Chapter 4 Boolean Algebra and Logic Simplification

4.1 Boolean Operations and Expressions

- Variable: a symbol used to represent a logical quantity;
- Complement: the inverse of a variable
  - Be indicated by a bar over the variable: NOT operation
- Boolean Addition: OR operation
- Boolean Multiplication: equivalent to the AND operation.

Boolean Operations and Expressions

- 0和1元素的性质

OR	AND	Complement
$X + 0 = X$	$X \cdot 0 = 0$	$\overline{0} = 1$
$X + 1 = 1$	$X \cdot 1 = X$	$\overline{1} = 0$

二进制加法运算	二进制乘法运算	布尔代数或运算	布尔代数与运算
$0+0=0$	$0 \times 0=0$	$0+0=0$	$0 \bullet 0=0$
$0+1=1$	$0 \times 1=0$	$0+1=1$	$0 \bullet 1=0$
$1+0=1$	$1 \times 0=0$	$1+0=1$	$1 \bullet 0=0$
$1+1=10$	$1 \times 1=1$	$1+1=1$	$1 \bullet 1=1$

Boolean Operations and Expressions

- 1850s, George Boole
  - 将逻辑表述映射到符号
  - 采用数学的方法处理逻辑推理

"An investigation into the Laws of Thought"

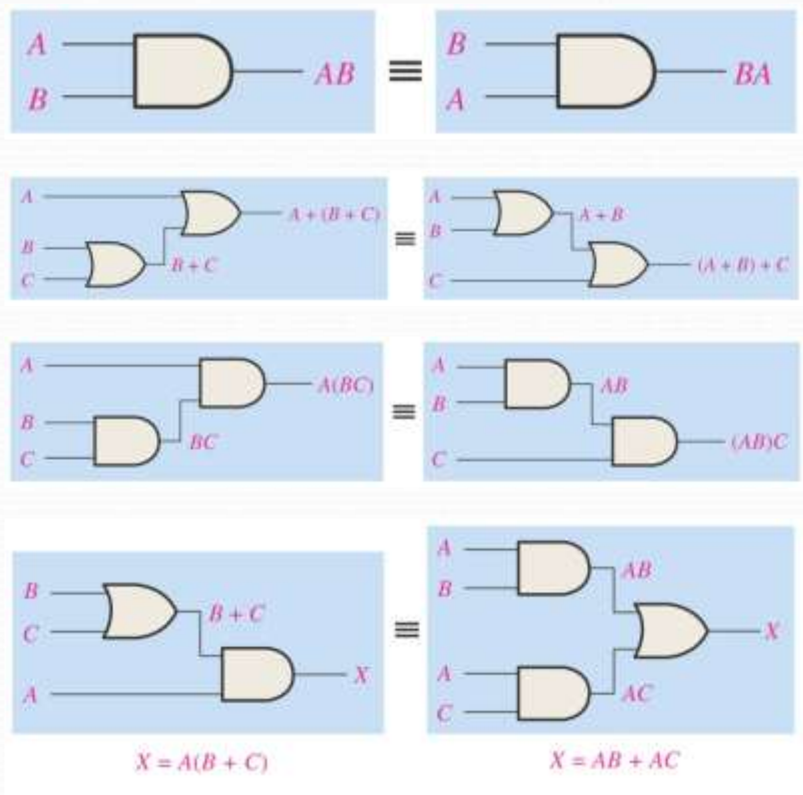
- *1938s*, Claude Elwood Shannon
  - 将布尔代数和硬件开关相联系
  - 第一次提出bit（比特）表示信息

"A Symbolic Analysis of Relay and Switching Circuits(1938)" Master thesis in MIT.

  - 1941香农加入AT&T Bell 实验室
  - *"The mathematical theory of communication"*, 1948

4.2 Laws and Rules of Boolean Algebra

- Laws of Boolean Algebra
  - **Commutative Laws**
    - A+B=B+A
    - AB=BA
  - **Associative Laws**
    - A+(B+C)=(A+B)+C
    - A(BC)=(AB)C
  - **Distributive Law**
    - A(B+C)=AB+AC



Rules of Boolean Algebra

$$0 \bullet A = 0$$

$$1 \bullet A = A$$

$$A \bullet A = A$$

$$A \bullet \overline{A} = 0$$

$$A \bullet B = B \bullet A$$

$$A \bullet (B \bullet C) = (A \bullet B) \bullet C$$

$$A \bullet (B + C) = A \bullet B + A \bullet C$$

$$\overline{\overline{A}} = A$$

$$\overline{1} = 0; \overline{0} = 1$$

$$1 + A = 1$$

$$0 + A = A$$

$$A + A = A$$

$$A + \overline{A} = 1$$

$$A + B = B + A$$

$$A + (B + C) = (A + B) + C$$

$$A + A \bullet B = A$$

$$A + \overline{A} \bullet B = A + (A + \overline{A}) \bullet B = A + B$$

$$A \bullet B + A \bullet \overline{B} = A$$

$$(A + B) \bullet (A + \overline{B}) = A$$

$$A \bullet (A + B) = A$$

$$A + B \bullet C = (A + B) \bullet (A + C)$$

$$A \bullet B + \overline{A} \bullet C + B \bullet C = A \bullet B + \overline{A} \bullet C + (A + \overline{A}) \bullet B \bullet C$$

$$= A \bullet B + \overline{A} \bullet C$$

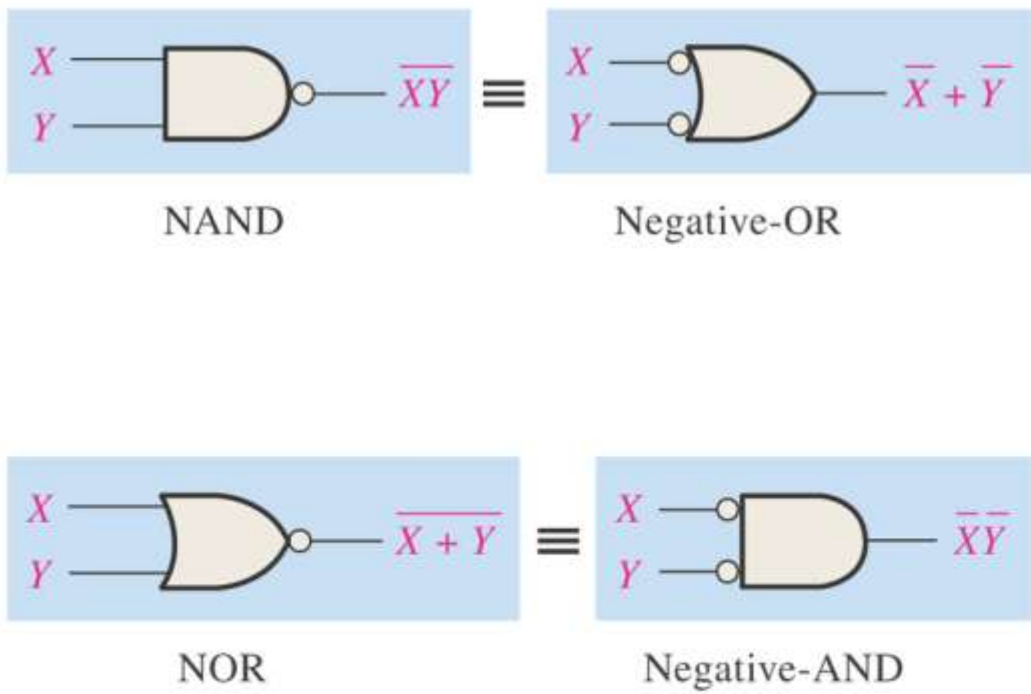
4.3 Demorgan’s Theorems

- The complement of a product of variables is equal to the sum of the complements of the variables
- The complement of a sum of variables is equal to the product of the complements of the variables

$$\overline{A \bullet B} = \overline{A} + \overline{B}$$

$$\overline{A + B} = \overline{A} \bullet \overline{B}$$

Figure 4–15 Gate equivalencies and the corresponding truth tables that illustrate DeMorgan's theorems. Notice the equality of the two output columns in each table. This shows that the equivalent gates perform the same logic function.



Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Example: Apply DeMorgan's theorems to the following expressions

$$\overline{XYZ} = ?$$

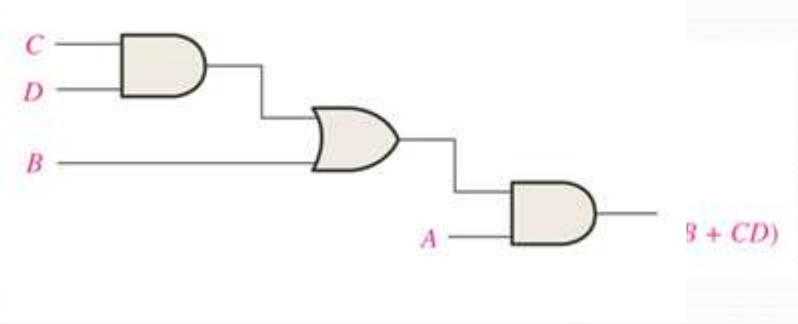
$$\overline{X + Y + Z} = ?$$

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

4.4 Boolean Analysis of Logic Circuits

- Boolean Expression for a Logic Circuit
- Constructing a Truth Table for a Logic Circuit



A	B	C	D	A(B+CD)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

4.5 Simplification Using Boolean Algebra

Examples:

$$Y_1 = A\overline{\overline{B}CD} + A\overline{B}CD$$

$$Y_2 = A\overline{B} + ACD + \overline{A}\,\overline{B} + \overline{A}CD$$

$$Y_3 = \overline{A}B\overline{C} + A\overline{C} + \overline{B}\,\overline{C}$$

$$Y_4 = B\overline{C}D + BC\overline{D} + B\overline{C}\,\overline{D} + BCD$$

$$A + \overline{A} = 1$$

Solutions:

$$Y_1 = A\overline{\overline{B}CD} + A\overline{B}CD = A(\overline{\overline{B}CD} + \overline{B}CD) = A$$

$$Y_2 = A\overline{B} + ACD + \overline{A}\,\overline{B} + \overline{A}CD = A(\overline{B} + CD) + \overline{A}(\overline{B} + CD) = \overline{B} + CD$$

$$Y_3 = \overline{A}B\overline{C} + A\overline{C} + \overline{B}\,\overline{C} = \overline{A}B\overline{C} + (A + \overline{B})\overline{C} = (\overline{A}B)\overline{C} + (\overline{\overline{A}B})\overline{C} = \overline{C}$$

$$Y_4 = B\overline{C}D + BC\overline{D} + B\overline{C}\,\overline{D} + BCD = B(\overline{C}D + C\overline{D}) + B(\overline{C}\,\overline{D} + CD)$$

$$= B(C \oplus D) + B(\overline{C \oplus D}) = B$$

Examples:

$A + AB = A$

$$Y_1 = (\overline{\overline{A}B} + C)\ ABD + AD$$

$$Y_2 = AB + AB\overline{C} + ABD + AB(\overline{C} + \overline{D})$$

$$Y_3 = A + \overline{\overline{A} \bullet \overline{BC}}(\overline{A} + \overline{\overline{B}\,\overline{C} + D}) + BC$$

$Solutions:$

$$Y_1 = (\overline{\overline{A}B}+C)\ ABD + AD = \left[(\overline{\overline{A}B}+C)\ B\right]AD + AD = AD$$

$$Y_2 = AB + AB\overline{C} + ABD + AB(\overline{C}+\overline{D}) = AB + AB\left[\overline{C} + D + (\overline{C}+\overline{D})\right] = AB$$

$$Y_3 = A + \overline{\overline{A}\bullet\overline{BC}}(\overline{A} + \overline{\overline{B}\,\overline{C}+D}) + BC$$

$$= (A+BC) + (A+BC)\left(\overline{A} + \overline{\overline{B}\,\overline{C}+D}\right) = A + BC$$

Examples :

$$Y_1 = \overline{B} + ABC$$

$$Y_2 = A\overline{B} + B + \overline{A}B$$

$$Y_3 = AC + \overline{A}D + \overline{C}D$$

$$A + \overline{A}B = A + B$$

$Solution:$

$$Y_1 = \bar{B} + ABC = \bar{B} + AC$$

$$Y_2 = A\bar{B} + B + \bar{A}B = A + B + \bar{A}B = A + B$$

$$Y_3 = AC + \bar{A}D + \bar{C}D = AC + (\bar{A} + \bar{C})\ D = AC + \overline{AC}D$$

$$= AC + D$$

$Example:$

$$Y = A\overline{B} + \overline{A}B + B\overline{C} + \overline{B}C$$

$A + \overline{A} = 1$

$Solution:$

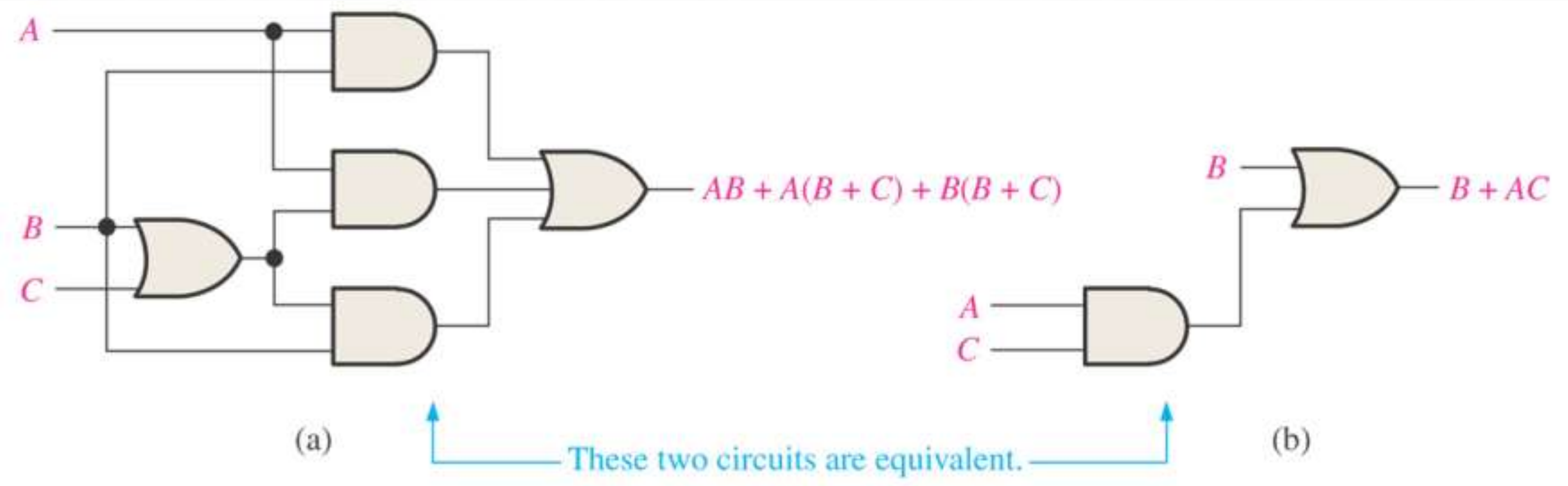
$$Y = A\bar{B} + \bar{A}B(C + \bar{C}) + B\bar{C} + (A + \bar{A})\bar{B}C$$

$$= A\bar{B} + \bar{A}BC + \bar{A}B\bar{C} + B\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$$

$$= (A\bar{B} + A\bar{B}C) + (B\bar{C} + \bar{A}B\bar{C}) + (\bar{A}BC + \bar{A}\bar{B}C)$$

$$= A\bar{B} + B\bar{C} + \bar{A}C$$

Figure 4–17 Gate circuits for Example 4–8



$$AB + A(B+C) + B(B+C) = A(B+B+C) + B(B+C)$$

$$= A(B+C) + B(B+C) = AB + AC + B + BC$$

$$= B(A+1) + AC + B(1+C)$$

$$= B + AC$$

4.6 Standard Forms of Boolean Expressions

- The sum-of-products (SOP) form （乘积之和）
  - A single overbar cannot extend over more than one variable;
  - More than one variable in a term can have an overbar
- The product-of-sums (POS) form （和之乘积）
  - A single overbar cannot extend over more than one variable
  - More than one variable in a term can have an overbar

Example:

$$A\overline{B}\,\overline{C}D + \overline{A}CD + AC$$

$$(A + B)(B + C + D)(A + C).$$

- Conversion of a General Expression to SOP Form
  - Applying the distributive law
- The standard SOP Form
  - **All the variables in the domain appear in each product term in the expression**

$$Y = A\overline{B}\,\overline{C}D + \overline{A}BCD + \overline{A}\,\overline{B}CD + ABCD$$

- Converting Product Terms to Standard SOP
  - Using $A + \overline{A} = 1$

$Example:$ Convert the following expression to the standard SOP form

$$Y = A\overline{B}\,\overline{C}D + \overline{A}CD + AC$$

$Solution:$

$$Y = A\overline{B}\,\overline{C}D + \overline{A}(B+\overline{B})\ CD + A(B+\overline{B})\ C$$

$$= A\overline{B}\,\overline{C}D + \overline{A}BCD + \overline{A}\,\overline{B}CD + ABC(D+\overline{D}) + A\overline{B}C(D+\overline{D})$$

$$= A\overline{B}\,\overline{C}D + \overline{A}BCD + \overline{A}\,\overline{B}CD + ABCD + ABC\overline{D} + A\overline{B}CD + A\overline{B}C\overline{D}$$

$$= \sum_i m_i\,(i = 3,7,9,10,11,14,15)$$

Minterm Form for Standard SOP

e.g., 3 variables:

