Personalized Recommendation System Simulation Evaluation Technical Guidance Document

Haoda Fu's draft, please comments and contribute ***

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Abstract

Key words and Phrases: *** ***.
Short title: ***

1 Introduction

Using data to generate actionable insights to guide patients, physicians, and payers to maximize their outcomes is a core topic for real world evidence research, and digital health solutions. Various solutions have been proposed, and it is crucial to understand their performance before applying it to make significant decisions for patients. This document provides guidance on how to design a simulation platform for such evaluation.

The evaluation is based on the potential outcome framework, i.e. the Rubin causal model **?**. The following of this document is organized as below. In section 2, we provide an overview on how the data is generated. In section 3, we put data generation model in a unified framework. In section 4, we explain examples of training and testing data to explain in details on how to use them. In section 5, we provide details on how we evaluate the model performance. In the end, this document also provides some simulation settings as a reference.

2 Data Generate Process

Data for personalized recommendation system should contain 3 parts: contextual information ($X \in \mathcal{X}$), contextual decision/action ($A \in \mathcal{X}$), and reward ($Y \in \mathbb{R}$). The research question is how we can learn from such data to develop a rule $\mathcal{D}(\cdot)$ which is a map from $\mathcal{X}$ to $\mathcal{A}$. The ideal system recommends the optimal decisions for each patients, i.e.,

$$\mathcal{D}^*(X) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, E\{Y|X, A = a\}.$$

To evaluate the model performance, each simulation generates training and testing data sets. The training data are used for model building. Users can further split training data for some tuning/validating purpose. The testing data sets are used to evaluate the model performance. The testing data sets contain the potential outcomes, and details will be provided in section 4.

There are three parts of data to generate in a sequence: X, A, and Y.

For contextual information X, they contains three types of variables: continuous variables (e.g. age, BMI, blood glucose values), ordinal categorical variables (e.g. cancer stage, severity of disease), or nominal categorical variables (e.g. gender, race) . The action A can be category variables, such as treatment, types of actions, or it can be continuous, such as dose. The reward/response Y can be continuous or ordinal categorical variable.

The action A can be depend on X, or it can be independent on X. For randomized control trials, treatment A is independent with X. For observational studies, treatment A often depends on X.

The reward/response Y is simulated based on different data generation models, and

details will be discussed in section 3. We use $Y(a)$ to denote potential outcome when a subject takes action a.

We have the following assumptions when we generate data,

- Stable unit treatment value assumption (SUTVA): $Y = \sum_{a=1}^{k} Y(a)I(A = a)$.
- No unmeasured confounder: $A \perp \{Y(a) : a \in \mathcal{A}\}|X$.

For the randomized control trials, the unmeasured confoundering assumptions are automatically satisfied.

3 Data Generation Models

Generalized Linear Model: When the responses are from exponential family, we generate data from the following GLM,

$$\ell\{E(Y|X)\} \quad = \quad \beta_0 + g(X) + t(A) + d(X, A),$$

where $\ell(\cdot)$ is a monotone link function. This model covers responses from various distributions including normal, binomial, Poisson, gamma, inverse gamma, and multinomial distributions.

Transformed Response Model: The transformed response model has the following form,

$$\tau(Y) \quad = \quad \beta_0 + g(X) + t(A) + d(X, A) + \epsilon,$$

where τ is a monotone transformation function, and $\epsilon \sim (0, \sigma^2)$ which can be nonparametric, i.e. mean equal to 0 with a finite second moment. This model covers the semi-parametric accelerated failure time model to generate time to event outcomes, and it also covers some often used PK/PD analytic models.

Intensity Function Model: When responses are time to event or recurrent events, the hazard or intensity function is often modeled as,

$$\lambda_i(t) \quad = \quad \lambda_0(t)\gamma_i \exp\{g(X_i) + t(A) + d(X_i, A)\},$$

where $\lambda_0(t)$ is a baseline hazard or intensity function, and γ_i is a frailty term with $\gamma_i \sim \text{Gamma}(1, \sigma^2)$. This model covers the Cox model, Anderson-Gail model, gamma-frailty model etc.

Remarks:

- $g(X)$ represents the prognostic effects. It can be parametric, e.g. $g(X) = X\beta$, semi-parametric, nonparametric additive model etc..

- $t(A)$ is the treatment effects.
- $d(X, A)$ models the predictive effects. Similarly, it can be parametric, semiparametric, or nonparametric models.

005_t03o1linear_Training_1.csv:

- 005: Dataset ID
- t03: 3 treatments
- o1: data from 1 observational study. (o: observational study, r:randomized control trial, m: mixture, e.g. m3 means that data from 3 studies, and some of them are observational studies and some of them are randomized control trials, study ID is considered as a covariate in X).
- linear: brief description of decision rule.

