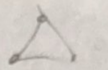


QUIZ 12, NOVEMBER 17.

Question. *The complement of a simple graph (V, E, f) is a graph with the same set of vertices V and with set of edges al the edges that do not appear in E. A graph is self complementary if it is isomorphic to its complement. Denote by n the number of vertices of the graph.*

(a) *Show that if a graph is self complementary, either n or $n-1$ are divisible by 4.* Hint: What is the number of edges of K_n.

there are $\frac{n(n-1)}{2}$ edges of K_n, but if a graph is self complementary then they must have same num of edges and therefore

$2c = \frac{n(n-1)}{2}$, c in each graph $= \frac{n(n-1)}{4}$, $\frac{n(n-1)}{4}$ is even, so if $n(n-1)$ is even, meaning n, $n-1$ or both

true for $n \geq 4$

are even, so $\frac{n}{4}$, $\frac{n-1}{4}$ or both $\in \mathbb{Z}$

(b) *Assume that a graph, is self complementary and write $d_1 \leq d_2 \leq \cdots \leq d_{n-1} \leq d_n$ for the degrees of its vertices. Show that $d_i + d_{n+1-i} = n-1$*

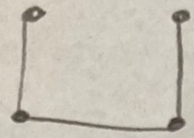
$G + \bar{G} = K_n$, meaning when we take a vertex with degree d, the vertex in $\bar{V}$ has degree $n-1-d$, as it loses those edges, but gains the missing ones, so $n-1-d+d = n-1$

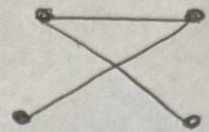
(c) *Assume that a graph, is self complementary and write $d_1 \leq d_2 \leq \cdots \leq d_{n-1} \leq d_n$ for the degrees of its vertices. Show that $d_n = d_{n-1}$.* Hint: the edge between two vertices is in the graph or in its complement.

For a self-complementary graph, a vertex the max degree, $V + \bar{V} = K_n$ degree is d_n and d_1 is the minimum degree, so $d_1 + d_n = n-1$. If $d_n \neq d_{n-1}$, suppose in G, d_n and d_1 are connected. In $\bar{G}$ that connection doesn't exist and d_1 swaps degree with d_1. Since $d_{n-1} \neq d_n$, $d_2 \neq d_1$ no vertices can satisfy degree adjacency in G, meaning it can't be isomorphic

Scanned with CamScanner

(d) *Find (up to isomorphism) all self complementary graphs with 4 vertices.*





(e) *Find (up to isomorphism) all self complementary graphs with 5 vertices.* $(d_1, d_2, 2, 4-d_2, 4-d_1)$