Tufts University
Department of Mathematics

Math 136 **Class work for the Week of March 6, 2023** [1] **Spring, 2023**

Due date: 11:59 pm, Monday, March 14, 2023

Roughly every week we will work in small groups to learn to write proofs and to solve problems. The problems will be long enough that you might need to talk with your group outside of class. Todd will grade the group work.

- *Scribe:* each week, someone in the group will volunteer to submit to Gradescope the group's answer *and* to enter all the group member names in Gradescope when uploading. This role should rotate through the group.
- *Respect:* when discussing problems, please make sure that everyone feels comfortable speaking and that all feedback is supportive and encouraging. For example:
  - Please make sure everyone has a chance to talk.
  - To start the discussion, you could have everyone suggest topics, theorems and ideas that relate to the question.
  - If you talk at the board please make sure everyone has a piece of chalk so they can add to the discussion.

Problem:

Theorem 1 (Riemann's Condition for Integrability) *Let $f:[a,b]\to\mathbb{R}$ be bounded. Then f is integrable if and only if*

$$\text{for each } \epsilon>0, \text{ there is a partition } P \text{ of } [a,b] \text{ such that } U(f,P)-L(f,P)<\epsilon. \tag{1}$$

Now, you will do the proof.

1. First, assume $f:[a,b]\to\mathbb{R}$ is integrable. Prove for each $\epsilon>0$ there is a partition P of $[a,b]$ satisfying the Riemann condition (1).
   (a) Why does there exist an Archimedean sequence of partitions $\{P_n\}$ for f?
   (b) Use an Archimedean sequence for f, $\{P_n\}$ to prove (1).
2. Now, assume $f:[a,b]\to\mathbb{R}$ satisfies Riemann's condition. Prove f is integrable.