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Turbulent Flame Speed and Self-Similar Propagation of Expanding Premixed Flames

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A unified scaling of experimental turbulent flame speed data, measured in constant-pressure expanding turbulent premixed flames, propagating in nearly homogenous isotropic turbulence in a dual-chamber fan-stirred vessel is presented. While the cold flow is characterized by high speed particle image velocimetry, the flame propagation rate is obtained by tracking high speed Schlieren images of unity Lewis number methane-air flames over wide ranges of pressure and turbulence intensity. It is found that the normalized turbulent flame speed as a function of the average radius scales as a turbulent Reynolds number to the one-half power, where the average radius is the length scale and thermal diffusivity is the transport property, thus showing self-similar propagation. Utilizing this dependence and recent theoretical results obtained by the spectral closure of the G-equation, it is found that the turbulent flame speeds from expanding flames and those from Bunsen geometries are scaled by the same one half power dependency of the turbulent Reynolds number with appropriate choice of length scales.

1. Introduction

The turbulent flame speed is a topic of wide interest in combustion and turbulence research as evidenced by the large volume of analytical [1-8], experimental [9-15], computational [16-17] and review literature [18-21] that has emerged in the past few decades. Besides being a fundamental problem of considerable complexity, its practical relevance is no less significant as the turbulent flame speed, being a measure of flame surface density, can be correlated to the volumetric heat release rate in a turbulent reacting flow: ubiquitous in any energy conversion device utilizing combustion. From a different perspective, turbulent propagation of expanding flames is also a problem of astrophysical interest as it is suggested that the phenomena of supernova explosion is characterized by the transition of a subsonic, turbulent deflagration wave to a supersonic, detonation wave. The complexity of the problem is compounded by the disagreement between theories, the high degree of scatter of the experimental turbulent flame speeds and their sensitivity on the geometry and type of the burner used in the investigation [19], – a major hindrance that has prevented its utilization as a meaningful physical quantity for experimental and computational turbulent combustion research.

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Fundamentally, under the long held assumption that the turbulent flame speed is a meaningful physical quantity and that it would demonstrate a unified scaling behavior under special flow conditions, say in homogenous isotropic turbulence, it behooves us to persist to seek such a scaling. The most obvious choice of flame parameters for such a scaling would be the planar laminar flame speed $S_L \sim \left(D\omega_b^0\right)^{0.5}$ and the corresponding laminar flame thickness $\delta_L \sim \left(D/\omega_b^0\right)^{0.5}$, where D and ω_b^0 are the characteristic thermal diffusivity and reaction rate respectively [22]. Thus from a hydrodynamics point of view, S_L and δ_L can characterize a premixed flame even in turbulence if a local laminar structure exists. Then the problem of turbulent flame propagation can be considered as a geometric problem in which the effect of turbulence is to wrinkle the flame at a multitude of length scales without perturbing the inner flame structure. Such a problem was considered analytically in [23] and it was shown that for a statistically planar flame propagating in homogenous isotropic turbulence, the turbulent flame speed normalized by the corresponding laminar flame speed for large turbulence Reynolds number $\left(\mathrm{Re}_T\right)$ is given to the leading order by

$$S_T / S_L \sim \left\langle \sqrt{1+\nabla g \cdot \nabla g} \right\rangle \sim \left[1+\int_{k_I}^{\infty} k^2 \Gamma(k) dk\right]^{1/2} \sim \left[\left(u_{rms}/S_L\right)\left(\lambda_I/\delta_L\right)\right]^{1/2} \tag{1}$$

where g is the fluctuating flame surface height from the mean surface, k the wavenumber, $\Gamma(k)$ the flame surface spectrum obtained from spectral closure of the G-equation [26] , u_{rms} the root mean square of velocity fluctuations and λ_I the velocity integral length scale which was assumed to be the flame hydrodynamic length scale. This approximation was shown to be valid when $\partial g/\partial x_i$ follows Gaussian distribution or $\left(\partial g/\partial x_i\right)\left(\partial g/\partial x_i\right)$ follows log-normal distribution as is expected for scalar gradients in homogenous isotropic turbulence. Equation (1) also shows the explicit dependence of the turbulent flame speed on the integral length scale which has been considered to be equivalent to the flame hydrodynamic length scale, thus pointing towards self similarity.

