

**Problem**

Let  $f_n(x) = ae^{-nax} - be^{-bnx}$  for  $x > 0$  and  $0 < a < b$ .

- (a) Prove that  $\sum_{n=1}^{\infty} \int_0^{\infty} |f_n(x)| dx = \infty$ .
- (b) Prove that  $\sum_{n=1}^{\infty} \int_0^{\infty} f_n(x) dx = 0$ .
- (c) Prove that  $\sum_{n=1}^{\infty} f_n(x) \in L^1([0, \infty))$ , and  $\int_0^{\infty} \sum_{n=1}^{\infty} f_n(x) dx = \ln(b/a)$ .

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