Homework 3

Graded

Student

Scott A. Fullenbaum

Total Points

16.5 / 20 pts

Question 1

Arc length

5 / 5 pts

- ✔ **– 0 pts** Correct.
- **– 0.75 pts** Incorrect derivatives part a.
- **– 0.75 pts** Incorrect derivatives part b.
- **– 0.75 pts** Incorrect arc length formula part a.
- **– 0.75 pts** Incorrect arc length formula part b.
- **– 0.5 pts** Minor calculation error part a.
- **– 0.5 pts** Minor calculation error part b.
- **– 0.75 pts** Integration error part a.
- **– 0.75 pts** Integration error part b.
- **– 1.25 pts** Show work for part b.
- **– 0.5 pts** Show more work.
- **– 1 pt** Simplify integral in part b to perform integration without computing resource.

Question 2

Plane equation and normal vector

3.5 / 5 pts

- ✔ **+ 0.5 pts** Found two vectors lying the plane
- ✔ **+ 1.5 pts** Found correct normal vector
- **+ 1.5 pts** Scaled normal to unit length
- ✔ **+ 1.5 pts** Found equation to the plane
- **– 1 pt** Did not scale normal to unit length
- **– 1 pt** Some calculation errors

Question 3

Dihedral angle and line of intersection

2 / 2 pts

- ✔ **– 0 pts** Significant attempt and/or completed.
- **– 1 pt** Minimal attempt and/or left uncompleted.
- **– 2 pts** Nearly zero to zero attempt.

Question 4

Reasoning about planes

3 / 3 pts

- ✔ **– 0 pts** Full attempt and mostly correct answer.
- **– 0.5 pts** Stated that only one plane could be found, did not discuss the point D on the plane, or stated that only a finite number of planes could be found.
- **– 1.5 pts** Stated that no planes could be found.
- **– 2 pts** Minimal attempt at problem.
- **– 3 pts** Nearly no attempt or no attempt.

Question 5

Sketching curves/surfaces

3 / 5 pts

- **– 0 pts** Good work!
- ✔ **– 1 pt** Incorrect interpretation (a)
- ✔ **– 1 pt** Incorrect interpretation (b)
- **– 1 pt** Incorrect interpretation (c)
- **– 1 pt** Insufficient detail in plots/unable to determine surface or traces in plots
- **– 0.5 pts** Equation/picture for trace or cross-section incorrect

What kind of surfaces do you get for (a) and (b)? Unclear from sketches. What is the xz trace in (a)? What value is chosen to determine equation for cross sections?

Question 6

Separation of level surfaces

0 / 0 pts

- ✔ **– 0 pts** Ungraded problem.

Question assigned to the following page: 1

Math 42 Homework 3

1 a) $\vec{r}(t) = \langle \sin t, \cos t, t\sqrt{3} \rangle \quad t \in [0, 5]$

$$L = \int_0^5 \sqrt{(\cos t)^2 + (-\sin t)^2 + (\sqrt{3})^2}\, dt$$

$$L = \int_0^5 \sqrt{\cos^2 t + \sin^2 t + 3}\, dt$$

$$L = \int_0^5 2\, dt = 2t\Big|_0^5 = 10$$

b) $\vec{r}(t) = \langle \frac{1}{2}t^2, \frac{2\sqrt{2}}{3}\pi^{1/4} t^{3/2}, t\sqrt{\pi} \rangle \quad t \in [0, \pi]$

$$L = \int_0^\pi \sqrt{(t)^2 + (\sqrt{2}\,\pi^{1/4} t^{1/2})^2 + (\sqrt{\pi})^2}\, dt$$

$$L = \int_0^\pi \sqrt{t^2 + 2\sqrt{\pi}\,t + \pi}\, dt \quad (t^2 + 2\sqrt{\pi}\,t + \pi) = (t + \sqrt{\pi})^2$$

$$L = \int_0^\pi \sqrt{(t + \sqrt{\pi})^2}\, dt$$

since integral is on 0 to 5, it's always positive

$$L = \int_0^\pi |t + \sqrt{\pi}|\, dt$$

$$L = \frac{t^2}{2} + \sqrt{\pi}\,t\Big|_0^\pi = \frac{\pi^2}{2} + \pi^{3/2}$$

Scanned with CamScanner

Question assigned to the following page: 2

2 a)