In this paper we present experimental turbulent flame speed data measured in constant-pressure expanding flames, propagating in nearly homogenous isotropic turbulence. Utilizing the self-similar property of turbulent flame speeds evolving from the theory and experimental data presented, we shall show in due course that the turbulent flame speeds from the present spherically expanding flames, as well as those from literature data on Bunsen flames, can be scaled by a single parameter: a turbulence Reynolds number based on the geometric and transport properties of the flame. A unified, configurationally independent description of turbulent flame propagation in homogenous isotropic turbulence is thus proposed.

2. Experiments

The experiments were conducted in a nearly constant-pressure apparatus that has been extensively employed in the study of laminar flames [27]. Briefly, the apparatus consists of an inner chamber situated within an outer chamber of much larger volume. The two chambers can be opened to each other by rotating a sleeve that otherwise covers a matrix of holes connecting the two chambers. The inner chamber is filled with the test combustible gas while the outer chamber with an inert gas that has the same density as that of the reactive gas. At the instant that the inner gas is ignited, say by a spark, the two chambers are connected such that the propagating flame is automatically quenched upon contacting the outer chamber. The propagation is therefore basically isobaric because of the small volume of the inner chamber, hence preventing any perturbation by global pressure rise on the local flame structure. Another advantage of the design is that experiments can be conducted under high initial pressures, up to 60 bars as in the studies of Refs. [27]-[28], while preserving the integrity of the optical windows.

In the present investigation, turbulence is generated by four high speed fans as in [9], with the resulting cold flow field characterized by high speed particle image velocimetry (HS-PIV). It was found that the root mean square velocity u_{rms} was a factor of two to three larger than the unavoidable radially inward mean flow $\langle U_r \rangle$ in the PIV measurement plane. Moreover, $\langle U_r \rangle$ $\leq 25\%$ of the mean flame propagation rate for all cases studied. It was ensured that the statistic $\langle u_x u_y \rangle \sim 0$, one of the necessary conditions for the turbulence being isotropic. The experiments were conducted at pressures of 1, 2, 3 and 5 atm and with u_{rms} ranging from 1.34 to 6m/s. The $u_{rms}(\langle R \rangle)$, obtained at a given radius R by averaging over all θ, over a set of five hundred PIV vector fields in the non-reacting flow, is a weakly increasing function with $\langle R \rangle$. As recognized in [9], an increase in $\langle R \rangle$ implies an increase in the total kinetic energy experienced by the flame, with $\langle R \rangle$ being a measure of the smallest wavenumber of the eddy capable of wrinkling the flame. In addition a slight non-homogeneity in the actual, u_{rms} which is $\leq \pm 10\%$ over the domain used for measuring the turbulent flame speed, also contributes to the increase of $u_{rms}(\langle R \rangle)$ with $\langle R \rangle$.

The domain of experimentation was suitably chosen to avoid ignition and wall effects at the initial and final stages of flame propagation respectively. Several turbulent flame propagation experiments were also conducted at different ignition energies to ensure that the flame propagation features observed are not artifacts of spark ignition.

3. Results and Discussion

Figure 1 shows a set of Schlieren images of methane−air turbulent premixed flames at an equivalence ratio of $\phi = 0.9$, corresponding to unity Lewis number (*Le*) with respect to inert, at different u_{rms} and pressures, and at nearly the same $\langle R \rangle$ realized at different instants of their propagation. Before presenting quantitative results, we discuss some key features of these images, which were obtained with a Phantom V7.3 high speed camera at 8510 fps. Specifically, it is observed that with increasing u_{rms}, the flame propagates faster on average, which is a well known feature, and slightly finer scale structures emerge due to the reduction of the Kolmogorov length scale. Furthermore, with increasing pressure, the flame also propagates faster, and the flame overall appears very different due to emergence of very small scale structures. These fine structures indicate reduction of the thickness of the laminar flamelets with increasing pressure, which also allows flame surface wrinkling at progressively smaller scales. We ask whether as a consequence of all these observations, the global propagation rate of these different flames can be appropriately scaled, as only then there can be a unified turbulent flame speed quantity as a function of the flow and flame parameters.

