問題 1

求極限
$$\lim_{x \to 0} \frac{e^{\tan x} - e^x}{x - \sin x}$$

解. Maclaurin Expansion: $e^{\tan x} = 1 + \tan x + o(\tan x)$, $e^x = 1 + x + o(x)$.

$$
\begin{aligned}
与式 &= \lim_{x \to 0} \frac{1 + \tan x + o(\tan x) - 1 - x - o(x)}{x - \sin x} \\
&= \lim_{x \to 0} \frac{\tan x - x + o(x)}{x - \sin x} \\
&= \lim_{x \to 0} \frac{\tan x - x}{x - \sin x} \\
&= \lim_{x \to 0} \frac{1 + \tan^2 x - 1}{1 - \cos x} \\
&= \lim_{x \to 0} \frac{2 \tan x \sec^2 x}{\sin x} \\
&= \lim_{x \to 0} \frac{2}{\cos x} \\
&= 2 \qquad\qquad\qquad\qquad\qquad\qquad\qquad \square
\end{aligned}
$$

問題 2

求極限
$$\lim_{x \to 0} \frac{\sqrt{1 - x^2} - \cos 3x}{e^x - x - 1}$$

解.

$$
\begin{aligned}
与式 &= \lim_{x \to 0} \frac{\frac{1}{2} \left(1 - x^2\right)^{-\frac{1}{2}} (-2x) + 3 \sin 3x}{e^x - 1} \\
&= \lim_{x \to 0} \frac{\frac{1}{2} \left(-\frac{1}{2}\right) \left(1 - x^2\right)^{-\frac{3}{2}} (-2x)^2 + (-2)\left[\frac{1}{2}\left(1 - x^2\right)^{-\frac{1}{2}}\right] + 9 \cos 3x}{e^x} \\
&= 8 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \square
\end{aligned}
$$