Adaptive Model Predictive Control: Robustness and Parameter Estimation

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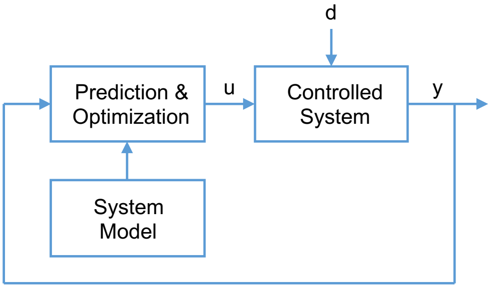
Motivation

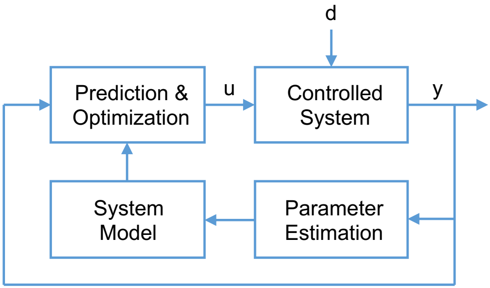
Robust MPC paradigm:



- Uncertainty in model & disturbances affects performance
- Large effort (time & money) spent on model identification offline

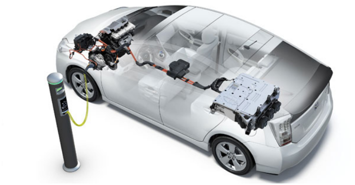
Motivation

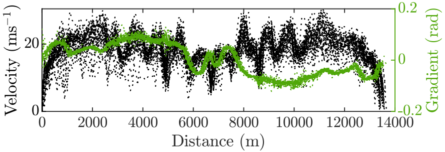
Adaptive MPC paradigm:

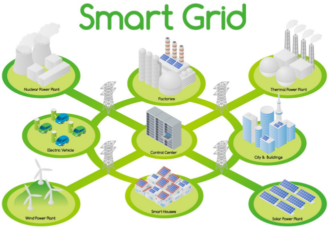


- Identify model online
- Require: robust constraint satisfaction
  closed loop stability & performance guarantees
  parameter convergence

Applications







- Uncertain parameters, uncertain demand
- Networks of interacting autonomous agents

Overview

An idea with a long history: e.g. self-tuning control, DMC, GPC . . .
[Clarke, Tuffs, Mohtadi, 1987]

Revisited with new tools:

- Set membership estimation
  [Bai, Cho, Tempo, 1998]
- Robust tube MPC
  [Langsson, Chryssochoos, Rakovic, Mayne, 2004]
- Dual adaptive/predictive control
  [Lee & Lee, 2009]

Overview

Recent work on MPC with model adaptation

- Online learning & identification:
  - Persistency of Excitation constraints
    [Marafioti, Bitmead, Hovd, 2014]
  - Kalman filter-based parameter estimation with covariance matrix in cost
    [Heirung, Ydstie, Foss, 2017]
  - Gaussian process regression, particle filtering
    [Klenske, Zeilinger, Scholkopf, Hennig, 2016]
    [Bayard & Schumitzky, 2010]
- Robust constraint satisfaction and performance:
  - Constraints based on prior uncertainty set, online update of cost only
    [Aswani, Gonzalez, Sastry, Tomlin, 2013]
  - Set-based identification, stable FIR plant model
    [Tanaskovic, Fagiano, Smith, Morari, 2014]

Overview

This talk considers how to

- ensure robust constraint satisfaction;
- update constraints & costs online via set-membership & point estimates;
- enforce parameter convergence via persistency of excitation conditions.

Outline:

1. Set membership parameter estimation
2. Polytopic tube robust MPC
3. Convex constraints ensuring persistency of excitation

Parameter set estimate

Plant model with unknown parameter vector $\theta^\star$ and disturbance w:

$$x_{k+1} = A(\theta^\star)x_k + B(\theta^\star)u_k + w_k$$

Assumption 1: model is affine in unknown parameters

$$x_{k+1} = D_k\theta^\star + d_k + w_k \qquad \begin{cases} D_k = D(x_k, u_k) \\ d_k = A_0 x_k + B_0 u_k \end{cases}$$

Assumption 2: stochastic disturbance $w_k \in \mathcal{W}$
disturbance set $\mathcal{W} \ni 0$ is compact and convex

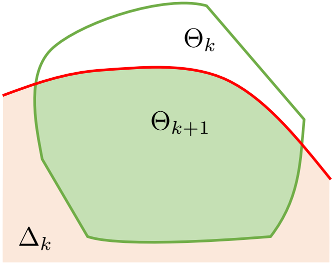
Unfalsified set: If x_{k+1}, x_k, u_k are known, then $\theta^\star \in \Delta_k$

$$\Delta_k = \{\theta : x_{k+1} = D_k\theta + d_k + w,\ w \in \mathcal{W}\}$$

Minimal parameter set estimate

Minimal parameter set update:

$$\Theta_{k+1} = \Theta_k \cap \Delta_k$$



Assumption 3: disturbance set is 'tight', i.e. for all $w^0 \in \partial\mathcal{W}$ and $\epsilon > 0$

$$\Pr\{\|w_k - w^0\| < \epsilon\} \geq p_w(\epsilon)$$

where $p_w(\epsilon) > 0 \ \forall \epsilon > 0$

Assumption 4: disturbance is persistently exciting, i.e. $\exists\ \alpha,\ \beta,\ N$ such that

$$\left.\begin{aligned} &\|D_k\| \leq \alpha \\ &\sum_{j=k}^{k+N-1} D_j^\top D_j \succeq \beta I \end{aligned}\right. \qquad \text{for all } k$$

