**Tufts University**
**Department of Mathematics**

**Math 136** **Class work for the Week of March 13, 2023** [1] **Spring, 2023**

**Due date: 11:59 pm, Tuesday, March 28, 2023**

Roughly every week we will work in small groups to learn to write proofs and to solve problems. The problems will be long enough that you might need to talk with your group outside of class. Todd will grade the group work.

- *Scribe:* each week, someone in the group will volunteer to submit to Gradescope the group's answer *and* to enter all the group member names in Gradescope when uploading. This role should rotate through the group.
- *Respect:* when discussing problems, please make sure that everyone feels comfortable speaking and that all feedback is supportive and encouraging. For example:
  - Please make sure everyone has a chance to talk.
  - To start the discussion, you could have everyone suggest topics, theorems and ideas that relate to the question.
  - If you talk at the board please make sure everyone has a piece of chalk so they can add to the discussion.

1. (10 points) Let $\mathbb{I} = [0,1] \times [0,1]$ and let $f : \mathbb{I} \to \mathbb{R}$ be a bounded function. Let $c \in \mathbb{R}$ and assume $f(\mathbf{x}) = c$ for all $\mathbf{x} \in \text{int}(\mathbb{I})$. Prove $f$ is integrable using Riemann's Condition.

   *HINT:* Let $M > 0$ such that $\forall (x,y) \in [0,1]^2$, $-M \leq f(x,y) \leq M$.

   Let $\delta \in (0, 1/2)$ and let $\mathbb{P}_\delta$ be the partition $\mathbb{P}_\delta = (\{0, \delta, 1-\delta, 1\}, \{0, \delta, 1-\delta, 1\})$.

   Let $\epsilon > 0$. Find a condition on $\delta$ for which $U(f, \mathbb{P}_\delta) - L(f, \mathbb{P}_\delta) < \epsilon$.

[1] 