Math 166 HW6

1 a) Let's find B at some points under $\alpha = 0.05$, $H_0 = \mu = 60$, $H_1\ \mu \neq 60$

$B = P(\text{reject } H_0 \mid H_1 \text{ true})$

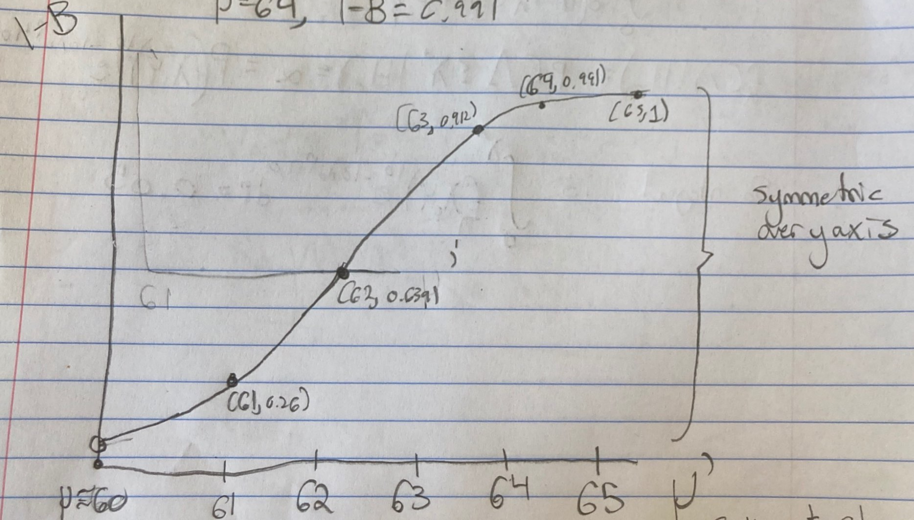
under H_0, the interval on which we want is $P(Z > \frac{\bar{\mu}-60}{4/\sqrt{16}}) = 0.05$, $\bar{\mu} - 60 = Z_{0.025} = -1.96$
$+60$ $\quad \mu = 60 + 1.96 = 58.04$

Since this is 2 sided, can add 1.96 to get upper bound of $(58.04, 61.96)$

So, for some $\mu' \neq 60$ want $P(X < 58.04) + P(X > 61.96)$

Can use table/calculator to solve, w/ knowing $\sigma = \frac{\sigma_{\bar{x}}}{\sqrt{n}} = 1$

Under $\mu' = 61$, $1-B = 0.26$ ~~$B = 0.83$~~
Under $\mu' = 63$, $1-B = 0.912$
$\mu' = 65$, $1-B \neq 1$ ~~$1-B = 0$~~ Symmetrical around $\mu = 60$
$\mu' = 62$, $1-B = 0.639$
$\mu = 64$, $1-B = 0.991$



- Isn't defined at $\mu = 60$, but $\lim_{\mu' \to 60} 1-B = \alpha$, as we get closer to $\mu = 60$, our intervals would overlap.

2 GLRT: pdf: $\lambda e^{-\lambda y}$ $y > 0$
0 $y < 0$

$$GLRT = \frac{\max_{\lambda \in \omega} L(\lambda)}{\max_{\lambda \in \Omega} L(\lambda)}$$

From book, MLE of exponential $\rightarrow$ see below
pdf is $\lambda_0^n e^{-n\lambda_0 \bar{x}}$

For denominator for λ, have $\prod_{j=1}^{n} \lambda e^{-\lambda y_j}$, get
solution of $\lambda = \frac{1}{\bar{x}}$, so for
the denominator for λ is $\begin{cases} \frac{1}{\sum y_j} & \text{if } \frac{1}{\sum y_j} < \lambda_0 \\ \lambda_0 & \text{otherwise} \end{cases}$

$x_j \neq 0$ so ok

$\Lambda = GLRT = \{ (\lambda_0 \varphi)^{10} e^{-10\lambda_0 \varphi + 10}$, if $\frac{1}{\varphi} < \lambda_0$

Want to find $P(0 < GLRT \leq \lambda^* \mid H_0 \text{ is true}) = \alpha$

$$\alpha = \int_0^{\lambda^*} d\lambda \, f_\Lambda(\lambda \mid H_0) =$$

$$f_\Lambda(\lambda \mid H_0) = P(\Lambda \leq \lambda^* \mid H_0) = \alpha = P\left((\lambda_0 \varphi)^{10} e^{-10\lambda_0 \varphi + 10} \leq \lambda^*\right)$$

So integral is $\int_0^{\lambda^*} (\lambda_0 \varphi)^{10} e^{-10\lambda_0 \varphi + 10} \, d\varphi = 0.05$

3 Want $E(e^{tx}) = \frac{1}{2^{n/2}\Gamma(\frac{n}{2})}\int_0^\infty e^{tx} \cdot x^{\frac{n}{2}-1} e^{-\frac{x}{2}}\,dx$

$$E(e^{tx}) = \frac{1}{2^{n/2}\Gamma(\frac{n}{2})}\int_0^\infty x^{\frac{n}{2}-1} e^{x(t-\frac{1}{2})}\,dx$$

if $t-\frac{1}{2}>0$, it diverges, so $t<\frac{1}{2}$ $\quad u = x(\frac{1}{2}-\frac{t}{2})$, $x = \frac{u}{t-\frac{1}{2}}$

$du = \frac{1}{2}-\frac{t}{2}$

$$E(e^{tx}) = \frac{(\frac{1}{2}-t)^{-n/2}}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})}\int_0^\infty u^{\frac{n}{2}-1} e^{-u}\,du$$

$$= \frac{(1-2t)^{-n/2}}{\Gamma(\frac{n}{2})}\int_0^\infty \underbrace{u^{\frac{n}{2}-1} e^{-u}\,du}_{\Gamma(\frac{n}{2})}$$

$$M_x(t) = (1-2t)^{-n/2}$$

$$M'_x(t) = n(1-2t)^{-\frac{n}{2}-1}, \quad M'_x(0) = n$$

$$M''_x(t) = (-n-2)(-n)(1-2t)^{-\frac{n}{2}-2}; \quad M''_x(0) = n^2+$$

$$Var(X^2) = E(X^2) - [E(X)]^2$$

$$= n^2+2n-n^2 = 2n.$$

So $\boxed{\begin{array}{l} M(t) = (1-2t)^{-n/2} \\ Var(X^2) = 2n \end{array}}$

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