

Numerical Analysis
homework 12: RLC Circuit

Due on Tuesday, May 22, 2017

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1 Introduction

For a simple RLC circuit as Figure 1:

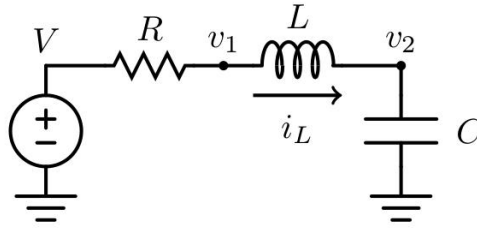


Figure 1: RLC circuit

The system consists of several ordinary differential equations:

$$\frac{v_1 - V}{R} + i_L = 0 \quad (1)$$

$$\frac{dv_2}{dt} = \frac{i_L}{C} \quad (2)$$

$$\frac{di_L}{dt} = \frac{v_1 - v_2}{L} \quad (3)$$

In this homework, we will implement three algorithm to solve the ODE above.

1.1 Forward Euler

Forward Euler is to model ODE as

$$x(t + h) = x(t) + h * f(t)$$

and the system can be derived as

$$v_1(t + h) = V + Ri_L(t + h) \quad (4)$$

$$v_2(t + h) = v_2(t) + \frac{h}{C}i_L(t) \quad (5)$$

$$i_L(t + h) = i_L(t) + \frac{h}{L}(v_1(t) - v_2(t)) \quad (6)$$

1.2 Backward Euler

Backward Euler is to model ODE as

$$x(t + h) = x(t) + h * f(t + h)$$

and the system can be derived as

$$v_1(t + h) = V + Ri_L(t + h) \quad (7)$$

$$v_2(t + h) = v_2(t) + \frac{h}{C}i_L(t + h) \quad (8)$$

$$i_L(t + h) = i_L(t) + \frac{h}{L}(v_1(t + h) - v_2(t + h)) \quad (9)$$

After some mathematical tricks, $i_L(t + h)$ can be derived as

$$i_L(t + h) = \frac{i_L(t) - \frac{h}{L}v_2(t) + \frac{hV}{L}}{1 + \frac{h^2}{LC} + \frac{hR}{L}} \quad (10)$$

1.3 Trapezoidal

Trapezoidal is to model ODE as

$$x(t+h) = x(t) + h * \frac{f(t+h) + f(t)}{2}$$

and the system can be derived as

$$v_1(t+h) = V + R \frac{i_L(t+h) + i_L(t)}{2} \quad (11)$$

$$v_2(t+h) = v_2(t) + \frac{h}{2C} (i_L(t+h) + i_L(t)) \quad (12)$$

$$i_L(t+h) = i_L(t) + \frac{h}{2L} (v_1(t+h) - v_2(t+h) + v_1(t) - v_2(t)) \quad (13)$$

After some mathematical tricks, $i_L(t+h)$ can be derived as

$$i_L(t+h) = \frac{(1 - \frac{h^2}{4LC})i_L(t) + \frac{h}{2L}(V + v_1(t) - 2v_2(t))}{1 + \frac{h^2}{4LC} + \frac{hR}{2L}} \quad (14)$$

