

# PCA, CA and Clustering

Fujie Mei Sergio Delgado Mario Wang

April 29, 2023

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# 1 Loading data and deleting columns

We will delete the columns we said that won't contained too many errors to be analyzeable.

```
df<-read.csv2("clean_data.csv")
df$X<-NULL
df$pdays<-NULL
df$previous<-NULL
df$errVar<-NULL
names(df)
```

```
## [1] "age"          "job"          "marital"
## [4] "education"    "housing"      "loan"
## [7] "contact"      "month"        "day_of_week"
## [10] "duration"     "campaign"     "poutcome"
## [13] "emp.var.rate" "cons.price.idx" "cons.conf.idx"
## [16] "euribor3m"    "nr.employed"  "y"
## [19] "Age_group"    "Campaign_contacts" "mout"
```

```
vars_con = c("age","campaign","emp.var.rate","cons.price.idx","cons.conf.idx","euribor3m","nr.employed")
vars_dis = c("job","marital","education","housing","loan","contact","month","day_of_week")
vars_res= c("y","duration")
```

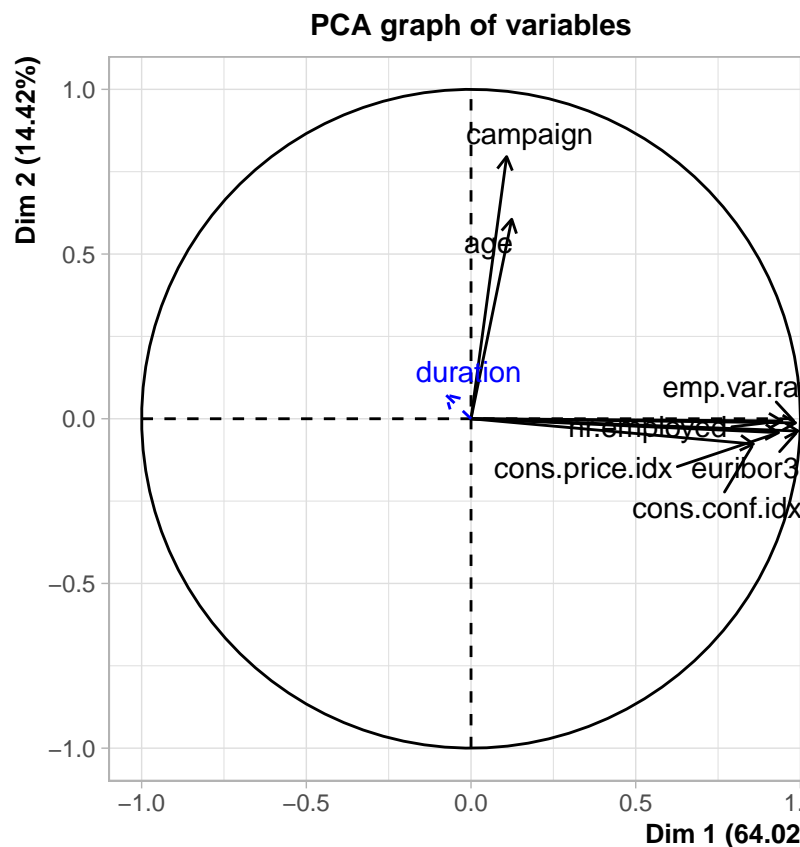
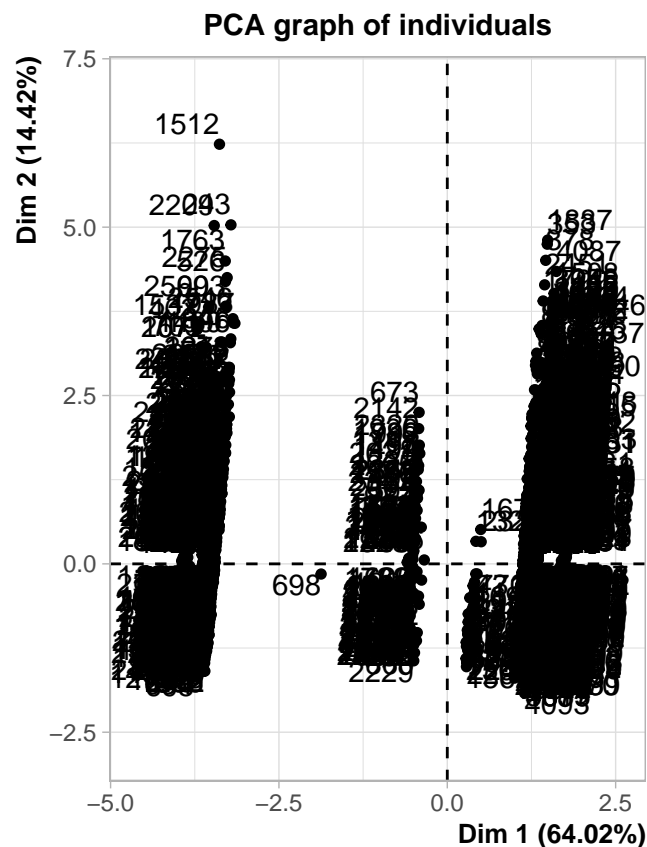
## 2 Principal Component Analysis (PCA)

We are going to do a PCA analysis in our numerical variables from our dataset, from the PCA graph we can see that the target variable, duration, has little effect and the rest of the variable are very contributive to their respective axes. As we can see, they are very near to the axes and their length are very long.

### 2.1 Eigenvalues and dominant axes analysis. How many axes we have to interpret according to Kayser and Elbow's rule?

From the Kayser's rule, all eigenvalues  $>1$ , we should consider 2 dimensions, on the other hand, with the Elbow's rule 4 dimensions is the most suitable. In our case we will take Kayser's rule into consideration because it's least number of components and the cumulative variation is almost 80%.

```
res.pca <- PCA(df[,c("duration",vars_con)],quantile.sup=c(1))
```



```
summary(res.pca)
```

```
##
## Call:
## PCA(X = df[, c("duration", vars_con)], quanti.sup = c(1))
##
##
## Eigenvalues
##
```

	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5	Dim.6	Dim.7
## Variance	4.481	1.009	0.978	0.338	0.167	0.018	0.009
## % of var.	64.016	14.419	13.965	4.831	2.381	0.264	0.124
## Cumulative % of var.	64.016	78.435	92.400	97.231	99.612	99.876	100.000

```
##
## Individuals (the 10 first)
##
```

	Dist	Dim.1	ctr	cos2	Dim.2	ctr	cos2	Dim.3
## 1	1.905	1.185	0.006	0.387	0.650	0.008	0.116	-0.511
## 2	3.706	-3.569	0.057	0.928	0.579	0.007	0.024	-0.792
## 3	2.215	1.208	0.007	0.298	-1.289	0.033	0.339	-0.359
## 4	3.640	-3.508	0.055	0.929	-0.389	0.003	0.011	-0.826
## 5	2.007	1.083	0.005	0.291	-0.613	0.007	0.093	-0.923
## 6	3.682	-3.625	0.059	0.969	-0.328	0.002	0.008	0.517
## 7	2.790	1.810	0.015	0.421	1.432	0.041	0.263	-0.642
## 8	2.328	-0.474	0.001	0.042	0.551	0.006	0.056	1.837
## 9	1.968	1.072	0.005	0.296	-0.917	0.017	0.217	-0.071
## 10	1.609	-0.611	0.002	0.144	-0.522	0.005	0.105	-0.833

```
##
##          ctr  cos2
## 1      0.005 0.072 |
```

```

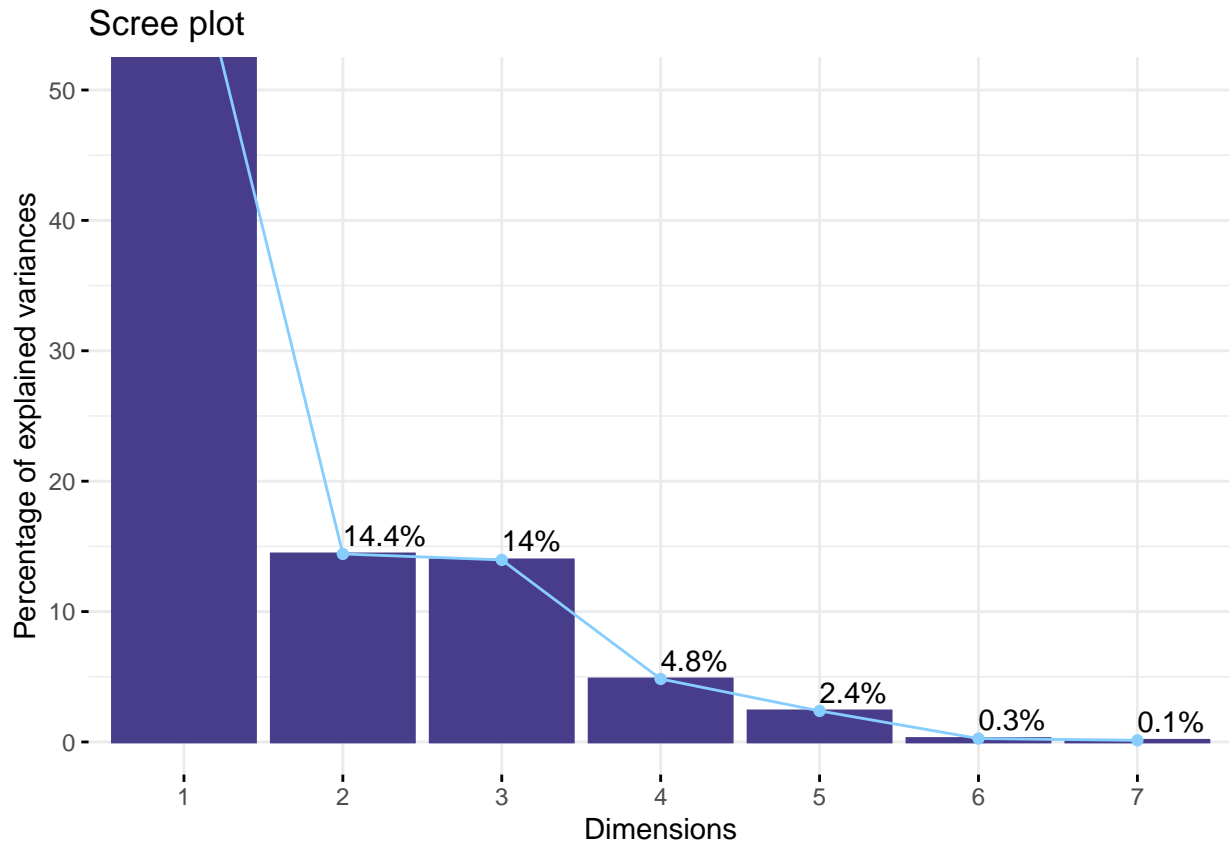
## 2          0.013  0.046 |
## 3          0.003  0.026 |
## 4          0.014  0.051 |
## 5          0.017  0.211 |
## 6          0.005  0.020 |
## 7          0.008  0.053 |
## 8          0.069  0.623 |
## 9          0.000  0.001 |
## 10         0.014  0.268 |
##
## Variables
##          Dim.1    ctr   cos2    Dim.2    ctr   cos2    Dim.3    ctr
## age          |  0.124  0.341  0.015 |  0.606 36.341  0.367 |  0.786 63.161
## campaign     |  0.108  0.261  0.012 |  0.796 62.742  0.633 | -0.593 35.973
## emp.var.rate |  0.985 21.671  0.971 | -0.012  0.015  0.000 | -0.027  0.073
## cons.price.idx |  0.934 19.453  0.872 | -0.042  0.176  0.002 | -0.004  0.002
## cons.conf.idx |  0.857 16.403  0.735 | -0.077  0.583  0.006 |  0.068  0.472
## euribor3m    |  0.993 21.988  0.985 | -0.037  0.138  0.001 | -0.009  0.009
## nr.employed  |  0.944 19.882  0.891 | -0.007  0.005  0.000 | -0.055  0.311
##          cos2
## age          0.617 |
## campaign     0.352 |
## emp.var.rate  0.001 |
## cons.price.idx 0.000 |
## cons.conf.idx  0.005 |
## euribor3m    0.000 |
## nr.employed   0.003 |
##
## Supplementary continuous variable
##          Dim.1   cos2    Dim.2   cos2    Dim.3   cos2
## duration      | -0.076  0.006 |  0.072  0.005 | -0.116  0.013 |

```

```

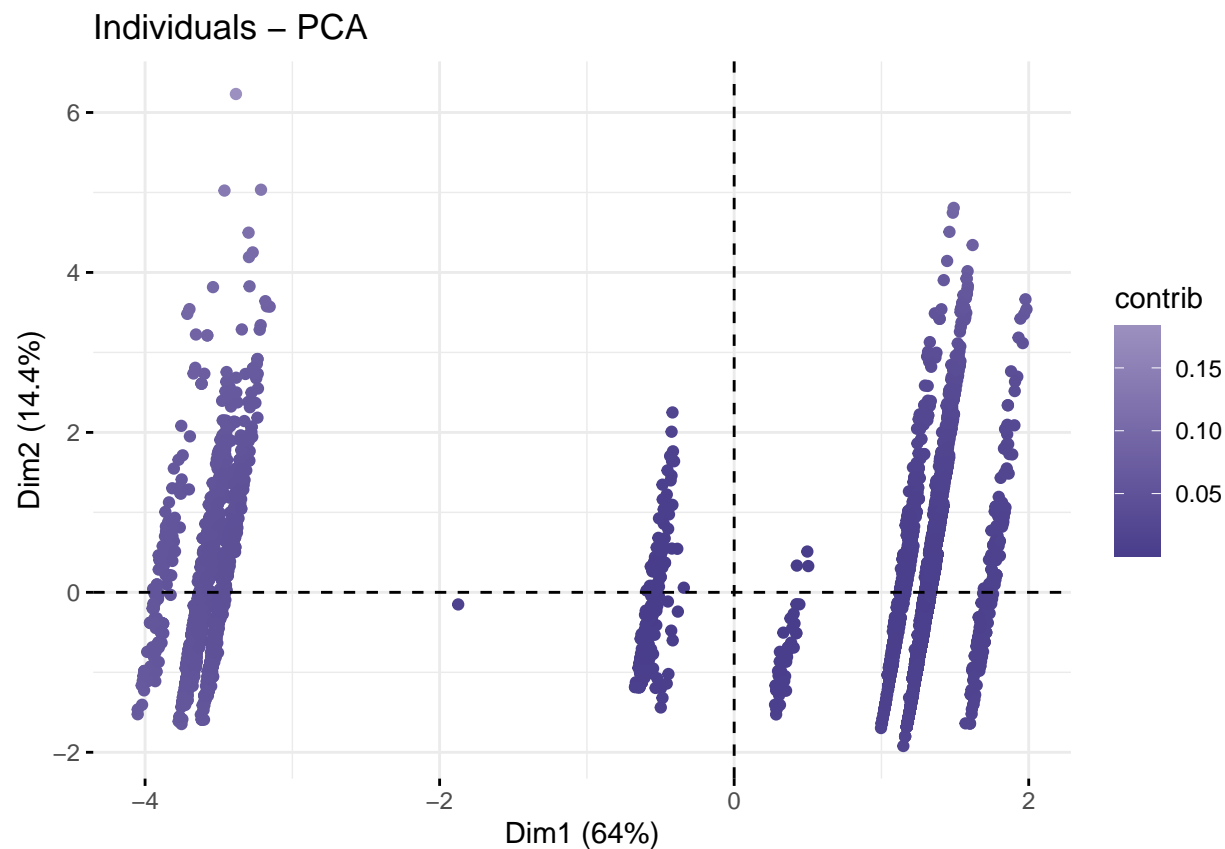
fviz_screplot(
  res.pca,
  addlabels=TRUE,
  ylim=c(0,50),
  barfill="darkslateblue",
  barcolor="darkslateblue",
  linecolor = "skyblue1"
)

```



## Individuals point of view: Are they any individuals “too contributive”? From what we can see in the graph of individuals, none is “too contributive”, as we can see contributions values that ranges from 0 to 0.20 more or less. So we can say in this part that almost all individuals contribute the same.

```
# head(res.pca$ind$contrib) # contribution of individuals to the princial components
fviz_pca_ind(res.pca, col.ind="contrib", geom = "point") +
scale_color_gradient2(low="darkslateblue", mid="white",
                      high="red", midpoint=0.40)
```



