(\cap f(w))(g(w)) : \phi(w') = 1$,
where f is a modal base an g is an ordering source

The set of assumptions is determined by the conversational backgrounds supplied by context.

(5) Epistemic conversational background: in view of what we know...

(6) Deontic conversational background: in view of goals/ideals...

The standard theory (Kratzer 1981) is too rational

There is no way in which ‘Jack must be an engineer’ follows from what we know about Jack.

Rather, based on the information that the interviewees consist of 30 engineers and 70 lawyers, one is more likely to infer the opposite.

Lassiter's (2011) alternative theory fares no better

Lassiter's theory significantly differs from the standard theory in that the entire theory operates on top of probability calculus.

(7) Epistemic necessity
should/*ought* ϕ is true iff $Pr(\phi) \geq \theta$

(8) Deontic necessity
should/*ought* ϕ is true iff $EU(\phi) \geq \theta$

Lassiter's theory cannot explain the problematic phenomena because again, it is a theory of rationality.

What tempts you to make weird inferences??

Are people just making dumb mistakes?

or

Is there a linguistic factor that tempts people to make those judgments?

Our explanation

Reasoning about (comparative) likelihoods is a reasoning strategy used by humans. This is reflected in the semantics of modals.

Our expected value-based modal semantics compares:

> (deontic) the expected utilities of contextually salient alternatives
>
> (epistemic) the $\underbrace{\text{explanatory values}}_{\text{irrational measure}}$ of contextually salient alternatives

Empirical consequence:

Modal semantics facilitates rational decision making, but the very same mechanism is the source of irrationality in assessing comparative likelihood!

Linguistic evidence: Korean

We derive the modal semantics in an entirely transparent manner.

In Korean, modal concepts are typically expressed in terms of a conditional and an evaluative predicate *toy* ‘EVAL’ (Chung 2019).

(9) Conditional evaluative construction

John-un	cip-ey	iss-∅-eya	toy-n-ta.
John-TOP	home-DAT	COP-PRES-only.if	EVAL-PRES-DECL

‘(Lit.) Only if John is home, it suffices.’
‘Jack must/should/ought to be home.’

Road map

- ◇ Introduce expected value-based semantics
- ◇ Case study:

  lawyers and engineers, the conjunction fallacy, the miners paradox
- ◇ Compositionally derive the semantics from Korean modal expressions

An expected value-based theory of modality

What it means to measure

We can measure various things:

i. The measured height of Lebron James is 206cm.

ii. The measured monetary value of iPhone SE is 550,000 KRW.

To interpret modal expressions, we will measure the value of a proposition (i.e., a set of worlds).

But what kind of value is being measured, and how?

Measuring the value of a world: μ_{EVAL}

The measure function μ_{EVAL} takes a world and returns the measured value of the world argument w.r.t. a conversational background R_w.

(10) $\mu_{\text{EVAL},w} = \lambda w'.\,|\,\{r \in R_w \mid r(w') = 1\}\,|$,

where R_w is the set of relevant propositions at w

As in Kratzer's (1981) standard theory, the conversational background determines the flavor of the modal.

Deontic measure

For deontics, the measure function employs a deontic conversational background, i.e., the set of relevant rules or ideals.

(11) Deontic interpretation of μ_{EVAL} at w

$$\mu_{\text{EVAL},w} = \lambda w'.\, |\, \{d \in R_{D,w} \mid d(w') = 1\}\, |\,,$$

where $R_{D,w}$ is the set of relevant rules/ideals at w

The return value of $\mu_{\text{EVAL},w}$ can be understood as the utility value of the given world argument if we assume that the utility of a world is solely determined by the information provided by R_D.

Epistemic measure

For epistemics, the measure function employs an epistemic conversational background, i.e., the set of relevant known facts (i.e., pieces of evidence).

(12) Epistemic interpretation of μ_{EVAL} at w

$$\mu_{\text{EVAL},w} = \lambda w'. \mid \{e \in R_{E,w} \mid e(w') = 1\} \mid,$$

where $R_{E,w}$ is the set of relevant known facts at w

Measuring the value of a proposition

Definition.

The measured valued of a proposition ϕ is the probability weighted average of the measured value of each ϕ-world normalized w.r.t. the probability of ϕ.

