Problem Session March 25th Worksheet

Math 70

March 25, 2021

(1) Consider the linear transformation T from $M_{2\times 2}$ to $\mathbb{P}_2$ defined by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = b + (c+d)t - (c+d)t^2.$$

Describe the kernel of T, and find a basis for the kernel. What is the dimension of the kernel?

(2) Short answer questions

(a) If $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$, and $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, find what $\vec{x}$ is (in standard coordinates).

(b) Is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in the null space of $\begin{bmatrix} 1 & -3 \\ -4 & 12 \end{bmatrix}$?

(c) Suppose the matrix product BC is defined and that BC is a 5×4 matrix. How many rows does B have?

(d) What is the determinant of the matrix $\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$?

(e) True/False: Every $n \times n$ invertible matrix is row equivalent to the identity matrix I_n.

(3) Recall that the union $H_1 \cup H_2$ of two subspaces H_1 and H_2 is the set of a vectors which are either in H_1 or H_2 (or both). Find two subspaces H_1 and H_2 in $\mathbb{R}^2$ whose union is *not a subspace* of $\mathbb{R}^2$. Prove that this union is not a subspace. (You should use specific numerical vectors and show that at least one of the properties in the definition of a subspace is not satisfied.)

(4) Consider the matrix

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -6 & 1 \\ 0 & 8 & 4 \end{bmatrix}.$$

(a) Find a basis $\mathcal{B}$ for Col A.

(b) Show that $\vec{x} = \begin{bmatrix} 5 \\ 12 \\ -8 \end{bmatrix}$ is in Col A.

(c) Find a basis for Nul A.