Math 145 Practicum

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Question 1. Find the orbits of the following actions of $\mathbb{R}$ on the torus $\mathbb{R}/\mathbb{Z} \times \mathbb{R}/\mathbb{Z}$:

1. $t \cdot (e^{ix}, e^{iy}) = (e^{i(x+t)}, e^{iy})$
2. $t \cdot (e^{ix}, e^{iy}) = (e^{i(x+t)}, e^{i(y+t)})$
3. $t \cdot (e^{ix}, e^{iy}) = (e^{i(x+t)}, e^{i(y+t\sqrt{2})})$

Question 2. Show that if G is a group acting on a set X and $x, y \in X$ are in the same orbit then the stabilizers of x and y are conjugate subgroups of G.

Question 3. Show that if G is a group acting on a set X and $g, h \in G$ are conjugate in G then then there is a bijection between X^g and X^h.

Question 4. Draw line segments on the faces of a cube such that the lines on opposite faces are parallel and the lines on adjacent faces do not intersect. What are all the ways to paint these lines each red or green, without double counting the ways related by symmetries of the cube?

Question 5. Prepare a short presentation on the proof of the Counting Theorem (page 98 of the textbook).