$\vec{AB} = \langle 3, -4, -6 \rangle$

$\vec{AC} = \langle 5, -2, -4 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -4 & -6 \\ 5 & -2 & -4 \end{vmatrix} = \begin{vmatrix} -4 & -6 \\ -2 & -4 \end{vmatrix} \vec{i} - \begin{vmatrix} 3 & -6 \\ 5 & -4 \end{vmatrix} \vec{j} + \begin{vmatrix} 3 & -4 \\ 5 & -2 \end{vmatrix} \vec{k}$$

$$\vec{n} = 4\vec{i} - 18\vec{j} + 14\vec{k}$$

$\vec{n} = \langle 4, -18, 14 \rangle$

$$\vec{u}_n = \frac{\vec{n}}{|\vec{n}|} = \frac{\langle 4, -18, 14 \rangle}{\sqrt{4^2 + (-18)^2 + 14^2}} = \left\langle \frac{4}{\sqrt{536}}, \frac{-18}{\sqrt{536}}, \frac{14}{\sqrt{536}} \right\rangle.$$

b) $A = (-2, 4, 3)$ $\quad \vec{AA_x} = \langle x+2, y-4, z-3 \rangle$

$\vec{n} = \langle 4, -18, 14 \rangle$

$-\vec{AA_x} \cdot \vec{n} = 0$

$\langle \cancel{-x}+2, y-4, z-3 \rangle \cdot \langle 4, -18, 14 \rangle = 0$

$4(x+2) - 18(y-4) + 14(z-3) = 0$

$4x + 8 - 18y + 72 + 14z - 42 = 0$

$$\frac{4x - 18y + 14z + 38 = 0}{2}$$

$$2x - 9y + 7z + 19 = 0$$

Question assigned to the following page: 3

3 a) Dihedral angle $= \theta$, $\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$, n_1 and n_2 are normal vectors

$\vec{n}_1 = \langle 4, -18, 14 \rangle$ $\quad |\vec{n}_1| = \sqrt{536}$

$\vec{n}_2 = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$ $\quad |\vec{n}_2| = 1$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{\sqrt{536}} = \frac{\frac{4}{\sqrt{3}} - \frac{18}{\sqrt{3}} + \frac{14}{\sqrt{3}}}{\sqrt{536}} = 0$$

$\cos\theta = 0$, $\theta = \frac{\pi}{2}$

Dihedral angle $= \frac{\pi}{2}$

b) Line is $\vec{n}_1 \times \vec{n}_2$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -18 & 14 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{vmatrix} = \begin{vmatrix} -18 & 14 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{vmatrix} \vec{i} - \begin{vmatrix} 4 & 14 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{vmatrix} \vec{j} + \begin{vmatrix} 4 & -18 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{vmatrix} \vec{k}$$

$$n_I = \vec{n}_1 \times \vec{n}_2 = \frac{-32}{\sqrt{3}} \vec{i} + \frac{10}{\sqrt{3}} \vec{j} + \frac{22}{\sqrt{3}} \vec{k}$$

unit vector $n_I = \dfrac{\langle \frac{-32}{\sqrt{3}}, \frac{10}{\sqrt{3}}, \frac{22}{\sqrt{3}} \rangle}{\sqrt{\frac{32^2}{3} + \frac{10^2}{3} + \frac{22^2}{3}}}$

$$= \frac{\langle \frac{-32}{\sqrt{3}}, \frac{10}{\sqrt{3}}, \frac{-14}{\sqrt{3}} \rangle}{\sqrt{536}}$$

$$= \langle \frac{-32}{\sqrt{1608}}, \frac{10}{\sqrt{1608}}, \frac{-14}{\sqrt{1608}} \rangle$$

Question assigned to the following page: 4

4 If such a plane exists, then their normal vectors would be orthogonal. This means that the dot product of the normal vectors equals 0.

