Math 165 HW3

I worked w/ David on HW3

1a) (I)f = person of unknown gender, use weights

so $P(TB) = \frac{1}{9000}\left(\frac{1}{2}\right) + \frac{1}{15000}\underbrace{\left(\frac{1}{2}\right)}_{P(\text{female}) \text{ weight}}$

$P(\text{person contracts TB}) = \boxed{\frac{1}{18000} + \frac{1}{30000}}$

b) $P(\text{male} \mid \text{contracts TB}) = \frac{P(\text{male} \cap \text{contracts TB})}{P(\text{contracts TB})}$

$= \frac{\frac{1}{2}\left(\frac{1}{9000}\right)}{\frac{1}{18000} + \frac{1}{30000}} = \frac{\frac{1}{18000}}{\frac{1}{18000} + \frac{1}{30000}} = 62.5\%$

2a) R_i = "plane in R_i" E = "plane found in R_i"

$P(R_i) = 1/3$ Use Bayes's theorem

$i=1$ $P(R_i \mid E) = \frac{P(E \mid R_i) \cdot P(R_i)}{P(E \mid R_1)P(R_1) + P(E \mid R_2)P(R_2) + P(E \mid R_3)P(R_3)}$

For $i=1$ $P(R_1 \mid E) = \frac{p_1 \cdot \frac{1}{3}}{p_1 \cdot \frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = \frac{\frac{1}{3}p_1}{\frac{1}{3}(p_1+2)} = \boxed{\frac{p_1}{p_1+2}}$

For $i=2$ $P(R_2 \mid E) = \frac{\frac{1}{3}}{\frac{1}{3}p_1 + \frac{1}{3} + \frac{1}{3}} = \frac{1}{p_1+2}$

For $i=3$ $P(R_3 \mid E) = \frac{\frac{1}{3}}{\frac{1}{3}p_1 + \frac{1}{3} + \frac{1}{3}} = \frac{1}{p_1+2}$

b) prior $P(R_1) = 1/3$

posterior is $P(R_1 \mid E) = \frac{p_1}{p_1+2}$

c) For $i=2$ $P(R_2) = \frac{1}{3}$

prior $P(R_2) = 1/3$

posterior is $P(R_2 \mid E) = \frac{1}{p_1+2}$

For $i=3$ $P(R_3) = \frac{1}{3}$

prior is $P(R_3) = \frac{1}{3}$

posterior is $P(R_3 \mid E) = \frac{1}{p_1+2}$

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3 a) $P(\text{good chip} \mid \text{pass cheap test}) = \dfrac{P(\text{pass cheap test} \mid \text{good chip}) \cdot P(\text{good chip})}{P(\text{pass cheap test})}$

$= \dfrac{1 \times 0.8}{0.8 + .1(.2)} = \boxed{0.98}$

b) $1 - 0.98 = 0.02$ ~~of~~ $=$ 2% of chips sold will be bad

4 a) $P(\text{positive}) = .99(.05) + .01(.8)$
$= \boxed{0.0575}$

b) $P(\text{has disease} \cap \text{tests positive}) = 0.01 \cdot .2 = \boxed{0.002}$

c) $P(\text{healthy} \cap \text{negative test}) = .99(.95) = \boxed{0.9405}$

d) $P(\text{has disease} \mid \text{test positive}) = \dfrac{P(\text{disease} \cap \text{tests positive})}{P(\text{tests positive})}$

$= \dfrac{.8 \times .01}{.99(.05) + .01(.8)} = \boxed{0.139}$

e) Yes, as probabilities apply as your population tends to infinity.

5 a) $P(A_i) = 1 - P(A_i^c) \rightarrow$ face i is never rolled

$\boxed{P(A_i) = 1 - (1 - p_i)^k}$

b) $P(A_i \cup A_J)$, where $i \neq J$, so looking for 2 faces instead

Find $P(A_i \cup A_J)^c = P(\text{neither } i \text{ or } J \text{ are rolled})$

$\boxed{P(A_i \cup A_J) = 1 - (1 - p_i - p_J)^k}$ $p_J = p$ j face is shown

6 a) $p(\text{share w/ at least 1}) = 1 - p(\text{share w/ } 0)$

$= \boxed{1 - \left(\frac{364}{365}\right)^{n-1}}$

b) $\frac{1}{2} = 1 - \left(\frac{364}{365}\right)^{n-1}$

$\log\left(\frac{1}{2} = \left(\frac{364}{365}\right)^{n-1}\right)$ $\log \frac{1}{2} = (n-1) \log \frac{364}{365}$

$\dfrac{-\log 2}{\log \frac{364}{365}} = n - 1$, $n = 253.65$ $\boxed{n \geq 254}$

c) birthday problem is any 2 students sharing, this q. is you share w/ someone else.

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7) Since A, B, and C are independent, we can assume pairwise independence, meaning that A and B are independent, A and C are independent, and B and C are independent. The same can be said about A^c and B, B and C^c, and A^c and C^c are independent.

$$P(A^c \cap B \cap C^c) = P((A^c \cap B) \cap C^c)$$

A^c is independent from C^c so is B, meaning $A^c \cap B$ independent from C^c

$$P(A^c \cap B \cap C^c) = P(A^c \cap B) P(C^c)$$

$$P(A^c \cap B \cap C^c) = P(A^c) P(B) P(C^c)$$

8a) X be random variable on which toss 1st head is on.

$$P(X=5) = \boxed{\underbrace{(1-p)^4}_{\text{4 failures}} \underbrace{p}_{\text{1 success}}}$$

b) $P(X=10 \mid X \neq 5) = \dfrac{P(X=10 \cap X \neq 5)}{P(X \neq 5)} \leftarrow = P(X=10)$

$$= \frac{(1-p)^9 \cdot p}{(1-p)^5} = \boxed{(1-p)^4 p}$$

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