☞ expected value of $\mu_{\mathrm{EVAL},w}$ conditioned on ϕ:

(13) The measured value of a proposition ϕ at w:

$$\begin{aligned}\frac{1}{Pr(\phi)}\sum_{w_j\in\phi}\mu_{\mathrm{EVAL},w}(w_j)*Pr(\{w_j\}) &= \sum_{w_j\in\phi}\mu_{\mathrm{EVAL},w}(w_j)*\frac{Pr(\{w_j\})}{Pr(\phi)}\\ &= \sum_{w_j\in\phi}\mu_{\mathrm{EVAL},w}(w_j)*Pr(\{w_j\}\mid\phi)\\ &= \mathbb{E}_w[\mu_{\mathrm{EVAL},w}\mid\phi]\end{aligned}$$

Deontic measure = expected utility

Given that $\mu_{\text{EVAL},w}$ returns the utility value of the world argument, the expected deontic value of ϕ is by definition the (evidential) expected utility of ϕ:

(13) $$\mathbb{E}_w[\mu_{\text{EVAL},w} \mid \phi] = \sum_{w_j \in \phi} \mu_{\text{EVAL},w}(w_j) * Pr(\{w_j\} \mid \phi) = EU(\phi)$$

Epistemic measure = likelihoood-based confirmation

For epistemics, we find it more intuitive to reformulate the measure function:

(14)

$$\begin{aligned}\mu_{\text{EVAL},w} &= \lambda w'. \mid \{e \in R_{E,w} \mid e(w') = 1\} \mid \\ &= \lambda w'. \sum_{i=1}^{n} e_i(w'), \qquad \text{where } R_{E,w} = \{e_1, ..., e_n\}\end{aligned}$$

Epistemic measure = likelihoood-based confirmation

The expected epistemic value of ϕ is the sum over the likelihoods (inverse probabilities) of ϕ with respect to each relevant known fact e_i.

(15) The explanatory value of ϕ

$$\sum_{w_j \in \phi} \mu_{\text{EVAL},w}(w_j) * Pr_w(\{w_j\} \mid \phi) = \sum_{w_j \in \phi} \sum_{i=1}^{n} e_i(w_j) * Pr_w(\{w_j\} \mid \phi)$$

$$= \sum_{i=1}^{n} \sum_{w_j \in \phi} e_i(w_j) * Pr_w(\{w_j\} \mid \phi)$$

$$= \sum_{i=1}^{n} Pr_w(e_i \mid \phi),$$

We will call this measured value the explanatory value of ϕ

Proposal

(16) Expected value-based semantics of modal necessity

$$[\![\text{must/should/ought } \phi]\!]^w = (\underbrace{\mathbb{E}_w[\mu_{\text{EVAL},w} \mid \phi]}_{\text{expected value of } \phi} > \theta) \wedge \forall \psi \in \mathit{Alt}(\phi) : (\underbrace{\mathbb{E}_w[\mu_{\text{EVAL},w} \mid \psi]}_{\text{expected value of } \psi} \leq \theta)$$

Deontic reading:

ϕ is the choice with a high expected utility.

Epistemic reading:

ϕ is the hypothesis with a high explanatory value.

Comparison with Kratzer (1981, 1991)

Our theory maintains the desirable feature of Kratzer's theory, namely that modal expressions, regardless of the flavor, share a common semantic core.

This context-sensitivity nicely captures the crosslinguistic generalization that the majority of modal expressions are ambiguous in flavor.

(17) John must leave. (deontic)

(18) It must be raining. (epistemic)

However, our theory utilizes probability calculus, calculating the expected measured value of the modal prejacent and its alternatives.

Comparison with Lassiter (2011, 2017)

Our theory shares one of the key features of Lassiter's theory, namely that the interpretation of modal expressions involves probabilistic reasoning.

Despite its cheritable features, the innovation in Lassiter's theory comes at the cost of giving up the cherished feature of the standard theory, namely that modals share a common semantic core.

	Deontic measure	Epistemic measure	Unified semantics?
Lassiter	Expected utility	(Posterior) probability	No
This work	Expected utility	Explanatory value	Yes

An alternative (but not quite equivalent) formulation

(19) $$\llbracket \text{ ought}_{\text{ALT}}\ \phi\ \rrbracket^{w}$$
$$= (\forall \psi \in \mathit{Alt}(\phi) : \mathbb{E}_{w}[\mu_{\text{EVAL},w} \mid \phi] - \mathbb{E}_{w}[\mu_{\text{EVAL},w} \mid \psi] > \theta),$$
$$\text{where } C = \left\{\mathbb{E}_{w}[\mu_{\text{EVAL},w} \mid \psi] \mid \psi \in \mathit{Alt}(\phi)\right\}$$

The explanatory value/expected utility of ϕ is significantly greater than the explanatory value/expected utility of its alternatives

An alternative (but not quite equivalent) formulation

Formal epistemologists have been interested in numerically representing the degree to which a given piece of evidence confirms a hypothesis.

