Tufts University
Department of Mathematics

Math 136 **Homework 1** [1] **Spring, 2023**

Due date: 11:59 pm, Sunday, January 29, 2023 on Gradescope.

You are encouraged to work on problems with other Math 136 students and to talk with Todd and Wentao, but your answers must be in your own words.

This homework covers sections 4.1–4.3 and 13.1, and we will have covered the material for problem 2 on 1/19.

Reading assignment: Read sections 4.1–4.3 and sections 13.1-13.3 of Fitzpatrick by January 25.

Problems:

1 (15 points) Let $f : [a,b] \to \mathbb{R}$ be continuous on $[a,b]$ and differentiable on (a,b). Answer the following without using the Fundamental Theorem of Calculus.

(a) Assume $f'(x) = 0$ for all $x \in (a,b)$. What can you conclude about f on $[a,b]$? Prove your result.

(b) Now assume $f'(x) = 3$ for all $x \in (a,b)$. What can you conclude about f on $[a,b]$? Prove your result.

2 (20 points) Suppose that f is differentiable on (a,b), f' is continuous on (a,b), and $f'(x) \neq 0$ for all $x \in (a,b)$.

(a) Prove that f is one-to-one on (a,b).

(b) Prove that f maps (a,b) onto some open interval I NOTE: unbounded open intervals are allowed.

* The results of part (a) and (b) and the definition of inverse function imply that f^{-1} is defined from domain I onto (a,b). By Theorem 3.29, $f^{-1} : I \to (a,b)$ is continuous.

(c) Prove that the derivative $(f^{-1})'$ exists and is continuous on I. *Note:* in your proof, explain why you do not divide by zero.

3 (15 points) Let $A \subset \mathbb{R}^n$. Prove that A is closed if and only if A contains all its limit points.

NOTE: this is a standard theorem but please don't look the proof up. You'll learn more if you prove this using what you learned in 135 as well as discussions with other students in the class, Todd, and Wentao.

4 (20 points) Decide whether the following limits exist and prove your result. If the limit exists, also determine the value of the limit.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\cos(x^3+y^6)}{3x^3+3y^6}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6+3y^2}$

5 (10 points) Section 4.3, p. 110, Fitzpatrick: # 11.

6 (10 points) Section 4.3, p. 110, Fitzpatrick: # 21.

7 (10 points) Section 13.1, p. 352, Fitzpatrick: # 7. Is the result of this problem true if g is unbounded. Why or why not?