Math 65 HW2

1 a) Let n be an integer and $n+1$ the successive integer. The sum, $K_0 = 2n+1$, which is the definition of an odd number.
Starting from $K = 2n+1$, $n, K \in \mathbb{N}$, $K = n+n+1$, $K = n + (n+1)$, showing sum of 2 consecutive integers.

b) No, 4 is even, but can be written as $2+2=4$, $0+4=4$, which doesn't contain 2 consecutive even integers.

2 If $\frac{bc}{a} \notin \mathbb{Z}$ then $\frac{b}{a} \notin \mathbb{Z}$. Contrapositive statement would be $\frac{b}{a} \in \mathbb{Z}$, and $\frac{bc}{a} \in \mathbb{Z}$. $\frac{b}{a} \in \mathbb{Z}$, meaning $\frac{b}{a} \in \mathbb{Z}$ for any multiple of a. $c \in \mathbb{Z}$, so ca is multiple of a, so $\frac{bc}{a} \in \mathbb{Z}$. Since the contrapositive is true, the original statement is true.

3

P	Q	$P \wedge Q$	$\neg Q$	$P \wedge \neg Q$	$(P \wedge Q) \vee (P \wedge \neg Q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	F	F
F	F	F	T	F	F

Based off table P is logically equivalent to $(P \wedge Q) \vee (P \wedge \neg Q)$

4 a) $\forall K \in \mathbb{Z}$, K is ~~prime~~ 2 factors and is

b) $\exists K \in \mathbb{Z}$ that is neither prime nor composite

c) $\forall x \in \mathbb{Z}$, $\exists y \in \mathbb{Z}$, such that $xy = 1$.

d) $\exists x \in \mathbb{Z}$, such that $\forall y \in \mathbb{Z}$ $xy = 1$

e) $\forall x \in \mathbb{Z}$, $\forall y \in \mathbb{Z}$, $x+y = y+x$

f) $\exists\, x \in \mathbb{Z}$ and $\exists\, y \in \mathbb{Z}$ such that $\frac{x}{y} \in \mathbb{Z}$

5a) $\forall\, k \in \mathbb{Z}$, k is not prime

b) $\exists\, k \in \mathbb{Z}$, k is prime and composite.

c) $\forall x \in \mathbb{Z}$, $\exists\, y \in \mathbb{Z}$ such that $xy \neq 1$

d) $\exists\, x \in \mathbb{Z}$, ~~$\exists y \in \mathbb{Z}$~~, such that $xy \neq 1$

e) $\forall x \in \mathbb{Z}$ and $\forall y \in \mathbb{Z}$, $x+y \neq y+x$.

f) $\exists\, x \in \mathbb{Z}$ and $\exists\, y \in \mathbb{Z}$ such that $\frac{x}{y} \notin \mathbb{Z}$

6a) True, $x = \pm 2$, but $x = -2 \notin \mathbb{N}$ so ~~$x = 2 \in \mathbb{N}$~~ unique solution

b) False $x^2 = 3$, $x = \pm\sqrt{3}$, $\sqrt{3} \notin \mathbb{N}$ so

c) False, $x = \pm 2$, $-2 \in \mathbb{Z}$, $2 \in \mathbb{Z}$.

d) True, $x = 0$. ~~$xy = x$, $xy - x = 0$~~ $x(y-1) = 0$, ~~so the~~ $= 0$, $x = 0$ or $y = 1$. $x = 0$ is a unique x value, while $y = 1$ is not.

e) True, $x = 1$, $xy = y$ $xy - y = 0$, $y(x-1) = 0$, meaning $y = 0$ or $x = 1$. Asked for unique x, which is 1

7 If $ax^2 + bx + c = 0$ has a rational root, then both roots are rational, meaning we can factor $ax^2 + bx + c = 0$ into $(Ax + B)(Cx + D)$, $A, B, C, D \in \mathbb{Z}$. Distributing $AC = a$ and $c = BD$. Since a, c are odd, by rules of multiplication, A, B, C, D are odd. However, $b = BC + AD$, BC and AD are odd, meaning that b is even, showing a contradiction and there is no rational solution