Problem Set 3

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Exercise 5.1

If $T = 1$, the maximization problem to choose consumption is:

$$\max_{c_1 \in [0, W_1]} u(c_1) \quad s.t. \quad W_2 = W_1 - c_1$$

Since the individual will not live in $T = 2$, he/she only cares about the amount of cake he/she can consume in period 1, so the optimal choice is to consume all he/she has in $T = 1$ since the utility function is increasing. Therefore, the optimal value of c_1 is $c_1 = W_1$. Equivalently, we can write the problem as:

$$\max_{W_2 \in [0, W_1]} u(W_1 - W_2)$$

W_2 is the amount of cake the individual saves in period 1. Since $u(c)$ is an increasing function, the optimal value of W_2 is 0. That is, the individual will not save any cake for period 2.

Exercise 5.2

If $T = 2$, the optimization problem is:

$$\max_{W_{t+1} \in [0, W_t]} u(W_1 - W_2) + \beta u(W_2 - W_3)$$

Therefore, the optimal amount of cake to save for period 3 is $W_3 = 0$ since $u(c)$ is increasing in c (so the lifetime utility is decreasing in W_3). For W_2, using the fact that $W_3 = 0$ at the optimal, we could write the condition for W_2:

$$-\frac{du(W_1 - W_2)}{dc} + \beta \frac{du(W_2)}{dc} = 0$$

Exercise 5.3

If $T=3$, the optimization problem is:

$$\max_{W_{t+1}\in[0,W_t]} u(W_1-W_2)+\beta u(W_2-W_3)+\beta^2 u(W_3-W_4)$$

Similar to the previous questions, the condition for W_4 is $W_4=0$. Then, we could write the condition for W_3:

$$-\beta\frac{du(W_2-W_3)}{dc}+\beta^2\frac{du(W_3)}{dc}=0$$

The condition for W_2 is:

$$-\frac{du(W_1-W_2)}{dc}+\beta\frac{du(W_2-W_3)}{dc}-\beta\frac{du(W_2-W_3)}{dc}\frac{dW_3}{dW_2}+\beta^2\frac{du(W_3)}{dc}\frac{dW_3}{dW_2}=0$$

Using the condition for W_3, we get:

$$-\frac{du(W_1-W_2)}{dc}+\beta\frac{du(W_2)}{dc}=0$$

If $W_1=1, \beta=0.9$, and $u(c_t)=ln(c_t)$, then we can get $W_4=0$,

$$-\frac{1}{W_2-W_3}+\frac{0.9}{W_3}=0$$

Or,

$$W_3=\frac{9W_2}{19}$$

And,

$$-\frac{1}{W_1-W_2}+\frac{0.9}{W_2-\frac{9W_2}{19}}=0$$

The solution of W_2 is:

$$W_2=\frac{171W_1}{271}=\frac{171}{271}\approx 0.631$$

Substituting the solution of W_2 into the condition for W_3, we get $W_3=\frac{1539}{5149}\approx 0.2989$

Therefore, $W_1=1, W_2=0.631, W_3=0.2989, W_4=0$

Using $c_t=W_t-W_{t+1}$, we can get $c_1=0.3690, c_2=0.3321, c_3=0.2989$

Both c and W decrease with time.

Exercise 5.4

The optimal solution for W_{T+1} is 0, so the problem can be written as:

$$V_{T-1}(W_{T-1}) \equiv \max_{W_T} u(W_{T-1} - W_T) + \beta u(W_T)$$

Differentiating it w.r.t. W_T and let $W_T = \psi_{T-1}(W_{T-1})$, we get:

$$-\frac{du(W_{T-1} - \psi_{T-1}(W_{T-1}))}{dc} + \beta\frac{du(\psi_{T-1}(W_{T-1}))}{dc} = 0$$

We can write V_{T-1} in terms of $\psi_{T-1}(W_{T-1})$:

$$V_{T-1}(W_{T-1}) = u(W_{T-1} - \psi_{T-1}(W_{T-1})) + \beta u(\psi_{T-1}(W_{T-1}))$$

Exercise 5.5

Since the individual will not live in period $T+1$, $\psi_T(\overline{\mathrm{W}}) = 0$

Then if $u(c) = ln(c)$, we have $V_T = ln(\overline{\mathrm{W}})$

Then, according to the answers in Exercise 5.4, we can get:

$$\psi_{T-1}(\overline{\mathrm{W}}) = \frac{\beta\overline{\mathrm{W}}}{\beta+1}$$

$$V_{T-1}(\overline{\mathrm{W}}) = \beta ln(\frac{\beta\overline{\mathrm{W}}}{\beta+1}) + ln\frac{\overline{\mathrm{W}}}{1+\beta}$$

Therefore, $V_{T-1}(\overline{\mathrm{W}})$ does not equal $V_T(\overline{\mathrm{W}})$, and $\psi_{T-1}(\overline{\mathrm{W}})$ does not equal to $\psi_T(\overline{\mathrm{W}})$

Exercise 5.6

The finite horizon Bellman equation for the function at $T-2$ is:

$$V_{T-2}(W_{T-2}) \equiv \max_{W_{T-1}} ln(W_{T-2} - W_{T-1}) + \beta V_T(W_{T-1})$$

Using the envelope theorem, we get:

$$-\frac{du(W_{T-2} - \psi_{T-2}(W_{T-2}))}{dc} + \beta\frac{du(\psi_{T-1}(W_{T-1}) - \frac{\beta\psi_{T-1}(W_{T-1})}{\beta+1})}{dc} = 0$$

Or,

$$-\frac{1}{W_{T-2} - \psi_{T-2}(W_{T-2})} + \frac{\beta}{(\psi_{T-1}(W_{T-1}) - \frac{\beta\psi_{T-1}(W_{T-1})}{\beta+1})} = 0$$

The analytical solution is:

$$W_{T-1} = \frac{(\beta^2 + \beta)W_{T-2}}{\beta^2 + \beta + 1}$$

And:

$$V_{T-2}(W_{T-2}) = ln\frac{W_{T-2}}{1+\beta+\beta^2} + \beta ln\frac{\beta W_{T-2}}{1+\beta+\beta^2} + \beta^2 ln\frac{\beta^2 W_{T-2}}{1+\beta+\beta^2}$$

Exercise 5.7

The analytical solutions for $\psi_{T-s}(W_{T-s})$ and $V_{T-s}(W_{T-s})$ are:

$$\psi_{T-s}(W_{T-s}) = \frac{[\sum_{i=1}^{s}\beta^i]W_{T-s}}{\sum_{i=0}^{s}\beta^i}$$

$$V_{T-s}(W_{T-s}) = \sum_{i=0}^{s}\beta^i ln[\frac{\beta^i W_{T-s}}{\sum_{i=0}^{s}\beta^i}]$$

We could take the limit $s \to \infty$:

$$\lim_{s\to\infty} \psi_{T-s}(W_{T-s}) = \beta W_{T-s}$$

$$\lim_{s\to\infty} V_{T-s}(W_{T-s}) = \frac{\beta ln\beta}{(1-\beta)^2} + \frac{1}{1-\beta}ln[(1-\beta)W_{T-s}]$$

Exercise 5.8

When the horizon is infinite, the Bellman equation is:

$$V_{T-s}(W_{T-s}) = \max_{W_{T-s+1}\in[0,W_{T-s}]} u(W_{T-s} - W_{T-s+1}) + \beta V_{T-s+1}(W_{T-s+1})$$

We can omit the subscript when T is infinite. That is:

$$V(W) = \max_{W' \in [0,W]} u(W - W') + \beta V(W')$$

Where W' is the amount of cake in the next period

Exercise 5.9

In [29]:

```
import numpy as np
w_max=1
w_min=0.01
n=100
w=np.linspace(w_min,w_max,n)
w
```

Out[29]:

```
array([0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1 , 0
.11,
       0.12, 0.13, 0.14, 0.15, 0.16, 0.17, 0.18, 0.19, 0.2 , 0.21, 0
.22,
       0.23, 0.24, 0.25, 0.26, 0.27, 0.28, 0.29, 0.3 , 0.31, 0.32, 0
.33,
       0.34, 0.35, 0.36, 0.37, 0.38, 0.39, 0.4 , 0.41, 0.42, 0.43, 0
.44,
       0.45, 0.46, 0.47, 0.48, 0.49, 0.5 , 0.51, 0.52, 0.53, 0.54, 0
.55,
       0.56, 0.57, 0.58, 0.59, 0.6 , 0.61, 0.62, 0.63, 0.64, 0.65, 0
.66,
       0.67, 0.68, 0.69, 0.7 , 0.71, 0.72, 0.73, 0.74, 0.75, 0.76, 0
.77,
       0.78, 0.79, 0.8 , 0.81, 0.82, 0.83, 0.84, 0.85, 0.86, 0.87, 0
.88,
       0.89, 0.9 , 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0
.99,
       1.  ])
```

