# Introduction to Level-k Thinking

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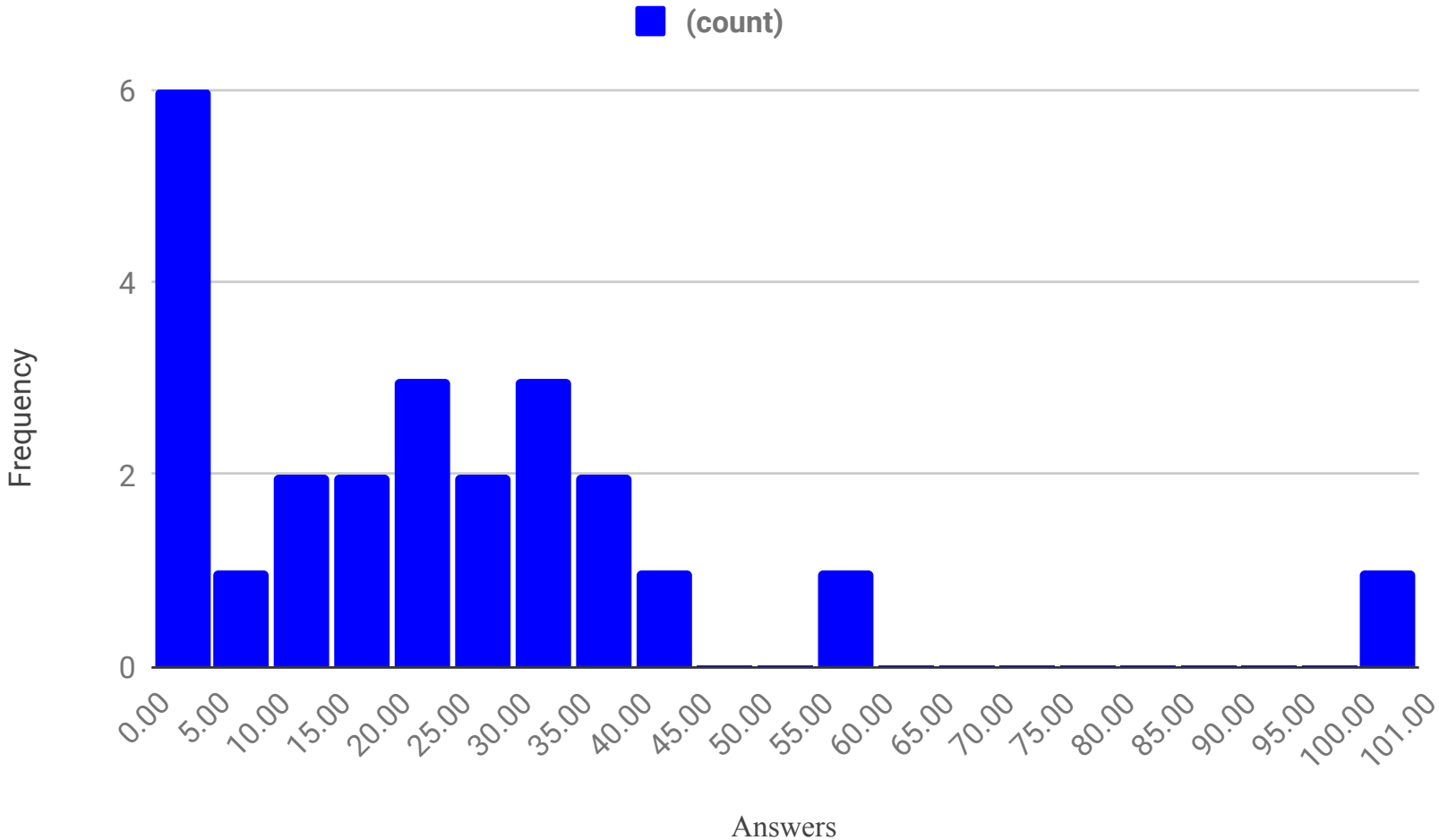
# Introduction

- Today's lecture will be a mixture of game theory, experimental economics and behavioral economics.
- You have been required to do an experiment before the class.
- The rules of the game:
  - You have to guess an integer number between 0 and 100 (the numbers 0 and 100 are included).
  - The person whose guess is closest to the $2/3$ of the average of the guesses of the whole class will be the winner.