Minimal parameter set estimate

If Assumptions 1-4 hold, then $\Theta_k \to \{\theta^*\}$ as $k \to \infty$ w.p. 1

This follows from:

A. For any $\theta^0 \in \Theta_k$, if $\|\theta^* - \theta^0\| \geq \epsilon$, then
$$\Pr\{\theta^0 \notin \Delta_j\} \geq p_w\bigl(\epsilon\sqrt{\beta/N}\bigr)$$
for all k, all $\epsilon > 0$, and for some $j \in \{k, \ldots, k+N-1\}$

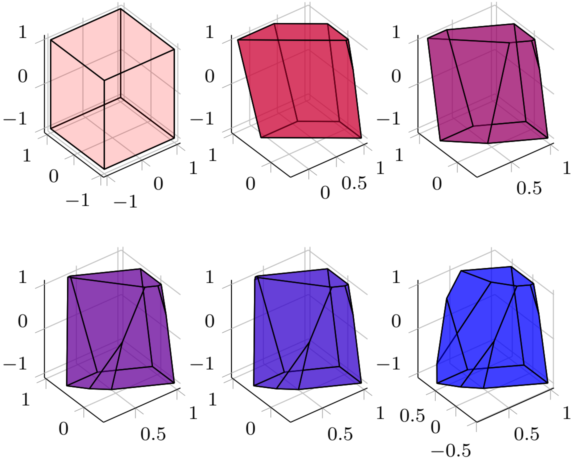
B. For any $\theta^0 \in \Theta_0$ such that $\|\theta^0 - \theta^*\| \geq \epsilon$,
$$\Pr\{\theta^0 \in \Theta_k\} \leq \Bigl[1 - p_w\bigl(\epsilon\sqrt{\beta/N}\bigr)\Bigr]^{\lfloor k/N \rfloor}$$
for all k and all $\epsilon > 0$

C. Borel-Cantelli lemma applied to $\displaystyle\sum_{k=0}^{\infty} \Pr\Bigl\{\max_{\theta\in\Theta_k} \|\theta - \theta^*\| \geq \epsilon\Bigr\}$
for all $\epsilon > 0$

Minimal parameter set estimate

The complexity of Θ_k is unbounded in general

e.g. Minimal parameter set Θ_k for $k = 1, \ldots, 6$ with polytopic $\mathcal{W}$ and Θ_0



Fixed complexity polytopic parameter set estimate

Define $\Theta_k = \{\theta : H_\Theta \theta \leq h_k\}$ for fixed $H_\Theta \in \mathbb{R}^{n_\Theta \times n}$

Update Θ_{k+1} by solving the LPs:

$$[h_{k+1}]_i = \max_{\substack{w_0 \in \mathcal{W}, \dots, w_{N-1} \in \mathcal{W} \\ \theta \in \Theta_k}} [H_\Theta]_i \theta$$

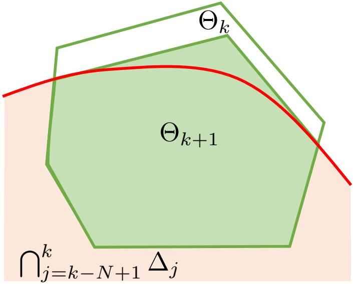
subject to

$$x_{k-N+2} = D_{k-N+1}\theta + d_{k-N+1} + w_0$$

$$\vdots$$

$$x_{k+1} = D_k \theta + d_k + w_{N-1}$$

for all $i \in \{1, \dots, n_\Theta\}$



then $\qquad \Theta_{k+1} \supseteq \Theta_k \cap \bigcap_{j=k-N+1}^{k} \Delta_j$

and $\qquad \Theta_{k+1} \subseteq \Theta_k \subseteq \cdots \subseteq \Theta_0$

Fixed complexity polytopic parameter set estimate

If Assumptions 1-4 hold, then $\Theta_k \to \{\theta^*\}$ as $k \to \infty$ w.p. 1

This follows from:

A. If $[h_k]_i - [H]_i\theta^* \geq \epsilon$, then

$$\Pr\Big\{\{\theta : [H]_i\theta = [h_k]_i\} \cap \bigcap_{j=k-N+1}^{k} \Delta_j = \emptyset\Big\} \geq \Big[p_w\Big(\frac{\epsilon\beta}{\alpha N}\Big)\Big]^N$$

for all i, k, and all $\epsilon > 0$

B. For all $\epsilon > 0$ and all k,

$$\Pr\Big\{[h_k]_i - [H]_i\theta^* \geq \epsilon\Big\} \leq \Big\{1 - \Big[p_w\Big(\frac{\epsilon\beta}{N\alpha}\Big)\Big]^N\Big\}^{\lfloor k/N \rfloor}$$

