MATH 70
Problem Session Worksheet 3
2/25/2021

1 Prove the contrapositive of the following: Let A be a 2×2 matrix. If $A\mathbf{x} = \mathbf{0}$ for all $\mathbf{x} \in \mathbb{R}^2$, then A is the zero matrix.

2 Using a proof by contradiction, prove that if $\mathbf{u}$ and $\mathbf{v}$ are not scalar multiples of each other, then the set $\{\mathbf{u}, \mathbf{v}\}$ is linearly independent.

3 Find a counterexample to disprove the following: if $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are nonzero vectors in $\mathbb{R}^3$, and none is a scalar multiple of the others, then $\text{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\} = \mathbb{R}^3$.

4 Determine if the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1/2 \\ 0 \\ x_1 + 4x_2 \end{bmatrix}$$

is a linear transformation. If yes, prove it. If not, use explicit numbers to prove it isn't.

5 Let $\{v_1, v_2, v_3\}$ be a linearly independent set in $\mathbb{R}^3$.

(a) Prove that the set $\{v_1, v_1 + v_2, v_2 + v_3\}$ is linearly independent.

(b) Prove that the set $\{v_1 - v_2, v_2 - v_3, v_3 - v_1\}$ is linearly dependent.