Learning quantum fuzzy orbits from coherent data

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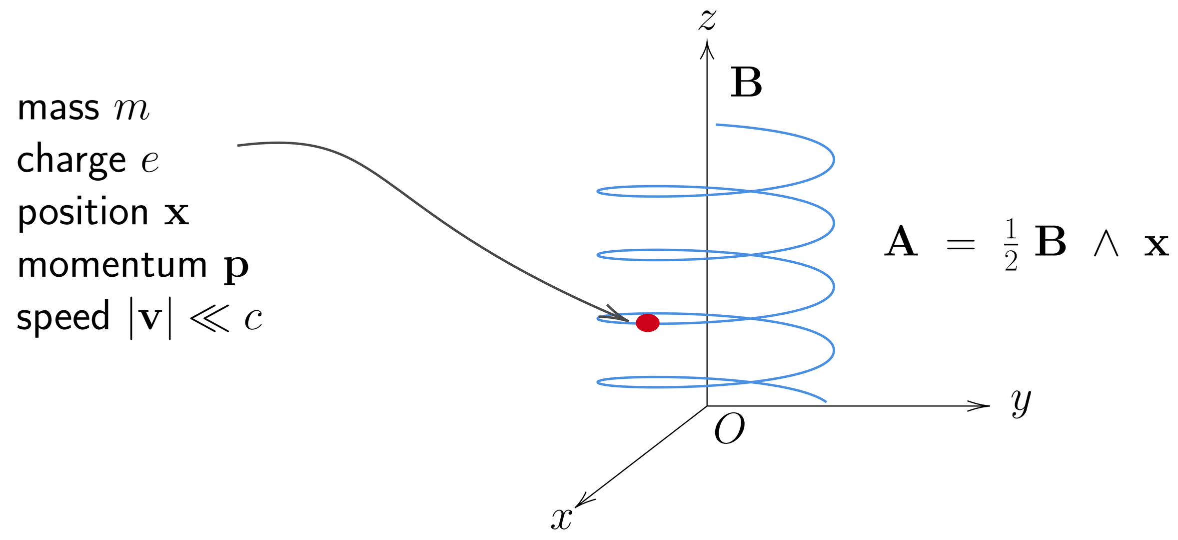
Classical Motion



For a harmonic potential $V(\mathbf{x}) = \frac{\alpha}{2m}\mathbf{x}^2$ and constant external forces $\mathbf{F}$ in the Ox and Oy directions, the Hamilton's equations of motion for the system

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{m}\begin{pmatrix} p_x + \beta y \\ p_y - \beta x \end{pmatrix}, \quad \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} p_x \\ p_y \end{pmatrix} = \frac{1}{m}\begin{pmatrix} \beta p_y - \tilde{\beta}^2 x \\ -\beta p_x - \tilde{\beta}^2 y \end{pmatrix} + \begin{pmatrix} F_x \\ F_y \end{pmatrix}, \tag{1}$$

where $\beta := eB/2c$, $\tilde{\beta}^2 := \beta^2 + \alpha$ and $\omega = 2\beta/m$ is the cyclotron frequency.

Parameterisation via UDE [1]

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = f(\mathbf{u},\ G(\mathbf{u})), \quad \mathbf{u}_0 = \mathbf{u}(t_0)$$

Parameterised equations of motion

Neural Network

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{m}\begin{pmatrix} p_x + \beta y \\ p_y - \beta x \end{pmatrix}, \quad \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} p_x \\ p_y \end{pmatrix} = \frac{1}{m}\begin{pmatrix} \beta p_y - G_1(x, \beta^2, \alpha, F_x) \\ -\beta p_x - G_2(y, \beta^2, \alpha, F_y) \end{pmatrix}. \tag{2}$$

