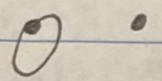
Math 65 Hw 11

1 a) Σ degrees $= 2 \cdot m$, m is number of edges.
Σ degrees of vertices w/even degree $= 2K$, $K \in \mathbb{N}$ as sum of even numbers is even.
Σ degrees $= \Sigma$ deg of vertices w/ even degrees $+ \Sigma$ deg of vertices w/ odd deg

$$2m = 2K + J$$

$$2m - 2K = J, \quad 2(m-K) = J,$$ meaning sum of degrees w/ odd vertices is even. Since sum of 2 odd numbers is even, and J is even, there must be an even amount of vertices w/ odd degree.

2a) In a simple graph, each vertice can be of degree 0 to n-1, where n is the number of vertices. However, you cannot have a vertice of degree 0 and one of n-1, as that means one vertice is connected to every other vertice, while one vertice is connected to no vertices so there are n-1 options for n vertices, so two of them must be the same.

b)  No, this graph has 2 vertices, but no vertices share degree.

3 Let n be number of ~~vertices~~ vertices, then $\frac{nr}{2}$ edges, as each vertice has r edges, and we double count so must divide by 2

4 a) If $V = \{v_1 \ldots v_n\}$ then subgraphs in total is $P(V)$, which is 2^n. An isomorphic subgraph of K_n has same number of vertices and edges, so non isomorphisms map to different number of vertices so total is number of equivalence classes which is n+1.

b) K_n has number of spanning subgraphs $= |P(\text{edges})|$ and K_n has $\binom{n}{2}$ edges, as for each edge, choose two vertices it's connected
So $|P(\text{edges})| = 2^{\binom{n}{2}}$

5a) Transitive:

G_1 forms a bijection with itself, meaning G_1 isomorphic to itself.

Symmetric

If G_1 forms a bijection with G_2, then by definition of a bijection, G_2 forms a bijection w/ G_1 so G_2 is isomorphic to G_1

Transitive:

$\sim \Rightarrow$ being isomorphic

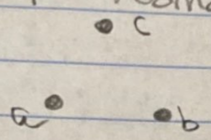
$G_1 \sim G_2 \quad G_2 \sim G_3$

$G_{V1}: V_1 \to V_2 \quad G_{E1}: E_1 \to E_2 \quad G_{V2}: V_2 \to V_3 \quad G_{E2}: E_2 \to E_3$

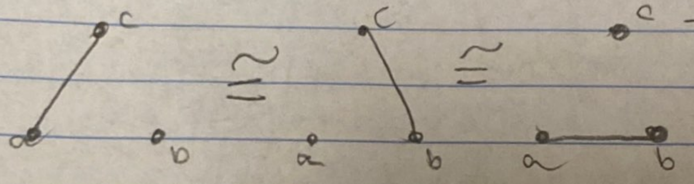
We then have $G_V: V_1 \to V_2 \to V_3$ and $G_E: e_1 \to e_2 \to e_3$ and since G_E and G_V are bijections, $G_1 \sim G_3$ and G_1 is isomorphic to G_3

b) Has 4 isomorphic classes, one w/ each num. of edges

0 edges

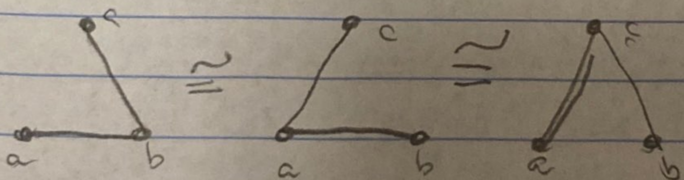


1 edge

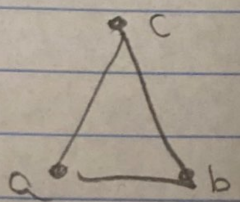


can rotate to get other graphs of same edges.

2 edges



3 edges



These all fulfill properties of isomorphism