A	B	C	m_0 $\mathbf{\overline{A}\,\overline{B}\,\overline{C}}$	m_1 $\mathbf{\overline{A}\,\overline{B}C}$	m_2 $\mathbf{\overline{A}B\overline{C}}$	m_3 $\mathbf{\overline{A}BC}$	m_4 $\mathbf{A\overline{B}\,\overline{C}}$	m_5 $\mathbf{A\overline{B}C}$	m_6 $\mathbf{AB\overline{C}}$	m_7 $\mathbf{ABC}$
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

$$Example \quad A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

$$A\overline{B}C = A\overline{B}C\left(D + \overline{D}\right) = A\overline{B}CD + A\overline{B}C\overline{D}$$

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}\left(C + \overline{C}\right)\left(D + \overline{D}\right) = \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$$

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

$$= A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

- The Standard POS Form
  - **All the variables in the domain appear in each sum term in the expression.**
  - Converting a Sum Term to Standard POS
  - Use

$$A\overline{A} = 0 \qquad A + BC = (A+B)(A+C)$$

Example

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})$$

$$=(A+\overline{B}+C+D\overline{D})(A\overline{A}+\overline{B}+C+\overline{D})$$

$$=(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})$$

Converting standard SOP to standard POS

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC \qquad \text{(standard SOP form)}$$

$$= ? \qquad \text{(standard POS form)}$$

$$\therefore \overline{Y} = \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + AB\overline{C}$$

$$Y = \overline{\overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + AB\overline{C}} = \left(A + B + \overline{C}\right)\left(\overline{A} + B + C\right)\left(\overline{A} + \overline{B} + C\right)$$

4.7 Boolean Expressions and Truth Tables

Truth Table

Input			Output
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Boolean Expression

Y=?

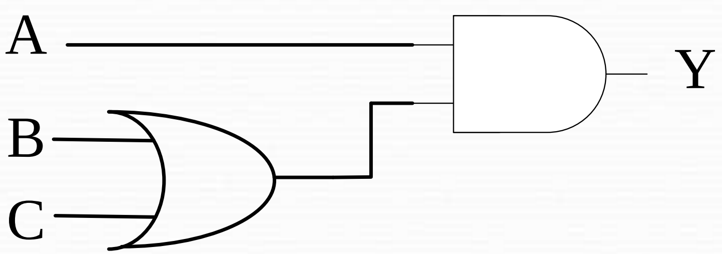
Truth Table

Input			Output
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Boolean Expression

$$Y=A(B+C)$$

Implementation



From Truth Table to Boolean Expression

Input			Output
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1 → $A\overline{B}C$
1	1	0	1 → $AB\overline{C}$
1	1	1	1 → ABC

$$Y = A\overline{B}C + AB\overline{C} + ABC$$
$$= A\overline{B}C + ABC + AB\overline{C} + ABC$$
$$= AC(\overline{B} + B) + AB(\overline{C} + C)$$
$$= AC + AB$$
$$= A(B + C)$$

SOP, POS expressions and truth table

Inputs			Output	P-Term	S-Term
A	B	C	X		
0	0	0	0		$(A+B+C)$
0	0	1	1	$\overline{A}\,\overline{B}C$	
0	1	0	0		$(A+\overline{B}+C)$
0	1	1	0		$(A+\overline{B}+\overline{C})$
1	0	0	1	$A\overline{B}\,\overline{C}$	
1	0	1	0		$(\overline{A}+B+\overline{C})$
1	1	0	0		$(\overline{A}+\overline{B}+C)$
1	1	1	1	ABC	

$$X = ?$$

SOP, POS expressions and truth table

$$X = \boxed{\overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC}$$

$$= \left(A+B+C\right)\left(A+\overline{B}+C\right)\left(A+\overline{B}+\overline{C}\right)\left(\overline{A}+B+\overline{C}\right)\left(\overline{A}+\overline{B}+C\right)$$

4.8 The Karnaugh Map

Example

Simplify the following Boolean expression

$$Y = AC + \overline{B}C + B\overline{D} + C\overline{D} + A\left(B + \overline{C}\right) + \overline{A}BC\overline{D} + A\overline{B}DE$$

$$Y = AC + \overline{B}C + B\overline{D} + C\overline{D} + A\left(B + \overline{C}\right) + \overline{A}BC\overline{D} + A\overline{B}DE$$

$$= AC + \overline{B}C + B\overline{D} + C\overline{D} + A\left(B + \overline{C}\right) + A\overline{B}DE$$

$$= AC + \overline{B}C + B\overline{D} + C\overline{D} + A\overline{\overline{B}C} + A\overline{B}DE$$

$$= AC + \overline{B}C + B\overline{D} + C\overline{D} + A + A\overline{B}DE$$

$$= A + \overline{B}C + B\overline{D} + C\overline{D}$$

$$= A + \overline{B}C + B\overline{D}$$

$$C\overline{D} + \overline{A}BC\overline{D} = C\overline{D}\left(1 + \overline{A}B\right) = C\overline{D}$$

$$A\left(B + \overline{C}\right) = A\overline{\overline{B}C}$$

$$\overline{B}C + A\overline{\overline{B}C} = \overline{B}C + A$$

$$AC + A + A\overline{B}DE = A$$

$$\overline{B}C + B\overline{D} + C\overline{D} = \overline{B}C + B\overline{D}$$

How do you think about this method?

- Many Boolean rules are used
- A little hard
- …
- Are there any other methods?

Karnaugh Map

- A systematic method for simplifying Boolean expressions
- Produce the simplest SOP expression
- Presents all of the possible values of input variables
  - An array of cells
  - Each cell represents a binary value of the input variables
  - Adjacency in position equivalents to adjacency in Boolean algebra

The Construction of Karnaugh Map

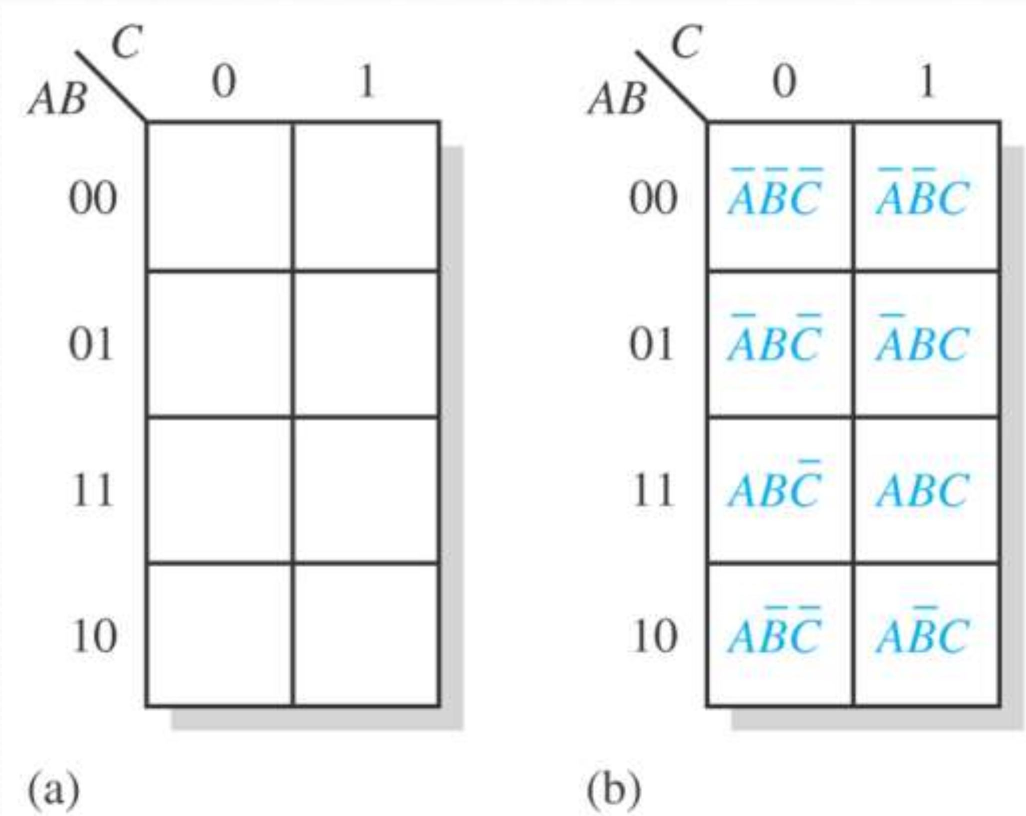


Figure 4–21 A ***3-variable Karnaugh map*** showing product terms.

