MATH 42 HOMEWORK 7

This homework is due at 11:59 p.m. (Eastern Time) on Wednesday, November 4. Scan the completed homework and upload it **as one pdf file** to Gradescope. The Canvas module "Written Assignments" has instructions for how to upload the assignment to Gradescope. This assignment covers §16.5−16.7.

Be sure to show work (integration by parts, substitutions, etc.) when calculating integrals. Unless stated in the problem, it is insufficient to simply respond with a numerical evaluation of definite integrals or an antiderivative of a non-standard integrand.

(1) Find the volume of the solid bounded by the cylinders $r = 1$ and $r = 2$ and the cones $\phi = \pi/6$ and $\phi = \pi/3$.

(2) Suppose the temperature inside a three dimensional ball is proportional to the square root of the distance from the center. Find a formula for the average temperature in the ball. What is the limit of the average temperature as the ball gets larger?

(3) Let $f(x, y, z) = \frac{1}{(x^2+y^2+z^2)}$ be the charge density (Coulomb per unit volume) for the solid inside the sphere $x^2 + y^2 + z^2 = 40$ and outside the cone $z^2 = 9(x^2 + y^2)$. Set up the integral (including bounds) in both cylindrical and spherical coordinates, but compute only one of them.

(4) Find the mass of the parallelepiped given by $0 \le x + y + z \le 10, 0 \le y + z \le 5$, and $0 \le z \le 2$, with density $f(x, y, z) = yz + z^2$, using a change of variables. (Note that the density is nonnegative since $yz + z^2 = z(y + z)$, and neither component may be less than 0.)

For each of the following,

- Sketch the original region of integration.
- Choose a useful change of variables based on the region of integration and the integrand (more than one choice might work here). Remember, your goal is to simplify both the region (i.e. endpoints on the integrals) and the integrand, so you get something easier to evaluate.
- Sketch the new region based on your change of variables.
- Transform the integrand to the new variables.
- Compute the Jacobian of your transformation.
- Write the integral in the new coordinate system.
- Evaluate the integral.

(5) $\iint_R (3x + 6y)^2 dA$, where R is the diamond bounded by $x = 2 - 2y, x = -2 + 2y, x = -2 - 2y, x = 2 + 2y$. (Hint: factor the integrand)

(6) $\iint_R xy dA$ where R is the interior of the ellipse $9x^2 + 4y^2 = 36$.