Problem Set 5

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1

a.

In [245]:

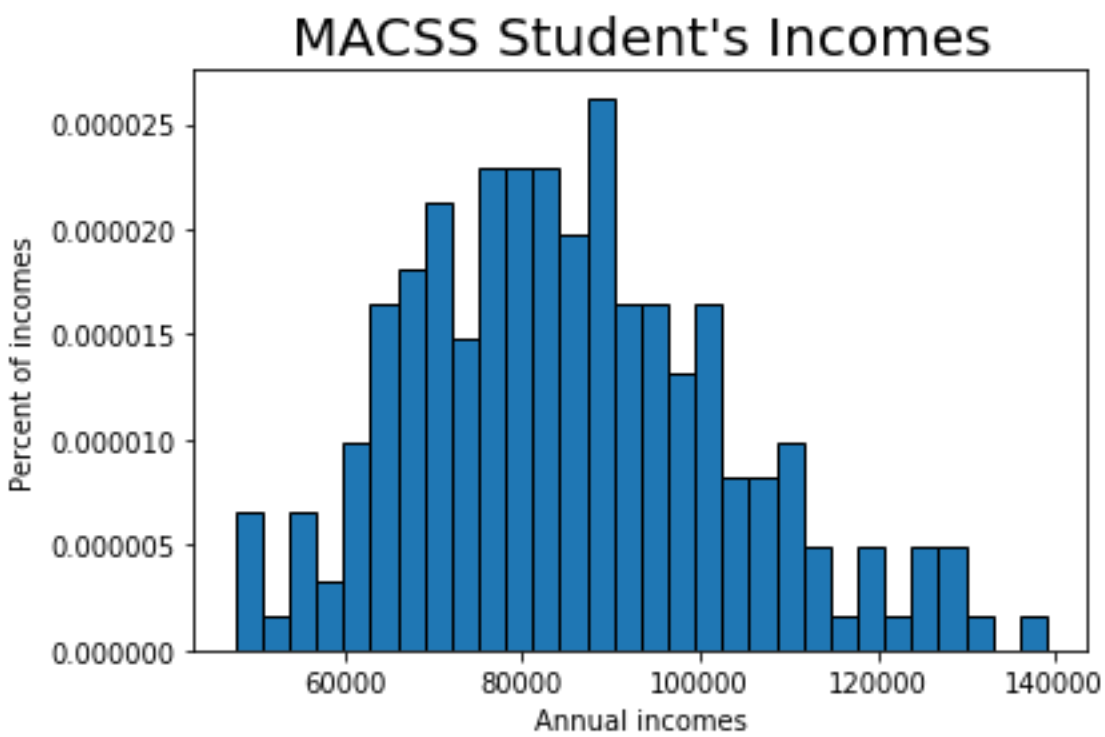
```
import numpy as np
import matplotlib.pyplot as plt
income=np.loadtxt('/Users/fulinguo/Desktop/persp-model-econ_W19/ProblemSets/PS5/data/incomes.txt')
```

In [246]:

```
num=30
plt.hist(income, num, density=True,edgecolor='k')
plt.title("MACSS Student's Incomes", fontsize=20)
plt.xlabel('Annual incomes')
plt.ylabel('Percent of incomes')
```

Out[246]:

```
Text(0, 0.5, 'Percent of incomes')
```



b.

In [247]:

```
import scipy.stats
import math
import scipy.integrate
import scipy.optimize as opt
def logpdf(x,mu,sigma):
    ans=(1/(sigma*np.sqrt(2*np.pi)*x))*np.exp(-(((np.log(x)-mu)**2)/(2*(sigma**2
))))
    return ans
def data_m(x):
    mean=x.mean()
    std=x.std()
    return mean,std
def model_m(mu,sigma):
    # xfx=lambda x: x*logpdf(x,mu,sigma)
    #(mean,m_err)=scipy.integrate.quad(xfx,0,+np.inf)
    mean=math.exp(mu+0.5*sigma**2)
    #x2fx=lambda x:((x - mean)**2)*logpdf(x,mu,sigma)
    #(var,v_err)=scipy.integrate.quad(x2fx,0,+np.inf)
    std=mean*math.sqrt(math.exp(sigma**2)-1)
    return mean, std
def err(x,mu,sigma,simple):
    m_data,v_data=data_m(x)
    moms_data=np.array([[m_data],[v_data]])
    m_model, v_model=model_m(mu,sigma)
    moms_model=np.array([[m_model],[v_model]])
    if simple:
        err_vec=moms_model-moms_data
    else:
        err_vec=(moms_model-moms_data)/moms_data
    return err_vec
def crit(params, *args):
    mu=params[0]
    sigma=params[1]
    x,W=args
    error=err(x,mu,sigma,simple=False)
    crit_val=error.T @ W @ error
    return crit_val
```

