MATH 42 HOMEWORK 4

This homework is due at 11:59 p.m. (Eastern Time) on Wednesday, October 7. Scan the completed homework and upload it **as one pdf file** to Gradescope. The Canvas module "Written Assignments" has instructions for how to upload the assignment to Gradescope. This assignment covers §15.3 − 5.

(1) Consider the surface given by $z = x^2 + 3y^2$. Find parametric equations for the tangent line to the curve of intersection of the surface and the plane $y = 1$ at the point $(1, 1, 4)$. (Hint: First, find the slope of that line.)

(2) Assume that the equation $e^{xyz} = \sin(x^2 + y^2 + z^2)$ implicitly defines z as a function of x, y. We will find $\frac{\partial z}{\partial x}$ using two methods:

(a) Go through and implicitly take partial derivatives with respect to x on both sides of the given equation, treating z as a function of x and treating y as constant, and remembering to use the chain rule when necessary. Then, solve for $\frac{\partial z}{\partial x}$. This is the method you learned in Calc I.

(b) (i) Ignore the above equation for a moment, and consider the general case where some equation $F(x, y, z) = 0$ implicitly defines z as a function of x, y. Use the multivariable chain rule to find a general formula for $\frac{\partial z}{\partial x}$. Hint: draw a tree.

(ii) Now, use the formula you found to find $\frac{\partial z}{\partial x}$ for the above example.

(c) Make sure your two answers match (otherwise, find your mistake). Which method do you prefer? Use your preferred method to find $\frac{\partial z}{\partial y}$ for the above example.

(3) Find the domain of the function $f(x, y) = \frac{\sqrt{x^2-y^2-9}}{y-x}$. Describe the domain of f algebraically. Sketch a graph of this domain in the plane labeling any curves involved and indicating which curves are included or excluded.

(4) Compare the level curves for the three functions:
$f(x, y) = x - y^2$, $g(x, y) = (x - y^2)^2$, $h(x, y) = (x - y^2)^3$.
Draw some sketches in the plane. In what ways are they similar and in what ways do they differ? Additionally, give the equation for the level curve of f intersecting the point $x = 2, y = 1$.

(5) Assume $g(x, y, z) = 0$ is a smooth surface and $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$ is a smooth curve on that surface. Use the multivariable chain rule to prove that the vector $\langle g_x, g_y, g_z\rangle$ is orthogonal to the curve $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$ at each point of the curve. (Note: being orthogonal to a curve means being orthogonal to its tangent vector / tangent line.)

(6) Let $f(x, y) = 1 - x^2/4 - y^2/16$. The point $(1, 2)$ lies in the level curve $f(x, y) = 1/2$, which is an ellipse.

(a) Find the gradient $\nabla f(1, 2)$.

(b) Find an equation of the tangent line to the ellipse $f(x, y) = 1/2$ at the point $(1, 2)$.