Question 1:

HW1:

Question 1: $Pr(a_1,\dots,a_n \mid \beta) = Pr(a_1 \mid a_2,\dots,a_n,\beta)\, Pr(a_2 \mid a_3 \dots a_n \beta) \dots Pr(a_n \mid$

1. $Pr(a_1 \dots a_n \mid \beta) = \underline{Pr(a_1,\dots,a_n,\beta)} / Pr(\beta)$

Proof by Bayes Conditioning
$Pr(a \mid \beta) = Pr(a \wedge \beta) / Pr(\beta)$

2. $Pr(a_1,\dots,a_n,\beta) = Pr(a_1 \mid a_2,\dots,a_n,\beta)\, Pr(a_2 \mid a_3,\dots,a_n,\beta) \dots Pr(a_n \mid \beta)\, Pr(\beta)$

Proof By Chain rule.

3. $= \dfrac{Pr(a_1 \mid a_2,\dots,a_n,\beta) \dots Pr(a_n \mid \beta)\, \cancel{Pr(\beta)}}{\cancel{Pr(\beta)}}$

Plug back into 1. and cancle out $Pr(\beta)$

Therefore:

$\therefore \boxed{Pr(a_1,\dots a_n \mid \beta) = Pr(a_1 \mid a_2,\dots,a_n,\beta)\, Pr(a_2 \mid a_3,\dots,a_n,\beta) \dots Pr(a_n \mid \beta).}$

Question 2:

Pr(Oil) = 0.5 Pr(Natural Gas) = 0.2 Pr(Neither) = 0.3

Pr(Pos | Oil) = 0.9 Pr(Pos | Natural gas) = 0.3 Pr(Pos | Neither) = 0.1

Pr(Pos) = Pr(Pos | Oil) * Pr(Oil) + Pr(Pos | ¬Oil) * Pr(¬Oil)
= Pr(Pos | Oil) * Pr(Oil) + Pr(Pos | Natural Gas) * Pr(Natural Gas) + Pr(Pos | Neither) * Pr(Neither)

→ Pr(Pos) = (0.9) * (0.5) + (0.3) * (0.2) + (0.1) * (0.3) = 0.54

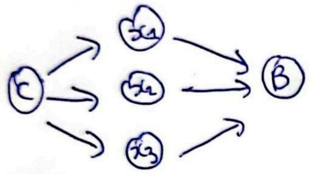
Bayes Rule: P(Oil | Pos) = (Pr(Pos | Oil) * Pr(Oil)) / Pr(Pos)

→ P(Oil | Pos) = (0.9 * 0.5) / 0.54 = 0.833

When the test is positive, the probability that oil is present is **0.833**.

Question 3:

Bayesian Network:



(c)
Coins: a, b, c

Outcomes: x_1, x_2, x_3 of either Heads or Tails

(B) Bell: on or off

CPT:

1\.

Coin (c)	Pr(c)
a	1/3
b	1/3
c	1/3

Probability of drawing a coin

2\.

Coin (c)	Pr(H|c)	Pr(T|c)
a	0.2	0.8
b	0.4	0.6
c	0.8	0.2

Probability of each flip given C.

Probability of bell ringing given x_1, x_2, x_3 →

3\.

x_1	x_2	x_3	Pr(on | $x_1 x_2 x_3$)	Pr(off | $x_1 x_2 x_3$)
H	H	H	1	0
H	H	T	0	1
H	T	H	0	1
T	T	T	1	0
T	H	H	0	1
T	H	T	0	1

Question 4:

Question 4:

a. I(A, ∅, BE) I(D, AB, CE) I(G, F, ABCDEH)

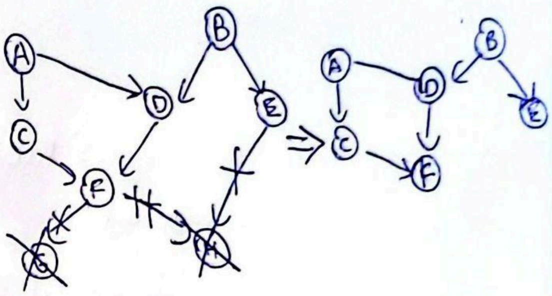
I(B, ∅, AC) I(E, B, ACDFG) I(H, EF, ABCDG)

I(C, A, BDE) I(F, CD, ABE)

b. d_sep(A, F, E)
(x z Y)

1. remove leaf node not in X ∪ Y ∪ Z
2. Delete edges outgoing from Z
3. Check if X is disconnected from Y
   if yes then dsep

resulting graph:



False, A & E are not disconnected.