Exercise 5.10

In [30]:

```
beta=0.9
v=np.zeros(100)
w1=np.linspace(w_min,w_max,n) # w1 is in [0.01,1], as the question mentions.
w_broad=np.tile(w.reshape(n,1),(1,n))
w1_broad=np.tile(w1,(n,1))
c_broad=w_broad-w1_broad
v_broad=np.tile(v,(n,1))
nega=(c_broad<=0)
c_broad[nega]=10**(-10)
v_broad[nega]=-10*(10)
v1=np.max(np.log(c_broad)+beta*v_broad,axis=1)
psi=[]
for i in range(n):
    psi.append(w1_broad[i][np.argmax(np.log(c_broad)+beta*v_broad,axis=1)[i]])
new_w=np.array(psi)
print('The resulting policy function is:\n', new_w)
print('The resulting value function is:\n', v1)
```

```
The resulting policy function is:
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0
.01
 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.
01
 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.
01
 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.
01
 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.
01
 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.
01
 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.
01
 0.01 0.01]
The resulting value function is:
 [-1.13025851e+02 -4.60517019e+00 -3.91202301e+00 -3.50655790e+00
 -3.21887582e+00 -2.99573227e+00 -2.81341072e+00 -2.65926004e+00
 -2.52572864e+00 -2.40794561e+00 -2.30258509e+00 -2.20727491e+00
 -2.12026354e+00 -2.04022083e+00 -1.96611286e+00 -1.89711998e+00
 -1.83258146e+00 -1.77195684e+00 -1.71479843e+00 -1.66073121e+00
 -1.60943791e+00 -1.56064775e+00 -1.51412773e+00 -1.46967597e+00
 -1.42711636e+00 -1.38629436e+00 -1.34707365e+00 -1.30933332e+00
 -1.27296568e+00 -1.23787436e+00 -1.20397280e+00 -1.17118298e+00
 -1.13943428e+00 -1.10866262e+00 -1.07880966e+00 -1.04982212e+00
 -1.02165125e+00 -9.94252273e-01 -9.67584026e-01 -9.41608540e-01
 -9.16290732e-01 -8.91598119e-01 -8.67500568e-01 -8.43970070e-01
 -8.20980552e-01 -7.98507696e-01 -7.76528789e-01 -7.55022584e-01
 -7.33969175e-01 -7.13349888e-01 -6.93147181e-01 -6.73344553e-01
 -6.53926467e-01 -6.34878272e-01 -6.16186139e-01 -5.97837001e-01
 -5.79818495e-01 -5.62118918e-01 -5.44727175e-01 -5.27632742e-01
 -5.10825624e-01 -4.94296322e-01 -4.78035801e-01 -4.62035460e-01
 -4.46287103e-01 -4.30782916e-01 -4.15515444e-01 -4.00477567e-01
 -3.85662481e-01 -3.71063681e-01 -3.56674944e-01 -3.42490309e-01
 -3.28504067e-01 -3.14710745e-01 -3.01105093e-01 -2.87682072e-01
 -2.74436846e-01 -2.61364764e-01 -2.48461359e-01 -2.35722334e-01
 -2.23143551e-01 -2.10721031e-01 -1.98450939e-01 -1.86329578e-01
 -1.74353387e-01 -1.62518929e-01 -1.50822890e-01 -1.39262067e-01
 -1.27833372e-01 -1.16533816e-01 -1.05360516e-01 -9.43106795e-02
 -8.33816089e-02 -7.25706928e-02 -6.18754037e-02 -5.12932944e-02
 -4.08219945e-02 -3.04592075e-02 -2.02027073e-02 -1.00503359e-02]
```

Exercise 5.11

In [31]:

```
def distance(v,v1):
    norm=np.sum((v1-v)*(v1-v))
    return norm
norm1=distance(v,v1)
print('The distance measure delta(t) is:',norm1)
```

```
The distance measure delta(t) is: 12953.769089096852
```

Exercise 5.12

In [32]:

```
v=v1.copy()
v_broad=np.tile(v,(n,1))
v_broad[nega]=-10*(10)
v1=np.max(np.log(c_broad)+beta*v_broad,axis=1)
psi=[]
for i in range(n):
    psi.append(w1_broad[i][np.argmax(np.log(c_broad)+beta*v_broad,axis=1)[i]])
new_w=np.array(psi)
norm2=distance(v,v1)
print('The resulting policy function is:\n', new_w)
print('The resulting value function is:\n', v1)
print('The distance measure delta(t-1) is:',norm2)
print('delta(t) - delta(t-1) =:',norm1-norm2)
```

```
The resulting policy function is:
 [0.01 0.01 0.02 0.02 0.03 0.03 0.04 0.04 0.05 0.05 0.06 0.06 0.07 0
.07
 0.08 0.08 0.09 0.09 0.1  0.1  0.1  0.11 0.11 0.12 0.12 0.13 0.13 0.
14
 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.18 0.18 0.19 0.19 0.19 0.2  0.
2
 0.21 0.21 0.22 0.22 0.23 0.23 0.24 0.24 0.25 0.25 0.26 0.26 0.27 0.
27
 0.28 0.28 0.28 0.29 0.29 0.3  0.3  0.31 0.31 0.32 0.32 0.33 0.33 0.
34
 0.34 0.35 0.35 0.36 0.36 0.37 0.37 0.37 0.38 0.38 0.39 0.39 0.4  0.
4
 0.41 0.41 0.42 0.42 0.43 0.43 0.44 0.44 0.45 0.45 0.46 0.46 0.46 0.
47
 0.47 0.48]
The resulting value function is:
 [-113.02585093 -106.32843602   -8.74982335   -8.05667617   -7.43284
371
   -7.0273786    -6.66246      -6.37477793   -6.11586407   -5.892720
52
   -5.69189132   -5.50956976   -5.34548036   -5.19132968   -5.052594
07
   -4.91906268   -4.79888442   -4.68110139   -4.57509666   -4.469736
```

```
14
   -4.37442596   -4.2796015    -4.19259012   -4.10681096   -4.026768
25
   -3.94845801   -3.87435004   -3.8023116    -3.73331873   -3.666621
56
   -3.60208303   -3.53998945   -3.47936483   -3.42128016   -3.364121
75
   -3.30955959   -3.25549236   -3.20404979   -3.1527565    -3.103966
33
   -3.05530583   -3.00878582   -2.96262185   -2.91817009   -2.874258
94
   -2.83169933   -2.78983132   -2.74900932   -2.70900273   -2.669782
02
   -2.63147837   -2.59373804   -2.55699824   -2.5206306    -2.485331
96
   -2.45024064   -2.41627434   -2.38237279   -2.34958297   -2.316852
09
   -2.28510339   -2.2535212    -2.22274954   -2.19223815   -2.162385
19
   -2.13287434   -2.10388681   -2.07531298   -2.0471421    -2.019447
61
   -1.99204864   -1.96518097   -1.93851272   -1.91242394   -1.886448
45
   -1.86109466   -1.83577685   -1.81108424   -1.78642517   -1.762327
61
   -1.73832619   -1.71479569   -1.69141776   -1.66842824   -1.645642
21
   -1.62316935   -1.600946     -1.5789671    -1.5572793    -1.535773
1
   -1.51459565   -1.49354224   -1.47285167   -1.45223238   -1.432006
81
   -1.41180411   -1.39200148   -1.37222046   -1.35280238   -1.333446
79]
The distance measure delta(t-1) is: 10863.950952565217
delta(t) - delta(t-1) =: 2089.818136531634
```

We could see that the distance measure decreases from T to $T-1$.