In [248]:

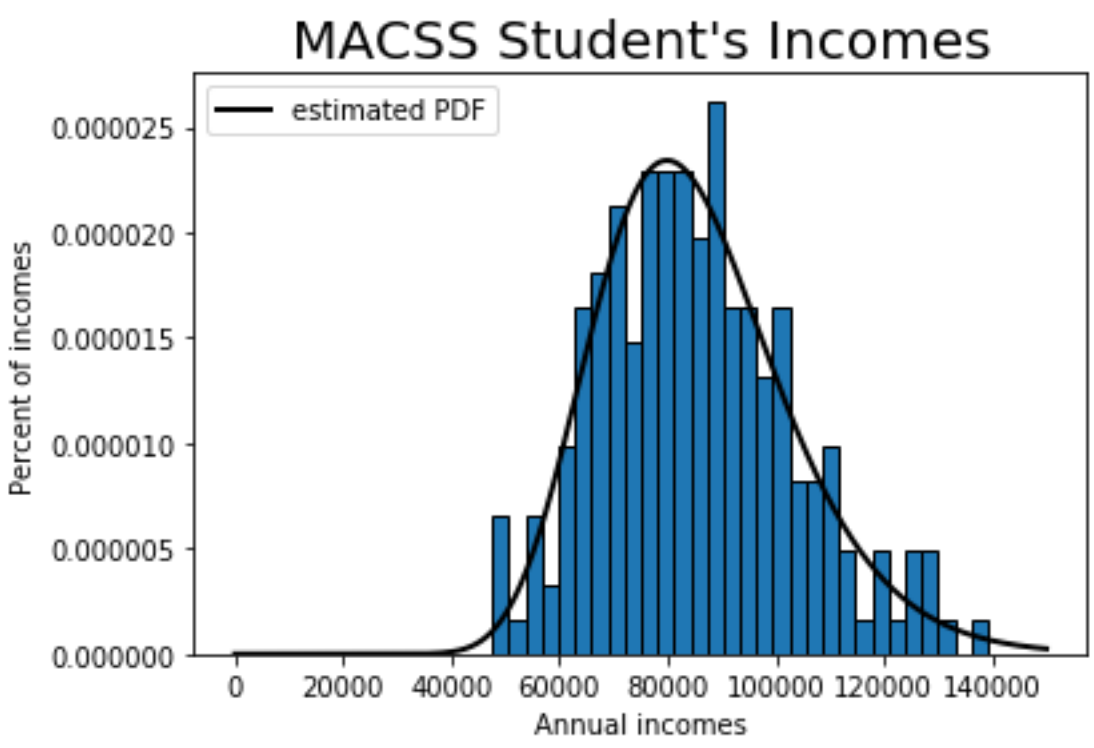
```
mu_init=10
sig_init=0.5
params_init=[mu_init, sig_init]
W_hat=np.eye(2)
gmm_args =(income, W_hat)
results = opt.minimize(crit, params_init, args=(gmm_args),method='L-BFGS-B', bou
nds=((None, None), (1e-10, None)))
mu_GMM1,sig_GMM1 = results.x
```

In [251]:

```
num_bins = 30
count, bins, ignored = plt.hist(income, num_bins, density=True,
                                edgecolor='k')
plt.title("MACSS Student's Incomes", fontsize=20)
plt.xlabel('Annual incomes')
plt.ylabel('Percent of incomes')
dist_pts = np.linspace(0.0001, 150000, 150000)
plt.plot(dist_pts, logpdf(dist_pts,mu_GMM1,sig_GMM1),
         linewidth=2, color='k', label='estimated PDF')
plt.legend(loc='upper left')
```

