Numerical Analysis
homework 09: Spline Interpolations

Due on Tuesday, May 2, 2017

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1 Introduction

In previous homework, we have used Lagrange to get the interpolated values of f301.dat. However, when using Lagrange, we can find that the error around the two side of support points is quite large. As a result, to reduce the interpolated error, we will introduce **Spline Interpolation** in this project.

1.1 Spline Interpolation

For a given support points set and their second derivative M, the interpolated value is

$$y(x) = \alpha_i + \beta_i(x - x_{i-1}) + \gamma_i(x - x_{i-1})^2 + \delta_i(x - x_{i-1})^3 \tag{1}$$
$$\alpha_i = y_{i-1} \tag{2}$$
$$\beta_i = \frac{y_i - y_{i-1}}{h_i} - \frac{h_i}{6}(M_i + 2M_{i-1}) \tag{3}$$
$$\gamma_i = \frac{M_{i-1}}{2} \tag{4}$$
$$\delta_i = \frac{M_i - M_{i-1}}{6h_i} \tag{5}$$

1.2 Moment

For calculate moment, we will model the problem into a linear system.

$$\begin{bmatrix} 2 & \lambda_0 & 0 & 0 & ... & 0 \\ \mu_1 & 2 & \lambda_1 & 0 & ... & 0 \\ 0 & \mu_2 & 2 & \lambda_2 & ... & 0 \\ . & & ... & & & \\ . & & ... & & 2 & \lambda_{n-1} \\ 0 & & ... & & \mu_n & 2 \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ M_2 \\ ... \\ M_{n-1} \\ M_n \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ ... \\ d_{n-1} \\ d_n \end{bmatrix} \tag{6}$$
$$\mu_i = \frac{h_i}{h_i + h_{i+1}} \tag{7}$$
$$\lambda_i = \frac{h_{i+1}}{h_i + h_{i+1}} \tag{8}$$
$$d_i = \frac{6}{h_i + h_{i+1}}\left(\frac{y_{i+1} - y_i}{h_{i+1}} - \frac{y_i - y_{i-1}}{h_i}\right) \tag{9}$$

In this project, I use **zero boundary condition**

$$\lambda_0 = 0$$
$$d_0 = 0$$
$$\mu_n = 0$$
$$d_n = 0$$

then we can claculate all M_i.

2 Discussion

In this section, we will discuss the following topic:

1. The maximum error between f3.dat...f21.dat and f301.dat
2. Comparison the result of Spline and Lagrange Interpolation

2.1 Maximum Error

Like the experiment in homework08, we interpolated the value from the same data(f3.data....f21.dat) and find their maximum error. Table 1 is the result of Spline Interpolation and Table 2 is result of Lagrange Interpolation.

Spline	f3	f5	f7	f13	f21
N	3	5	7	13	21
max_error	354.947	190.82	73.7436	29.0448	19.6417

Table 1: Error of Spline Interpolation

Lagrange	f3	f5	f7	f13	f21
N	3	5	7	13	21
Max Error(all)	372.866858	248.340631	379.107286	1283.448899	16728.564779
Max Error(550 700)	372.866858	233.364371	148.890794	39.618945	17.803983

Table 2: Error of Lagrange Interpolation

In the previous homework, we have already known that the error at the two side of Lagrange can be quite large. However, from Table 1 we can find that Spline doesn't have such problem, which means Spline Interpolation can be more accurate at the two side of interpolation and it can be closer to origin data when we use more support points.

2.2 Comparison

In this section, I will compare the two algorithm with the waveform for f3.dat to f21.dat. Figure 1 and 2 is the result of f3.dat of Spline and Lagrange, respectively.