Exercise 5.13

In [33]:

```
v=v1
v_broad=np.tile(v,(n,1))
v_broad[nega]=-10*(10)
v1=np.max(np.log(c_broad)+beta*v_broad,axis=1)
psi=[]
for i in range(n):
    psi.append(w1_broad[i][np.argmax(np.log(c_broad)+beta*v_broad,axis=1)[i]])
new_w=np.array(psi)
norm3=np.sum((v1-v)*(v1-v))
print('The resulting policy function is:\n', new_w)
print('The resulting value function is:\n', v1)
print('The distance measure delta(t-2)=',norm3)
print('Therefore, we have: delta(t)=', norm1, 'delta(t-1)=', norm2, 'delta(t-2)=
', norm3 )
```

```
The resulting policy function is:
 [0.01 0.01 0.02 0.03 0.03 0.04 0.05 0.05 0.06 0.07 0.07 0.08 0.09 0
.09
 0.1  0.1  0.11 0.12 0.12 0.13 0.14 0.14 0.15 0.16 0.16 0.17 0.18 0.
18
 0.19 0.19 0.2  0.2  0.21 0.22 0.22 0.23 0.24 0.24 0.25 0.26 0.26 0.
27
 0.28 0.28 0.29 0.29 0.3  0.31 0.31 0.32 0.33 0.33 0.34 0.35 0.35 0.
36
 0.36 0.37 0.37 0.38 0.39 0.39 0.4  0.41 0.41 0.42 0.43 0.43 0.44 0.
45
 0.45 0.46 0.46 0.47 0.48 0.48 0.49 0.5  0.5  0.51 0.52 0.52 0.53 0.
53
 0.54 0.55 0.55 0.56 0.56 0.57 0.58 0.58 0.59 0.6  0.6  0.61 0.62 0.
62
 0.63 0.64]
The resulting value function is:
 [-113.02585093 -106.32843602 -100.30076261  -12.4800112   -11.78686
402
  -11.16303156  -10.60158234  -10.19611724   -9.83119864   -9.502771
9
   -9.21508983   -8.95617596   -8.72315349   -8.50000993   -8.299180
74
   -8.11685918   -7.9361129    -7.7720235    -7.61787282   -7.470192
36
   -7.33145675   -7.19792536   -7.07306331   -6.95288505   -6.835102
02
   -6.72694159   -6.62093686   -6.51557634   -6.42017208   -6.324861
9
   -6.23003744   -6.14302606   -6.0572469    -5.97190488   -5.891862
18
   -5.81355194   -5.73635069   -5.66224272   -5.59020428   -5.519725
07
   -5.45073219   -5.38403502   -5.31920043   -5.25466191   -5.192568
32
   -5.1319437    -5.07191624   -5.01383157   -4.95667316   -4.900788
```

```
93
   -4.84622677   -4.79215955   -4.73988335   -4.68844078   -4.637147
48
   -4.58804154   -4.53925138   -4.49059088   -4.44407086   -4.397772
55
   -4.35160858   -4.30715682   -4.26324567   -4.21945122   -4.176891
61
   -4.13502359   -4.09347602   -4.05265403   -4.01264744   -3.973127
41
   -3.9339067    -3.89560304   -3.85786272   -3.8201815    -3.783441
71
   -3.74707406   -3.71106814   -3.67576949   -3.64067817   -3.606204
89
   -3.57223859   -3.53833704   -3.50527122   -3.4724814    -3.439750
52
   -3.40798174   -3.37623305   -3.34465086   -3.3138792    -3.283309
53
   -3.25279814   -3.22294517   -3.19343433   -3.16397654   -3.134989
01
   -3.10641518   -3.07799121   -3.04982033   -3.02212584   -2.994665
58]
The distance measure delta(t-2)= 8977.756436435935
Therefore, we have: delta(t)= 12953.769089096852 delta(t-1)= 10863.9
50952565217 delta(t-2)= 8977.756436435935
```

We could see that the distance measure decreases from T to $T-2$.

Exercise 5.14

In [34]:

```
tole=10**(-9)
v1=np.zeros(n) # iterate from s=0, so I let v1 equal to 100 zeros.
s=0
norm=100
while norm>=tole:
    v=v1.copy()
    v_broad=np.tile(v,(n,1))
    v_broad[nega]=-10*(10)
    v1=np.max(np.log(c_broad)+beta*v_broad,axis=1)
    psi=[]
    for i in range(n):
        psi.append(w1_broad[i][np.argmax(np.log(c_broad)+beta*v_broad,axis=1)[i]
])
    new_w=np.array(psi)
    norm=np.sum((v1-v)*(v1-v))
    s+=1
    print(s,'th interation, the distance measure is:', norm)
```

```
1 th interation, the distance measure is: 12953.769089096852
2 th interation, the distance measure is: 10863.950952565217
3 th interation, the distance measure is: 8977.756436435935
4 th interation, the distance measure is: 7382.24901708053
5 th interation, the distance measure is: 6053.64947290758
6 th interation, the distance measure is: 4955.534353993658
7 th interation, the distance measure is: 4051.647563915583
8 th interation, the distance measure is: 3309.681757444844
9 th interation, the distance measure is: 2701.8629074114037
10 th interation, the distance measure is: 2204.5706301672863
11 th interation, the distance measure is: 1798.174632137887
12 th interation, the distance measure is: 1466.3178829746114
13 th interation, the distance measure is: 1195.575424435967
14 th interation, the distance measure is: 974.71746178734
15 th interation, the distance measure is: 794.737562830258
16 th interation, the distance measure is: 648.0805260065758
17 th interation, the distance measure is: 528.5903875840728
18 th interation, the distance measure is: 431.34527830206116
19 th interation, the distance measure is: 352.1861565489469
20 th interation, the distance measure is: 287.718266936896
21 th interation, the distance measure is: 235.21435239743894
22 th interation, the distance measure is: 192.54551696247194
23 th interation, the distance measure is: 157.8744733634087
24 th interation, the distance measure is: 129.01572545703957
25 th interation, the distance measure is: 104.97646943197154
26 th interation, the distance measure is: 85.06947581985516
27 th interation, the distance measure is: 68.59074781384093
28 th interation, the distance measure is: 55.05314639552278
29 th interation, the distance measure is: 43.95567883532289
30 th interation, the distance measure is: 34.949963414220754
31 th interation, the distance measure is: 27.680456819191267
32 th interation, the distance measure is: 21.894943989059666
33 th interation, the distance measure is: 17.342012162342982
34 th interation, the distance measure is: 13.838966463003064
35 th interation, the distance measure is: 11.158221225947582
36 th interation, the distance measure is: 9.237215110380395
37 th interation, the distance measure is: 7.97040076899211
38 th interation, the distance measure is: 6.739157995241672
39 th interation, the distance measure is: 5.653989273049574
40 th interation, the distance measure is: 4.779943278771013
41 th interation, the distance measure is: 3.9714681752727925
42 th interation, the distance measure is: 3.277364683773252
43 th interation, the distance measure is: 2.6813529330202153
44 th interation, the distance measure is: 2.1554343044038156
45 th interation, the distance measure is: 1.6951258345254605
46 th interation, the distance measure is: 1.2928654964914037
47 th interation, the distance measure is: 0.9436459038098374
48 th interation, the distance measure is: 0.6341970927127146
49 th interation, the distance measure is: 0.3645484356975179
50 th interation, the distance measure is: 0.12751446830056473
51 th interation, the distance measure is: 0.0011771192772362061
52 th interation, the distance measure is: 0.0
```

In [35]:

```
print('It takes', s, 'iterations')
```

```
It takes 52 iterations
```

In [36]:

```
print('The resulting value function is:\n', v1)
print('The resulting policy function is:\n', new_w)
```

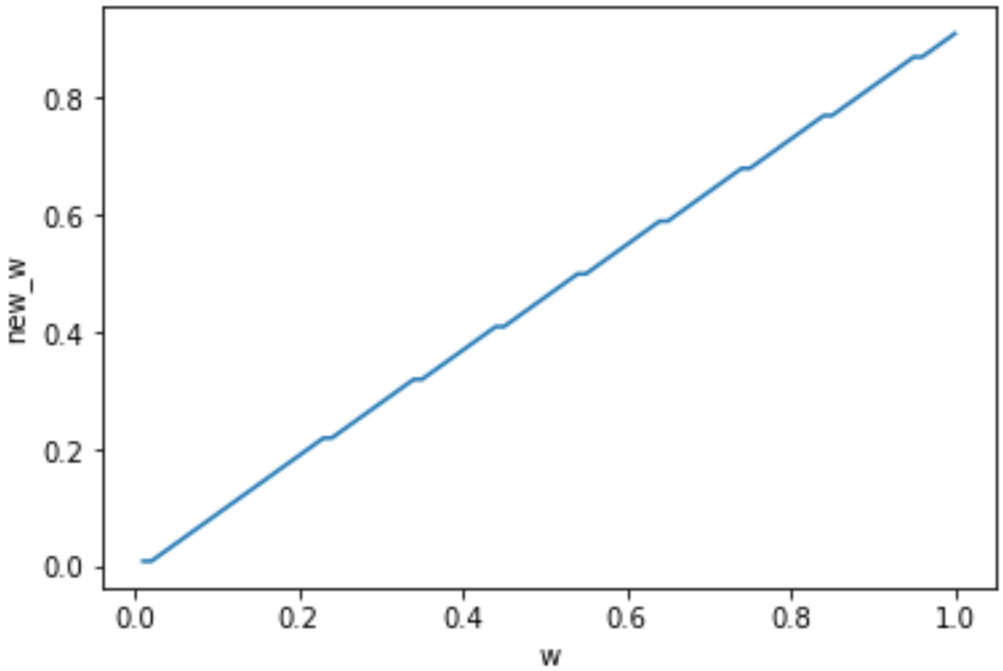
```
The resulting value function is:
 [-113.02585093 -106.32843602 -100.30076261  -94.87585653  -89.99344
106
  -85.59926714  -81.64451062  -78.08522974  -74.88187695  -71.998859
44
  -69.40414368  -67.0688995   -64.96717974  -63.07563195  -61.373238
94
  -59.84108523  -58.4621469   -57.22110239  -56.10416234  -55.098916
29
  -54.19419485  -53.37994555  -52.64712118  -51.953974    -51.294432
07
  -50.67059961  -50.07701187  -49.51556265  -48.98133369  -48.476029
39
  -47.99522332  -47.54044946  -47.107724    -46.69842752  -46.292962
41
  -45.90350949  -45.53514266  -45.17022406  -44.81971644  -44.488186
29
  -44.15975956  -43.84430269  -43.54592556  -43.2503415   -42.962659
43
  -42.67874825  -42.41020883  -42.14418317  -41.88526931  -41.629749
25
  -41.38806377  -41.14864068  -40.9156182   -40.68565015  -40.462506
6
  -40.24498967  -40.02950889  -39.81978865  -39.61281741  -39.411988
21
  -39.21622297  -39.02229027  -38.83354206  -38.64726794  -38.464946
38
  -38.28420011  -38.10801139  -37.93347196  -37.76359857  -37.595951
86
  -37.43186246  -37.26919081  -37.11062097  -36.95353548  -36.799384
8
  -36.64649875  -36.49561671  -36.34793625  -36.20153177  -36.058818
91
  -35.91744197  -35.77870636  -35.64110891  -35.50531508  -35.371783
69
  -35.23887127  -35.10710724  -34.97866566  -34.85142642  -34.726564
37
  -34.60272667  -34.48051222  -34.36033396  -34.24071279  -34.122125
16
  -34.00434212  -33.88874471  -33.77422938  -33.66185354  -33.550399
61]
The resulting policy function is:
```

```
 [0.01 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1  0.11 0.12 0
.13
 0.14 0.15 0.16 0.17 0.18 0.19 0.2  0.21 0.22 0.22 0.23 0.24 0.25 0.
26
 0.27 0.28 0.29 0.3  0.31 0.32 0.32 0.33 0.34 0.35 0.36 0.37 0.38 0.
39
 0.4  0.41 0.41 0.42 0.43 0.44 0.45 0.46 0.47 0.48 0.49 0.5  0.5  0.
51
 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.59 0.59 0.6  0.61 0.62 0.63 0.
64
 0.65 0.66 0.67 0.68 0.68 0.69 0.7  0.71 0.72 0.73 0.74 0.75 0.76 0.
77
 0.77 0.78 0.79 0.8  0.81 0.82 0.83 0.84 0.85 0.86 0.87 0.87 0.88 0.
89
 0.9  0.91]
```

