

Question 1. Consider the two lines k and ℓ on the hexagon below. The symmetry group of the regular hexagon D_6 is generated by rotation of 60° and a reflection. Does it matter which of k or ℓ is used for that reflection?

Question 2. Let k, ℓ be two lines in the plane, r the reflection across k and s the reflection across ℓ . Find the order of rs in each of the following cases:

- (a) k and ℓ intersect at angle $\frac{p}{q}\pi$ radians where p, q are positive integers
- (b) k and ℓ intersect at any other angle
- (c) k and ℓ are parallel

Question 3. (a) Fix a positive integer n and let $G_n = \{z \in \mathbb{C} \mid z^n = 1\}$. Show that G_n forms a group under multiplication of complex numbers.

(b) Now let n vary; show G is a group under multiplication complex numbers where:

$$G = \bigcup_{n=1}^{\infty} \{z \in \mathbb{C} \mid z^n = 1\}$$

Question 4. (a) Which of the following sets form a group under multiplication modulo 14:

- $\{1, 3, 5\}$
- $\{1, 3, 5, 7\}$
- $\{1, 7, 13\}$
- $\{1, 9, 11, 13\}$

(b) Show that if a subset of $\{1, 2, \dots, 21\}$ contains an even number or the number 11 then it cannot form a group under multiplication modulo 22.

Question 5. Prove that if p is a prime number then the set $\{1, 2, \dots, p-1\}$ with multiplication modulo p is a group.

Question 6. Let G be a group and $x, y, g \in G$

- (a) Show that x and gxg^{-1} have the same order.
- (b) Show that xy and yx have the same order.

Question 7. Let $G = \{x \in \mathbb{Q} \mid 0 \leq x < 1\}$ and define

$$x +_1 y = \begin{cases} x + y, & 0 \leq x + y < 1 \\ x + y - 1, & x + y \geq 1 \end{cases}$$

Show G with $+_1$ is a infinite group and every element of G has finite order.