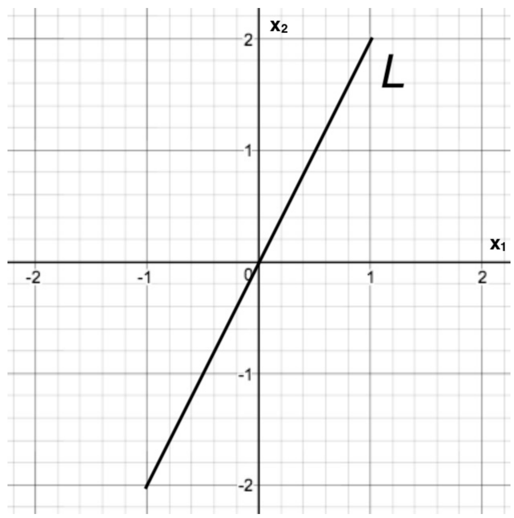
Eigenthings

Math 70

Eigenvalues, eigenvectors, and eigenspaces: they can be confusing the first time (or second or third time) you learn about them. Sure, we learn algorithms to compute them, but why should we care? What can they teach us or do for us?

(1) **First, what is the geometric intuition behind an eigenvector? An eigenvector points in a direction which gets either stretched ($\lambda > 1$), fixed ($\lambda = 1$), contracted ($0 \leq \lambda < 1$), or flipped ($\lambda < 0$), when multiplied by a matrix A.**

Let A be the matrix of the linear transformation on $\mathbb{R}^2$ that reflects points across the line L. You should NOT find the standard matrix A for the transformation.



(a) Find 2 vectors (in 2 different, linearly independent directions) which are either stretched, fixed, contracted, or flipped by this reflection.

(b) Write two eigenspaces as the spans of the two vectors you found above.

(c) Find the eigenvalue associated to each eigenspace. Is each space stretched, fixed, contracted, or flipped?

(2) **Consider** $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$.

(a) What are the eigenvalues of A and B?

(b) Find the eigenspaces for A. Is A diagonalizable? Is A invertible?

(c) Find the eigenspaces for B. Is B diagonalizable? Is B invertible?

(d) Are A and B similar matrices? Are they row equivalent?

(e) Students often times confuse being similar with being row equivalent. Note that these are distinct concepts.

(f) Students often confuse being diagonalizable with being invertible. Note that these are distinct concepts.