Exercise 5.15

In [37]:

```
from matplotlib import pyplot as plt
plt.plot(w,new_w)
plt.xlabel('w')
plt.ylabel('new_w')
plt.show()
```



Exercise 5.16

In [38]:

```
import scipy.stats
sigma=0.5
m=7
u=4*0.5
epsilon=np.linspace(u-3*sigma,u+3*sigma,m)

gamma=np.array([0.0]*m)
for i in range(m):
    gamma[i]=scipy.stats.norm.pdf(epsilon[i],u,sigma)

print('epsilon:\n',epsilon)
print('gamma:\n',gamma)
```

```
epsilon:
 [0.5 1.  1.5 2.  2.5 3.  3.5]
gamma:
 [0.0088637  0.10798193 0.48394145 0.79788456 0.48394145 0.10798193
 0.0088637 ]
```

Exercise 5.17

In [39]:

```
beta=0.9
w=np.linspace(w_min,w_max,n)
w1=np.linspace(w_min,w_max,n) # w1 is in [0.01, 1], as the question says
v1=np.zeros((n,m))
w_broad=np.tile(np.tile(w.reshape(n,1),(1,m)),(1,1,n)).reshape((n,m,n))
ep_broad=np.tile(np.tile(epsilon.reshape(m,1),(1,n)),(n,1,1)).reshape(n,m,n)
w1_broad=np.tile(w1,(n,m,1))
v=v1.copy()
ev=np.zeros(n)
for i in range(m):
    ev+=v.reshape(m,n)[i]*gamma[i]
ev_broad=np.tile(ev,(n,m,1))
v_broad=np.tile(v.reshape(m,n),(n,1,1))
c_broad=w_broad-w1_broad
nega=(c_broad<=0)
c_broad[nega]=10**(-10)
ev_broad[nega]=-10*(10)
v1=np.max(ep_broad*(np.log(c_broad))+beta*ev_broad,axis=2)
psi=np.zeros((n,m))
for i in range(n):
    for j in range(m):
        psi[i][j]=w1_broad[i][j][np.argmax(ep_broad*np.log(c_broad)+beta*ev_broa
d,axis=2)][i][j]
new_w=np.array(psi)
new_w
print('The resulting policy function is:\n',new_w)
```

```
The resulting policy function is:
 [[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
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 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
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 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
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 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
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 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
```

```
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
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 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
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 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]]
```