# Results


(count)
Frequency
6
4
2
0
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00
55.00
60.00
65.00
70.00
75.00
80.00
85.00
90.00
95.00
100.00
101.00
Answers

# Introduction

- Game Theory is the study of strategic decision making
  - Your payoff depends on your own actions and the actions of others.
- Standard tool for prediction: Nash Equilibrium.
  - In Nash Equilibrium, no player has incentive to deviate given the choices of other players.
- Nash Equilibrium sometimes fails to predict the behavior in real life. For example, in this experiment...
  - Because NE assumes a high degree of rationality .
  - NE also assumes that players assume a high degree of rationality of their opponents.

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# Formal model: Beauty contest game

- Players: n Players
- Strategy set: Each player choose his/her strategy $s_i \in \{0, 1, 2, ..., 100\}$
- Payoff rule: The winner will be the one whose answer is closest to p times the average choice.
- $p \in (0, 1)$
- Utility function: $u_i(s_i, \{s_1, ..., s_{i-1}, s_{i+1}, ..., s_n\}) = u_i(s_i, s_{-i})$

# Nash Equilibrium

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- How to find the Nash Equilibrium?
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# Iterated elimination of dominated strategies

- The first round: Any number that is higher than $100 * p$ is a dominated strategy(never a best response).
- The second round: Any number that is higher than $100 * p^2$ is a dominated strategy(never a best response).
- ...
- The kth round: Any number that is higher than than $100 * p^k$ is a dominated strategy(never a best response).
- As k becomes large enough, only the choice $0$ survive the iterated eliminated of dominated strategy.
- By elimination of dominated strategies, we get the Nash Equilibrium which is 0.

# Assumptions of Nash Equilibrium

- Rationality and common knowledge of rationality.
- Rationality: Player know how to choose the optimal strategy given his/her belief.
- Common knowledge of rationality:
  - Players knows that their opponents are rational.
  - Players know that their opponents know that other players are rational.
  - and so on...

# Alternative to CKR: Depth of reasoning (Level-k thinking)

In the case of beauty contest game in our experiment:

- I believe other players will choose 50, so I will best response to 50, which is 33.
- I believe other players believe everyone will choose 50 and so will choose 33. I will therefore choose the best response to 33, which is 22.
- I think that other players will initially think that everyone will choose 50, and will consider choosing 33. However, they will think that others have done the same reasoning, and will therefore choose 22. I will best respond to this and play 15.
- and so on...

# Depth of reasoning (level-k thinking)

In a formal manner:

- I assume that other players will choose $\bar{s}$, so I will choose $s_i^1 \in \arg\max_{s \in S_i} u_i(s, \bar{s})$
- I assume that other players will choose the best response to $\bar{s}$ and choose $s_j^1 \in \arg\max_{s \in S_j} u_j(s, \bar{s})$, so I will choose $s_i^2 \in \arg\max_{s \in S_i} u_i(s, s_j^1)$
- I assume that other players will choose the best response to $s_j^1$ and choose $s_j^2 \in \arg\max_{s \in S_j} u_j(s, s_i^1)$, so I will choose $s_i^3 \in \arg\max_{s \in S_i} u_i(s, s_j^2)$
- ...

# Level-k thinking and Nash Equilibrium

- Notice that a Nash Equilibrium is a fixed point of this type of reasoning.
  - I assume that other players will best respond to $s_i^*$ and so play $s_j^* \in \arg\max_{s \in S_j} u_j(s, s_i^*)$.I therefore will choose $s_i^* \in \arg\max_{s \in S_i} u_i(s, s_j^*)$
- In the case of Beauty contest game, this type of reasoning will converge to Nash Equilibrium.
- But this is not always true.