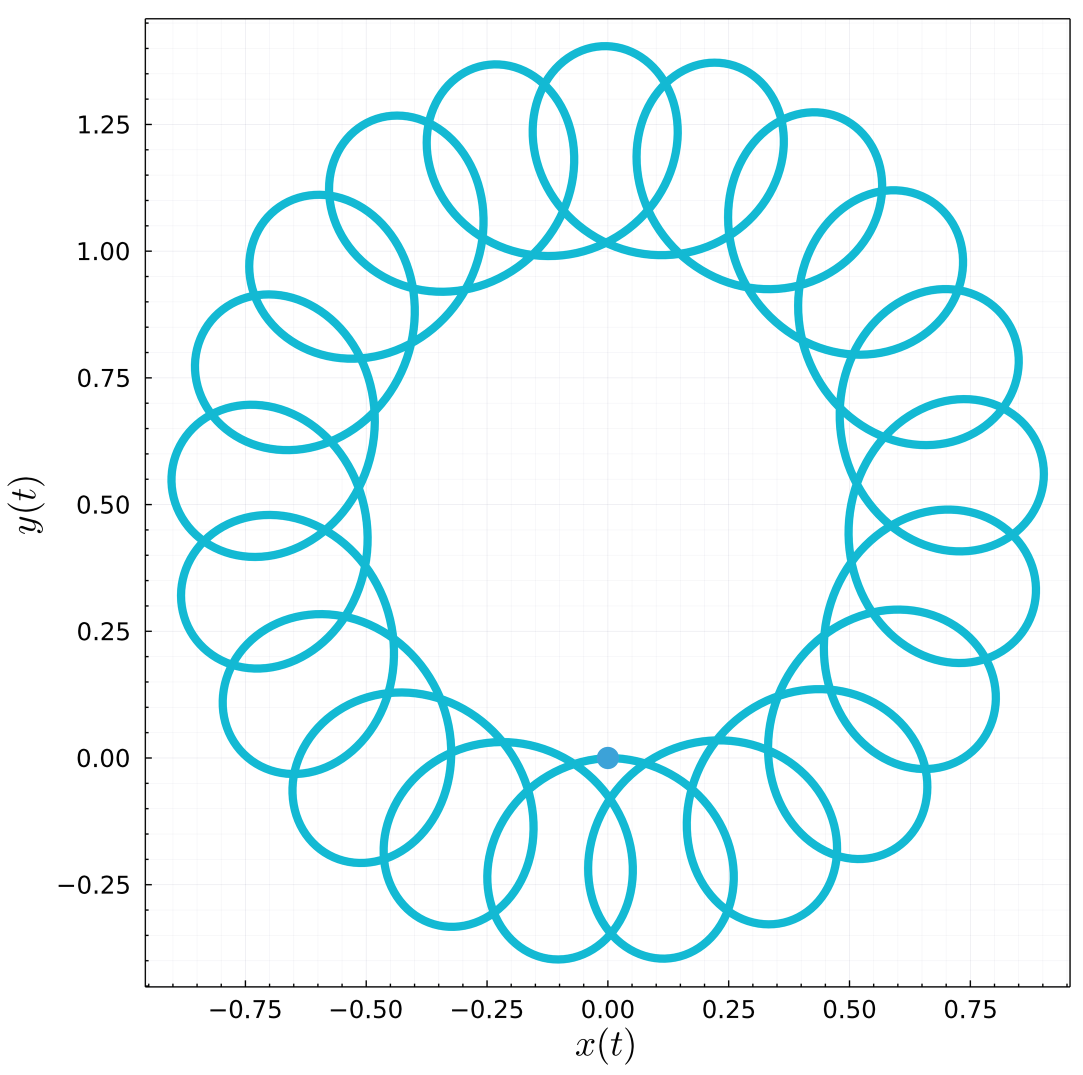


Figure 1. Exact solution to Eqs. (1) for a charged particle with initial conditions $x = y = 0, p_x = 3/2, p_y = 0$. The particle moves in an elastic field $\beta = 3$, the strength of the harmonic potential $\alpha = 2$ and $F_x = 0, F_y = 1$.

