Problem Session April 1st Worksheet

Math 70

April 1, 2021

(1) True or False:

(a) If Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\} = \mathbb{R}^3$, then $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is a basis for $\mathbb{R}^3$.

(b) Any vector space is isomorphic to $\mathbb{R}^n$ for some n.

(c) If A is an invertible, $n \times n$ matrix, then so is A^2.

(d) If A is $m \times m$ and rank $A = m$, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.

(e) If A is $m \times n$, then Row A and Nul A are both subspaces of $\mathbb{R}^m$.

(2) Suppose that A is a 3×12 matrix. Find the following:

(a) The maximum possible value of rank A.

(b) The minimum possible value of rank A.

(c) The maximum possible value of $\dim(\text{Nul}A)$.

(d) The minimum possible value of $\dim(\text{Nul } A)$.

(3) Suppose H_1 and H_2 are both subspaces of a vector space V. The intersection of H_1 and H_2, denoted $H_1 \cap H_2$, is the set of all vectors which are contained in both H_1 and H_2. Prove that $H_1 \cap H_2$ is a subspace of V.