In [40]:

```
print('The resulting value function is:\n',v1)
```

```
The resulting value function is:
```

```
[[-1.01512925e+02 -1.13025851e+02 -1.24538776e+02 -1.36051702e+02
  -1.47564627e+02 -1.59077553e+02 -1.70590478e+02]
 [-2.30258509e+00 -4.60517019e+00 -6.90775528e+00 -9.21034037e+00
  -1.15129255e+01 -1.38155106e+01 -1.61180957e+01]
 [-1.95601150e+00 -3.91202301e+00 -5.86803451e+00 -7.82404601e+00
  -9.78005751e+00 -1.17360690e+01 -1.36920805e+01]
 [-1.75327895e+00 -3.50655790e+00 -5.25983685e+00 -7.01311579e+00
  -8.76639474e+00 -1.05196737e+01 -1.22729526e+01]
 [-1.60943791e+00 -3.21887582e+00 -4.82831374e+00 -6.43775165e+00
  -8.04718956e+00 -9.65662747e+00 -1.12660654e+01]
 [-1.49786614e+00 -2.99573227e+00 -4.49359841e+00 -5.99146455e+00
  -7.48933068e+00 -8.98719682e+00 -1.04850630e+01]
 [-1.40670536e+00 -2.81341072e+00 -4.22011608e+00 -5.62682143e+00
  -7.03352679e+00 -8.44023215e+00 -9.84693751e+00]
 [-1.32963002e+00 -2.65926004e+00 -3.98889006e+00 -5.31852007e+00
  -6.64815009e+00 -7.97778011e+00 -9.30741013e+00]
 [-1.26286432e+00 -2.52572864e+00 -3.78859297e+00 -5.05145729e+00
  -6.31432161e+00 -7.57718593e+00 -8.84005026e+00]
 [-1.20397280e+00 -2.40794561e+00 -3.61191841e+00 -4.81589122e+00
  -6.01986402e+00 -7.22383683e+00 -8.42780963e+00]
 [-1.15129255e+00 -2.30258509e+00 -3.45387764e+00 -4.60517019e+00
  -5.75646273e+00 -6.90775528e+00 -8.05904783e+00]
 [-1.10363746e+00 -2.20727491e+00 -3.31091237e+00 -4.41454983e+00
  -5.51818728e+00 -6.62182474e+00 -7.72546220e+00]
 [-1.06013177e+00 -2.12026354e+00 -3.18039530e+00 -4.24052707e+00
  -5.30065884e+00 -6.36079061e+00 -7.42092238e+00]
 [-1.02011041e+00 -2.04022083e+00 -3.06033124e+00 -4.08044166e+00
  -5.10055207e+00 -6.12066249e+00 -7.14077290e+00]
 [-9.83056428e-01 -1.96611286e+00 -2.94916928e+00 -3.93222571e+00
  -4.91528214e+00 -5.89833857e+00 -6.88139500e+00]
 [-9.48559992e-01 -1.89711998e+00 -2.84567998e+00 -3.79423997e+00
  -4.74279996e+00 -5.69135995e+00 -6.63991995e+00]
 [-9.16290732e-01 -1.83258146e+00 -2.74887220e+00 -3.66516293e+00
  -4.58145366e+00 -5.49774439e+00 -6.41403512e+00]
 [-8.85978421e-01 -1.77195684e+00 -2.65793526e+00 -3.54391368e+00
  -4.42989210e+00 -5.31587053e+00 -6.20184895e+00]
 [-8.57399214e-01 -1.71479843e+00 -2.57219764e+00 -3.42959686e+00
  -4.28699607e+00 -5.14439528e+00 -6.00179450e+00]
 [-8.30365603e-01 -1.66073121e+00 -2.49109681e+00 -3.32146241e+00
  -4.15182802e+00 -4.98219362e+00 -5.81255922e+00]
 [-8.04718956e-01 -1.60943791e+00 -2.41415687e+00 -3.21887582e+00
  -4.02359478e+00 -4.82831374e+00 -5.63303269e+00]
 [-7.80323874e-01 -1.56064775e+00 -2.34097162e+00 -3.12129550e+00
  -3.90161937e+00 -4.68194324e+00 -5.46226712e+00]
 [-7.57063866e-01 -1.51412773e+00 -2.27119160e+00 -3.02825547e+00
  -3.78531933e+00 -4.54238320e+00 -5.29944706e+00]
 [-7.34837985e-01 -1.46967597e+00 -2.20451396e+00 -2.93935194e+00
  -3.67418993e+00 -4.40902791e+00 -5.14386590e+00]
 [-7.13558178e-01 -1.42711636e+00 -2.14067453e+00 -2.85423271e+00
  -3.56779089e+00 -4.28134907e+00 -4.99490724e+00]
 [-6.93147181e-01 -1.38629436e+00 -2.07944154e+00 -2.77258872e+00
  -3.46573590e+00 -4.15888308e+00 -4.85203026e+00]
 [-6.73536824e-01 -1.34707365e+00 -2.02061047e+00 -2.69414730e+00
```

```
 -3.36768412e+00 -4.04122094e+00 -4.71475777e+00]
[-6.54666660e-01 -1.30933332e+00 -1.96399998e+00 -2.61866664e+00
 -3.27333330e+00 -3.92799996e+00 -4.58266662e+00]
[-6.36482838e-01 -1.27296568e+00 -1.90944851e+00 -2.54593135e+00
 -3.18241419e+00 -3.81889703e+00 -4.45537987e+00]
[-6.18937178e-01 -1.23787436e+00 -1.85681153e+00 -2.47574871e+00
 -3.09468589e+00 -3.71362307e+00 -4.33256025e+00]
[-6.01986402e-01 -1.20397280e+00 -1.80595921e+00 -2.40794561e+00
 -3.00993201e+00 -3.61191841e+00 -4.21390482e+00]
[-5.85591491e-01 -1.17118298e+00 -1.75677447e+00 -2.34236596e+00
 -2.92795745e+00 -3.51354894e+00 -4.09914044e+00]
[-5.69717142e-01 -1.13943428e+00 -1.70915142e+00 -2.27886857e+00
 -2.84858571e+00 -3.41830285e+00 -3.98801999e+00]
[-5.54331312e-01 -1.10866262e+00 -1.66299394e+00 -2.21732525e+00
 -2.77165656e+00 -3.32598787e+00 -3.88031919e+00]
[-5.39404831e-01 -1.07880966e+00 -1.61821449e+00 -2.15761932e+00
 -2.69702415e+00 -3.23642898e+00 -3.77583381e+00]
[-5.24911062e-01 -1.04982212e+00 -1.57473319e+00 -2.09964425e+00
 -2.62455531e+00 -3.14946637e+00 -3.67437744e+00]
[-5.10825624e-01 -1.02165125e+00 -1.53247687e+00 -2.04330250e+00
 -2.55412812e+00 -3.06495374e+00 -3.57577937e+00]
[-4.97126137e-01 -9.94252273e-01 -1.49137841e+00 -1.98850455e+00
 -2.48563068e+00 -2.98275682e+00 -3.47988296e+00]
[-4.83792013e-01 -9.67584026e-01 -1.45137604e+00 -1.93516805e+00
 -2.41896007e+00 -2.90275208e+00 -3.38654409e+00]
[-4.70804270e-01 -9.41608540e-01 -1.41241281e+00 -1.88321708e+00
 -2.35402135e+00 -2.82482562e+00 -3.29562989e+00]
[-4.58145366e-01 -9.16290732e-01 -1.37443610e+00 -1.83258146e+00
 -2.29072683e+00 -2.74887220e+00 -3.20701756e+00]
[-4.45799060e-01 -8.91598119e-01 -1.33739718e+00 -1.78319624e+00
 -2.22899530e+00 -2.67479436e+00 -3.12059342e+00]
[-4.33750284e-01 -8.67500568e-01 -1.30125085e+00 -1.73500114e+00
 -2.16875142e+00 -2.60250170e+00 -3.03625199e+00]
[-4.21985035e-01 -8.43970070e-01 -1.26595511e+00 -1.68794014e+00
 -2.10992518e+00 -2.53191021e+00 -2.95389525e+00]
[-4.10490276e-01 -8.20980552e-01 -1.23147083e+00 -1.64196110e+00
 -2.05245138e+00 -2.46294166e+00 -2.87343193e+00]
[-3.99253848e-01 -7.98507696e-01 -1.19776154e+00 -1.59701539e+00
 -1.99626924e+00 -2.39552309e+00 -2.79477694e+00]
[-3.88264395e-01 -7.76528789e-01 -1.16479318e+00 -1.55305758e+00
 -1.94132197e+00 -2.32958637e+00 -2.71785076e+00]
[-3.77511292e-01 -7.55022584e-01 -1.13253388e+00 -1.51004517e+00
 -1.88755646e+00 -2.26506775e+00 -2.64257904e+00]
[-3.66984588e-01 -7.33969175e-01 -1.10095376e+00 -1.46793835e+00
 -1.83492294e+00 -2.20190753e+00 -2.56889211e+00]
[-3.56674944e-01 -7.13349888e-01 -1.07002483e+00 -1.42669978e+00
 -1.78337472e+00 -2.14004966e+00 -2.49672461e+00]
[-3.46573590e-01 -6.93147181e-01 -1.03972077e+00 -1.38629436e+00
 -1.73286795e+00 -2.07944154e+00 -2.42601513e+00]
[-3.36672277e-01 -6.73344553e-01 -1.01001683e+00 -1.34668911e+00
 -1.68336138e+00 -2.02003366e+00 -2.35670594e+00]
[-3.26963234e-01 -6.53926467e-01 -9.80889701e-01 -1.30785293e+00
 -1.63481617e+00 -1.96177940e+00 -2.28874264e+00]
```

```
 [-3.17439136e-01 -6.34878272e-01 -9.52317409e-01 -1.26975654e+00
  -1.58719568e+00 -1.90463482e+00 -2.22207395e+00]
 [-3.08093070e-01 -6.16186139e-01 -9.24279209e-01 -1.23237228e+00
  -1.54046535e+00 -1.84855842e+00 -2.15665149e+00]
 [-2.98918500e-01 -5.97837001e-01 -8.96755501e-01 -1.19567400e+00
  -1.49459250e+00 -1.79351100e+00 -2.09242950e+00]
 [-2.89909248e-01 -5.79818495e-01 -8.69727743e-01 -1.15963699e+00
  -1.44954624e+00 -1.73945549e+00 -2.02936473e+00]
 [-2.81059459e-01 -5.62118918e-01 -8.43178377e-01 -1.12423784e+00
  -1.40529730e+00 -1.68635675e+00 -1.96741621e+00]
 [-2.72363588e-01 -5.44727175e-01 -8.17090763e-01 -1.08945435e+00
  -1.36181794e+00 -1.63418153e+00 -1.90654511e+00]
 [-2.63816371e-01 -5.27632742e-01 -7.91449113e-01 -1.05526548e+00
  -1.31908186e+00 -1.58289823e+00 -1.84671460e+00]
 [-2.55412812e-01 -5.10825624e-01 -7.66238436e-01 -1.02165125e+00
  -1.27706406e+00 -1.53247687e+00 -1.78788968e+00]
 [-2.47148161e-01 -4.94296322e-01 -7.41444483e-01 -9.88592644e-01
  -1.23574080e+00 -1.48288897e+00 -1.73003713e+00]
 [-2.39017900e-01 -4.78035801e-01 -7.17053701e-01 -9.56071602e-01
  -1.19508950e+00 -1.43410740e+00 -1.67312530e+00]
 [-2.31017730e-01 -4.62035460e-01 -6.93053189e-01 -9.24070919e-01
  -1.15508865e+00 -1.38610638e+00 -1.61712411e+00]
 [-2.23143551e-01 -4.46287103e-01 -6.69430654e-01 -8.92574205e-01
  -1.11571776e+00 -1.33886131e+00 -1.56200486e+00]
 [-2.15391458e-01 -4.30782916e-01 -6.46174374e-01 -8.61565832e-01
  -1.07695729e+00 -1.29234875e+00 -1.50774021e+00]
 [-2.07757722e-01 -4.15515444e-01 -6.23273166e-01 -8.31030888e-01
  -1.03878861e+00 -1.24654633e+00 -1.45430405e+00]
 [-2.00238783e-01 -4.00477567e-01 -6.00716350e-01 -8.00955133e-01
  -1.00119392e+00 -1.20143270e+00 -1.40167148e+00]
 [-1.92831240e-01 -3.85662481e-01 -5.78493721e-01 -7.71324962e-01
  -9.64156202e-01 -1.15698744e+00 -1.34981868e+00]
 [-1.85531841e-01 -3.71063681e-01 -5.56595522e-01 -7.42127363e-01
  -9.27659203e-01 -1.11319104e+00 -1.29872288e+00]
 [-1.78337472e-01 -3.56674944e-01 -5.35012416e-01 -7.13349888e-01
  -8.91687360e-01 -1.07002483e+00 -1.24836230e+00]
 [-1.71245154e-01 -3.42490309e-01 -5.13735463e-01 -6.84980618e-01
  -8.56225772e-01 -1.02747093e+00 -1.19871608e+00]
 [-1.64252033e-01 -3.28504067e-01 -4.92756100e-01 -6.57008134e-01
  -8.21260167e-01 -9.85512201e-01 -1.14976423e+00]
 [-1.57355372e-01 -3.14710745e-01 -4.72066117e-01 -6.29421490e-01
  -7.86776862e-01 -9.44132235e-01 -1.10148761e+00]
 [-1.50552546e-01 -3.01105093e-01 -4.51657639e-01 -6.02210186e-01
  -7.52762732e-01 -9.03315278e-01 -1.05386782e+00]
 [-1.43841036e-01 -2.87682072e-01 -4.31523109e-01 -5.75364145e-01
  -7.19205181e-01 -8.63046217e-01 -1.00688725e+00]
 [-1.37218423e-01 -2.74436846e-01 -4.11655269e-01 -5.48873691e-01
  -6.86092114e-01 -8.23310537e-01 -9.60528960e-01]
 [-1.30682382e-01 -2.61364764e-01 -3.92047146e-01 -5.22729528e-01
  -6.53411910e-01 -7.84094292e-01 -9.14776674e-01]
 [-1.24230680e-01 -2.48461359e-01 -3.72692039e-01 -4.96922719e-01
  -6.21153398e-01 -7.45384078e-01 -8.69614758e-01]
 [-1.17861167e-01 -2.35722334e-01 -3.53583500e-01 -4.71444667e-01
```

```
  -5.89305834e-01 -7.07167001e-01 -8.25028167e-01]
 [-1.11571776e-01 -2.23143551e-01 -3.34715327e-01 -4.46287103e-01
  -5.57858878e-01 -6.69430654e-01 -7.81002430e-01]
 [-1.05360516e-01 -2.10721031e-01 -3.16081547e-01 -4.21442063e-01
  -5.26802578e-01 -6.32163094e-01 -7.37523610e-01]
 [-9.92254694e-02 -1.98450939e-01 -2.97676408e-01 -3.96901877e-01
  -4.96127347e-01 -5.95352816e-01 -6.94578286e-01]
 [-9.31647891e-02 -1.86329578e-01 -2.79494367e-01 -3.72659156e-01
  -4.65823945e-01 -5.58988735e-01 -6.52153524e-01]
 [-8.71766936e-02 -1.74353387e-01 -2.61530081e-01 -3.48706774e-01
  -4.35883468e-01 -5.23060161e-01 -6.10236855e-01]
 [-8.12594647e-02 -1.62518929e-01 -2.43778394e-01 -3.25037859e-01
  -4.06297324e-01 -4.87556788e-01 -5.68816253e-01]
 [-7.54114449e-02 -1.50822890e-01 -2.26234335e-01 -3.01645779e-01
  -3.77057224e-01 -4.52468669e-01 -5.27880114e-01]
 [-6.96310337e-02 -1.39262067e-01 -2.08893101e-01 -2.78524135e-01
  -3.48155168e-01 -4.17786202e-01 -4.87417236e-01]
 [-6.39166858e-02 -1.27833372e-01 -1.91750057e-01 -2.55666743e-01
  -3.19583429e-01 -3.83500115e-01 -4.47416800e-01]
 [-5.82669081e-02 -1.16533816e-01 -1.74800724e-01 -2.33067633e-01
  -2.91334541e-01 -3.49601449e-01 -4.07868357e-01]
 [-5.26802578e-02 -1.05360516e-01 -1.58040773e-01 -2.10721031e-01
  -2.63401289e-01 -3.16081547e-01 -3.68761805e-01]
 [-4.71553397e-02 -9.43106795e-02 -1.41466019e-01 -1.88621359e-01
  -2.35776699e-01 -2.82932038e-01 -3.30087378e-01]
 [-4.16908045e-02 -8.33816089e-02 -1.25072413e-01 -1.66763218e-01
  -2.08454022e-01 -2.50144827e-01 -2.91835631e-01]
 [-3.62853464e-02 -7.25706928e-02 -1.08856039e-01 -1.45141386e-01
  -1.81426732e-01 -2.17712079e-01 -2.53997425e-01]
 [-3.09377019e-02 -6.18754037e-02 -9.28131056e-02 -1.23750807e-01
  -1.54688509e-01 -1.85626211e-01 -2.16563913e-01]
 [-2.56466472e-02 -5.12932944e-02 -7.69399416e-02 -1.02586589e-01
  -1.28233236e-01 -1.53879883e-01 -1.79526530e-01]
 [-2.04109973e-02 -4.08219945e-02 -6.12329918e-02 -8.16439890e-02
  -1.02054986e-01 -1.22465984e-01 -1.42876981e-01]
 [-1.52296037e-02 -3.04592075e-02 -4.56888112e-02 -6.09184150e-02
  -7.61480187e-02 -9.13776225e-02 -1.06607226e-01]
 [-1.01013537e-02 -2.02027073e-02 -3.03040610e-02 -4.04054146e-02
  -5.05067683e-02 -6.06081220e-02 -7.07094756e-02]
 [-5.02516793e-03 -1.00503359e-02 -1.50755038e-02 -2.01006717e-02
  -2.51258396e-02 -3.01510076e-02 -3.51761755e-02]]
```

