

問題 1

求極限

$$\lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x}{x - \sin x}$$

解. Maclaurin Expansion: $e^{\tan x} = 1 + \tan x + o(\tan x)$, $e^x = 1 + x + o(x)$.

$$\begin{aligned} \text{与式} &= \lim_{x \rightarrow 0} \frac{1 + \tan x + o(\tan x) - 1 - x - o(x)}{x - \sin x} \\ &= \lim_{x \rightarrow 0} \frac{\tan x - x + o(x)}{x - \sin x} \\ &= \lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x} \\ &= \lim_{x \rightarrow 0} \frac{1 + \tan^2 x - 1}{1 - \cos x} \\ &= \lim_{x \rightarrow 0} \frac{2 \tan x \sec^2 x}{\sin x} \\ &= \lim_{x \rightarrow 0} \frac{2}{\cos x} \\ &= 2 \end{aligned}$$

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問題 2

求極限

$$\lim_{x \rightarrow 0} \frac{\sqrt{1-x^2} - \cos 3x}{e^x - x - 1}$$

解.

$$\begin{aligned} \text{与式} &= \lim_{x \rightarrow 0} \frac{\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) + 3 \sin 3x}{e^x - 1} \\ &= \lim_{x \rightarrow 0} \frac{\frac{1}{2}\left(-\frac{1}{2}\right)(1-x^2)^{-\frac{3}{2}}(-2x)^2 + (-2)\left[\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\right] + 9 \cos 3x}{e^x} \\ &= 8 \end{aligned}$$

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