
問題 1

求極限

$$\lim_{x\to 0}\frac{\mathrm{e}^{\tan x}-\mathrm{e}^{x}}{x-\sin x}$$

解. Maclaurin Expansion: $\mathrm{e}^{\tan x}=1+\tan x+o(\tan x)$, $\mathrm{e}^{x}=1+x+o(x)$.

$$\begin{aligned}
\text{与式} &= \lim_{x\to 0}\frac{1+\tan x+o(\tan x)-1-x-o(x)}{x-\sin x}\\
&= \lim_{x\to 0}\frac{\tan x-x+o(x)}{x-\sin x}\\
&= \lim_{x\to 0}\frac{\tan x-x}{x-\sin x}\\
&= \lim_{x\to 0}\frac{1+\tan^2 x-1}{1-\cos x}\\
&= \lim_{x\to 0}\frac{2\tan x\sec^2 x}{\sin x}\\
&= \lim_{x\to 0}\frac{2}{\cos x}\\
&= 2
\end{aligned}$$

□

問題 2

求極限

$$\lim_{x\to 0}\frac{\sqrt{1-x^2}-\cos 3x}{\mathrm{e}^{x}-x-1}$$

解.

$$\begin{aligned}
\text{与式} &= \lim_{x\to 0}\frac{\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x)+3\sin 3x}{\mathrm{e}^{x}-1}\\
&= \lim_{x\to 0}\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(1-x^2\right)^{-\frac{3}{2}}(-2x)^2+(-2)\left[\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}\right]+9\cos 3x}{\mathrm{e}^{x}}\\
&= 8
\end{aligned}$$

□