Exercise 5.18

In [41]:

```
def norm(v,v1):
    vec=(v-v1).reshape(n*m,1)
    norm=sum(vec*vec)[0]
    return norm
d0=norm(v,v1)
print('The distance metric delta(t)=:', norm(v,v1))
```

```
The distance metric delta(t)=: 139544.20160321452
```

Exercise 5.19

In [42]:

```
v=v1.copy()
ev=np.zeros(n)
for i in range(m):
    ev+=v.reshape(m,n)[i]*gamma[i]
ev_broad=np.tile(ev,(n,m,1))
v_broad=np.tile(v.reshape(m,n),(n,1,1))
c_broad=w_broad-w1_broad
nega=(c_broad<=0)
c_broad[nega]=10**(-10)
ev_broad[nega]=-10*(10)
v1=np.max(ep_broad*np.log(c_broad)+beta*ev_broad,axis=2)
psi=np.zeros((n,m))
for i in range(n):
    for j in range(m):
        psi[i][j]=(w1_broad[i][j][np.argmax(ep_broad*np.log(c_broad)+beta*ev_bro
ad,axis=2)[i][j]])
new_w=np.array(psi)
print('The resulting policy function is:\n',new_w)
```

```
The resulting policy function is:
 [[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.09 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.09 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.09 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.11 0.09 0.02 0.02 0.02 0.02 0.02]
 [0.11 0.09 0.02 0.02 0.02 0.02 0.02]
 [0.11 0.09 0.02 0.02 0.02 0.02 0.02]
 [0.11 0.09 0.09 0.02 0.02 0.02 0.02]
```

```
 [0.11 0.11 0.09 0.02 0.02 0.02 0.02]
 [0.11 0.11 0.09 0.02 0.02 0.02 0.02]
 [0.11 0.11 0.09 0.02 0.02 0.02 0.02]
 [0.11 0.11 0.11 0.09 0.02 0.02 0.02]
 [0.11 0.11 0.11 0.09 0.02 0.02 0.02]
 [0.11 0.11 0.11 0.09 0.02 0.02 0.02]
 [0.11 0.11 0.11 0.09 0.02 0.02 0.02]
 [0.11 0.11 0.11 0.11 0.09 0.02 0.02]
 [0.11 0.11 0.11 0.11 0.09 0.02 0.02]
 [0.11 0.11 0.11 0.11 0.09 0.02 0.02]
 [0.11 0.11 0.11 0.11 0.11 0.09 0.02]
 [0.11 0.11 0.11 0.11 0.11 0.09 0.02]
 [0.11 0.11 0.11 0.11 0.11 0.09 0.02]
 [0.11 0.11 0.11 0.11 0.11 0.09 0.02]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.09]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.09]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.09]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.09]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
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 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
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 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.18 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
```

```
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.25 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.32 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.32 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.32 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.32 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.32 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.32 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.32 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.32 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.32 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.39 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.39 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.39 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.39 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.39 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.39 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.39 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.39 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.46 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.46 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.46 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.46 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.46 0.18 0.11 0.11 0.11 0.11 0.11]
 [0.46 0.18 0.11 0.11 0.11 0.11 0.11]
 [0.46 0.18 0.11 0.11 0.11 0.11 0.11]]
```

In [43]:

```
print('The resulting value function is:\n',v1)
```

```
The resulting value function is:
 [[-101.51292546 -113.02585093 -124.53877639 -136.05170186 -147.5646
2732
  -159.07755279 -170.59047825]
 [  -7.28069861   -9.5832837   -11.8858688   -14.18845389  -16.49103
898
   -18.79362408  -21.09620917]
 [  -6.00770672   -8.31029181  -10.61287691  -12.80215953  -14.75817
103
   -16.71418254  -18.67019404]
 [  -5.66113313   -7.61714463   -9.57315614  -11.52916764  -13.48517
914
   -15.44119064  -17.25106616]
 [  -5.45840058   -7.21167953   -8.96495847  -10.71823742  -12.47151
637
   -14.22479532  -15.97807427]
 [  -5.31455954   -6.92399745   -8.53343537  -10.14287328  -11.75231
119
```

```
   -13.3617491   -14.97118702]
 [  -5.20298777   -6.7008539    -8.19872004   -9.69658618  -11.19445
231
   -12.69231845  -14.19018459]
 [  -5.11182699   -6.51853235   -7.9252377    -9.33194306  -10.73864
842
   -12.14535378  -13.55205914]
 [  -5.03475165   -6.36438167   -7.69401168   -9.0236417   -10.35327
172
   -11.68290174  -13.01253176]
 [  -4.96798595   -6.23085027   -7.4937146    -8.75657892  -10.01944
324
   -11.28230756  -12.54517188]
 [  -4.71362275   -6.11306724   -7.31704004   -8.52101285   -9.72498
565
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```

In [44]:

```
d1=norm(v,v1)
print('The distance metric delta(t-1)=:', norm(v,v1))
print('delta(t)-delta(t-1)=',d0-d1)
```