2 Implementation

Algorithm 1 Ordinary Differential Equation

```

t = start
while t < end_time do
    Compute x(t+h)
    t += h
end while

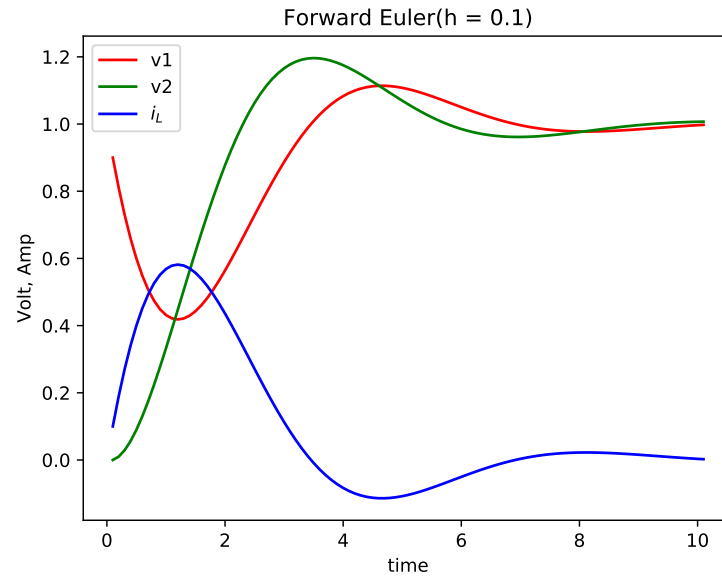
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3 Discussion

In this section, we will discuss the result for different h and algorithm.

3.1 Forward Euler

When $h = 0.1$, Figure 2 shows v_1 , v_2 and i_L for $h \leq t \leq 10$

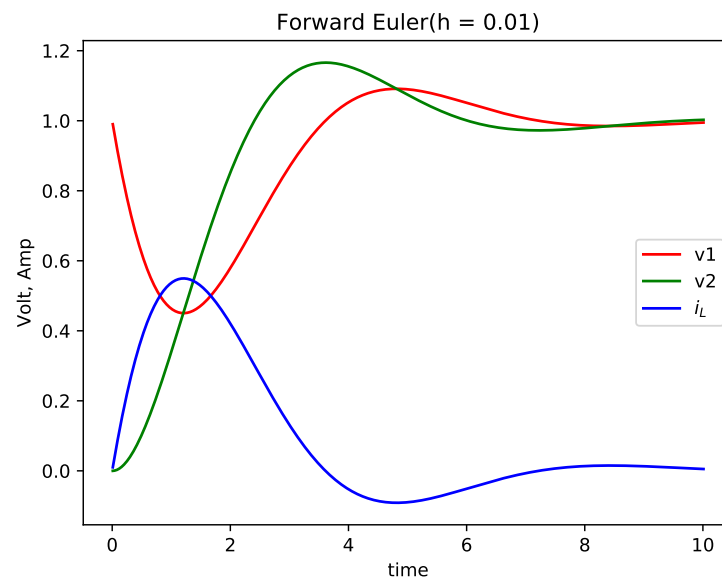
Figure 2: Forward Euler($h = 0.1$)

For the maximum and minimum value of v_1 , v_2 and i_L , Table 1 shows the result.

Forward	V1	V2	iL
Max	1.1139	1.19627	0.581654
Min	0.418346	0	-0.113905

Table 1: Max/Min of Forward Euler($h = 0.1$)

When $h = 0.01$, Figure 3 shows v_1 , v_2 and i_L for $h \leq t \leq 10$

Figure 3: Forward Euler($h = 0.01$)

For the maximum and minimum value of v_1 , v_2 and i_L , Table 2 shows the result.

Forward	V1	V2	iL
Max	1.09125	1.16602	0.549617
Min	0.450383	0	-0.0912488

Table 2: Max/Min of Forward Euler($h = 0.01$)

3.2 Backward Euler

When $h = 0.1$, Figure 4 shows v_1 , v_2 and i_L for $h \leq t \leq 10$

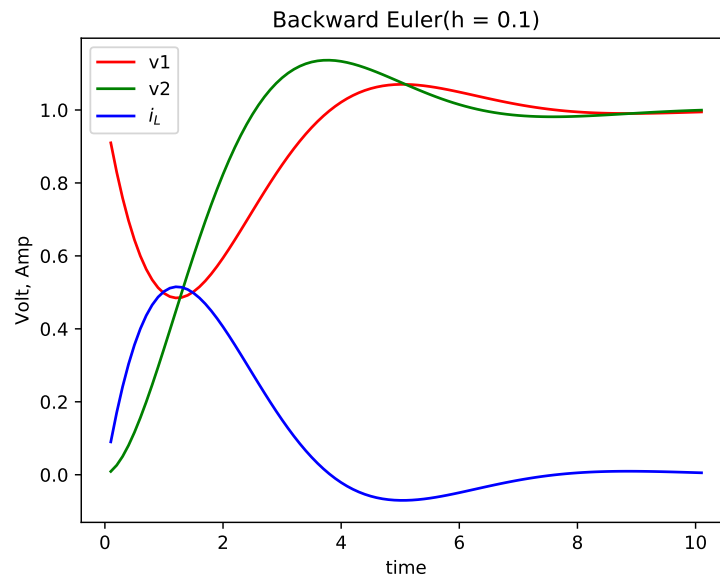


