Numerical Analysis
homework 12: RLC Circult

Due on Tuesday, May 22, 2017

102061149 Fu-En Wang

1 Introduction

For a simple RLC circuit as Figure 1:

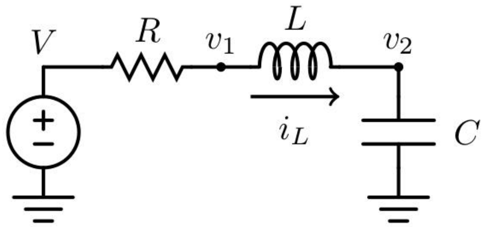

Figure 1: RLC circuit

The system consists of several ordinary differential equations:

$$\frac{v_1 - V}{R} + i_L = 0 \tag{1}$$

$$\frac{dv_2}{dt} = \frac{i_L}{C} \tag{2}$$

$$\frac{di_L}{dt} = \frac{v_1 - v_2}{L} \tag{3}$$

In this homework, we will implement three algorithm to solve the ODE above.

1.1 Forward Euler

Forward Euler is to model ODE as

$$x(t+h) = x(t) + h * f(t)$$

and the system can be derived as

$$v_1(t+h) = V + Ri_L(t+h) \tag{4}$$

$$v_2(t+h) = v_2(t) + \frac{h}{C} i_L(t) \tag{5}$$

$$i_L(t+h) = i_L(t) + \frac{h}{L}(v_1(t) - v_2(t)) \tag{6}$$

1.2 Backward Euler

Backward Euler is to model ODE as

$$x(t+h) = x(t) + h * f(t+h)$$

and the system can be derived as

$$v_1(t+h) = V + Ri_L(t+h) \tag{7}$$

$$v_2(t+h) = v_2(t) + \frac{h}{C} i_L(t+h) \tag{8}$$

$$i_L(t+h) = i_L(t) + \frac{h}{L}(v_1(t+h) - v_2(t+h)) \tag{9}$$

After some mathematical tricks, $i_L(t+h)$ can be derived as

$$i_L(t+h) = \frac{i_L(t) - \frac{h}{L} v_2(t) + \frac{hV}{L}}{1 + \frac{h^2}{LC} + \frac{hR}{L}} \tag{10}$$

1.3 Trapezoidal

Trapezoidal is to model ODE as

$$x(t+h) = x(t) + h * \frac{f(t+h) + f(t)}{2}$$

and the system can be derived as

$$v_1(t+h) = V + R\frac{i_L(t+h) + i_L(t)}{2} \tag{11}$$

$$v_2(t+h) = v_2(t) + \frac{h}{2C}(i_L(t+h) + i_L(t)) \tag{12}$$

$$i_L(t+h) = i_L(t) + \frac{h}{2L}(v_1(t+h) - v_2(t+h) + v_1(t) - v_2(t)) \tag{13}$$

After some mathematical tricks, $i_L(t+h)$ can be derived as

$$i_L(t+h) = \frac{(1 - \frac{h^2}{4LC})i_L(t) + \frac{h}{2L}(V + v_1(t) - 2v_2(t))}{1 + \frac{h^2}{4LC} + \frac{hR}{2L}} \tag{14}$$

2 Implementation

Algorithm 1 Ordinary Differential Equation

```
t = start
while t < end_time do
    Compute x(t + h)
    t += h
end while
```

3 Discussion

In this section, we will discuss the result for different h and algorithm.