```
The distance metric delta(t-1)=: 7030.872195469327
delta(t)-delta(t-1)= 132513.3294077452
```

Therefore, delta(t-1) is smaller than delta(t). The distance decreases.

Exercise 5.20

In [45]:

```
v=v1.copy()
ev=np.zeros(n)
for i in range(m):
    ev+=v.reshape(m,n)[i]*gamma[i]
ev_broad=np.tile(ev,(n,m,1))
v_broad=np.tile(v.reshape(m,n),(n,1,1))
c_broad=w_broad-w1_broad
nega=(c_broad<=0)
c_broad[nega]=10**(-10)
ev_broad[nega]=-10*(10)
v1=np.max(ep_broad*np.log(c_broad)+beta*ev_broad,axis=2)
psi=np.zeros((n,m))
for i in range(n):
    for j in range(m):
        psi[i][j]=(w1_broad[i][j][np.argmax(ep_broad*np.log(c_broad)+beta*ev_bro
ad,axis=2)[i][j]])
new_w=np.array(psi)
new_w
print('The resulting policy function is:\n',new_w)
```

```
The resulting policy function is:
 [[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
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 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
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 [0.53 0.25 0.11 0.11 0.11 0.11 0.11]
 [0.53 0.25 0.11 0.11 0.11 0.11 0.11]
 [0.53 0.25 0.11 0.11 0.11 0.11 0.11]
 [0.53 0.25 0.11 0.11 0.11 0.11 0.11]
 [0.53 0.25 0.11 0.11 0.11 0.11 0.11]
 [0.53 0.25 0.11 0.11 0.11 0.11 0.11]
 [0.53 0.25 0.11 0.11 0.11 0.11 0.11]]
```

In [46]:

```
print('The resulting value function is:\n',v1)
```

```
The resulting value function is:
 [[-101.51292546 -113.02585093 -124.53877639 -136.05170186 -147.5646
2732
  -159.07755279 -170.59047825]
 [ -13.13308318  -15.43566827  -17.73825336  -20.04083846  -22.34342
355
   -24.64600864  -26.94859374]
 [ -11.40502069  -13.70760578  -16.01019087  -18.31277596  -20.61055
56
   -22.5665671   -24.5225786 ]
 [ -11.0584471   -13.0144586   -14.9704701   -16.9264816   -18.88249
311
   -20.83850461  -22.79451611]
 [ -10.85571454  -12.60899349  -14.36227244  -16.11555139  -17.86883
034
   -19.62210928  -21.37538823]
 [ -10.7118735   -12.32131142  -13.93074933  -15.54018724  -17.14962
515
   -18.75906307  -20.36850098]
 [ -10.54223325  -12.09816787  -13.596034    -15.09390014  -16.59176
628
   -18.08963241  -19.58749855]
 [ -10.39839221  -11.91584631  -13.32255167  -14.72925703  -16.13596
238
   -17.54266774  -18.9493731 ]
 [ -10.28682044  -11.76169563  -13.09132565  -14.42095567  -15.75058
568
   -17.0802157   -18.40984572]
 [ -10.19565966  -11.60236502  -12.89102856  -14.15389288  -15.41675
72
   -16.67962153  -17.94248585]
 [ -10.11858432  -11.44821434  -12.71435401  -13.91832681  -15.12229
```

```
961
   -16.32627242  -17.53024522]
 [  -9.92357015  -11.31468294  -12.55631323  -13.70760578  -14.85889
832
   -16.01019087  -17.16148342]
 [  -9.67933069  -11.19689991  -12.40087271  -13.51698542  -14.62062
288
   -15.72426033  -16.82789779]
 [  -9.47659814  -11.09153939  -12.24283194  -13.34296266  -14.40309
443
   -15.4632262   -16.52335797]
 [  -9.3327571   -10.94219501  -12.09986667  -13.18287725  -14.20298
766
   -15.22309808  -16.24320849]
 [  -9.22118532  -10.71905146  -11.9693496   -13.02948137  -14.01771
773
   -15.00077416  -15.98383059]
 [  -9.13002455  -10.5367299   -11.84928554  -12.86939596  -13.84523
555
   -14.79379555  -15.74235554]
 [  -9.05294921  -10.38257922  -11.71220924  -12.72118001  -13.68388
925
   -14.60017998  -15.51647072]
 [  -8.98618351  -10.24904783  -11.51191215  -12.58319427  -13.53175
426
   -14.41830612  -15.30428454]
 [  -8.92729199  -10.1312648   -11.3352376   -12.45411723  -13.37040
796
   -14.24683088  -15.10423009]
 [  -8.87461173  -10.02590428  -11.17719683  -12.32848937  -13.21884
64
   -14.08462921  -14.91499482]
 [  -8.82695664   -9.9305941   -11.03423156  -12.13786901  -13.07595
037
   -13.93074933  -14.73546829]
 [  -8.78345096   -9.84358272  -10.90371449  -11.96384626  -12.94078
232
   -13.77114792  -14.56470271]
 [  -8.7434296    -9.76354002  -10.78365043  -11.80376084  -12.81254
908
   -13.61726804  -14.40188266]
 [  -8.70637562   -9.68943204  -10.67248847  -11.6555449   -12.63860
133
   -13.47089754  -14.24630149]
 [  -8.67187918   -9.62043917  -10.56899917  -11.51755916  -12.46611
915
   -13.3313375   -14.08840136]
 [  -8.63960992   -9.55590065  -10.47219138  -11.38848212  -12.30477
285
   -13.19798221  -13.93282019]
 [  -8.60929761   -9.49527603  -10.38125445  -11.26723287  -12.15321
129
   -13.03918971  -13.78386154]
```

```
 [  -8.5807184    -9.43811762  -10.29551683  -11.15291604  -12.01031
526
   -12.86771447  -13.64098456]
 [  -8.55368479   -9.38405039  -10.214416    -11.0447816   -11.87514
72
   -12.70551281  -13.50371207]
 [  -8.52803814   -9.3327571   -10.13747606  -10.94219501  -11.74691
397
   -12.55163293  -13.35635188]
 [  -8.50364306   -9.28396694  -10.06429081  -10.84461468  -11.62493
856
   -12.40526243  -13.18558631]
 [  -8.48038305   -9.23744692   -9.99451079  -10.75157465  -11.50863
852
   -12.26570239  -13.02276625]
 [  -8.45815717   -9.19299516   -9.92783314  -10.66267113  -11.39750
911
   -12.1323471   -12.86718508]
 [  -8.43687737   -9.15043554   -9.86399372  -10.5775519   -11.29111
008
   -12.00466825  -12.71822643]
 [  -8.41646637   -9.10961355   -9.80276073  -10.49590791  -11.18905
509
   -11.88220227  -12.57534945]
 [  -8.39685601   -9.07039284   -9.74392966  -10.41746648  -11.09100
331
   -11.76454013  -12.43807696]
 [  -8.37798585   -9.03265251   -9.68731917  -10.34198583  -10.99665
249
   -11.65131915  -12.30598581]
 [  -8.35980203   -8.99628486   -9.6327677   -10.26925054  -10.90573
338
   -11.54221622  -12.17869905]
 [  -8.34225637   -8.96119354   -9.58013072  -10.1990679   -10.81800
508
   -11.43694226  -12.05587943]
 [  -8.32530559   -8.92729199   -9.52927839  -10.1312648   -10.73325
12
   -11.3352376   -11.937224  ]
 [  -8.30891068   -8.89450217   -9.48009366  -10.06568515  -10.65127
664
   -11.23686813  -11.82245962]
 [  -8.29303633   -8.86275347   -9.43247061  -10.00218775  -10.57190
49
   -11.14162204  -11.71133918]
 [  -8.2776505    -8.83198181   -9.38631312   -9.94064444  -10.49497
575
   -11.04930706  -11.60363837]
 [  -8.26272402   -8.80212885   -9.34153368   -9.88093851  -10.42034
334
   -10.95974817  -11.499153  ]
 [  -8.24823025   -8.77314131   -9.29805237   -9.82296344  -10.34787
45
```