The Construction of Karnaugh Map

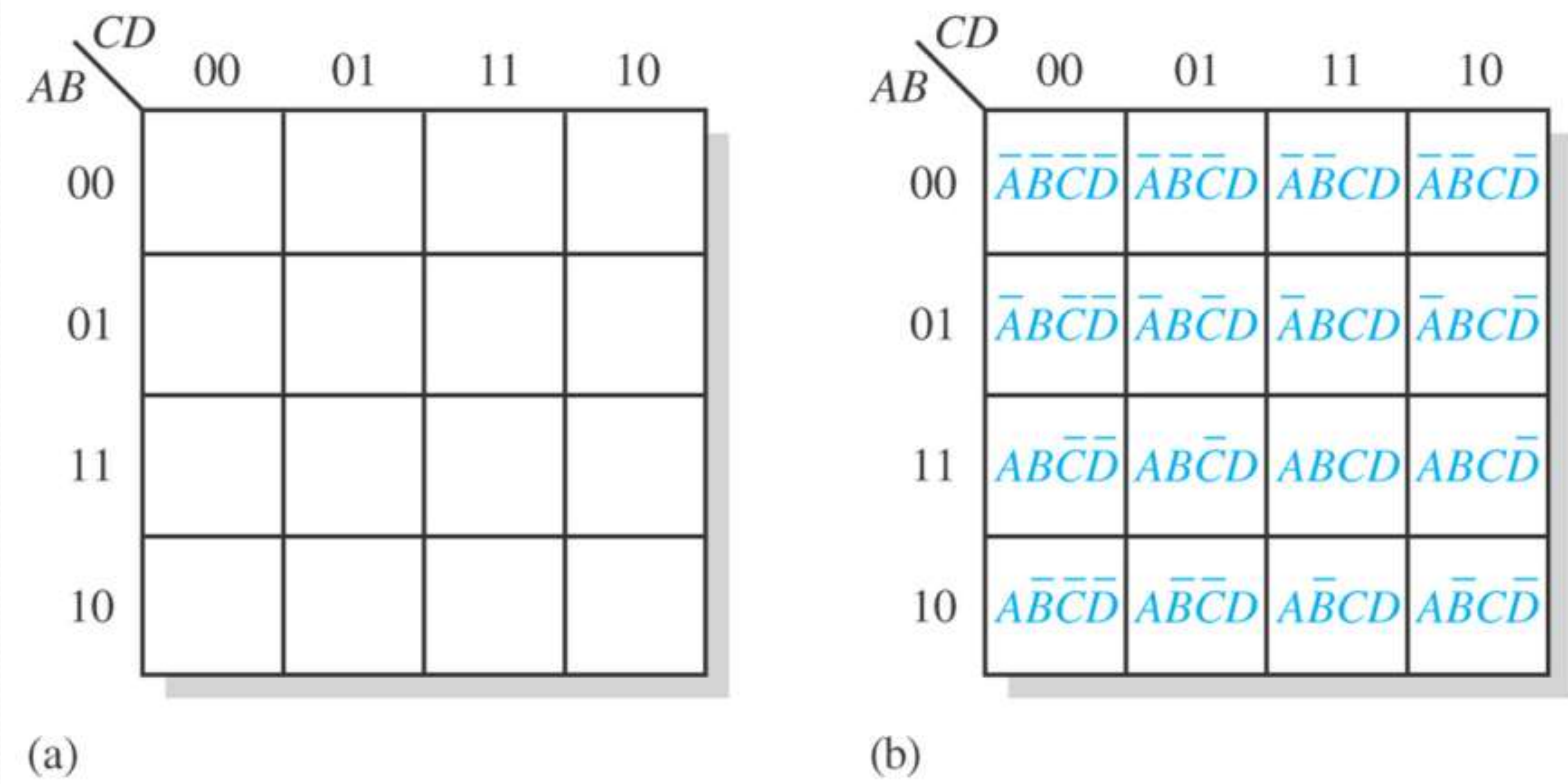


Figure 4–22 A ***4-variable Karnaugh*** map.

Cell Adjacency

- Adjacency (in logic): a single-variable change
  - Cells that differ by only one variable are adjacency
  - Cells that differ by more than one variable are not adjacency
- Adjacency (in location)
  - Cells locate next to others

Cell Adjacency

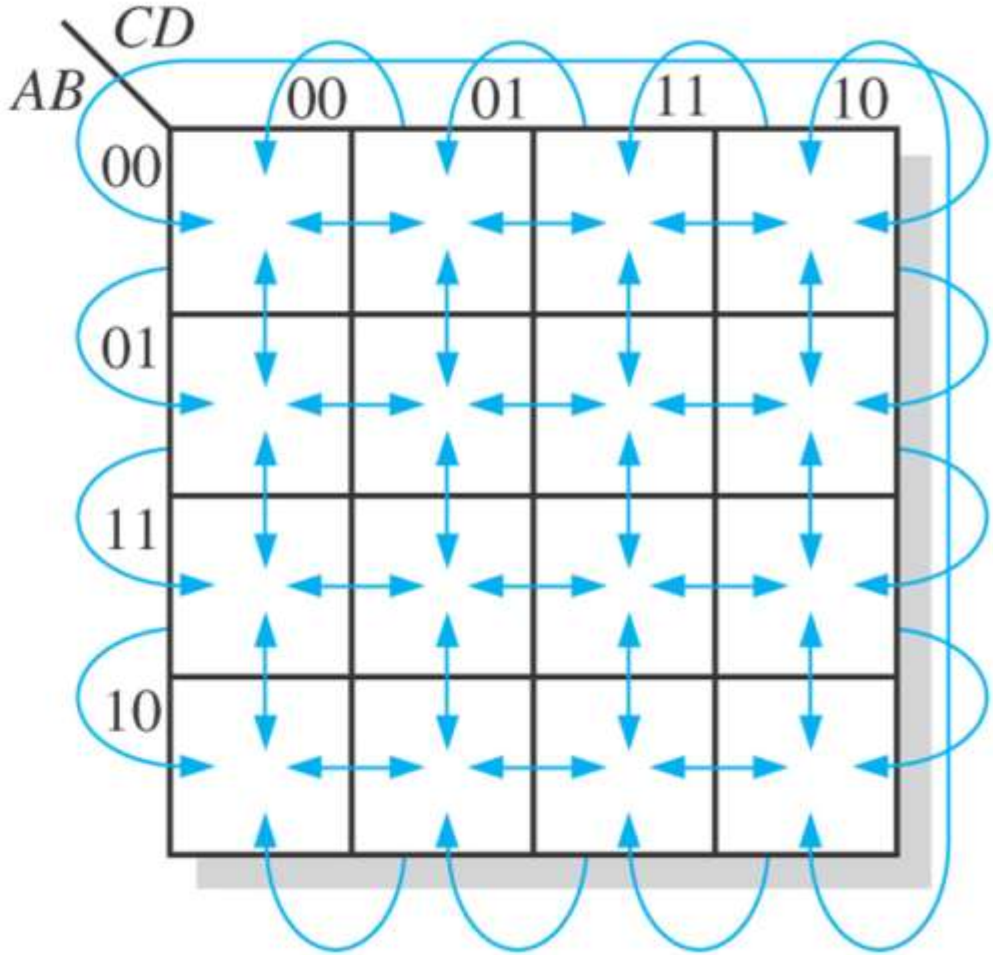


Figure 4–23 Adjacent cells on a Karnaugh map are those that differ by only one variable. Arrows point between adjacent cells.