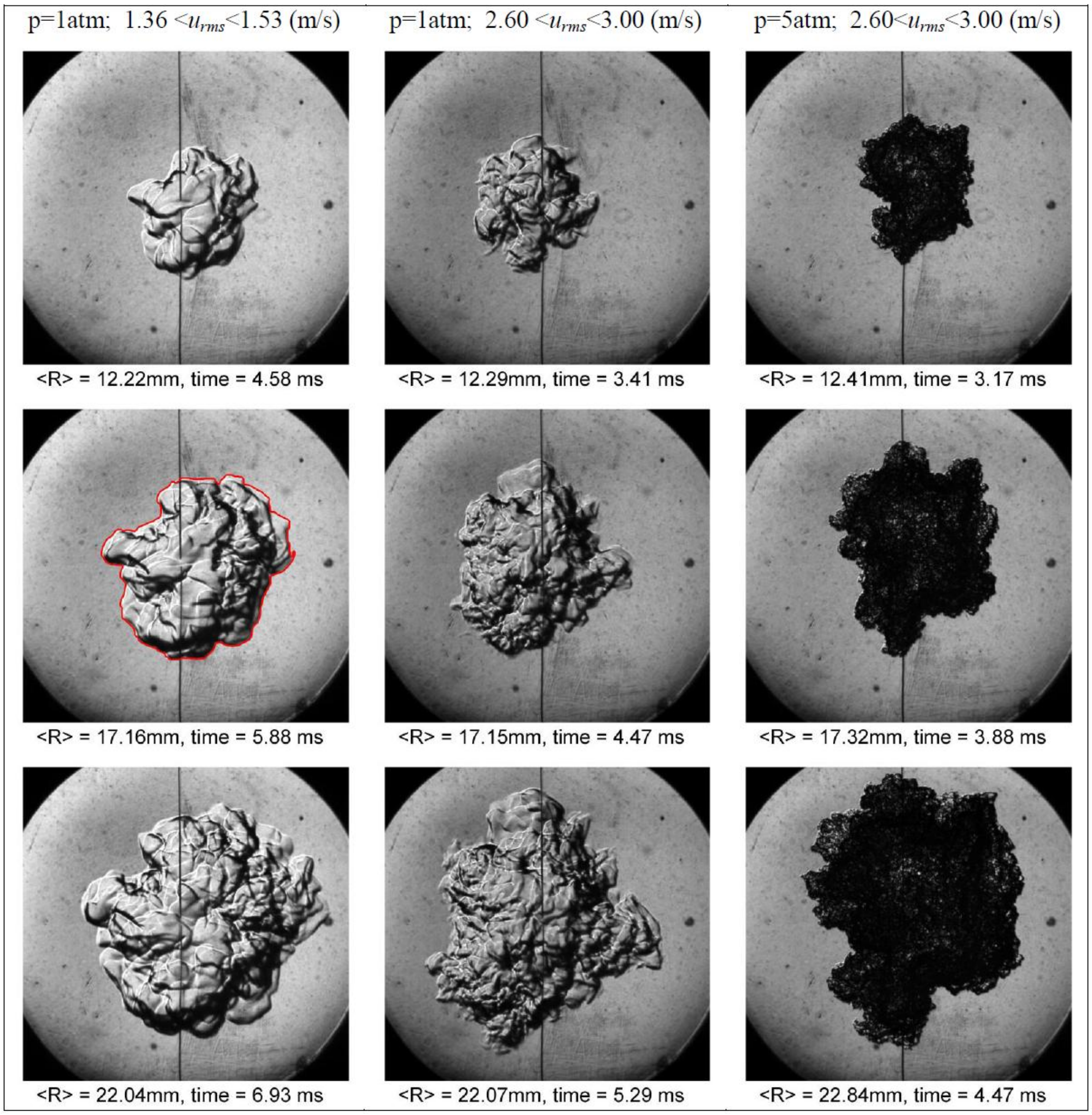

Figure 1: High speed Schlieren imaging at different u_{rms} and pressure, but at nearly same $\langle R \rangle$ showing the emergence of fine scale structures and associated increase in average propagation rate. The u_{rms} range specified above is the range experienced by the flame due to change in the largest length scale of the flame itself. Within the range 1-5 atm, $S_L^u \sim p^{-0.38}$, and $\delta_L \sim p^{-0.69}$ [31-32] resulting in $d\langle R \rangle / dt \sim p^{0.16}$.

