$a\overline{)b}$ $3\overline{)10}$ $3\frac{10}{3}$

$3+\frac{1}{3}$

QUIZ 14, DEC 8

Question. (a) *Show that every rational number has a decimal expression that either terminates or repeats.* Hint: A rational number is the quotient of two integers, use long division. What are the possible remainders?

$n \in Q$ such that $n = \frac{a}{b}$ where $a, b \in Q$

if $b\overline{)a}$ ... $r_1, r_2 \ldots r_n < b$ so eventually banded

If remainder $r_0 \neq 0$ then by to repeat a value of r_i and n

n as ... with it's not 0, then it will repeat in a cycle

(b) *Show that* $x = 3.746746746\ldots$ *is a rational number, that is the quotient of two integers.* Hint Take a look first at $y = x - 3$. Compute $1000y - y$

$y = x - 3$ $\quad y = .746746746$

$$\frac{1000y - y}{999} = 746.746746 - .746 = \frac{746}{999} =$$

$$\frac{746}{999} = y$$

$$\boxed{\frac{746}{999} + 3 = X}$$

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(c) *Show that a decimal expression that either terminates or repeats is a rational number.* $x = a_n a_{n-1} \ldots a_1 a_0 \cdot b_1 b_2 \ldots b_m b_1 b_2 \ldots b_m b_1 b_2 \ldots b_m \ldots$

WLFoC assume that $a_n a_{n-1} \ldots a_1 a_0$ is the nonrepeating part of x. ~~assume $x<1$~~ but if it's shift by constant greater than 0.

$y = x - a_n a_{n-1} \ldots a_1 a_0$ + shift is necessary

$10^m \cdot y - y \in \mathbb{Z}$ as remains 999 repeating part

$$\frac{10^m \cdot y - y}{10^m - 1} \in \mathbb{Z}$$

$$x = y + a_n a_{n-1} \ldots a_1 a_0$$

$$x = \frac{10^m y - y}{10^m - 1} + a_n a_{n-1} \ldots a_1 a_0$$

(d) *Show that every rational number is the limit of a sequence of decimal numbers that terminate*

$Q = \frac{m}{k}$, $m, k \in \mathbb{Z}$ $Q \in \mathbb{Q}$ $a_1, a_2 \ldots a_n$

$a_0 \ldots a_n$ are digits

$$Q = .a_0 + 0.0a_1 + 0.00a_2 + \ldots + 0.000 a_n = 0.a_0 a_1 a_2 \ldots a_n$$

$$Q = .a_0, \rightarrow .a_0 a_1, \quad .a_0 a_1 \ldots a_n$$

$a_n = a_0 a_1 a_2 \ldots a_n$

$\lim_{n \to \infty} |a_n - a_0 a_1 \ldots a_n| < \frac{1}{10^n} \Leftrightarrow \in$ meaning? if converges? $\ln 10$ Since

If ~~it~~ since ~~repeats~~ ~~therefore~~ has a nonzero nth remainder

~~w/ higher~~ ~~represented from~~ w. fraction terminate and converges