Numerical Analysis
homework 09: Spline Interpolations

Due on Tuesday, May 2, 2017

102061149 Fu-En Wang

1 Introduction

In previous homework, we have used Lagrange to get the interpolated values of f301.dat. However, when using Lagrange, we can find that the error around the two side of support points is quite large. As a result, to reduce the interpolated error, we will introduce **Spline Interpolation** in this project.

1.1 Spline Interpolation

For a given support points set and their second derivative M, the interpolated value is

$$y(x) = \alpha_i + \beta_i(x - x_{i-1}) + \gamma_i(x - x_{i-1})^2 + \delta_i(x - x_{i-1})^3 \tag{1}$$

$$\alpha_i = y_{i-1} \tag{2}$$

$$\beta_i = \frac{y_i - y_{i-1}}{h_i} - \frac{h_i}{6}(M_i + 2M_{i-1}) \tag{3}$$

$$\gamma_i = \frac{M_{i-1}}{2} \tag{4}$$

$$\delta_i = \frac{M_i - M_{i-1}}{6h_i} \tag{5}$$

1.2 Moment

For calculate moment, we will model the problem into a linear system.

$$\begin{bmatrix} 2 & \lambda_0 & 0 & 0 & ... & 0 \\ \mu_1 & 2 & \lambda_1 & 0 & ... & 0 \\ 0 & \mu_2 & 2 & \lambda_2 & ... & 0 \\ . & & ... & & & \\ . & & ... & & 2 & \lambda_{n-1} \\ 0 & & ... & & \mu_n & 2 \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ M_2 \\ ... \\ M_{n-1} \\ M_n \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ ... \\ d_{n-1} \\ d_n \end{bmatrix} \tag{6}$$

$$\mu_i = \frac{h_i}{h_i + h_{i+1}} \tag{7}$$

$$\lambda_i = \frac{h_{i+1}}{h_i + h_{i+1}} \tag{8}$$

$$d_i = \frac{6}{h_i + h_{i+1}}\left(\frac{y_{i+1} - y_i}{h_{i+1}} - \frac{y_i - y_{i-1}}{h_i}\right) \tag{9}$$

In this project, I use **zero boundary condition**

$$\lambda_0 = 0$$
$$d_0 = 0$$
$$\mu_n = 0$$
$$d_n = 0$$

then we can claculate all M_i.