Homework 4

● Graded

53 Minutes Late

Student

Scott A. Fullenbaum

Total Points

17.5 / 20 pts

Question 1

Tangent line to the curve

5 / 5 pts

✔ **– 0 pts** Correct

– 0.5 pts Minor algebraic error

– 1 pt Algebraic error (e.g. incorrect determinant)

– 1 pt Minor conceptual error

– 2 pts Conceptual error

Question 2

Find partial derivatives

5 / 5 pts

✔ **– 0 pts** Correct

– 1.5 pts Miscalculations in "calc 1" method

– 1.5 pts Incorrect general formula

– 1.5 pts Miscalculations with derived formula

– 1.5 pts Incorrect $\partial z / \partial y$

– 1.5 pts No or incorrect derivation of formula

Question 3

Domain of a multivariable function

4.5 / 5 pts

– 0 pts Correct.

✔ **– 0.25 pts** Did not label axes on plot.

– 0.25 pts Did not indicate where the boundary of the region intersects the abscissa or indicated incorrect values.

✔ **– 0.25 pts** Missed clearly indicating on the plot that the hyperbola boundary of the region is included in the domain.

– 0.75 pts Arithmetic mistake.

– 1.75 pts Incorrect algebraic statement of domain.

– 1.75 pts Incorrect or very unclear plot of two-dimensional domain.

– 0.75 pts Incorrectly stated that hyperbola boundary (or part of boundary) ought to be excluded from domain. Should be x^2 - y^2 >= 9.

– 1 pt Somewhat incorrect plot of domain and/or somewhat incorrect algebraic form.

– 0.5 pts Confusing shading or no shading to indicate included region.

– 2.5 pts Did not plot domain.

– 5 pts No response, nearly no response, or a response that was not applicable to the question posed.

Question 4

Compare level curves

1 / 1 pt

✔ **– 0 pts** Problem attempted

– 1 pt No response given

Question 5

Orthogonality

0 / 2 pts

✔ **– 0 pts** Problem earnestly attempted

– 2 pts Not attempted

✔ **– 2 pts** Late assignment

Question 6

Gradient and tangent line

2 / 2 pts

✔ **– 0 pts** Problem attempted

– 2 pts Problem not answered

Questions assigned to the following page: 1 and 2

Math 42 HW 4

1) $x^2+3y^2-z=0$

$\nabla F(x,y,z) = \langle 2x, 6y, -1 \rangle |_{(1,1,4)} = \langle 2, 6, -1 \rangle$

$\vec{n} = \langle 0, 1, 0 \rangle$

$\nabla f(2,6,-1) \times \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 6 & -1 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 6 & -1 \\ 1 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & -1 \\ 0 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & 6 \\ 0 & 1 \end{vmatrix}\vec{k}$

$= \vec{i} - 0\vec{j} + 2\vec{k}$

$= \langle 1, 0, 2 \rangle$

$f = \langle 1, 0, 2 \rangle t + \langle 1, 1, 4 \rangle$

$f = \langle t+1, 1, 2t+4 \rangle$

$x(t) = t+1$
$y(t) = 1$
$z(t) = 2t+4$

2 a) $\frac{d}{dx}\left(e^{xyz} = \sin(x^2+y^2+z^2)\right)$

$e^{xyz}\left(yz + xy\frac{dz}{dx}\right) = \cos(x^2+y^2+z^2)\left(2x + 2z\frac{dz}{dx}\right)$

$yze^{xyz} + xye^{xyz}\frac{dz}{dx} = 2x\cos(x^2+y^2+z^2) + 2z\cos(x^2+y^2+z^2)\frac{dz}{dx}$

$xye^{xyz}\frac{dz}{dx} - 2z\cos(x^2+y^2+z^2)\frac{dz}{dx} = 2x\cos(x^2+y^2+z^2) - yze^{xyz}$

$\frac{dz}{dx}\left(xye^{xyz} - 2z\cos(x^2+y^2+z^2)\right) = 2x\cos(x^2+y^2+z^2) - yze^{xyz}$

$$\frac{dz}{dx} = \frac{2x\cos(x^2+y^2+z^2) - yze^{xyz}}{xye^{xyz} - 2z\cos(x^2+y^2+z^2)}$$

Scanned with CamScanner

Question assigned to the following page: 2

2 b.i) $0 = \sin(x^2+y^2+z^2) - e^{xyz}$

$F(x,y,z)$

$x \quad y \quad z$ where $x = x(t)$, $y = y(t)$, $z = z(x,y)$

$x \; y$

$$0 = \frac{dF}{d\cancel{x}} = \frac{\partial F}{\partial x}\frac{dx}{d\cancel{x}} + \frac{\partial F}{\partial y}\frac{\cancel{dy}}{d\cancel{x}} + \frac{\partial F}{\partial z}\cdot\frac{\partial z}{\partial x}$$

$$0 = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial x}$$

$$\frac{-F_x}{F_z} = \frac{\partial z}{\partial x}$$

b.ii) $F_x = 2x\cos(x^2+y^2+z^2) - yze^{xyz}$

$F_z = 2z\cos(x^2+y^2+z^2) - xye^{xyz}$

$$\frac{\partial z}{\partial x} = \frac{yze^{xyz} - 2x\cos(x^2+y^2+z^2)}{2z\cos(x^2+y^2+z^2) - xye^{xyz}}$$

c) They match, I prefer chain rule method.

