Formulation of 2D shallow water model of turbidity currents

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$$\frac{\partial h}{\partial t} + U\frac{\partial h}{\partial x} + V\frac{\partial h}{\partial y} = e_w\sqrt{U^2+V^2} - h\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) \tag{1}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y} = -RgC\frac{\partial \eta}{\partial x}-\frac{1}{2}Rgh\frac{\partial C}{\partial x}-RgC\frac{\partial h}{\partial x}-\frac{u_*^2}{h}-\frac{e_w U\sqrt{U^2+V^2}}{h}+\nu_t\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y} = -RgC\frac{\partial \eta}{\partial y}-\frac{1}{2}Rgh\frac{\partial C}{\partial y}-RgC\frac{\partial h}{\partial y}-\frac{v_*^2}{h}-\frac{e_w V\sqrt{U^2+V^2}}{h}+\nu_t\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} + V\frac{\partial C}{\partial y} = \frac{w_s(e_s - r_0 C)}{h} - \frac{e_w C\sqrt{U^2+V^2}}{h} \tag{4}$$

$$\frac{d\eta}{dt} = \frac{w_s(r_0 C - e_s)}{1-\lambda_p} \tag{5}$$