C. Borel-Cantelli lemma applied to $\displaystyle\sum_{k=0}^{\infty} \Pr\big\{[h_k]_i - [H]_i\theta^* \geq \epsilon\big\}$

for all i and all $\epsilon > 0$

Example: fixed complexity parameter set estimate

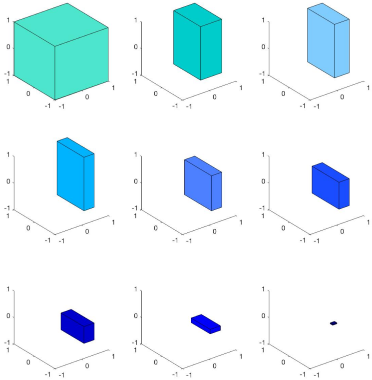


Figure: Parameter set Θ_k at time steps $k \in \{0, 1, 2; 10, 25, 50; 100, 500, 5000\}$

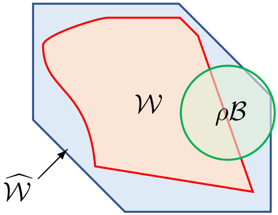
Θ set	Volume (%)	Cost*
Θ_0	100	62.22
Θ_1	26.1	61.13
Θ_2	18.3	61.03
Θ_{10}	12.7	60.96
Θ_{25}	8.3	60.93
Θ_{50}	6.3	60.77
Θ_{100}	3.4	59.45
Θ_{500}	0.7	57.94
Θ_{5000}	0.0089	53.95
$\theta^\star$	-	52.70

Table: Volume of Θ_k as $\Theta_k/\Theta_0 \times 100\%$; Cost* with same initial x_0 and constraints

Inexact disturbance bounds

What if $\mathcal{W}$ is not exactly known?

Suppose a bounding set $\widehat{\mathcal{W}}$ is known



Assumption 5: $\widehat{\mathcal{W}}$ is compact and convex, and $\mathcal{W} \subseteq \widehat{\mathcal{W}} \subseteq \mathcal{W} \oplus \rho\mathcal{B}$ for some $\rho \geq 0$, and $\mathcal{B} = \{x : \|x\| \leq 1\}$

Replace $\mathcal{W}$ with $\widehat{\mathcal{W}}$ in the fixed complexity polytopic parameter set update

then $\theta^\star \in \widehat{\Delta}_k = \{\theta : x_{k+1} = D_k\theta + d_k + w,\ w \in \widehat{\mathcal{W}}\}$, and

if Assumptions 1-5 hold, then $\Theta_k \to \{\theta^*\} \oplus \rho\sqrt{N/\beta}\mathcal{B}$ as $k \to \infty$ w.p. 1

Noisy measurements

Let $y_k = x_k + m_k$ be an estimate of x_k

Assumption 6: stochastic measurement noise $m_k \in \mathcal{M}$
where $\mathcal{M} \ni 0$ is a compact, convex polytope

Assumption 7: the noise bound is tight, i.e. for all $m^0 \in \partial\mathcal{M}$ and $\epsilon > 0$

$$\Pr\left\{\|m_k - m^0\| < \epsilon\right\} \geq p_m(\epsilon)$$

where $p_m(\epsilon) > 0 \ \forall \epsilon > 0$

Then $\mathcal{M} = \mathrm{co}\{m^{(j)}, \, j = 1, \ldots, n_{\mathcal{M}}\}$ implies $\theta^\star \in \mathrm{co}\{\widehat{\Delta}_k^{(j)}, \, j = 1, \ldots, n_{\mathcal{M}}\}$

$$\widehat{\Delta}_k^{(j)} = \left\{\theta : y_{k+1} = D(y_k - m^{(j)}, u_k)\theta + d(y_k - m_k^{(j)}, u_k) + w, \ w \in \widehat{\mathcal{W}}\right\}$$

If Assumptions 1-7 hold, then $\Theta_k \to \{\theta^*\} \oplus \rho\sqrt{N/\beta}\mathcal{B}$ as $k \to \infty$ w.p. 1

Parameter point estimate

To ensure closed loop l^2 stability, we define the MPC cost in terms of a point estimate $\hat{\theta}_k$ of $\theta^\star$, computed using a LMS filter

- Given a parameter estimate $\hat{\theta}_k$, let $\hat{x}_{1|k} = A(\hat{\theta}_k)x_k + B(\hat{\theta}_k)u_k$
- Then for a given parameter update gain $\mu > 0$ satisfying

$$1/\mu > \sup_{(x,u)\in\mathcal{Z}} \|D(x,u)\|^2$$

the point estimate $\hat{\theta}_k$ is defined

$$\begin{aligned} \tilde{\theta}_k &= \hat{\theta}_{k-1} + \mu D^\top(x_{k-1}, u_{k-1})(x_k - \hat{x}_{1|k-1}) \\ \hat{\theta}_k &= \Pi_{\Theta_k}(\tilde{\theta}_k) \end{aligned}$$

where Π_{Θ_k} is the Euclidean projection onto Θ_k

Here $\mathcal{Z}$ is the joint state and control constraint set (assumed bounded) and the point estimate update is simply a projection onto Θ_k if $\mu \to 0$

