1. Quiz 3, Monday September 20.

Instructions: SHOW YOUR WORK in order to receive credit. Include proofs and justification for your assertions, with words and equations.

Question 1. (a) *Let $a \in \mathbb{R} - \{1\}$. Show that* $S_n = 1 + a + a^2 + \cdots + a^{n-1} + a^n = \frac{a^{(n+1)}-1}{a-1}$.

1 $n=0$, $S_0 = 1 = \frac{(a^{0+1}-1}{a-1} = \frac{a-1}{a-1} = 1$

Prove for $n-1$, $S_{n-1} = \frac{a^{(n-1+1)}-1}{a-1} = \frac{a^n-1}{a-1}$

$S_{n-1} + a^n = S_n = \frac{a^n-1}{a-1} + a^n\left(\frac{a-1}{a-1}\right) = \frac{a^n-1}{a-1} + \frac{a^{n+1}-a^n}{a-1}$

$= \frac{a^n - 1 + a^{n+1} - a^n}{a-1}$

$S_n = \frac{a^{n+1}-1}{a-1}$

$2^{n-1}+2^{n-1}$ $2^{n-1}(1+1)=2^n$

(b) *The towers of Hanoi is a game consisting of a board with three pivots and a collection of n disks all of different sizes with a hole that fits in the pivots. You start with all the disks stacked in one pivot, all in decreasing order and must move them to one of the other pivots by moving one piece at a time and never placing a disk above a smaller one. What is the minimum number of moves that you need to make to solve the towers of Hanoi puzzle? Prove your answer.*

Assume minimum number of moves is 2^n-1 where $n \in N > 0$ because who wants a game with 0 disks.

Step 1 For $n=1$, $2^n-1 = 2-1 = 1$. It takes 1 move as you just move one disk to a new location

Step 2. For $n-1$ takes $2^{n-1}-1 = k$ moves where $\in N \neq 0$

$S_n = 1+2+4+ \ldots +2^{n-2}+2^{n-1} = 2^n-1$ S_n = total moves

For $n=1$, $S_1 = 1$, $2^1-1=1$.

$S_{n-1} = 2^{n-1}-1+2^n = S_n$ $S_n = 2(2^{n-1})-1 = 2^n-1$

$S_n = 2^n-1$ which agrees w/ our initial assumption

with our assumption

(c) (extra credit) *If your pieces start on pivot 1 and you want your final tower to be in pivot 3, where should you place your top piece in the first move?*

First move to 3rd pivot, because it takes 2^n-1 moves to complete it so