Wednesday, January 18

Monday, January 2, 2023 20:55

Please feel free to call me Todd or anything I'll recognize that is not dirty ;-) .

Temporary student hours: 1:30-3:00 JCC 575 every Friday!

I will survey you for permanent office hours.

MATHEMATICAL CONTEST IN MODELING: February 16-20, 2023. TEAMS OF THREE UNDERGRADS

https://www.contest.comap.com/undergraduate/contests/

In order to give you more information and allow you to meet other students who may be interested in forming a team, we will hold an information session on Wednesday, January 25th at 6:00pm. ROOM 574 JCC

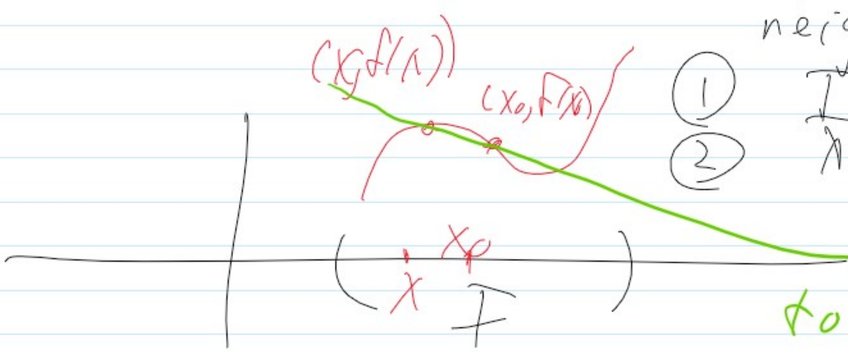
They will briefly discuss the format and some sample questions that have been given in the past. Please RSVP to either Arkadz Kirshtein (Arkadz.Kirshtein@tufts.edu) or James Adler (james.adler@tufts.edu) as soon as you can just so we can get an idea of interest and start a email list for students to find teammates.

Introduction

Derivatives for $f: I \to \mathbb{R}$

I open $x_0 \in I$ (I is called a nbd of x_0)

neighborhood



(1) I open
(2) $x_0 \in I$

secant line to graph at $(x, f(x))$, & $(x_0, f(x_0))$

slope is $\dfrac{f(x) - f(x_0)}{x - x_0}$

goal: get slope of curve at $(x_0, f(x_0))$

to get slope of curve take limit as $x \to x_0$

Defn $x_0 \in \mathbb{R}$, I nbd of x_0, $f: I \to \mathbb{R}$

f is diff at x_0 (differentiable) if

$$\frac{d}{dx}f(x_0) = f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \quad \underline{\text{exists}}$$

alternate form of limit: $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$, $h = x - x_0$

Thm $n \in \mathbb{Z}$ then $f(x) = x^n$ is diff on its domain and

$$\frac{d}{dx} x^n = n x^{n-1}$$

pf $n \geq 0$ pf

$n < 0$ pf.

now $n \geq 0$

$$\frac{d}{dx} x^n = \lim_{h \to 0} \frac{(x+h)^n - (x)^n}{h} \quad \text{if limit exists}$$

$$= \lim_{h \to 0} \frac{\cancel{x^n} + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \cdots + \binom{n}{n} h^n - x_n}{h}$$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ $\binom{n}{n} = 1$ $\binom{n}{1} = n$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad \binom{n}{n} = 1 \quad \binom{n}{1} = n$

$$= \lim_{h \to 0} \frac{h}{h}\left(nx^{n-1} + \binom{n}{2}x^{n-2}h + \dots + h^{n-1}\right)$$

$\to 0$

$$= nx^{n-1}.$$

Thm (diff fns are cont)

I open interval $x_0 \in I$ $f: I \to \mathbb{R}$

f is diff at x_0. Then f is cont at x_0.

if f is cont at x_0 is f diff at x_0: Nope

Pf f diff at x_0 so

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \quad \text{exists}$$

Goal: prove f is cont. at x_0

as den $\to 0$ prove num $\to 0$

f is cont at x_0 iff

$$\lim_{x \to x_0} f(x) - f(x_0) = 0$$

$$\left(\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}\right) \lim_{x \to x_0} (x - x_0) = f'(x_0) \cdot 0 = 0$$

$$0 = \lim_{x \to x_0} \left(\frac{f(x) - f(x_0)}{x - x_0}\right)(x - x_0)$$

$$= \lim \left(f(x) - f(x_0)\right) \to f \text{ cont at } x_0.$$

$$= \lim_{x \to x_0} (f(x) - f(x_0)) \to f \text{ cont at } x_0.$$

Thm $I \subseteq \mathbb{R}$ nbd of x_0, $f: I \to \mathbb{R}$, $g: I \to \mathbb{R}$ diff at x_0, then