Parameter point estimate

The closed loop l^2 gain property is based on the following result

If $\sup_{k\in\mathbb{N}} \|x_k\| < \infty$ and $\sup_{k\in\mathbb{N}} \|u_k\| < \infty$, then $\hat{\theta}_k \in \Theta_k$ for all k and

$$\sup_{T\in\mathbb{N}, w_k\in\mathcal{W}, \hat{\theta}_0\in\Theta_0} \frac{\sum_{k=0}^{T} \|\tilde{x}_{1|k}\|^2}{\frac{1}{\mu}\|\hat{\theta}_0 - \theta^\star\|^2 + \sum_{k=0}^{T} \|w_k\|^2} \leq 1$$

where $\tilde{x}_{1|k} = A(\theta^\star)x_k + B(\theta^\star)u_k - \hat{x}_{1|k}$ is the 1-step prediction error

Control Problem

Consider robust regulation of the system

$$x_{k+1} = A(\theta)x_k + B(\theta)u_k + w_k$$

with $\theta \in \Theta_k$, $w_k \in \mathcal{W}$, subject to the state and control constraints

$$Fx_k + Gu_k \leq \mathbf{1} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}^\top$$

Assumption (Robust stabilizability):
There exists a set $\mathcal{X} = \{x : Vx \leq \mathbf{1}\}$ and feedback gain K such that $\mathcal{X}$ is λ-contractive for some $\lambda \in [0, 1)$, i.e.

$$V\Phi(\theta)x \leq \lambda\mathbf{1}, \quad \text{for all } x \in \mathcal{X}, \theta \in \Theta_0.$$

where $\Phi(\theta) = A(\theta) + B(\theta)K$.

Control Problem

State and control input sequences predicted at time k: $u_{i|k}, x_{i|k}$, $i = 0, 1, \ldots$ are expressed in terms of decision variables $\mathbf{v} = (v_{0|k}, \ldots, v_{N|k})$:

$$u_{i|k} = \begin{cases} Kx_{i|k} + v_{i|k} & i = 0, 1, \ldots \\ Kx_{i|k} & \end{cases}$$

The regulation cost is defined in terms of point estimate $\hat{\theta}_k$:

$$J_N(x_k, \hat{\theta}_k, \mathbf{v}_k) = \sum_{i=0}^{N-1} \left(\|\hat{x}_{i|k}\|_Q^2 + \|\hat{u}_{i|k}\|_R^2 \right) + \|\hat{x}_{N|k}\|_P^2$$

where $\hat{x}_{i|k}$, $\hat{u}_{i|k}$ are defined by $\hat{x}_{0|k} = x_k$ and

$$\begin{aligned} \hat{x}_{i+1|k} &= A(\hat{\theta}_k)\hat{x}_{i|k} + B(\hat{\theta}_k)\hat{u}_{i|k} \\ \hat{u}_{i|k} &= K\hat{x}_{i|k} + v_{i|k} \end{aligned}$$

and where $P \succeq \Phi^\top(\theta) P \Phi(\theta) + Q + K^\top RK$ for all $\theta \in \Theta_0$

Tube MPC

A sequence of sets (a "tube") is constructed to bound the predicted state $x_{i|k}$, with ith cross section, $\mathcal{X}_{i|k}$:

$$\mathcal{X}_{i|k} = \{x : Vx \leq \alpha_{i|k}\}$$

where V is determined offline and $\alpha_{i|k}$ are online decision variables

- **A** For robust satisfaction of $x_{i|k} \in \mathcal{X}_{i|k}$, we require

  $$V\Phi(\theta)x + VB(\theta)v_{i|k} + \bar{w} \leq \alpha_{i+1|k} \quad \text{for all } x \in \mathcal{X}_{i|k},\ \theta \in \Theta_k$$

  where $[\bar{w}]_i = \max_{w \in \mathcal{W}} [V]_i w$

- **B** For robust satisfaction of $Fx_{i|k} + Gu_{i|k} \leq \mathbf{1}$, we require

  $$(F + GK)x + Gv_{i|k} \leq \mathbf{1} \quad \text{for all } x \in \mathcal{X}_{i|k}$$

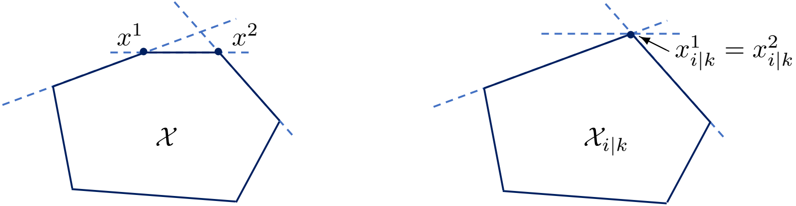
Condition (A) is bilinear in x and θ, but it can be expressed in terms of linear inequalities using a vertex representation of either $\mathcal{X}_{i|k}$ or Θ_k

Tube MPC

We generate the vertex representation:

$$\mathcal{X}_{i|k} = \mathrm{co}\{x^1_{i|k}, \dots x^m_{i|k}\}$$

using the property that $\{x : [V]_r x \leq [\alpha_{i|k}]_r\}$ is a supporting hyperplane of $\mathcal{X}_{i|k}$ for each r:



