

AdaBoost

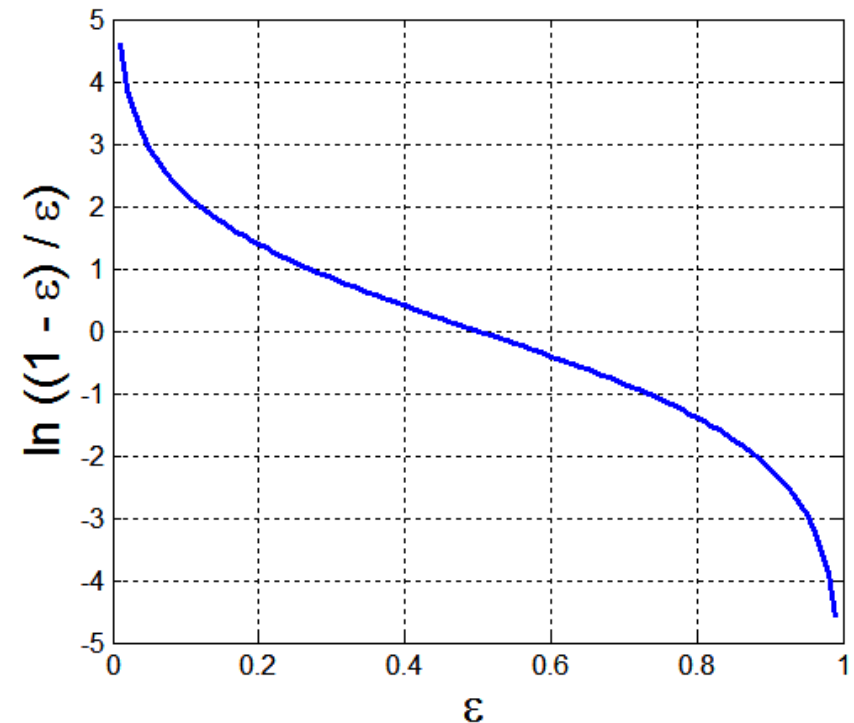
□ Base classifiers: C_1, C_2, \dots, C_T

□ Error rate:

$$\varepsilon_i = \frac{1}{N} \sum_{j=1}^N w_j \delta(C_i(x_j) \neq y_j)$$

□ Importance of a classifier:

$$\alpha_i = \frac{1}{2} \ln \left(\frac{1 - \varepsilon_i}{\varepsilon_i} \right)$$



AdaBoost Algorithm

- Weight update:

$$w_i^{(j+1)} = \frac{w_i^{(j)}}{Z_j} \begin{cases} \exp^{-\alpha_j} & \text{if } C_j(x_i) = y_i \\ \exp^{\alpha_j} & \text{if } C_j(x_i) \neq y_i \end{cases}$$

where Z_j is the normalization factor

- If any intermediate rounds produce error rate higher than 50%, the weights are reverted back to $1/n$ and the resampling procedure is repeated
- Classification:

$$C^*(x) = \arg \max_y \sum_{j=1}^T \alpha_j \delta(C_j(x) = y)$$

AdaBoost Algorithm

Algorithm 5.7 AdaBoost Algorithm

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1:  $\mathbf{w} = \{w_j = 1/n \mid j = 1, 2, \dots, n\}$ .    {Initialize the weights for all  $n$  instances.}
2: Let  $k$  be the number of boosting rounds.
3: for  $i = 1$  to  $k$  do
4:   Create training set  $D_i$  by sampling (with replacement) from  $D$  according to  $\mathbf{w}$ .
5:   Train a base classifier  $C_i$  on  $D_i$ .
6:   Apply  $C_i$  to all instances in the original training set,  $D$ .
7:    $\epsilon_i = \frac{1}{n} [\sum_j w_j \delta(C_i(x_j) \neq y_j)]$     {Calculate the weighted error}
8:   if  $\epsilon_i > 0.5$  then
9:      $\mathbf{w} = \{w_j = 1/n \mid j = 1, 2, \dots, n\}$ .    {Reset the weights for all  $n$  instances.}
10:    Go back to Step 4.
11:   end if
12:    $\alpha_i = \frac{1}{2} \ln \frac{1-\epsilon_i}{\epsilon_i}$ .
13:   Update the weight of each instance according to equation (5.88).
14: end for
15:  $C^*(\mathbf{x}) = \arg \max_y \sum_{j=1}^T \alpha_j \delta(C_j(\mathbf{x}) = y)$ .
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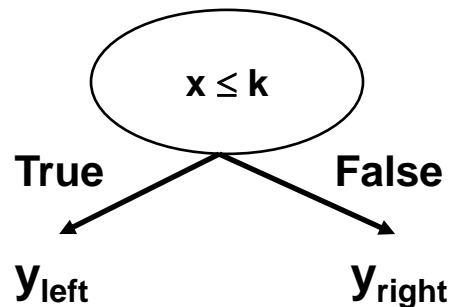
AdaBoost Example

- Consider 1-dimensional data set:

Original Data:

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	1	1	1	-1	-1	-1	-1	1	1	1

- Classifier is a decision stump
 - Decision rule: $x \leq k$ versus $x > k$
 - Split point k is chosen based on entropy



AdaBoost Example

- Training sets for the first 3 boosting rounds:

Boosting Round 1:

x	0.1	0.4	0.5	0.6	0.6	0.7	0.7	0.7	0.8	1
y	1	-1	-1	-1	-1	-1	-1	-1	1	1

Boosting Round 2:

x	0.1	0.1	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3
y	1	1	1	1	1	1	1	1	1	1

Boosting Round 3:

x	0.2	0.2	0.4	0.4	0.4	0.4	0.5	0.6	0.6	0.7
y	1	1	-1	-1	-1	-1	-1	-1	-1	-1

- Summary:

Round	Split Point	Left Class	Right Class	alpha
1	0.75	-1	1	1.738
2	0.05	1	1	2.7784
3	0.3	1	-1	4.1195

AdaBoost Example

□ Weights

Round	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1.0
1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
2	0.311	0.311	0.311	0.01	0.01	0.01	0.01	0.01	0.01	0.01
3	0.029	0.029	0.029	0.228	0.228	0.228	0.228	0.009	0.009	0.009

□ Classification

Round	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1.0
1	-1	-1	-1	-1	-1	-1	-1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	-1	-1	-1	-1	-1	-1	-1
Sum	5.16	5.16	5.16	-3.08	-3.08	-3.08	-3.08	0.397	0.397	0.397
Sign	1	1	1	-1	-1	-1	-1	1	1	1

Predicted
Class