Out[251]:

```
<matplotlib.legend.Legend at 0x14d41a0b8>
```



In [252]:

```
mean_data,var_data= data_m(income)
mean_model,var_model=model_m(mu_GMM1,sig_GMM1)
err1 = err(income,mu_GMM1, sig_GMM1, True).reshape(2,)
print('mu_GMM1:',mu_GMM1,'sig_GMM1:',sig_GMM1)
print('the value of the GMM criterion function at the estimated parameter values
is:', results.fun[0][0])
print('average income from data=', mean_data, ', standard deviation of income fr
om data=', var_data)
print('average income from GMM=', mean_model, ', standard deviation of income fr
om GMM=', var_model)
print('Error vector=', err1)
```

```
mu_GMM1: 11.3318808543205 sig_GMM1: 0.20869663038572484
the value of the GMM criterion function at the estimated parameter v
alues is: 2.2184833798327347e-14
average income from data= 85276.82360625811 , standard deviation of
income from data= 17992.542128046523
average income from GMM= 85276.8239494969 , standard deviation of in
come from GMM= 17992.53944911269
Error vector= [ 0.00034324 -0.00267893]
```

We could see that the average income from data is 85276.82 and from GMM estimation is also 85276.82. The standard deviation of income from data is 17992.54 and from the GMM estimation is also 17992.54. Therefore, the data moments and the model moments are very close.

c.

In [253]:

```
import numpy.linalg as lin
def get_Err_mat2(income,mu,sigma,simple=False):
    R=2
    N=len(income)
    Err_mat = np.zeros((R, N))
    mean_model, var_model = model_m(mu, sigma)
    if simple:
        Err_mat[0, :]=income-mean_model
        Err_mat[1, :]=((mean_data-income)**2)-var_model
    else:
        Err_mat[0, :]=(income - mean_model)/ mean_model
        Err_mat[1, :]=(((mean_data-income)**2)-var_model)/var_model

    return Err_mat
```

In [254]:

```
Err_mat=get_Err_mat2(income,mu_GMM1,sig_GMM1,False)
VCV2=(1/income.shape[0])*(Err_mat @ Err_mat.T)
W_hat2=lin.inv(VCV2)
print(VCV2)
print(W_hat2)
```

```
[[4.45167061e-02 1.68385311e+03]
 [1.68385311e+03 9.54184674e+08]]
[[ 2.40701668e+01 -4.24767094e-05]
 [-4.24767094e-05  1.12297396e-09]]
```

In [256]:

```
params_init=np.array([mu_GMM1, sig_GMM1])
gmm_args=(income,W_hat2)
results_2=opt.minimize(crit, params_init, args=(gmm_args),
                       method='L-BFGS-B', bounds=((None, None), (1e-10, None)))
mu_GMM2,sig_GMM2=results.x
print('mu_GMM2=',mu_GMM2,' sig_GMM2=',sig_GMM2)
print('the value of the GMM criterion function at the estimated parameter values
is:', results_2.fun[0][0])
```

