Numerical Analysis
homework 06: Matrix Condition Numbers

Due on Tuesday, April 11, 2017

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1   Introduction

In this homework, we will implement the Power Method algorithm to find certain eigenvalues of a given matrix A.

1.1   Termination Condition

To calculate error of each iteration, we will use four kind of error fomula:

1. $\epsilon_1 = |V^{(k+1)} - V^k|$

2. $\epsilon_2 = \|q^{(k+1)} - q^k\|_2$

3. $\epsilon_3 = \|r^{(k+1)}\|$

4. $\epsilon_4 = \frac{\|r^{(k+1)}\|}{|(W^k)^T q^k|}$

where $r^k = Aq^k - V^k q^k$ and $W^k = \frac{(q^k)^T A}{\|(q^k)^T A\|_2}$. In this project, we need to use the four error to test Power Method and find out which error we prefer.

1.2   Power Method

We will implement three Power Method algorithm to find eigenvalue.

1. **Power Method**(to find largest eigenvalue).

2. **Inverse Power Method**(to find smallest eigenvalue).

3. **Inverse Power Method with Shift**(to find eigenvalue closest to ω).

1.3   Condition Numbers

Condition Numbers is defined as

$$k = \frac{\lambda_1}{\lambda_2}$$

We need to find the condition numbers of the following resistor network.

1. 2×2 resistor network

2. 4×4 resistor network

3. 10×10 resistor network

4. 20×20 resistor network

5. 40×40 resistor network

6. 50×50 resistor network

2   Algorithm

Algorithm 1 Power Method

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for each it $\in \{1, ..., \text{maxIter}\}$ **do**
   $q^{(k+1)} = \frac{q^k = A^k q^0}{\|q^k = A^k q^0\|}$
   $V^k = (q^k)^T A q^k$
end for