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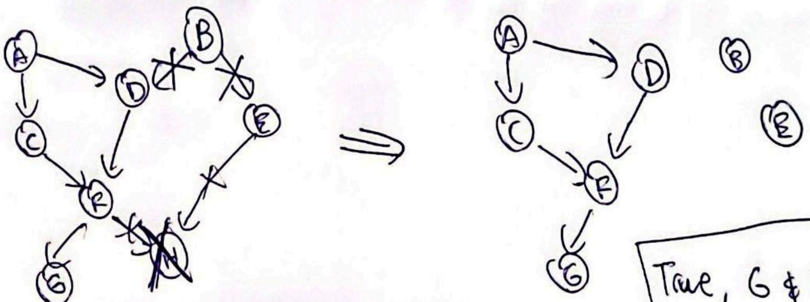
d_sep(G, B, E)
(x z Y)



True, G & E are disconnected

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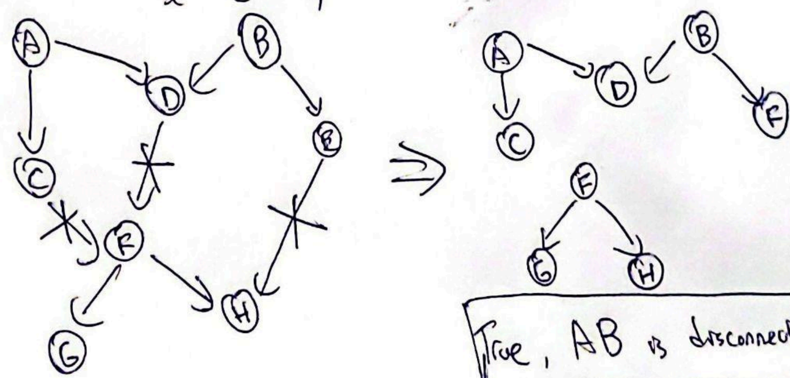
d_sep(AB, CDE, GH)
(x z Y)



True, AB is disconnected from GH

c. $Pr(a,b,c,d,e,f,g,h) =$

$Pr(a) = Pr(a)$

$Pr(b|a) = Pr(b)$

$Pr(c|a,b) = Pr(c|a)$

$Pr(d|a,b,c) = Pr(d|a,b)$

$\Rightarrow \boxed{Pr(a)\cdot Pr(b)\cdot Pr(c|a)\cdot Pr(d|a,b)\cdot Pr(e|b)\cdot Pr(f|c,d)\cdot Pr(g|f)\cdot Pr(h|e,f)}$

d. • $Pr(A,B) = Pr(A)Pr(B) \Rightarrow Pr(A=1)\,Pr(B=1) = (0.2)(0.7) = \boxed{0.14}$

Since A & B are independent.

• $Pr(E|A) = \sum_{b\in\{0,1\}} Pr(E=0|B=b)\cdot Pr(B=b) \Rightarrow Pr(E=0|B=0)\cdot Pr(B=0) + Pr(E=0|B=1)\cdot Pr(B=1)$

$\Rightarrow (0.1)\cdot(0.3) + (0.9)(0.7) = \boxed{0.66}$

Since E only depends on B, and not A, we use law of total Probability.

Question 5:

5.

a. models of α : $\omega_0, \omega_2, \omega_3$

b. $Pr(\alpha) = Pr(\omega_0) + Pr(\omega_2) + Pr(\omega_3)$

$= 0.3 + 0.1 + 0.4$

$= \boxed{0.8}$

c.

	A	B	Pr(A,B)	Pr(AB|α)
ω_0	T	T	0.3	0.3/0.8 = 0.375
ω_1	T	F	0.2	0
ω_2	F	T	0.1	0.1/0.8 = 0.125
ω_3	F	F	0.4	0.4/0.8 = 0.5

d. $Pr(A \Rightarrow \neg B \mid \alpha)$, $A \Rightarrow \neg B$: $\neg A \vee \neg B$, $A \Rightarrow B$: $\neg A \vee B$

	A	B	¬A∨¬B	¬A∨B
ω_0	T	T	F	T
ω_1	T	F	T	F
ω_2	F	T	T	T
ω_3	F	F	T	T

$$Pr(A \Rightarrow \neg B \mid \alpha) = \frac{Pr(\omega_2) + Pr(\omega_3)}{P(\alpha)}$$

$$= \frac{0.1 + 0.4}{0.8} = \boxed{0.625}$$