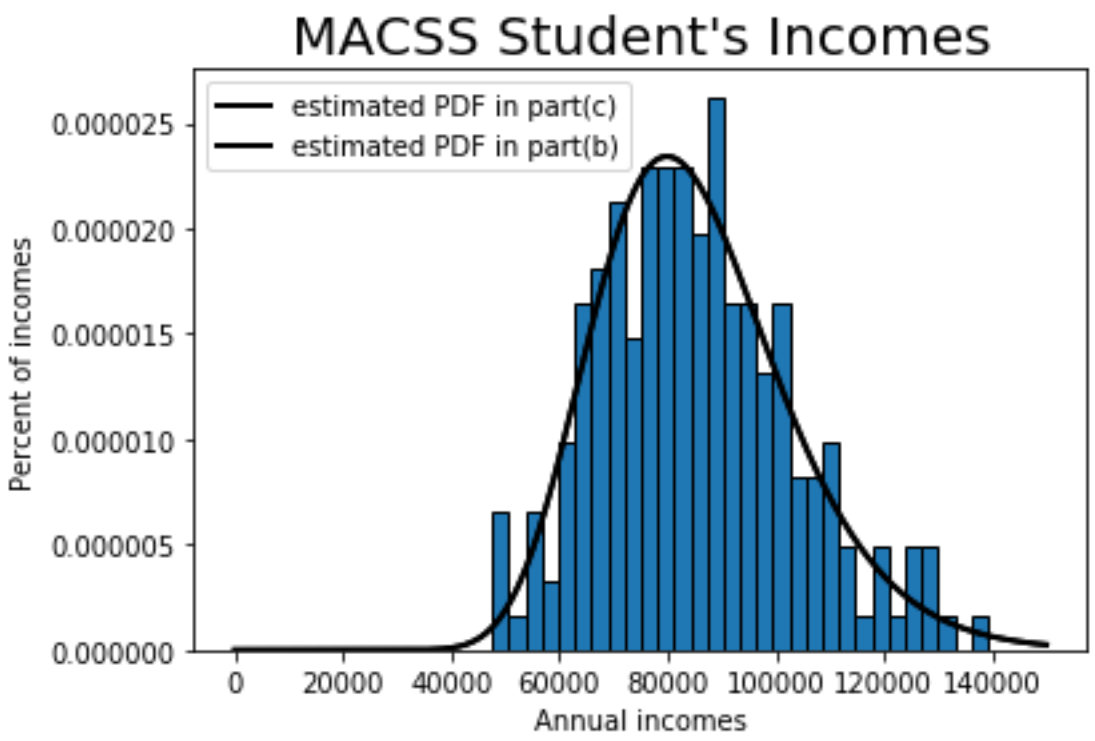
```
mu_GMM2= 11.3318808543205  sig_GMM2= 0.20869663038572484
the value of the GMM criterion function at the estimated parameter v
alues is: 3.9000171460277093e-16
```

In [257]:

```
num_bins = 30
count, bins, ignored = plt.hist(income, num_bins, density=True,
                                edgecolor='k')
plt.title("MACSS Student's Incomes", fontsize=20)
plt.xlabel('Annual incomes')
plt.ylabel('Percent of incomes')
dist_pts = np.linspace(0.0001, 150000, 150000)
plt.plot(dist_pts, logpdf(dist_pts,mu_GMM2,sig_GMM2),
         linewidth=2, color='k', label='estimated PDF in part(c)')
plt.plot(dist_pts, logpdf(dist_pts,mu_GMM1,sig_GMM1),
         linewidth=2, color='k', label='estimated PDF in part(b)')
plt.legend(loc='upper left')
plt.legend(loc='upper left')
```

Out[257]:

```
<matplotlib.legend.Legend at 0x14d141da0>
```



In [259]:

```
print('mu_GMM2=',mu_GMM2,' sig_GMM2=',sig_GMM2)
mean_data2,var_data2= data_m(income)
mean_model2,var_model2=model_m(mu_GMM2,sig_GMM2)
err2= err(income,mu_GMM2, sig_GMM2, True).reshape(2,)
print('the value of the GMM criterion function at the estimated parameter values
is:', results_2.fun[0][0])
print('average income from data=', mean_data2, ', standard deviation of income f
rom data=', var_data2)
print('average income from GMM=', mean_model2, ', standard deviation of income f
rom GMM=', var_model2)
print('Error vector=', err2)
```

```
mu_GMM2= 11.3318808543205  sig_GMM2= 0.20869663038572484
the value of the GMM criterion function at the estimated parameter v
alues is: 3.9000171460277093e-16
average income from data= 85276.82360625811 , standard deviation of
income from data= 17992.542128046523
average income from GMM= 85276.8239494969 , standard deviation of in
come from GMM= 17992.53944911269
Error vector= [ 0.00034324 -0.00267893]
```

We could see that the average income from data is about 85276.82 and from GMM estimation is also about 85276.82. The standard deviation of income from data is about 17992.54 and from the GMM estimation is also about 17992.54. Therefore, the data moments and the model moments are very close.

d.