$\vec{n}_D = \langle a, b, c \rangle$ and $\vec{n} = \langle 4, -18, 14 \rangle$

$\vec{n}_D \cdot \vec{n} = 4a - 18b + 14c = 0$

This equation has infinitely many solutions, so there are infinitely many normal vectors and therefore infinitely many planes that are orthogonal to the plane in problem 2 and pass through point D in problem 2.

Scanned with CamScanner

Question assigned to the following page: 5

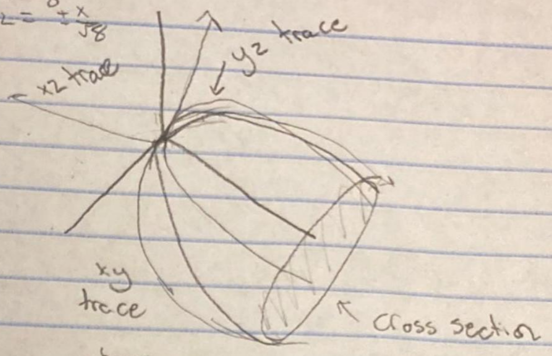
5 a) xy trace. $y=\frac{1}{4}x^2$

xz trace $2z^2=\frac{1}{4}x^2$

yz trace $y=-2z^2$

$z^2=\frac{x^2}{8}$

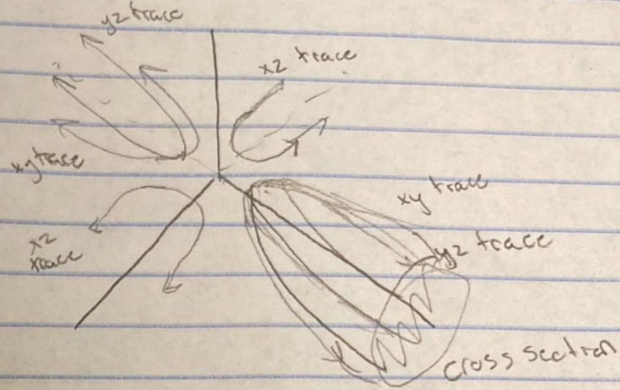
$z=\pm\frac{x}{\sqrt{8}}$



b) xy trace: $x^2=2+2y^2$

xz trace $x^2=2+4z^2$

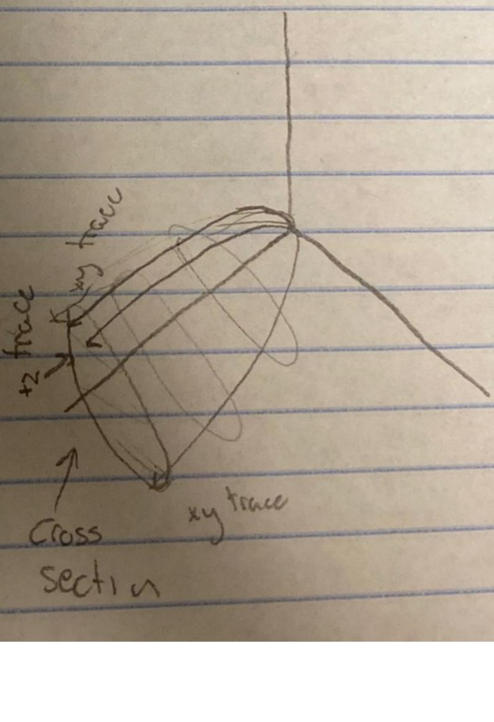
yz trace DNE



c) xy-trace $x=y^2$

xz-trace $x=z^2$

yz trace: $(0,0)$



Question assigned to the following page: 6

6 $\left(K=\sqrt{(x-3)^2+(y+4)^2+z^2}\right)^2$

$$K^2=(x-3)^2+(y+4)^2+z^2$$

The level surfaces of $f(x,y,z)$ forms a sphere with radius K, where K is the constant $f(x,y,z)$ equals. The sphere is centered at $(3,-4,0)$