Numerical Analysis
homework 08: Polynomial Interpolations

Due on Tuesday, April 25, 2017

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1 Introduction

When recording experiment data, we usually have no time to record densely. However, to get the value of certain range we skip, we need to use interpolation to find it out. In this homework, we will use **Lagrange Interpolation** to recover the waveform and compare it with the groundtruth data.

1.1 Lagrange Interpolation

Give a set of support points $\{(x_i, y_i), 0 \le i \le n\}$, then

$$F(x) = \sum_{i=0}^{n} y_i \prod_{k=0, k\neq i}^{n} \frac{x - x_k}{x_i - x_k} \tag{1}$$

where $F(x)$ is interpolated value of location x.

2 Implementation

Algorithm 1 Non-recursive Nevile's Algorithm

XS, YS is support data, x is location to interpolate.
NS = YS
for each k ∈ {1,...., n-1} **do**
  for each j ∈ {0,..., n-k-1} **do**
    NS[j] = ((x-XS[j])*NS[j+1]-(x-XS[k+j])*NS[j]) / (XS[j+k]-XS[j])
  end for
end for
return NS[0]

2.1 Complexity

From Algorithm 1, the double for-loop make the whole process a $\boldsymbol{O(n^2)}$ problem.

3 Discussion

In this section, we will discussion the folowing topics:

1. Experiment result of **f3.dat, f5,dat, f7,dat, f13.dat, f21.dat**
2. Maximum error against **f301.dat**, which is our groundtruth
3. Maximum error(x=550 to 700) against **f301.dat**, which is our groundtruth

3.1 Maximum Error

Table 1 shows the result of all dat.

	f3	f5	f7	f13	f21
Max Error(all)	372.866858	248.340631	379.107286	1283.4489	16728.5648
Max Error(550 to 700)	372.866858	233.364371	148.890794	39.618945	17.803983

Table 1: Max error of all dat

In Table 1, Max Error(550 to 700) decreases when using more support points. However, Max Error(all) increases when using more support points. Because the range of interpolation is from 475 to 775, so the maximum error should occut at the 475 to 549 or 701 to 775.

3.2 Experiment Result

In this section we will plot the interpolated data along with the original data to find out some problems of **Lagrange Interpolation**. The following figures show the experiment result of f3.dat to f21.dat.

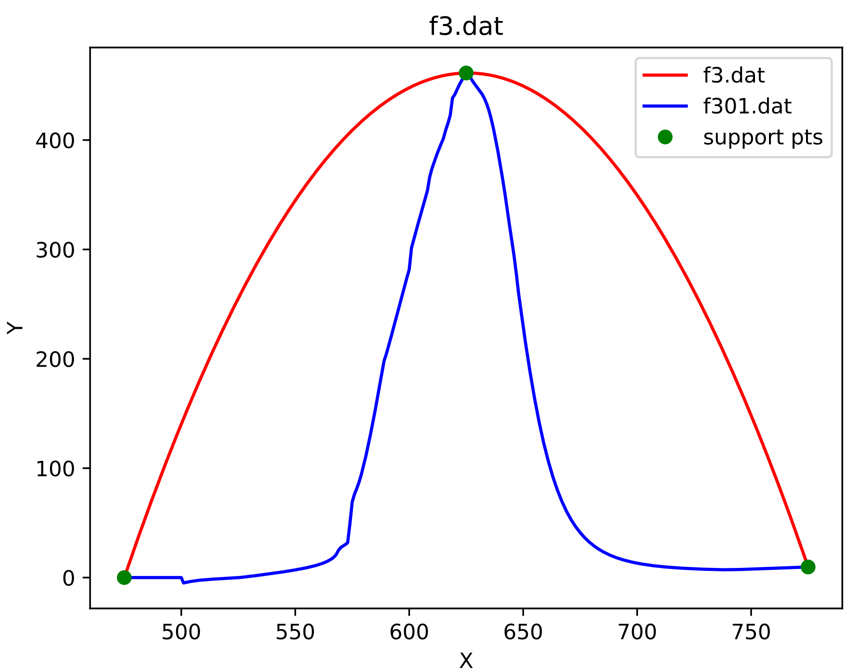

Figure 1: Result of f3.dat

The number of support points in Figure 1 is 3, so the interpolated data will be a second order polynomial and has a huge error with groundtruth.

