Tufts University
Department of Mathematics

Math 235 **Groupwork #4 : Due on 10/6**[1] **Fall, 2023**

Problem

Suppose that $f : \mathbb{R} \to \mathbb{R}$ is function and let

$$\phi_n(x) = \sum_{k=-\infty}^{\infty} f(\tfrac{k}{n})\chi_{[\frac{k}{n}, \frac{k+1}{n})}(x)$$

(a) Prove that ϕ_n is measurable for each $n \geq 1$, and that for each $x_0 \in \mathbb{R}$, at which f is continuous, $\lim_{n\to\infty} \phi_n(x_0) = f(x_0)$.

(b) Conclude that if f is continuous almost everywhere, then it is a measurable.