Part III assignment 1!

● Graded

Student

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Total Points

20 / 20 pts

Question 1

17.2

5 / 5 pts

✔ **– 0 pts** Correct

– 1 pt $\frac{1}{3}$ can be reflected across $\frac{1}{2}$ to get $\frac{2}{3}$, so the orbit of $\frac{1}{3}$ is $(\frac{1}{3}+\mathbb{Z})\cup(\frac{2}{3}+\mathbb{Z})$. Similarly the orbit of $\frac{1}{2}$ is $\frac{1}{2}+\mathbb{Z}$.

– 1 pt 1 and $\frac{1}{2}$ are both the centers of reflections in D_∞ so they each have one non-trivial reflection in their stabilizer. $\frac{1}{3}$ is not the center of a reflection so its stabilizer is trivial.

Question 2

17.4

5 / 5 pts

✔ **– 0 pts** Correct

Question 3

17.9

5 / 5 pts

✔ **– 0 pts** Correct

– 0 pts In (b) $t=2\pi$ sends every point back where it started so orbits are circles and there are infinitely many.

– 0 pts In (c) no value of t except 0 sends any point back to itself and a single orbit is dense in the torus.

– 0 pts Unclear answer

1 It is dense in $C\times C$ but does not cover all of it. This is admittedly a toplogy thing more than a group theory thing.

Question 4

example of surjection not inducing a product structure

5 / 5 pts

✔ **– 0 pts** Correct

– 1 pt Major logical issue.

– 0 pts Minor logical issue.

Questions assigned to the following page: 1 and 2

M145 HW P3 I

17.2) $D_\infty \curvearrowright \mathbb{R}$

So $\text{Orb}(1) = \mathbb{Z}$ as just rotate it

$\text{Stab}(1) = \langle rsr^{-1} \rangle$ as sends 1 to self, no other unique way

$\text{Orb}(\frac{1}{2}) = \mathbb{Z} + \frac{1}{2}$ as rotating gives $\mathbb{Z} + \frac{1}{2}$, and invariant under reflection

Now by same idea as above

$\text{Stab}(\frac{1}{2}) = \langle rsr^{-1} \rangle$

$\text{Orb}(2/3) = (\mathbb{Z} + 2/3) \cup (\mathbb{Z} + \frac{1}{3})$

as $2/3 - 1/3 \equiv -1/3$ so, and for $\text{Stab}(2/3)$ as not midpoint of $[0,1]$ it's going to just be $\{e\}$ as reflecting puts a point on $\mathbb{Z} + 2/3 \Rightarrow \mathbb{Z} + 1/3$ and vice versa.

So: $\text{Orb}(1) = \mathbb{Z}$ $\quad \text{Stab}(1) = \langle rsr^{-1} \rangle$

$\text{Orb}(\frac{1}{2}) = \mathbb{Z} + \frac{1}{2}$ $\quad \text{Stab}(\frac{1}{2}) = \langle rsr^{-1} \rangle$

$\text{Orb}(2/3) = (\mathbb{Z} + \frac{1}{3}) \cup (\mathbb{Z} + 2/3)$

$\text{Stab}(2/3) = \{e\}$

17.4)

$\Rightarrow$ $G \curvearrowright X$ Let $g \in G$, and $y = gx$

Now by thm 17.1, points in same orbit have same conjugate stabilizers so

$gG_xg^{-1} = G_x \Rightarrow gG_x = G_xg$ meaning G_x is normal $\square$

$\Leftarrow$ Let $y \in G(x)$, $h \in G_y \Leftrightarrow g^{-1}hg \in G_x$

So, $G_y = gG_xg^{-1}$, but as G_x is normal $G_x = G_y$

So same stabilizers

$\square$

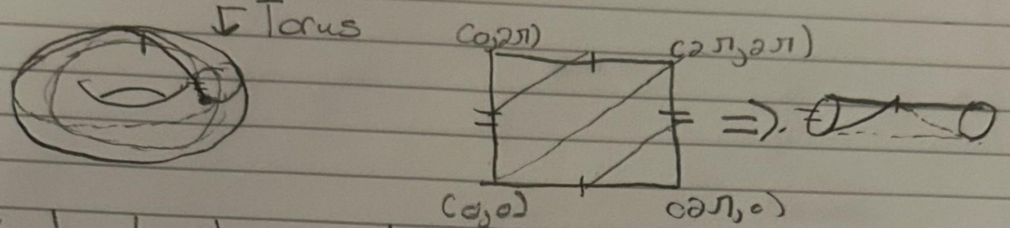
Question assigned to the following page: 3

17.9)

a) As $e^{i(x+t)}$ periodic and e^{iy} fixed get one circle so orbit is:

$Orb(t) = C \times e^{iy}$

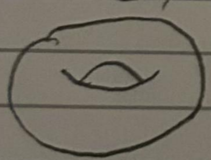
b)



It's hard to visualize but path we get where we cross over each circle once and then return to the start making this a periodic path:

So $orbit(t) = (e^{i(x+t)}, e^{i(y+t)})$ and get just a path.

c) So,



Given x, y and $e^{i(x+t)}, e^{i(y+t\sqrt{2})}$ find orbit.

So WLOG, $x = y = 0$ so $(e^{it}, e^{it\sqrt{2}})$

Can do as $e^{ix+it} = e^{ix}e^{it}$

So Clearly as $t \in \mathbb{R}$ e^{it} will give C.

W.t.s. what $e^{it\sqrt{2}}$ gives:

So first for $t = j$ or $t = k$ where $j \neq k$ $e^{ij\sqrt{2}} \neq e^{ik\sqrt{2}}$ as implies $j\sqrt{2} = k\sqrt{2} \pm 2\pi$ which won't work. We also never return to the start as like sliding by $\sqrt{2}$ and don't get back to start. So, as $t \in \mathbb{R}$, overall t, will get essentially all of C so

$orbit(t) = C \times C$

Question assigned to the following page: 4

4) $G = \mathbb{Z}$ $H = \mathbb{Z}/5\mathbb{Z}$

$\phi: G \to H$

$x \longrightarrow x \bmod n$

ϕ is clearly surjective, to show homomorphism:

$\phi(xy) = (x+y) \bmod n = x \bmod n + y \bmod n$

$= \phi(x)\phi(y)$

So homomorphism

Now, we know previous HW, the only subgroups of $\mathbb{Z}$ are $n\mathbb{Z}$ where $n \in \mathbb{Z}$

However, $|n\mathbb{Z}| = \infty$, and $|\mathbb{Z}/5\mathbb{Z}| = 5$

So $n\mathbb{Z} \not\cong \mathbb{Z}/5\mathbb{Z}$

To show for any K, $G \not\cong H \times K$

well, as $G \cong \mathbb{Z}$ is cyclic + infinite, and H is finite:

K must be infinite, cyclic so $K \cong \mathbb{Z}$, but violates fact that for direct product that $\mathbb{Z} \cap (\mathbb{Z}/5\mathbb{Z}) = \{e\}$

So $\nexists K$ s.t $G \cong H \times K$