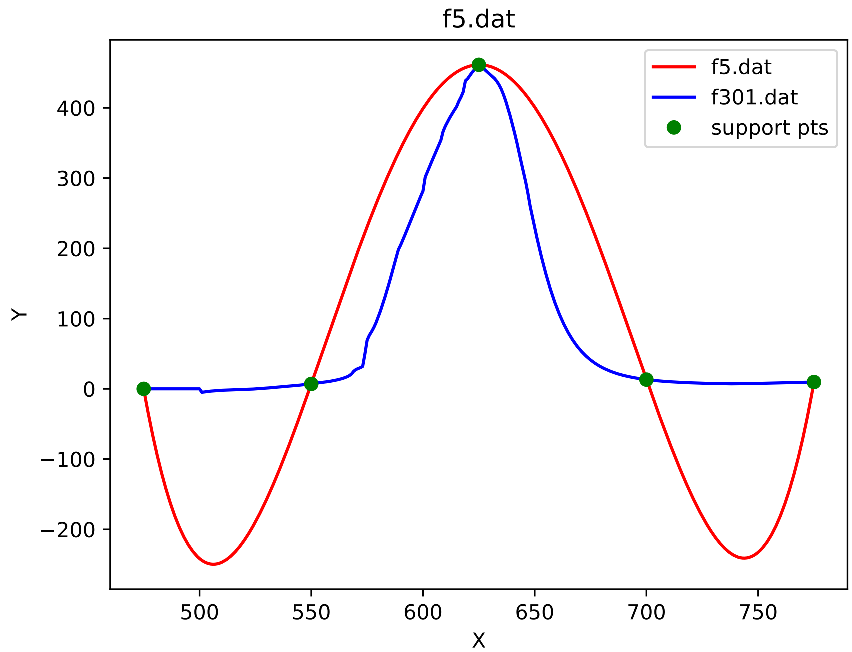

Figure 2: Result of f5.dat

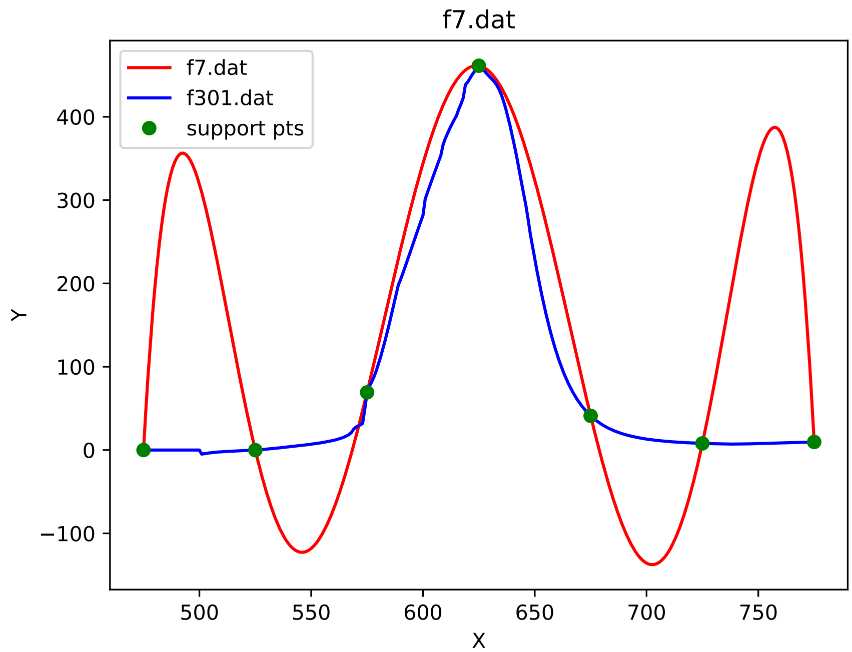

Figure 3: Result of f7.dat

In Figure 2 and 3, the order of polynomial become 4 and 6 and our interpolated data match the support points. But the number of support points seems still not enough to recover the origin data.

