Exponential

$$\int \exp\left(-\alpha x\right) dx = \frac{1}{\alpha}\exp(-\alpha x)$$

$$\int_0^\infty \exp\left(-\alpha x\right) dx = \frac{1}{\alpha}$$

Polynomial and exponential

1st order

$$\int x \exp\left(-\alpha x\right) dx = \left(\frac{1}{\alpha^2} - \frac{x}{\alpha}\right)\exp(-\alpha x)$$

$$\int_0^\infty x \exp\left(-\alpha x\right) dx = \frac{1}{\alpha^2}$$

2nd order

$$\int x^2 \exp\left(-\alpha x\right) dx = \left(\frac{2}{\alpha^3} - \frac{2x}{\alpha^2} + \frac{x^2}{\alpha}\right)\exp(-\alpha x)$$

$$\int_0^\infty x^2 \exp\left(-\alpha x\right) dx = \frac{2}{\alpha^3}$$

3rd order

$$\int x^3 \exp\left(-\alpha x\right) dx = \left(\frac{6}{\alpha^4} - \frac{6x}{\alpha^3} + \frac{3x^2}{\alpha^2} - \frac{x^3}{\alpha}\right)\exp(-\alpha x)$$

$$\int_0^\infty x^3 \exp\left(-\alpha x\right) dx = \frac{6}{\alpha^4}$$

n-th order

$$\int_0^\infty x^n \exp\left(-\alpha x\right) dx = \frac{n!}{\alpha^{n+1}}$$

Related with Black-body Radiation

3rd order

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

List of Gaussian Basis Integrals

Basic integrals

$$\int_0^u e^{-x^2} \mathrm{d}x = \frac{\sqrt{\pi}}{2} \mathrm{erf}(u)$$

$$\int_0^u e^{-\alpha x^2} \mathrm{d}x = \frac{\sqrt{\pi}}{2\sqrt{\alpha}} \mathrm{erf}(\sqrt{\alpha} u)$$

Gaussian basis integrals

1st order

$$\int_0^u x e^{-\alpha x^2} \mathrm{d}x = \frac{1}{2\alpha} \left[1 - e^{-\alpha u^2} \right]$$

$$\int_0^\infty x e^{-\alpha x^2} \mathrm{d}x = \frac{1}{2\alpha}$$

$$\int_\infty^\infty x e^{-\alpha x^2} \mathrm{d}x = 0$$

2nd order

$$\int_0^u x^2 e^{-\alpha x^2} \mathrm{d}x = \frac{1}{2\alpha\sqrt{\alpha}} \left[\frac{\sqrt{\pi}}{2} \mathrm{erf}(\sqrt{\alpha} u) - \sqrt{\alpha} u e^{\alpha u^2} \right]$$

$$\int_0^\infty x^2 e^{-\alpha x^2} \mathrm{d}x = \frac{\sqrt{\pi}}{4\alpha\sqrt{\alpha}}$$

$$\int_{-\infty}^\infty x^2 e^{-\alpha x^2} \mathrm{d}x = \frac{\sqrt{\pi}}{2\alpha\sqrt{\alpha}}$$

3rd order

$$\int_0^u x^3 e^{-\alpha x^2} \mathrm{d}x = \frac{1}{2\alpha\sqrt{\alpha}} \left[1 - (1 + \alpha u^2) e^{-\alpha u^2}\right]$$

$$\int_0^\infty x^3 e^{-\alpha x^2} \mathrm{d}x = \frac{1}{2\alpha^2}$$

$$\int_{-\infty}^\infty x^3 e^{-\alpha x^2} \mathrm{d}x = 0$$

List of formulas related to Cholesky decomposition

Definition

The Cholesky factor of positive-definite symmetric matrix X is

$$X = LL^\top$$

where L is a lower triangular matrix.

Trace

Theorem

The trace of X is

$$Tr(X) = \sum_{i,j} L_{i,j}^2$$

Proof

$$Tr(X) = \sum_i X_{i,i} = \sum_i \left(\sum_j L_{i,j} L_{j,i}^\top \right) = \sum_{i,j} L_{i,j}^2$$

Determinant

Theorem

The determinant of X is

$$\det(X) = \prod_i L_{i,i}^2$$

Proof

$$\det(X) = \det(L) \times \det(L^\top) = \prod_i L_{i,i} \times \prod_i L_{i,i} = \prod_i L_{i,i}^2$$