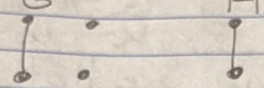
Math 65 HW 10

1a) False

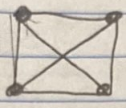
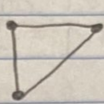


$\alpha(G) = 3$, $\alpha(H) = 2$

$\alpha(G) \geq \alpha(H)$

b) $\alpha(G)$ is largest independent set, meaning for any subgraph H, H can't have an independent set larger than G, as $H \subseteq G$, so $\alpha(G) \geq \alpha(H)$.

c) False

G:  H: 

$w(G) = 4$, $w(H) = 3$

$w(G) \geq w(H)$

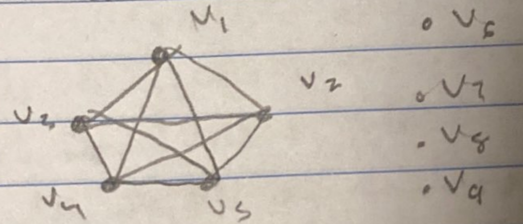
d) $w(G)$ largest clique $\in G$, since all $H \subseteq G$, $\max(H) \leq \max(G)$, meaning that $w(G) \geq w(H)$

2 $w(G) = \alpha(G) = 5$. Need at least 9 vertices

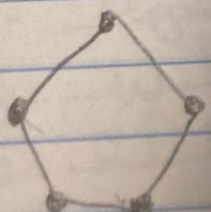
Since $w(G) = 5$, there must be at least 5 vertices

Since $\alpha(G) = 5$, can pick 4 unconnected vertices, then pick a vertex included in $w(G) = 5$. Meaning need at least 9 vertices

$w(G) = 5$, $\alpha(G) = 5$

G: 

3 $w(G) = \alpha(G) = 2$

G: 

$w(G) = 2$, $\alpha(G) = 2$

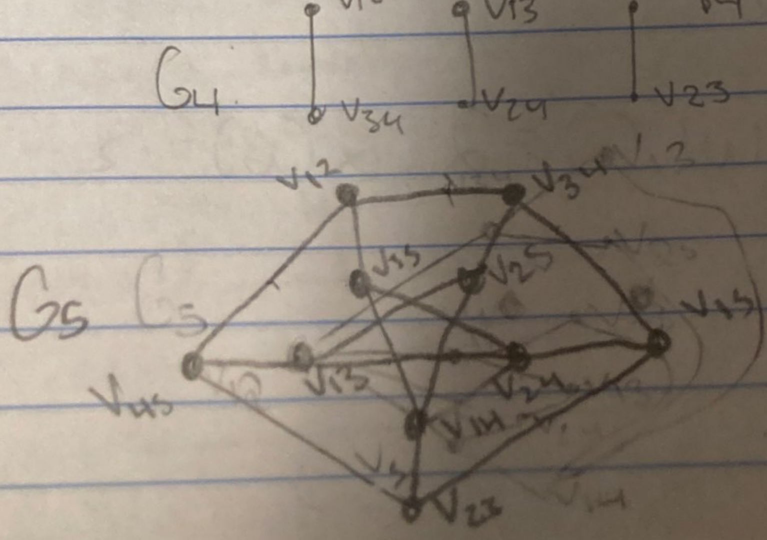
4 Assume G is disconnected.
Take $u, v \in \bar{G}$ w/no edge in G. Meaning edge for u, v exists in $\bar{G}$. Pick vertex w in G such that uw and vw are not edges of G, meaning uw, vw are edges in $\bar{G}$, meaning can construct path from u to v, uw, wv meaning a path exists in $\bar{G}$, meaning if G is disconnected, $\bar{G}$ is disconnected, so G or $\bar{G}$ are connected

5 a) Proof by induction
for $n=2$, $\binom{1}{2}+1=1$ •—• is connected

Assume true for up to graphs w/ n vertices, so show holds for $n+1$.
edges for n vertices $= \binom{n-1}{2}+1$.
When we add another vertex, it must be connected to every vertex, which is $\binom{n-1+1}{2} = \binom{n}{2}+1$ to account for all vertices

b) For $\binom{n-1}{2}$ that connects $n-1$ vertices together but one vertex is left unconnected.
G is not connected

6 a)



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6 b) G_n has $\binom{n}{2}$ vertices, each degree being $\binom{n-2}{2}$, as connected to all edges that don't contain either number

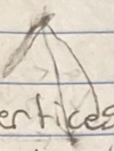
c) edges = $\frac{1}{2}$ vertices · degrees = $\frac{1}{2}\binom{n}{2}\binom{n-2}{2}$

d) Looking at 6a, number of connected components

G_2: 1 component
G_3: 3 components
G_4: 3 components
G_5: 1 component.

For $n \geq 5$, there is 1 connected component.

Proof:
WLOG, to go from $V_{12} \to V_{1b}$, $b \neq 2$ can take path $V_{12} \to V_{xy} \to V_{1b}$, where $x \neq y$, $x, y \neq 1, 2, b$. For this happen, must be at least 5 unique values, so at least 5 vertices.



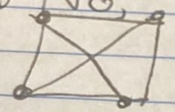
vertices
doublecounted
edges cannot

7 a) If G is noneulerian, we can break G into a bunch of subgraphs. H_1, H_2 ... H_n. Where H_1 is longest path formed by going over each edge once, H_2 2nd largest, and so on, until no edges or vertices are left. Then, take a new v' and draw an edge e_1 from v' to a vertex in H_1, that when H_1 is seperated from G, is of degree 1. Go to end of H_1, then draw edge e_2 from this point back to v', then go from v' to a point on H_2 with degree 1. You can repeat this process for H_1 ... H_n with the circuit complete when you go from end of H_n back along edge e_{2n} to v'.

2n edges

7b) Like in a), let's split G into $H_1, \ldots, H_n$. Trace from point with degree 1 to end of H_1, then draw an edge from end of H_1 to a point considered start of H_2, then repeat this until we reach end of H_n meaning we should need n-1 edges, where n is number of trees that can be created

c) No, take the simple connected noneulerian graph K_4.



Since it is a complete graph, it is impossible to draw another edge and keep it simple without adding new vertices. Therefore we need a loop to make K_4 eulerian.