(1) Let $X = \{1, 2, 3\}$. Work with your group to find an example of a topology on X other than the discrete topology containing at least 4 open sets. Prove that your example is a topology.

(2) Let $X = \{1, 2\}$. There are four functions $X \to X$, namely

x	$f_{11}(x)$	$f_{12}(x)$	$f_{21}(x)$	$f_{22}(x)$
1	1	1	2	2
2	1	2	1	2

We have been thinking about three topologies on X:

$$\tau_d = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$
$$\tau_s = \{\emptyset, \{1\}, \{1, 2\}\}$$
$$\tau_i = \{\emptyset, \{1, 2\}\}.$$

Given the following topologies on the domain and codomain, decide whether each function $X \to X$ is continuous. Use any shortcuts you can. Do you see any patterns?

Domain topology	Codomain topology	f_{11}	f_{12}	f_{21}	f_{22}
τ_d	τ_d				
τ_d	τ_s				
τ_d	τ_i				
τ_s	τ_d				
τ_s	τ_s				
τ_s	τ_i				
τ_i	τ_d				
τ_i	τ_s				
τ_i	τ_i				

(3) (The finite complement topology) Let X be any set. The collection of subsets

$$\tau = \{U \subseteq X \mid U^c \text{ is finite }\}$$

is called the **finite complement topology**.

(a) Show that the finite complement topology is a topology.

(b) How does the finite complement topology on $\mathbb{R}$ compare with the usual topology on $\mathbb{R}$? Justify your answer.

(4) Let Z be a finite subset of $\mathbb{R}$. What is the subspace topology on Z?

(5) Let $Z = \{0\} \cup \{\frac{1}{n} \mid n \in \mathbb{Z}_{>0}\}$. Give Z the subspace topology it inherits from $\mathbb{R}$. Does Z have the discrete topology?

(6) (Open subsets in the finite complement topology are big.) Show that any two subsets of $\mathbb{R}$ that are open in the finite complement topology have a non-empty intersection.

(7) Let τ be the finite complement topology on $\mathbb{R}$. What are the continuous functions $f : (\mathbb{R}, \tau) \to (\mathbb{R}, \tau)$? Give an answer in terms of the preimages of points.