Introduction to Level-k Model

Haihan Yu

Multi-Agent Lab, Kyushu University

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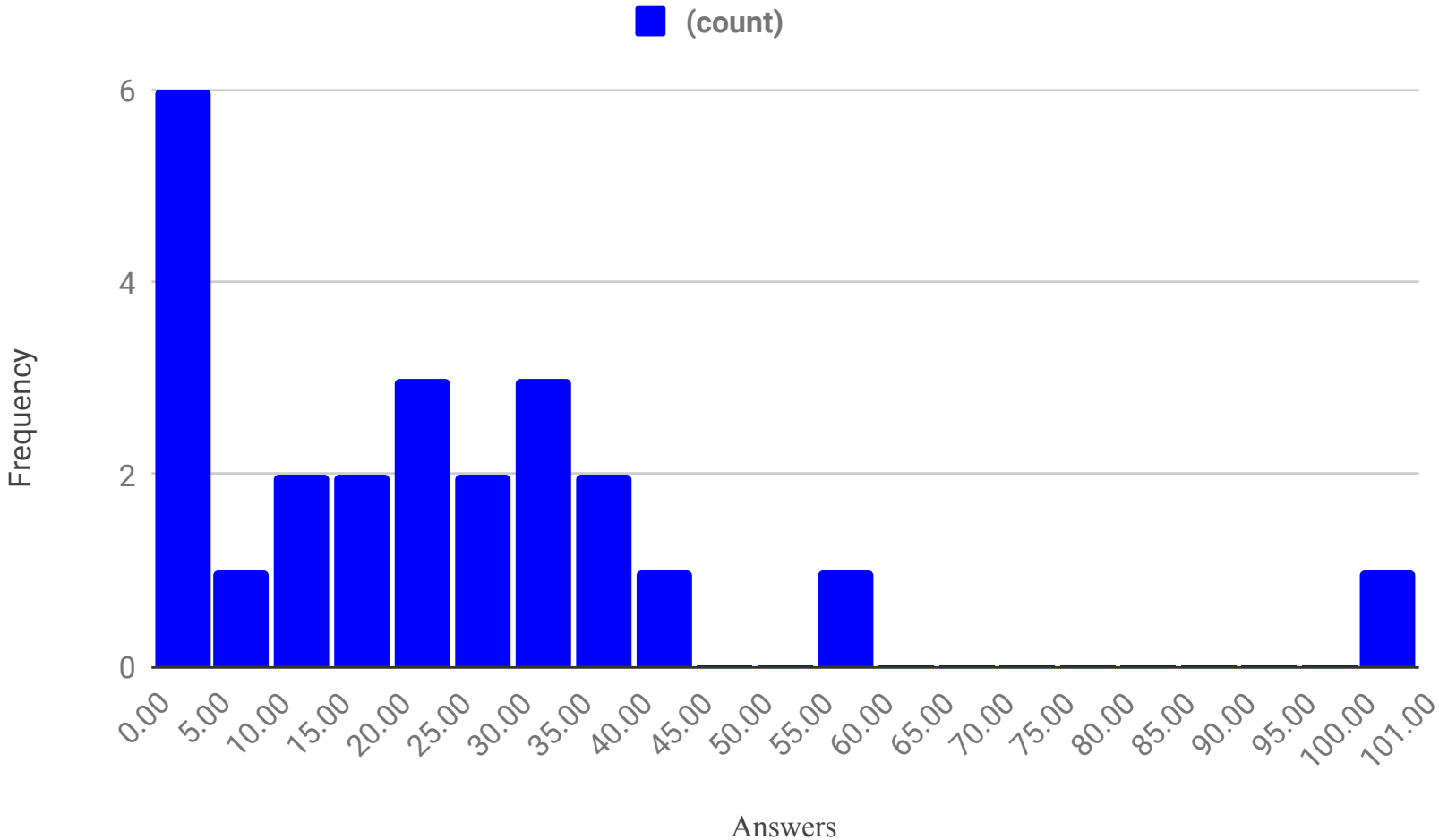
Introduction

Results


(count)
Frequency
6
4
2
0
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00
55.00
60.00
65.00
70.00
75.00
80.00
85.00
90.00
95.00
100.00
101.00
Answers

Formal model: Beauty contest game

- Players: n Players
- Strategy set: Each player choose his/her strategy $s_i \in \{0, 1, 2, ..., 100\}$
- Payoff rule: The winner will be the one whose answer is closest to p times the average choice.
- $p \in (0, 1)$
- Utility function: $u_i(s_i, \{s_1, ..., s_{i-1}, s_{i+1}, ..., s_n\}) = u_i(s_i, s_{-i})$

Nash Equilibrium

- Nash Equilibrium: A strategy profile $\{s_1^*, ..., s_n^*\}$ that no plyaer has the incentive to deviate unilaterally.
- How to find the Nash Equilibrium?
- By iterated elimination of dominated strategies.

Iterated elimination of dominated strategies

- The first round: Any number that is higher than $100 * p$ is a dominated strategy(never a best response).
- The second round: Any number that is higher than $100 * p^2$ is a dominated strategy(never a best respnose).
- ...
- The kth round: Any number that is higher than than $100 * p^k$ is a dominated strategy(never a best response).
- As k becomes large enough, only the choice 0 survive the iterated eleminated of dominated strategy.
- By elimination of dominated strategies, we get the Nash Equibliurm which is $s_i = 0$

Assumptions of Nash Equilibrium

- Rationality and common knowledge of rationality.
- Rationality: Player know how to choose the optimal strategy given his/her belief.
- Common knowledge of rationality:
  - Players knows that their opponents are rational.
  - Players know that their opponents know that other players are rational.
  - and so on...