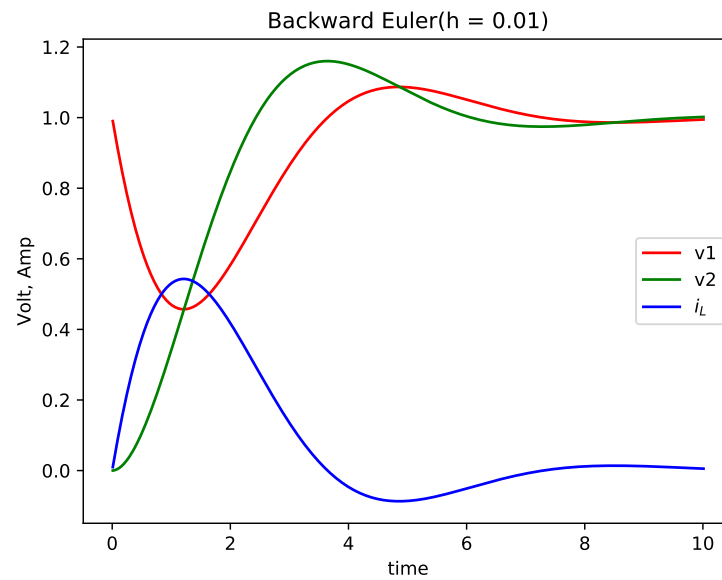
Figure 4: Backward Euler($h = 0.1$)

For the maximum and minimum value of v_1 , v_2 and i_L , Table 3 shows the result.

Backward	V1	V2	iL
Max	1.07026	1.13651	0.515275
Min	0.484725	0	-0.0702564

Table 3: Max/Min of Backward Euler($h = 0.1$)

When $h = 0.01$, Figure 5 shows v_1 , v_2 and i_L for $h \leq t \leq 10$

Figure 5: Backward Euler($h = 0.01$)

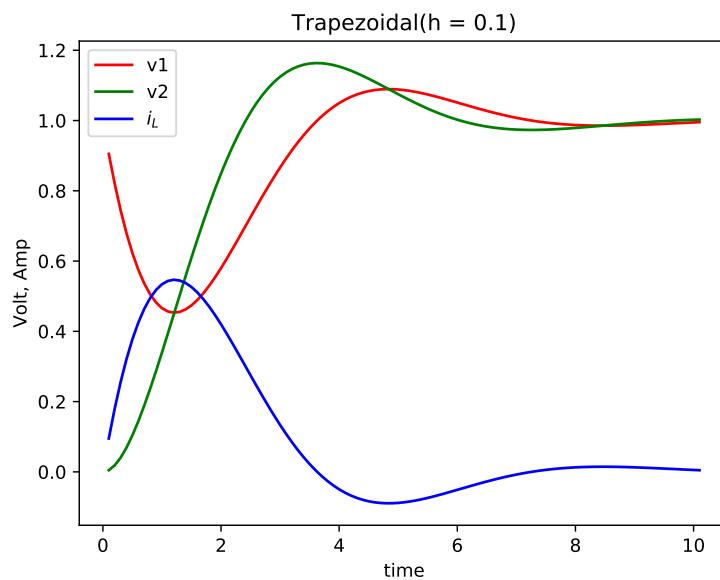
For the maximum and minimum value of v_1 , v_2 and i_L , Table 4 shows the result.

Backward	V1	V2	iL
Max	1.08694	1.16011	0.543012
Min	0.456988	0	-0.0869399

Table 4: Max/Min of Backward Euler($h = 0.01$)

3.3 Trapezoidal

When $h = 0.1$, Figure 6 shows v_1 , v_2 and i_L for $h \leq t \leq 10$

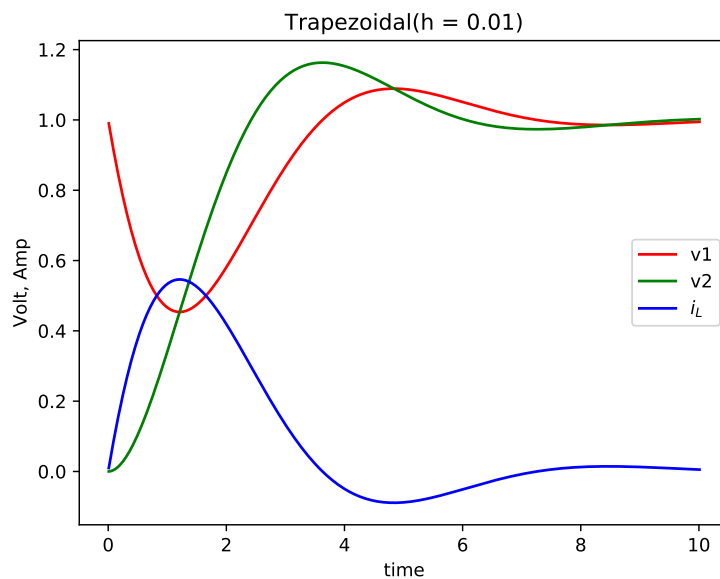
Figure 6: Trapezoidal($h = 0.1$)

For the maximum and minimum value of v_1 , v_2 and i_L , Table 5 shows the result.

Trap	V1	V2	iL
Max	1.08936	1.16346	0.546816
Min	0.453184	0	-0.0893592

Table 5: Max/Min of Trapezoidal($h = 0.1$)

When $h = 0.01$, Figure 7 shows v_1 , v_2 and i_L for $h \leq t \leq 10$

Figure 7: Trapezoidal($h = 0.01$)