It is recognized that all these observations are not affected by any intrinsic flame instability as it was demonstrated by laminar flame propagation experiments that over the pressure range explored, essentially no DL instability develops. Furthermore, the $Le \sim 1$ condition prohibits any onset of diffusive-thermal instability and the effect of mean curvature due to statistical sphericity of the flame should be negligible on the mean propagation rate.

In the data reduction, $\langle R \rangle$ is defined as $\langle R \rangle = \sqrt{A/\pi}$, where A is the area enclosed by the flame edge tracked from the high speed Schlieren imaging using fully automated image processing Matlab routines utilizing Canny edge detection [29], representatively shown in the 2nd row 1st column of Fig. 1. It is found from quantitative measurements that the average radius $\langle R \rangle$ is a monotonic but nonlinear function of time, which implies that the flame propagation rate $d\langle R \rangle / dt$ is not a constant.

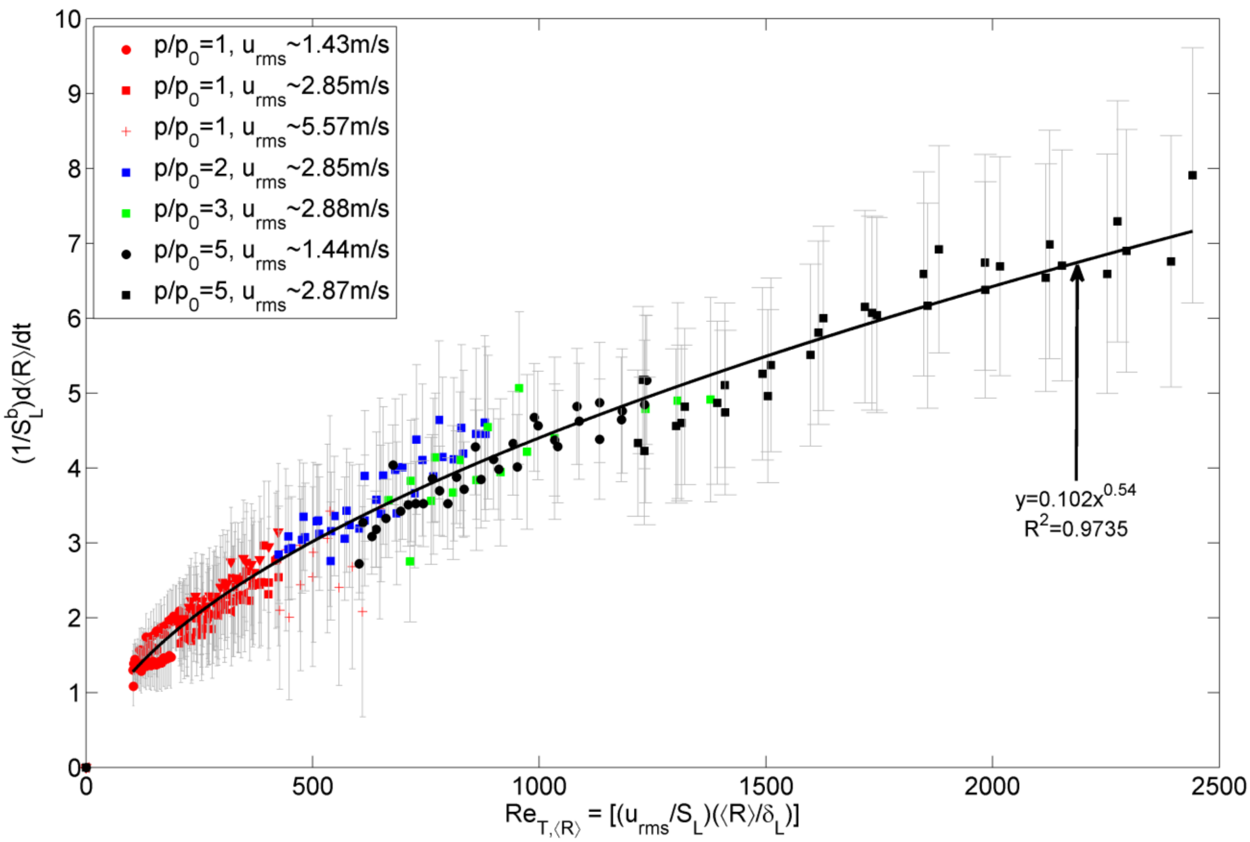

Figure 2: Plot of flame propagation rate normalized by laminar burnt flame speed with respect to turbulent Re with average radius as length scale and thermal diffusivity as transport property. Not included in the fitting is the $p/p_0 = 1$, $u_{rms} \sim 5.57$m/s case as the Schlieren images show widespread local extinction in such flames and evident from their lower propagation rates. u_{rms} appearing in the abscissa is the $u_{rms}(\langle R \rangle)$, obtained by integrating over all θ . $u_{rms} \sim$ denotes the average u_{rms} experienced by the flames during the propagation event. The error bars indicate the error that could be caused by the mean flow.

Equation (1) provides insight for self-similar propagation if λ_I is replaced with the relevant largest flame length scale, i.e. hydrodynamic length scale of flame surface fluctuations which should be a linear function of $2\pi\langle R\rangle$ or simply $\langle R\rangle$ itself. Consequently, in Fig. 2 we have plotted $d\langle R\rangle/dt$, obtained from the experimentally obtained $\langle R\rangle$, normalized by the laminar flame speed with respect to the burnt gas (S_L^b), as a function of $\left[(u_{rms}/S_L)(\langle R\rangle/\delta_L)\right]$, with the integral length scale being replaced by $\langle R\rangle$. Since the thermal diffusivity $D \sim S_L\delta_L$, the term $\left[(u_{rms}/S_L)(\langle R\rangle/\delta_L)\right] = \mathrm{Re}_{T,\langle