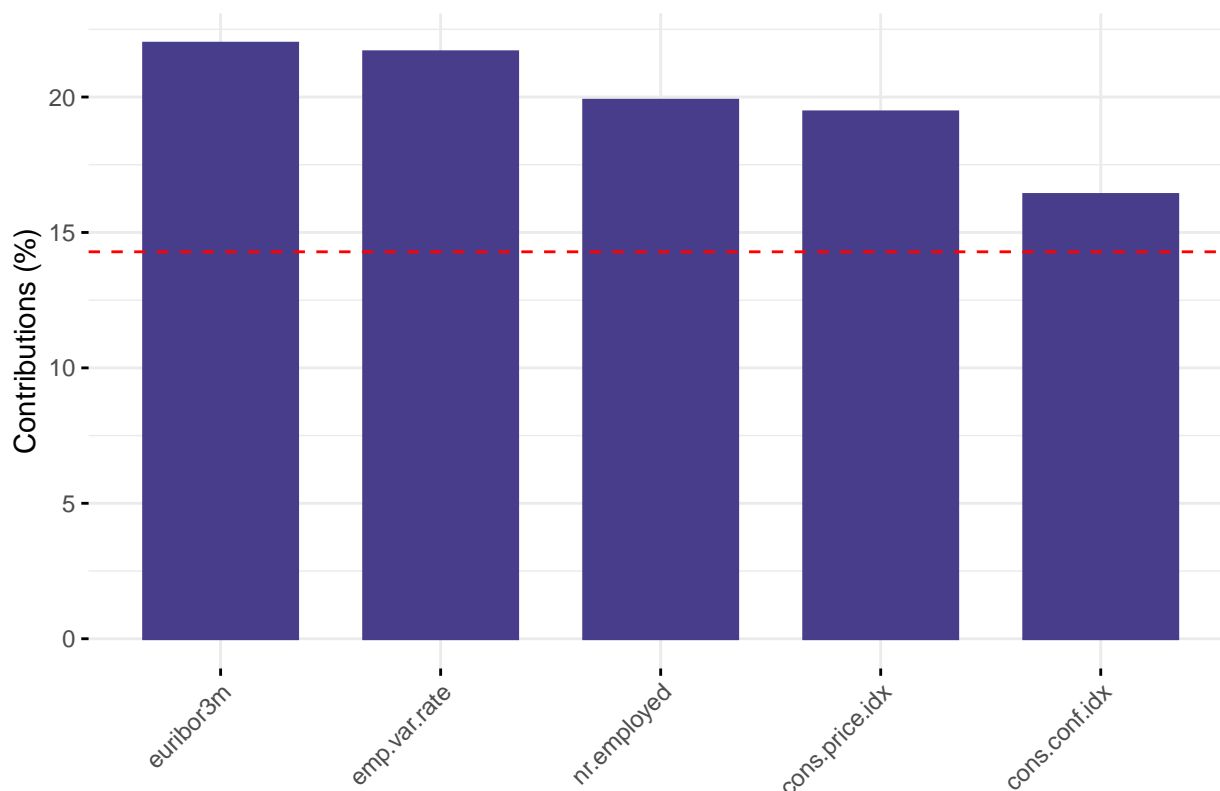
## Interpreting the axes

### 2.1.1 Dim1

We see that in the first dimension, the variables: euribor, emp.var.rate,nr.employed,cons.price.idx and cons.conf.idx are very contributive to the dimension and we can see that all relates to the economy, so we should name the ax as economic status. We can see that all of them are positively correlated such that as all of their values grow, the other variables will follow.

```
res.des<-dimdesc(res.pca)
fviz_contrib( # contributions of variables to PC1
  res.pca,
  fill = "darkslateblue",
  color = "darkslateblue",
  choice = "var",
  axes = 1,
  top = 5)
```

## Contribution of variables to Dim-1



```
res.des$Dim.1
```

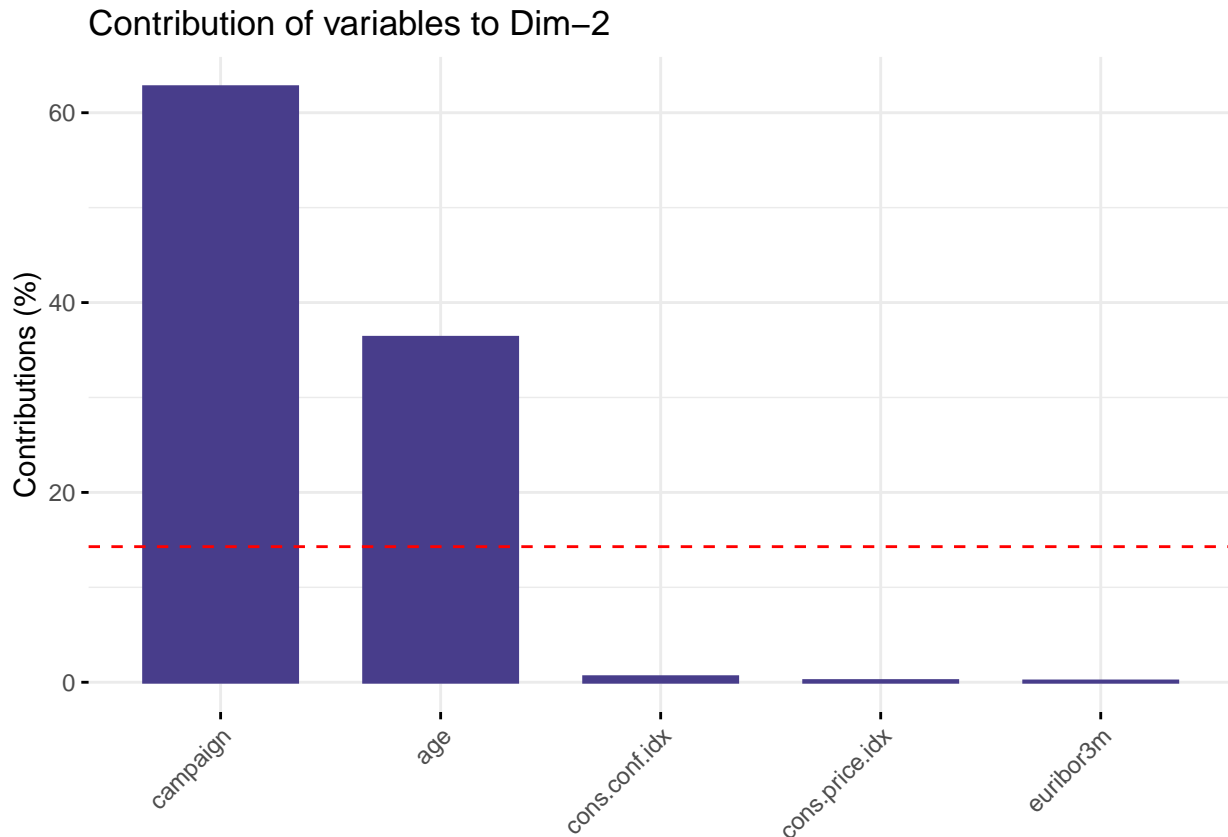
```
##
## Link between the variable and the continuous variables (R-square)
## =====
##          correlation      p.value
## euribor3m      0.99263021 0.000000e+00
## emp.var.rate   0.98545330 0.000000e+00
## nr.employed    0.94389664 0.000000e+00
## cons.price.idx 0.93366222 0.000000e+00
## cons.conf.idx  0.85735341 0.000000e+00
## age            0.12353703 1.838123e-18
## campaign       0.10813127 1.766000e-14
## duration       -0.07640653 6.326626e-08
```

### 2.1.2 Dim2

In this dimension we see that the only variables that contribute significantly are campaign and age, since we think that campaign is the most relevant feature, we should name this as campaign calls, and it tells us that the older the person the more calls the person will receive.

```
res.des<-dimdesc(res.pca)
fviz_contrib( # contributions of variables to PC1
  res.pca,
  fill = "darkslateblue",
  color = "darkslateblue",
  choice = "var",
  axes = 2,
```

```
top = 5)
```



```
res.des$Dim.2
```

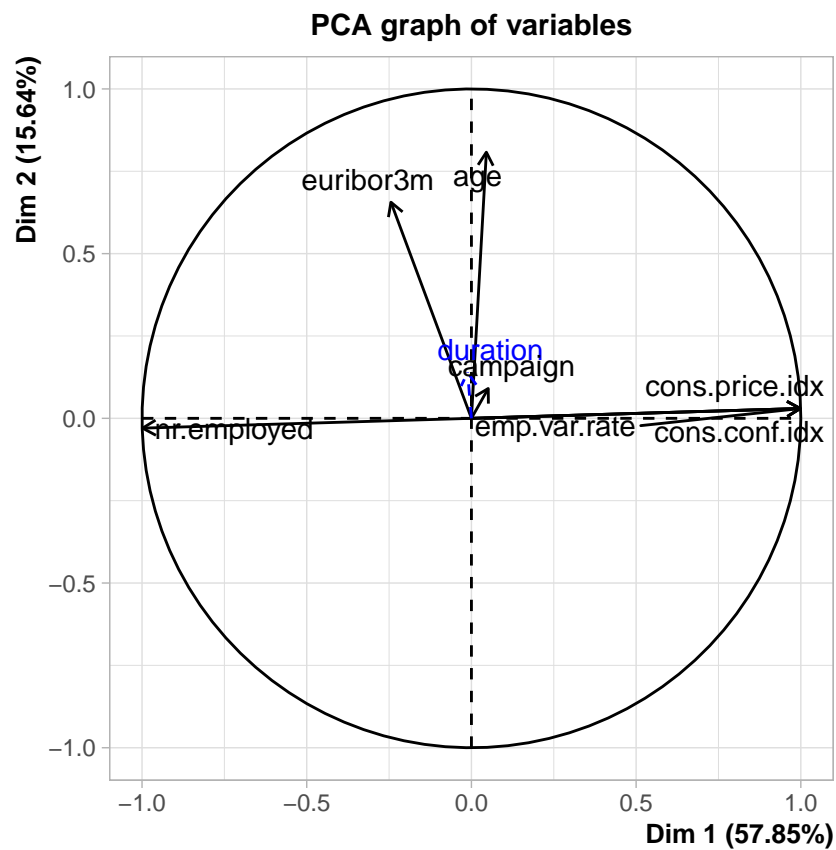
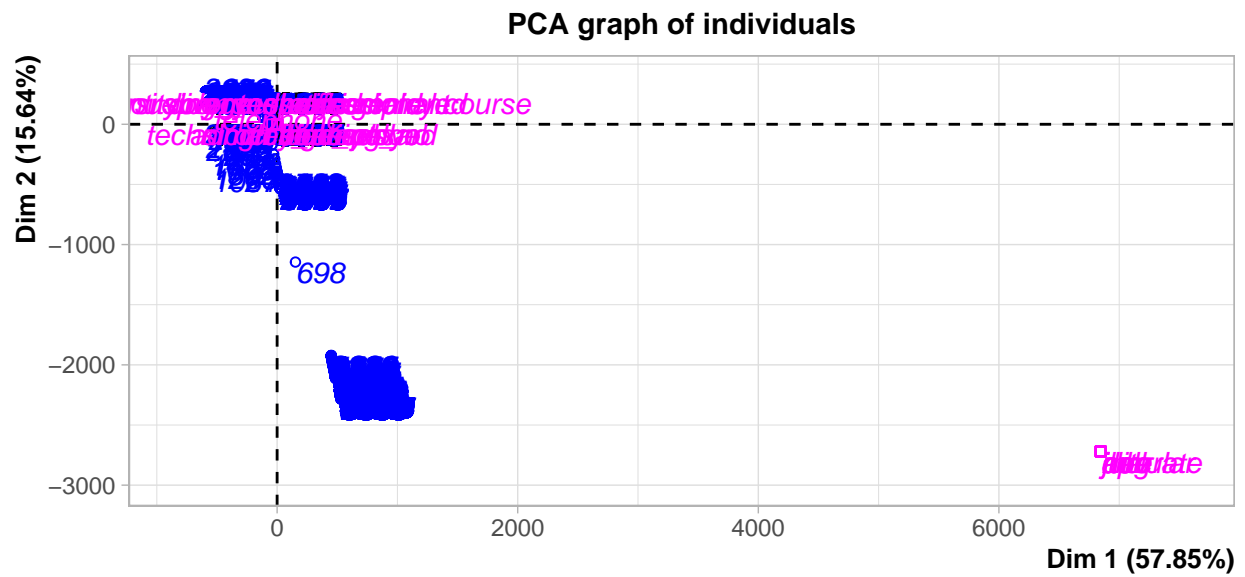
```
##
## Link between the variable and the continuous variables (R-square)
## =====
## correlation      p.value
## campaign        0.79578843 0.000000e+00
## age              0.60564701 0.000000e+00
## duration         0.07248939 2.878338e-07
## euribor3m        -0.03729857 8.347880e-03
## cons.price.idx   -0.04214843 2.873808e-03
## cons.conf.idx    -0.07673524 5.552160e-08
```

## 2.2 Perform a PCA taking into account also supplementary variables the supplementary variables can be quantitative and/or categorical

We will perform the PCA taking into account all the supplementary variables. The first we've gotten doesn't really make any sense, if we take a look at the PCA graph, we see that age is strongly correlated to euribor but we know for a fact that the euribor rates are totally independent from individuals, we can also see that the nr.employed are completely negatively correlated to all the other economic variables, which doesn't make a lot of sense either, for example, it is counterintuitive to think that the higher the number of employed people, the lower the employment variation rate.

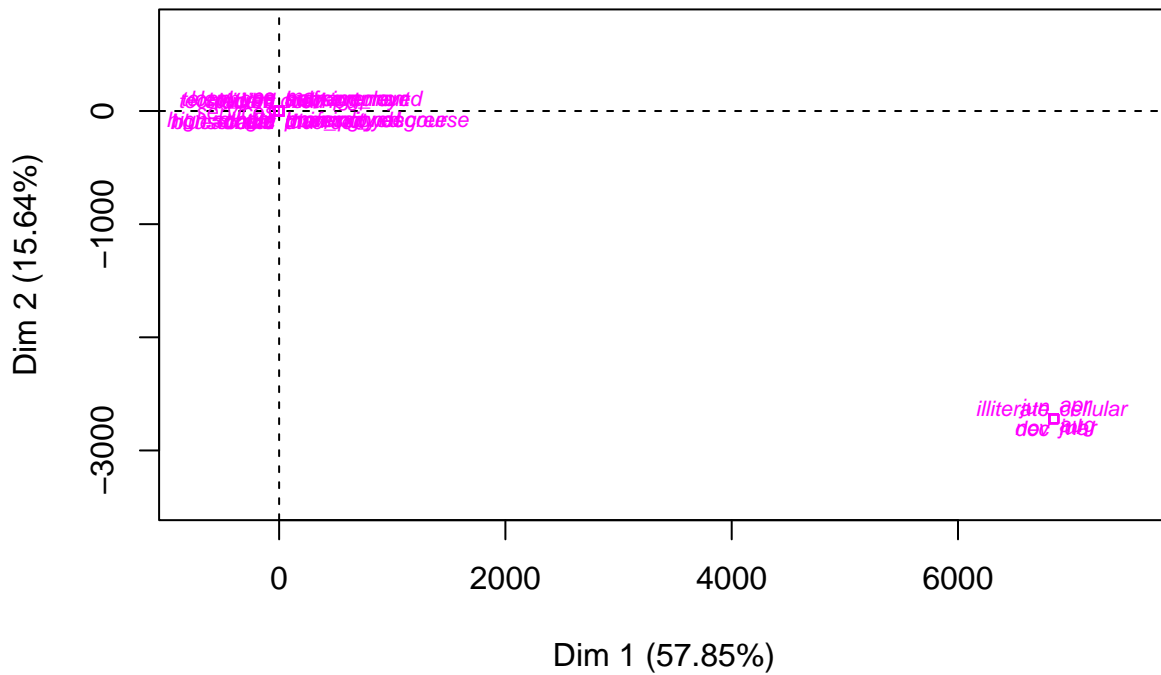
```
ll <- which( df$mout == "YesMOut")
res.pca_sup<-PCA(df[,c(vars_res, vars_con,vars_dis)],quali.sup=c(1,10:17),quanti.sup= c(2), ind.sup = 1)
```





```
plot(res.pca_sup, choix="ind",invisible=c("ind","ind.sup"), cex=0.7, graph.type = "classic")
```

