

Types of Ensemble Methods

- Manipulate data distribution
 - Example: bagging, boosting
- Manipulate input features
 - Example: random forests
- Manipulate class labels
 - Example: error-correcting output coding

Bagging

□ Sampling with replacement

Original Data	1	2	3	4	5	6	7	8	9	10
Bagging (Round 1)	7	8	10	8	2	5	10	10	5	9
Bagging (Round 2)	1	4	9	1	2	3	2	7	3	2
Bagging (Round 3)	1	8	5	10	5	5	9	6	3	7

□ Build classifier on each bootstrap sample

□ Each sample has probability $(1 - 1/n)^n$ of being selected

Bagging Algorithm

Algorithm 5.6 Bagging Algorithm

- 1: Let k be the number of bootstrap samples.
 - 2: for $i = 1$ to k do
 - 3: Create a bootstrap sample of size n , D_i .
 - 4: Train a base classifier C_i on the bootstrap sample D_i .
 - 5: end for
 - 6: $C^*(x) = \arg \max_y \sum_i \delta(C_i(x) = y)$, $\{\delta(\cdot) = 1$ if its argument is true, and 0 otherwise. $\}$
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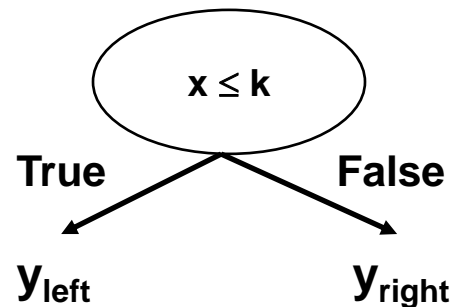
Bagging Example

- Consider 1-dimensional data set:

Original Data:

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	1	1	1	-1	-1	-1	-1	1	1	1

- Classifier is a decision stump
 - Decision rule: $x \leq k$ versus $x > k$
 - Split point k is chosen based on entropy



Bagging Example

Bagging Round 1:

x	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.9	0.9
y	1	1	1	1	-1	-1	-1	-1	1	1

$x \leq 0.35 \rightarrow y = 1$

$x > 0.35 \rightarrow y = -1$

Bagging Example

Bagging Round 1:

x	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.9	0.9
y	1	1	1	1	-1	-1	-1	-1	1	1

$x \leq 0.35 \rightarrow y = 1$

$x > 0.35 \rightarrow y = -1$

Bagging Round 2:

x	0.1	0.2	0.3	0.4	0.5	0.5	0.9	1	1	1
y	1	1	1	-1	-1	-1	1	1	1	1

$x \leq 0.7 \rightarrow y = 1$

$x > 0.7 \rightarrow y = 1$

Bagging Round 3:

x	0.1	0.2	0.3	0.4	0.4	0.5	0.7	0.7	0.8	0.9
y	1	1	1	-1	-1	-1	-1	-1	1	1

$x \leq 0.35 \rightarrow y = 1$

$x > 0.35 \rightarrow y = -1$

Bagging Round 4:

x	0.1	0.1	0.2	0.4	0.4	0.5	0.5	0.7	0.8	0.9
y	1	1	1	-1	-1	-1	-1	-1	1	1

$x \leq 0.3 \rightarrow y = 1$

$x > 0.3 \rightarrow y = -1$

Bagging Round 5:

x	0.1	0.1	0.2	0.5	0.6	0.6	0.6	1	1	1
y	1	1	1	-1	-1	-1	-1	1	1	1

$x \leq 0.35 \rightarrow y = 1$

$x > 0.35 \rightarrow y = -1$

Bagging Example

Bagging Round 6:

x	0.2	0.4	0.5	0.6	0.7	0.7	0.7	0.8	0.9	1
y	1	-1	-1	-1	-1	-1	-1	1	1	1

$x \leq 0.75 \rightarrow y = -1$
 $x > 0.75 \rightarrow y = 1$

Bagging Round 7:

x	0.1	0.4	0.4	0.6	0.7	0.8	0.9	0.9	0.9	1
y	1	-1	-1	-1	-1	1	1	1	1	1

$x \leq 0.75 \rightarrow y = -1$
 $x > 0.75 \rightarrow y = 1$

Bagging Round 8:

x	0.1	0.2	0.5	0.5	0.5	0.7	0.7	0.8	0.9	1
y	1	1	-1	-1	-1	-1	-1	1	1	1

$x \leq 0.75 \rightarrow y = -1$
 $x > 0.75 \rightarrow y = 1$

Bagging Round 9:

x	0.1	0.3	0.4	0.4	0.6	0.7	0.7	0.8	1	1
y	1	1	-1	-1	-1	-1	-1	1	1	1

$x \leq 0.75 \rightarrow y = -1$
 $x > 0.75 \rightarrow y = 1$

Bagging Round 10:

x	0.1	0.1	0.1	0.1	0.3	0.3	0.8	0.8	0.9	0.9
y	1	1	1	1	1	1	1	1	1	1

$x \leq 0.05 \rightarrow y = 1$
 $x > 0.05 \rightarrow y = 1$

Bagging Example

□ Summary of Training sets:

Round	Split Point	Left Class	Right Class
1	0.35	1	-1
2	0.7	1	1
3	0.35	1	-1
4	0.3	1	-1
5	0.35	1	-1
6	0.75	-1	1
7	0.75	-1	1
8	0.75	-1	1
9	0.75	-1	1
10	0.05	1	1

Bagging Example

- Assume test set is the same as the original data
- Use majority vote to determine class of ensemble classifier

Round	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1.0
1	1	1	1	-1	-1	-1	-1	-1	-1	-1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	-1	-1	-1	-1	-1	-1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1
5	1	1	1	-1	-1	-1	-1	-1	-1	-1
6	-1	-1	-1	-1	-1	-1	-1	1	1	1
7	-1	-1	-1	-1	-1	-1	-1	1	1	1
8	-1	-1	-1	-1	-1	-1	-1	1	1	1
9	-1	-1	-1	-1	-1	-1	-1	1	1	1
10	1	1	1	1	1	1	1	1	1	1
Sum	2	2	2	-6	-6	-6	-6	2	2	2
Predicted Class	1	1	1	-1	-1	-1	-1	1	1	1