

Problem Suppose that g is a continuous function on the closed bounded interval $[a, b]$ and let $G(x) = \int_a^x g(t)dt$ for $x \in [a, b]$.

1. Prove that G is differentiable everywhere on $[a, b]$, and that $G'(x) = g(x)$ for all $x \in [a, b]$.
2. Prove $G \in C^1[a, b]$, and conclude that G has bounded variation on $[a, b]$.

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