## PCA graph of individuals



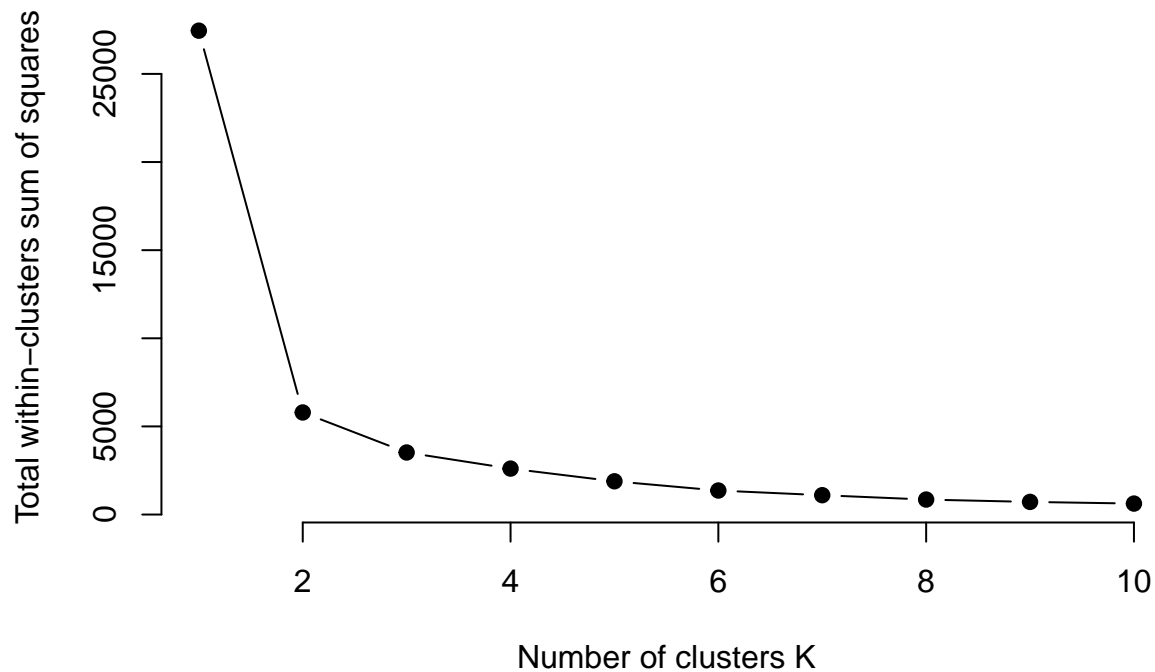
### 3 KMEANS

From this graph, we apply Elbow's method which reveals that the optimal number of clusters is 4. So we will proceed to model and interpret the kMEANS with 4 clusters.

```
dclu<- res.pca$ind$coord[,1:2]; # los dos ejes
k.max <- 10
wss <- sapply(1:k.max,
              function(k){kmeans(dclu, k, nstart=50,iter.max = 15 )$tot.withinss})
wss
```

```
## [1] 27452.2639 5791.5810 3520.8784 2606.4789 1886.5163 1362.8296
## [7] 1097.9665 849.8887 717.2235 622.5091
```

```
plot(1:k.max, wss,
     type="b", pch = 19, frame = FALSE,
     xlab="Number of clusters K",
     ylab="Total within-clusters sum of squares")
```

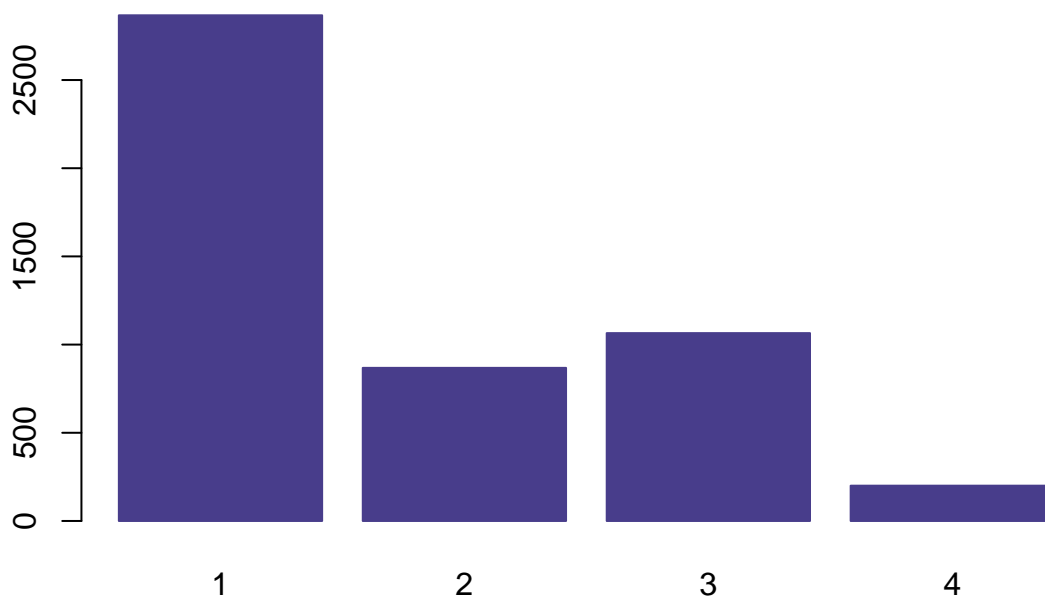


```
# coordinates are real - Euclidean metric
dist<-dist(dclu)
kc<-kmeans(dist, 4) #calculate the distances, it turns into a matrix
```

We can see in this graph the distribution of individuals within each cluster.

```
df$claKM<-0
df$claKM<-kc$cluster
df$claKM<-factor(df$claKM)
barplot(table(df$claKM),col="darkslateblue",border="darkslateblue",main="[k-means]#observations/cluster")
```

**[k-means]#observations/cluster**



## 3.1 Interpret the results of the classification

### 3.1.1 The description of the clusters by the variables

- Cluster 1:
  - These are the people who will say yes to the campaign and being contacted by cellular, these people are young around the ages of 20-30 and are still students.
- Cluster 2:
  - These are people who are more likely to say yes when they are being contacted on november by cellular.
- Cluster 3:
  - These are people who are being contacted by telephone and their response will be a no for the campaign, these people are more towards adults profiles from ages 30-50 and don't have higher degrees.
- Cluster 4:
  - These are people who are frequently contacted by campaign and are around the ages of 40-60 who will say no to a campaign.

We can see that there are two groups within the clusters, the cluster of people who will say yes and a cluster of people who will say no, in clusters 1-2 and 3-4 respectively. So the campaign will be more successful if it focuses on people of cluster's 1-2.

## 4 Hierarchical clustering

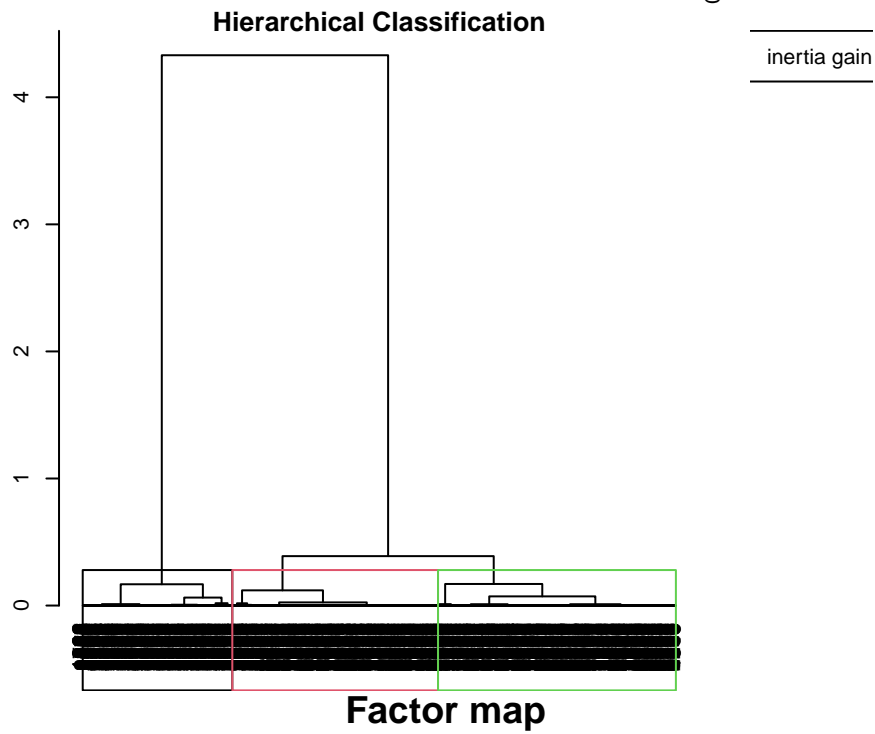
We've decided that number of clusters is the one that the algorithm gives us, with `nb.clust=-1`. ## Description of Clusters

- Cluster 1:
  - These are the people who will say yes to the campaign and being contacted by cellulars, and mostly single university graduates. Also, they are being called during the months of april and may, which are nearing summer seasons and these kind of people tend to have money saved.
- Cluster 2:
  - These are people who are more likely to say no when they are being contacted on november by cellular, cluster similar to the one in KMEANS. These people are divorced and don't have any housing loan.
- Cluster 3:
  - These are people who are married and retired which will most likely say no, and are most usually contacted by telephone. We see that these are people who have their life together already and aren't interested in these kind of campaigns anymore.

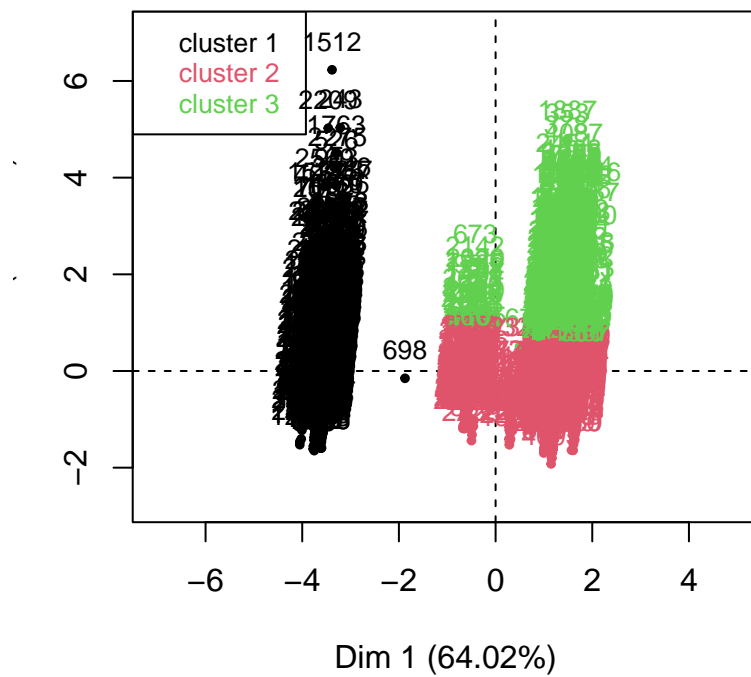
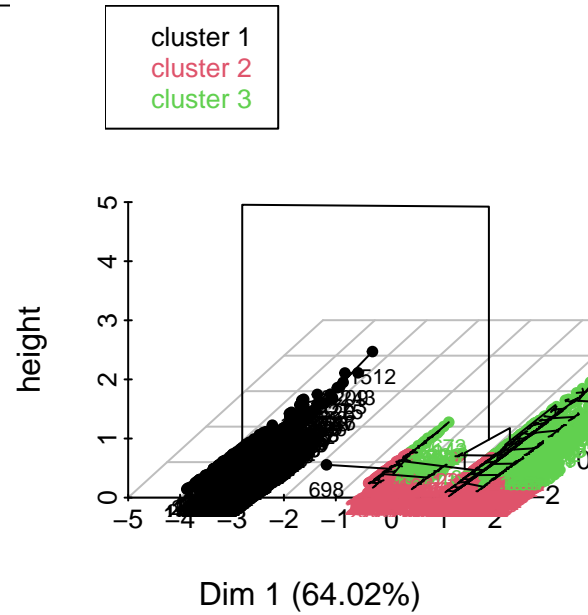
Following these trends, the company should focus specifically on people with characteristics similar to that of cluster 1. We can see something in common with these two methods which is that younger educated people are more likely to say yes.

```
res.pca<-PCA(df[,c('duration',vars_con,vars_dis,"y")],quanti.sup=1,quali.sup = c(9:17), ncp=2, graph=FALSE)
res.hcpc<-HCPC(res.pca,order=TRUE, nb.clust = -1)
```

## Hierarchical Clustering



## Hierarchical clustering on the



```
attributes(res.hcpc)
```

```
## $names
```

```
## [1] "data.clust" "desc.var" "desc.axes" "desc.ind" "call"
##
## $class
## [1] "HCPC"
```

```
summary(res.hcpc$data.clust)
```

```
##      duration      age      campaign      emp.var.rate
## Min.   : 4.0    Min.   :18.00    Min.   :1.000    Min.   : -2.9000
## 1st Qu.:175.0    1st Qu.:32.00    1st Qu.:1.000    1st Qu.: -1.8000
## Median :342.0    Median :38.00    Median :2.000    Median : 1.1000
## Mean   :479.8    Mean   :39.96    Mean   :2.006    Mean   : 0.3275
## 3rd Qu.:686.0    3rd Qu.:47.00    3rd Qu.:2.033    3rd Qu.: 1.1000
## Max.   :4199.0    Max.   :88.00    Max.   :8.000    Max.   : 1.4000
##
## cons.price.idx cons.conf.idx      euribor3m      nr.employed
## Min.   :92.76    Min.   : -50.00    Min.   :1.244    Min.   :5076
## 1st Qu.:93.08    1st Qu.: -42.70    1st Qu.:1.811    1st Qu.:5099
## Median :93.99    Median : -36.40    Median :4.856    Median :5191
## Mean   :93.68    Mean   : -39.81    Mean   :3.965    Mean   :5174
## 3rd Qu.:93.99    3rd Qu.: -36.40    3rd Qu.:4.857    3rd Qu.:5191
## Max.   :94.47    Max.   : -36.10    Max.   :5.045    Max.   :5228
##
##      job      marital      education
## blue-collar:1248 divorced: 530      basic      :1736
## admin.      :1214 married :3103      high.school :1169
## technician : 757 single  :1367      illiterate  : 2
## services   : 517      professional.course: 631
## management : 380      university.degree :1462
## retired    : 192
## (Other)     : 692
##      housing      loan      contact      month      day_of_week
## housing_no :2442    loan_no :4278    cellular :2007    may      :3164    fri: 849
## housing_yes:2558    loan_yes: 722    telephone:2993    apr      : 442    mon:1107
##                                     jul      : 407    thu:1029
##                                     jun      : 357    tue:1073
##                                     aug      : 271    wed: 942
##                                     nov      : 190
##                                     (Other): 169
##      y      clust
## y_no :2400    1:1262
## y_yes:2600    2:2477
##                                     3:1261
##
##
##
##
```

```
attributes(res.hcpc$desc.var)
```

```
## $names
## [1] "test.chi2" "category" "quanti.var" "quanti" "call"
##
## $class
## [1] "catdes" "list"
```

```
# Factors globally related to clustering partition
res.hcpc$desc.var$test.chi2
```

```
##                p.value df
## contact      0.000000e+00  2
## month        0.000000e+00 16
## y            0.000000e+00  2
## job          4.367799e-56 20
## marital      1.393485e-55  4
## education    5.162811e-17  8
## day_of_week  6.022360e-14  8
## housing      1.331316e-11  2
```

```
# Categories over/under represented in each cluster
res.hcpc$desc.var$category
```

```
## $`1`
##                Cla/Mod  Mod/Cla Global      p.value
## y=y_yes           48.538462 100.000000  52.00 0.000000e+00
## contact=cellular   60.139512  95.641838  40.14 0.000000e+00
## month=apr          100.000000  35.023772   8.84 1.337796e-294
## month=mar          100.000000   9.984152   2.52 3.535437e-78
## marital=single     37.820044  40.966719  27.34 1.776329e-34
## month=jun          47.338936  13.391442   7.14 6.958630e-21
## job=student        67.647059   5.467512   2.04 1.214952e-19
## education=university.degree 32.831737  38.034865  29.24 5.437955e-15
## housing=housing_yes 29.124316  59.033281  51.16 8.970164e-11
## day_of_week=thu    32.555879  26.545166  20.58 2.862929e-09
## job=admin          31.136738  29.952456  24.28 8.689279e-08
## job=retired        39.062500   5.942948   3.84 1.759644e-05
## job=management     21.052632   6.339144   7.60 4.795407e-02
## day_of_week=fri    22.497055  15.134707  16.98 4.206311e-02
## job=housemaid      15.384615   1.426307   2.34 9.955469e-03
## day_of_week=tue    21.528425  18.304279  21.46 1.411157e-03
## job=services       19.535783   8.003170  10.34 1.282339e-03
## month=oct           0.000000   0.000000   0.84 4.663341e-06
## housing=housing_no 21.171171  40.966719  48.84 8.970164e-11
## job=blue-collar    18.028846  17.828843  24.96 4.079496e-12
## education=basic    19.297235  26.545166  34.72 8.369367e-13
## marital=married    20.270706  49.841521  62.06 1.241676e-24
## month=nov           0.000000   0.000000   3.80 2.848071e-25
## month=aug           0.000000   0.000000   5.42 4.405850e-36
## month=jul           0.000000   0.000000   8.14 9.888672e-55
## month=may          16.561315  41.521395  63.28 9.308480e-75
## y=y_no             0.000000   0.000000  48.00 0.000000e+00
## contact=telephone   1.837621   4.358162  59.86 0.000000e+00
##                v.test
## y=y_yes                Inf
## contact=cellular       Inf
## month=apr              36.683515
## month=mar              18.717943
## marital=single        12.245476
## month=jun               9.374376
## job=student            9.067754
```

```

## education=university.degree    7.816344
## housing=housing_yes            6.483361
## day_of_week=thu                5.939271
## job=admin.                     5.352197
## job=retired                    4.293391
## job=management                 -1.977775
## day_of_week=fri                -2.032895
## job=housemaid                 -2.577372
## day_of_week=tue                -3.192359
## job=services                  -3.219903
## month=oct                      -4.579390
## housing=housing_no             -6.483361
## job=blue-collar                -6.934400
## education=basic                -7.154966
## marital=married                -10.245355
## month=nov                      -10.386780
## month=aug                      -12.541851
## month=jul                      -15.580430
## month=may                      -18.293586
## y=y_no                         -Inf
## contact=telephone              -Inf
##