Learning and reconstructing the classical motion

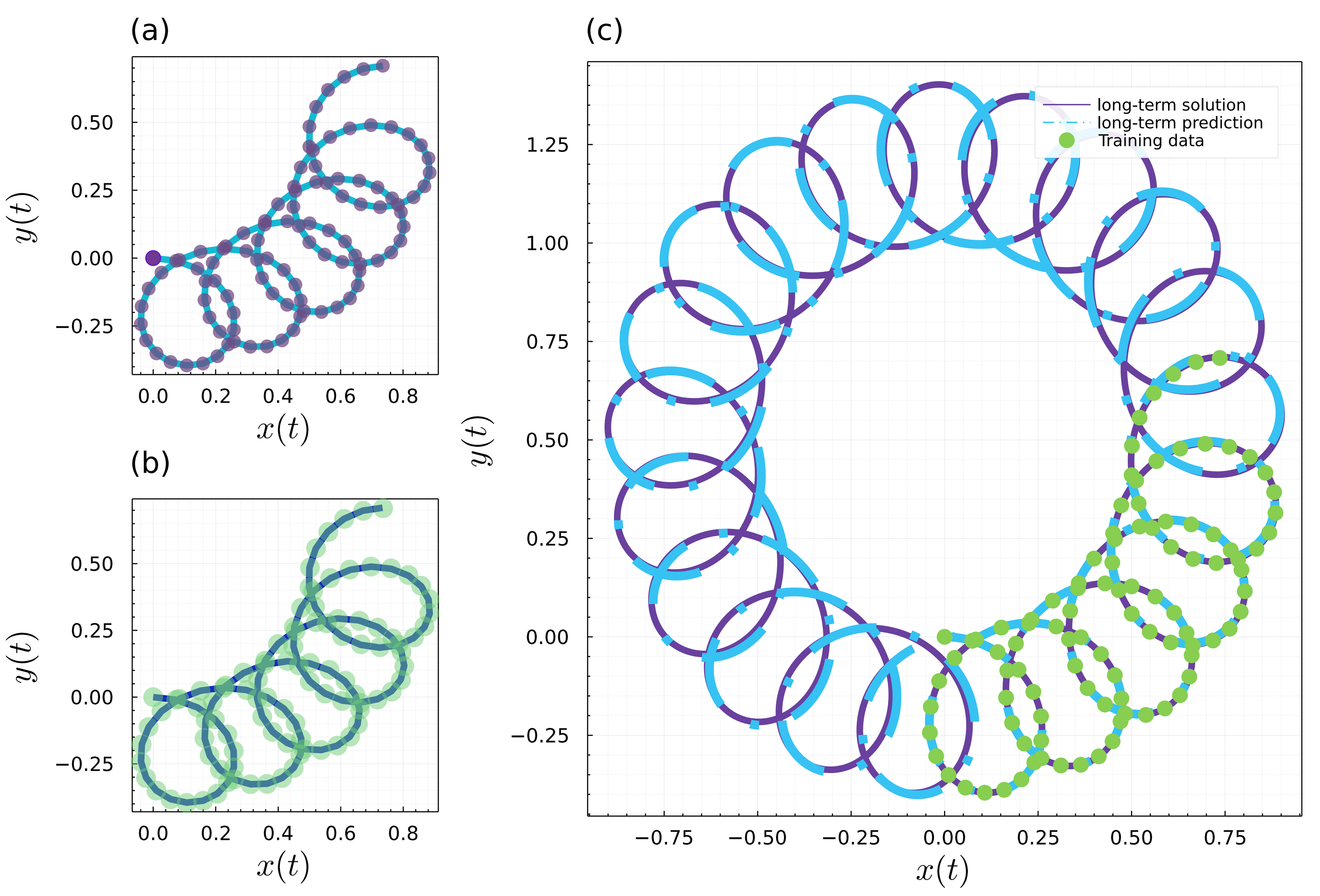


Figure 2. Learning classical orbits from noisy data through model (2). (a) Noisy data in the interval of integration $t \in [0, 5]$. (b) Learned portion of orbit through model (2). (c) Long-term prediction using model (2) in the interval of integration $t \in [0, 20]$, which closes the loop.

Quantum motion in magnetic ion-traps

Heisenberg

$$\frac{\mathrm{d}U(t,t_0)}{\mathrm{d}t} = \frac{i}{\hbar}[H(t),\ U(t,t_0)]$$

$$\mathbf{q}(t) = U^\dagger(t,t_0)\mathbf{q}U(t,t_0) = u(t)\mathbf{q}$$

$$\mathbf{A}(t) = \frac{\beta(t)}{2}\begin{pmatrix} -y \\ x \end{pmatrix}$$

$$U(t,t_0) = U_{\mathsf{osc}}(t,t_0)U_{\mathsf{rot}}(t,t_0)$$

$$[U_{\mathsf{osc}}(t,t_0), U_{\mathsf{rot}}(t,t_0)] = 0$$

$$U_{\mathsf{rot}}(t,t_0) = \exp\left(\frac{i}{\hbar}L_z \int_0^t \mathrm{d}t'\ \beta(t')\right)$$

$$U^\dagger_{\mathsf{osc}}(t,t_0)\ \mathbf{q}U_{\mathsf{osc}}(t,t_0) = b(t)\ \mathbf{q}$$

The matrix $b(t)$ forms a symplectic algebra and satisfies the evolution equation (see [2,3]):

$$\frac{\mathrm{d}b(t)}{\mathrm{d}t} = \Lambda(t)b(t); \quad \Lambda(t) = \begin{pmatrix} & 1 \\ -\beta^2(t) & \end{pmatrix}, \tag{3}$$

a possible parameterisation of this equation is:

$$\frac{\mathrm{d}b(t)}{\mathrm{d}t} = \begin{pmatrix} b_{21}(t) & b_{22}(t) \\ -G_1(b_{11}(t), \beta(t)) & -G_2(b_{12}(t), \beta(t)) \end{pmatrix}. \tag{4}$$

