Homework 3

Graded

Student

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Total Points

6 / 6 pts

Question 1

5.4 **1** / 1 pt

✔ **– 0 pts** Correct

– 0.3 pts The main thing distinguishing the odd and even cases is that if n is odd then r is generated by r^2 so $\langle r^2, r^2 s\rangle$ is all of D_n while if n is even then $\langle r^2, r^2 s\rangle$ is not all of D_n and is instead isomorphic to $D_{n/2}$

Question 2

5.5 **1** / 1 pt

✔ **– 0 pts** Correct

– 0.5 pts You need to show that if $x \in H$ then $x^{-1} \in H$

Question 3

5.7 **1** / 1 pt

✔ **– 0 pts** Correct

– 0.1 pts Minor algebra issues but correct reasoning

– 0.5 pts Insufficient evidence of being a subgroup

Question 4

5.8 **1** / 1 pt

✔ **– 0 pts** Correct

– 0.5 pts Elements of finite order *generate* a subgroup but that subgroup contains elements of infinite order so the finite order elements do not *form* a subgroup because they are not closed

– 0.5 pts There are additional elements of finite order that you did not list.

– 0.5 pts Composing two reflections yields a translation, so the set is not closed and does not form a subgroup.

Question 5

6.2 **1** / 1 pt

✔ **– 0 pts** Correct

– 0.25 pts Incorrect computation in part (b)

– 0.25 pts Incorrect computation in part (c)

– 0.1 pts Did not say which permutations are even

Question 6

6.3

1 / 1 pt

✔ **– 0 pts** Correct

– 0.5 pts Permutations sending $\{2, 5, 7\}$ amongst themselves may permute $\{1, 3, 4, 6, 8, 9\}$ so there are more such permutations than you considered.

– 0.2 pts Computational algebra issue

– 0.3 pts Did not find the order of the subgroup.

Question 7

6.7

0 / 0 pts

✔ **– 0 pts** Submitted

Questions assigned to the following page: 1 and 2

Math 145 HW3

5.4) First, let n be odd. Then,
$s = (r^2)^{n-1} r^2 s = r^{2n-2} r^2 s = (r^n)^2 s = e \cdot s$
$r =$
As n is odd, $\exists k > 0$ s.t $k \bmod n = 1$.
$k = n+1$, and as n is odd, $n+1$ is even so k is even $\exists m > 0$ s.t $k = 2m$. Therefore,
$(r^2)^m = r^{2m} = r^{n+1} = r^n r = r$

As we have representations for r, s in terms of $r^2, r^2 s$, then $\langle r^2, r^2 s\rangle = D_n$ for odd n.

If n is even, still have $s = (r^2)^{n-1} r^2 s$
However, no representation of r.
Using only r^2, get $\{e, r^2, r^4, \dots r^{n-2}\}$
Can combine this w/ $r^2 s$ to add $\{r^2 s, r^4 s, \dots r^{n-2} s\}$
By symmetry, also get $\{sr^2, sr^4, \dots sr^{n-2}\}$
Show as $r^2 s r^{2k} = r(sr) r^{2k-1} = rsr^{2k-1} = r(sr)r^{2k-1}$
$= sr^{k-2}$

Therefore for even n:
$\langle r^2, r^2 s\rangle = \{e, r^2, r^4, \dots r^{n-2}, r^2 s, r^4 s, \dots$
$\dots r^{n-2} s, s, sr^2, sr^4, \dots sr^{n-2}\}$

5.5)
$\Rightarrow$ As H is a subgroup it is closed under the operation, so $x, y \in H \Rightarrow xy \in H$

$\Leftarrow$ As H is finite, nonempty, let $x \in H$.
Then $x^2 \in H, x^3 \in H, \dots x^n \in H$ where n is finite.
Let $|x| = n$ and $\exists k > n$ s.t. $x^k = x^n$.
So $x^{k-n} = e$, and $e \in H$.

Questions assigned to the following page: 2, 3, and 4

To show inverse, for $x \in H$, as $|x| = n$
then $x(x^{n-1}) = x^n = e$ as x is order n,
also $x^{n-1} \cdot x = x^n = e$, so inverse is
in H.

H is closed under operation as $x, y \in H \Rightarrow$
$xy \in H$, so H is a subgroup of G.