3.1 Forward Euler

When h = 0.1, Figure 2 shows v1, v2 and i_L for $h \leq t \leq 10$

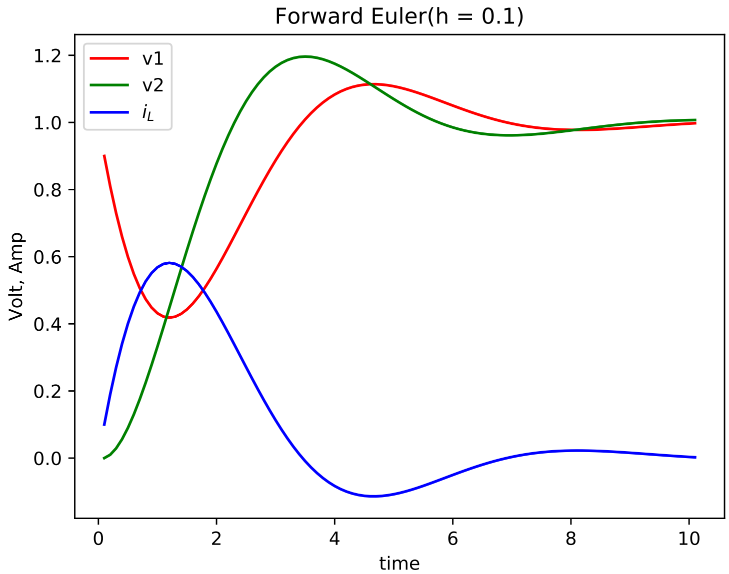

Figure 2: Forward Euler(h = 0.1)

For the maximum and minimum value oof v1, v2 and i_L, Table 1 shows the result.

Forward	V1	V2	iL
Max	1.1139	1.19627	0.581654
Min	0.418346	0	-0.113905

Table 1: Max/Min of Forward Euler(h = 0.1)

When h = 0.01, Figure 3 shows v1, v2 and i_L for $h \leq t \leq 10$

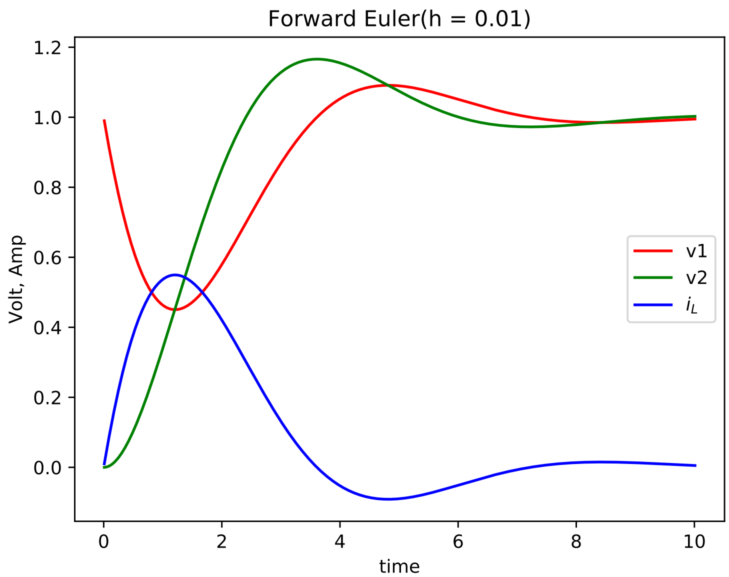

Figure 3: Forward Euler(h = 0.01)

For the maximum and minimum value oof v1, v2 and i_L, Table 2 shows the result.

Forward	V1	V2	iL
Max	1.09125	1.16602	0.549617
Min	0.450383	0	-0.0912488

Table 2: Max/Min of Forward Euler(h = 0.01)

