MATH 42 HOMEWORK 2

Topics covered: Lines & Curves; Differentiation & Integration in $\mathbb{R}^3$; § 13.4-13.5, 14.1 - 14.3

This homework is due at 11:59 pm (Eastern Time) on Wednesday, September 23. You will need to scan your completed homework and upload it **as one pdf file** to Gradescope. Please see the Canvas module "Written Assignments" for instructions on how to upload your assignment to Gradescope.

(1) Let $\vec{a} = \langle 3, 2, 1 \rangle$ $\vec{b} = \langle 2, 2, 2 \rangle$ $\vec{c} = \langle 4, 0, 1 \rangle$. Compute the following:
   (a) $(\vec{c} \cdot \vec{b})(\vec{a} + \vec{b} + \vec{c})$
   (b) $(\vec{b} \times \vec{c}) \cdot \vec{b}$
   (c) $\vec{a} + (\vec{b} \times \vec{a})$
   (d) $\vec{b} \cdot (\vec{a} \times \vec{c})$
   (e) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{b})$

(2) For each of the following parametrizations, describe and sketch the curve it represents. Assume for all of the below that $-\infty < t < \infty$. You can check your answers on a computer graphing utility - playing around with a 3D graphing utility is a great way to build some intuition.
   (a) $\vec{r}(t) = \langle 2\cos(t), 2\sin(t), 5 \rangle$
   (b) $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$
   (c) $\vec{r}(t) = \langle t\cos(t), t\sin(t) \rangle$
   (d) $\vec{r}(t) = \langle t\cos(t), t\sin(t), t \rangle$
   (e) $\vec{r}(t) = \langle 3t^2, 4t^2, 5t^2 \rangle$

(3) Let $\vec{r}(t)$ be the position vector of a moving point P. Find its velocity, acceleration, and speed at the given time t.
   (a) $\vec{r}(t) = \langle \frac{2}{t}, \frac{3}{t+1} \rangle$; $t = 2$
   (b) $\vec{r}(t) = \langle e^{2t}, e^{-t} \rangle$; $t = 4$
   (c) $\vec{r}(t) = e^t \langle \cos t, \sin t, 1 \rangle$; $t = \pi/2$
   (d) $\vec{r}(t) = \langle 1 + t, 2t, 2 + 3t \rangle$; $t = 2$
   (e) $\vec{r}(t) = \langle 2t, e^{-t^2} \rangle$; $t = 1$

(4) Evaluate the following integrals.
   (a) $\displaystyle\int_0^2 6t^2 \mathbf{i} - 4t\mathbf{j} + 3\mathbf{k}\, dt$
   (b) $\displaystyle\int_{-1}^1 -5t\mathbf{i} + 8t^3 \mathbf{j} + -3t^2 \mathbf{k}\, dt$
   (c) $\displaystyle\int \sin t\mathbf{i} + \cos t\mathbf{j} + \tan t\mathbf{k}\, dt$
   (d) $\displaystyle\int_0^1 te^{t^2} \mathbf{i} + \sqrt{t}\mathbf{j} + (t^2 + 1)^{-1}\mathbf{k}\, dt$

(5) Suppose that a particle has acceleration vector at time t given by
$$\vec{a}(t) = \langle e^t, t^2 + t, \sin t \cos t \rangle$$

If the particle has initial velocity $\vec{v}(0) = \langle 1, 0, 2 \rangle$ and initial position, $\vec{r}(0) = \langle 0, 0, 0 \rangle$, find the expressions for $\vec{v}(t)$ and $\vec{r}(t)$.