Math 166 HW 4

1a) Consistent when $\lim_{n\to\infty} P(|\hat{\lambda}_n-\lambda|<\epsilon)=1$
for $\lambda_1 = Y_1$

$$P(Y_1>2\lambda)=\int_{2\lambda}^{\infty}\lambda e^{-\lambda y}\,dy=e^{-2\lambda^2}$$

$$P(|Y_1-\lambda|<\tfrac{\lambda}{2})<1-e^{-2\lambda^2}<1$$

so not consistent

b) $\hat{\lambda}_n=\sum_{i=1}^{n} Y_i$, $P(\sum Y_i>2\lambda)\geq P(Y_1>2\lambda)=e^{-2\lambda^2}$
sum of n $Y_1 \dots Y_n$ $ne^{-2\lambda^2}<1$, so
not consistent

2 $P^*(A_J)+\sum_{i=1}^{n}P^*(A_K)=1$

$$\frac{(1-P(B_J|A_J))P(A_J)}{1-P(B_J|A_J)P(A_J)}+\frac{\sum P(A_K)}{1-P(B_J|A_J)P(A_J)}=1$$

$$\frac{P(A_J)+\sum P(A_K)-P(A_J)P(B_J|A_J)}{(1-P(B_J|A_J)P(A_J))}=1$$

$$=\frac{1-P(B_J|A_J)P(A_J)}{1-P(B_J|A_J)P(A_J)}=1$$

So new poseteriors for $j\neq K$, for $P(A_K)$)
is $\frac{P(A_K)}{1-P(B_J|A_J)P(A_J)}\to P(A_K)$

3 T = treasure in region r_j, R = event region searched and treasure found

$P(R_1|T_1)=0.9$ $P(R_2|T_2)=0.7$ $P(R_3|T_3)=0.9$
$P(T_1)=0.4$ $P(T_2)=0.1$ $P(T_3)=0.5$
Start in region 3, using Bayesian search strategy
$P^1(T_3)=0.091$ $P^1(T_1)=0.727$ $P^1(T_2)=.182$
Search r_1
$P^2(T_1)=0.218$ $P^2(T_2)=.527$ $P^2(T_3)=0.2634$
Search r_2

$P^3(T_2) = .251$ $P^3(T_1) = 0.333$ $P^3(T_3) = .416$

Search r_3

$P^4(T_3) = 0.067$ $P^4(T_1) = .534$ $P^4(T_2) = .402$

Search r_1

$P^5(T_1) = .103$ $P^5(T_2) = .774$ $P^5(T_3) = .129$

Any probabilities not adding to 1 are due to rounding/lack of precision in my answers.

4 Following the continuous definition

$$f_\theta(\theta | X=k) = \frac{p_X(k|\theta) g_\theta(\theta)}{\int_{-\infty}^{\infty} p_X(k|\theta) g_\theta(\theta)\, d\theta}$$

$$p_X(k|\theta) g_\theta = (1-\theta)^{k-1}\theta \cdot \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} \theta^{r-1}(1-\theta)^{s-1}$$

$$= \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} \theta^{r}(1-\theta)^{s+k-2}$$

This is a Beta distribution with parameters $r+1$ and $s+k-1$

That means this is the pdf of $g_\theta(\theta | X=k)$