Hence each vertex $x^j_{i|k}$ is given by the intersection of hyperplanes corresponding to a fixed set of rows of V, and

$$x^j_{i|k} = U^j \alpha_{i|k}$$

for some U^j, determined offline from the vertices of $\mathcal{X} = \{x : Vx \leq \mathbf{1}\}$

Tube MPC

In terms of both hyperplane and the vertex descriptions of $\mathcal{X}_{i|k}$, the robust tube constraints become

- A $V\Phi(\theta)U^j\alpha_{i|k} + VB(\theta)v_{i|k} + \bar{w} \leq \alpha_{i+1|k}$ for all $\theta \in \Theta_k$, $j = 1, \ldots, m$
- B $(F + GK)U^j\alpha_{i|k} + Gv_{i|k} \leq \mathbf{1}$, $j = 1, \ldots, m$

Now condition (B) is linear and (A) can be equivalently written as linear constraints using

Polyhedral set inclusion lemma

Let $\mathcal{P}_i = \{x : F_i x \leq f_i\} \subset \mathbb{R}^n$ for $i = 1, 2$. Then $\mathcal{P}_1 \subseteq \mathcal{P}_2$ iff

$$\exists \Lambda \geq 0 \text{ such that } \Lambda F_1 = F_2 \text{ and } \Lambda f_1 \leq f_2$$

Robust MPC online optimization problem

Summary of constraints in the online MPC optimization at time k:

$$
\begin{aligned}
& V x_k \leq \alpha_{0|k} \\
\\
& \Lambda^j_{i|k} H_\Theta = V D(U^j \alpha_{i|k}, K U^j \alpha_{i|k} + v_{i|k}) \\
& \Lambda^j_{i|k} h_k \leq \alpha_{i+1|k} - V d(u^j \alpha_{i|k}, K U^j \alpha_{i|k} + v_{i|k}) - \bar{w} \\
& \Lambda^j_{i|k} \geq 0 \\
\\
& (F + GK) U^j \alpha_{i|k} + G v_{i|k} \leq \mathbf{1} \\
\\
& \Lambda^j_{N|k} H_\Theta = V D(U^j \alpha_{N|k}, K U^j \alpha_{N|k}) \\
& \Lambda^j_{N|k} h_k \leq \alpha_{N|k} - V d(u^j \alpha_{N|k}, K U^j \alpha_{N|k}) - \bar{w} \\
& \Lambda^j_{N|k} \geq 0 \\
\\
& (F + GK) U^j \alpha_{N|k} \leq \mathbf{1}
\end{aligned}
$$

for $i = 0, \ldots, N-1$, $j = 1, \ldots, m$

Let $\mathcal{F}(x_k, \Theta_k)$ be the feasible set for the decision variables $\mathbf{v}_k, \boldsymbol{\alpha}_k, \boldsymbol{\Lambda}_k$

Robust adaptive MPC algorithm

Offline: Choose Θ_0, $\mathcal{X}$, feedback gain K, and compute P

Online, at each time $k = 1, 2, \ldots$:

1. Given x_k, update the set (Θ_k) and point ($\hat{\theta}_k$) parameter estimates
2. Compute the solution $(\mathbf{v}_k^*, \boldsymbol{\alpha}_k^*, \boldsymbol{\Lambda}_k^*)$ of the QP:
$$\min_{\mathbf{v}_k, \boldsymbol{\alpha}_k, \boldsymbol{\Lambda}_k} J(x_k, \hat{\theta}_k, \mathbf{v}_k)$$
$$\text{subject to } (\mathbf{v}_k, \boldsymbol{\alpha}_k, \boldsymbol{\Lambda}_k) \in \mathcal{F}(x_k, \Theta_k)$$
3. Apply the control law $u_k^* = K x_k + v_{0|k}^*$

Robust adaptive MPC algorithm

The MPC algorithm has the following closed loop properties:

If $\theta^\star \in \Theta_0$ and $\mathcal{F}(x_0, \Theta_0) \neq \emptyset$, then for all $k > 0$:

1. $\theta^\star \in \Theta_k$
2. $D(x_k, \Theta_k) \neq \emptyset$
3. $Fx_k + Gu_k \leq \mathbf{1}$

and the closed loop system is finite-gain l^2-stable,
i.e. there exist constants $c_0, c_1, c_2 > 0$ such that for all T:

$$\sum_{k=0}^{T} \|x_k\|^2 \leq c_0 \|x_0\|^2 + c_1 \|\hat{\theta}_0 - \theta^\star\|^2 + c_2 \sum_{k=0}^{T} \|w_k\|^2$$

Numerical example

Second-order linear system with

$$(A(\theta), B(\theta)) = (A_0, B_0) + \sum_{i=1}^{3} (A_i, B_i)\theta_i$$

$$A_0 = \begin{bmatrix} 0.5 & 0.2 \\ -0.1 & 0.6 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.042 & 0 \\ 0.072 & 0.03 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.015 & 0.019 \\ 0.009 & 0.035 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 \\ -0 & 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.0397 \\ 0.059 \end{bmatrix}.$$

- ▷ true parameter $\theta^\star = [0.8 \;\; 0.2 \;\; -0.5]^\top$, initial set $\Theta_0 = \{\theta : \|\theta\|_\infty \leq 1\}$.
- ▷ disturbance uniformly distributed on $\mathcal{W} = \{w \in \mathbb{R}^2 : \; \|w\|_\infty \leq 0.1\}$, w_k
- ▷ state and input constraints: $[x]_2 \geq -0.3$ and $u_k \leq 1$.