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

(a)

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

(b)

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

(c)

AB \ CDE	000	001	011	010	110	111	101	100
00	m_0	m_1	m_3	m_2	m_6	m_7	m_5	m_4
01	m_8	m_9	m_{11}	m_{10}	m_{14}	m_{15}	m_{13}	m_{12}
11	m_{24}	m_{25}	m_{27}	m_{26}	m_{30}	m_{31}	m_{29}	m_{28}
10	m_{16}	m_{17}	m_{19}	m_{18}	m_{22}	m_{23}	m_{21}	m_{20}

(d)

5 – variable

$2^5 = 32$ cells

DE \ ABC	000	001	011	010	110	111	101	100
00	0	4	12	8	24	28	20	16
01	1	5	13	9	25	29	21	17
11	3	7	1	11	27	31	23	19
10	2	6	14	10	26	30	22	18

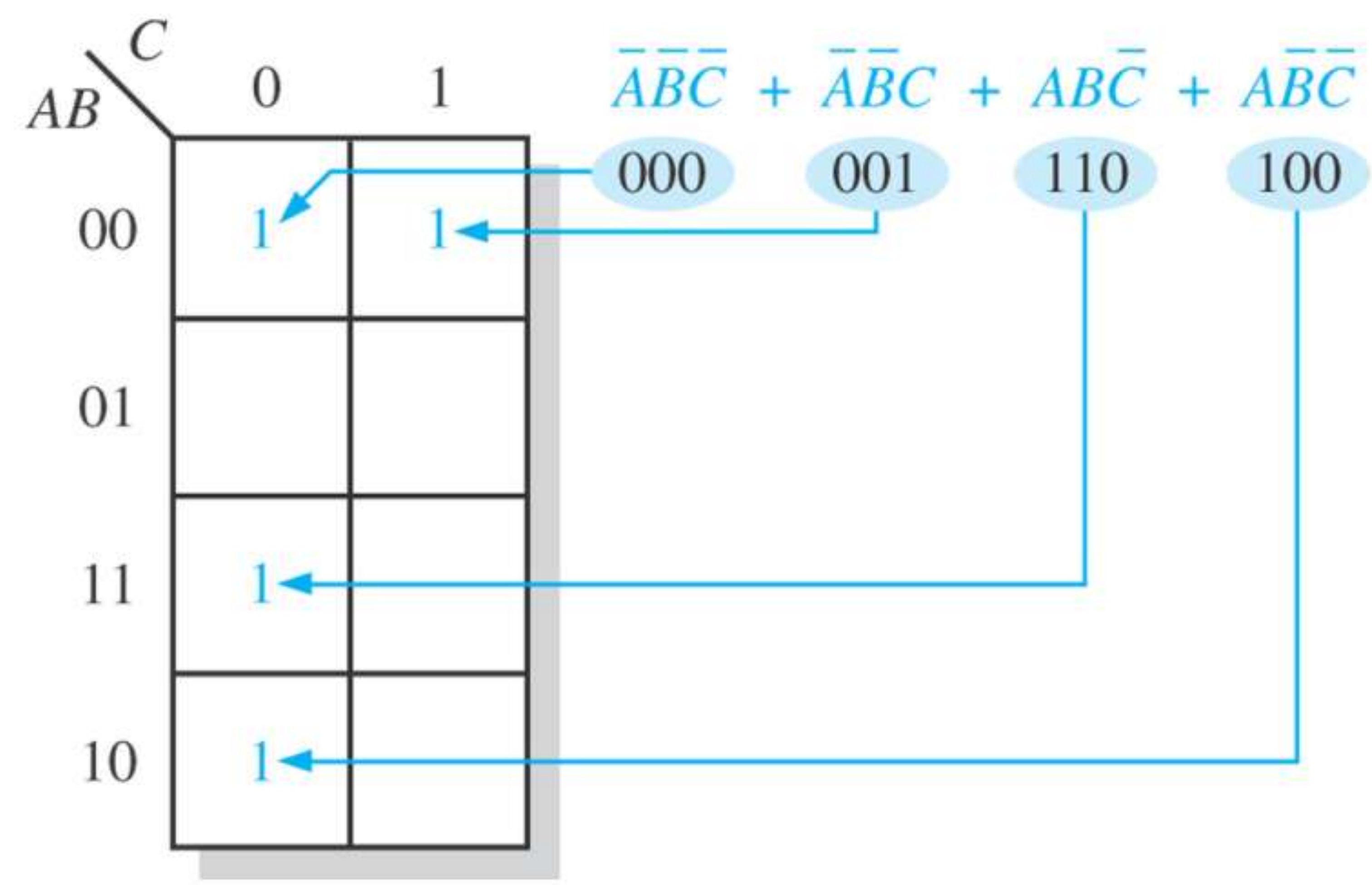
The adjacent cells include the cell located in symmetric place.（相邻格包括对称位置）