$$0 = \frac{\partial F}{\partial x}\frac{dx}{dx} + \frac{\partial F}{\partial y}\frac{dy}{dy} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial y}$$

$$\frac{-F_y}{F_z} = \frac{\partial z}{\partial y}$$

$F_y = 2y\sin(x^2+y^2+z^2) - xze^{xyz}$

$F_z = 2z\cos(x^2+y^2+z^2) - xye^{xyz}$

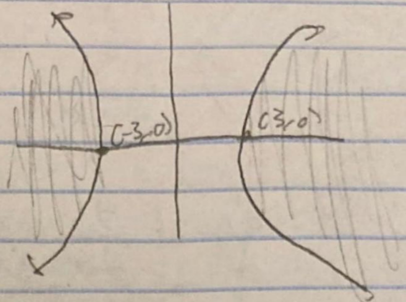
$$\frac{-F_y}{F_z} = \frac{xze^{xyz} - 2y\sin(x^2+y^2+z^2)}{2z\cos(x^2+y^2+z^2) - xye^{xyz}} = \frac{\partial F}{\partial y}$$

Scanned with CamScanner

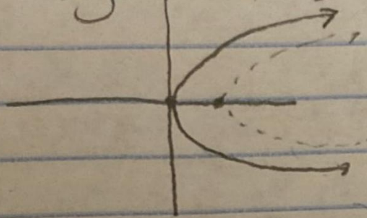
Questions assigned to the following page: 4 and 3

3 $x^2-y^2-9\geq 0$

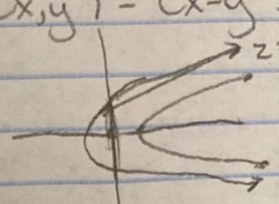
$x^2-y^2\geq 9 \rightarrow$



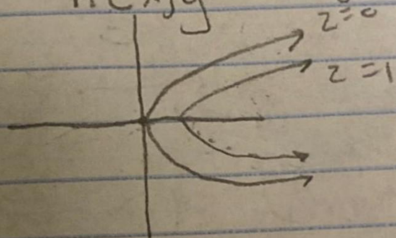
4 $f(x,y)=x-y^2$



$g(x,y)=(x-y^2)^2$



$h(x,y)=(x-y^2)^3$



For $z=0$, all graphs model $y=\pm\sqrt{x}$.
As z increases, $g(x,y)$ splits into 2 graphs
while $f(x,y)$ and $h(x,y)$ ~~are~~ 1. As z increases,
all functions shift to the right.

$f(2,1)=1$ The level curve intercepting (
is $1=x-y^2$

Question assigned to the following page: 5

5

$g(x, y, z)$

$x \quad y \quad z$

t

$$\frac{dg}{dt} = \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt} + \frac{\partial g}{\partial z}\frac{dz}{dt}$$

$$\frac{dg}{dt} = g_x \frac{dx}{dt} + g_y \frac{dy}{dt} + g_z \frac{dz}{dt} =$$

$\vec{r}(t)$

$$\frac{dg}{dt} = \langle g_x, g_y, g_z \rangle \cdot \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle,$$

$$\frac{d}{dt}(G(x, y, z) = 0) = \frac{dG}{dt} = 0$$

$$0 = \langle g_x, g_y, g_z \rangle \cdot \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$$

meaning these 2 vectors are orthogonal, and therefore $\langle g_x, g_y, g_z \rangle$ is also orthogonal to $\langle x(t), y(t), z(t) \rangle$ as $\vec{r}(t)$ is the tangent vector of $\vec{r}(t)$.

Question assigned to the following page: 6

6 a) $\nabla f(x,y) = \langle -\frac{x}{2}, -\frac{y}{8} \rangle$

$\nabla f(1,2) = \langle -\frac{1}{2}, -\frac{1}{4} \rangle$

b) $F_x(1,2) = -\frac{1}{2}$ $F_y(1,2) = -\frac{1}{4}$

$-\frac{1}{2}(x-1) + -\frac{1}{4}(y-2) + \frac{1}{2} = 0$

$-\frac{x}{2} + \frac{1}{2} - \frac{y}{4} + \frac{1}{2} + \frac{1}{2} = 0$

$-\frac{x}{2} - \frac{y}{4} + \frac{3}{2} = 0$

$4\left(-\frac{y}{4} = \frac{x}{2} - \frac{3}{2}\right)$

$\frac{-y = 2x - 6}{-1}$

$y = 6 - 2x$