5.7)
As $|e| = 1$, $e \in H$. Let $a \in G$, and
$|a| = n$. So $a \in H$. $a^n = e$, then $(a^{-1})^n = (a^n)^{-1} = e^{-1} = e$
so a^{-1} has finite order so $a^{-1} \in H$.

Next, to show closed under operation.
Let $a, b \in G$ w/ $|a| = m$ and $|b| = n$. Next, let
$k = mn$.
$(ab)^k = a^k b^k$, as Abelian
$\Rightarrow a^{mn} b^{mn} = (a^m)^n (b^n)^m = e^n e^m = e$
So $|ab| \leq k$ which is finite, so $ab \in H$

Therefore, H is a subgroup of G.

5.8) Let $n \in \mathbb{Z}$, then $(t^n s)^2 = e$. To
show, $(t^n s)(t^n s) = t^n (s t^n) s = t^n (st) t^{n-1} s$
$\Rightarrow t^{n-1} s t^{n-1} s = (t^{n-1} s)^2$. (book says $st = t^{-1} s$)
We follow this pattern n times and get:
$(t^{n-n} s)^2 = (e s)^2 = s^2 = e$.

So $|t^n s| = 2$, $\forall n \in \mathbb{Z}$ also $|e| = 1 = e$ so in
the set of finite order $= \{e, t^{-n} s, \dots, t^{-2} s, t^{-1} s, s, t s, t^2 s, \dots t^n s\}$

The set H isn't a subgroup of D_∞, proof
on next page

5.8 (con)

By def, $s \in H$, $t^n s \in H$ $\forall n \in \mathbb{Z}$. Let $n \neq 0$.

If H is a subgroup, then $st^n s \in H$

$st^n s = (st)(t^{n-1}s) = t^{-1}s\,t^{n-1}s$ follow this n times

$$\vdots$$

$$= t^{-n}s^2 = t^{-n}$$

However, $t^{-n} \notin H$, especially as t^{-n} is not finite order, and every element in H has finite order, so

Question assigned to the following page: 5

6.2)

So Disjoint: (1734)(26)(58)

All Transpositions: (43)(73)(17)(26)(58)

Not in A_8 as odd # transpositions.

b) (4568)(1245)

$$\begin{array}{c c l} & \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 3 & 5 & 1 & 6 & 7 & 8 \\ 2 & 5 & 3 & 6 & 1 & 8 & 7 & 4 \end{bmatrix} & \begin{array}{l} \text{original} \\ (1245) \\ (4568) \end{array} \end{array}$$

match for res

So matching first and third rows get cycles as:

(125)(468)

As transpositions: (25)(12)(68)(46)

This is in A_8 as # transpositions is even.

c)

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 5 & 4 & 6 & 7 & 8 \\ 1 & 2 & 3 & 5 & 4 & 8 & 6 & 7 \\ 1 & 5 & 2 & 3 & 4 & 8 & 6 & 7 \\ 1 & 5 & 4 & 3 & 6 & 8 & 2 & 7 \end{bmatrix} \begin{array}{l} \text{original} \\ (45) \\ (687) \\ (253) \\ (624) \end{array}$$

match for res

So matching original and final get disjoint cycles:

(25687)(34)

As transpositions:

(67)(68)(56)(25)(34)

Not in A_8 as # transpositions is odd

Question assigned to the following page: 6

6.3) Subgroup denoted as H
So e is sending every element to itself, so 2, 5, 7 sent among themselves and $e \in H$.

For inverses: A permutation is a bijective map, so any $f \in G$ has an inverse. However, we need to show for $f \in H$, $f^{-1} \in H$

Consider WLOG and ignoring numbers other than 2, 5, 7) the cycle (257)
So 2 5 7 and can construct this
2 5 7
part of f^{-1} as (527). As this still sends 2, 5, 7 among themselves, then $f^{-1} \in H$.
↑
We can ignore 1, 3, 4, 6, 8, 9 for this argument.

Closed: Composition of permutations is closed generally and again, show for within H as looking in 2, 5, 7 the composition of 2 permutations that send them among themselves, still send them among themselves.

For order: There are $3!$ ways to order 2, 5, 7 as 3 options for 1^{st} slot, then 2 for second, and one for third.
There are then $6!$ ways to order the remaining 1, 3, 4, 6, 8, 9, using similar logic

So, it follows $|H| = 3! \cdot 6!$, as both cycles disjoint