Question 1:

a. **P ⇒ ¬Q, Q ⇒ ¬P**

P	Q	¬P	¬Q	P ⇒ ¬Q	Q ⇒ ¬P
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

It is equivalent

b. **P ⇔ ¬Q, ((P ∧ ¬Q) ∨ (¬P ∧ Q))**

P	Q	¬P	¬Q	P ⇔ ¬Q	((P ∧ ¬Q) ∨ (¬P ∧ Q))
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

It is equivalent

Question 2:

a. **(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)**

Smoke	Fire	Smoke ⇒ Fire	¬Smoke ⇒ ¬Fire	(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Neither

b. (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

Smoke	Heat	Fire	Smoke ⇒ Fire	Smoke v Heat	(Smoke v Heat) ⇒ Fire	(Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
T	T	T	T	T	T	T
T	T	F	F	T	F	T
T	F	T	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Neither

c. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

S	H	F	S ∧ H	(S ∧ H) ⇒ F)	(S ⇒ F)	(H ⇒ F)	(S ⇒ F) ∨ (H ⇒ F)	Whole
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Valid

Question 3:

a. **Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).**

M: Mythical
I: Immortal
L: Mammal
H: Horned
G: Magical

1. $M \Rightarrow I$
2. $\neg M \Rightarrow (\neg I \land L)$
3. $(I \lor L) \Rightarrow H$
4. $H \Rightarrow G$

b. **Convert the knowledge base into CNF.**

1. $M \Rightarrow I$
   $\neg M \lor I$

2. $\neg M \Rightarrow (\neg I \land L)$
   $(M \lor \neg I)$
   $(M \lor L)$

3. $(I \lor L) \Rightarrow H$
   $(\neg I \lor H)$
   $(\neg L \lor H)$

4. $H \Rightarrow G$
   $\neg H \lor G$

c. **Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?**

1. **Unicorn is Mythical**
   $KB \models M$
   1. $\{\neg M, I\}$ (KB)
   2. $\{M, L\}$ (KB)
   3. $\{M, \neg I\}$ (KB)
   4. $\{\neg I, H\}$ (KB)
   5. $\{\neg L, H\}$ (KB)
   6. $\{\neg H, G\}$ (KB)

7. {¬M} (¬(M))
8. {L} (2, 7)
9. {¬I} (3, 7)
10. {H} (4, 9)
11. {G} (6, 10)

Since there is no contradiction, we cannot prove that the unicorn is mythical

2. **Unicorn is Magical**

KB ⊨ G

1. {¬M, I}	(KB)
2. {M, L}	(KB)
3. {M, ¬I}	(KB)
4. {¬I, H}	(KB)
5. {¬L, H}	(KB)
6. {¬H, G}	(KB)
7. {¬G}	(¬(G))
8. {¬H}	(6, 7)
9. {¬L}	(5, 8)
10. {M}	(2, 9)
11. {I}	(1, 10)
12. {H}	(4, 11)
13. {}	(8, 12)

Since there is contradiction, we can prove that the unicorn is Magical

3. **Unicorn is Horned**

KB ⊨ H

1. {¬M, I}	(KB)
2. {M, L}	(KB)
3. {M, ¬I}	(KB)
4. {¬I, H}	(KB)
5. {¬L, H}	(KB)
6. {¬H, G}	(KB)
7. {¬H}	(¬(H))
8. {¬L}	(5, 7)
9. {M}	(2, 8)
10. {I}	(1, 9)
11. {H}	(4, 10)
12. {}	(7, 11)

Since there is contradiction, we can prove that the unicorn is Horned

Question 4:

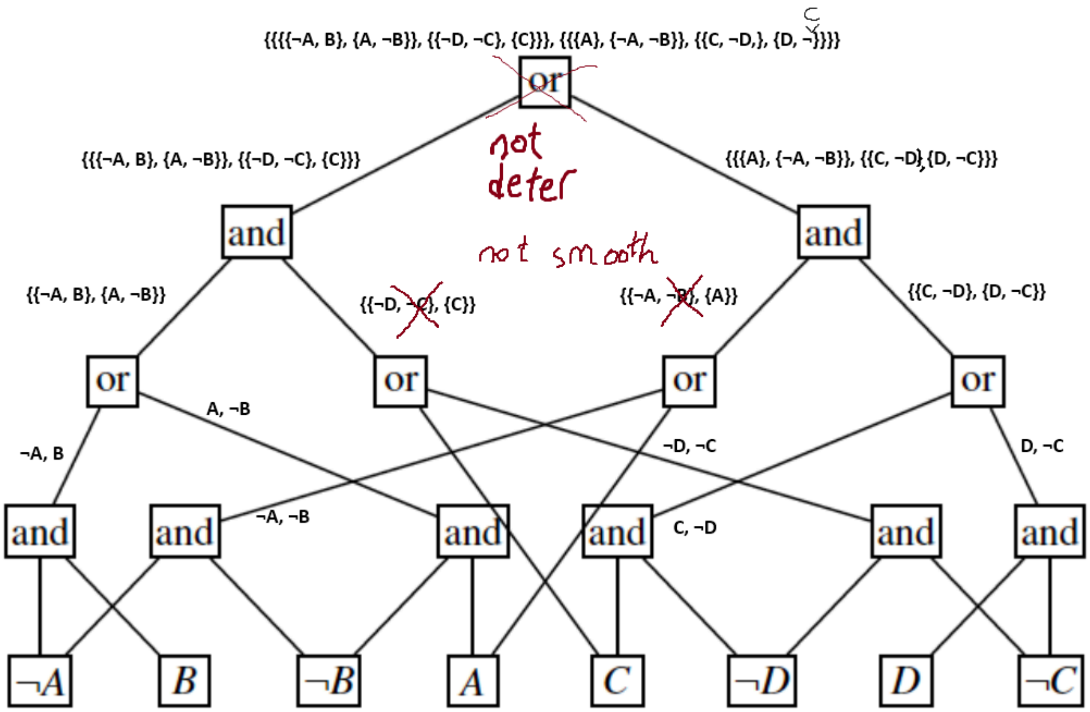

Figure 1

Figure one is decomposable, because variables don't overlap in each “and”. Not deterministic, because the top “or” are not mutually exclusive. And not smooth, because in the middle “or” that I highlighted, they do not share the same variable to be smooth.

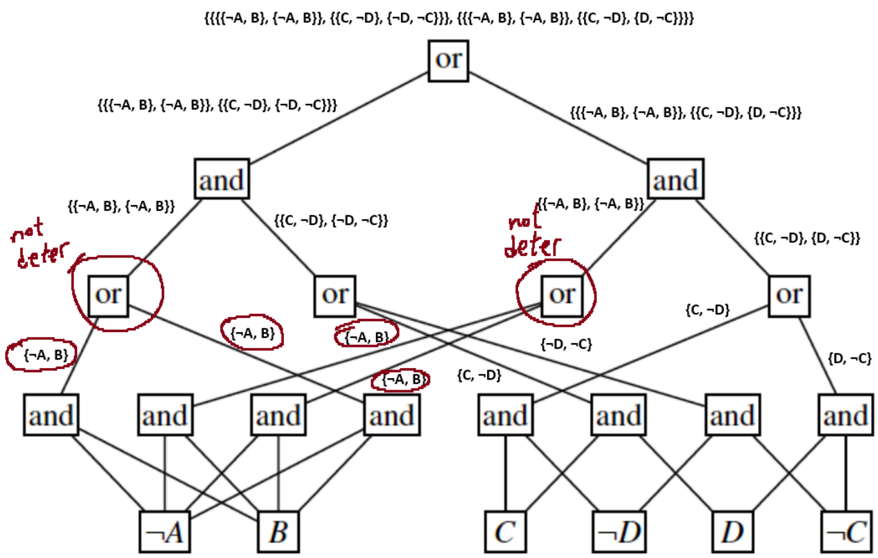

Figure 2

Figure two is decomposable, because variables don't overlap in each “and”. Smooth because both sides on “or” use the same variables. Not deterministic, because in the highlighted “or”, both values are set to true, where we have to have only one value set to true to be deterministic.

Question 5:

ω(A) = 0.1, ω(¬A) = 0.9, ω(B) = 0.3, ω(¬B) = 0.7, ω(C) = 0.5, ω(¬C) = 0.5, ω(D) = 0.7, ω(¬D) = 0.3.

a. **Compute the Weighted Model Count for formula (¬A∧B)∨(¬B∧A) by enumerating its models, computing their weights, then adding them up.**

A	**B**	**(¬A∧B)**	**(¬B∧A)**	**(¬A∧B)∨(¬B∧A)**
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	F	F	F

(¬A∧B)∨(¬B∧A) → ((0.9) * (0.3)) + ((0.7) * (0.1)) = 0.34

b. **What is the relation between the count on the root with the Weighted Model Count for the formula?**

Since the figure 3 is decomposable, deterministic, and smooth, the count on the root is equal to the WMC of the formula, which is 0.34

c. **Compute the Weighted Model Count for the formula associated with the decomposable, deterministic and smooth NNF circuit in Figure 4.**

$(((0.9 * 0.3) + (0.7 * 0.1)) * ((0.5 * 0.7) + (0.5 * 0.3))) + (((0.9 * 0.7) + (0.3 * 0.1)) * ((0.5 * 0.3) + (0.7 * 0.5))) = 0.5$