Numerical example: constraint satisfaction

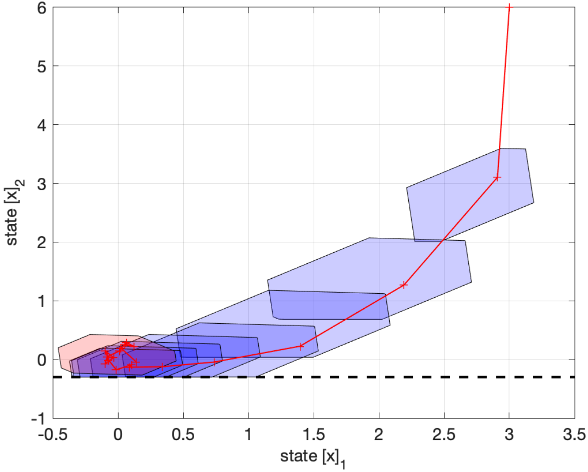


Figure: Realized closed-loop trajectory from initial condition $x_0 = [3\ 6]^\top$ (red line), predicted state tube at time $k = 0$ (tube cross-sections: blue, terminal set: pink)

Numerical example: constraint satisfaction

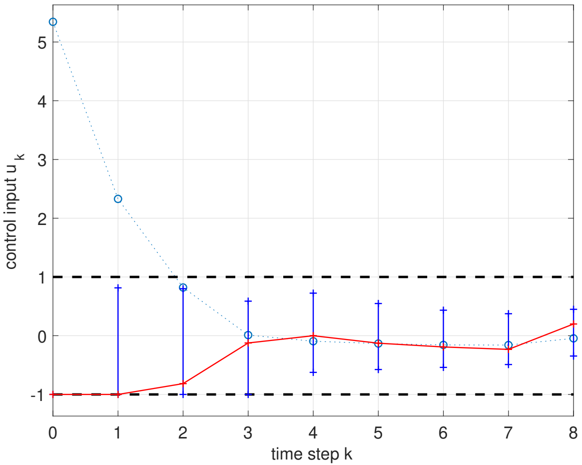


Figure: Realized closed-loop trajectory from initial condition $x_0 = [3\ 6]^\top$ (red line), predicted control tube at time $k = 0$ (tube cross-sections: blue)

Persistent excitation

The PE condition evaluated over a future horizon is nonconvex in $u_{i|k}, x_{i|k}$:

$$\text{(PE):} \qquad \sum_{i=0}^{N-1} D^\top(x_{i|k}, u_{i|k})\, D(x_{i|k}, u_{i|k}) \succeq \beta I$$

Let $u_{i|k} = \bar{u}_{i|k} + \tilde{u}_{i|k}$ and $x_{i|k} = \bar{x}_{i|k} + \tilde{x}_{i|k}$, where $\bar{x}_{0|k} = x_k$ and

$$\bar{u}_{i|k} = K\bar{x}_{i|k} + v^*_{i+1|k-1}$$
$$\bar{x}_{i+1|k} = A(\hat{\theta}_k)\bar{x}_{i|k} + B(\hat{\theta}_k)\bar{u}_{i|k}$$

then $D_{i|k} = \bar{D}_{i|k} + \tilde{D}_{i|k}$, $\bar{D}_{i|k} = D(\bar{x}_{i|k}, \bar{u}_{i|k})$, $\tilde{D}_{i|k} = D(\tilde{x}_{i|k}, \tilde{u}_{i|k})$

$$\begin{aligned} D_{i|k}^\top D_{i|k} &= \tilde{D}_{i|k}^\top \bar{D}_{i|k} + \bar{D}_{i|k}^\top \tilde{D}_{i|k} + \bar{D}_{i|k}^\top \bar{D}_{i|k} + \tilde{D}_{i|k}^\top \tilde{D}_{i|k} \\ &\succeq \tilde{D}_{i|k}^\top \bar{D}_{i|k} + \bar{D}_{i|k}^\top \tilde{D}_{i|k} + \bar{D}_{i|k}^\top \bar{D}_{i|k} \end{aligned}$$

so $\tilde{D}_{i|k}^\top \bar{D}_{i|k} + \bar{D}_{i|k}^\top \tilde{D}_{i|k} + \bar{D}_{i|k}^\top \bar{D}_{i|k} \succeq \beta I$ implies $D_{i|k}^\top D_{i|k} \succeq \beta I$

Persistent excitation

A sufficient condition for $\sum_{i=0}^{N-1} D_{i|k}^\top D_{i|k} \succeq \beta_k I$ is an LMI in $\tilde{x}_{i|k}, \tilde{u}_{i|k}, \beta_k$

(PE-LMI): $$\sum_{i=0}^{N-1} \bigl(\tilde{D}_{i|k}^\top \bar{D}_{i|k} + \bar{D}_{i|k}^\top \tilde{D}_{i|k} + \bar{D}_{i|k}^\top \bar{D}_{i|k} \bigr) \succeq \beta_k I$$