3.2 Backward Euler

When h = 0.1, Figure 4 shows v1, v2 and i_L for $h \leq t \leq 10$

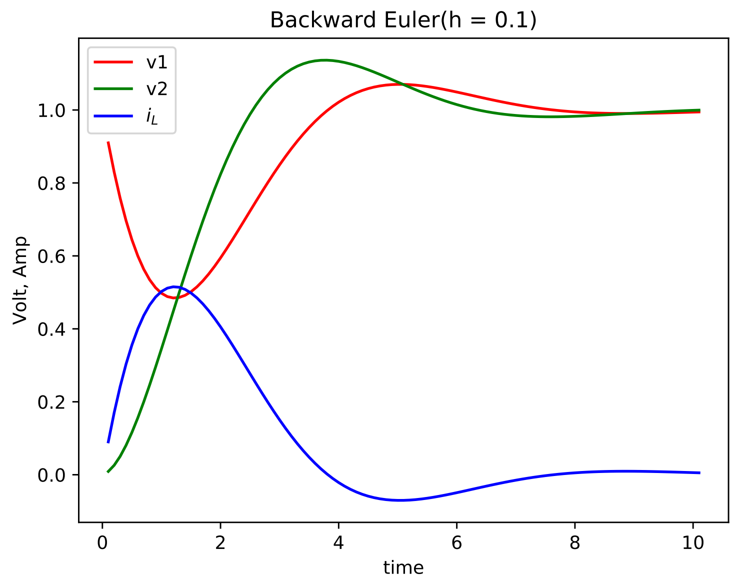

Figure 4: Backward Euler(h = 0.1)

For the maximum and minimum value oof v1, v2 and i_L, Table 3 shows the result.

Backward	V1	V2	iL
Max	1.07026	1.13651	0.515275
Min	0.484725	0	-0.0702564

Table 3: Max/Min of Backward Euler(h = 0.1)

When h = 0.01, Figure 5 shows v1, v2 and i_L for $h \leq t \leq 10$

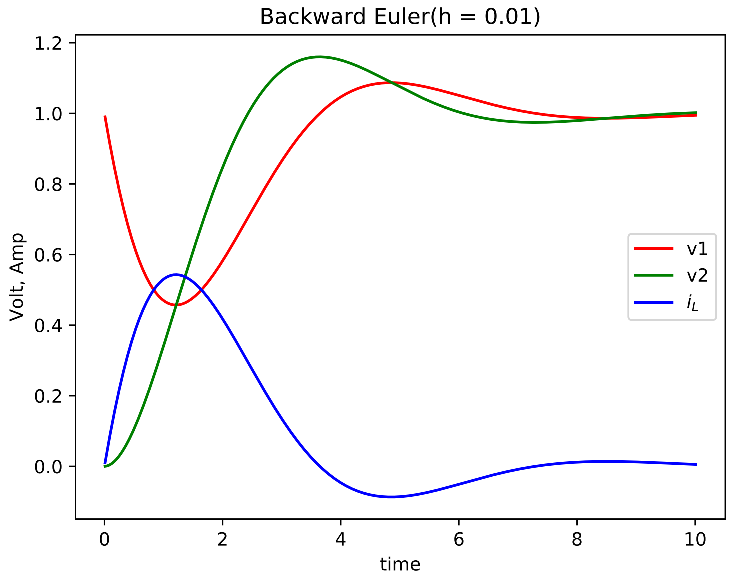

Figure 5: Backward Euler(h = 0.01)

For the maximum and minimum value oof v1, v2 and i_L, Table 4 shows the result.

Backward	V1	V2	iL
Max	1.08694	1.16011	0.543012
Min	0.456988	0	-0.0869399

Table 4: Max/Min of Backward Euler(h = 0.01)

3.3 Trapezoidal

When h = 0.1, Figure 6 shows v1, v2 and i_L for $h \leq t \leq 10$

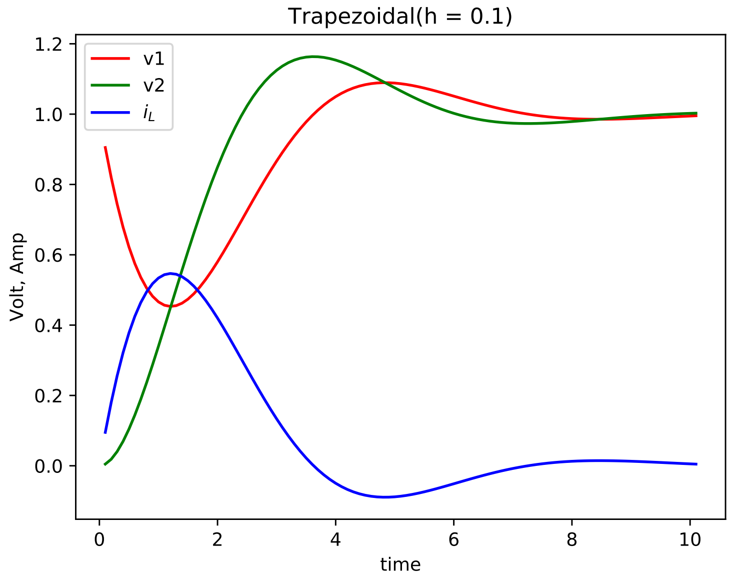

Figure 6: Trapezoidal(h = 0.1)

