QUIZ 10, NOVEMBER 3

Question. *There are 4 kinds of donuts available in a store Boston cream, pumpkin spice, glazed and chocolate. Half a dozen donuts are picked at random.*

(a) *Do you think that the the events "at least 2 are chocolate" and " at most 1 is pumpkin spice," are independent? Do not make any computations yet, just discuss it with your peers.*

Not independent choosing more of one type affects ability to choose another.

(b) *What is the probability that at least 2 are chocolate?*

C = number of chocolate donuts

$P(C \geq 2) = \binom{4+4-1}{4-1} = \binom{7}{3}$ combinations of

$= 1 - \binom{7}{3}$

$P(C \geq 2) = \frac{\binom{7}{3}}{\binom{4+4-1}{4-1}} = \frac{\binom{7}{3}}{\binom{9}{3}}$

(c) *What is the probability when choosing 6 donuts that at most 1 is pumpkin spice?*

P = number of pumpkin donuts

$P(P \leq 1) = 1 - \text{answer from b}$ ← can substitute in as donuts have same chance of being picked

$= 1 - \frac{\binom{7}{3}}{\binom{1}{3}}$

Use variables defined from first page.

(d) *What is the probability that at least 2 are chocolate and most 1 is pumpkin spice?*

$P(C \geq 2 \cap P \leq 1) = \frac{\binom{7}{3} - \binom{5}{3}}{\binom{9}{3}}$

$P(P \leq 1)$ means

at least 2 chocolate

and at least 2 pumpkin

(e) *If at least 2 are chocolate, what is the probability that at most 1 is pumpkin spice?*

$$P(P \leq 1 \mid C \geq 2) = \frac{P(P \leq 1 \cap C \geq 2)}{P(C \geq 2)}$$

$$= \frac{\frac{\binom{7}{3} - \binom{5}{3}}{\binom{9}{3}}}{\frac{\binom{7}{3}}{\binom{9}{3}}}$$

$$= \frac{\binom{7}{3} - \binom{5}{3}}{\binom{7}{3}}$$

(f) *Are the events "at least 2 are chocolate" and " at most 1 is pumpkin spice" independent?*

No, if they were,

$P(P \leq 1 \cap C \geq 2) = P(P \leq 1)\, P(C \geq 2)$

$\frac{\binom{7}{3}\binom{5}{3}}{\binom{9}{3}} \neq \left(1 - \frac{\binom{7}{3}}{\binom{9}{3}}\right)\left(\frac{\binom{7}{3}}{\binom{9}{3}}\right)$

meaning they aren't independent.

$\frac{5!}{3!2!} \cdot \frac{4!}{2!2!} = \frac{5!4!}{2!2!}$

$\frac{\binom{5}{3}}{\binom{9}{3}}$