Math 165 HW 5

I worked w/ David on HW5

1 $-5 \le X - Y \le 5$

$X-Y$	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X-Y)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

$$E(X-Y) = \sum_{\text{all } x,y} (X-Y)\,P(X-Y) = 0$$

2 Want $E(X)$ for one group where X is number of tests.

$E(X) = \sum x P(X=x)$, either 1/11 tests and P need 1 test $= \left(\frac{9}{10}\right)^{10}$, $P(X=11) = 1-\left(\frac{9}{10}\right)^{10}$

$$E(X) = 11\left(1-\left(\tfrac{9}{10}\right)^{10}\right) + 1(.35) = 7.5$$

Total tests $= 10E(X) = \boxed{75 \text{ expected tests}}$

3 a) For $I_A = P(A)$ $I_B = P(B)$

$I_A = \begin{cases} 1 & \text{if } P(A) \\ 0 & \text{for } 1-P(A) \end{cases}$ & $I_B = \begin{cases} 1 & \text{if } P(B) \\ 0 & \text{for } 1-P(B) \end{cases}$

$(I_A + I_B)^2 = 0, 1, 4$ as $I_A + I_B = 0, 1, 2$

$P(I_A + I_B)^2 = 0 = (1-P(A))($ $P(A)(1-P(B)) + P(B)(1-P(A))$

$P(A) - P(A)P(B) + P(B) -$

$(I_A+I_B)^2$	0	1	4
$P((I_A+I_B)^2)$	$(1-P(A))(1-P(B))$	$P(A)+P(B)-P(A)P(B)$	$P(A)P(B)$

b) $E[(I_A + I_B)^2] = E(I_A^2 + 2I_AI_B + I_B^2) = P(A) + P(B)$

$(P(A) + P(B))^2 = P(A)^2 + 2P(A)P(B) + P(B)^2$

$E = 0(1-P(A))(1-P(B)) + 1(P(A) + P(B) - P(A)P(B)) + 4P(A)P(B)$

$E = P(A) + P(B) - P(A \cap B) + 4P(A \cap B)$

$E = P(A) + P(B) + 3P(A \cap B)$

4 Each person has $\frac{1}{10}$ chance to get out at each floor.
Forms binomial distribution w/ np for people getting out per floor, $\mu = 12(\frac{1}{10}) = 1.2$
$\frac{10}{1.2} = 8.33$ floors which rounds up to 9

5 a) Since $P(X_1=k) = P(X_2=k)$ can just find $P(X_1=k)$ which Total $P(X_1=k)$, [illegible] by [illegible] k, og is k nonaces and 1 ace so have formula:
$$P(X_i=k) = \prod_{i=0}^{k} \frac{48-i}{52-i} \cdot \frac{4}{52-k}$$

b) $E(X_i) = \mu = X_1 \frac{\text{num. of [illegible]}}{\text{num. of outcomes}} = \frac{48}{5} = \boxed{9.6}$

c) $X_1 \ldots X_5$ are not pairwise independent since $(X_1 = X_2 = X_3 \ldots X_5)$ have identical distributions they could be considered same random variable and a random variable is only independent w/ itself if $P(X)=1$ or $P(X)=0$, this isn't the case, so they aren't pairwise independant.

6 a) $\mu = np = 100(\frac{1}{2}) = \boxed{50}$

b) $SD = \sqrt{np(1-p)} = 5$ X random var of how many have disease
Using Chebychev $P(|X-E(X)| \geq k\,SD(X)) \leq \frac{1}{k^2}$
75 is 5 SD's away meaning $P \leq \frac{1}{5^2}$ = and $P(X \geq 75) = .04$

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Outcome	Same color	Diff. color
P(Outcome)	4/9	5/9
Winnings	1	-1.1

Expected value $= \frac{4}{9}(1) + \frac{5}{9}(-1.1) = \boxed{\$-0.067}$