

OMGTSP

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Problem Statement

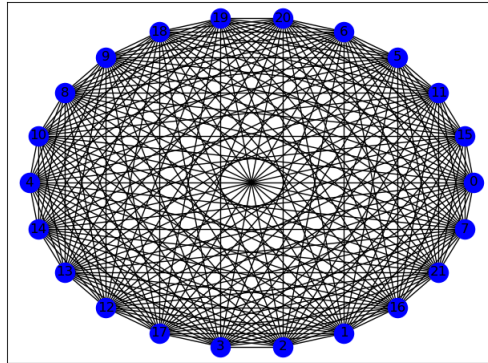
Given a complete weighted graph $G = (V, E, w)$, return a cycle containing all $|V|$ nodes with minimal weight.

Why the TSP?

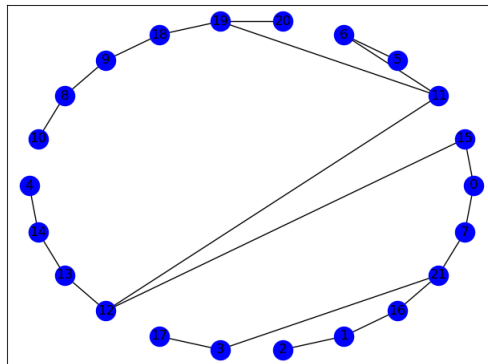
I've chosen to study the TSP because it lies at the intersection of the many diverse fields I encountered while an undergraduate student. Understanding this problem and building a good solver requires a subtle understanding of optimization, linear algebra, graph theory and computer science. Furthermore, at the time of writing, there is no efficient solution to the TSP. It is what's known as a NP-Hard problem (More on that later). Even so, there exists modern software that can solve the TSP quite quickly. The TSP lies on the boundary of what is and is not possible to do with a modern computer. For all these reasons and more, I believe the TSP is an excellent starting point for any student wanting to learn more about mathematics and CS.

Part 1: The Heuristic

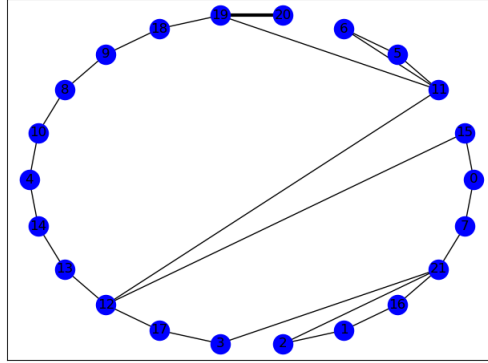
Christofides Algorithm is currently the best known polynomial time algorithm for approximating the solution to the TSP. The algorithm consists of 4 stages. Consider the complete graph $ulysses_{22}$.



First, we construct a minimum spanning tree.



By the handshaking lemma, we know that this MST will have an even number of nodes that are adjacent to an odd number of edges. These nodes are identified below.



Fourth and finally, we transform the Euclidean tour into a Hamiltonian tour (a tour in which every node is visited once). We do this by shortcutting. For example, in the above tour one can see that 21 is visited twice. Once in the path $[7,21,16]$ and another in $[3,21,2]$. To short cut, we simply take all but one of these paths and cut out 21. If we keep $[7,21,16]$, then $[3,21,2]$ becomes $[3,2]$. Since 3 and 2 were once connected to 21 but are now connected to each other, they maintain the same number of edges. 21 however is adjacent to two less edges, so every node in the graph has either the same number or less edges than before shortcutting. We repeat this process until every edge is visited exactly once.