For each model, we have two data sets for testing and evaluation purposes. These two data sets are matched by patient ID. The first data set contains patients covariates, and it is labeled as "`nnn_xxxx_Testing_1.csv`". The second data set contains outcomes for evaluation which is labeled as "`nnn_xxxx_Testing_2.csv`".

[*Put Table 2 about here*]

[*Put Table 4 about here*]

4 Data Sets

We have 3 data sets, and they are training dataset M_1, testing data covariates M_2, testing data results M_3.

Example of M_1:

[*Put Table 1 about here*]

Example of M_2:

[*Put Table 2 about here*]

Example of M_3:

[*Put Table 4 about here*]

5 Evaluation Process

Suppose, we would like to evaluate a model which is a decision rule to map patient covariates into a treatment, i.e. $\mathcal{D}(\cdot) : X \mapsto A$.

Step 1: The input is M_1, and the output is a trained decision rule $\widehat{\mathcal{D}}(\cdot)$.

Step 2: The inputs are $\widehat{\mathcal{D}}(\cdot)$ and M_2, and the output is a vector $\widehat{A}$ with treatment assignment for each subject.

Step 3: The inputs are $\widehat{A}$ and M_3, and the outputs are the scores based on different evaluation criteria.

6 Evaluation Criteria

This subsection describes a high level evaluation process.

Primary criteria: The average benefit $N^{-1}\sum_{i=1}^{N}\sum_{a=1}^{k} Y_i(a) I\{a = \widehat{A}_i\}$.

Secondary criteria 1: Proportion of misclassification $N^{-1}\sum_{i=1}^{N} I\{A_i^o \neq \widehat{A}_i\}$.

Secondary criteria 2: Average of proportion of misclassification

$$k^{-1}\sum_{a=1}^{k}\left\{\sum_{i=1}^{N} I(A_i^o = a)\right\}^{-1}\sum_{i=1}^{N} I\{A_i^o \neq \widehat{A}_i, A_i^o = a\}.$$

Appendix A: ***

Simulation Settings and True Parameters (Confidential)

6.1 t2r1dichotomized

6.2 t2o1dicchotomized

6.3 t2o1linear

6.4 t2o1parabola

6.5 t2o1ring

6.6 t3r1dichotomized

6.7 t3r1ring

6.8 t3o1ring

6.9 t9r1bump

6.10 t9o1bump

Table 1: *An illustration training dataset*

ID	Y	A	X_1	X_2	X_3	$\cdots$
1	1.5	1	0	26	7.8	$\cdots$
2	1.2	2	1	28	8.2	$\cdots$
3	0.3	3	1	31	8.9	$\cdots$
4	0.9	2	0	35	9.4	$\cdots$
5	1.7	1	1	22	7.3	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$
n	1.6	2	0	29	8.1	$\cdots$

Table 2: *An illustration testing dataset: nnn_xxxx_Testing_1.csv.*

ID	X_1	X_2	X_3	$\cdots$
1	0	26	7.8	$\cdots$
2	1	28	8.2	$\cdots$
3	1	31	8.9	$\cdots$
4	0	35	9.4	$\cdots$
5	1	22	7.3	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$
N	0	29	8.1	$\cdots$

Table 3: *An illustration testing dataset: nnn_xxxx_Testing_2.csv.* $Y(a)$ is the potential outcome taking treatment a, A is the observed treatment assignment, and A^o is the oracle optimal treatment assignment based on $d(X, A)$.

ID	$Y(1)$	$Y(2)$	$\cdots$	$Y(k)$	A	A^o
1	1.2	1.5	$\cdots$	1.3	3	2
2	1.3	1.1	$\cdots$	1.4	2	3
3	0.9	0.8	$\cdots$	1.7	1	3
4	1.8	1.6	$\cdots$	1.2	1	1
5	1.4	1.4	$\cdots$	1.5	2	2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
N	1.7	1.4	$\cdots$	1.1	3	1

Table 4: *An illustration testing dataset: nnn_xxxx_Testing_2.csv.* The observed benefit is $V = 1/N \sum(1.2 + 1.1 + 0.9 + 1.6 + 1.4 + \cdots + 1.4)$, and theoretical optimal value is $V^o = 1/N \sum(1.5 + 1.3 + 0.8 + 1.8 + 1.4 + \cdots + 1.7)$, and the estimated value is $\widehat{V} = 1/N \sum(1.5 + 1.3 + 0.9 + 1.6 + 1.4 + \cdots + 1.7)$.

ID	$Y(1)$	$Y(2)$	A	A^o	$\widehat{A}$
1	1.2	1.5	1	2	2
2	1.3	1.1	2	1	1
3	0.9	0.8	1	2	1
4	1.8	1.6	2	1	2
5	1.4	1.4	2	2	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N	1.7	1.4	2	1	1