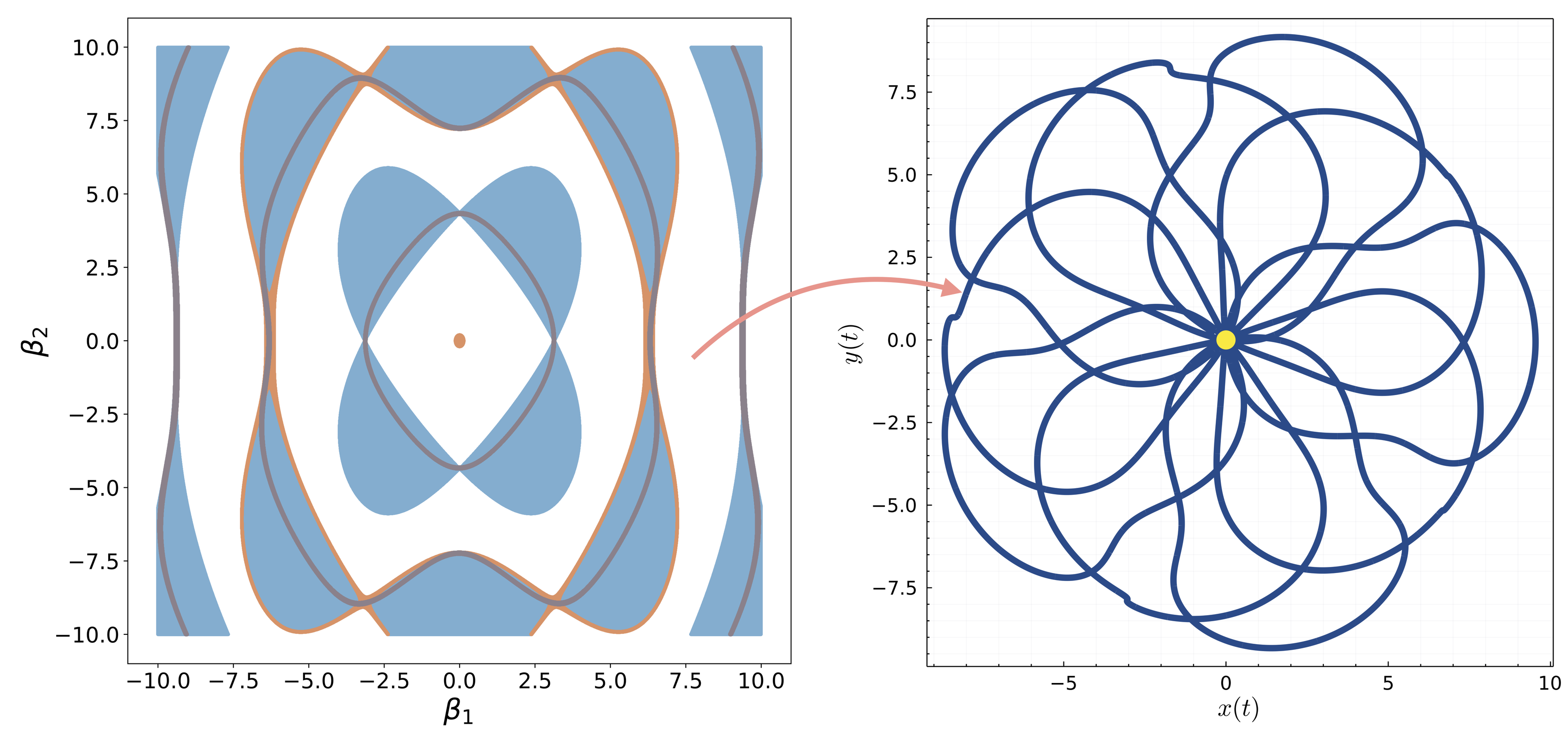


Figure 3. Left panel: Ince-Strutt diagram of the parameter space spanned by $\{\beta_1, \beta_2\}$. The clear areas (coloured) areas correspond to stable (unstable) motion. Right panel: Numerical solution to the Heisenberg's evolution problem (Mielnik's evolution loop). The quantum particle has initial conditions $x = y = 0, p_x = 10, p_y = -5$ and recovers its basal state after 25 cycles. The elastic potential is in the stability region, $\beta(t) = \beta_1 + \beta_2 \sin(\omega t)$. In the example, $(\beta_1, \beta_2) = (-1, 1)$.