```

```
## $`2`
```

	Cla/Mod	Mod/Cla	Global	p.value	v.test
## contact=telephone	65.72001	79.4105773	59.86	1.675288e-177	28.406616
## y=y_no	67.20833	65.1190957	48.00	8.887554e-130	24.237822
## month=may	56.03666	71.5785224	63.28	1.075478e-33	12.098508
## month=nov	82.10526	6.2979411	3.80	2.811309e-21	9.469521
## month=oct	97.61905	1.6552281	0.84	5.879586e-12	6.882537
## housing=housing_no	53.93120	53.1691562	48.84	1.286627e-09	6.069058
## job=blue-collar	55.52885	27.9773920	24.96	1.031806e-06	4.885474
## month=jul	60.19656	9.8909972	8.14	7.028006e-06	4.492839
## day_of_week=tue	54.98602	23.8191361	21.46	5.678910e-05	4.025770
## job=services	57.25338	11.9499394	10.34	2.111521e-04	3.705286
## education=high.school	53.63559	25.3128785	23.38	1.380871e-03	3.198620
## month=aug	55.35055	6.0557126	5.42	4.947934e-02	1.964438
## marital=divorced	44.33962	9.4872830	10.60	1.133578e-02	-2.532174
## day_of_week=mon	44.98645	20.1049657	22.14	5.936734e-04	-3.434488
## education=university.degree	45.07524	26.6047638	29.24	4.914743e-05	-4.059645
## job=student	29.41176	1.2111425	2.04	3.389517e-05	-4.145582
## housing=housing_yes	45.34793	46.8308438	51.16	1.286627e-09	-6.069058
## month=jun	31.37255	4.5215987	7.14	6.194346e-13	-7.196135
## job=retired	11.97917	0.9285426	3.84	2.957509e-29	-11.228411
## month=mar	0.00000	0.0000000	2.52	7.630513e-39	-13.036054
## y=y_yes	33.23077	34.8809043	52.00	8.887554e-130	-24.237822
## month=apr	0.00000	0.0000000	8.84	3.785117e-141	-25.293301
## contact=cellular	25.41106	20.5894227	40.14	1.675288e-177	-28.406616

```
## $`3`
```

	Cla/Mod	Mod/Cla	Global	p.value
## contact=telephone	32.442366	77.00237906	59.86	4.062364e-49
## y=y_no	32.791667	62.41078509	48.00	1.642002e-32
## marital=married	29.745408	73.19587629	62.06	9.704644e-22
## job=retired	48.958333	7.45440127	3.84	5.519471e-13



```

## month=aug          44.649446  9.59555908  5.42 9.154379e-13
## month=jul          39.803440 12.84694687  8.14 1.437399e-11
## day_of_week=mon    31.707317 27.83505155 22.14 3.140557e-08
## education=basic    29.550691 40.68199841 34.72 3.357623e-07
## month=may          27.402023 68.75495638 63.28 2.638969e-06
## marital=divorced   33.773585 14.19508327 10.60 3.121665e-06
## job=management     32.368421  9.75416336  7.60 1.125170e-03
## job=housemaid      35.897436  3.33068993  2.34 9.519927e-03
## month=nov          17.894737  2.69627280  3.80 1.500201e-02
## job=unemployed     15.702479  1.50674068  2.42 1.146364e-02
## education=high.school 21.813516 20.22204600 23.38 1.984436e-03
## education=university.degree 22.093023 25.61459159 29.24 9.774983e-04
## job=admin.         21.087315 20.30134814 24.28 1.151868e-04
## month=oct          2.380952  0.07930214  0.84 7.697880e-05
## day_of_week=thu    19.144801 15.62252181 20.58 2.716028e-07
## job=student         2.941176  0.23790642  2.04 7.924325e-10
## month=mar          0.000000  0.00000000  2.52 7.272218e-17
## y=y_yes            18.230769 37.58921491 52.00 1.642002e-32
## marital=single     11.631309 12.60904044 27.34 1.547322e-46
## contact=cellular   14.449427 22.99762094 40.14 4.062364e-49
## month=apr          0.000000  0.00000000  8.84 1.396587e-59
##                      v.test
## contact=telephone   14.731231
## y=y_no              11.872641
## marital=married      9.579998
## job=retired          7.211855
## month=aug            7.142657
## month=jul            6.754085
## day_of_week=mon      5.533415
## education=basic      5.102180
## month=may            4.697088
## marital=divorced     4.662648
## job=management       3.257200
## job=housemaid        2.592796
## month=nov            -2.432330
## job=unemployed       -2.528239
## education=high.school -3.092552
## education=university.degree -3.296924
## job=admin.           -3.856150
## month=oct            -3.953616
## day_of_week=thu      -5.142156
## job=student          -6.146436
## month=mar            -8.342523
## y=y_yes              -11.872641
## marital=single       -14.324094
## contact=cellular     -14.731231
## month=apr            -16.278765

```

```
### desc.ind ###
```

```
### C. The description of the clusters by the individuals ###
```

```
res.hcpc$desc.var$category
```

```
## $`1`
```

```
##                      Cla/Mod    Mod/Cla Global      p.value
```

## y=y_yes	48.538462	100.000000	52.00	0.000000e+00
## contact=cellular	60.139512	95.641838	40.14	0.000000e+00
## month=apr	100.000000	35.023772	8.84	1.337796e-294
## month=mar	100.000000	9.984152	2.52	3.535437e-78
## marital=single	37.820044	40.966719	27.34	1.776329e-34
## month=jun	47.338936	13.391442	7.14	6.958630e-21
## job=student	67.647059	5.467512	2.04	1.214952e-19
## education=university.degree	32.831737	38.034865	29.24	5.437955e-15
## housing=housing_yes	29.124316	59.033281	51.16	8.970164e-11
## day_of_week=thu	32.555879	26.545166	20.58	2.862929e-09
## job=admin.	31.136738	29.952456	24.28	8.689279e-08
## job=retired	39.062500	5.942948	3.84	1.759644e-05
## job=management	21.052632	6.339144	7.60	4.795407e-02
## day_of_week=fri	22.497055	15.134707	16.98	4.206311e-02
## job=housemaid	15.384615	1.426307	2.34	9.955469e-03
## day_of_week=tue	21.528425	18.304279	21.46	1.411157e-03
## job=services	19.535783	8.003170	10.34	1.282339e-03
## month=oct	0.000000	0.000000	0.84	4.663341e-06
## housing=housing_no	21.171171	40.966719	48.84	8.970164e-11
## job=blue-collar	18.028846	17.828843	24.96	4.079496e-12
## education=basic	19.297235	26.545166	34.72	8.369367e-13
## marital=married	20.270706	49.841521	62.06	1.241676e-24
## month=nov	0.000000	0.000000	3.80	2.848071e-25
## month=aug	0.000000	0.000000	5.42	4.405850e-36
## month=jul	0.000000	0.000000	8.14	9.888672e-55
## month=may	16.561315	41.521395	63.28	9.308480e-75
## y=y_no	0.000000	0.000000	48.00	0.000000e+00
## contact=telephone	1.837621	4.358162	59.86	0.000000e+00
##	v.test			
## y=y_yes	Inf			
## contact=cellular	Inf			
## month=apr	36.683515			
## month=mar	18.717943			
## marital=single	12.245476			
## month=jun	9.374376			
## job=student	9.067754			
## education=university.degree	7.816344			
## housing=housing_yes	6.483361			
## day_of_week=thu	5.939271			
## job=admin.	5.352197			
## job=retired	4.293391			
## job=management	-1.977775			
## day_of_week=fri	-2.032895			
## job=housemaid	-2.577372			
## day_of_week=tue	-3.192359			
## job=services	-3.219903			
## month=oct	-4.579390			
## housing=housing_no	-6.483361			
## job=blue-collar	-6.934400			
## education=basic	-7.154966			
## marital=married	-10.245355			
## month=nov	-10.386780			
## month=aug	-12.541851			
## month=jul	-15.580430			

```

## month=may -18.293586
## y=y_no -Inf
## contact=telephone -Inf
##
## $`2`
## Cla/Mod Mod/Cla Global p.value v.test
## contact=telephone 65.72001 79.4105773 59.86 1.675288e-177 28.406616
## y=y_no 67.20833 65.1190957 48.00 8.887554e-130 24.237822
## month=may 56.03666 71.5785224 63.28 1.075478e-33 12.098508
## month=nov 82.10526 6.2979411 3.80 2.811309e-21 9.469521
## month=oct 97.61905 1.6552281 0.84 5.879586e-12 6.882537
## housing=housing_no 53.93120 53.1691562 48.84 1.286627e-09 6.069058
## job=blue-collar 55.52885 27.9773920 24.96 1.031806e-06 4.885474
## month=jul 60.19656 9.8909972 8.14 7.028006e-06 4.492839
## day_of_week=tue 54.98602 23.8191361 21.46 5.678910e-05 4.025770
## job=services 57.25338 11.9499394 10.34 2.111521e-04 3.705286
## education=high.school 53.63559 25.3128785 23.38 1.380871e-03 3.198620
## month=aug 55.35055 6.0557126 5.42 4.947934e-02 1.964438
## marital=divorced 44.33962 9.4872830 10.60 1.133578e-02 -2.532174
## day_of_week=mon 44.98645 20.1049657 22.14 5.936734e-04 -3.434488
## education=university.degree 45.07524 26.6047638 29.24 4.914743e-05 -4.059645
## job=student 29.41176 1.2111425 2.04 3.389517e-05 -4.145582
## housing=housing_yes 45.34793 46.8308438 51.16 1.286627e-09 -6.069058
## month=jun 31.37255 4.5215987 7.14 6.194346e-13 -7.196135
## job=retired 11.97917 0.9285426 3.84 2.957509e-29 -11.228411
## month=mar 0.00000 0.0000000 2.52 7.630513e-39 -13.036054
## y=y_yes 33.23077 34.8809043 52.00 8.887554e-130 -24.237822
## month=apr 0.00000 0.0000000 8.84 3.785117e-141 -25.293301
## contact=cellular 25.41106 20.5894227 40.14 1.675288e-177 -28.406616
##
## $`3`
## Cla/Mod Mod/Cla Global p.value
## contact=telephone 32.442366 77.00237906 59.86 4.062364e-49
## y=y_no 32.791667 62.41078509 48.00 1.642002e-32
## marital=married 29.745408 73.19587629 62.06 9.704644e-22
## job=retired 48.958333 7.45440127 3.84 5.519471e-13
## month=aug 44.649446 9.59555908 5.42 9.154379e-13
## month=jul 39.803440 12.84694687 8.14 1.437399e-11
## day_of_week=mon 31.707317 27.83505155 22.14 3.140557e-08
## education=basic 29.550691 40.68199841 34.72 3.357623e-07
## month=may 27.402023 68.75495638 63.28 2.638969e-06
## marital=divorced 33.773585 14.19508327 10.60 3.121665e-06
## job=management 32.368421 9.75416336 7.60 1.125170e-03
## job=housemaid 35.897436 3.33068993 2.34 9.519927e-03
## month=nov 17.894737 2.69627280 3.80 1.500201e-02
## job=unemployed 15.702479 1.50674068 2.42 1.146364e-02
## education=high.school 21.813516 20.22204600 23.38 1.984436e-03
## education=university.degree 22.093023 25.61459159 29.24 9.774983e-04
## job=admin. 21.087315 20.30134814 24.28 1.151868e-04
## month=oct 2.380952 0.07930214 0.84 7.697880e-05
## day_of_week=thu 19.144801 15.62252181 20.58 2.716028e-07
## job=student 2.941176 0.23790642 2.04 7.924325e-10
## month=mar 0.000000 0.00000000 2.52 7.272218e-17
## y=y_yes 18.230769 37.58921491 52.00 1.642002e-32

```

```
## marital=single      11.631309 12.60904044 27.34 1.547322e-46
## contact=cellular    14.449427 22.99762094 40.14 4.062364e-49
## month=apr           0.000000 0.00000000 8.84 1.396587e-59
##                      v.test
## contact=telephone    14.731231
## y=y_no               11.872641
## marital=married      9.579998
## job=retired          7.211855
## month=aug            7.142657
## month=jul            6.754085
## day_of_week=mon      5.533415
## education=basic      5.102180
## month=may            4.697088
## marital=divorced     4.662648
## job=management       3.257200
## job=housemaid        2.592796
## month=nov            -2.432330
## job=unemployed       -2.528239
## education=high.school -3.092552
## education=university.degree -3.296924
## job=admin.           -3.856150
## month=oct            -3.953616
## day_of_week=thu      -5.142156
## job=student          -6.146436
## month=mar            -8.342523
## y=y_yes              -11.872641
## marital=single       -14.324094
## contact=cellular     -14.731231
## month=apr            -16.278765
```

## 5 CA

We will cut the duration, which is the numerical target into 8 levels. We will study the CA obtained from the Duration-Age\_group and then Duration-education. We want to see this because in the clustering findings we discovered that young educated people are more likely to say yes, so we want to see if it affects the duration as well.