In [260]:

```
def data_moments3(x):
    perc1=x[x<75000].shape[0]/ x.shape[0]
    perc2=(x[(x>=75000)&(x<100000)].shape[0]/x.shape[0])
    perc3=(x[(x>=100000)].shape[0]/x.shape[0])
    return perc1,perc2,perc3
def model_moments3(mu,sigma):
    xfx = lambda x: logpdf(x,mu,sigma)
    (bpct_1_mod, bp_1_err)=scipy.integrate.quad(xfx, 0, 75000)
    (bpct_2_mod, bp_2_err)=scipy.integrate.quad(xfx, 75000, 100000)
    (bpct_3_mod, bp_3_err)=scipy.integrate.quad(xfx, 100000, +np.inf)
    return bpct_1_mod, bpct_2_mod, bpct_3_mod
def err_vec3(x,mu,sigma,simple):
    bpct_1_dat, bpct_2_dat, bpct_3_dat=data_moments3(x)
    moms_data=np.array([[bpct_1_dat], [bpct_2_dat], [bpct_3_dat]])
    bpct_1_mod, bpct_2_mod, bpct_3_mod=model_moments3(mu,sigma)
    moms_model=np.array([[bpct_1_mod], [bpct_2_mod], [bpct_3_mod]])
    if simple:
        err_vec=moms_model - moms_data
    else:
        err_vec=(moms_model - moms_data) / moms_data
    return err_vec
def crit3(params, *args):
    mu,sigma=params
    x,W=args
    err=err_vec3(x,mu,sigma,simple=False)
    crit_val = err.T @ W @ err
    return crit_val
```

In [261]:

```
mu_init=11
sig_init=0.2
params_init = np.array([mu_init, sig_init])
W_hat1_3 = np.eye(3)
gmm_args = (income,W_hat1_3)
results_3 = opt.minimize(crit3, params_init, args=(gmm_args),
                         method='L-BFGS-B', bounds=((None, None), (1e-10, None)))
mu_GMM1_3, sig_GMM1_3 = results_3.x
print('mu_GMM1_3=', mu_GMM1_3, ' sig_GMM1_3=', sig_GMM1_3)
```

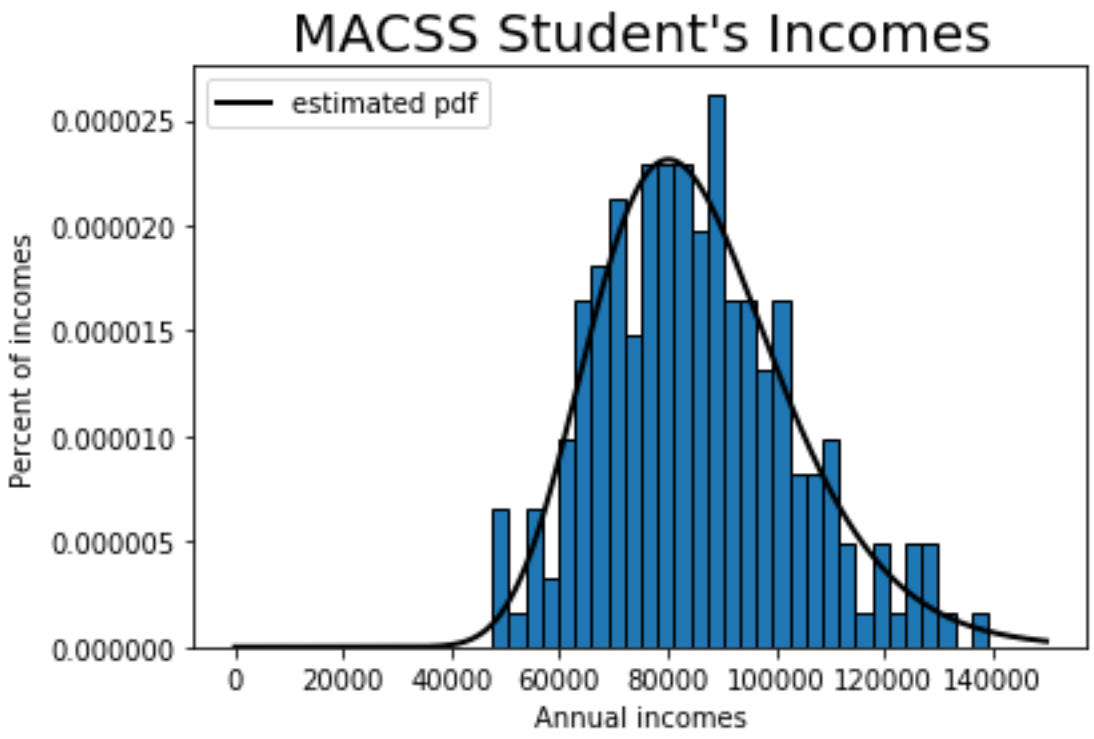
```
mu_GMM1_3= 11.335681325682913  sig_GMM1_3= 0.21059845527059384
```

