Tufts University
Department of Mathematics
Spring 2022

MA 166: Statistics

Solutions to Homework 5 (v1.1) [1]

Assigned Monday 21 February 2022
Due Monday 28 February 2022 at 11:59 pm EDT.

1. **Larsen & Marx, Section 6.2, Problem 6.2.2, page 352: An herbalist is experimenting with juices extracted from berries and roots that may have the ability to affect the Stanford-Binet IQ scores of students afflicted with mild cases of attention deficit disorder (ADD). A random sample of twenty-two children diagnosed with the condition have been drinking Berry Smart daily for two months. Past experience suggests that children with ADD score an average of 95 on the IQ test with a standard deviation of 15. If the data are to be analyzed using the $\alpha = 0.06$ level of significance, what values of $\overline{y}$ would cause H_0 to be rejected? Assume that H_1 is two-sided.**

   We suppose that the null hypothesis H_0 is that Berry Smart has no effect and will result in an average IQ of $\mu = 95$ with $\sigma = 15$. Since we are told to assume that H_1 is two-sided, we also suppose that H_0 will be rejected if $\overline{y}$ is more than $z_{\alpha/2}$ n-sample standard deviations away from μ. Then the probability of making a Type I error is

   $$\alpha = P\left(H_0 \text{ is rejected} \mid H_0 \text{ is true}\right) = P\left(\left|\frac{\overline{y}-\mu}{\sigma/\sqrt{n}}\right| > z_{\alpha/2} \mid \mu = 95\right).$$

   It follows that H_0 will be rejected if

   $$\left|\frac{\overline{y}-95}{15/\sqrt{22}}\right| > z_{0.03} = 1.88.$$

   or

   $$|\overline{y}-95| > 6.012.$$

   Hence, H_0 will be rejected if either $\boxed{\overline{y} < 88.99}$ or if $\boxed{\overline{y} > 101.01}$.

2. **Larsen & Marx, Section 6.2, Problem 6.2.4, page 353: Company records show that drivers get an average of 32,500 miles on a set of Road Hugger All-Weather radial tires. Hoping to improve that figure, the company has added a new polymer to the rubber that should help protect the tires from deterioration caused by extreme temperatures. Fifteen drivers who tested the new tires have reported getting an average of 33,800 miles. Can the company claim that the polymer has produced a statistically significant increase in tire mileage? Test $H_0:\ \mu = 32,500$ against a one-sided alternative at the $\alpha = 0.05$ level. Assume that the standard deviation (σ) of the**

tire mileages has not been affected by the addition of the polymer and is still four thousand miles.

We suppose that the null hypothesis H_0 is that the new polymer has no effect and will result in an average distance traveled of $\mu = 32,500$ miles before the tires wear out, with $\sigma = 4000$ miles. Since we are told to assume that H_1 is one-sided, we also suppose that H_0 will be rejected if $\overline{y}$ is more than z_α n-sample standard deviations greater then μ. Then the probability of making a Type I error is

$$\alpha = P\left(H_0 \text{ is rejected} \mid H_0 \text{ is true}\right) = P\left(\frac{\overline{y} - \mu}{\sigma/\sqrt{n}} > z_\alpha \mid \mu = 32,500\right).$$

It follows that H_0 will be rejected if

$$\left|\frac{\overline{y} - 32,500}{4000/\sqrt{15}}\right| > z_{0.05} = 1.645.$$

or

$$\overline{y} - 32,500 > 1698.95.$$

Hence, H_0 will be rejected if $\boxed{\overline{y} > 34,199}$. Since the measured value of $\overline{y}$ was $33,800 < 34,199$, we are unable to reject the null hypothesis.

3. **Larsen & Marx, Section 6.2, Problem 6.2.10, page 353: As a class research project, Rosaura wants to see whether the stress of final exams elevates the blood pressures of freshmen women. When they are not under any untoward duress, healthy eighteen-year-old women have systolic blood pressures that average 120 mm Hg with a standard deviation of 12 mm Hg. If Rosaura finds that the average blood pressure for the fifty women in Statistics 101 on the day of the final exam is 125.2, what should she conclude? Set up and test an appropriate hypothesis.**

Since Rosaura wants to know if the stress of the final exams *elevates* blood pressure, and presumably does not care if it decreases blood pressure, a one-sided test is called for. The null hypothesis H_0 will be that the exam has no effect on blood pressure.