Alternative to CKR: Depth of reasoning (Level-k thinking)

In the case of beauty contest game in our experiment:

- I believe other players will choose 50, so I will best response to 50, which is 33.
- I believe other players believe everyone will choose 50 and so will choose 33. I will therefore choose the best response to 33, which is 22.
- I think that other players will initially think tink that everyone will choose 50, and will consider choosing 33. However, they will think that others have done the same reasoning, and will therefore choose 22. I will best respond to this and play 15.
- and so on...

Depth of reasoning (level-k thinking)

In a formal manner:

- I assume that other players will choose $\bar{s}$, so I will choose $s_i^1 \in \arg\max_{s \in S_i} u_i(s, \bar{s})$
- I assume that other players will choose the best response to $\bar{s}$ and choosing $s_j^1 \in \arg\max_{s \in S_j} u_j(s, \bar{s})$, so I will choose $s_i^2 \in \arg\max_{s \in S_i} u_i(s, s_j^1)$
- I assume that other players will choose the best response to s_j^1 and choosing $s_j^2 \in \arg\max_{s \in S_j} u_j(s, s_i^1)$, so I will choose $s_i^3 \in \arg\max_{s \in S_i} u_i(s, s_j^2)$
- ...

Level-k thinking and Nash Equilibrium

- Notice that a Nash Equilibrium is a fixed poinnt of this type of reasoning.
  - I assume that other players will best respond to s_i^* and so play $s_j^* \in \arg\max_{s \in S_j} u_j(s, s_i^*)$.I therefore will choose $s_i^* \in \arg\max_{s \in S_i} u_i(s, s_j^*)$
- In the case of Beauty contest game, this type of reasoning will coverge to Nash Equilibrium.
- But this is not always true.

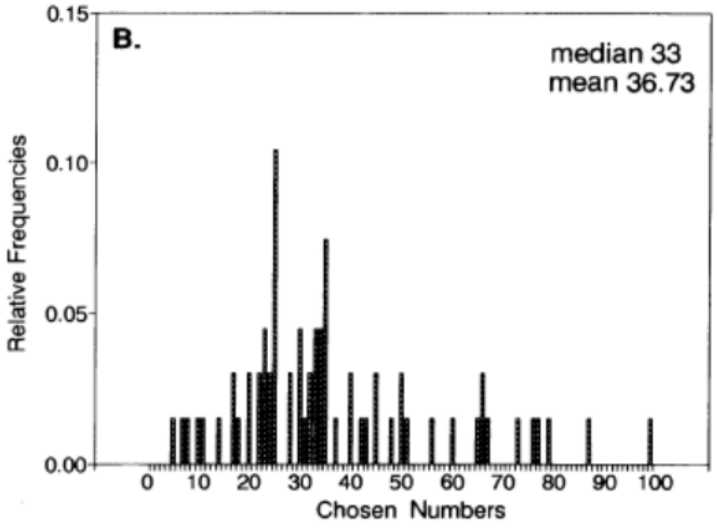
Level-k thinking

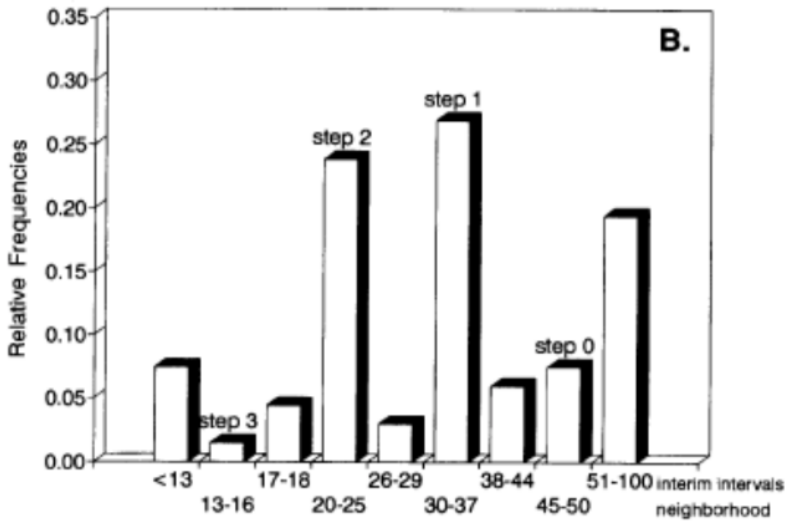
- Human beings have constraints in the depth of their reasoning.
- The depth of reasoning (measured by how many steps of reasoning they can conduct).
- There are types of players:
  - Level 0: Non-strategic players (play random)
  - Level 1: Best respond to level 0
  - Level 2: Best respond to level 1
  - Level 3: Best respond to level 3
  - ...
- There is a distribution of types in the population: π_i(probability of level i).
- Generally assumed that $\pi_0 = 0$
  - Level 0 is anchor for the higher level players

Level-k thinking

- What is implication for the data in our Beauty Contest game
- We should see spikes of response at the following level:
  - Level 0: 50
  - Level 1: 33
  - Level 2: 22
  - Level 3: 15

Results of Nagel 1995





Class experiment resutls