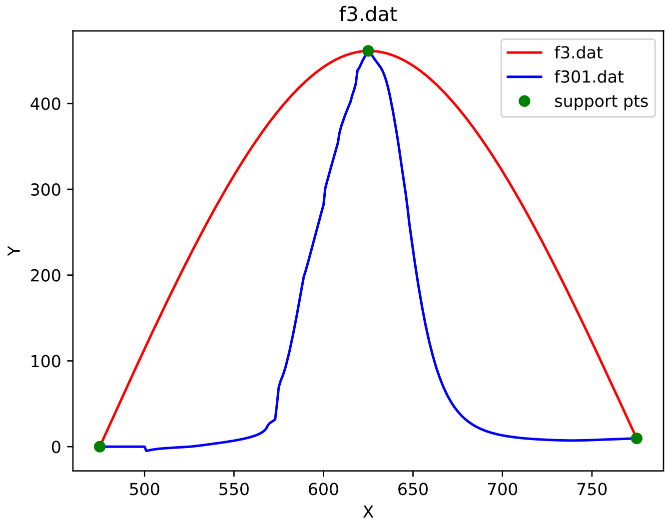

Figure 1: Result of f3.dat(Spline)

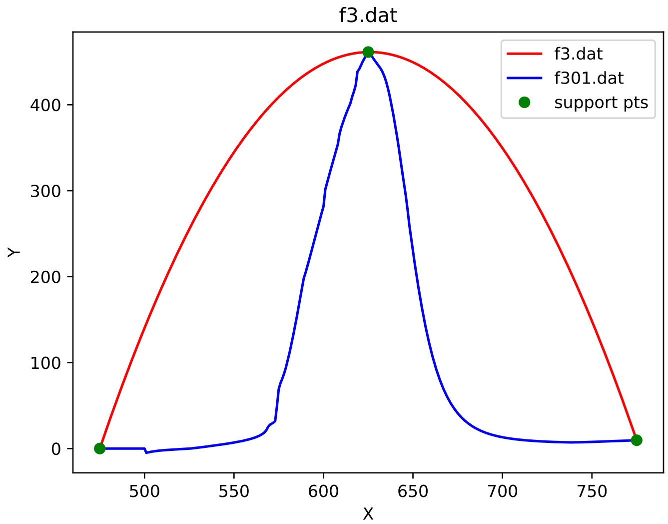

Figure 2: Result of f3.dat(Lagrange)

With 3 support points, the result is quite similar. Both of them are second order polynomial. Figure 3 and 4 is the result of f5.dat.

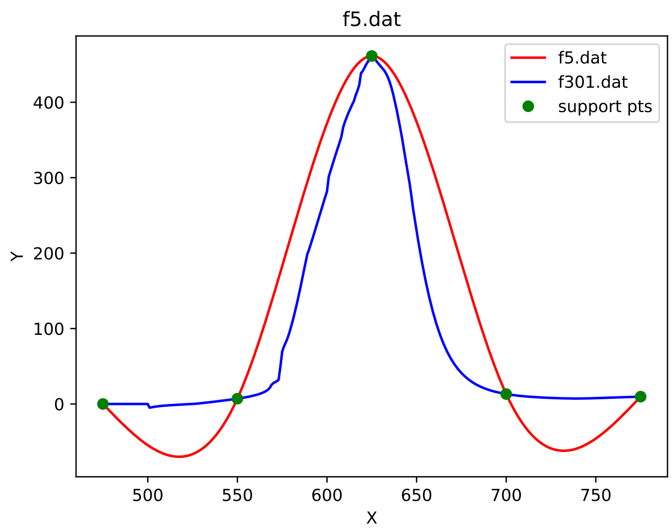

Figure 3: Result of f5.dat(Spline)

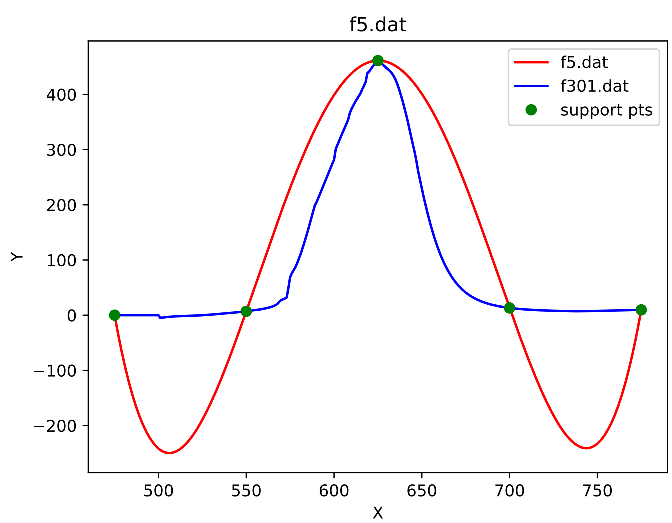

Figure 4: Result of f5.dat(Lagrange)

From the two figures, we can find that the oscillation at the two side is more obvious in Langrange and it seems that the result of Spline fit the answer better than Lagrange.