For the maximum and minimum value oof v1, v2 and i_L, Table 5 shows the result.

Trap	V1	V2	iL
Max	1.08936	1.16346	0.546816
Min	0.453184	0	-0.0893592

Table 5: Max/Min of Trapezoidal(h = 0.1)

When h = 0.01, Figure 7 shows v1, v2 and i_L for $h \leq t \leq 10$

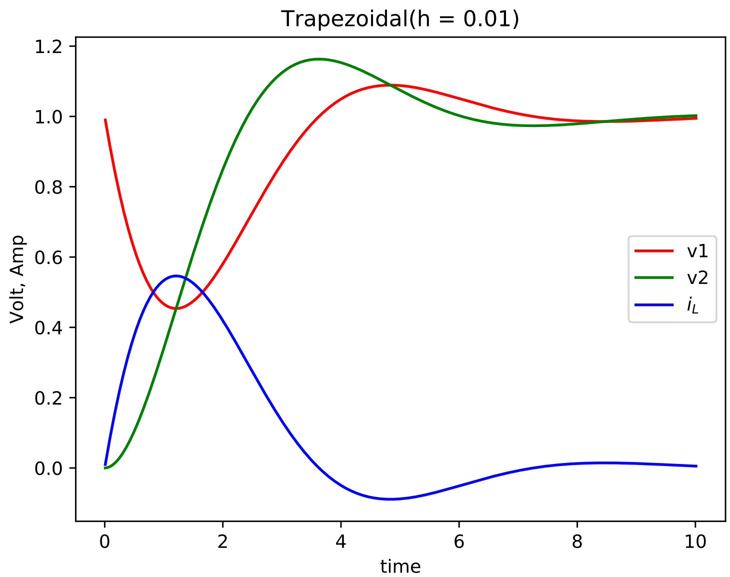

Figure 7: Trapezoidal(h = 0.01)

For the maximum and minimum value oof v1, v2 and i_L, Table 6 shows the result.

Trap	V1	V2	iL
Max	1.08907	1.16304	0.546298
Min	0.453702	0	-0.0890672

Table 6: Max/Min of Trapezoidal(h = 0.01)

3.4 Comparison

For convenient purpose, I merge the four tables above as Table 7 and 8

Forward	V1	V2	iL
Max	1.1139	1.19627	0.581654
Min	0.418346	0	-0.113905
Backward	V1	V2	iL
Max	1.07026	1.13651	0.515275
Min	0.484725	0	-0.0702564
Trap	V1	V2	iL
Max	1.08936	1.16346	0.546816
Min	0.453184	0	-0.0893592

Table 7: Max/Min(h = 0.1)

Forward	V1	V2	iL
Max	1.09125	1.16602	0.549617
Min	0.450383	0	-0.0912488
Backward	V1	V2	iL
Max	1.08694	1.16011	0.543012
Min	0.456988	0	-0.0869399
Trap	V1	V2	iL
Max	1.08907	1.16304	0.546298
Min	0.453702	0	-0.0890672

Table 8: Max/Min(h = 0.01)

From the two tables, we can find that the value of Trapezoidal is between that of Forward and Backward Euler. Recall from Section 1.3, the formula of Trapezoidal is:

$$x(t+h) = x(t) + h * \frac{f(t+h) + f(t)}{2}$$

The part $\frac{f(t+h)+f(t)}{2}$ is the mean value of that of Forward and Backward. As a result, it seems reasonable we got a value between the two method.