(20) The *L* confirmation measure (Crupi et al. 2008, Tentori et al. 2013)

$$L(h, e) = log\left(\frac{Pr(e \mid h)}{Pr(e \mid \neg h)}\right)$$

(21) Log-based alternative analysis (assuming $Alt(\phi) = \{\neg\phi\}$)

$[\![\text{ ought}_{\text{LOG,ALT}}\ \phi\ ]\!]^{w}$

$= (\forall \psi \in Alt(\phi) : log(\mathbb{E}_{w}[\mu_{\text{EVAL},w} \mid \phi]) - log(\mathbb{E}_{w}[\mu_{\text{EVAL},w} \mid \psi]) > \theta)$

$= (log(Pr_{w}(e \mid \phi)) - log(Pr_{w}(e \mid \neg\phi)) > \theta)$

$= (log\left(\frac{Pr_{w}(e|\phi)}{Pr_{w}(e|\neg\phi)}\right) > \theta)$

$= (L(\phi, e) > \theta)$

Case study

Analysis: lawyers and engineers

In short:

explanatory value of **engineer** > explanatory value of **lawyer**

The math:

(22) Reasonable probability assignments

a. $Pr_w(\textbf{not-political-social} \mid \textbf{engineer}) = 0.78$
b. $Pr_w(\textbf{enjoys-mathematical-puzzles} \mid \textbf{engineer}) = 0.55$
c. $Pr_w(\textbf{not-political-social} \mid \textbf{lawyer}) = 0.35$
d. $Pr_w(\textbf{enjoys-mathematical-puzzles} \mid \textbf{lawyer}) = 0.28$

Analysis: lawyers and engineers

'Jack ought to be an engineer' is true iff

$\mathbb{E}_w[\mu_{\text{EVAL},w} \mid \textbf{engineer}]$ is significantly greater than $\mathbb{E}_w[\mu_{\text{EVAL},w} \mid \textbf{lawyer}]$

(23) $\mathbb{E}_w[\mu_{\text{EVAL},w} \mid \textbf{engineer}] = 0.78 + 0.55 = 1.33$

(24) $\mathbb{E}_w[\mu_{\text{EVAL},w} \mid \textbf{lawyer}] = 0.35 + 0.28 = 0.63$

Analysis: the conjunction fallacy

The conjunction fallacy concerns people's intuitions about (naive) likelihood, but what we have presented in this paper is a theory of modal necessity.

However, there is a natural extension of the theory that allows us to say a word about the conjunction fallacy.

Proposal:

Define comparative likelihood in terms of explanatory values. ϕ is a better possibility than ψ iff the explanatory value of ϕ is greater than that of ψ.

If this is on the right track, we predict that people calculate the probabilities of the description being true given that Linda is a feminist bank teller or given that she is a bank teller.

(25) $\mathbb{E}_{w}[\mu_{\text{EVAL},w} \mid \textbf{teller} \wedge \textbf{feminist}] > \mathbb{E}_{w}[\mu_{\text{EVAL},w} \mid \textbf{teller}]$

The miners paradox (Kolodny and MacFarlane 2010)

(26) Ten miners are trapped either in shaft A or in shaft B, but we do not know which. Flood waters threaten to flood the shafts. We have enough sandbags to block one shaft, but not both. If we block one shaft, all the water will go into the other shaft, killing any miners inside it. If we block neither shaft, both shafts will fill halfway with water, and just one miner, the lowest in the shaft, will be killed.

a. We ought to block neither shaft.

b. If the miners are in shaft A, we ought to block shaft A.

c. If the miners are in shaft B, we ought to block shaft B.

The miners paradox: summary of possible consequences

Action	If miners in A	If miners in B
Block shaft A	All saved	All drowned
Block shaft B	All drowned	All saved
Block neither shaft	One drowned	One drowned

Failure of modus ponens?

