Math 165 HW 2

I worked with David O. on the HW

1 $P(\text{rain tomorrow} \mid \text{rain today}) = \dfrac{P(\text{rains today and tomorrow})}{P(\text{rains today})} = \dfrac{0.3}{.4} = \boxed{0.75}$

2 a) A and B are mutually exclusive if $P(A \cap B) = 0$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.5 + P(B) - 0$, $\boxed{P(B) = 0.3}$

b) A and B are independent if $P(A \cap B) = P(A)P(B)$

$P(A) + P(B) - P(A \cup B) = P(A \cap B)$

$P(A) + P(B) - P(A \cup B) = P(A)P(B)$, $P(B) = x$

$0.5 + x - 0.8 = 0.5x$

$\dfrac{-0.3 = -0.5x}{-0.5}$, $x = \boxed{P(B) = 0.6}$

3 $P(\text{white on bottom} \mid \text{black on top}) = \dfrac{P(\text{white on bottom} \cap \text{black on top})}{P(\text{black on top})}$

top bottom

$\Omega = \{WW, BW, WB, BB\}$

30% 25% 25% 20%

$= \dfrac{0.25}{0.25 + 0.2} = \dfrac{.25}{.45} = \boxed{5/9}$

4 a) 13 cards in each suit so 1st is unique, but

$P(2^{nd} \text{ unique from } 1^{st}) = \dfrac{51-12}{51} = \boxed{39/51}$

b) $P(3^{rd} \text{ different suit} \mid \text{first 2 share suit}) = \dfrac{P(3^{rd} \text{ different} \cap \text{first 2 same})}{P(\text{first two same})}$

suit

$P(1^{st} \text{ two same}) = 12/51$

$P(3^{rd} \text{ different}) = 12/51 \cdot \frac{51-12}{51-1} = \frac{39}{50}$

$= \dfrac{\frac{12}{51} \times \frac{39}{50}}{12/51} = \boxed{\dfrac{39}{50}}$

c) $P(\underbrace{\text{third different suit}}_{A} \mid \underbrace{\text{first two diff. suit}}_{B}) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{\cancel{39/51} \cdot 26/50}{\cancel{39/51}} = \boxed{\dfrac{26}{50}}$

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d) $P(\underbrace{4^{th} \text{ is unique suit}}_{A} \mid \underbrace{1^{st} \text{ three are unique suits}}_{B})$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{13/49 \cdot \cancel{\frac{26}{50}}}{\cancel{\frac{26}{50}}} = \boxed{\frac{13}{49}}$$

e) w/ all 4 different suits, first A is unique

so $P = 1 \times \frac{39}{51} \times \frac{26}{50} \times \frac{13}{49} = .105 = \boxed{0.105}$

5 a) $P(\text{all work}) = P_1 \cdot P_2 \cdot P_3 = 0.2 \times 0.3 \times 0.6 = \boxed{0.036}$

independent, so use fact $P(A \cap B) = P(A)P(B)$

b) $P(\text{none work}) = \underset{1^c}{0.8} \times \underset{2^c}{0.4} \times \underset{3^c}{0.6} = \boxed{0.224}$

c) $P_1 \cdot P_2 \cdot P_3{}^c = 0.2 \times 0.3 \times 0.4 = \boxed{0.024}$

d) there are 3 combinations:

P_1, P_2 work P_3 doesn't, P_1, P_3 work, P_2 doesn't, P_1, P_2, P_3 work, P_1 - doesn't.

$$\sum P(x=2) = 0.2 \times 0.3 \times 0.4 + 0.2 \times 0.6 \times 0.7 + 0.3 \times 0.6 \times 0.8$$
$$= 0.024 + 0.084 + 0.144 = \boxed{0.252}$$

e) answer is p from d + combination where all 3 work

$0.252 + \underset{\substack{\uparrow \\ \text{from A}}}{0.036} = \boxed{0.288}$

6 If A is independent of itself, then $P(A \cap A) = P(A) \cdot P(A)$

$P(A \cap A) = P(A)$ as A is one event, so $P(A) = P(A) \cdot P(A)$,

$P(A) = x$, $x = x^2$, $0 = x^2 - x$, $0 = x(x-1)$, $x = 0, 1$ meaning $P(A) = 0$

or $P(A) = 1$ if A is independent of itself.

