MATH 42 HOMEWORK 1

Topics covered: Geometry review, § 13.1-4

This homework is due at 11:59 pm (Eastern Time) on Wednesday, September 16. You will need to scan your completed homework and upload it **as one pdf file** to Gradescope. Please see the Canvas module "Written Assignments" for instructions on how to upload your assignment to Gradescope.

(1) Using the Pythagorean Theorem, we showed in class that the distance between two points $(a_1, a_2), (b_1, b_2)$ in the plane is given by

$$\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

Show that the distance between two points $(a_1, a_2, a_3), (b_1, b_2, b_3)$ in 3-space is given by

$$\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

(2) Look at the equation $x^2 + y^2 + z^2 = a$. If a is negative, the equation has no solutions. If $a = 0$ the equation has a unique point as solution namely $(0, 0, 0)$. If $a > 0$, the set of solutions form a sphere.

Look now at the equation $x^2 + y^2 + z^2 - 4x + 8y - 10z = a$. Classify the set of solutions depending on the value of a.

(3) Given the vectors

$$\vec{u} =< 3, 3, 4 >, \quad \vec{v} =< 2, 0, 1 >$$

find vectors $\vec{x}, \vec{y}$ such that $\vec{x}$ is parallel to $\vec{v}$, $\vec{y}$ is perpendicular to $\vec{v}$, and $\vec{u} = \vec{x} + \vec{y}$.

(4) For the given vectors $\vec{u}$ and $\vec{v}$, calculate the $\text{proj}_{\vec{v}}\,\vec{u}$ and $\text{scal}_{\vec{v}}\,\vec{u}$:

(a) $\vec{u} =< -1, 4 >$ and $\vec{v} =< -4, 2 >$
(b) $\vec{u} =< 3, 3, -3 >$ and $\vec{v} =< 1, -1, 2 >$
(c) $\vec{u} = 5\vec{i} + \vec{j} - 5\vec{k}$ and $\vec{v} = -\vec{i} + \vec{j} - 2\vec{k}$

(5) Find the volume of the parallelepiped (i.e. the slanted box) such that its edges are parallel to the vectors

$$\vec{u} = \langle 1, 0, 3 \rangle, \quad \vec{v} = \langle 0, 2, 1 \rangle, \quad \vec{w} = \langle 1, 1, 1 \rangle.$$