Satanya@張睿 (2020年10月10日)

.問題 1

求不定積分

$$\int \frac{x+5}{x^2-6x+13} \, \mathrm{d}x$$

解.

与式 =
$$\int \frac{x+5}{x^2+2\cdot 3x+3^2+4} \, \mathrm{d}x$$

= $\int \frac{x-3+8}{(x-3)^2+4} \, \mathrm{d}(x-3)$
= $\int \frac{x-3}{(x-3)^2+4} \, \mathrm{d}(x-3) + 8 \int \frac{1}{(x-3)^2+4} \, \mathrm{d}(x-3)$
= $\frac{1}{2} \int \frac{1}{(x-3)^2+4} \, \mathrm{d}[(x-3)^2+4] + 8 \int \frac{1}{(x-3)^2+2^2} \, \mathrm{d}(x-3)$
= $\frac{1}{2} \log|x^2-6x+13| + 8 \cdot \frac{1}{2} \arctan \frac{x-3}{2} + C$
= $\frac{1}{2} \log(x^2-6x+13) + 4 \arctan \frac{x-3}{2} + C$

問題 2_

求不定積分

$$\int \frac{\mathrm{d}x}{x(x-1)^2}$$

解. 設

与式 =
$$\int \frac{A}{x} + \frac{Bx + C}{x^2 - 2x + 1} \, \mathrm{d}x$$

則

$$\begin{cases} A+B=0\\ -2A+C=0\\ A=1 \end{cases} \Rightarrow \begin{cases} A=1\\ B=-1\\ C=2 \end{cases}$$

即

与式 =
$$\int \frac{1}{x} + \frac{-x+2}{x^2 - 2x + 1} dx$$

= $\int \frac{1}{x} dx - \int \frac{x-1-1}{(x-1)^2} d(x-1)$
= $\int \frac{1}{x} dx - \int \frac{1}{x-1} d(x-1) + \int \frac{1}{(x-1)^2} d(x-1)$
= $\log |x| - \log |x-1| - \frac{1}{x-1} + C$

問題 3_

求不定積分

$$\int \frac{\cos 2x - \sin 2x}{\cos x + \sin x} \, \mathrm{d}x$$

解.

$$\exists \vec{x} = \int \frac{\cos^2 x - \sin^2 x - 2\sin x \cos x}{\cos x + \sin x} dx$$

$$= \int \frac{(\cos x + \sin x)(\cos x - \sin x) - 2\sin x \cos x}{\cos x + \sin x} dx$$

$$= \int \cos x - \sin x dx - \int \frac{2\sin x \cos x}{\cos x + \sin x} dx$$

$$= \sin x + \cos x - \int \frac{1 + 2\sin x \cos x - 1}{\cos x + \sin x} dx$$

$$= \sin x + \cos x - \int \frac{(\cos x + \sin x)^2 - 1}{\cos x + \sin x} dx$$

$$= \sin x + \cos x - \int \cos x + \sin x - \frac{1}{\cos x + \sin x} dx$$

$$= \sin x + \cos x - \int \cos x + \sin x - \frac{1}{\cos x + \sin x} dx$$

$$= \sin x + \cos x - \sin x + \cos x - \int \frac{1}{\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)} dx$$

$$= 2\cos x - \frac{\sqrt{2}}{2} \int \csc\left(x + \frac{\pi}{4}\right) d\left(x + \frac{\pi}{4}\right)$$

$$= 2\cos x - \frac{\sqrt{2}}{2} \log\left|\csc(x + \frac{\pi}{4}) - \cot(x + \frac{\pi}{4})\right| + C$$