### 5.1 Eigenvalues and dominant axes(1)

We can see that independence test fails to refute  $H_0$  since the  $p$ -value = 0.3263 > 0.05, so there is no independence between duration and age. We can see that the farthest value is 10-20 from age which makes sense since teens aren't likely to be contacted. Since all the other values are around the center we can see that the duration is dependent on the age group (mostly).

```
aux2<-c(5,60,120,150,180,240,300,1200,2100)
duration_fact<-factor(cut(df$duration,breaks=aux2,include.lowest=T))
table(duration_fact)
```

```
## duration_fact
##           [5,60]      (60,120]      (120,150]      (150,180]
##              175          493          321          330
##      (180,240]      (240,300]      (300,1.2e+03] (1.2e+03,2.1e+03]
##              543          422          2415          274
```

```
levels(duration_fact)<-paste0("duration-",levels(duration_fact))
df$duration_fact<-duration_fact
```

```
tt<-table(df[,c("Age_group","duration_fact")])
```

```
chisq.test(tt, simulate.p.value = TRUE) #to see if the rows and columns are independents. H0: Rows and
```

```
##
```

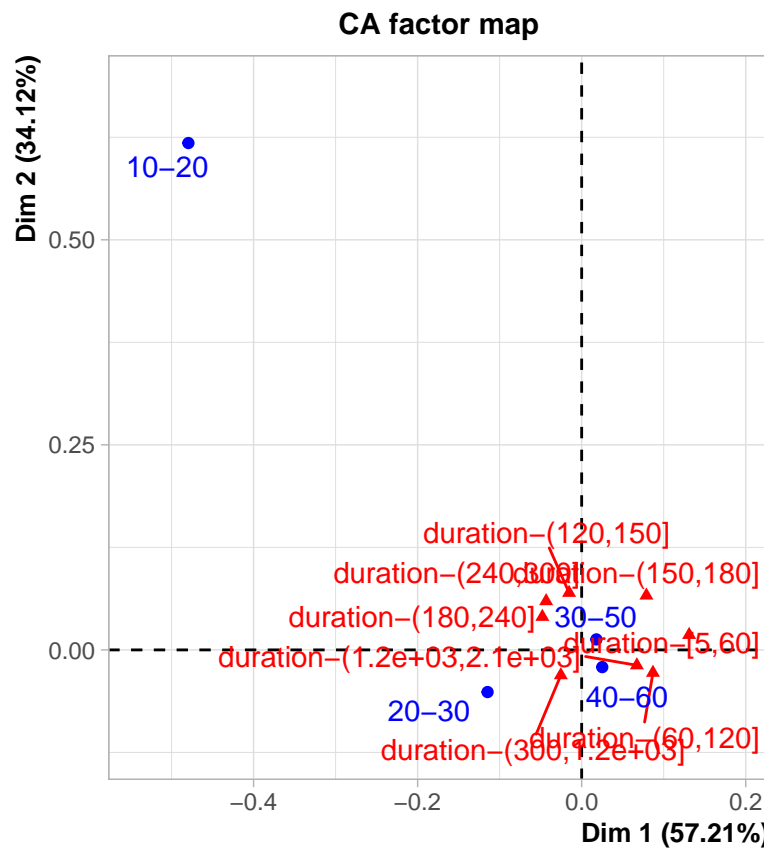
```
## Pearson's Chi-squared test with simulated p-value (based on 2000
## replicates)
```

```
##
```

```
## data: tt
```

```
## X-squared = 23.401, df = NA, p-value = 0.3048
```

```
res.ca <- CA(tt)
```



The mean of eigenvalues = 0.001606341 making that only the first 2 dimensions satisfies Kaiser's criteria. So the dominant axes are 1 and 2 with a cumulative variance of 91.3%.

```
mean(res.ca$eig[,1])
```

```
## [1] 0.001606341
```

```
summary(res.ca)
```

```
##
```

```
## Call:
```

```
## CA(X = tt)
```

```
##
```

```
## The chi square of independence between the two variables is equal to 23.40117 (p-value = 0.3229672)
```

```
##
## Eigenvalues
##          Dim.1   Dim.2   Dim.3
## Variance      0.003   0.002   0.000
## % of var.     57.211  34.123   8.666
## Cumulative % of var. 57.211  91.334 100.000
##
## Rows
##          Iner*1000   Dim.1   ctr   cos2   Dim.2   ctr
## 10-20      |      1.800 | -0.479 24.043  0.368 |  0.618 66.952
## 20-30      |      2.121 | -0.115 63.923  0.831 | -0.051 21.491
## 30-50      |      0.402 |  0.018  8.018  0.550 |  0.013  6.843
## 40-60      |      0.496 |  0.025  4.017  0.223 | -0.021  4.713
##          cos2   Dim.3   ctr   cos2
## 10-20      0.612 |  0.112  8.717  0.020 |
## 20-30      0.167 | -0.006  1.200  0.002 |
## 30-50      0.280 | -0.010 16.440  0.171 |
## 40-60      0.156 |  0.042 73.642  0.620 |
##
## Columns
##          Iner*1000   Dim.1   ctr   cos2   Dim.2   ctr
## duration-[5,60]      |      0.628 |  0.131 22.057  0.968 |  0.018  0.722
## duration-(60,120]    |      0.987 |  0.087 27.301  0.763 | -0.028  4.702
## duration-(120,150]   |      0.327 | -0.015  0.543  0.046 |  0.070 18.757
## duration-(150,180]   |      0.777 |  0.079 14.749  0.524 |  0.066 17.466
## duration-(180,240]   |      0.437 | -0.048  8.975  0.566 |  0.040 10.664
## duration-(240,300]   |      0.582 | -0.043  5.814  0.275 |  0.059 18.359
## duration-(300,1.2e+03] |      0.790 | -0.025 11.396  0.398 | -0.031 28.135
## duration-(1.2e+03,2.1e+03] |      0.291 |  0.067  9.165  0.869 | -0.019  1.195
##          cos2   Dim.3   ctr   cos2
## duration-[5,60]      0.019 | -0.015  1.906  0.013 |
## duration-(60,120]    0.078 |  0.040 37.519  0.159 |
## duration-(120,150]   0.942 |  0.008  0.974  0.012 |
## duration-(150,180]   0.370 | -0.036 19.833  0.107 |
## duration-(180,240]   0.401 | -0.011  3.388  0.032 |
## duration-(240,300]   0.518 |  0.037 28.764  0.206 |
## duration-(300,1.2e+03] 0.585 | -0.005  3.230  0.017 |
## duration-(1.2e+03,2.1e+03] 0.068 | -0.018  4.385  0.063 |
```

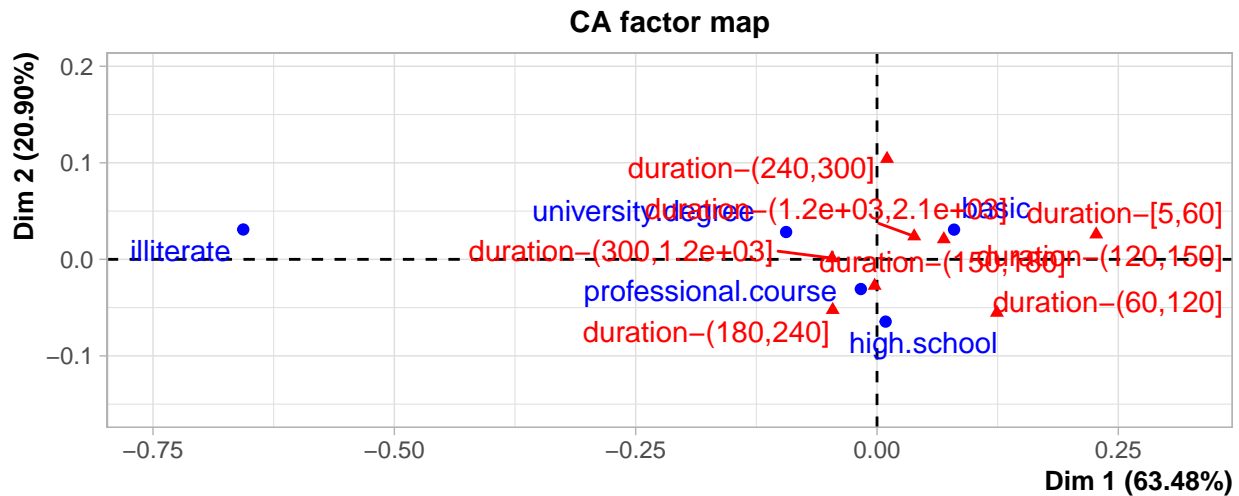
## 5.2 Eigenvalues and dominant axes(2)

We can see that independence test fails to refute  $H_0$  since the  $p$ -value=0.09445>0.05, so there is no independence between duration and education. From the factor map we can see that the farthest value is illiterate and the other values are really near from each other indicating that there is some dependence between them.

```
tt<-table(df[,c("education","duration_fact")])
chisq.test(tt, simulate.p.value = TRUE) #to see if the rows and colum
```

```
##
## Pearson's Chi-squared test with simulated p-value (based on 2000
## replicates)
##
## data:  tt
## X-squared = 39.394, df = NA, p-value = 0.09345
```

```
res.ca <- CA(tt)
```



The mean of eigenvalues = 0.001980396 making that only the first 3 dimensions satisfies Kaiser's criteria. So the dominant axes are 1 and 2 with a cummulative variance of 97.7%.

```
mean(res.ca$eig[,1])
```

```
## [1] 0.001980396
```

```
summary(res.ca)
```

```
##
```

```
## Call:
```

```
## CA(X = tt)
```

```
##
```

```
## The chi square of independence between the two variables is equal to 39.39403 (p-value = 0.0747902)
```

```
##
```

```
## Eigenvalues
```

	Dim.1	Dim.2	Dim.3	Dim.4
## Variance	0.005	0.002	0.001	0.000
## % of var.	63.476	20.902	13.305	2.317
## Cumulative % of var.	63.476	84.378	97.683	100.000

```
##
```

```
## Rows
```

	Iner*1000	Dim.1	ctr	cos2	Dim.2	ctr
## basic	2.551	0.080	44.065	0.868	0.031	19.904
## high.school	1.165	0.009	0.372	0.016	-0.065	58.748
## illiterate	0.426	-0.657	3.447	0.407	0.031	0.023
## professional.course	0.938	-0.017	0.705	0.038	-0.031	7.232
## university.degree	2.841	-0.094	51.412	0.910	0.028	14.092

```
##
```

	cos2	Dim.3	ctr	cos2
## basic	0.129	-0.004	0.437	0.002
## high.school	0.835	-0.027	16.255	0.147
## illiterate	0.001	-0.476	8.636	0.214
## professional.course	0.128	0.078	73.242	0.823
## university.degree	0.082	-0.007	1.430	0.005

```
##
```

```
## Columns
```

	Iner*1000	Dim.1	ctr	cos2	Dim.2	ctr
## duration-[5,60]	1.927	0.227	36.100	0.942	0.026	1.420

```
## duration-(60,120] | 1.931 | 0.124 30.423 0.792 | -0.055 18.306
## duration-(120,150] | 0.352 | 0.069 6.154 0.879 | 0.021 1.698
## duration-(150,180] | 0.273 | -0.002 0.008 0.001 | -0.027 2.992
## duration-(180,240] | 0.900 | -0.046 4.575 0.256 | -0.052 18.103
## duration-(240,300] | 1.189 | 0.010 0.182 0.008 | 0.104 55.541
## duration-(300,1.2e+03] | 1.205 | -0.047 20.930 0.874 | 0.001 0.046
## duration-(1.2e+03,2.1e+03] | 0.145 | 0.039 1.627 0.563 | 0.024 1.893
##
## cos2 Dim.3 ctr cos2
## duration-[5,60] 0.012 | -0.049 8.165 0.045 |
## duration-(60,120] 0.157 | 0.029 7.706 0.042 |
## duration-(120,150] 0.080 | -0.010 0.616 0.018 |
## duration-(150,180] 0.182 | -0.046 13.112 0.506 |
## duration-(180,240] 0.333 | 0.057 33.651 0.394 |
## duration-(240,300] 0.774 | 0.055 24.568 0.218 |
## duration-(300,1.2e+03] 0.001 | -0.015 10.991 0.096 |
## duration-(1.2e+03,2.1e+03] 0.216 | -0.015 1.191 0.086 |
```

## 5.3 Conclusions

All in all, we can see that the findings of CA relative to duration-age and duration-education are very linked to the findings of the clustering, so we can really say with a certain confidence that the age and education of an individual is really impactful on the target variables.

# 6 MCA

## 6.1 Eigenvalues and dominant axes analysis

We consider, according to the generalized Kaiser theorem, all those dimensions such that their eigenvalue is greater than the mean. We see that the average gives us 0.125. Therefore, we will take up to dimension 15, which represents the 60% of the sample.

```
mean(res.mca$eig[,1])
```

```
## [1] 0.125
```

```
res.mca$eig
```

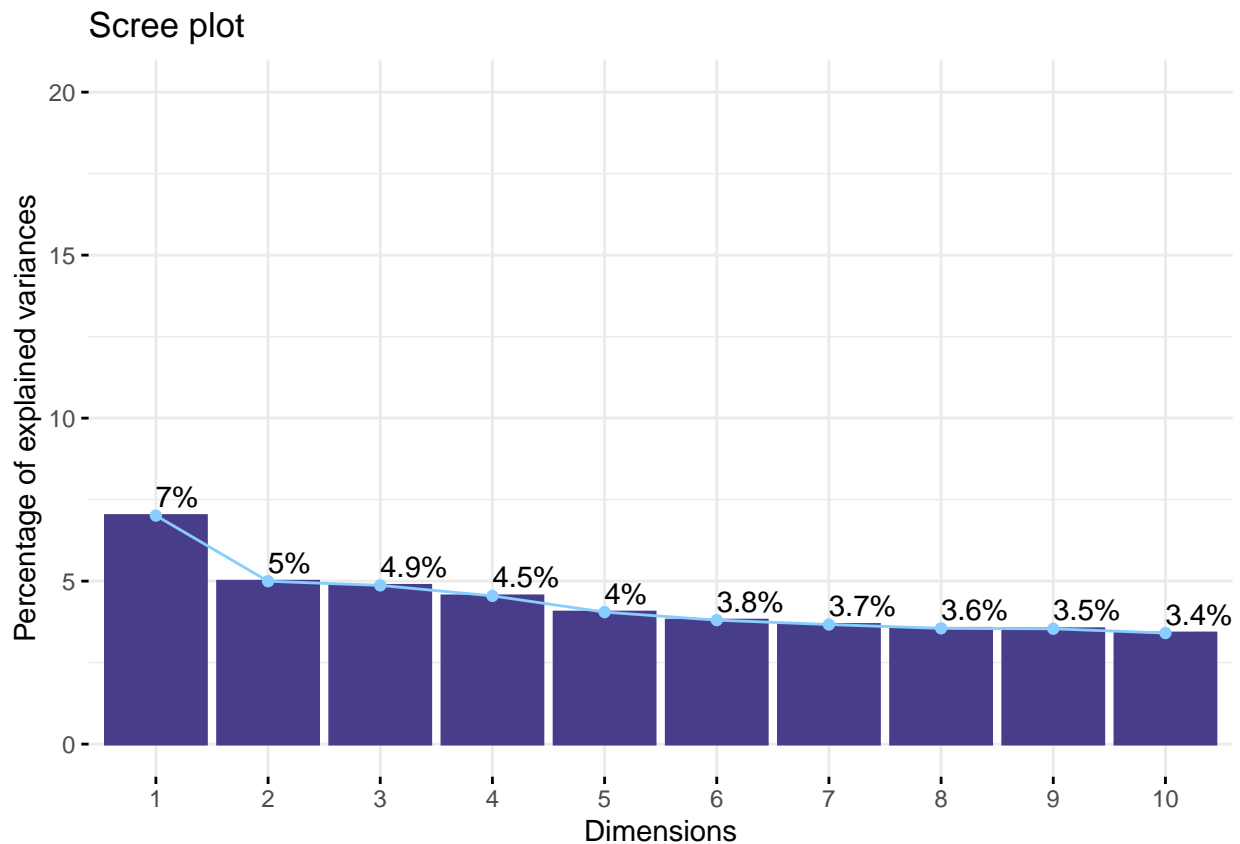
```
##      eigenvalue percentage of variance cumulative percentage of variance
## dim 1 0.27166749          7.0107741          7.010774
## dim 2 0.19359439          4.9959843          12.006758
## dim 3 0.18867610          4.8690607          16.875819
## dim 4 0.17618335          4.5466671          21.422486
## dim 5 0.15693106          4.0498338          25.472320
## dim 6 0.14736533          3.8029761          29.275296
## dim 7 0.14216283          3.6687181          32.944014
## dim 8 0.13759710          3.5508930          36.494907
## dim 9 0.13708679          3.5377236          40.032631
## dim 10 0.13205455          3.4078594          43.440490
## dim 11 0.13098810          3.3803382          46.820828
## dim 12 0.12938254          3.3389042          50.159733
## dim 13 0.12721618          3.2829981          53.442731
## dim 14 0.12612378          3.2548073          56.697538
## dim 15 0.12500640          3.2259716          59.923509
## dim 16 0.12417242          3.2044496          63.127959
## dim 17 0.12346130          3.1860981          66.314057
```



```
## dim 18 0.12142900      3.1336516      69.447709
## dim 19 0.12056091      3.1112492      72.558958
## dim 20 0.11866219      3.0622501      75.621208
## dim 21 0.11547654      2.9800397      78.601248
## dim 22 0.11277008      2.9101955      81.511443
## dim 23 0.11020115      2.8439007      84.355344
## dim 24 0.10911690      2.8159200      87.171264
## dim 25 0.10703430      2.7621755      89.933439
## dim 26 0.09307584      2.4019571      92.335397
## dim 27 0.08887914      2.2936552      94.629052
## dim 28 0.07182627      1.8535813      96.482633
## dim 29 0.05912249      1.5257418      98.008375
## dim 30 0.04113547      1.0615606      99.069935
## dim 31 0.03604000      0.9300646      100.000000
```

We can also visualize the percentages of inertia explained by each MCA dimensions:

