

問題 1

求不定積分

$$\int \frac{x+5}{x^2-6x+13} dx$$

解.

$$\begin{aligned} \text{与式} &= \int \frac{x+5}{x^2+2\cdot 3x+3^2+4} dx \\ &= \int \frac{x-3+8}{(x-3)^2+4} d(x-3) \\ &= \int \frac{x-3}{(x-3)^2+4} d(x-3) + 8 \int \frac{1}{(x-3)^2+4} d(x-3) \\ &= \frac{1}{2} \int \frac{1}{(x-3)^2+4} d[(x-3)^2+4] + 8 \int \frac{1}{(x-3)^2+2^2} d(x-3) \\ &= \frac{1}{2} \log |x^2-6x+13| + 8 \cdot \frac{1}{2} \arctan \frac{x-3}{2} + C \\ &= \frac{1}{2} \log (x^2-6x+13) + 4 \arctan \frac{x-3}{2} + C \end{aligned}$$

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問題 2

求不定積分

$$\int \frac{dx}{x(x-1)^2}$$

解. 設

$$\text{与式} = \int \frac{A}{x} + \frac{Bx+C}{x^2-2x+1} dx$$

則

$$\begin{cases} A+B=0 \\ -2A+C=0 \\ A=1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \\ C=2 \end{cases}$$

即

$$\begin{aligned}\text{与式} &= \int \frac{1}{x} + \frac{-x+2}{x^2-2x+1} dx \\ &= \int \frac{1}{x} dx - \int \frac{x-1-1}{(x-1)^2} d(x-1) \\ &= \int \frac{1}{x} dx - \int \frac{1}{x-1} d(x-1) + \int \frac{1}{(x-1)^2} d(x-1) \\ &= \log|x| - \log|x-1| - \frac{1}{x-1} + C \\ &= \log\left|\frac{x}{x-1}\right| - \frac{1}{x-1} + C\end{aligned}\quad \square$$

問題 3

求不定積分

$$\int \frac{\cos 2x - \sin 2x}{\cos x + \sin x} dx$$

解.

$$\text{与式} = \text{unimplemented}$$

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