Learning quantum dynamics

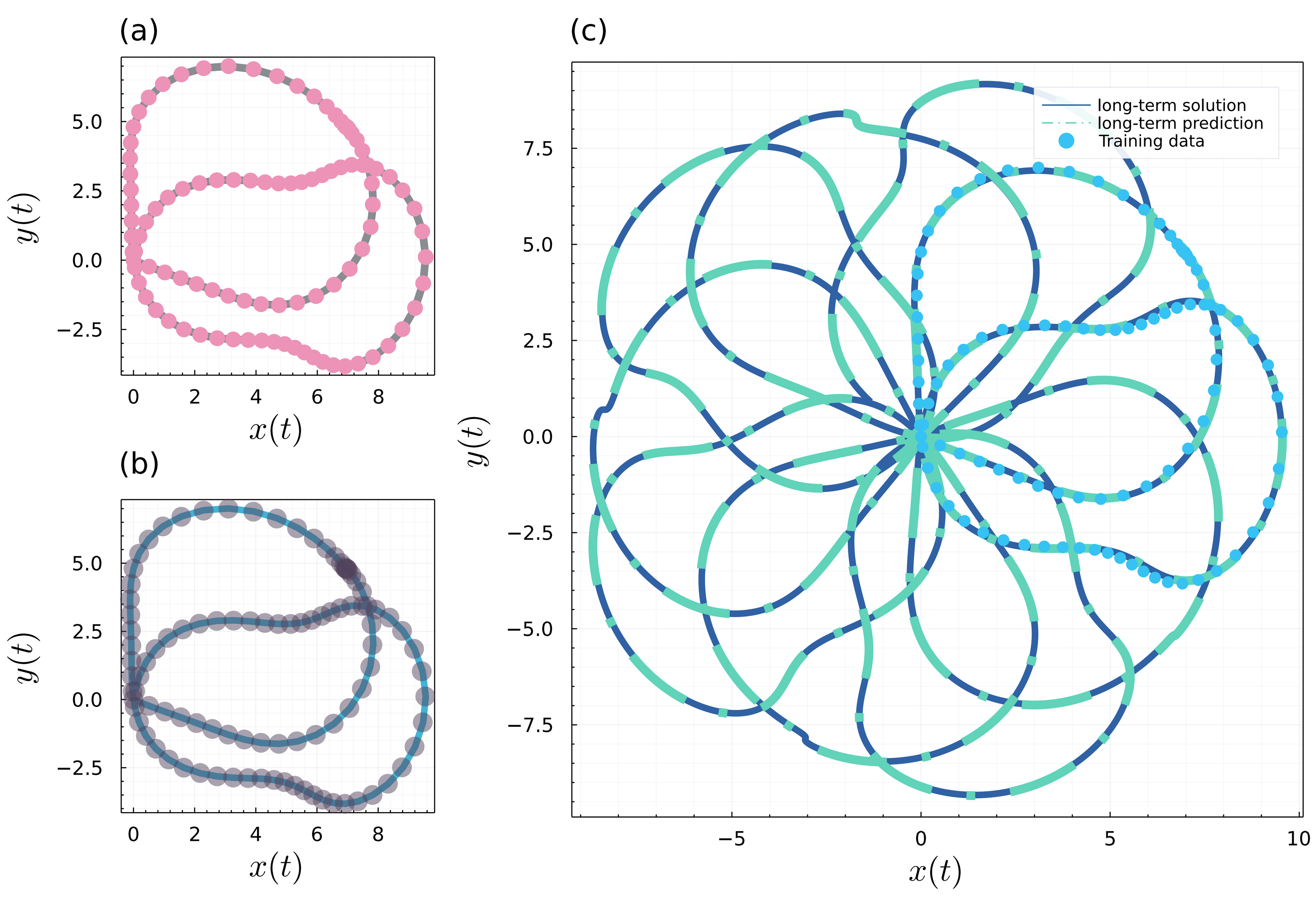


Figure 4. Reconstruction of fuzzy orbits using the symplectic model (4). **(a)** Noisy data corresponding to the eigenvalues of the position operators X and Y. **(b)** The parameterisation allows to learn from noisy data and reconstruct a portion of the fuzzy orbits. **(c)** Long-term prediction using the oscillatory model (4). The parameters β_1 and β_2 could be identified from the learning data leading to stable motion.

References

1 C. Rackauckas et al., *Universal Differential Equations for Scientific Machine Learning*, arXiv:2001.04385 (2021).

2 J. Fuentes, *Quantum control operations with fuzzy evolution trajectories based on polyharmonic magnetic fields*, Sci. Rep. 10, 22256 (2020).

3 B. Mielnik & A. Ramírez, *Ion traps: some semiclassical observations*, Phys. Scr. 82 055002 (2010).

* Code available on https://github.com/fuentesigma/cyclotronReconstruction