Worksheet 1 Math 70

Question 1) If $Ax=b$ has no solutions, then the augmented matrix has a row of 0's with a non zero pivot as the last value. The last value that's a part of x is b, meaning A has rows with all zeroes, meaning not every row has a pivot.

Question 2a) $2(\vec{x}+\vec{y}) = 2\vec{y}+2\vec{x}$

$$2\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}\right) = 2\begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \\ x_4+y_4 \end{bmatrix} = \begin{bmatrix} 2x_1+2y_1 \\ 2x_2+2y_2 \\ 2x_3+2y_3 \\ 2x_4+2y_4 \end{bmatrix}$$

In this step, I added $\vec{x}$ and $\vec{y}$ together and multiplied them by the scalar 2.

$$2\vec{y}+2\vec{x} = 2\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + 2\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2y_1+2x_1 \\ 2y_2+2x_2 \\ 2y_3+2x_3 \\ 2y_4+2x_4 \end{bmatrix}$$

. Here, I added $2\vec{y}+2\vec{x}$ by definitions of scalar multiplication and vector addition

$$\begin{bmatrix} 2x_1+2y_1 \\ 2x_2+2y_2 \\ 2x_3+2y_3 \\ 2x_4+2y_4 \end{bmatrix} = \begin{bmatrix} 2y_1+2x_1 \\ 2y_2+2x_2 \\ 2y_3+2x_3 \\ 2y_4+2x_4 \end{bmatrix}$$

by the Commutativity property of vectors

Question 2b) $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ $-\vec{x} = \begin{bmatrix} -x_1 \\ -x_2 \\ -x_3 \\ -x_4 \end{bmatrix}$ as $\vec{x}$ is multiplied by a scalar of -1

$$\vec{x} + -\vec{x} = \begin{bmatrix} x_1-x_1 \\ x_2-x_2 \\ x_3-x_3 \\ x_4-x_4 \end{bmatrix} = \vec{0}$$

, by vector addition and the additive inverse property of vectors

Question 3.a.1) $-x_1 + x_2 = 12$
$2x_1 + x_2 = 18$
$3x_1 + x_2 = 0$

Question 3.a.2)
$$\left[\begin{array}{cc|c} -1 & 1 & 12 \\ 2 & 1 & 18 \\ 3 & 1 & 0 \end{array}\right]$$

$r_i = r_i \times -1$
$$\left[\begin{array}{cc|c} 1 & -1 & -12 \\ 2 & 1 & 18 \\ 3 & 1 & 0 \end{array}\right]$$

$r_{ii} = r_{ii} - 2r_i$
$$\left[\begin{array}{cc|c} 1 & -1 & -12 \\ 0 & 3 & 42 \\ 3 & 1 & 0 \end{array}\right]$$

$r_{III} = r_{III} - 3r_I$
$$\left[\begin{array}{cc|c} 1 & -1 & 12 \\ 0 & 3 & 42 \\ 0 & 4 & -36 \end{array}\right]$$

At this step, you can conclude the matrix has no solution as the system at this point is $x_1 - x_2 = 12$, $3x_2 = 42$, $4x_2 = -36$, meaning $x_2 = 14$ and -9, which is impossible so the matrix has no solution.

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Question 3.b.1)
$x_1 - x_2 + x_3 = 12$
$4x_1 + 2x_2 + x_3 = 18$
$9x_1 + 3x_2 + x_3 = 0$

Question 3.b.2)
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 12 \\ 4 & 2 & 1 & 18 \\ 9 & 3 & 1 & 0 \end{array}\right]$$

$r_{ii} = r_{ii} - 4r_i$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 12 \\ 0 & 6 & -3 & -30 \\ 9 & 3 & 1 & 0 \end{array}\right]$$

$r_{iii} = r_{III} - 9r_I$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 12 \\ 0 & 6 & -3 & -30 \\ 0 & 12 & -8 & -108 \end{array}\right]$$

$r_{II} = \frac{1}{6} r_{II}$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 12 \\ 0 & 1 & -\frac{1}{2} & -5 \\ 0 & 12 & -8 & -108 \end{array}\right]$$

$r_{III} = r_{III} - 12r_{II}$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 12 \\ 0 & 1 & -\frac{1}{2} & -5 \\ 0 & 0 & -2 & -48 \end{array}\right]$$

which gives the linear system:
$$\begin{cases} x_1 - x_2 + x_3 = 12 \\ x_2 - \frac{x_3}{2} = -5 \\ -2x_3 = -48 \end{cases}$$

This system can be solved ~~with~~ with $x_3 = 24$, $x_2 = 7$, and $x_1 = -5$, meaning the equation of the quadratic is: $-5x^2 + 7x + 24 = b$

Question 3.c.1) $-x_1 + x_2 - x_3 + x_4 = 12$
$8x_1 + 4x_2 + 2x_3 + x_4 = 18$
$27x_1 + 9x_2 + 3x_3 + x_4 = 0$

Question 3.c.2)
$$\left[\begin{array}{cccc|c} -1 & 1 & -1 & 1 & 12 \\ 8 & 4 & 2 & 1 & 18 \\ 27 & 9 & 3 & 1 & 0 \end{array}\right]$$

$r_{II} = r_{II} + 8r_I$
$$\left[\begin{array}{cccc|c} -1 & 1 & -1 & 1 & 12 \\ 0 & 12 & -6 & 9 & 114 \\ 27 & 9 & 3 & 1 & 0 \end{array}\right]$$

$r_{III} = r_{III} + 27r_I$
$$\left[\begin{array}{cccc|c} -1 & 1 & -1 & 1 & 12 \\ 0 & 12 & -6 & 9 & 18 \\ 0 & 36 & -24 & 28 & 324 \end{array}\right]$$

$r_{II} = \frac{1}{12} r_{II}$
$$\left[\begin{array}{cccc|c} -1 & 1 & -1 & 1 & 12 \\ 0 & 1 & -\frac{1}{2} & \frac{3}{4} & \frac{3}{2} \\ 0 & 36 & -24 & 28 & 324 \end{array}\right]$$

$r_{III} = r_{III} - 36r_{II}$
$$\left[\begin{array}{cccc|c} -1 & 1 & -1 & 1 & 12 \\ 0 & 1 & -\frac{1}{2} & \frac{3}{4} & \frac{3}{2} \\ 0 & 0 & -6 & 1 & -18 \end{array}\right]$$ Gives the linear system.

$$\begin{cases} -x_1 + x_2 - x_3 + x_4 = 12 \\ x_2 - \frac{x_3}{2} + \frac{3}{4}x_4 = \frac{3}{2} \\ -6x_3 + x_4 = -18 \end{cases}$$

x_1, x_2, and x_3 are basic variables, but x_4 is a free variable, meaning that the matrix/system has infinitely many solutions

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