In [264]:

```
num_bins=30
count, bins, ignored = plt.hist(income, num_bins, density=True,edgecolor='k')
plt.title("MACSS Student's Incomes",fontsize=20)
plt.xlabel('Annual incomes')
plt.ylabel('Percent of incomes')
dist_pts = np.linspace(0.0001, 150000, 150000)
plt.plot(dist_pts, logpdf(dist_pts,mu_GMM1_3,sig_GMM1_3),
         linewidth=2, color='k', label='estimated pdf')
plt.legend(loc='upper left')
```

Out[264]:

```
<matplotlib.legend.Legend at 0x14f8d4f98>
```



In [265]:

```
print('mu_GMM3=',mu_GMM1_3,' sig_GMM3=',sig_GMM1_3)
m13,m23,m33= data_moments3(income)
m_mode13,m_mode23,m_mode33=model_moments3(mu_GMM1_3,sig_GMM1_3)
err3= err_vec3(income,mu_GMM1_3, sig_GMM1_3, True).reshape(3,)
print('the value of the GMM criterion function at the estimated parameter values
is:', results_3.fun[0][0])
print('the first data moment=',m13, ', the second data moment=',m23,', the third
data moment=',m33)
print('the first model moment=',m_mode13, ', the second model moment=',m_mode23,
', the third model moment=',m_mode23)
print('Error vector=', err3)
```

```
mu_GMM3= 11.335681325682913  sig_GMM3= 0.21059845527059384
the value of the GMM criterion function at the estimated parameter v
alues is: 3.238799432760124e-15
the first data moment= 0.3 , the second data moment= 0.5 , the third
data moment= 0.2
the first model moment= 0.30000000785030917 , the second model momen
t= 0.5000000022181954 , the third model moment= 0.5000000022181954
Error vector= [ 7.85030918e-09  2.21819541e-09 -1.00685043e-08]
```

The three data moments are 0.3, 0.5, 0.2, and the three model moments are all very close to their corresponding data moment. We could also see that all elements in the error vector are below 10^{-8}, so the three data moments and three model moments are close.

e.

In [266]:

```
def get_Err_mat3(income, mu, sigma,simple=False):
    R = 3
    N = len(income)
    Err_mat = np.zeros((R, N))
    p1, p2, p3 = model_moments3(mu, sigma)
    if simple:
        pd1=(income<75000)
        Err_mat[0, :]=pd1-p1
        pd2=(income>=75000)&(income<100000)
        Err_mat[1, :]=pd2-p2
        pd3=(income>=1000000)
        Err_mat[2, :]=pd3-p3
    else:
        pd1=(income<75000)
        Err_mat[0, :] = (pd1- p1) / p1
        pd2=(income>=75000)&(income<100000)
        Err_mat[1, :] = (pd2-p2) /p2
        pd3=(income>=100000)
        Err_mat[2, :] = (pd3-p3)/p3
    return Err_mat
```

In [267]:

```
Err_mat3=get_Err_mat3(income,mu_GMM1_3,sig_GMM1_3,True)
VCV2_3=(1/income.shape[0])*(Err_mat3 @ Err_mat3.T)
W_hat2_3=lin.pinv(VCV2_3)
print(VCV2_3)
print(W_hat2_3)
```

```
[[ 2.10000000e-01 -1.50000000e-01  1.57006174e-09]
 [-1.50000000e-01  2.50000000e-01  4.43639066e-10]
 [ 1.57006174e-09  4.43639066e-10  3.99999960e-02]]
[[ 8.33333333e+00  5.00000000e+00 -3.82551117e-07]
 [ 5.00000000e+00  7.00000000e+00 -2.73894581e-07]
 [-3.82551117e-07 -2.73894581e-07  2.50000025e+01]]
```