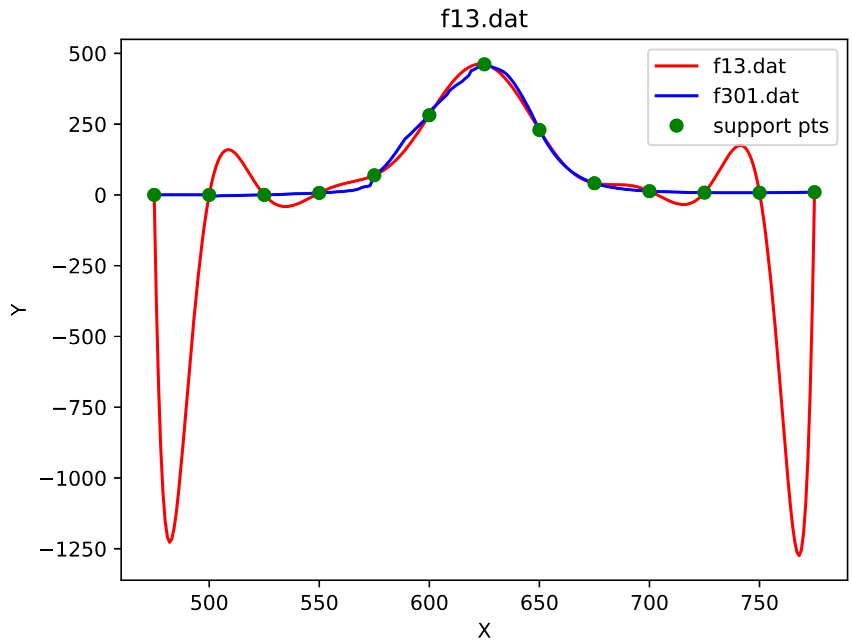

Figure 4: Result of f13.dat

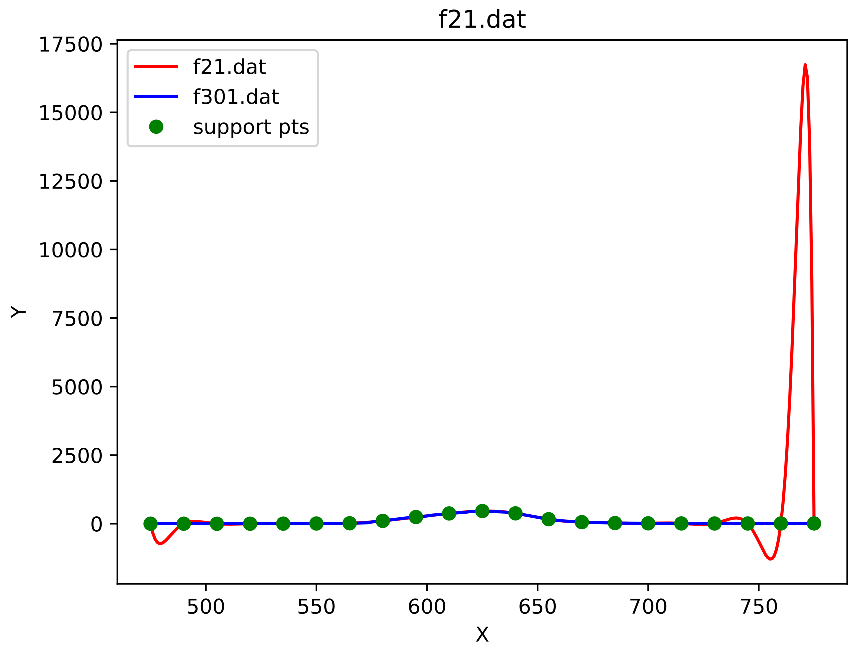

Figure 5: Result of f21.dat

In Figure 4 and 5, the mid part of interpolated value is very close to the origin one, but the left and right side have a great error. Because when number of support points is large, the interpolated polynomial will have a large order and x^n will cause a huge error. As a result, **round-off error** will be very large at the two side of interpolation. So when we use Lagrange Interpolation, we have to prevent ourself from getting value at the neighbor of two side. In addition, we should also prevent extrapolation, because Largrange also has large error at location out of support point range.

4 Conclusion

From the result in Figure 4 and 5, Lagrange Interpolation can be pretty accurate at the mid part of data but has huge error at the two side. So if we need to get data at the two side, we should prevent ourself from using this algorithm.