1. QUIZ 4, MONDAY, SEPTEMBER 27

Instructions: SHOW YOUR WORK in order to receive credit. Include proofs and justification for your assertions, with words and equations.

Question 1.1. (a) *Define what it means for a function $f : A \to B$ to be one to one.*

f is one-to-one (injective) if $\forall a_1, a_2 \in A$, $a_1 \neq a_2$, and $f(a_1) \neq f(a_2)$. By contrapositive f is also one-to-one if $\forall a_1, a_2 \in A$, $f(a_1) = f(a_2)$, $a_1 = a_2$

(b) *Define what it means for a function $f : A \to B$ to be onto.*

f is onto if $\forall b \in B \; \exists a \in A \; f(a) = b$

(c) *Consider the function* $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$, $(a, b) \to (3a + 2b, a + b)$ *Prove or disprove that f is one to one.*

$\mathbb{Z} \to \mathbb{Z}$

For f to be one-to-one, $\forall a_1, b_1, a_2, b_2 \in \mathbb{Z} \times \mathbb{Z}$, $f(a_1, b_1) = f(a_2, b_2)$ then $(a_1, b_1) = (a_2, b_2)$

$(3a_1 + 2b_1, a_1 + b_1) = (3a_2 + 2b_2, a_2 + b_2)$

$3a_1 + 2b_1 = 3a_2 + 2b_2$ $\quad$ $3a_1 + 2(a_2 + b_2 - a_1) = 3a_2 + 2b_2$

$a_1 + b_1 = a_2 + b_2 - a_1$ $\quad$ $3a_1 + 2a_2 + 2b_2 - 2a_1 = 3a_2 + 2b_2$

$-a_1$ $\quad$ $a_1 = a_2$

Since $a_1 = a_2$, $\cancel{a_1} + b_1 = \cancel{a_2} + b_2$, $b_1 = b_2$ therefore by contrapositive f is one-to-one.

(d) *Prove or disprove that the function f defined in (c) is onto.*

$\mathbb{Z}\times\mathbb{Z} \to \mathbb{Z}\times\mathbb{Z}$

$(a,b) \to (3a+2b, a+b)$

$f(a,b) = (3a+2b, a+b) = (c,d) \quad (c,d) \in \mathbb{Z}\times\mathbb{Z}$

$3a+2b = c \quad , \quad a+b = d$

$b = d-a$

$3a+2(d-a) = c$

$a+2d = c$

$a = c-2d \qquad c-2d+b = d$

$b = 3d-c$

$f(a,b) = f(c-2d, 3d-c)$

So for every output $(c-2d, 3d-c)$ there is an input (a,b) meaning that f is onto.