This can be expressed in terms of the MPC optimization variables using

$$\begin{aligned} \tilde{x}_{i|k} &\in \mathcal{X}_{i|k} - \{\bar{x}_{i|k}\} \\ \tilde{u}_{i|k} &\in K(\mathcal{X}_{i|k} - \{\bar{x}_{i|k}\}) + \{v_{i|k}\} - \{v^*_{i+1|k-1}\} \end{aligned}$$

Hence

$$\tilde{D}_{i|k} \in \mathrm{co}\Bigl\{ D\bigl(U^j \alpha_{i|k} - \bar{x}_{i|k}, K(U^j \alpha_{i|k} - \bar{x}_{i|k}) + v_{i|k} - v^*_{i+1|k-1}\bigr) \Bigr\}$$

so (PE-LMI) is equivalent to an LMI in optimization variables $\mathbf{v}_k, \boldsymbol{\alpha}_k, \beta_k$

Robust adaptive MPC algorithm with PE condition

Offline: Choose Θ_0, $\mathcal{X}$, γ, feedback gain K, and compute P

Online, at each time $k = 1, 2, \ldots$:

1. Given x_k, update set (Θ_k) and point $(\hat{\theta}_k)$ parameter estimates, and compute $\bar{x}_{i|k}, \bar{u}_{i|k}$, $i = 0, \ldots, N-1$

2. Compute the solution $(\mathbf{v}_k^*, \boldsymbol{\alpha}_k^*, \boldsymbol{\Lambda}_k^*)$ of the semidefinite program

$$\min_{\mathbf{v}_k, \boldsymbol{\alpha}_k, \boldsymbol{\Lambda}_k, \beta_k} J(x_k, \hat{\theta}_k, \mathbf{v}_k) - \gamma \beta_k$$

subject to $(\mathbf{v}_k, \boldsymbol{\alpha}_k, \boldsymbol{\Lambda}_k) \in \mathcal{F}(x_k, \Theta_k)$ and (PE-LMI)

3. Apply the control law $u_k^* = K x_k + v_{0|k}^*$

PE condition: numerical example

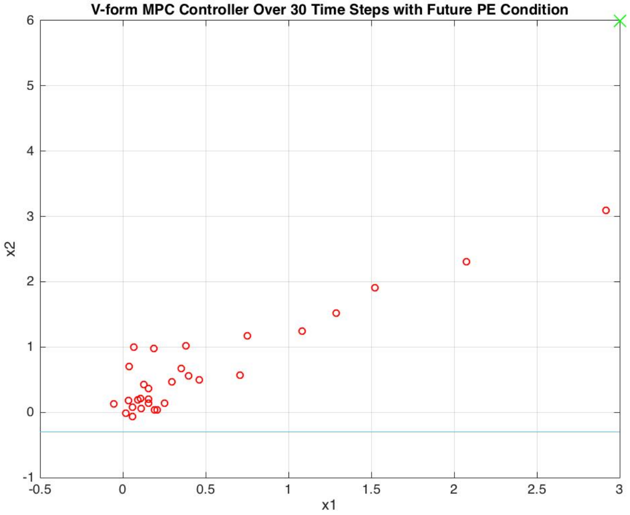


Figure: Evolution of closed loop system state x_k with PE weighting ($\gamma = 10^3$)

PE condition: numerical example

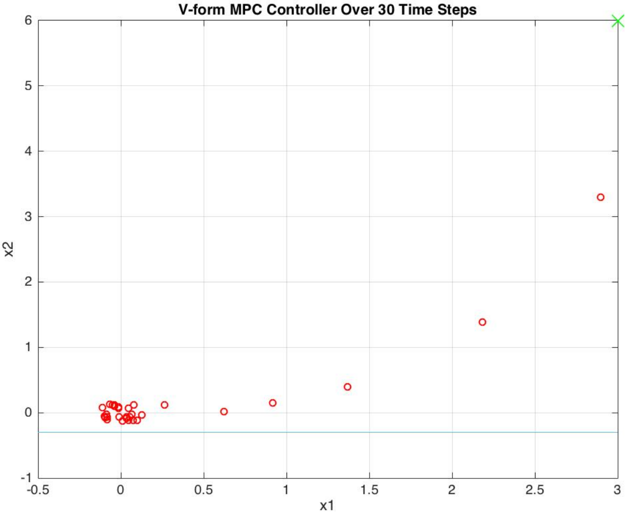


Figure: Evolution of closed loop system state x_k without PE weighting ($\gamma = 0$)

PE condition: numerical example

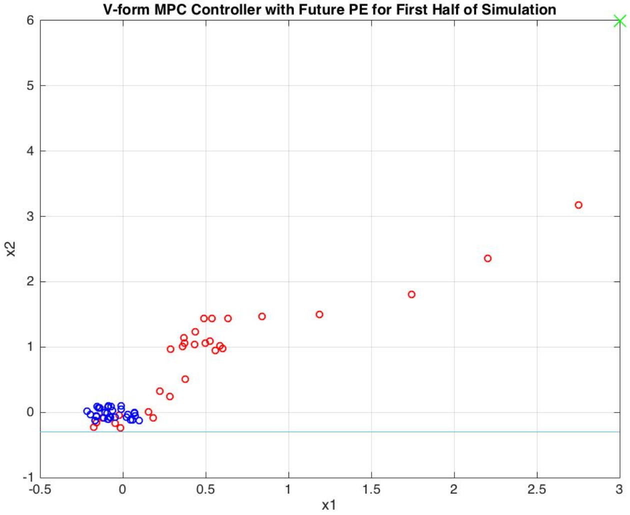


Figure: Evolution of closed loop system state x_k with time-varying PE weighting

Time-varying parameters

Assumption (time-varying parameters)

