

Composing permutations.

Notation: α a permutation in S_n

That is, α is a bijection from $\{1, \dots, n\}$ to $\{1, \dots, n\}$.

Disjoint cycles

notation: $\alpha = (ijkl)(zw)$ is the permutation which is the function.

$$\begin{aligned}\alpha(i) &= j \\ \alpha(j) &= k \\ \alpha(k) &= l \\ \alpha(l) &= i \\ \alpha(z) &= w \\ \alpha(w) &= z\end{aligned}$$

all other numbers
are fixed.

We can also write a permutation as.

$$\begin{bmatrix} 1 & 2 & 3 & \dots & n \\ \alpha(1) & \alpha(2) & \alpha(3) & \dots & \alpha(n) \end{bmatrix}$$

Ex of composition of functions:

$$(243)(13)$$

$\swarrow \quad \searrow$
 $\alpha(3)=2$

$$\text{let } \alpha = (243) \quad + \quad \beta = (13)$$

$$\text{then } \alpha\beta(1) = \alpha(3) = 2$$

$$\alpha\beta(2) = \alpha(2) = 4$$

\vdots

So $(243)(13) = (1243)$

In the column notation:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 3 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{bmatrix}$$

$$\alpha \circ \beta = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{bmatrix}$$

computing
 $\alpha(\beta(i))$