Part II, assignment 1

● Graded

Student

Scott A. Fullenbaum

Total Points

16 / 16 pts

Question 1

Subgroup of S_5 isomorphic to Z_6

4 / 4 pts

✔ **– 0 pts** Correct

– 1 pt Use Cayley's method - look at the group acting on itself by left multiplication

Question 2

8.3

4 / 4 pts

✔ **– 0 pts** Correct

– 2 pts Incorrect permutations

Question 3

8.6

4 / 4 pts

✔ **– 0 pts** Correct

– 1 pt Use Cayley's method - look at the group acting on itself by left multiplication

– 1 pt On the right track but missing a step

Question 4

8.11

4 / 4 pts

✔ **– 0 pts** Correct

– 1 pt Did not describe the symmetry q or given symmetry does not induce the permutation (12)

– 1 pt Cayley's theorem is not applicable in this case.

– 1 pt Showing a bijection between the symmetries and permutations does not quite show an isomorphism; you need some explanation of why it preserves the group structure

Questions assigned to the following page: 1 and 2

Q1) So $\mathbb{Z}_5$

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

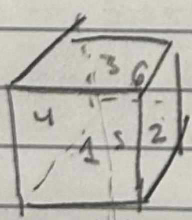
So as $S_5 = \{1, 2, 3, 4, 5\}$ we see that $\mathbb{Z}_5$ is just permuting it's row as a cycle.
So take $H = \langle (1, 2\ 3\ 4\ 5) \rangle$. and $H \cong \mathbb{Z}_5$ as have map $\phi : \mathbb{Z}_5 \to H$

$0 \longrightarrow (12345)$ H is clearly
$1 \longrightarrow (12345)$ subgroup of S_5
$2 \longrightarrow (12345)^2$
$3 \longrightarrow (12345)^3$
$4 \longrightarrow (12345)^4$

This is clearly bijective, and to show $\phi(a+b) = \phi(a)\phi(b)$

$$\phi(a+b) = (12345)^{a+b \bmod 5} = (12345)^a (12345)^b = \phi(a)\phi(b)$$

So have an isomorphism between subgroup of S_5 and $\mathbb{Z}_5$.

8.3) 

front: 1 right: 2
top: 3 left: 4
bottom: 5 back: 6

Done by rotating die

So r: | s: | t:

r	s	t
$1 \to 2$	$1 \to 5$	$1 \to 5$
$2 \to 6$	$2 \to 1$	$2 \to 4$
$3 \to 3$	$3 \to 4$	$3 \to 6$
$4 \to 1$	$4 \to 6$	$4 \to 2$
$5 \to 5$	$5 \to 2$	$5 \to 1$
$6 \to 4$	$6 \to 3$	$6 \to 3$

Questions assigned to the following page: 4 and 3

So have $r = (1264)$
$s = (152)(346)$
$t = (15)(24)(36)$

8.6) So Cayley table is this for D_3

	e	r	r^2	s	rs	r^2s
e	e	r	r^2	s	rs	r^2s
r	r	r^2	e	rs	r^2s	s
r^2	r^2	e	r	r^2s	s	rs
s	s	r^2s	rs	e	r^2	r
rs	rs	s	r^2s	r	e	r^2s
r^2s	r^2s	rs	s	r^2	r	e

Then we make the 'map'

$$\phi \begin{cases} e \to 1 \\ r \to 2 \\ r^2 \to 3 \\ s \to 4 \\ rs \to 5 \\ r^2s \to 6 \end{cases}$$

So applying Cayley's Theorem we get

$$\left.\begin{aligned} L_e &= e \\ L_r &= (231)(564) \\ L_{r^2} &= (132)(465) \\ L_s &= (14)(26)(35) \\ L_{rs} &= (15)(24)(36) \\ L_{r^2s} &= (16)(25)(34) \end{aligned}\right\} \text{These form a subgroup}$$

To see closed, like multiplying all of these by each other and get element in set.

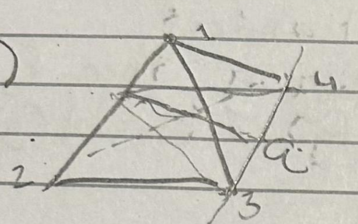
For inverse, $L_r^{-1} = L_r^2$ and then $L_s^{-1} = L_s$, $L_{rs}^{-1} = L_{rs}$ and $L_{r^2s}^{-1} = L_{r^2s}$, and $L_e^{-1} = L_e$.

Identity is $L_e = e$ clearly

Questions assigned to the following page: 4 and 3

Therefore we have subgroup H of S_6, w. and using Cayley's thm, have constructed an isomorphism so have shown $H \cong D_3$

8.11)



q is reflecting around the plane that has (34) in it and perpendicular bisector to edge (34) and goes through midpoint of edge (12)
- This will hold 3 and 4 in place but swap 1, 2

Now then $qr \Rightarrow$
$$\begin{array}{c} \\ r \\ q \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix} = (1234)$$

So $qr = (1234)$

This isn't a rotation as $r = (234)$ is even, but (1234) is odd. It's not a reflection as any reflection swaps 2 points holding the other 2 in place, and (1234) is swapping every point to a new location

However, $(1234) = (12)(13)(14)$ and these are reflections, so (1234) is the product of 3 reflections.

Now, I'm going to prove $G \cong S_4$

Question assigned to the following page: 4

Let $\phi: T \to S_4$
$f \to \sigma$

Now first show ϕ is a homomorphism
so for $f,g \in T$ $\phi(fg) = \phi(f)\phi(g)$

So ~~fg~~ fg is a composition of functions so we're doing g then f,
So it follows that $\phi(fg) = \phi(f)\phi(g)$.

Now, show ϕ is bijective:
Surjective:
Let $\sigma \in S_4$ w.t.s $\exists f \in T$ s.t $\phi(f) = \sigma$
$\sigma = \alpha_1 \alpha_2 \alpha_3 \alpha_4$ and as it's a permutation can be written as transpositions
now we know that we can write transpositions with different reflections
So $\exists q_1, q_2, q_3, q_4 \in T$ that do this
and if ~~WLOG~~ $\sigma = (\alpha_1 \alpha_2)(\alpha_1 \alpha_3)(\alpha_1 \alpha_4)$
then take:

Homomorphism

$$\phi(q_1 \cdot q_2 \cdot q_3) = \phi(q_1)\phi(q_2)\phi(q_3)$$
$$= (\alpha_1, \alpha_2)(\alpha_1 \alpha_3)(\alpha_1 \alpha_4)$$
$$= \sigma$$

So, have shown $\forall \sigma$, some combo of q's that $\phi(q_1 \dots q_n) = \sigma$ so onto.

Injective:
WTS $\phi(f) = \phi(g)$ then $f = g$
As f, g are isometrics, then are combinatorally the same so $f = g$

$\square$

So $G \cong S_4$ meaning # symmetries of $T \Rightarrow |G| = |S_4| = 24$ so 24 symmetries of T

$\square$