

1 Find a value of h such that the system

$$\begin{aligned}3x_1 - 9x_2 &= 4 \\ x_1 - 3x_2 &= h\end{aligned}$$

is consistent. How many solutions are there in this case?

2

- (a) What is the largest possible number of pivots a 4×6 matrix can have? Why?
- (b) What is the largest possible number of pivots a 6×4 matrix can have? Why?
- (c) How many solutions does a consistent linear system of 3 equations and 4 unknowns have? Why?
- (d) Let A be a 4×4 matrix with exactly 3 pivot columns. Does the system $A\mathbf{x} = \mathbf{b}$ have a solution for every \mathbf{b} ?

3

- (a) Is there a line through the points $(0, -3)$, $(2, 1)$, and $(1.5, 0)$? If so, find it. If not, explain why not.
- (b) Is there a quadratic through the points $(2, 3)$ and $(2, 2)$. If so, find it. If not, explain why not.
- (c) (Possibly hard and more for fun than anything else): Let a be a real number and consider the points $(a, 0)$ and $(a, 1)$. Is there any value of a for which some quadratic contains these two points? If so, give an example (i.e., a particular value of a like $a = 7$, together with the equation of a quadratic passing through $(7, 0)$ and $(7, 1)$.) If not, why not?

4 Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ be vectors in \mathbb{R}^3 . Using the definitions of vector addition and scalar multiplication, show carefully that

$$(-1)(\mathbf{x} + (-1)\mathbf{y}) = \mathbf{y} + (-1)\mathbf{x}$$