Numerical Analysis

Homework 4. Linear Iterative Methods

Due: March 28, 2017

Linear iterative methods are easy to implement, in this homework you'll implement three basic iterative solution methods: Jacobi, Gauss-Seidel and symmetric Gauss-Seidel. To enable reuse of these methods, you should add the following function declarations to the `MAT.h` header file and their implementations to the `MAT.cpp` file.

```
int jacobi(MAT &A,VEC b,VEC &x,int maxIter,double tol);
int gaussSeidel(MAT &A,VEC b,VEC &x,int maxIter,double tol);
int sgs(MAT &A,VEC b,VEC &x,int maxIter,double tol);
```

All functions return an integer that is the number of iterations to reach the solution accuracy less than `tol`. Thus, if it returns `maxIter` then the solution `x` is not convergent to the desired accuracy yet. And all these functions have 5 arguments. Their meanings are

`A`: the matrix of the linear system,
`b`: the right hand side vector of the linear system,
`x`: an initial guess for the iterative solution method and the final solution when function returns,
`maxIter`: maximum number of iterations allowed,
`tol`: accuracy tolerance of the iterative method.

The first 4 arguments should be easy to understand. The last argument checks for the error of the iterations. If the error of the current solution to the real solution is small enough, then the iterative process can be terminated. In most cases, the solution is not known beforehand. Thus, in practice one checks for the difference of solution vectors between iterations. If the difference is smaller than `tol` then the iteration is terminated. Since these three iterative methods are stationary with order 1 and have the following general form.

$$\mathbf{x}^{(k+1)} = \mathbf{B}\mathbf{x}^{(k)} + \mathbf{f}, \qquad k \geq 0. \tag{4.1}$$

Thus, we define the error between iterations to be

$$err = \|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\|_p \tag{4.2}$$

That is, we use a vector p-norm to calculate the error for a given iteration, $k+1$. Popular norms for this application are:

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|, \tag{4.3}$$

$$\|\mathbf{x}\|_2 = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}, \tag{4.4}$$

$$\|\mathbf{x}\|_\infty = \max_{i=1}^{n} |x_i|. \tag{4.5}$$

Please implement all three basic iterative methods and apply them to solve question 4 of homework 3.

1. Using 1-norm, $\| \cdot \|_1$, Eq. (4.3), for tolerance checking, please find the tolerance that enables solution accuracy of 10^{-7} volts, 0.1 μV, for those three corner voltages, as compared to the solution found in homework 4. Record the CPU time for each method. All initial guesses, $\mathbf{x}_0$, are assumed to be the zero vector, $\mathbf{0}$.
2. Using 2-norm, $\| \cdot \|_2$, Eq. (4.4), for tolerance checking, please find the tolerance that enables solution accuracy of 10^{-7} volts, 0.1 μV, for those three corner voltages, as compared to the solution found in homework 4. Record the CPU time for each method. All initial guesses, $\mathbf{x}_0$, are assumed to be the zero vector, $\mathbf{0}$.
3. Using ∞-norm, $\| \cdot \|_\infty$, Eq. (4.5), for tolerance checking, please find the tolerance that enables solution accuracy of 10^{-7} volts, 0.1 μV, for those three corner voltages, as compared to the solution found in homework 4. Record the CPU time for each method. All initial guesses, $\mathbf{x}_0$, are assumed to be the zero vector, $\mathbf{0}$.
4. Please compared the answers and CPU times you obtained. What are your observations? For your own applications, which norm would you prefer? Please keep this set of implementations in your `MAT.cpp` file for possible reuse later on.

Notes.

1. For this homework you need to turn in a set of `C++` source codes for question 3. That includes `hw04.cpp`, which solves the 40×40 resistor network, `MAT.h`, the new header file, `MAT.cpp`, which includes the three new functions for the linear iterative methods, and the `VEC.h` and `VEC.cpp` if these have also be modified.
2. A `pdf` file is also needed. Please name this file `hw04a.pdf`.
3. Submit your files on EE workstations. Please use the following command to submit your homework 4.

   ```
   $ ~ee407002/bin/submit hw04 hw04a.pdf hw04.cpp MAT.h MAT.cpp VEC.h VEC.cpp
   ```

   where `hw04` indicates homework 4.
4. Your report should be clearly written such that I can understand it. The writing, including English grammar, is part of the grading criteria.