There exists a constant r_θ *such that the parameter vector* $\theta_k^\star$ *satisfies* $\theta_k^\star \in \Theta_0$ *for all* k *and* $\|\theta_{k+1}^\star - \theta_k^\star\| \leq r_\theta$

Define the dilation operator:

$$R_j(\Theta) = \{\theta : H_\Theta \theta \leq h + j r_\theta \mathbf{1}\}$$

Then the minimal parameter set at $k+1$ is

$$\Theta_{k+1} = R_1(\Theta_k \cap \Delta_k) \cap \Theta_0$$

and Θ_k is replaced in the tube MPC constraints by

$$\Theta_{i|k} = R_i(\Theta_k) \cap \Theta_0$$

Robust adaptive MPC algorithm with time-varying parameters

Parameter estimate bounds and recursive feasibility properties are unchanged:

Theorem (Closed loop properties)

If $\theta^\star \in \Theta_0$ and $\mathcal{F}(x_0, \Theta_0) \neq \emptyset$, then for all $k > 0$:

1. $\theta^\star \in \Theta_k$
2. $D(x_k, \Theta_k) \neq \emptyset$
3. $Fx_k + Gu_k \leq \mathbf{1}$

But the LMS filter has an additional tracking error, which invalidates the l^2 properties, i.e. "certainty equivalence" no longer applies

However other performance measures can be used in this context, such as the min-max approach of [Lorenzen, Allgöwer, Cannon, 2017]

Numerical example: time-varying parameters

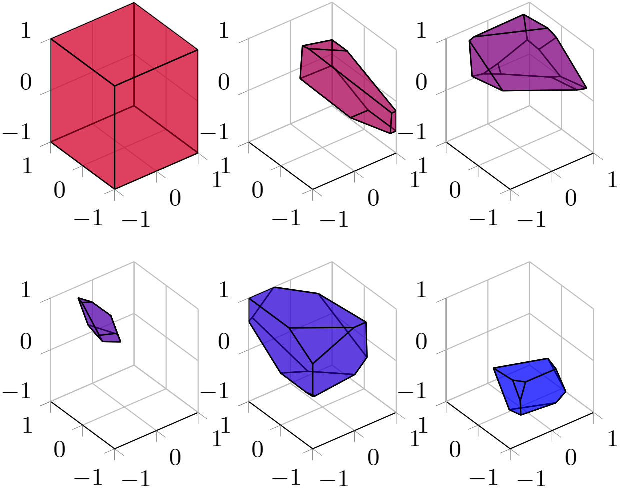


Figure: Parameter set Θ_k at times $k \in \{0, 100, 200, 300, 400, 500\}$ for the time-varying system with $r_\theta = 0.01$

Numerical example: time-varying parameters

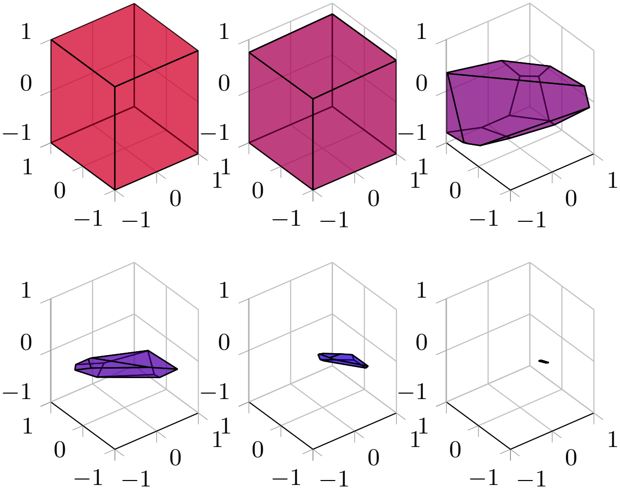


Figure: Parameter set Θ_k at times $k \in \{0, 5, 25, 70, 120, 500\}$ for the non-time-varying case for comparison

Conclusions & Outlook

Conclusions:

- Stable adaptive robust MPC is computationally tractable
- Set-membership parameter estimation and LMS point estimates are obvious choices for MPC cost functions and robust constraints
- Nonconvex PE conditions can be relaxed to convex sufficient conditions

Future work

- How to ensure recursive feasibility with PE constraints?
- How to balance requirements for performance and parameter convergence with PE penalty term in cost
- Alternative control objectives and stability properties
- Can we relax the assumption of bounded disturbances?

References:
– M. Lorenzen, M. Cannon, & F. Allgöwer, "Robust MPC with recursive model update" Automatica (2019)
– X. Lu, M. Cannon, "Robust adaptive tube model predictive control" ACC (2019)