8： 12, 9, 24, 0, 10

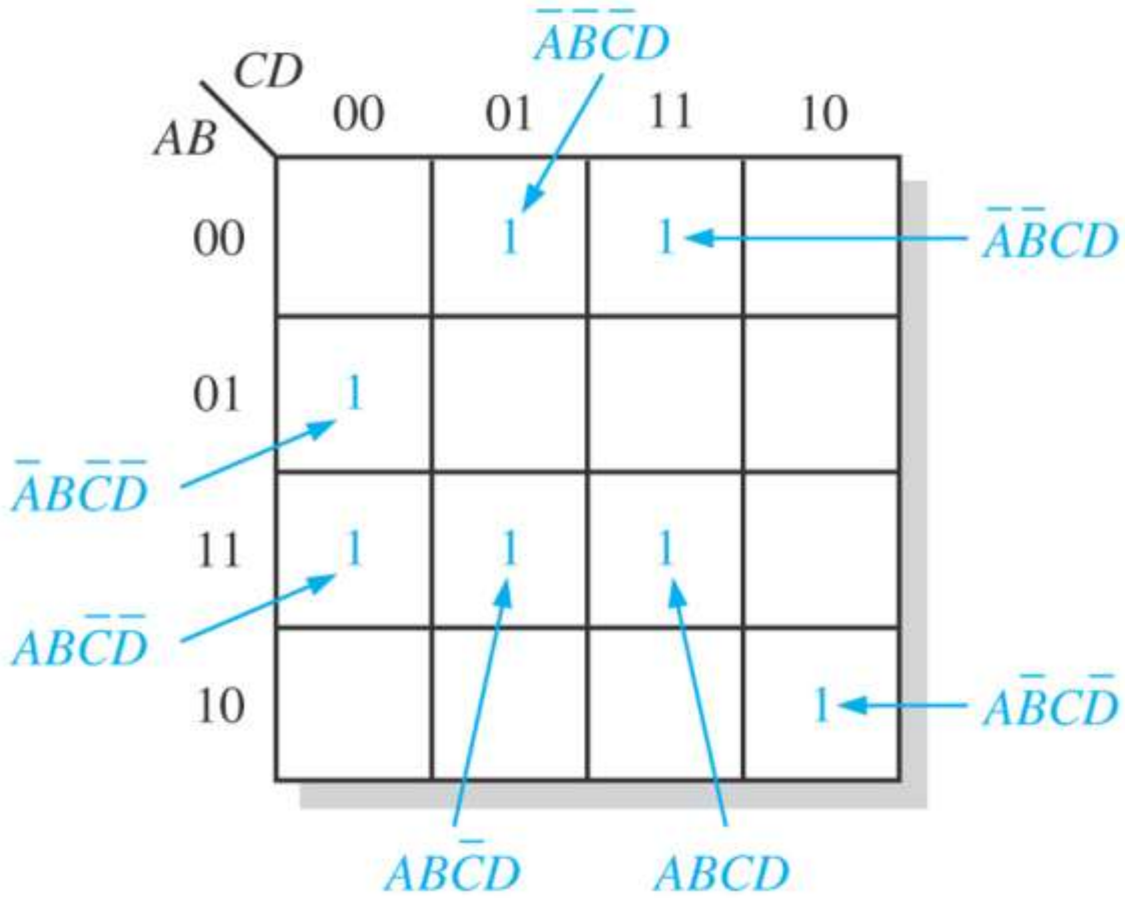
4.9 Karnaugh Map SOP Minization

- Mapping a SOP Expression
- Karnaugh Map Simplification of SOP Expressions

Example: Mapping a standard SOP expression



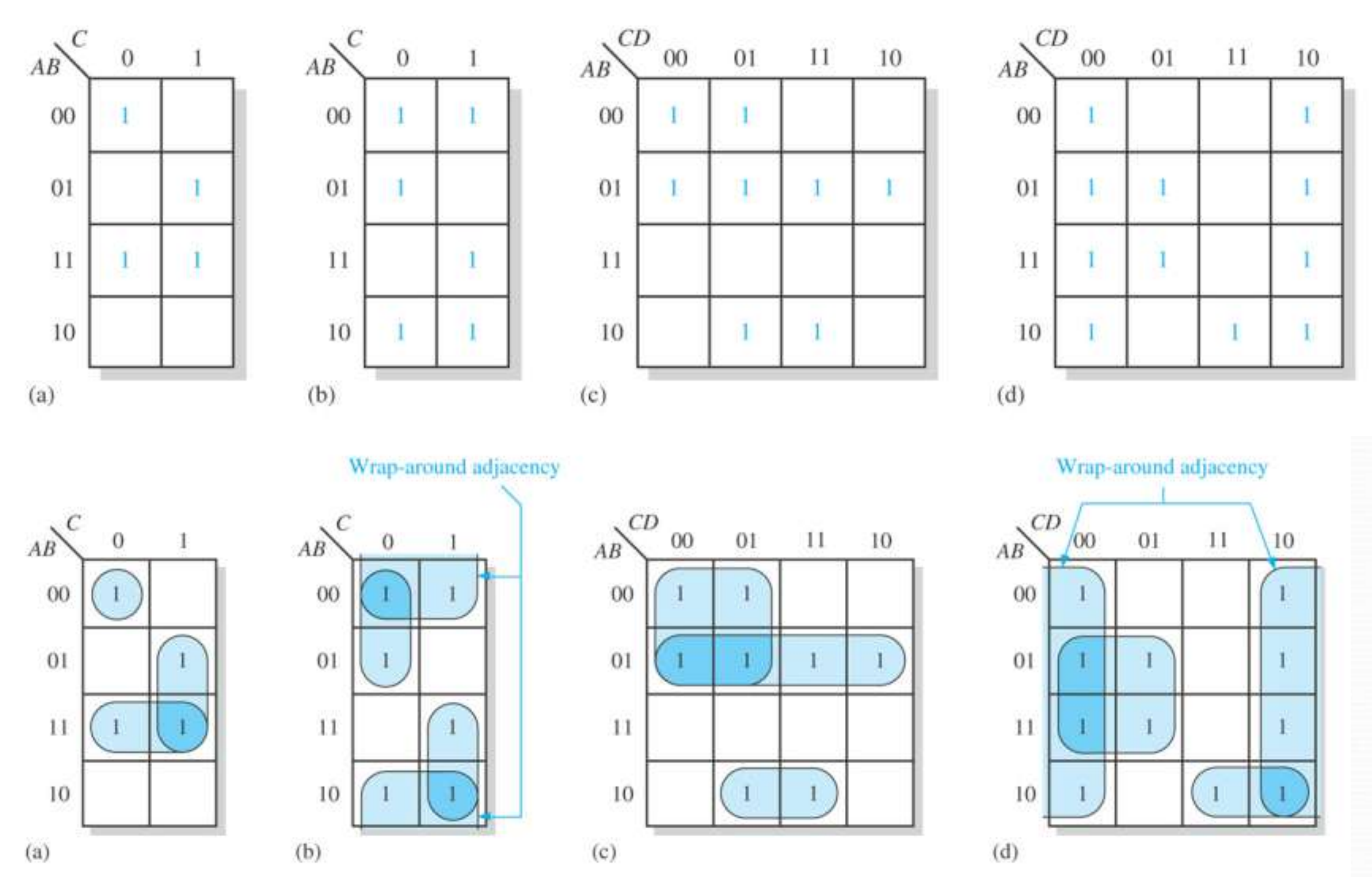
Another mapping example



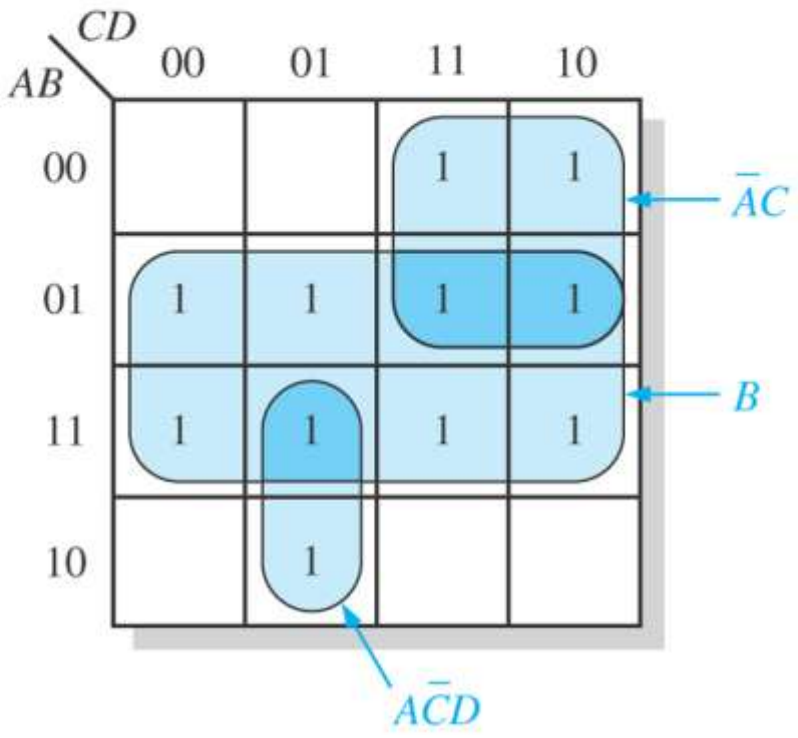
Karnaugh Map Simplification of SOP Expressions

- Group the 1s
  - Maximize the size of the groups
  - Minimize the number of groups
- Rules
  - A group must contain 2^n cells
  - Each cell must be adjacent to one or more cells in that group
  - Include 1s as much as possible
  - Each 1 on the map must be included at least one group
  - Cell with 1 can be included into more than one group

Example: Group the 1s in each of the Karnaugh maps in the following fi

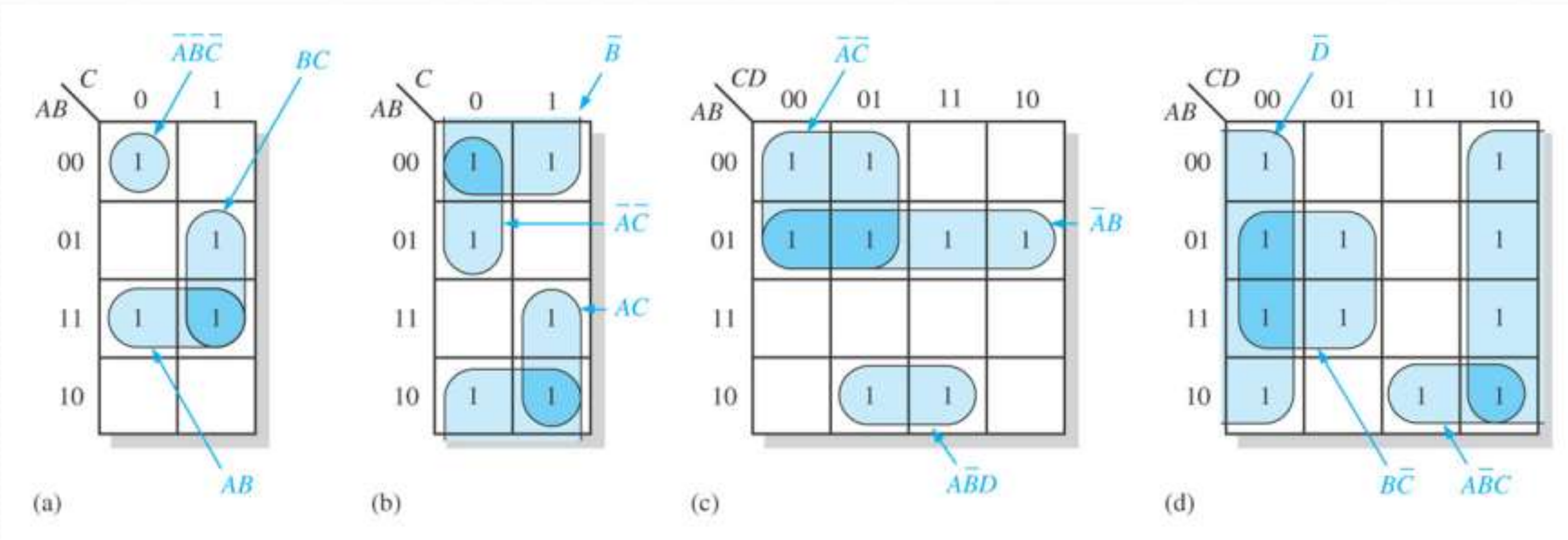


Example: Determine the product terms for the Karnaugh map in the following figure and write the resulting minimum SOP expression.