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7) $\Omega = \{HH\ ij,\ HT\ i,\ TH\ j,\ TT\}$ $1 \le i, j \le 6$

Sums to 4 are 1+3, 2+2 and 4+0

$P(HH \cap 13) = P(13 \mid HH)\,P(HH)$

$= P(13) = \frac{1}{36} \times P(HH) = \frac{1}{36} \times \frac{1}{4} = \frac{1}{144}$

$P(HH \cap 13) = P(HH \cap 31)$

$P(HH \cap 22) = P(22 \mid HH)\,P(HH)$

$= P(22) \times P(HH) = \frac{1}{36} \times \frac{1}{4} = \frac{1}{144}$

$P(HT \cap 40) = P(TH \cap 04)$

$P(HT \cap 40) = P(40 \mid HT)\,P(HT)$ $\quad P(HT) = 1/4$

$= \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

$P(HT \cap 40) = \frac{1}{24}$ $\quad P(TH \cap 04) = \frac{1}{24}$

$P(\text{total}) = 3\left(\frac{1}{144}\right) + 2\left(\frac{1}{24}\right) = \frac{3}{144} + \frac{1}{12} = \boxed{\frac{15}{144}}$

8) $\boxed{\begin{array}{l} A = \{3, 4\} \\ B = \{1, 4, 5\} \\ C = \{3, 4, 6\} \end{array}}$

$P(A) = 1/3$

$P(B) = 1/2$ $\quad P(A \cap B) = \frac{1}{6} = P(A) \cdot P(B)$

$P(C) = 1/2$ $\quad P(B \cap C) = 1/6 = P(B) \cap P(C)$

$P(A \cap C) = \frac{1}{3}$, $P(A) \cdot P(C) = 1/6$

9 a) $P(A \cap B \cap C) = \frac{P(A \cap (B \cap C))}{\cancel{P(B \cap C)}} \cdot \frac{\cancel{P(B \cap C)}}{\cancel{P(C)}} \cdot \cancel{P(C)}$

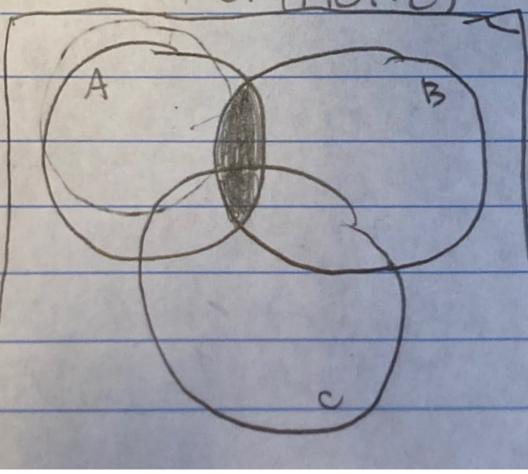
$P(A \cap B \cap C) = P(A \cap B \cap C)$

b) $P(A \cap B) = P(A \mid (B \cap C))P(B \mid C)P(C) + P(A \mid (B \cap C^c))P(B \mid C^c)P(C^c)$

$= P(A \cap B \cap C) + \frac{P(A \cap B \cap C^c)}{\cancel{P(B \cap C^c)}} \cdot \frac{\cancel{P(B \cap C^c)}}{\cancel{P(C^c)}} \cdot \cancel{P(C^c)}$

$= P(A \cap B \cap C) + P(A \cap B \cap C^c)$

LHS →



RHS →

LHS=RHS