```
   -10.87278556  -11.39769662]
 [  -8.23414481   -8.74497044   -9.25579606   -9.76662168  -10.27744
731
   -10.78827293  -11.29909855]
 [  -8.22044532   -8.71757146   -9.2146976    -9.71182373  -10.20894
987
   -10.70607601  -11.20320214]
 [  -8.2071112    -8.69090321   -9.17469523   -9.65848724  -10.14227
925
   -10.62607127  -11.10986328]
 [  -8.19201645   -8.66492773   -9.135732     -9.60653627  -10.07734
054
   -10.54814481  -11.01894908]
 [  -8.17663062   -8.63960992   -9.09775529   -9.55590065  -10.01404
602
   -10.47219138  -10.93033675]
 [  -8.16170414   -8.61491731   -9.06071637   -9.50651543   -9.95231
449
   -10.39811355  -10.84391261]
 [  -8.14721037   -8.59081976   -9.02457004   -9.45832032   -9.89207
061
   -10.32582089  -10.75957117]
 [  -8.13312494   -8.56728926   -8.98927429   -9.41125933   -9.83324
436
   -10.2552294   -10.67721443]
 [  -8.11942545   -8.54429974   -8.95479002   -9.36528029   -9.77577
057
   -10.18626084  -10.59675112]
 [  -8.10609133   -8.52182688   -8.92108073   -9.32033458   -9.71958
843
   -10.11884228  -10.51809612]
 [  -8.09310358   -8.49984798   -8.88811237   -9.27637677   -9.66464
116
   -10.05290556  -10.44116995]
 [  -8.07972793   -8.47834177   -8.85585306   -9.23336436   -9.61087
565
    -9.98838694  -10.36589823]
 [  -8.06480145   -8.45728836   -8.82427295   -9.19125754   -9.55824
213
    -9.92522671  -10.2922113 ]
 [  -8.05030768   -8.43666908   -8.79334402   -9.15001896   -9.50669
391
    -9.86336885  -10.2200438 ]
 [  -8.03622224   -8.41646637   -8.76303996   -9.10961355   -9.45618
714
    -9.80276073  -10.14933432]
 [  -8.02252276   -8.39666374   -8.73333602   -9.07000829   -9.40668
057
    -9.74335285  -10.08002512]
 [  -8.00918863   -8.37724566   -8.70420889   -9.03117212   -9.35813
536
    -9.68509859  -10.01206182]
 [  -7.99620089   -8.35819746   -8.6756366    -8.99307573   -9.31051
```

```
487
   -9.62795401  -9.94539314]
 [ -7.98354199  -8.33950533  -8.6475984   -8.95569147  -9.26378
454
   -9.57187761  -9.87997068]
 [ -7.97119568  -8.32115619  -8.62007469  -8.91899319  -9.21791
169
   -9.51683019  -9.81574869]
 [ -7.9591469   -8.30313768  -8.59304693  -8.88295618  -9.17286
543
   -9.46277467  -9.75268392]
 [ -7.9459485   -8.28543811  -8.56649756  -8.84755702  -9.12861
648
   -9.40967594  -9.6907354 ]
 [ -7.93224901  -8.26804636  -8.54040995  -8.81277354  -9.08513
713
   -9.35750071  -9.6298643 ]
 [ -7.91891489  -8.25095193  -8.5147683   -8.77858467  -9.04240
104
   -9.30621741  -9.57003379]
 [ -7.90592715  -8.23414481  -8.48955762  -8.74497044  -9.00038
325
   -9.25579606  -9.51120887]
 [ -7.89326824  -8.21761551  -8.46476367  -8.71191183  -8.95905
999
   -9.20620815  -9.45335631]
 [ -7.88092194  -8.20135499  -8.44037289  -8.67939079  -8.91840
869
   -9.15742659  -9.39644449]
 [ -7.86887316  -8.18535465  -8.41637238  -8.64739011  -8.87840
784
   -9.10942557  -9.3404433 ]
 [ -7.85710791  -8.16960629  -8.39274984  -8.61589339  -8.83903
694
   -9.0621805   -9.28532405]
 [ -7.84561315  -8.1541021   -8.36949356  -8.58488502  -8.80027
648
   -9.01566794  -9.23105939]
 [ -7.83323055  -8.13883463  -8.34659235  -8.55435008  -8.76210
78
   -8.96986552  -9.17762324]
 [ -7.8202428   -8.12379675  -8.32403554  -8.52427432  -8.72451
31
   -8.92475189  -9.12499067]
 [ -7.8075839   -8.10898167  -8.30181291  -8.49464415  -8.68747
539
   -8.88030663  -9.07313787]
 [ -7.79523759  -8.09438287  -8.27991471  -8.46544655  -8.65097
839
   -8.83651023  -9.02204207]
 [ -7.78318882  -8.07999413  -8.2583316   -8.43666908  -8.61500
655
   -8.79334402  -8.97168149]
```

```
 [  -7.77142357   -8.0658095    -8.23705465   -8.40829981   -8.57954
496
    -8.75079011   -8.92203527]
 [  -7.75992881   -8.05182325   -8.21607529   -8.38032732   -8.54457
936
    -8.70883139   -8.87308342]
 [  -7.74869238   -8.03781476   -8.19538531   -8.35274068   -8.51009
605
    -8.66745142   -8.82480679]
 [  -7.73770293   -8.02277688   -8.17497683   -8.32552937   -8.47608
192
    -8.62663447   -8.77718701]
 [  -7.72543717   -8.00796179   -8.1548423    -8.29868333   -8.44252
437
    -8.58636541   -8.73020644]
 [  -7.71309086   -7.99336299   -8.13497446   -8.27219288   -8.40941
13
    -8.54662972   -8.68384815]
 [  -7.70104209   -7.97897426   -8.11536633   -8.24604872   -8.37673
11
    -8.50741348   -8.63809586]
 [  -7.68927684   -7.96478962   -8.09601123   -8.22024191   -8.34447
259
    -8.46870327   -8.59293395]
 [  -7.67778208   -7.95080338   -8.07690269   -8.19476385   -8.31262
502
    -8.43048619   -8.54834736]
 [  -7.66654565   -7.93701006   -8.05803451   -8.16960629   -8.28117
807
    -8.39274984   -8.50432162]
 [  -7.6555562    -7.92340441   -8.03940073   -8.14476125   -8.25012
177
    -8.35548228   -8.4608428 ]
 [  -7.6448031    -7.90998138   -8.0209956    -8.12022107   -8.21944
653
    -8.318672     -8.41789747]
 [  -7.63392598   -7.8964603    -8.00281356   -8.09597834   -8.18914
313
    -8.28230792   -8.37547271]
 [  -7.62187721   -7.88207157   -7.98484927   -8.07202596   -8.15920
266
    -8.24637935   -8.33355604]
 [  -7.61011196   -7.86788693   -7.96709758   -8.04835705   -8.12961
651
    -8.21087598   -8.29213544]
 [  -7.5986172    -7.85390069   -7.94955352   -8.02496497   -8.10037
641
    -8.17578786   -8.2511993 ]
 [  -7.58738077   -7.84010737   -7.93221229   -8.00184332   -8.07147
436
    -8.14110539   -8.21073642]
 [  -7.57639132   -7.82650171   -7.91506925   -7.97898593   -8.04290
262
```

```
    -8.1068193    -8.17073599]
 [  -7.56563821   -7.81307869   -7.89811991   -7.95638682   -8.01465
373
    -8.07292064   -8.13118754]]
```

In [47]:

```python
d2=norm(v,v1)
print('The distance metric delta(t-1)=:', norm(v,v1))
print('Therefore, we have delta(t)=',d0,'delta(t-1)=',d1,'delta(t-2)=',d2)
```

```
The distance metric delta(t-1)=: 19297.944207674045
Therefore, we have delta(t)= 139544.20160321452 delta(t-1)= 7030.872
195469327 delta(t-2)= 19297.944207674045
```

delta(t-2) is larger than delta(t-1) , smaller than delta(t)

Exercise 5.21

In [48]:

```
tole=10**(-9)
v1=np.zeros((n,m))
distance=100
s=0
while distance>=tole:
    v=v1.copy()
    ev=np.zeros(n)
    for i in range(m):
        ev+=v.reshape(m,n)[i]*gamma[i]
    ev_broad=np.tile(ev,(n,m,1))
    v_broad=np.tile(v.reshape(m,n),(n,1,1))
    c_broad=w_broad-w1_broad
    nega=(c_broad<=0)
    c_broad[nega]=10**(-100)
    ev_broad[nega]=-10*(10)
    v1=np.max(ep_broad*np.log(c_broad)+beta*ev_broad,axis=2)
    distance=norm(v,v1)
    s+=1
    print(s,'th interation, distance:', distance)

psi=np.zeros((n,m))
for i in range(n):
    for j in range(m):
        psi[i][j]=(w1_broad[i][j][np.argmax(ep_broad*np.log(c_broad)+beta*ev_bro
ad,axis=2)[i][j]])
new_w=np.array(psi)
```

```
1 th interation, distance: 2498878.1959750284
2 th interation, distance: 8353.06987844887
3 th interation, distance: 20118.64410299783
4 th interation, distance: 64631.90875658161
5 th interation, distance: 209245.90583105315
6 th interation, distance: 677549.80213404
7 th interation, distance: 2193895.611379207
8 th interation, distance: 6751697.978498628
9 th interation, distance: 14985745.52556715
10 th interation, distance: 12422328.901031619
11 th interation, distance: 4179560.9709801567
12 th interation, distance: 874131.1466875821
13 th interation, distance: 89849.02420427458
14 th interation, distance: 487.2117055093519
15 th interation, distance: 0.0310050589493355
16 th interation, distance: 1.9730919419138106e-06
17 th interation, distance: 1.255631496451555e-10
```

In [49]:

```
print('It takes',s,'iterations.')
```

```
It takes 17 iterations.
```

Exercise 5.22

In [50]:

```
from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
fig=plt.figure()
ax=Axes3D(fig)
E,W=np.meshgrid(epsilon,w)
ax.plot_surface(W,E,new_w)
ax.set_xlabel('w')
ax.set_ylabel('epsilon')
ax.set_zlabel('new w')
plt.show()
```