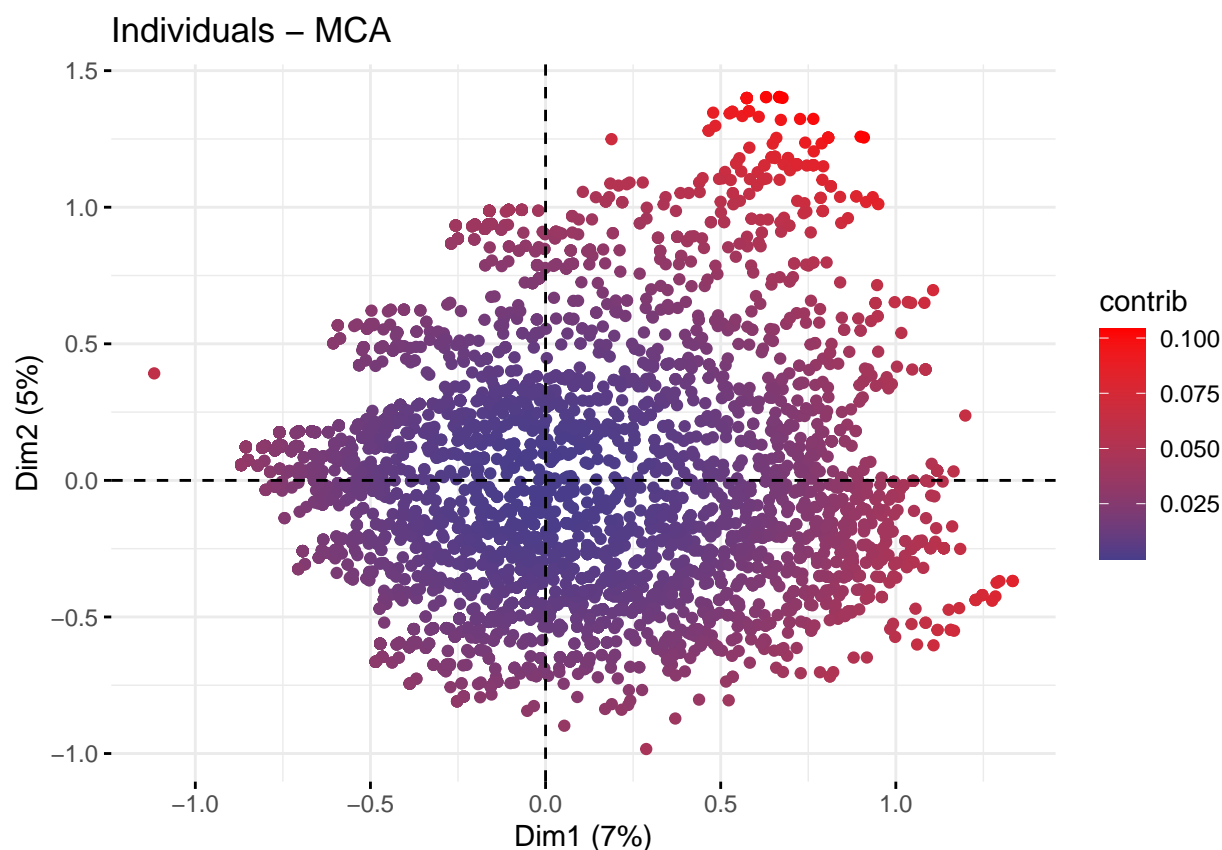
```
fviz_screplot(
  res.mca,
  addlabels=TRUE,
  ylim=c(0,20),
  barfill="darkslateblue",
  barcolor="darkslateblue",
  linecolor="skyblue1"
)
```



## 6.2 Individuals point of view

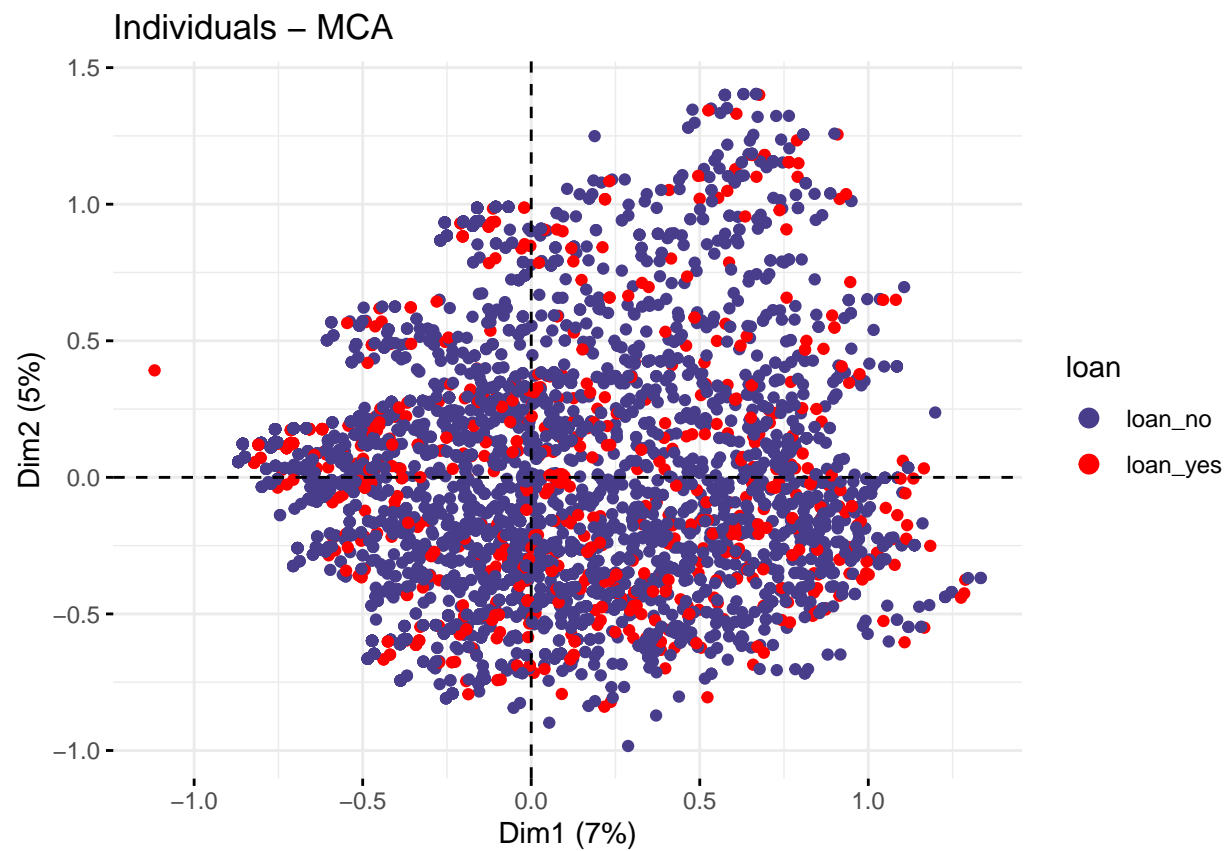
We can see in the legend that the contributions go from 0.025 to 0.1 so we can't say that there are individuals who are too contributive.

```
fviz_mca_ind(  
  res.mca,  
  geom=c("point"),  
  col.ind="contrib",  
  gradient.cols=c("darkslateblue", "red")  
)
```

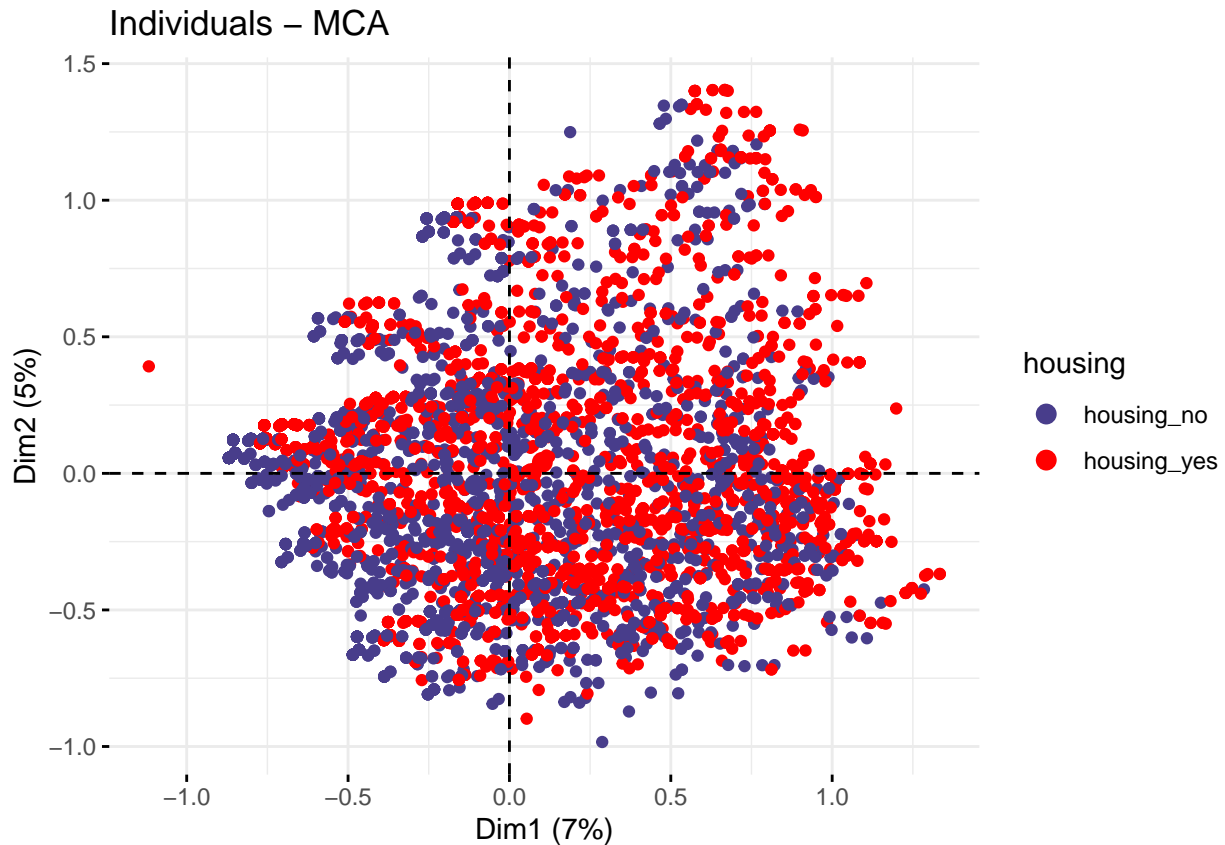


We've tried many variables but as we can see with these two they are mostly homogenous across the factorial map, that is, evenly distributed.

```
fviz_mca_ind(res.mca, label="none", habillage="loan", palette=c("darkslateblue", "red"))
```



```
fviz_mca_ind(res.mca, label="none", habillage="housing", palette=c("darkslateblue", "red"))
```

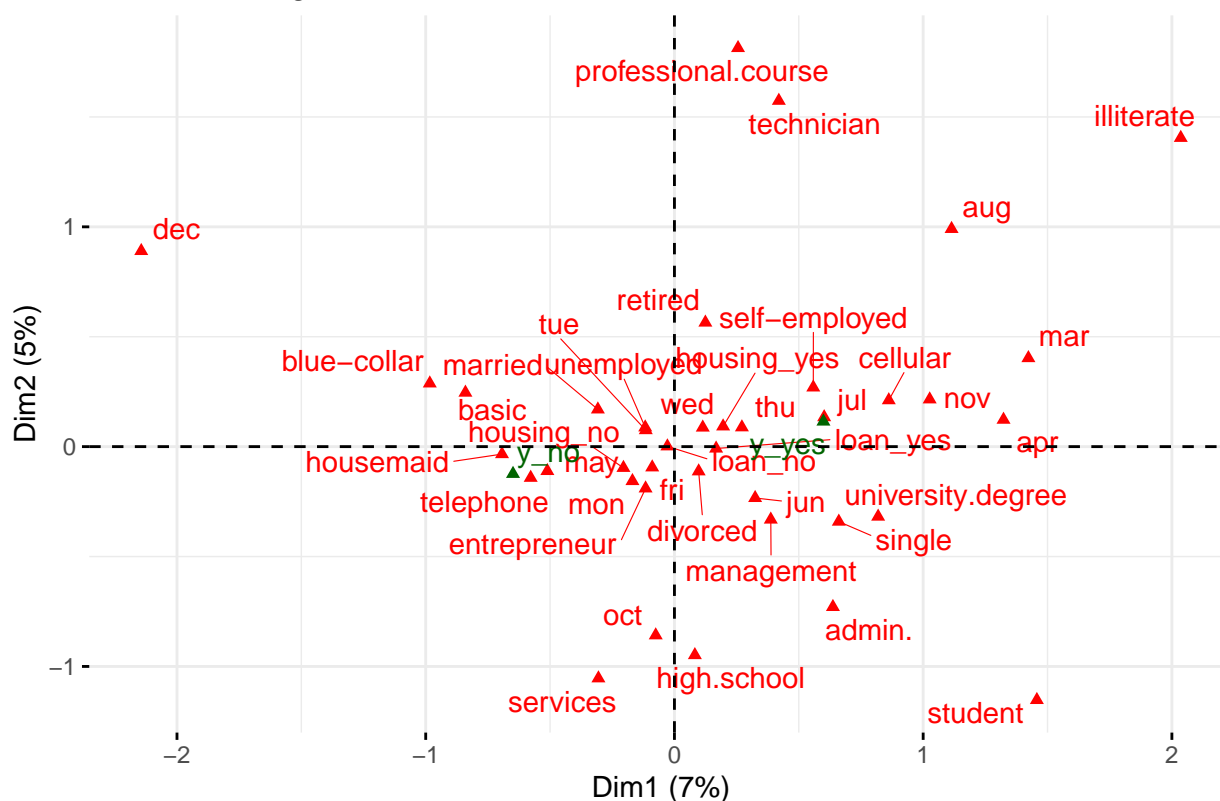


### 6.3 Interpreting map of categories: average profile versus extreme profiles (rare categories)

We can see that the month-december, education-illiterate are extreme profiles from the DIM1 and professional course and technician are extreme profiles from DIM2. All the remaining categories are all gravitating towards the center, we can clearly see the separation of categories respect to the variable “y”, the ones near “yes” will make it more likely that the individual with those characteristic will says yes, and the same logics is applied for no.

```
fviz_mca_var(res.mca, repel=TRUE)
```

## Variable categories – MCA



## 6.4 Interpreting the axes association to factor map

```
res.desc <- dimdesc(res.mca, axes = c(1,2))
```

### 6.4.1 Description of dimension 1

The first dimension of the MCA plot is primarily driven by the contact type, education, and whether or not the client subscribed to a term deposit. Clients who were contacted via cellular communication, had a university degree, and subscribed to a term deposit are more likely to be positively associated with this dimension, while those who were contacted via telephone, had a lower level of education, and did not subscribe to a term deposit are more likely to be negatively associated with this dimension.

```
res.desc[[1]]
```

```
##
## Link between the variable and the categorical variable (1-way anova)
## =====
##           R2      p.value
## y          0.388798633 0.000000e+00
## job         0.454019225 0.000000e+00
## education   0.452808059 0.000000e+00
## contact     0.498061874 0.000000e+00
## month       0.516277391 0.000000e+00
## marital     0.179201680 5.245561e-215
## housing     0.040126746 2.001070e-46
## day_of_week 0.028106884 8.505113e-30
## loan        0.004738099 1.105581e-06
```

```
##
## Link between variable abd the categories of the categorical variables
## =====
##               Estimate      p.value
## contact=cellular      0.375209177 0.000000e+00
## education=university.degree 0.181682749 0.000000e+00
## y=y_yes               0.325258697 0.000000e+00
## month=apr              0.511046411 3.415216e-204
## marital=single        0.266172640 4.043475e-197
## job=admin.             0.267302547 1.238292e-153
## month=aug              0.402184394 3.366440e-82
## month=mar              0.563181378 2.130686e-60
## job=student            0.694645091 4.436658e-51
## month=nov              0.356349157 4.177682e-48
## housing=housing_yes    0.104436599 2.001070e-46
## month=jul              0.135213586 2.341728e-37
## job=technician         0.153932743 1.385318e-36
## day_of_week=thu        0.140160364 1.035432e-22
## job=management         0.137101268 3.289527e-15
## job=self-employed      0.226136196 3.172369e-13
## loan=loan_yes          0.051035445 1.105581e-06
## day_of_week=wed        0.058158164 1.030385e-04
## education=illiterate    0.815840512 3.977185e-03
## month=dec              -1.295868353 3.204833e-02
## marital=divorced       -0.027470225 1.761693e-02
## day_of_week=fri        -0.047788642 4.270685e-03
## education=high.school  -0.202385914 1.329667e-03
## day_of_week=tue        -0.061292801 2.020357e-05
## loan=loan_no           -0.051035445 1.105581e-06
## month=jun              -0.009469419 1.858941e-10
## day_of_week=mon        -0.089237085 1.787286e-10
## education=professional.course -0.111933787 5.757979e-12
## job=services           -0.224117923 1.891021e-13
## job=housemaid          -0.426172378 2.782137e-14
## housing=housing_no     -0.104436599 2.001070e-46
## marital=married        -0.238702415 5.327177e-185
## month=may              -0.444877259 0.000000e+00
## contact=telephone      -0.375209177 0.000000e+00
## education=basic        -0.683203561 0.000000e+00
## job=blue-collar        -0.577497794 0.000000e+00
## y=y_no                 -0.325258697 0.000000e+00
```

