Numerical Analysis
homework 12: RLC Circult

Due on Tuesday, May 22, 2017

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1 Introduction

For a simple RLC circuit as Figure 1:

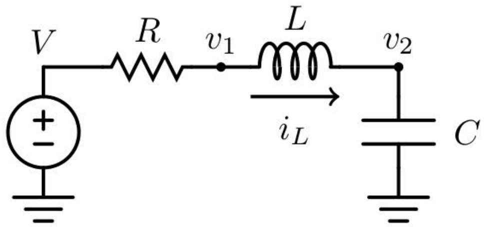

Figure 1: RLC circuit

The system consists of several ordinary differential equations:

$$\frac{v_1 - V}{R} + i_L = 0 \tag{1}$$

$$\frac{dv_2}{dt} = \frac{i_L}{C} \tag{2}$$

$$\frac{di_L}{dt} = \frac{v_1 - v_2}{L} \tag{3}$$

In this homework, we will implement three algorithm to solve the ODE above.

1.1 Forward Euler

Forward Euler is to model ODE as

$$x(t+h) = x(t) + h * f(t)$$

and the system can be derived as

$$v_1(t+h) = V + Ri_L(t+h) \tag{4}$$

$$v_2(t+h) = v_2(t) + \frac{h}{C} i_L(t) \tag{5}$$

$$i_L(t+h) = i_L(t) + \frac{h}{L}(v_1(t) - v_2(t)) \tag{6}$$

1.2 Backward Euler

Backward Euler is to model ODE as

$$x(t+h) = x(t) + h * f(t+h)$$

and the system can be derived as

$$v_1(t+h) = V + Ri_L(t+h) \tag{7}$$

$$v_2(t+h) = v_2(t) + \frac{h}{C} i_L(t+h) \tag{8}$$

$$i_L(t+h) = i_L(t) + \frac{h}{L}(v_1(t+h) - v_2(t+h)) \tag{9}$$

After some mathematical tricks, $i_L(t+h)$ can be derived as

$$i_L(t+h) = \frac{i_L(t) - \frac{h}{L} v_2(t) + \frac{hV}{L}}{1 + \frac{h^2}{LC} + \frac{hR}{L}} \tag{10}$$

1.3 Trapezoidal

Trapezoidal is to model ODE as

$$x(t+h) = x(t) + h * \frac{f(t+h) + f(t)}{2}$$

and the system can be derived as

$$v_1(t+h) = V + R\frac{i_L(t+h) + i_L(t)}{2} \tag{11}$$

$$v_2(t+h) = v_2(t) + \frac{h}{2C}(i_L(t+h) + i_L(t)) \tag{12}$$

$$i_L(t+h) = i_L(t) + \frac{h}{2L}(v_1(t+h) - v_2(t+h) + v_1(t) - v_2(t)) \tag{13}$$

After some mathematical tricks, $i_L(t+h)$ can be derived as

$$i_L(t+h) = \frac{(1 - \frac{h^2}{4LC})i_L(t) + \frac{h}{2L}(V + v_1(t) - 2v_2(t))}{1 + \frac{h^2}{4LC} + \frac{hR}{2L}} \tag{14}$$