R\rangle}$ represents a turbulence Reynolds number with $\langle R\rangle$ being the length scale and the thermal diffusivity replacing the kinematic viscosity. It is observed from Fig. 2 that all the data from different conditions of turbulence intensity and pressure, and at each instant of the propagation event, collapse reasonably well on a $\mathrm{Re}_{T,\langle R\rangle}^{\alpha}$ curve, with the exponent $\alpha = 0.54$ obtained by nonlinear least-square fitting over the entire data set. This result therefore suggests the possible validity of the one-half power scaling. More importantly, it suggests that the turbulent expanding flame propagation is self similar, at least in the domain of interrogation, as evident from the relationship

$$\left(S_L^b\right)^{-1} d\langle R\rangle/dt = O(1)\left[(u_{rms}/S_L)(\langle R\rangle/\delta_L)\right]^{1/2} \qquad (2)$$

The 1/2-power of Eqn. (2) also indicates that the turbulent flame is accelerating. This is due to the fact that as the flame expands during propagation, its smallest wavenumber decreases resulting in a continuous increase in the integral of the $k^2\Gamma$ spectrum or the flame surface scalar dissipation spectrum, as shown in Fig. 3.

It is important to note that the mechanism of flame acceleration proposed here is distinctly different than that proposed in [9], in which the flame acceleration is attributed to the increase of $u_{rms}(\langle R\rangle)$ as experienced by the flame. We have experimentally found that the θ-averaged $u_{rms}(\langle R\rangle) \sim \langle R\rangle^{0.25}$, which shows a much weaker dependence on $\langle R\rangle$ than the near-linear scaling required to explain Eqn. (2) and for collapsing all the data on a single curve as in Fig. 2. Theoretically, it can also be shown that

$$u_{rms}(\langle R\rangle) \sim \left(\int_{k(\langle R\rangle)}^{\infty} \varepsilon^{2/3} k^{-5/3} \exp\left(-C(k\eta)^{4/3}\right) dk\right)^{1/2} \sim \left(\langle R\rangle\right)^{1/3}$$

where ε is the mean kinetic energy dissipation rate and η the Kolmogorov length scale. In comparison, the global average flame surface dissipation rate, i.e. the integral in Eq. (1), is given by [23],

$$\int_{k(\langle R\rangle)}^{\infty} k^2\Gamma(k)dk \sim \left(\langle R\rangle\right); \text{ where } \Gamma(k) = Bk^{-5/3}\exp\left(-\tfrac{3}{4}(2\pi)^{4/3} c_2 Mk\left(\frac{u_{rms}}{S_L}\frac{\lambda_I}{\delta_L}\right)^{-1}(k/k_I)^{4/3}\right) \text{ and } Mk$$

