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問題 1

求極限

$$\lim_{x\to 0}\frac{e^{\tan x}-e^x}{x-\sin x}$$

解. Maclaurin Expansion: $e^{\tan x}=1+\tan x+o(\tan x)$, $e^x=1+x+o(x)$.

$$\begin{aligned}
\text{与式} &= \lim_{x\to 0}\frac{1+\tan x+o(\tan x)-1-x-o(x)}{x-\sin x}\\
&= \lim_{x\to 0}\frac{\tan x-x+o(x)}{x-\sin x}\\
&= \lim_{x\to 0}\frac{\tan x-x}{x-\sin x}\\
&= \lim_{x\to 0}\frac{1+\tan^2 x-1}{1-\cos x}\\
&= \lim_{x\to 0}\frac{2\tan x\sec^2 x}{\sin x}\\
&= \lim_{x\to 0}\frac{2}{\cos x}\\
&= 2
\end{aligned}$$

□

問題 2

求極限

$$\lim_{x\to 0}\frac{\sqrt{1-x^2}-\cos 3x}{e^x-x-1}$$

解.

$$\begin{aligned}
\text{与式} &= \lim_{x\to 0}\frac{\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x)+3\sin 3x}{e^x-1}\\
&= \lim_{x\to 0}\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(1-x^2\right)^{-\frac{3}{2}}(-2x)^2+(-2)\left[\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}\right]+9\cos 3x}{e^x}\\
&= 8
\end{aligned}$$

□