Orbit of Pallas

This problem considers estimating the trajectory of the asteroid Pallas using observational data in terms of right ascension and declination. Given the periodic nature of the movement of celestial bodies, the proposed function is a combination of trigonometric functions sines and cosines. To find the fitting parameters, the least square method is employed which gives reasonably good results. In addition to fitting, an interpolation problem is considered which finds a trigonometric polynomial that passes exactly through the data points. Given the number of observations, the tasks of least square fitting and interpolation were computationally easy as they require solving small sized linear systems. One could surmise that, if one seeks to fit to trigonometric functions with higher k, solving the linear system is expensive because the resulting matrix A is dense. However, it might be possible to design fast linear solvers leveraging the periodic property of trigonometric functions.

Implied volatility

The problem considers estimating implied volatility using the Black-Scholes method as a baseline and formulating the problem as that of a root finding. The root finding formulation is quite natural, and one could use any of the root-finding problems. For this problem, an appealing property is that Newton's method converges quadratically from an estimate which we have a closed form of. Calculating Newton's updates requires defining the Black-Scholes function and its derivative. Both these functions can be computed readily. Testing the idea on an actual data set shows that it yields reasonable values of implied volatility. Since Black-Scholes is not a perfect model, there is a minor variation in the volatility across the exercise prices. While this approach is useful, its practical merit relies on the assumptions of Black-Scholes and to what extent the option price can be well-estimated by the provided formula. However, irrespective of that, the proposed root finding method can be used to provide quick estimates of volatility and can be possibly combined or refined with more advanced option valuation techniques.