(26) a. We ought to block neither shaft.

b. If the miners are in shaft A, we ought to block shaft A.
(if **mA** then **bA**)

c. If the miners are in shaft B, we ought to block shaft B.
if (**mB** then **bB**)

d. Either the miners are in shaft A or shaft B.
(**mA** or **mB**)

e. From (26b–d), it follows by modus ponens:

We ought to block shaft A or shaft B.
(**bA** or **bB**)

Analysis: we ought to block neither shaft

In short:

‘We ought to block neither shaft’ is true iff
block-neither has the highest expected utility

(27) Cariani et al.’s (2013) deontic conversational background:

$$R_{D,w} = \left\{ \begin{array}{c} \text{1 miner is saved,} \\ \text{2 miners are saved,} \\ \vdots \\ \text{10 miners are saved} \end{array} \right\}$$

Analysis: we ought to block neither shaft

The context guarantees that 9 miners will be saved if we block neither shaft, so μ_{EVAL} returns 9 for every **block-neither**-world.

The expected utility of **block-neither** is 9:

(28)

$$\begin{aligned}
&\mathbb{E}_w[\mu_{\text{EVAL},w}\,|\textbf{block-neither}] \\
&\quad = \sum_{w_j \in \textbf{block-neither}} \mu_{\text{EVAL},w}(w_j) * Pr_w(\{w_j\} \mid \textbf{block-neither}) \\
&\quad = \sum_{w_j \in \textbf{block-neither}} 9 * Pr_w(\{w_j\} \mid \textbf{block-neither}) \\
&\quad = 9 * \sum_{w_j \in \textbf{block-neither}} Pr_w(\{w_j\} \mid \textbf{block-neither}) \\
&\quad = 9
\end{aligned}$$

Analysis: we ought to block neither shaft

$\mu_{\text{EVAL},w}$ returns 10 for each **block-A** $\wedge$ **miners-in-A**-world, and 0 for each **block-A** $\wedge$ **miners-in-B**-world.

(29)

$$
\begin{aligned}
&\mathbb{E}_w[\mu_{\text{EVAL},w} \mid \textbf{block-A}] \\
&= \sum_{w_i \in \textbf{block-A}} \mu_{\text{EVAL},w}(w_i) * Pr_w(\{w_i\} \mid \textbf{block-A}) \\
&= \sum_{w_j \in \textbf{block-A} \wedge \textbf{miners-in-A}} \mu_{\text{EVAL},w}(w_j) * Pr_w(\{w_j\} \mid \textbf{block-A}) \\
&\quad + \sum_{w_k \in \textbf{block-A} \wedge \textbf{miners-in-B}} \mu_{\text{EVAL},w}(w_k) * Pr_w(\{w_k\} \mid \textbf{block-A}) \\
&= \sum_{w_j \in \textbf{block-A} \wedge \textbf{miners-in-A}} 10 * Pr_w(\{w_j\} \mid \textbf{block-A}) \\
&\quad + \sum_{w_k \in \textbf{block-A} \wedge \textbf{miners-in-B}} 0 * Pr_w(\{w_k\} \mid \textbf{block-A})
\end{aligned}
$$

Analysis: we ought to block neither shaft

$$\begin{aligned}
&= 10 * \sum_{w_j \in \textbf{block-A} \wedge \textbf{miners-in-A}} Pr_w(\{w_j\} \mid \textbf{block-A}) \\
&= 10 * Pr_w(\textbf{block-A} \wedge \textbf{miners-in-A} \mid \textbf{block-A}) \\
&= 10 * Pr_w(\textbf{miners-in-A} \mid \textbf{block-A}) \\
&= 10 * Pr_w(\textbf{miners-in-A}) \\
&= 5
\end{aligned}$$

By the same logic:

(30) $\mathbb{E}_w[\mu_{\text{EVAL},w} \mid \textbf{block-A}] = 5$

Analysis: if the miners are in shaft A, we ought to block shaft A

In calculating expected utility, $Pr_w()$ additionally conditions on **miners-in-A.**

This doesn't change the expected utility of **block-neither.** However, it changes the expected utilities of **block-A** and **block-B**:

(31) $[\![$ if **miners-in-A** $\wedge$ **block-A**, then EVAL $]\!]^w$

$$
\begin{aligned}
&= \mathbb{E}_w[\mu_{\text{EVAL},w} \mid \textbf{miners-in-A} \wedge \textbf{block-A}] \\
&= \sum_{w_i \in \textbf{miners-in-A} \wedge \textbf{block-A}} \mu_{\text{EVAL},w}(w_i) * Pr_w(\{w_i\} \mid \textbf{miners-in-A} \wedge \textbf{block-A}) \\
&= \sum_{w_i \in \textbf{miners-in-A} \wedge \textbf{block-A}} 10 * Pr_w(\{w_i\} \mid \textbf{miners-in-A} \wedge \textbf{block-A}) \\
&= 10
\end{aligned}
$$

(32) $[\![$ if **miners-in-A** $\wedge$ **block-B**, then EVAL $]\!]^w = 0$

Deriving the semantics from linguistic data

Recall: Korean conditional evaluatives

(33) John-un cip-ey iss-∅-eya toy-n-ta.
John-TOP home-DAT COP-PRES-only.if EVAL-PRES-DECL
'(Lit.) Only if John is home, it suffices.'
'Jack must/should/ought to be home.'