In [270]:

```
params_init = np.array([mu_GMM1_3,sig_GMM1_3])
gmm_args = (income,W_hat2_3)
results2_3=opt.minimize(crit3, params_init, args=(gmm_args),
                        method='L-BFGS-B', bounds=((None, None), (1e-10, None)
))
mu_GMM2_3,sig_GMM2_3=results2_3.x
print('mu_GMM2_3=', mu_GMM2_3, ' sig_GMM2_3=', sig_GMM2_3)
print('the value of the GMM criterion function at the estimated parameter values
is:', results2_3.fun[0][0])
```

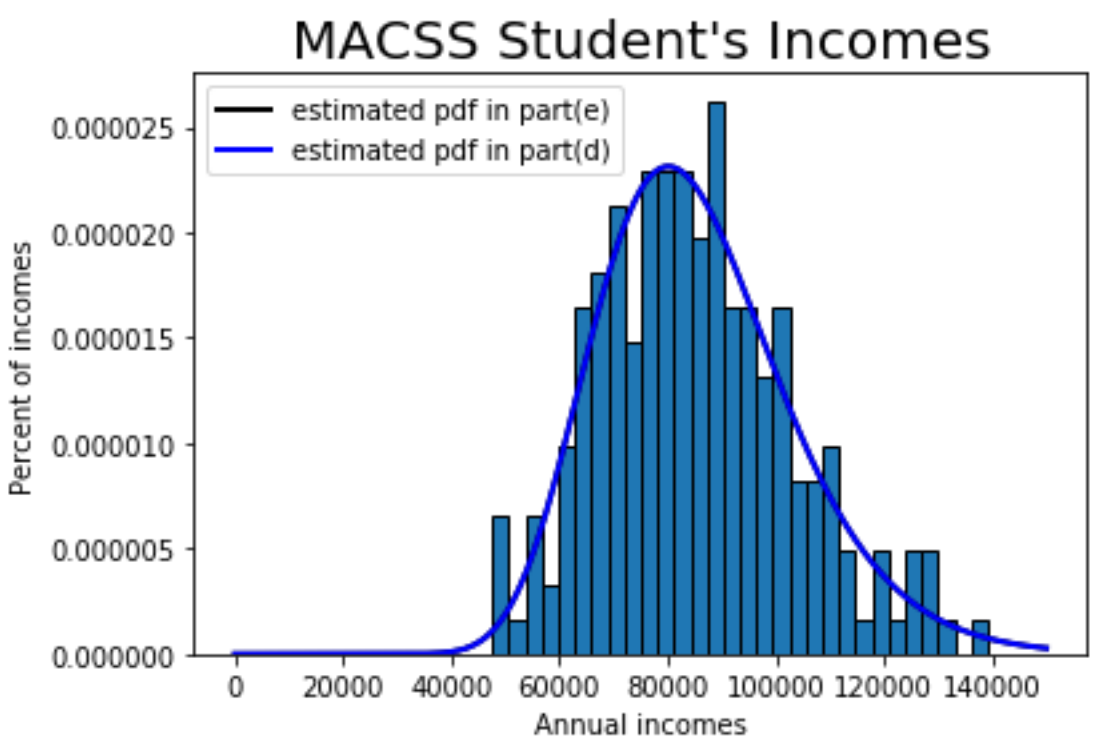
```
mu_GMM2_3= 11.335681325682913  sig_GMM2_3= 0.21059845527059384
the value of the GMM criterion function at the estimated parameter v
alues is: 7.036415349843903e-14
```

In [271]:

```
num_bins = 30
count, bins, ignored = plt.hist(income, num_bins, density=True,
                                edgecolor='k')
plt.title("MACSS Student's Incomes", fontsize=20)
plt.xlabel('Annual incomes')
plt.ylabel('Percent of incomes')
dist_pts = np.linspace(0.0001, 150000, 150000)
plt.plot(dist_pts, logpdf(dist_pts,mu_GMM2_4,sig_GMM2_4),
         linewidth=2, color='k', label='estimated pdf in part(e)')
plt.plot(dist_pts, logpdf(dist_pts,mu_GMM1_3,sig_GMM1_3),
         linewidth=2, color='b', label='estimated pdf in part(d)')
plt.legend(loc='upper left')
```

