Thursday, September 30

The Kermack-McKendrick ODEs are:

$$\frac{dS}{dt} = -\alpha IS \tag{1}$$
$$\frac{dI}{dt} = -\beta I + \alpha IS \tag{2}$$
$$\frac{dR}{dt} = \beta I \tag{3}$$

From (1) and (3), we concluded:

$$\frac{dS}{dR} = -\frac{\alpha}{\beta}S.$$

When $t = 0$, we assume $R = 0$, so "$S(0)$" is S_0, regardless of whether we take $S(0)$ to mean "S when $t = 0$" or "S when $R = 0$". We now conclude

$$S(R) = e^{-(\alpha/\beta)t}S_0.$$

That's amazing: S can be computed directly from R.

Is that true for the discrete model that the equations (1)–(3) came from? That model was:

$$\begin{aligned} S_{k+1} &= S_k - \alpha\Delta t I_k S_k \\ I_{k+1} &= I_k - \beta\Delta t I_k + \alpha\Delta t I_k S_k \\ R_{k+1} &= R_k + \beta\Delta t I_k \end{aligned}$$

First, note that

$$R_k = \beta\Delta t \sum_{j=0}^{k-1} I_j.$$

(Why is that?) Second, note that

$$S_k = \prod_{j=0}^{k=1}(1 - \alpha\Delta t I_j)S_0.$$

(Here $\prod$ stands for a product, just as $\sum$ stands for a sum.)

Explain why R_k does not determine S_k, but for very small Δt, it does approximately determine S_k.

I count that as an argument for using the ODEs: They simplify the analysis here.