Homework 11

● Graded

Student

Scott A. Fullenbaum

Total Points

20 / 20 pts

Question 1

Computing a Flux

5 / 5 pts

- ✔ **– 0 pts** Completion & Timeliness
- **– 2 pts** Late
- **– 2 pts** No attempt onQ1

Question 2

Divergence Theorem

5 / 5 pts

- ✔ **– 0 pts** Correct answer for part a.
- ✔ **– 0 pts** Correct answer for part b.
- **– 0.5 pts** Small mistake in part a.
- **– 0.5 pts** Small mistake part b.
- **– 1 pt** Medium mistake in part a.
- **– 1.25 pts** Incorrect integral in part a.
- **– 1.25 pts** Incorrect integral in part b.
- **– 1.75 pts** Part a incorrect.
- **– 1.75 pts** Part b incorrect.
- **– 2.5 pts** No answer for part a.

Question 3

Theorem of Stokes

5 / 5 pts

- ✔ **– 0 pts** Correct
- **– 1 pt** Incorrect parameterization or calculation errors in A
- **– 1.5 pts** Integration error in A
- **– 1 pt** Incorrect parameterization or calculation errors in B
- **– 1.5 pts** Integration error in B

Question 4

Theorem of Stokes 2

5 / 5 pts

- ✔ **– 0 pts** Correct
- **– 0.5 pts** Incorrect normal vector
- **– 0.25 pts** Incorrectly computed curl
- **– 0.25 pts** Minor algebraic error
- **– 0.5 pts** Incorrect integral bounds
- **– 0.5 pts** Transcription error (problem copied down incorrectly or error copying terms)
- **– 1 pt** Conceptual error
- **– 3 pts** Major conceptual error
- **– 0.5 pts** Major algebraic error

Question assigned to the following page: 1

Calc Hw 11

1 a) $S: x^2+y^2+z^2=1$

$\vec{r}(u,v) = \langle \sin u\cos v, \sin u \sin v, \cos u\rangle$

$\vec{r}_u = \langle \cos u\cos v, \cos u \sin v, -\sin u\rangle$

$\vec{r}_v = \langle -\sin u\sin v, \sin u \cos v, 0\rangle$

$0 \le u \le 2\pi$

$0 \le v \le 2\pi$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos u\cos v & \cos u\sin v & -\sin u \\ -\sin u\sin v & \sin u\cos v & 0 \end{vmatrix}$$

$$= \left\langle \begin{vmatrix} \cos u\sin v & -\sin u \\ \sin u\cos v & 0 \end{vmatrix}, -\begin{vmatrix} \cos u\cos v & -\sin u \\ -\sin u\sin v & 0 \end{vmatrix}, \begin{vmatrix} \cos u\cos v & \cos u\sin v \\ -\sin u\sin v & \sin u\cos v \end{vmatrix} \right\rangle$$

$$= \langle \sin^2 u\cos v, \sin^2 u \sin v, \cos^2 v \sin u\cos u + \sin^2 v \sin u\cos u \rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle \sin^2 u\cos v, \sin^2 u\sin v, \sin u\cos u\rangle$$

$$\iint_S F\cdot n\, dS = \int_0^{2\pi}\int_0^{2\pi} \langle \sin u\cos v, \sin u\sin v, \cos u\rangle \cdot \langle \sin^2 u\cos v, \sin^2 u\sin v, \sin u\cos u\rangle\, du\, dv$$

$$= \int_0^{2\pi}\int_0^{2\pi} \sin^3 u\cos^2 v + \sin^3 u \sin^2 v + \sin u\cos u\cos u\, du\, dv$$

$$= \int_0^{2\pi}\int_0^{\pi} \sin^3 u + \cos^2 u \sin u\, du\, dv$$

$$= 2\pi \int_0^{\pi} \sin u(\sin^2 u + \cos^2 u)\, du$$

$$= 2\pi \int_0^{\pi} \sin u\, du = \boxed{4\pi}$$

$$= \int_0^{2\pi} \frac{4}{3} \, d\theta = \boxed{\frac{8\pi}{3}}$$

1 b) $\iiint_D \nabla\cdot F\, dV \qquad \nabla\cdot F = 3 \langle 1, 1, 1\rangle$

$$= 3\int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = 6\pi \int_0^{\pi} \frac{1}{3}\sin\phi\, d\phi = \boxed{4\pi}$$

Questions assigned to the following page: 1 and 2

1 c) The answers are the same, which matches the divergence theorem. Using the divergence theorem was easier

2a) $\iint_S \vec{F}\cdot\vec{n}\,dS = \iiint_E \operatorname{div} F\,dV \quad E: x^2+y^2+z^2=6$