$$\frac{d}{dx}(f+g)(x_0) = f'(x_0) + g'(x_0)$$

$$c \in \mathbb{R} \quad ((cf)'(x_0) = c f'(x_0)$$

$$(f \cdot g)'(x_0) = f(x_0) g'(x_0) + g(x_0) f'(x_0)$$

$$\text{if } g(x_0) \neq 0 \quad \frac{d}{dx}\left(\frac{f}{g}\right)(x_0) = \frac{g(x_0) f'(x_0) - f(x_0) g'(x_0)}{g^2}$$

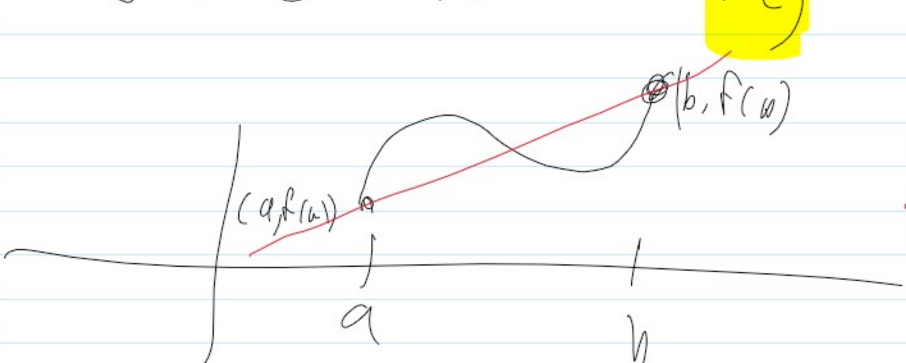
$$\frac{d}{dx}\left(\frac{hi}{ho}\right) = \frac{ho\, d\, hi - hi\, d\, ho}{ho\ ho}$$

Mean Value thm MVT

Thm (MVT) $f: [a,b] \to \mathbb{R}$ cont on $[a,b]$ diff on (a,b) then

$$\exists\, c \in (a,b) \text{ st } f'(c) = \underbrace{\frac{f(b) - f(a)}{b-a}}$$

slope of secant line from $(a, f(a))$ to $(b, f(b))$



local max & local min

local max & local min

Defn. $A \subset \mathbb{R}$ $\quad f: A \to \mathbb{R}$ $\quad x_0 \in A$

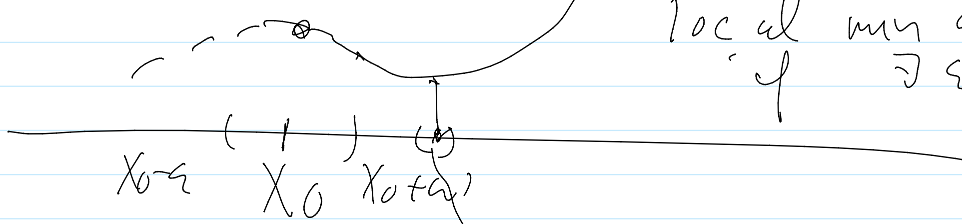
f has local max at x_0

if $\exists\, \varepsilon > 0$ st

$$f(x) \le f(x_0) \qquad x \in A \cap (x_0 - \varepsilon, x_0 + \varepsilon)$$

local min at x_0 if $\exists\, \varepsilon > 0$ st

$$f(x) \ge f(x_0) \qquad x \in A \cap (x_0 - \varepsilon, x_0 + \varepsilon)$$



Thm $f: I \to \mathbb{R}$, I nbhd of $x_0 \in \mathbb{R}$

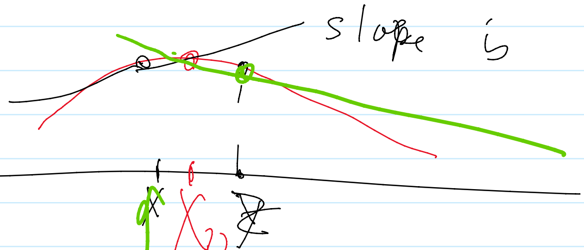
assume f is diff at x_0

and f has local extremum (max or min)

at x_0 then $f'(x_0) = 0$

pf for max

slope is



slope of sec line ≥ 0 $\qquad x < x_0$ x in nbhd

slope of sec line ≤ 0 $\qquad x > x_0$ x in nbhd

$f'(x_0) = \lim$ is 0?