

**Problem** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function.

- (a) Prove that the set  $C$  of all points  $x$  at which  $f$  is continuous is a measurable set. (**Hint:** Is  $C$  a  $G_\delta$  set?)
- (b) Use the first part to prove that there exists no function  $g : \mathbb{R} \rightarrow \mathbb{R}$  that is only continuous on the rationals.

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