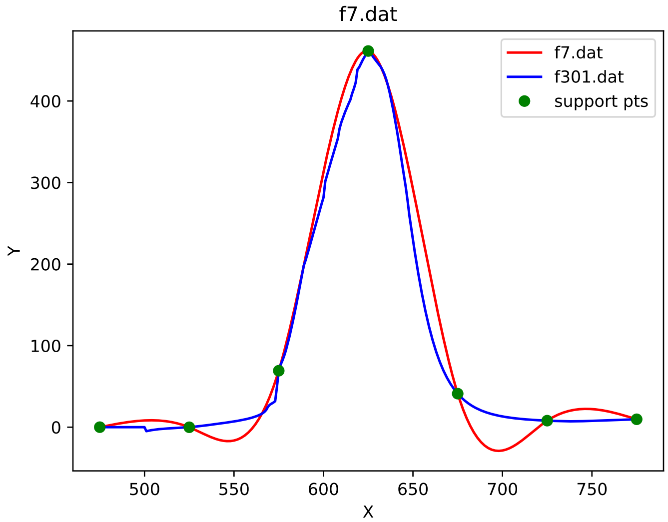

Figure 5: Result of f7.dat(Spline)

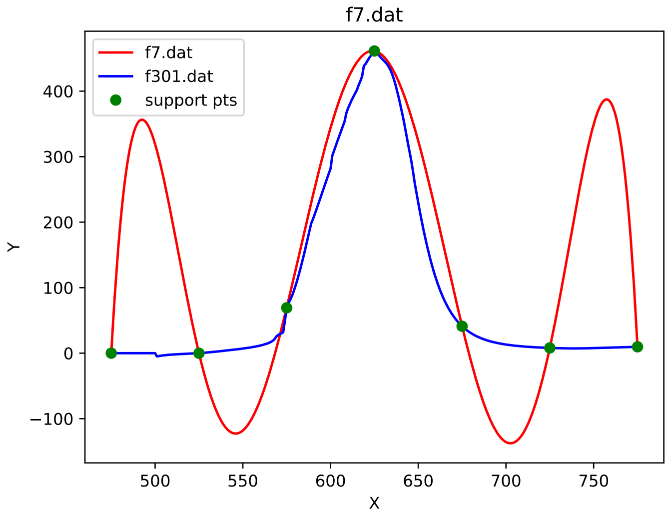

Figure 6: Result of f7.dat(Lagrange)

When we see the result from f7.dat(Figure 5 and 6), the result of Lagrange is a sixth order polunomial, so we can find 6 peak in Figure 5. However, the result of Spline is getting more closer to the answer.

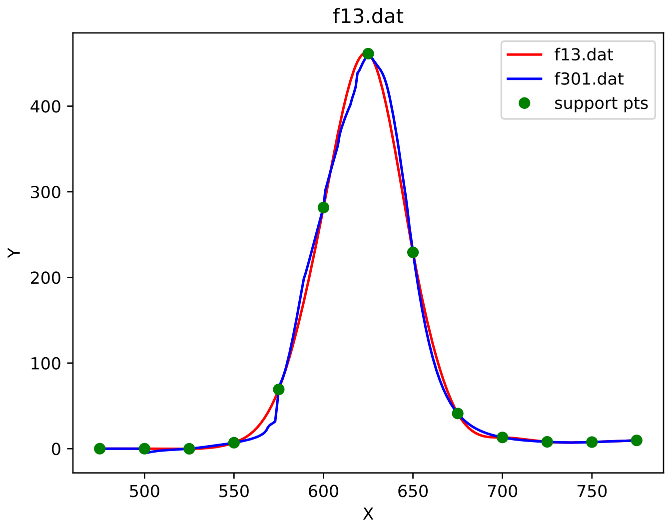

Figure 7: Result of f13.dat(Spline)