is the Markstein length. Physically the turbulent flame speed can be effectively increased by stretching and raising the $k^2\Gamma$ spectrum, as shown in Fig. 3, towards higher or lower wavenumbers by change of the largest or smallest flame length scales respectively. This mechanism is believed to be of significance in explaining the acceleration of such expanding flames, which by its definition implies decrease of its smallest wavenumber. Increase in pressure results in stretching the spectrum on the side of the higher wavenumber. The acceleration may eventually cease when the flame length scale $\langle R \rangle$ approaches the flow hydrodynamic length scale, at which the growth of the $k^2\Gamma$ spectrum is terminated.

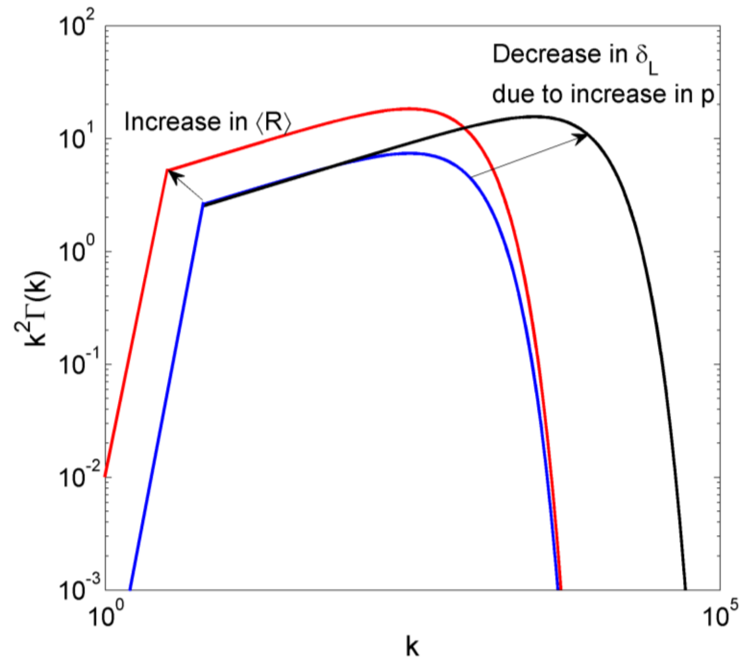

Figure 3: Schematic of flame surface dissipation spectrum showing the effect of increase of the largest flame length scale: $\langle R \rangle$ decrease of the smallest length scale i.e. δ_L, both causing increase in the area under the spectrum and thus increased turbulent flame speed ratio, specifically acceleration for an expanding flame.

4. Gas Expansion Effects and Unification of Turbulent Flame Speeds

It is important to recognize that $d\langle R \rangle / dt \neq S_{T,b}$ due to gas expansion effects. An expanding turbulent flame with zero mean flow necessitates zero mean burnt gas velocity boundary condition. This is satisfied by a radially outward gas expansion flow induced ahead of the flame given by $V_0 \sim (\Theta - 1) S_{T,u}$, where $\Theta = \rho_u / \rho_b$. According to measurements by Bradley et al. [30], $\langle R \rangle$ obtained by Schlieren imaging corresponds to the progress variable, $\langle c \rangle \sim 0.05 - 0.1$. This is reasonable as the Schlieren image, being a projection contains interference of large scale flame structures from planes other than the diametrical plane. Assuming that the location $\langle R \rangle$ lies predominantly in the unburnt gas and free from density fluctuations, it can be shown that:

$$
\begin{aligned}
&\langle \rho_0 \rangle 4\pi r_0^2 \left(-S_{T,0}\right) = \langle \rho_1 \rangle 4\pi r_1^2 \left(-S_{T,1}\right) = \langle \rho_{0.5} \rangle 4\pi r_{0.5}^2 \left(-S_{T,0.5}\right) \\
&\Rightarrow \langle \rho_0 \rangle 4\pi r_0^2 \mathbf{n} \cdot \left(\langle \mathbf{V_0} \rangle - d\langle \mathbf{R} \rangle / dt\right) = \langle \rho_1 \rangle 4\pi r_1^2 \left(-S_{T,1}\right) \\
&\Rightarrow d\langle R \rangle / dt = \Theta S_{T,0} = \left(r_1^2 / r_0^2\right) S_{T,1}
\end{aligned}
\tag{3}
$$

where the numerical subscript denotes the $\langle c \rangle$ value.