For the maximum and minimum value of v_1 , v_2 and i_L , Table 6 shows the result.

Trap	V1	V2	iL
Max	1.08907	1.16304	0.546298
Min	0.453702	0	-0.0890672

Table 6: Max/Min of Trapezoidal($h = 0.01$)

3.4 Comparison

For convenient purpose, I merge the four tables above as Table 7 and 8

Forward	V1	V2	iL
Max	1.1139	1.19627	0.581654
Min	0.418346	0	-0.113905
Backward	V1	V2	iL
Max	1.07026	1.13651	0.515275
Min	0.484725	0	-0.0702564
Trap	V1	V2	iL
Max	1.08936	1.16346	0.546816
Min	0.453184	0	-0.0893592

Table 7: Max/Min($h = 0.1$)

Forward	V1	V2	iL
Max	1.09125	1.16602	0.549617
Min	0.450383	0	-0.0912488
Backward	V1	V2	iL
Max	1.08694	1.16011	0.543012
Min	0.456988	0	-0.0869399
Trap	V1	V2	iL
Max	1.08907	1.16304	0.546298
Min	0.453702	0	-0.0890672

Table 8: Max/Min($h = 0.01$)

From the two tables, we can find that the value of Trapezoidal is between that of Forward and Backward Euler. Recall from Section 1.3, the formula of Trapezoidal is:

$$x(t+h) = x(t) + h * \frac{f(t+h) + f(t)}{2}$$

The part $\frac{f(t+h)+f(t)}{2}$ is the mean value of that of Forward and Backward. As a result, it seems reasonable we got a value between the two method.