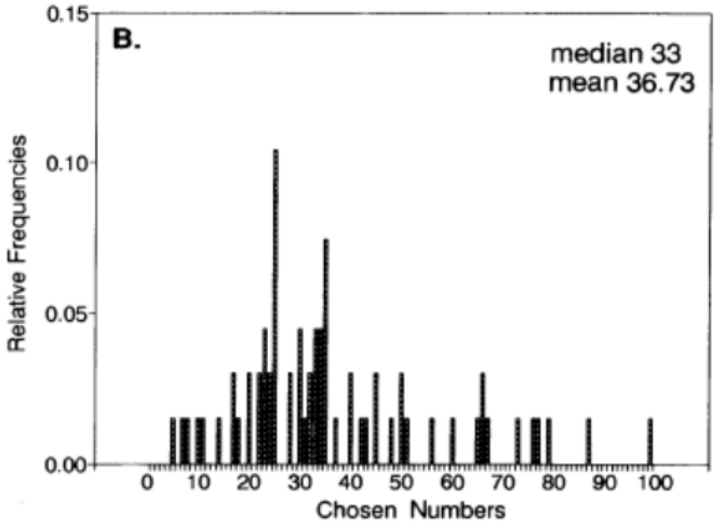
# Level-k thinking

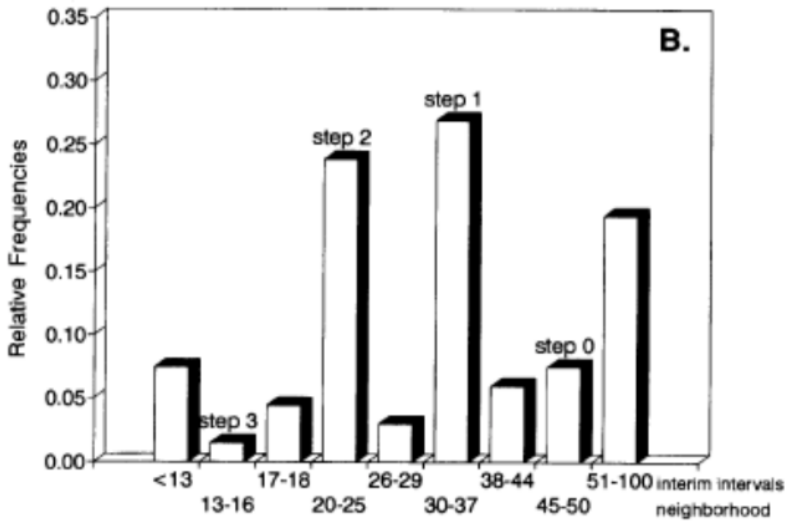
- Human beings have constraints in the depth of their reasoning.
- The depth of reasoning (measured by how many steps of reasoning they can conduct).
- There are types of players:
  - Level 0: Non-strategic players (play randomly)
  - Level 1: Best respond to level 0
  - Level 2: Best respond to level 1
  - Level 3: Best respond to level 2
  - ...
- There is a distribution of types in the population: $\pi_i$(probability of level i).
- Generally assumed that $\pi_0 = 0$
  - Level 0 is anchor for the higher level players