$$B + \overline{A}C + A\overline{C}D$$

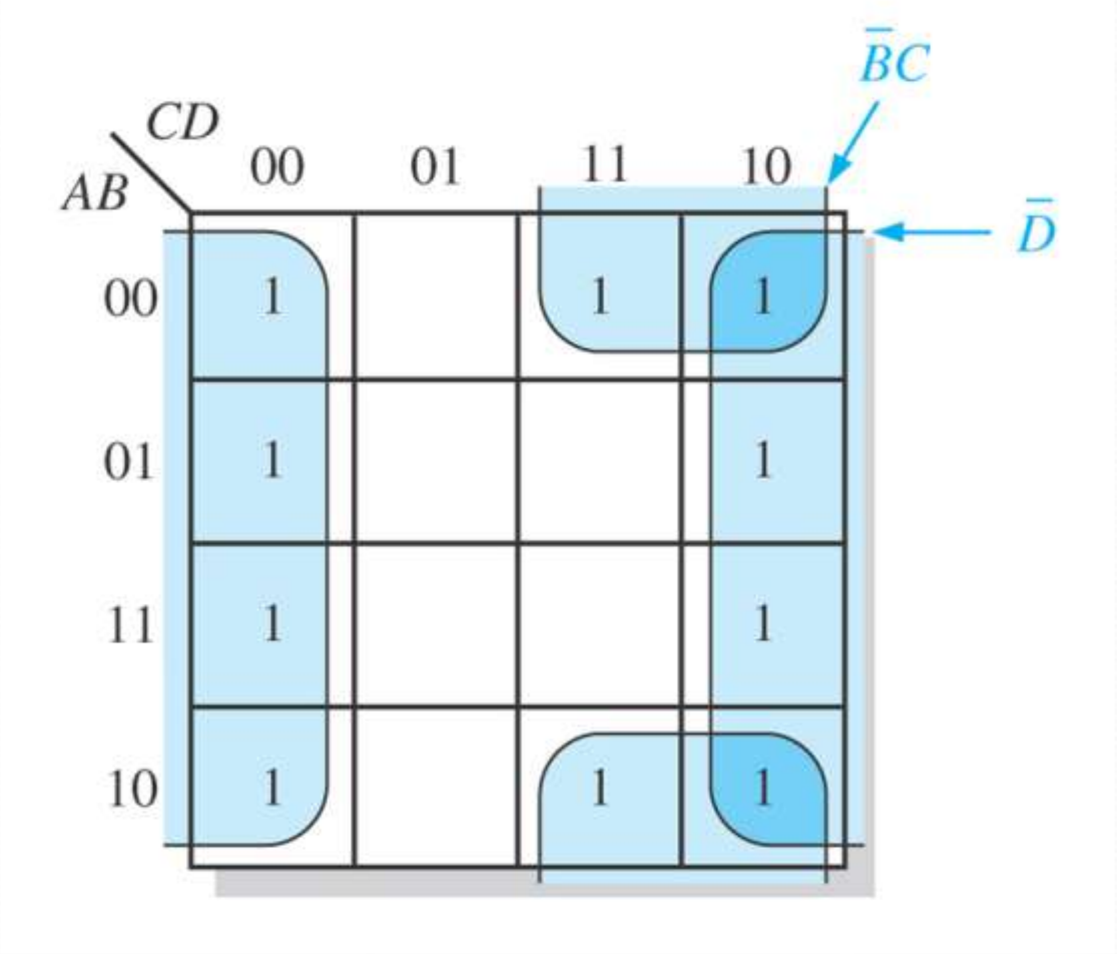
Example: Determine the product terms for the Karnaugh map in the following figure and write the resulting minimum SOP expression.



$$(a)\, AB + BC + \overline{A}\,\overline{B}\,\overline{C} \qquad (b)\, \overline{B} + \overline{A}\,\overline{C} + AC$$

$$(c)\, \overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D \qquad (d)\, \overline{D} + A\overline{B}C + B\overline{C}$$

Example: Determine the product terms for the Karnaugh map in the following figure and write the resulting minimum SOP expression.

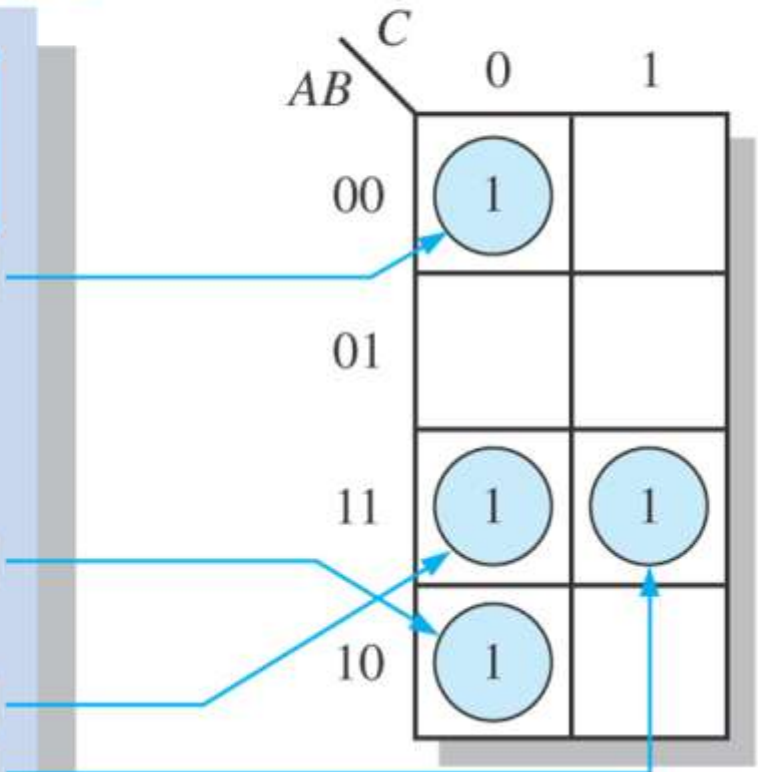


$$\overline{D} + \overline{B}C$$

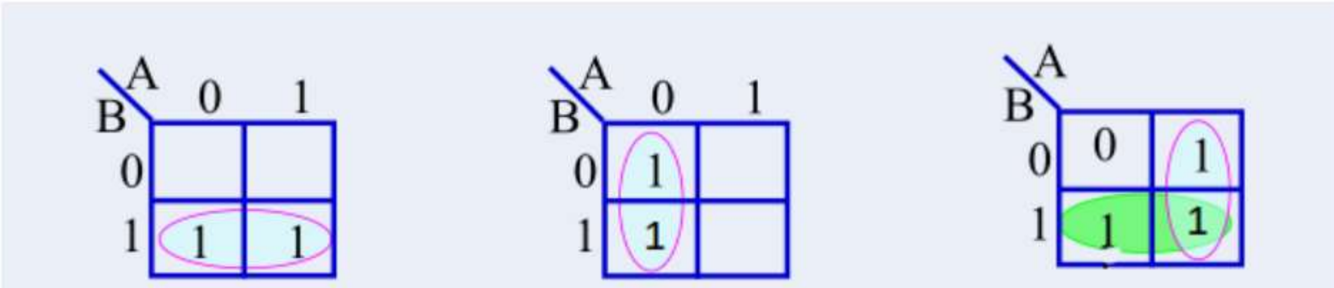
Figure 4–35 Example of mapping directly from a truth table to a Karnaugh map.

$$X = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

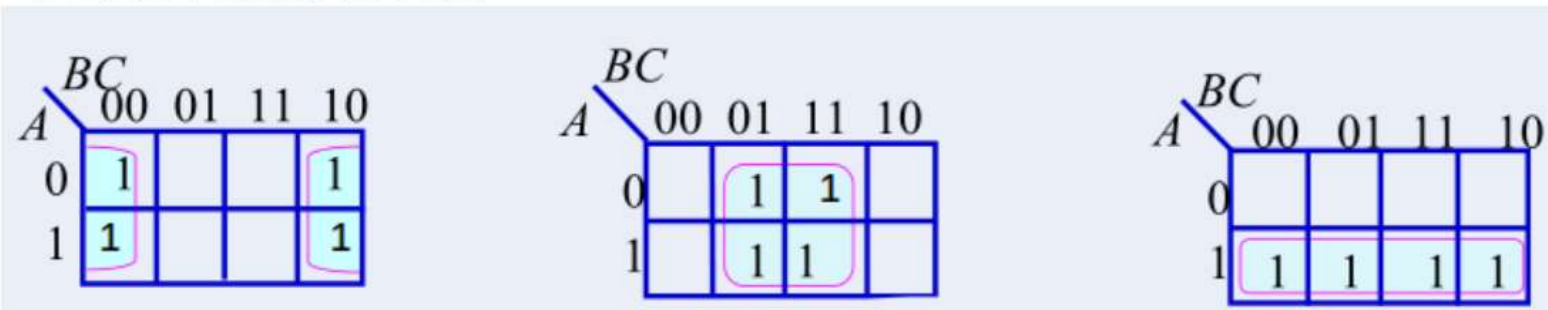
Inputs A	B	C	Output X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



