

1. (A function space) This problem is essentially the same as Th. 12.3, pp. 315-316.
2. (Topology of a metric space) Mimic the proofs for  $\mathbb{R}^n$ .
3. (Closure) (a) Use  $X = \text{int}A \cup \text{bd}A \cup \text{ext}A$ .  
(b) See Exam 2 Review Problem # 2(c).
4. (Pointwise convergence) Use the comparison test for uniform convergence.
5. (Topology of a function space) See example posted on Canvas under Readings.  
(a) Let  $f_k \in A$  and  $f_k \rightarrow f$ . Then  

$$-x^2 - 1 \leq f_k(x) \leq x^2 + 1.$$
Remember that the limit preserves  $\leq$  (Lemma 2.21).
6. (Cauchy sequences) In this problem,  $c$  is the contraction constant for  $T$ , so  $0 < c < 1$ . Show that  $\forall \varepsilon > 0, \exists K \in \mathbb{N}$  s.t.  

$$\forall k \geq K, \quad \left| \frac{c^k}{1-c} \right| < \frac{\varepsilon}{d(T(p_0), p_0)}.$$
7. (c)  $f(x, y) = (-y, x)$  is the counterclockwise rotation through  $90^\circ$  since  $\begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} -y \\ x \end{bmatrix} = -xy + yx = 0$ .
8. Use the comparison test for uniform convergence.

(End of hints)