

QUIZ 8, OCTOBER 27

Question. (a) Consider the assignment $f_1 : \mathbb{Z} \rightarrow \mathbb{Z}_4$ given by $f_1(z) = [z]_4$. Is this a well defined function? Why or why not?

$$f_1(z) = [z]_4$$

each $z \in \mathbb{Z}$ can only be defined in one equivalence class, meaning each $f_1(z)$ is mapped to only one equivalence class, making it well defined.

(b) Consider the assignment $f_2 : \mathbb{Z}_4 \rightarrow \mathbb{Z}$ given by $f_2([z]_4) = z$. Is this a well defined function? Why or why not?

~~$f_2([z]_4)$~~ is not well defined, as $[z]_4$ can map the same z to multiple places as the equivalence class can repeat.

Using ex.) from class, $[2]_4$ from $a-b=4k$, would contain all integers with remainder two, which is infinitely

many

(c) Consider the assignment $f_3 : \mathbb{Z}_8 \rightarrow \mathbb{Z}_4$ given by $f_3([z]_8) = [z]_4$. Is this a well defined function? Why or why not?

$$f_3([z]_8) = [z]_4$$

No $[z]_8$ has $[z]_8$ and $[z]_8$, and $[z]_4$ only has

$$[z]_8 \rightarrow \mathbb{Z}_4$$

$$[z]_8 \neq [z]_8 \Rightarrow [z]_4 = [z]_4 \quad 2z \neq z$$

$$[z_1]_8 = [z_2]_8 \Rightarrow [z_1]_4 = [z_2]_4$$

$$\exists k \in \mathbb{Z} \quad z_1 - z_2 = 8k \rightarrow \exists k' \quad z_1 - z_2 = 4k'$$

$$z_1 - z_2 = 4(2k) \quad \text{so } 2k = k'$$

There fore from here, you get

$$[z_1]_8 = [z_2]_8 \Rightarrow [z_1]_4 = [z_2]_4$$

and f_3 is well defined functional
well equation when $k' = 2k$, meaning f_3 is well defined
well defined function.

(d) Consider the assignment $f_4 : \mathbb{Z}_4 \rightarrow \mathbb{Z}_8$ given by $f_4([z]_4) = [z]_8$. Is this a well defined function? Why or why not?

$$f_4([1]_4) = [1]_8 = [5]_8 = [9]_8 = [13]_8 = [17]_8 = [21]_8 = [25]_8 = [29]_8 = [33]_8 = [37]_8 = [41]_8 = [45]_8 = [49]_8 = [53]_8 = [57]_8 = [61]_8 = [65]_8 = [69]_8 = [73]_8 = [77]_8 = [81]_8 = [85]_8 = [89]_8 = [93]_8 = [97]_8 = [101]_8 = [105]_8 = [109]_8 = [113]_8 = [117]_8 = [121]_8 = [125]_8 = [129]_8 = [133]_8 = [137]_8 = [141]_8 = [145]_8 = [149]_8 = [153]_8 = [157]_8 = [161]_8 = [165]_8 = [169]_8 = [173]_8 = [177]_8 = [181]_8 = [185]_8 = [189]_8 = [193]_8 = [197]_8 = [201]_8 = [205]_8 = [209]_8 = [213]_8 = [217]_8 = [221]_8 = [225]_8 = [229]_8 = [233]_8 = [237]_8 = [241]_8 = [245]_8 = [249]_8 = [253]_8 = [257]_8 = [261]_8 = [265]_8 = [269]_8 = [273]_8 = [277]_8 = [281]_8 = [285]_8 = [289]_8 = [293]_8 = [297]_8 = [301]_8 = [305]_8 = [309]_8 = [313]_8 = [317]_8 = [321]_8 = [325]_8 = [329]_8 = [333]_8 = [337]_8 = [341]_8 = [345]_8 = [349]_8 = [353]_8 = [357]_8 = [361]_8 = [365]_8 = [369]_8 = [373]_8 = [377]_8 = [381]_8 = [385]_8 = [389]_8 = [393]_8 = [397]_8 = [401]_8 = [405]_8 = [409]_8 = [413]_8 = [417]_8 = [421]_8 = [425]_8 = [429]_8 = [433]_8 = [437]_8 = [441]_8 = [445]_8 = [449]_8 = [453]_8 = [457]_8 = [461]_8 = [465]_8 = [469]_8 = [473]_8 = [477]_8 = [481]_8 = [485]_8 = [489]_8 = [493]_8 = [497]_8 = [501]_8 = [505]_8 = [509]_8 = [513]_8 = [517]_8 = [521]_8 = [525]_8 = [529]_8 = [533]_8 = [537]_8 = [541]_8 = [545]_8 = [549]_8 = [553]_8 = [557]_8 = [561]_8 = [565]_8 = [569]_8 = [573]_8 = [577]_8 = [581]_8 = [585]_8 = [589]_8 = [593]_8 = [597]_8 = [601]_8 = [605]_8 = [609]_8 = [613]_8 = [617]_8 = [621]_8 = [625]_8 = [629]_8 = [633]_8 = [637]_8 = [641]_8 = [645]_8 = [649]_8 = [653]_8 = [657]_8 = [661]_8 = [665]_8 = [669]_8 = [673]_8 = [677]_8 = [681]_8 = [685]_8 = [689]_8 = [693]_8 = [697]_8 = [701]_8 = [705]_8 = [709]_8 = [713]_8 = [717]_8 = [721]_8 = [725]_8 = [729]_8 = [733]_8 = [737]_8 = [741]_8 = [745]_8 = [749]_8 = [753]_8 = [757]_8 = [761]_8 = [765]_8 = [769]_8 = [773]_8 = [777]_8 = [781]_8 = [785]_8 = [789]_8 = [793]_8 = [797]_8 = [801]_8 = [805]_8 = [809]_8 = [813]_8 = [817]_8 = [821]_8 = [825]_8 = [829]_8 = [833]_8 = [837]_8 = [841]_8 = [845]_8 = [849]_8 = [853]_8 = [857]_8 = [861]_8 = [865]_8 = [869]_8 = [873]_8 = [877]_8 = [881]_8 = [885]_8 = [889]_8 = [893]_8 = [897]_8 = [901]_8 = [905]_8 = [909]_8 = [913]_8 = [917]_8 = [921]_8 = [925]_8 = [929]_8 = [933]_8 = [937]_8 = [941]_8 = [945]_8 = [949]_8 = [953]_8 = [957]_8 = [961]_8 = [965]_8 = [969]_8 = [973]_8 = [977]_8 = [981]_8 = [985]_8 = [989]_8 = [993]_8 = [997]_8$$

Not meaning that f_4 is not well defined
as an input has multiple outputs.