Out[271]:

```
<matplotlib.legend.Legend at 0x150dcbfd0>
```



In [272]:

```
print('mu_GMM3=',mu_GMM2_3,' sig_GMM3=',sig_GMM2_3)
m132,m232,m332= data_moments3(income)
m_mode132,m_mode232,m_mode332=model_moments3(mu_GMM2_3,sig_GMM2_3)
err32= err_vec3(income,mu_GMM2_3, sig_GMM2_3, True).reshape(3,)
print('the value of the GMM criterion function at the estimated parameter values
is:', results2_3.fun[0][0])
print('the first data moment=',m132, ', the second data moment=',m232,', the thi
rd data moment=',m332)
print('the first model moment=',m_mode132, ', the second model moment=',m_mode23
2,', the third model moment=',m_mode232)
print('Error vector=', err32)
```

```
mu_GMM3= 11.335681325682913  sig_GMM3= 0.21059845527059384
the value of the GMM criterion function at the estimated parameter v
alues is: 7.036415349843903e-14
the first data moment= 0.3 , the second data moment= 0.5 , the third
data moment= 0.2
the first model moment= 0.30000000785030917 , the second model momen
t= 0.5000000022181954 , the third model moment= 0.5000000022181954
Error vector= [ 7.85030918e-09  2.21819541e-09 -1.00685043e-08]
```

The three data moments are 0.3, 0.5, 0.2, and the three model moments are all very close to their corresponding data moment. We could also see that all elements in the error vector are below 10^{-8}, so the three data moments and three model moments are close.

f.

I think part (c) fits the data best because the value of the GMM criterion function at the estimated parameter values is the smallest (which is 3.9000171460277093e-16) in part (c) among the four parts.

2

a.

In [273]:

```
q2= open('/Users/fulinguo/Desktop/persp-model-econ_W19/ProblemSets/PS5/data/sick
.txt', 'r')
data=q2.readlines()
sick=[]
age=[]
children=[]
temp_winter=[]
for i in range(1,len(data)):
    sick.append(float(data[i].split(',')[0]))
    age.append(float(data[i].split(',')[1]))
    children.append(float(data[i].split(',')[2]))
    temp_winter.append(float(data[i].split(',')[3]))
sick=np.array(sick)
age=np.array(age)
children=np.array(children)
temp_winter=np.array(temp_winter)
```

In [274]:

```
def model2_m(age,children,temp_winter,beta0,beta1,beta2,beta3):
    return beta0+beta1*age+beta2*children+beta3*temp_winter
def err2(sick,age,children,temp_winter,beta0,beta1,beta2,beta3,simple):
    moms_data=sick
    moms_model=model2_m(age,children,temp_winter,beta0,beta1,beta2,beta3)
    if simple:
        err_vec=moms_model-moms_data
    else:
        err_vec=(moms_model-moms_data)/moms_data
    return err_vec
def crit2(params, *args):
    beta0,beta1,beta2,beta3 = params
    sick,age,children,temp_winter, W = args
    err = err2(sick,age,children,temp_winter, beta0,beta1,beta2,beta3,simple=Tru
e)
    crit_val = err.T @ W @ err
    return crit_val
```

In [275]:

```
beta0_init = 10.0
beta1_init = 0.0
beta2_init = 0.0
beta3_init = 0.0
params_init = np.array([beta0_init, beta1_init, beta2_init, beta3_init])
WW = np.eye(200)
gmm_args = (sick,age,children,temp_winter,WW)
results_4 = opt.minimize(crit2, params_init, args=(gmm_args),
                         method='L-BFGS-B', bounds=((None, None), (None, None),(No
ne, None), (None, None)))
beta0_GMM, beta1_GMM,beta2_GMM, beta3_GMM = results_4.x
print('beta0:',beta0_GMM, 'beta1:',beta1_GMM, 'beta2:',beta2_GMM, 'beta3:',beta3
_GMM)
print('The value of the GMM criterion function is:', results_4.fun)
```

```
beta0: 0.25164463318989333 beta1: 0.012933452102936l2 beta2: 0.40050
118318764755 beta3: -0.009991694856086237
The value of the GMM criterion function is: 0.0018212898101786223
```

Therefore, the estimated β_0 is 0.252, the estimated β_1 is 0.013, the estimated β_2 is 0.401, the estimated β_3 is -0.010, and the value of the GMM criterion function is 0.001821