

問題 1

設 $z = z(x, y)$ 是由 $x^2 - 6xy + 10y^2 - 2yz - z^2 + 18 = 0$ 確定的函數. 求 z 的極值.

解. 与式兩邊分別對 x 同 y 求偏導數:

$$2x - 6y - 2y \frac{\partial z}{\partial x} - 2z \frac{\partial z}{\partial x} = 0$$

$$-6x + 20y - 2z - 2y \frac{\partial z}{\partial y} - 2z \frac{\partial z}{\partial y} = 0$$

令 $\frac{\partial z}{\partial x} = 0, \frac{\partial z}{\partial y} = 0$, 又有題設方程, 整理得:

$$\begin{cases} 2x - 6y = 0 \\ -6x + 20y - 2z = 0 \\ x^2 - 6xy + 10y^2 - 2yz - z^2 + 18 = 0 \end{cases}$$

解得

□