## 6.4.2 Description of dimension 2

The dimension 2 appears to be strongly influenced by the type of job and level of education of the respondents, with some additional contribution from the month of last contact and marital status variables.

```
res.desc[[2]]
```

```
##
## Link between the variable and the categorical variable (1-way anova)
## =====
##               R2      p.value
## job           0.691224421 0.000000e+00
## education     0.676742542 0.000000e+00
```

```
## month      0.079818846 9.388966e-85
## marital    0.050964218 1.739981e-57
## contact    0.029940676 6.615411e-35
## y          0.014137106 3.303438e-17
## day_of_week 0.011153159 1.972311e-11
## housing    0.008898955 2.330139e-11
##
## Link between variable abd the categories of the categorical variables
## =====
##              Estimate      p.value
## education=professional.course  0.604588052 0.000000e+00
## job=technician                 0.720719739 0.000000e+00
## month=aug                      0.359914771 7.414765e-65
## marital=married                0.116201309 3.211613e-54
## contact=cellular               0.077658706 6.615411e-35
## job=blue-collar               0.154955587 3.277243e-32
## y=y_yes                       0.052356952 3.303438e-17
## job=retired                   0.276720638 1.240984e-15
## housing=housing_yes           0.041517652 2.330139e-11
## month=mar                     0.101335181 4.567219e-06
## job=self-employed             0.146859347 4.499815e-04
## day_of_week=thu               0.038775635 1.526750e-03
## month=nov                     0.018736054 2.495384e-03
## day_of_week=wed               0.038303385 3.045035e-03
## day_of_week=tue               0.033092483 5.489247e-03
## education=illiterate          0.424733650 4.691648e-02
## job=entrepreneur              -0.055252227 7.917238e-03
## month=apr                     -0.022002410 6.965792e-03
## marital=divorced              -0.007768646 6.241464e-03
## month=jul                     -0.016880733 4.732601e-03
## day_of_week=fri               -0.041388290 2.597441e-03
## month=jun                     -0.178674277 4.587300e-06
## month=oct                     -0.453580636 2.224987e-08
## day_of_week=mon               -0.068783213 3.484534e-09
## housing=housing_no            -0.041517652 2.330139e-11
## job=management                -0.117563670 1.602679e-11
## y=y_no                       -0.052356952 3.303438e-17
## month=may                     -0.124631199 5.180098e-25
## job=student                   -0.478780854 2.410154e-32
## contact=telephone             -0.077658706 6.615411e-35
## education=basic               -0.085245402 2.245510e-37
## education=university.degree  -0.333427014 2.157554e-48
## marital=single                -0.108432663 1.404283e-50
## job=services                  -0.435419880 5.021304e-151
## job=admin.                    -0.292540341 3.198180e-205
## education=high.school         -0.610649285 0.000000e+00
```

## 6.5 Perform a MCA taking into account also supplementary variables

### 6.5.1 Description of dimensions

```
res.desc <- dimdesc(res.mca_sup, axes = c(1,2))
```

**6.5.1.1 Description of dimension 1** The first dimension is positively correlated with the duration of the last contact, which means that clients who had longer contacts are more likely to be positioned towards the positive end of the first dimension. The first dimension is negatively correlated with the age and the economic indicators, such as the number of employees, employment variation rate, consumer price index, consumer confidence index, and the euribor 3 month rate. This means that older clients and clients with higher economic indicators are more likely to be positioned towards the negative end of the first dimension. The first dimension is negatively correlated with the binary variable that indicates whether the client subscribed to a term deposit or not. This means that clients who did not subscribe to a term deposit are more likely to be positioned towards the negative end of the first dimension.

```
res.desc[[1]]

##
## Link between the variable and the continuous variables (R-square)
## =====
##               correlation      p.value
## duration      0.2326336 1.997449e-62
## age           -0.1860164 3.645494e-40
## nr.employed   -0.3244683 6.414285e-123
## emp.var.rate  -0.4649930 9.507227e-267
## euribor3m     -0.4682708 5.380749e-271
## cons.conf.idx -0.5659700 0.000000e+00
## cons.price.idx -0.5734473 0.000000e+00
##
## Link between the variable and the categorical variable (1-way anova)
## =====
##               R2      p.value
## y             0.388798633 0.000000e+00
## job           0.454019225 0.000000e+00
## education     0.452808059 0.000000e+00
## contact       0.498061874 0.000000e+00
## month         0.516277391 0.000000e+00
## marital       0.179201680 5.245561e-215
## housing       0.040126746 2.001070e-46
## day_of_week   0.028106884 8.505113e-30
## loan          0.004738099 1.105581e-06
##
## Link between variable abd the categories of the categorical variables
## =====
##               Estimate      p.value
## contact=cellular      0.375209177 0.000000e+00
## education=university.degree 0.181682749 0.000000e+00
## y=y_yes               0.325258697 0.000000e+00
## month=apr             0.511046411 3.415216e-204
## marital=single        0.266172640 4.043475e-197
## job=admin.            0.267302547 1.238292e-153
## month=aug             0.402184394 3.366440e-82
## month=mar             0.563181378 2.130686e-60
## job=student           0.694645091 4.436658e-51
## month=nov             0.356349157 4.177682e-48
## housing=housing_yes   0.104436599 2.001070e-46
## month=jul             0.135213586 2.341728e-37
## job=technician        0.153932743 1.385318e-36
## day_of_week=thu       0.140160364 1.035432e-22
## job=management        0.137101268 3.289527e-15
```



```
## job=self-employed      0.226136196  3.172369e-13
## loan=loan_yes          0.051035445  1.105581e-06
## day_of_week=wed        0.058158164  1.030385e-04
## education=illiterate    0.815840512  3.977185e-03
## month=dec              -1.295868353  3.204833e-02
## marital=divorced       -0.027470225  1.761693e-02
## day_of_week=fri        -0.047788642  4.270685e-03
## education=high.school  -0.202385914  1.329667e-03
## day_of_week=tue        -0.061292801  2.020357e-05
## loan=loan_no           -0.051035445  1.105581e-06
## month=jun              -0.009469419  1.858941e-10
## day_of_week=mon        -0.089237085  1.787286e-10
## education=professional.course -0.111933787  5.757979e-12
## job=services           -0.224117923  1.891021e-13
## job=housemaid          -0.426172378  2.782137e-14
## housing=housing_no     -0.104436599  2.001070e-46
## marital=married        -0.238702415  5.327177e-185
## month=may              -0.444877259  0.000000e+00
## contact=telephone      -0.375209177  0.000000e+00
## education=basic        -0.683203561  0.000000e+00
## job=blue-collar        -0.577497794  0.000000e+00
## y=y_no                 -0.325258697  0.000000e+00
```

**6.5.1.2 Description of dimension 2** Age is weakly positively correlated with the second dimension of the MCA, meaning that it has some association with the categorical variables being analyzed. Duration has a weak positive correlation with the second dimension of the MCA, indicating that it also has some relationship with the categorical variables being analyzed. The number of employees and consumer confidence index have a weak positive and negative correlation, respectively, with the second dimension of the MCA, suggesting that they have some association with the categorical variables being analyzed. Education and job have the strongest association, with an R-squared value of around 0.67-0.69, followed by month, marital, contact, and housing. The variable “y” (indicating whether or not the client subscribed to a term deposit) has a relatively weak association with the categorical variables, with an R-squared value of 0.014. Among the categories of the categorical variables, several have a relatively strong association with the dimension, either positively or negatively. For example, professional course education, technician job, and August month are positively associated with the dimension, while illiterate education, entrepreneur job, and October month are negatively associated with the dimension.

```
res.desc[[2]]
```

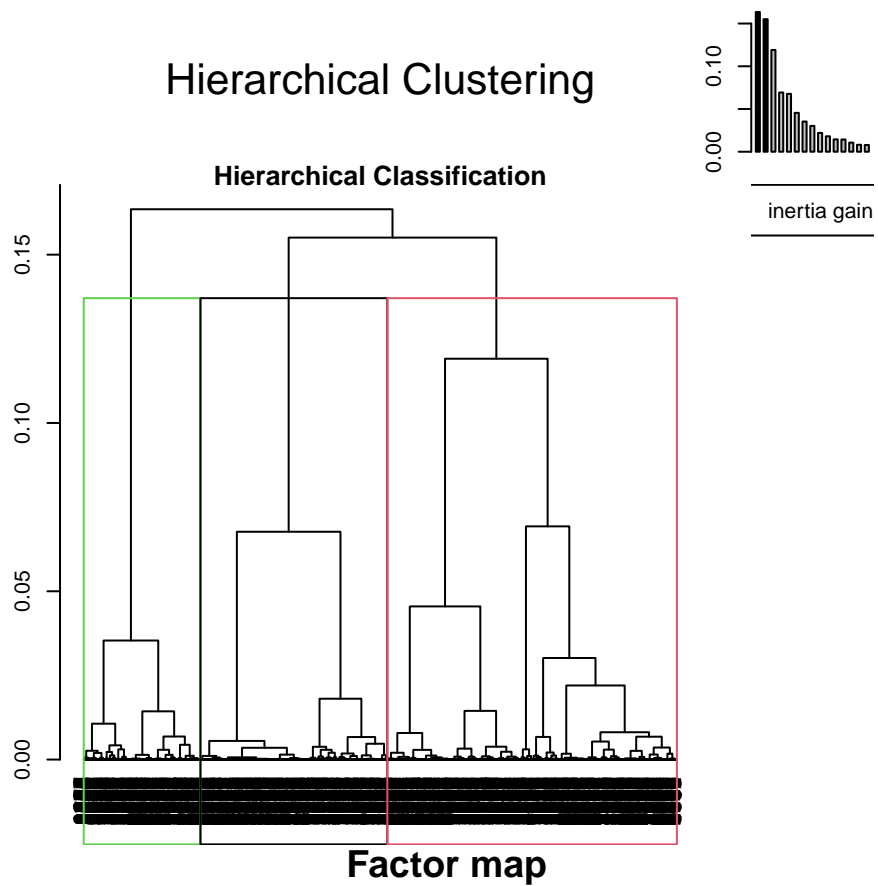
```
##
## Link between the variable and the continuous variables (R-square)
## =====
##          correlation      p.value
## age          0.16111055 1.992409e-30
## duration      0.08057452 1.161164e-08
## nr.employed   0.02812660 4.672916e-02
## cons.conf.idx -0.05189257 2.416841e-04
## cons.price.idx -0.10615595 5.241665e-14
##
## Link between the variable and the categorical variable (1-way anova)
## =====
##          R2      p.value
## job       0.691224421 0.000000e+00
## education 0.676742542 0.000000e+00
## month     0.079818846 9.388966e-85
```

```
## marital      0.050964218 1.739981e-57
## contact      0.029940676 6.615411e-35
## y            0.014137106 3.303438e-17
## day_of_week  0.011153159 1.972311e-11
## housing      0.008898955 2.330139e-11
##
## Link between variable abd the categories of the categorical variables
## =====
##              Estimate      p.value
## education=professional.course  0.604588052 0.000000e+00
## job=technician                 0.720719739 0.000000e+00
## month=aug                      0.359914771 7.414765e-65
## marital=married                0.116201309 3.211613e-54
## contact=cellular               0.077658706 6.615411e-35
## job=blue-collar               0.154955587 3.277243e-32
## y=y_yes                       0.052356952 3.303438e-17
## job=retired                   0.276720638 1.240984e-15
## housing=housing_yes           0.041517652 2.330139e-11
## month=mar                     0.101335181 4.567219e-06
## job=self-employed             0.146859347 4.499815e-04
## day_of_week=thu               0.038775635 1.526750e-03
## month=nov                     0.018736054 2.495384e-03
## day_of_week=wed               0.038303385 3.045035e-03
## day_of_week=tue               0.033092483 5.489247e-03
## education=illiterate          0.424733650 4.691648e-02
## job=entrepreneur              -0.055252227 7.917238e-03
## month=apr                     -0.022002410 6.965792e-03
## marital=divorced              -0.007768646 6.241464e-03
## month=jul                     -0.016880733 4.732601e-03
## day_of_week=fri               -0.041388290 2.597441e-03
## month=jun                     -0.178674277 4.587300e-06
## month=oct                     -0.453580636 2.224987e-08
## day_of_week=mon               -0.068783213 3.484534e-09
## housing=housing_no            -0.041517652 2.330139e-11
## job=management                -0.117563670 1.602679e-11
## y=y_no                       -0.052356952 3.303438e-17
## month=may                     -0.124631199 5.180098e-25
## job=student                   -0.478780854 2.410154e-32
## contact=telephone             -0.077658706 6.615411e-35
## education=basic               -0.085245402 2.245510e-37
## education=university.degree  -0.333427014 2.157554e-48
## marital=single                -0.108432663 1.404283e-50
## job=services                  -0.435419880 5.021304e-151
## job=admin.                    -0.292540341 3.198180e-205
## education=high.school         -0.610649285 0.000000e+00
```

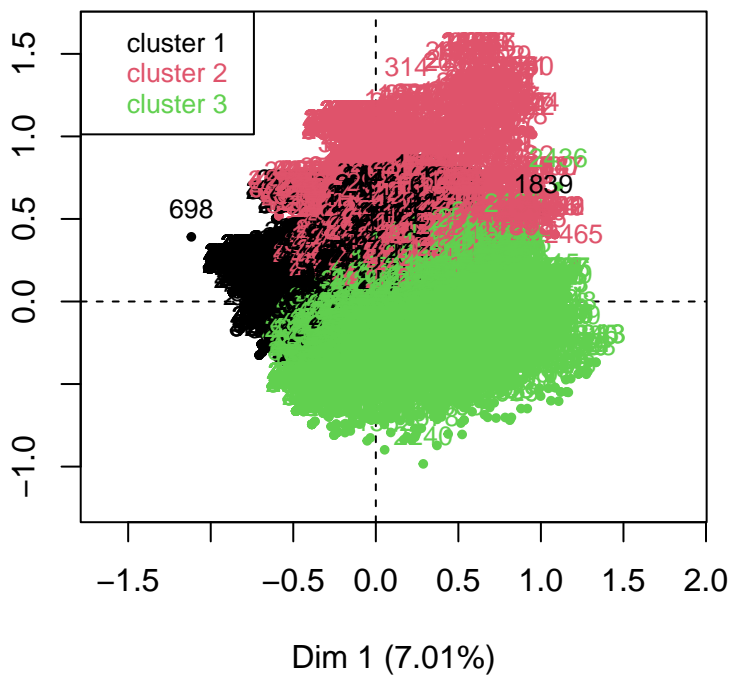
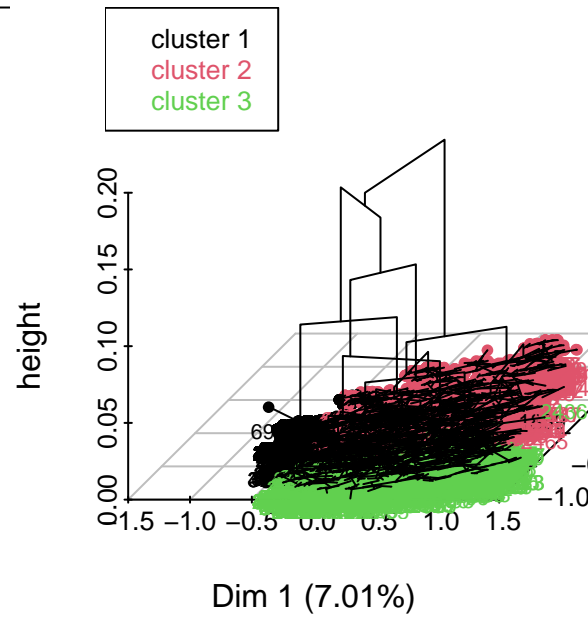
## 7 Hierarchical Clustering (from MCA)

We've decided that numbers of cluster is the one that the algorithm gives us, with `nb.clust=-1`.

```
res.hcpcMCA <- HCPC(res.mca,nb.clust = -1, order = TRUE)
```



## Hierarchical clustering on the



### 7.1 Description of clusters

- Cluster 1:

- The first cluster are people who are more likely to say no contacted via telephone and have a basic type of education and have a blue-collar kind of job and are married.
- Cluster 2:
  - The second cluster are people who are more likely to say yes being contacted by cellular and are educated from a professional course and are technicians. They are also married and young.
- Cluster 3:
  - The first cluster are people who are almost guaranteed to say yes, they are university educated and are working on more technical jobs such as management and administration, they are young and most likely single as well.