The probability of making a Type I error is then

$$\alpha = P\left(H_0 \text{ is rejected} \mid H_0 \text{ is true}\right) = P\left(\frac{\overline{y} - \mu}{\sigma/\sqrt{n}} > z_\alpha \mid \mu = 120\right).$$

Given that Rosaura has measured $\overline{y} = 125.2$, it follows that H_0 will be rejected if

$$\left|\frac{125.2 - 120}{12/\sqrt{50}}\right| > z_\alpha.$$

or

$$z_\alpha < 3.064,$$

which will be true for any α greater than 0.0011. We may conclude that, unless she is insisting on an extremely high confidence level (i.e., better than 99.89%), Rosaura should reject H_0. In other words, the exam really does seem to have an effect on stress levels.

4. **Larsen & Marx, Section 6.3, Problem 6.3.2, page 358: Efforts to find a genetic explanation for why certain people are right-handed and others left-handed have been largely unsuccessful. Reliable data are difficult to find because of environmental factors that also influence a child's "handedness." To avoid that complication, researchers often study the analogous problem of "pawedness" in animals, where both genotypes and the environment can be partially controlled. In one such experiment (30), mice were put into a cage having a feeding tube that was equally accessible from the right or the left. Each mouse was then carefully watched over a number of feedings. If it used its right paw more than half the time to activate the tube, it was defined to be "right-pawed." Observations of this sort showed that 67% of mice belonging to strain A/J are right-pawed. A similar protocol was followed on a sample of thirty-five mice belonging to strain A/HeJ. Of those thirty-five, a total of eighteen were eventually classified as right-pawed. Test whether the proportion of right-pawed mice found in the A/HeJ sample was significantly different from what was known about the A/J strain. Use a two-sided alternative and let 0.05 be the probability associated with the critical region.**

   The null hypothesis will be that $p_0 = 0.67$. We may then note

$$\begin{aligned} np_0 + 3\sqrt{np_0(1-p_0)} &= 31.7954 < 35 = n \\ np_0 - 3\sqrt{np_0(1-p_0)} &= 15.1046 > 0, \end{aligned}$$

   so that we may use large-sample statistics to analyze this problem.

   For the rest of this problem, we are going to use more than the usual precision, because, as will be seen, the final conclusion will be a very closely run affair. In particular, we will take $z_{0.05/2} = 1.95996398$, instead of the usual 1.96.

   For a double-sided test, we can then calculate

$$\alpha = P\left(H_0 \text{ is rejected} \mid H_0 \text{ is true}\right) = P\left(\left|\frac{\overline{k} - p_0}{\sqrt{p_0(1-p_0)/n}}\right| > z_{\alpha/2} \mid p_0 = 0.67\right).$$

   or

$$\left|\overline{k} - p_0\right| > z_{\alpha/2}\sqrt{\frac{p_0(1-p_0)}{n}} = 0.155778830879$$

   Hence, we should reject H_0 if either $\overline{k} < 0.67 - 0.1557788 = 0.5142211$ or if $\overline{k} > 0.67 + 0.1557788 = 0.8257788$. In our case $\overline{k} = \frac{18}{35} = 0.5142857$, so, while this is really a very close call, we fail to reject the null hypothesis.

5. **Larsen & Marx, Section 6.3, Problem 6.3.4, page 359: Suppose $H_0 : p = 0.45$ is to be tested against $H_1 : p > 0.45$ at the $\alpha = 0.14$ level of significance, where $p = P(i\text{th trial ends in success})$. If the sample size is two hundred, what is the smallest number of successes that will cause H_0 to be rejected?**

We have $n = 200$ and $p_0 = 0.45$, so we may note

$$\begin{aligned} np_0 + 3\sqrt{np_0(1-p_0)} =& 111.107 < 200 = n \\ np_0 - 3\sqrt{np_0(1-p_0)} =& 68.8931 > 0, \end{aligned}$$

so that we may use large-sample statistics to analyze this problem.

We have a one-sided test, and so we reject H_0 if

$$\frac{k - np_0}{\sqrt{np_0(1-p_0)}} > z_\alpha,$$

or

$$k > np_0 + z_\alpha\sqrt{np_0(1-p_0)}.$$

For the numbers given, this is $k > 97.60$, so we reject H_0 if $k > 98$.