# Level-k thinking

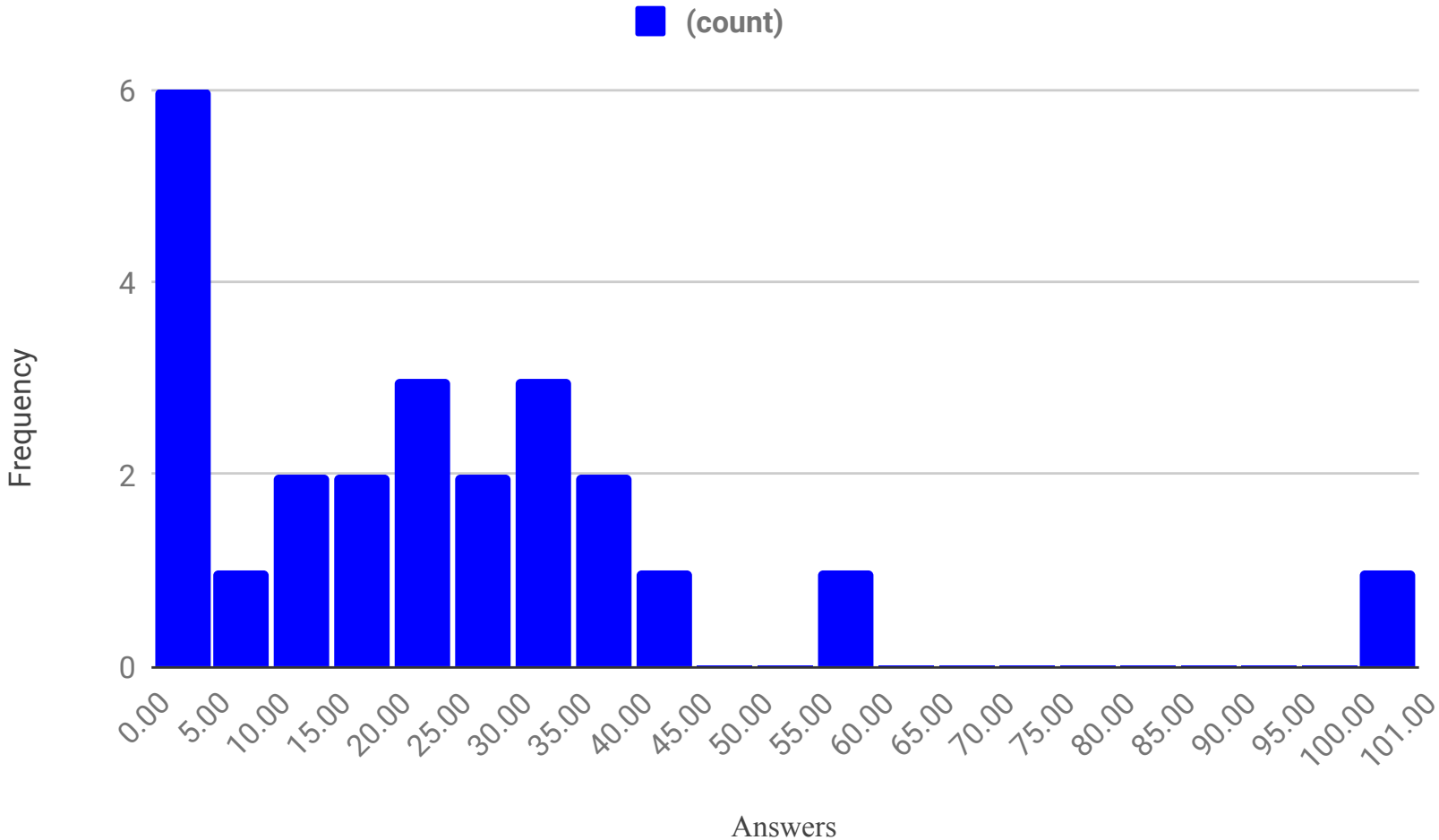
- What is implication for the data in our Beauty Contest game
- We should see spikes of response at the following level:
  - Level 0: 50
  - Level 1: 33
  - Level 2: 22
  - Level 3: 15

# Results of Nagel 1995



# Results of class experiment



# Results of class experiment

- You are more 'rational' than the sample of Nagel.
- And you have higher confidence for your classmates.
  - 22.6 vs. 36.7
- There are more people choose Nash Equilibrium in this class.
- Evidence that most people are approximately level 2 players.
- 2/3 of the mean: 15.06
- The winner's guess is 15.
- The winner is

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# References


- Nagel, Rosemarie. "Unraveling in guessing games: An experimental study." The American Economic Review 85.5 (1995): 1313-1326.
- Crawford, Vincent P., Miguel A. Costa-Gomes, and Nagore Iriberri. "Structural models of nonequilibrium strategic thinking: Theory, evidence, and applications." Journal of Economic Literature 51.1 (2013): 5-62.
- Kneeland, Terri. "Identifying Higher-Order Rationality." Econometrica 83.5 (2015): 2065-2079.

**Thank you!**

If you have any question or comment or you find any interesting real life examples of level-k thinking , please send email to haihanyu(at)agent.inf.kyushu-u.ac.jp