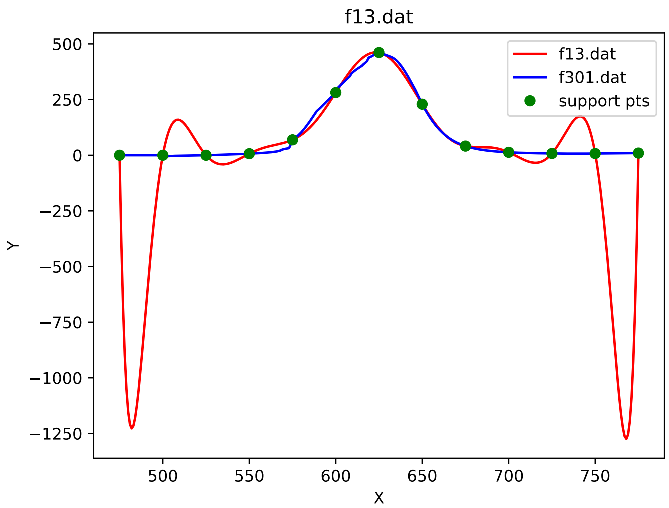

Figure 8: Result of f13.dat(Lagrange)

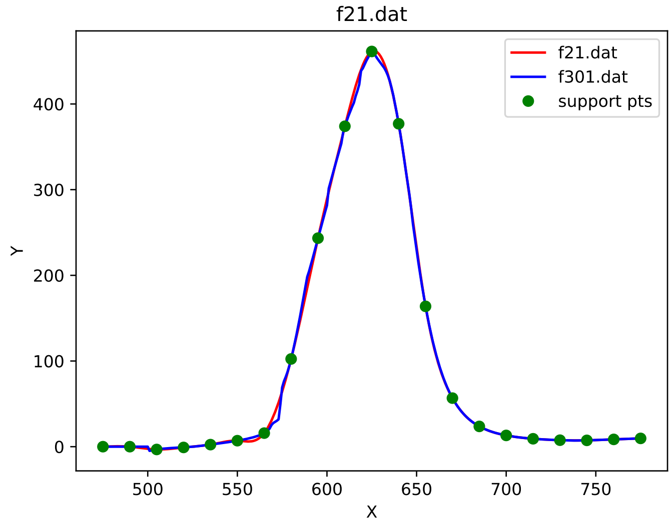

Figure 9: Result of f21.dat(Spline)

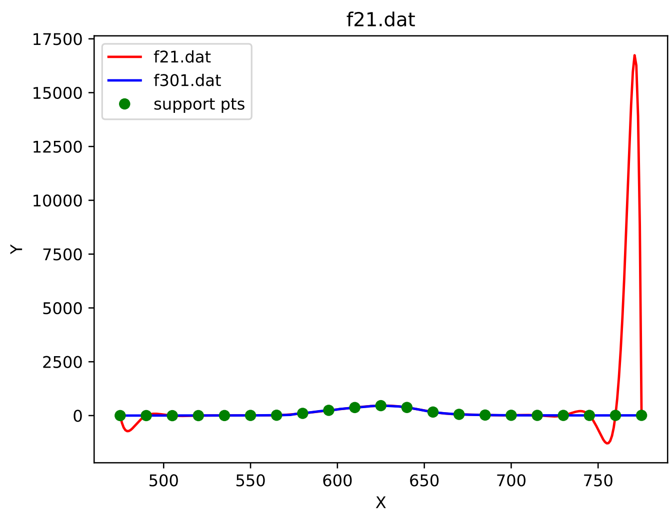

Figure 10: Result of f21.dat(Lagrange)

From Figure 7 and 9, Spline fit the answer very well, while Lagrange starts to have huge error at the two side in Figure 8 and 10.
From all figures above, I think Spline Interpolation have better accuracy than Lagrange Interpolation.