From this clustering analysis, we can see that the clusters aren't very different than the previous ones, young university graduates are still the people who are more likely to say yes.

res.hcpcMCA\$desc.var\$category      *# description of each cluster by the categories*

```
## $`1`
##          Cla/Mod   Mod/Cla Global      p.value
## education=basic   88.536866  93.4346505  34.72  0.000000e+00
## job=blue-collar   90.544872  68.6930091  24.96  0.000000e+00
## marital=married   43.022881  81.1550152  62.06  8.588297e-90
## month=may         39.475348  75.9270517  63.28  6.253971e-40
## contact=telephone 39.859673  72.5227964  59.86  1.923064e-38
## y=y_no            40.875000  59.6352584  48.00  7.647273e-31
## job=retired        64.062500  7.4772036   3.84  2.478293e-19
## job=housemaid      65.811966  4.6808511   2.34  2.179802e-13
## housing=housing_no 36.650287  54.4072948  48.84  3.509902e-08
## loan=loan_no       33.543712  87.2340426  85.56  1.763566e-02
## loan=loan_yes      29.085873  12.7659574  14.44  1.763566e-02
## month=jun          26.330532  5.7142857   7.14  5.442846e-03
## month=oct           11.904762  0.3039514   0.84  2.044557e-03
## job=self-employed  21.951220  2.1884498   3.28  1.829562e-03
## marital=divorced   24.339623  7.8419453  10.60  5.904849e-06
## month=mar          12.698413  0.9726444   2.52  1.551889e-07
## housing=housing_yes 29.319781  45.5927052  51.16  3.509902e-08
## month=aug           18.081181  2.9787234   5.42  2.320910e-08
## month=nov           15.263158  1.7629179   3.80  2.243860e-08
## month=apr           16.742081  4.4984802   8.84  1.588880e-15
## job=student         0.000000  0.0000000   2.04  1.266359e-18
## job=services        14.313346  4.4984802  10.34  4.919823e-24
## job=management      9.210526  2.1276596   7.60  2.317436e-29
## y=y_yes            25.538462  40.3647416  52.00  7.647273e-31
## contact=cellular    22.521176  27.4772036  40.14  1.923064e-38
## marital=single      13.240673  11.0030395  27.34  1.071892e-81
## education=professional.course 1.743265  0.6686930  12.62  8.741972e-99
## education=high.school 7.271172  5.1671733  23.38  9.106072e-121
## job=technician      1.453104  0.6686930  15.14  5.026176e-124
## job=admin.          4.036244  2.9787234  24.28  6.683809e-169
## education=university.degree 0.752394  0.6686930  29.24  1.822607e-288
##
##          v.test
## education=basic      Inf
## job=blue-collar      Inf
## marital=married      20.092469
## month=may            13.225476
## contact=telephone    12.965368
## y=y_no               11.546966
```

```

## job=retired      8.989734
## job=housemaid    7.337279
## housing=housing_no 5.513889
## loan=loan_no     2.373180
## loan=loan_yes    -2.373180
## month=jun        -2.779585
## month=oct        -3.083682
## job=self-employed -3.116589
## marital=divorced -4.529768
## month=mar        -5.246294
## housing=housing_yes -5.513889
## month=aug        -5.586201
## month=nov        -5.592064
## month=apr        -7.969834
## job=student      -8.808673
## job=services     -10.111357
## job=management   -11.249942
## y=y_yes          -11.546966
## contact=cellular -12.965368
## marital=single   -19.144683
## education=professional.course -21.095530
## education=high.school -23.367708
## job=technician   -23.686006
## job=admin.       -27.701574
## education=university.degree -36.296721
##
## $`2`
##
## Cla/Mod    Mod/Cla Global    p.value
## education=professional.course 98.256735 64.3153527 12.62 0.000000e+00
## job=technician 98.546896 77.3858921 15.14 0.000000e+00
## month=aug 33.948339 9.5435685 5.42 4.370508e-09
## contact=cellular 21.674141 45.1244813 40.14 4.695792e-04
## housing=housing_yes 20.914777 55.4979253 51.16 2.701465e-03
## y=y_yes 20.769231 56.0165975 52.00 5.442728e-03
## marital=single 21.433797 30.3941909 27.34 1.876250e-02
## day_of_week=tue 21.435228 23.8589212 21.46 4.520494e-02
## job=self-employed 13.414634 2.2821577 3.28 4.683164e-02
## marital=married 18.175959 58.5062241 62.06 1.172465e-02
## y=y_no 17.666667 43.9834025 48.00 5.442728e-03
## housing=housing_no 17.567568 44.5020747 48.84 2.701465e-03
## month=may 17.951960 58.9211618 63.28 1.890152e-03
## job=retired 10.416667 2.0746888 3.84 7.639167e-04
## contact=telephone 17.674574 54.8755187 59.86 4.695792e-04
## job=housemaid 5.982906 0.7261411 2.34 4.085151e-05
## job=student 2.941176 0.3112033 2.04 8.064158e-07
## job=entrepreneur 5.319149 1.0373444 3.76 2.337388e-08
## education=university.degree 12.722298 19.2946058 29.24 6.788955e-15
## job=management 4.473684 1.7634855 7.60 2.825998e-18
## job=services 5.996132 3.2157676 10.34 1.657293e-19
## education=high.school 7.784431 9.4398340 23.38 1.357278e-34
## job=blue-collar 4.086538 5.2904564 24.96 1.033624e-69
## job=admin. 3.377265 4.2531120 24.28 2.510399e-75
## education=basic 3.859447 6.9502075 34.72 9.461812e-110
##
## v.test

```

```

## education=professional.course      Inf
## job=technician                     Inf
## month=aug                          5.869523
## contact=cellular                   3.497535
## housing=housing_yes                2.999812
## y=y_yes                            2.779592
## marital=single                     2.350216
## day_of_week=tue                    2.002742
## job=self-employed                  -1.987820
## marital=married                    -2.520325
## y=y_no                             -2.779592
## housing=housing_no                 -2.999812
## month=may                          -3.106971
## job=retired                        -3.365548
## contact=telephone                  -3.497535
## job=housemaid                      -4.102610
## job=student                        -4.933808
## job=entrepreneur                   -5.584971
## education=university.degree        -7.788351
## job=management                     -8.718224
## job=services                       -9.033854
## education=high.school              -12.267285
## job=blue-collar                    -17.649115
## job=admin.                         -18.364872
## education=basic                    -22.257880
##
## $`3`
##
## Cla/Mod  Mod/Cla  Global      p.value
## job=admin.      92.586491 47.009619 24.28 1.130221e-317
## education=university.degree 86.525308 52.906734 29.24 5.568529e-293
## education=high.school      84.944397 41.530740 23.38 2.823746e-198
## job=management            86.315789 13.718110  7.60 9.214754e-60
## job=services              79.690522 17.231284 10.34 1.601752e-55
## marital=single            65.325530 37.348390 27.34 1.298027e-52
## job=student              97.058824  4.140527  2.04 1.769413e-28
## contact=cellular         55.804684 46.842325 40.14 2.043330e-20
## y=y_yes                  53.692308 58.385613 52.00 4.690501e-18
## month=apr               64.027149 11.836052  8.84 7.929581e-13
## job=self-employed        64.634146  4.433292  3.28 1.144098e-05
## month=nov               62.631579  4.976997  3.80 3.082353e-05
## job=entrepreneur         62.234043  4.893350  3.76 5.529699e-05
## marital=divorced         55.471698 12.296110 10.60 1.953103e-04
## month=mar               63.492063  3.345880  2.52 3.648824e-04
## month=jun               55.742297  8.322877  7.14 1.906125e-03
## housing=housing_yes      49.765442 53.241322 51.16 4.835333e-03
## month=oct               66.666667  1.171058  0.84 1.472746e-02
## day_of_week=tue         44.920783 20.158929 21.46 3.192193e-02
## housing=housing_no       45.782146 46.758678 48.84 4.835333e-03
## job=housemaid           28.205128  1.380176  2.34 1.320427e-05
## job=retired             25.520833  2.049352  3.84 1.204909e-10
## y=y_no                 41.458333 41.614387 48.00 4.690501e-18
## contact=telephone       42.465753 53.157675 59.86 2.043330e-20
## month=may              42.572693 56.336261 63.28 1.736625e-22
## marital=married         38.801160 50.355500 62.06 2.422605e-60

```

```

## education=professional.course 0.000000 0.000000 12.62 4.520998e-197
## job=technician 0.000000 0.000000 15.14 2.066279e-241
## job=blue-collar 5.368590 2.802175 24.96 4.677485e-309
## education=basic 7.603687 5.520703 34.72 0.000000e+00
## v.test
## job=admin. 38.103288
## education=university.degree 36.581805
## education=high.school 30.041448
## job=management 16.304193
## job=services 15.696359
## marital=single 15.265513
## job=student 11.069216
## contact=cellular 9.260052
## y=y_yes 8.660658
## month=apr 7.162367
## job=self-employed 4.387977
## month=nov 4.167295
## job=entrepreneur 4.032031
## marital=divorced 3.725006
## month=mar 3.564261
## month=jun 3.104482
## housing=housing_yes 2.817804
## month=oct 2.439012
## day_of_week=tue -2.145387
## housing=housing_no -2.817804
## job=housemaid -4.356692
## job=retired -6.438713
## y=y_no -8.660658
## contact=telephone -9.260052
## month=may -9.756126
## marital=married -16.385626
## education=professional.course -29.949094
## job=technician -33.180374
## job=blue-collar -37.579332
## education=basic -Inf

```

```
res.hcpcMCA$desc.var$test.chi2 # categorical variables which characterizes the clusters
```

```

## p.value df
## job 0.000000e+00 20
## education 0.000000e+00 8
## marital 6.642756e-90 4
## month 1.440972e-42 16
## contact 1.954621e-36 2
## y 6.175640e-30 2
## housing 1.237023e-07 2

```

## 7.2 Parangons and class-specific individuals.

```
res.hcpcMCA$desc.ind$para # representative individuals of each cluster
```

```

## Cluster: 1
## 234 2836 4098 1223 1121
## 0.09147034 0.09147034 0.09147034 0.14811486 0.16283767
## -----

```

```
## Cluster: 2
##      2942      478      2056      1367      660
## 0.2780758 0.3440311 0.3548909 0.3577262 0.4071450
## -----
## Cluster: 3
##      1938      1515      1389      31      566
## 0.1828545 0.2159257 0.2336711 0.2483546 0.2483755
```

What we obtain are the more representative individuals, paragons, for each cluster. We get the rownames of each paragon in every single cluster.

```
res.hcpcMCA$desc.ind$dist # individuals distant from each cluster
```

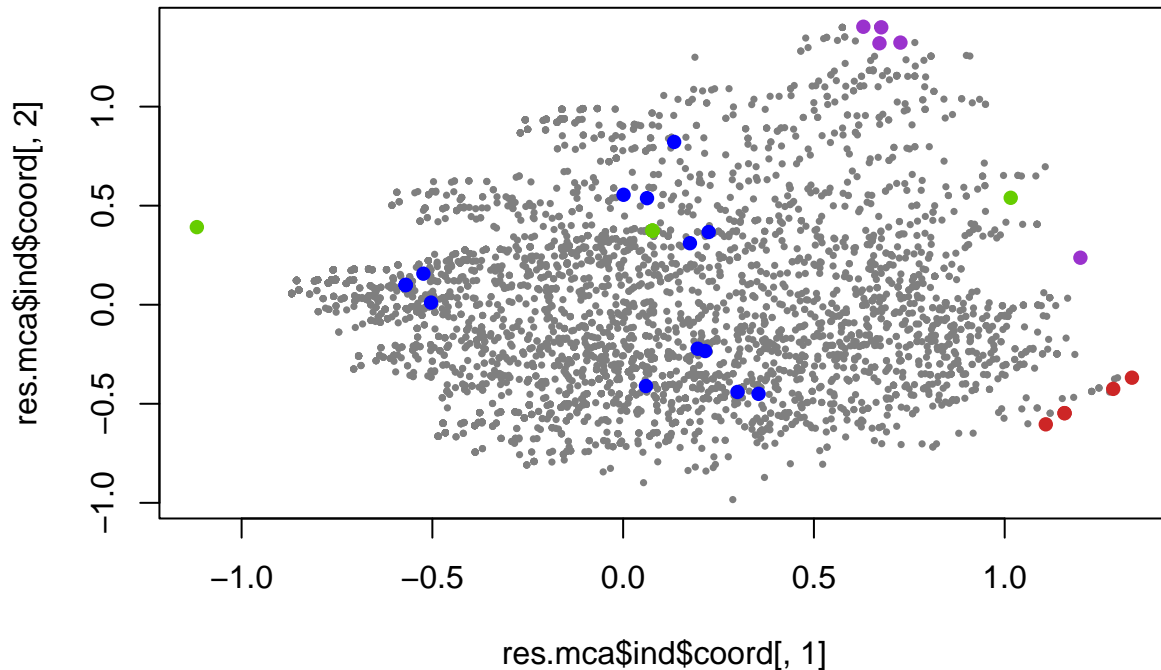
```
## Cluster: 1
##      698      1839      269      1541      2063
## 2.251399 2.244362 1.675209 1.675209 1.675209
## -----
## Cluster: 2
##      2465      1602      2162      23      94
## 2.296221 1.918623 1.891750 1.867893 1.866062
## -----
## Cluster: 3
##      76      144      586      1978      1813
## 2.574272 2.527872 2.524225 2.524225 2.492180
```

We get the graphical representation for the individuals that characterize classes (para and dist).

```
# characteristic individuals
para1<-which(rownames(res.mca$ind$coord)%in%names(res.hcpcMCA$desc.ind$para[[1]]))
dist1<-which(rownames(res.mca$ind$coord)%in%names(res.hcpcMCA$desc.ind$dist[[1]]))
para2<-which(rownames(res.mca$ind$coord)%in%names(res.hcpcMCA$desc.ind$para[[2]]))
dist2<-which(rownames(res.mca$ind$coord)%in%names(res.hcpcMCA$desc.ind$dist[[2]]))
para3<-which(rownames(res.mca$ind$coord)%in%names(res.hcpcMCA$desc.ind$para[[3]]))
dist3<-which(rownames(res.mca$ind$coord)%in%names(res.hcpcMCA$desc.ind$dist[[3]]))

plot(res.mca$ind$coord[,1],res.mca$ind$coord[,2],col="grey50",cex=0.5,pch=16)
points(res.mca$ind$coord[para1,1],res.mca$ind$coord[para1,2],col="blue",cex=1,pch=16)
points(res.mca$ind$coord[dist1,1],res.mca$ind$coord[dist1,2],col="chartreuse3",cex=1,pch=16)
points(res.mca$ind$coord[para2,1],res.mca$ind$coord[para2,2],col="blue",cex=1,pch=16)
points(res.mca$ind$coord[dist2,1],res.mca$ind$coord[dist2,2],col="darkorchid3",cex=1,pch=16)
points(res.mca$ind$coord[para3,1],res.mca$ind$coord[para3,2],col="blue",cex=1,pch=16)
points(res.mca$ind$coord[dist3,1],res.mca$ind$coord[dist3,2],col="firebrick3",cex=1,pch=16)
```





### 7.3 Comparison of clusters obtained after hierarchical clustering (based on PCA) on target duration and binary target.

Given the following description from clusters in MCA:

- Cluster 1:
  - The first cluster are people who are more likely to say no contacted via telephone and have a basic type of education and have a blue-collar kind of job and are married.
- Cluster 2:
  - The second cluster are people who are more likely to say yes being contacted by cellular and are educated from a professional course and are technicians. They are also married and young.
- Cluster 3:
  - The first cluster are people who are almost guaranteed to say yes, they are university educated and are working on more technical jobs such as management and administration, they are young and most likely single as well.

and then PCA:

- Cluster 1:
  - These are the people who will say yes to the campaign and being contacted by cellars, and mostly single university graduates. Also, they are being called during the months of april and may, which are nearing summer seasons and these kind of people tend to have money saved.
- Cluster 2:
  - The are people who are more likely to say no when they are being contacted on november by cellular, cluster similar to the one in KMEANS. These people are divorced and don't have any housing loan.
- Cluster 3:
  - These are people who are married and retired which will most likely say no, and are most usually contacted by telephone. We see that these are people who have their life together already and aren't interested in these kind of campaigns anymore.

We can compare the clusters, but we can't say anything about the duration but we can clearly see some trends on the binary target: \* In both methods we can see that the people who will say yes are young people, who are highly educated with most of them having university degrees and having good jobs and are contacted by

cellular, a clear indication they are young. And the people who say no are tending towards older people who are married and have their life together already, the majority of them being retired and are contacted with a telephone.