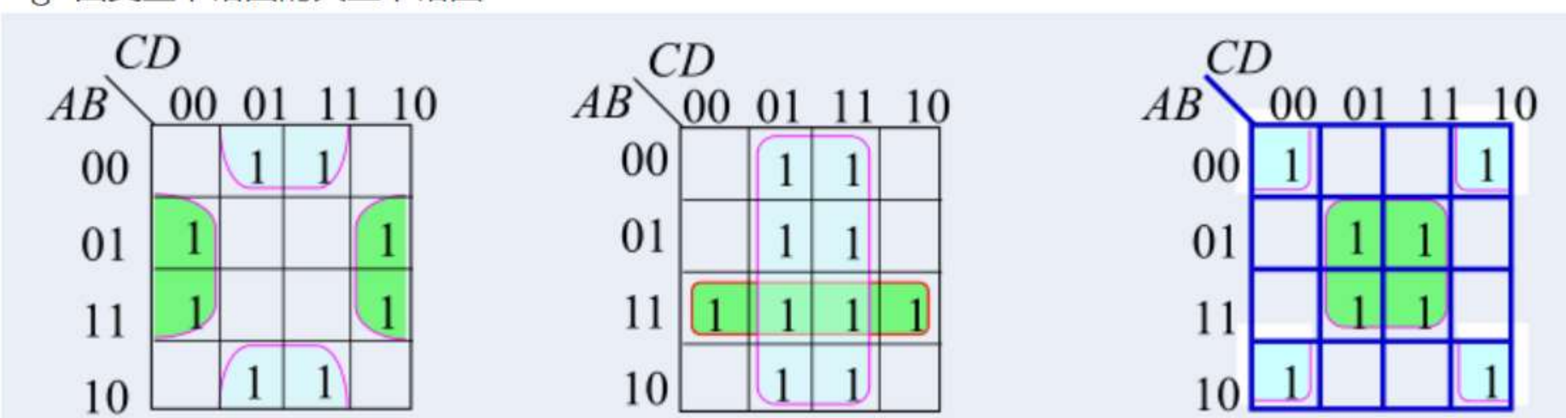
e.g. 二变量卡诺图的典型卡诺圈



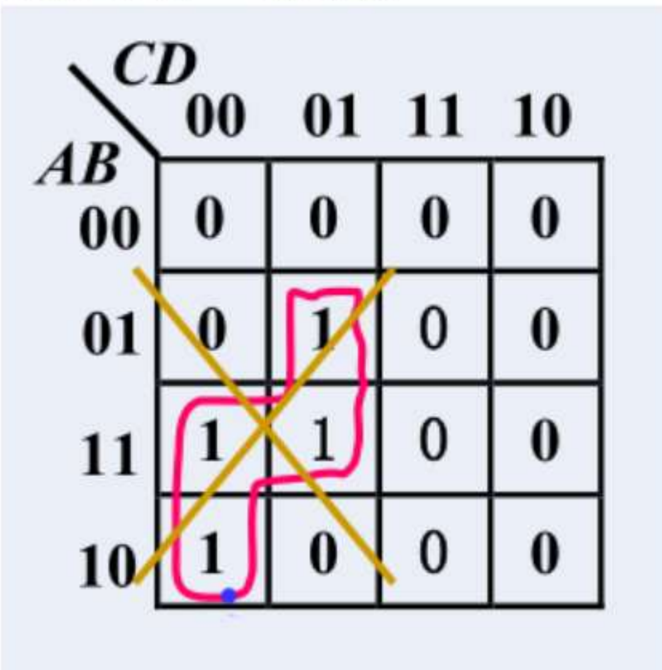
e.g. 三变量卡诺图的典型卡诺圈



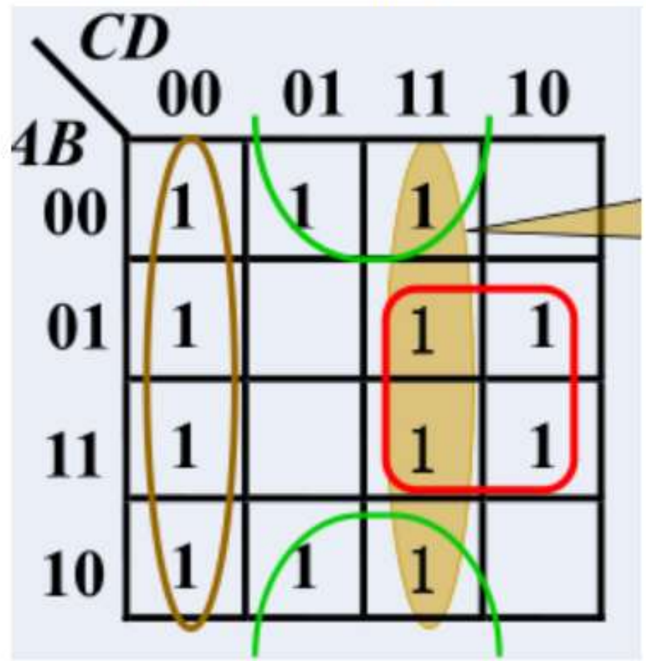
e.g. 四变量卡诺图的典型卡诺圈



无效圈示例1：不是矩形



无效圈示例2：没有新变量，无效圈



Example

Simplify the following Boolean expression

$$Y = AC + \overline{B}C + B\overline{D} + C\overline{D} + A\left(B + \overline{C}\right) + \overline{A}BC\overline{D} + A\overline{B}DE$$

$$Y = AC + \overline{B}C + B\overline{D} + C\overline{D} + A\left(B + \overline{C}\right) + \overline{A}BC\overline{D} + A\overline{B}DE$$

AC	$\overline{B}C$	$B\overline{D}$	$C\overline{D}$	$A\left(B + \overline{C}\right)$		$\overline{A}BC\overline{D}$	$A\overline{B}DE$
10100	00100	01000	00100	11000	10000	01100	10011
10101	00101	01001	00101	11001	10001	01101	10111
10110	00110	01100	01100	11010	10010		
10111	00111	01101	01101	11011	10011		
11100	10100	11000	10100	11100	11000		
11101	10101	11001	10101	11101	11001		
11110	10110	11100	11100	11110	11010		
11111	10111	11101	11101	11111	11011		

$$Y = AC + \overline{B}C + B\overline{D} + C\overline{D} + A\left(B + \overline{C}\right) + \overline{A}BC\overline{D} + A\overline{B}DE$$

AB \ CDE	000	001	011	010	110	111	101	100
00					1	1	1	1
01	1	1					1	1
11	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1

$$Y = A + \overline{B}C + B\overline{D}$$

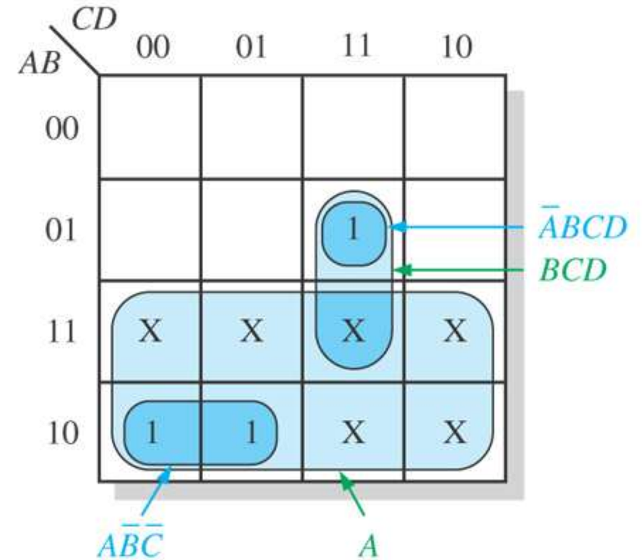
- Don’t care conditions （无关项）
  - Some input variable combinations are not allowed
    - These unallowed states will never occur in application
    - They can be treated as “don’t care” terms
  - “don’t care” terms either a 1 or 0 may be assigned to the output

Figure 4–36 Example of the use of ***"don't care" conditions*** to simplify an expression.

Inputs *ABCD*	Output *Y*
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	1
1 0 0 1	1
1 0 1 0	X
1 0 1 1	X
1 1 0 0	X
1 1 0 1	X
1 1 1 0	X
1 1 1 1	X

Don't cares

(a) Truth table



(b) Without "don't cares" $Y = A\overline{B}\overline{C} + \overline{A}BCD$
With "don't cares" $Y = A + BCD$

Summary

- Boolean Algebra
  - Variable
  - Operation
  - Laws and rules
- Simplification of Boolean expression
- SOP form and POS form
- Karnaugh maps

HW

- 11th， Page 157~159
  - 21
  - 33
  - 44
  - 45
- 10th， Page 142~144
  - 21
  - 33
  - 44
  - 45