$\operatorname{div} F = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \cdot \langle x, -2y, 3x \rangle = -1$

$$\iiint_E -1\,dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{6}} -\rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$$

$$= 2\pi \int_0^{\pi} -\frac{\rho^3}{3}\sin\phi \Big|_0^{\sqrt{6}}\,d\phi$$

$$= 2\pi \int_0^{\pi} -\frac{6^{3/2}}{3}\sin\phi\,d\phi$$

$$= -2\pi\left(-\frac{6^{3/2}}{3}\right)\cdot 2$$

$$= \boxed{-\frac{4\pi}{3}\cdot 6^{3/2}}$$

$$= \boxed{-8\sqrt{6}\pi}$$

2 b) $\iiint_E \operatorname{div} F\,dV$ where E is cube in text

$\operatorname{div} F = \langle \partial_x, \partial_y, \partial_z \rangle \cdot \langle x^2-z^2, 2xz, y^2 \rangle = 2x$

$$\int_0^1\int_0^1\int_0^1 2x\,dx\,dy\,dz$$

$$2\int_0^1\int_0^1 \frac{1}{2}\,dy\,dz = \int_0^1\int_0^1 dy\,dz = \boxed{1}$$

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Question assigned to the following page: 3

3 a) $\iint_S (\nabla\times F)\cdot\vec{n}\,dS =$ $S: z=\sqrt{25-x^2-y^2}$ $z\geq 3$

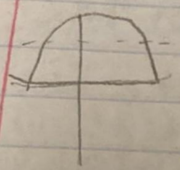
$\nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ d_x & d_y & d_z \\ x^2-z^2 & y^2 & xz \end{vmatrix} = \left\langle \begin{vmatrix} d_y & d_z \\ y^2 & xz \end{vmatrix}, -\begin{vmatrix} d_x & d_z \\ x^2-z^2 & xz \end{vmatrix}, \begin{vmatrix} d_x & d_y \\ x^2-z^2 & y \end{vmatrix} \right\rangle$

$\nabla\times\vec{F} = \langle 0,-3z,0\rangle$

$\vec{n} = \langle -2x,-2y,1\rangle$

$\vec{n} = \left\langle \frac{-x}{\sqrt{25-x^2-y^2}}, \frac{y}{\sqrt{25-x^2-y^2}}, 1\right\rangle$

$\iint_S (\nabla\times F)\cdot\vec{n}\,dS = \iint_S \langle 0,-3\sqrt{25-x^2-y^2},0\rangle\cdot\left\langle \frac{x}{\sqrt{25-x^2-y^2}}, \frac{y}{\sqrt{25-x^2-y^2}}, 1\right\rangle dS$



$= \iint_S -3y\,dA = -3\int_0^{2\pi}\int_0^4 r^2\sin\theta\,dr\,d\theta$

$= -3\int_0^{2\pi} \frac{64}{3}\sin\theta\,d\theta$

$= -64\int_0^{2\pi}\sin\theta\,d\theta$

$= -64(-\cos\theta)\Big|_0^{2\pi} = \boxed{0}$

3 b) $\iint_S (\nabla\times F)\cdot n\,dS = \int_C \vec{F}\cdot d\vec{r}$ $F=\langle -2,x,y\rangle$

$C: x^2+y^2=9,\ z=0$, $\vec{r}(t)=\langle 3\cos t, 3\sin t, 0\rangle$ $0\leq t\leq 2\pi$

$\vec{r}'(t) = \langle -3\sin t, 3\cos t, 0\rangle$

$\int_0^{2\pi} \langle 0, 3\cos t, 3\sin t\rangle\cdot\langle -3\sin t, 3\cos t, 0\rangle\,dt$

Questions assigned to the following page: 3 and 4

$$9\int_0^{2\pi} \langle \cos^2 t \rangle \, dt = 9\int_0^{2\pi} \frac{1+\cos(2t)}{2}\, dt$$

$$= 9\left(\frac{t}{2} + \frac{\sin(2t)}{4}\right)\Big|_0^{2\pi}$$

$$= 9(\pi) = \boxed{9\pi}$$

4 $\oint_C \vec{F}\cdot d\vec{r} = \iint_S (\nabla\times\vec{F})\cdot\vec{n}\, dS$

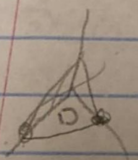
$$\nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2-y^2 & x & 2yz \end{vmatrix} = \left\langle \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 2yz \end{vmatrix}, -\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^2-y^2 & 2yz \end{vmatrix}, \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2-y^2 & x \end{vmatrix} \right\rangle$$

$\text{Curl}\, F = \langle 2z, 0, 2y+1 \rangle$

$\vec{n} = \langle -z_x, -z_y, 1\rangle$, $\vec{n} = \langle 2, 1, 1\rangle$

$$\iint_S \langle 2z, 0, 2y+1\rangle\cdot\langle 2,1,1\rangle\, dS = \iint_S 4z+2y+1\, dA$$

$6-2x-y$



D: $z=0$, $y=6-2x$

$$\int_0^3\int_0^{6-2x} 4(6-2x-y)+2y+1\, dy\, dx$$

$$= \int_0^3\int_0^{6-2x} 25-8x-2y\, dy\, dx$$

$$= \int_0^3 25(6-2x) - 8x(6-2x) - (6-2x)^2\, dx$$

$$= \int_0^3 114 - 74x + 12x^2\, dx$$

$$= 114x - 37x^2 + 4x^3\Big|_0^3 = \boxed{117}$$