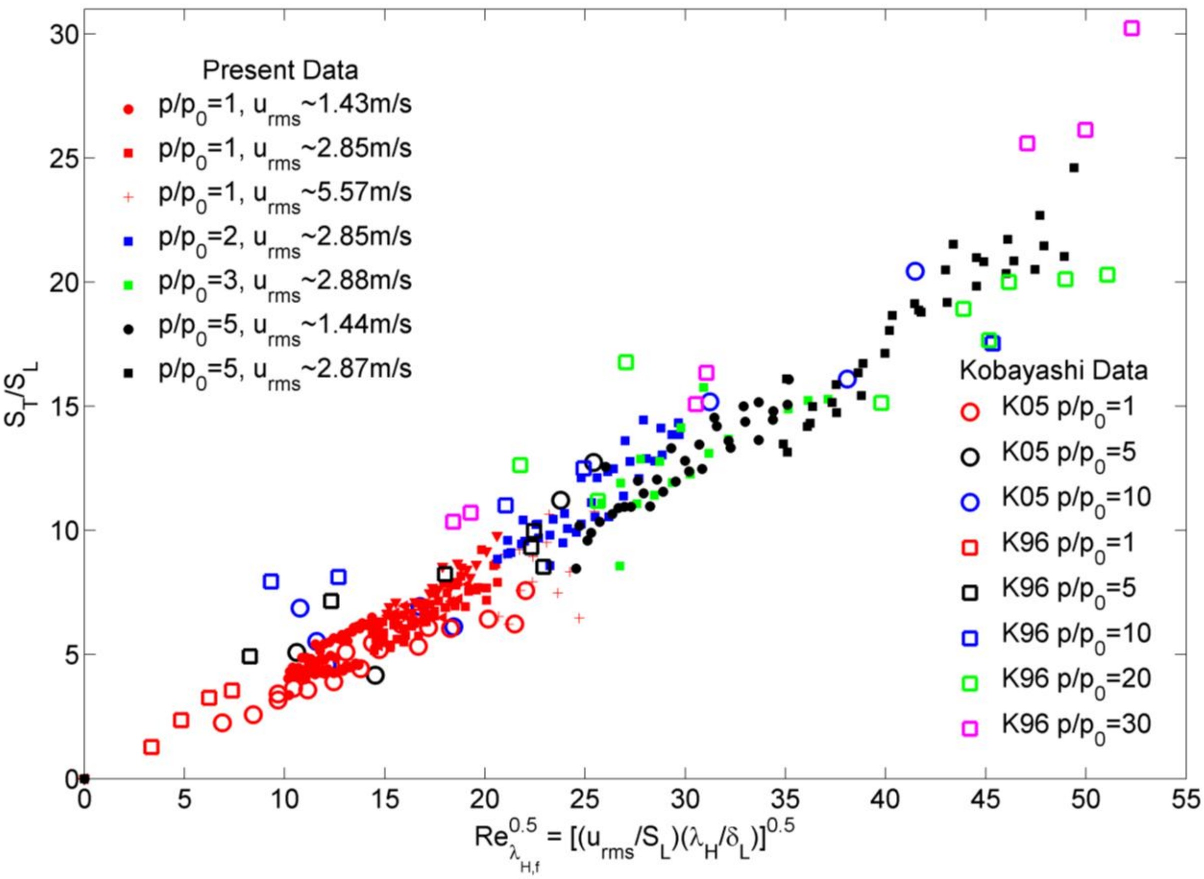

Figure 4: Unifying turbulent flame speed from expanding flames (data from Fig. 2) and that from Kobayashi`s experiments by a turbulent Reynolds number to the one half power scaling. The length scale is the instantaneous average flame radius in case of spherical flames and burner diameter for Bunsen flames. Kinematic viscosity is replaced by thermal diffusivity. The ordinate is $S_{T,0.5} / S_L^u$ **, chosen to appropriately represent the normalized turbulent flame speed.**