Break down of Korean conditional evaluatives:

i. if conditional
ii. evaluative predicate
iii. exhaustification

The semantics of *toy* ‘EVAL’

We analyze the evaluative predicate *toy* ‘EVAL’ as the measure function μ_{EVAL} presented earlier.

(34) $[\![\text{ EVAL }]\!]^{w} = \mu_{\text{EVAL},w} = \lambda w'. \mid \{r \in R_w \mid r(w') = 1\} \mid,$

where R_w is the set of relevant propositions at w

An expected value-based analysis of conditionals

We assume that conditionals denote the degree of support for the consequent, given the antecedent.

Technically, the value of ‘if ϕ then ψ’ is the expected value of ψ given ϕ.

(35) $[\![\text{ if } \phi, \text{ then } \psi]\!]^{w} = \mathbb{E}_{w}[\psi \mid \phi] = \sum_{w_j \in \phi} \psi(w_j) * Pr_w(\{w_j\} \mid \phi)$

Probability is a special case of expected value:

When the value of the consequent is either 0 (false) or 1 (true), the expected value reduces to the *probability* of the consequent given the antecedent.

cf. Adams (1965), Crupi and Andrea (2019), Douven (2008), Gibbard (1981), Jackson (1979), Kaufmann (2005), Lewis (1976), Pearl (2000)

Deriving the relevant measures

Simply plug the evaluative predicate *toy* 'EVAL' into the consequent!

(35) $[\![\text{ if } \phi, \text{ then } \psi]\!]^{w} = \mathbb{E}_{w}[\psi \mid \phi] = \sum_{w_j \in \phi} \psi(w_j) * Pr_w(\{w_j\} \mid \phi)$

(36) $[\![\text{ if } \phi, \text{ then } \textsc{eval}]\!]^{w} = \mathbb{E}_{w}[\mu_{\textsc{eval},w} \mid \phi] = \sum_{w_j \in \phi} \mu_{\textsc{eval},w}(w_j) * Pr(\{w_j\} \mid \phi)$

We use Lassiter's (2017) thresholding operator Θ to map a degree representation to a bivalent one.

(37) $\Theta([\![\text{ if } \phi, \text{ then } \textsc{eval}]\!]^{w}) = \mathbb{E}_{w}[\mu_{\textsc{eval},w} \mid \phi] > \theta$

Exhaustification

We simply assume that the exhaustification component of *-(e)ya* 'only if' takes a proposition ϕ and negates each of its alternatives, in addition to conveying that ϕ is true.

(38) The compositional semantics of Korean conditional evaluatives

$$
\begin{aligned}
&[\![\text{ only}_{-(e)ya}]\!]^{w}(\Theta([\![\text{ if } \phi, \text{ then } \textsc{eval}]\!]^{w}))\\
&= (\mathbb{E}_{w}[\mu_{\textsc{eval},w} \mid \phi] > \theta) \wedge \forall \psi \in \mathit{Alt}(\phi) : \neg(\mathbb{E}_{w}[\mu_{\textsc{eval},w} \mid \psi] > \theta)\\
&= \underbrace{(\mathbb{E}_{w}[\mu_{\textsc{eval},w} \mid \phi] > \theta) \wedge \forall \psi \in \mathit{Alt}(\phi) : (\mathbb{E}_{w}[\mu_{\textsc{eval},w} \mid \psi] \leq \theta)}_{\text{what we proposed as the semantics of necessity modals}}
\end{aligned}
$$

Take-home message

People make weird inferences, but for a linguistically motivated reason.

The way in which people reason about comparative likelihoods is reflected in the semantics of modals.

Modal semantics facilitates rational decision making, but the very same mechanism is the source of irrationality in assessing comparative likelihood.

Psychologists of reasoning should pay more attention to linguistic data!

Thank you!

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