(1) Consider the following subset of the real line:

$$S = \{0\} \cup (1,2) \cup (2,3) \cup (\mathbb{Q} \cap (4,5)).$$

Compute the following sets: (You may take for granted that $\overline{\mathbb{Q}} = \mathbb{R}$ and $\mathbb{Q}^\circ = \emptyset$.)

(a) $\overline{S}$

(b) $\left(\overline{S}\right)^\circ$

(c) $\overline{\left(\overline{S}\right)^\circ}$

(d) S°

(e) $\overline{S^\circ}$

(f) $\left(\overline{S^\circ}\right)^\circ$

Remark 1. S is an example of a **Kuratowski 14 set**.

(2) Let A and B be two subsets of a topological space X. Prove the following statements or find a counterexample:
   (a) If $A \subseteq B$, then $\operatorname{Int} A \subseteq \operatorname{Int} B$

   (b) $\operatorname{Int}(A \cap B) = \operatorname{Int} A \cap \operatorname{Int} B$

(c) $\operatorname{Int}(A \cup B) = \operatorname{Int} A \cup \operatorname{Int} B$

(d) $\overline{A \cap B} = \overline{A} \cap \overline{B}$.

(3) Let X and Y be topological spaces. We've seen that a function $f : X \to Y$ is continuous if and only if it is locally continuous. That is f is continuous if and only if there exists an open cover $\{U_i\}_{i \in I}$ such that $f|_{U_i} : U_i \to Y$ is continuous for all $i \in I$.

More generally a property $\mathcal{P}$ of functions is said to be **local** if for any open cover $\{U_i\}_{i \in I}$ of X, f has property $\mathcal{P}$ if and only if $f|_{U_i} : U_i \to Y$ has property $\mathcal{P}$ for all $i \in I$.

(a) Consider the property of being a constant function. Let $\{U_i\}_{i \in I}$ be an open cover of X and let $f : X \to Y$ be a function. Is it true that if $f : X \to Y$ is constant, then is $f|_{U_i} : U_i \to Y$ is constant for all $i \in I$?

(b) In the same situation, if $f|_{U_i} : U_i \to Y$ is constant for all $i \in I$, is it necessarily true that $f : X \to Y$ is constant? Prove or give a counterexample.

(4) Let X and Y be topological spaces. Recall that a function $f : X \to Y$ is said to be **open** if for all open subsets $V \subseteq X$, the image $f(V) \subseteq Y$ is an open subset of Y. Let's show that being an open function is a local property of functions.

(a) Let $\{U_i\}_{i \in I}$ be an open cover of X. Let $f : X \to Y$ be an open function. Show that for each $i \in I$, $f|_{U_i} : U_i \to Y$ is also an open function.

(b) Let $f : X \to Y$ be a function so that $f|_{U_i} : U_i \to Y$ is an open function for each $i \in I$. Show that f is also an open function.