Math 165 HW 6

I worked with David.

1a) $N = I_{A_1} + I_{A_2} + I_{A_3}$

b) $E(N) = E(I_{A_1} + I_{A_2} + I_{A_3})$

$= E(I_{A_1}) + E(I_{A_2}) + E(I_{A_3})$

$= P(A_1) + P(A_2) + P(A_3) = \frac{1}{5} + \frac{1}{4} + \frac{1}{3} = \boxed{\frac{47}{60}}$

c) For $A_1 \cap A_2 \cap A_3 = \emptyset$, meaning disjoint.

$Var(N) = E(N^2) - E(N)^2$

$E(N^2) = E(N)$ as N^2

	N	N^c
I	1	0
Prob.	47/60	13/60

So $E(N^2) = 1^2 \cdot \frac{47}{60} + 0^2 \cdot \frac{13}{60}$

$E(N^2) = \frac{47}{60}$

true since events don't overlap.

$Var(N) = E(N^2) - E(N)^2$

$= \frac{47}{60}\left(\frac{13}{60}\right) = \boxed{\frac{611}{3600}}$

d) If A_1, A_2, A_3 are independent, then $Var(N) = Var(I_{A_1}) + Var(I_{A_2}) + Var(I_{A_3})$

$Var(A_1) = E(I_{A_1}^2) - E(I_{A_1})^2$

$= \frac{1}{5} - \frac{1}{25} = \frac{4}{25}$

I_{A_1}	A_1	A_1^c
I	1	0
P	$P(A_1)$	$1-P(A_1)$

$P(A_1) = \frac{1}{5}$

$E(A_1) = \frac{1}{5}$

$E(A_1)^2 = 1^2 \cdot \frac{1}{5} = \frac{1}{5}$

repeat for A_2, A_3

$Var(A_2) = E(I_{A_2}^2) - E(I_{A_2})^2$

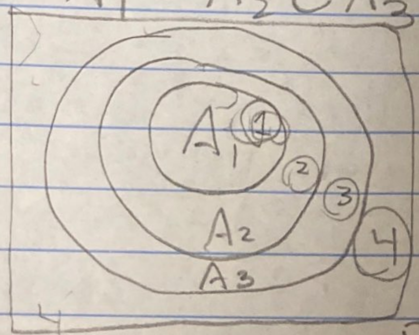
$= \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

$Var(A_3) = E(I_{A_3}^2) - E(I_{A_3})^2 =$

$= \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$

So $Var(N) = \frac{4}{25} + \frac{3}{16} + \frac{2}{9} = \boxed{\frac{2051}{3600}}$

c) $A_1 \subset A_2 \subset A_3$



then can think of it like: As move outwards N goes down as we enter a larger circle. as we move outside less likely events.

	1	2	3	4
N	3	2	1	0
$P(A_n)$	$1/5$	$(1/4 - 1/5)$	$(1/3 - 1/4)$	$1 - 1/3$

$P(A_n)$ → chance of event

$$E(N^2) = 3^2 \cdot \frac{1}{5} + 2^2\left(\frac{1}{4} - \frac{1}{5}\right) + 1^2\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$Var(N) = E(N^2) - \left(\frac{47}{60}\right)^2 = \boxed{\frac{5291}{3600}}$$

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2.a.i) Problem is equivalent to saying if we have one person right above $50,000, how many at the other end would make $0 for avg = 10,000.

$$\frac{-50000}{(X+1)} \quad 10000 \quad X = 4$$

, meaning that a person would be at least in top 20% w/ income over 50,000

b) If SD = 8000, then $50,000 is 5 SDs away, can use Chebyshev's equation

$$P(\text{income} > 5\,SDs) \leq \frac{1}{25} = .04 \quad \frac{1}{25}$$

meaning the upper band is now the top 4%.

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3 a) For case 1 have the following table for investing in one stock

Profit	200	100	0	-100
Probability	1/4	1/4	1/4	1/4
100·profit	20000	10000	0	-10000 ← invest all money in one stock

Two outcomes with profit > 8000, and the probability of either of these happening is 1/4, but are independent so $P(\text{profit} > 8000) = \frac{1}{2}$

b) We have 4 variables i, j, k, l.

$i \rightarrow$ times with profit of \$200

$j \rightarrow$ times with profit of \$100

$k \rightarrow$ times with profit of \$0

$l \rightarrow$ times w/ profit of \$-100

total profit = $200i + 100j + 0k - 100l$ and $i + j + k + l = 100$

Can ignore k and rewrite equation as $i + j + l \leq 100$

Want profit ≥ 8000, so outcome $8000 \leq 200i + 100j - 100l$

To find the probability, have trinomial sum: $\displaystyle\sum_{200i + 100j - 100l \geq 8000} \binom{100}{i, j, k} \left(\frac{1}{4}\right)^{100}$ ← all i, j, k are equally likely

4 a) $\Omega = (HHH, HHT, HTH, THH, HTT, THT, TTH, TTT)$
The game ends on 6 out comes out of 8, so $p(\text{game ends}) = \frac{\#\text{game enders}}{|\Omega|} = \frac{6}{8} = \boxed{0.75}$

b) This is a geometric distribution.
$P(X=r) = \left(\frac{1}{4}\right)^{r-1} \cdot \frac{3}{4}$

c) For a geometric distribution, the expected value is $\frac{1}{p}$ so expected duration of play is $\frac{1}{\frac{3}{4}} = \boxed{\frac{4}{3}}$

5 a) Let i be number of games until A wins
$P(A_{win} = i) = (q_A q_B)^{i-1} p_A q_B \rightarrow$ Both must fail until A succeeds and B doesn't
take the sum: $\sum_{i=1}^{\infty} p_A q_B (q_A q_B)^{i-1} = \boxed{\frac{p_A q_B}{1 - q_A q_B}}$

b) Let i be number of games until B wins
$P(B_{win} = i) = (q_A q_B)^{i-1} p_B q_A$ as both fail in the first i-1 games until B wins
Take the sum: $\sum_{i=1}^{\infty} (q_A q_B)^{i-1} p_B q_A = \boxed{\frac{p_B q_A}{1 - q_A q_B}}$

c) Let i be games until a draw, meaning 2 heads
both get tails until both get heads
$P(\text{Draw} = i) = (q_A q_B)^{i-1} p_A p_B$
$\sum_{i=1}^{\infty} (q_A q_B)^{i-1} p_A p_B = \boxed{\frac{p_A p_B}{1 - q_A q_B}}$

d) let it take K games, $q_A q_B$ is probability of the game continuing meaning $1 - q_A q_B$ is probability game ending

So $P(\text{games} = K) = (q_A q_B)^{K-1} (1 - q_A q_B)$

6 w/ $X = \#$, balls in box

want $P(95 \le X \le 105)$ Meaning want probability of being within $5/3$ Standard deviations of the mean.

Using Chebyschev, outside $5/3$ SD's has probability of $\frac{1}{(5/3)^2} = \frac{9}{25}$, so P of within $5/3$ SD's is $1 - \frac{9}{25} = \boxed{\frac{16}{25}}$