It has been verified by measuring the Schlieren radius at each θ that the flame brush thickness $\delta_T \sim \langle R \rangle / 2$, which implies that $S_{T,1} = 4\left(d\langle R \rangle / dt\right)$. Similarly, simple algebra yields

$$
S_{T,0.5} / S_L^u = \left(2\Theta / (\Theta + 1)\right)\left(r_0^2 / r_{0.5}^2\right)\left(S_L^b\right)^{-1}\left(d\langle R \rangle / dt\right) = (28/9)\left(S_L^b\right)^{-1}\left(d\langle R \rangle / dt\right),
$$

for $\Theta = 7$ and assuming $\langle \rho_{0.5} \rangle = \left(\langle \rho_0 \rangle + \langle \rho_1 \rangle\right)/2$, which can be considered to be the normalized turbulent flame speed as it is defined on the location of the mean flame front, $\langle c \rangle = 0.5$.

This result can then be compared with experimental turbulent flame speeds determined from other geometries such as the widely used Bunsen flame. Extensive data over large pressure and turbulence intensity ranges has been reported in Kobayashi *et al.* [14]-[15]. However the two papers present measurements of the turbulent flame speed at two different $\langle c \rangle$ locations, with the more recent paper [15] measuring at $\langle c \rangle = 0.1$ and the earlier one [14] at $\langle c \rangle = 0.5$. For $\langle c \rangle = 0.1$, the gas expansion velocity is negligible and the configuration is statistically steady, while at $\langle c \rangle = 0.5$ there are non-negligible gas expansion effects, resulting in turbulent flame speeds with identical scaling but different prefactors. According to Smallwood *et al.* [31] the correction factors for converting the turbulent flame speed at $\langle c \rangle = 0.1$ to that at $\langle c \rangle = 0.5$ should be ~1.2 to 1.5 for Bunsen flames at 1 atm pressure condition. Consequently we choose the correction factor to be the average of the two, 1.35. This prefactor should not depend on pressure as the flame brush is mainly controlled by large-scale eddies while pressure causes reduction in the flame thickness, which is a small scale effect. This data is shown in Fig. 4 along with that from the present experiments as S_T / S_L versus $\left(\mathrm{Re}_{\lambda H, f}\right)^{1/2}$, i.e. a turbulence Reynolds number with the flame hydrodynamic length scale and flame transport properties. It is seen that the two data sets collapse quite well, hence suggesting the possibility of a unified turbulent flame speed when it is appropriately scaled and corrected for gas expansion effects.

5. Remarks

Utilizing insights from recent theoretical studies, we have proposed and experimentally substantiated herein a unified scaling of the normalized turbulent flame speed with the turbulence Reynolds number to the one-half power; with the average radius being the length scale and thermal diffusivity the transport coefficient. The constant value of the power in the Reynolds number dependence thus suggests self-similar propagation.

Several points require further discussion. First, the above self-similar result was obtained by drawing correspondence between the results for a planar steady flame wrinkled by homogenous isotropic turbulence and those of an expanding spherical flame in near homogenous isotropic turbulence, by substituting the integral length scale of the former with the instantaneous average radius of the later. Such a change of the length scale would precede a question that the two cases are fundamentally different because one is a statistically planar steady flame while the other is an expanding and thus time varying phenomenon. In this regard we note that the very definition of turbulent flame speed is an ensemble average of the normalized flame surface per unit volume times the laminar flame speed. Thus by the *ergodicity* hypothesis it can be expected that after the initial spark ignition and stretch-affected period, statistically steady and expanding cases would yield identical results if the flow field is fully developed, as then the ensemble average converges to a time average and vice versa, when they are properly scaled and with unity *Le*.

Next we note that the self-similar propagation given by Eqn. (2) may imply a fractal structure i.e. hierarchical clustering [33]. Self similarity of hydrodynamically unstable expanding flames due to fractal structure was proposed in [34] and constitutes an important research issue.

Finally, the fact that the turbulent flame undergoes self-similar acceleration suggests it being a possible mechanism for the phenomenon of deflagration-to-detonation transition (DDT) in a turbulent flow field. Resolution of issues of such nature merits further study.

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