Tufts University
Department of Mathematics
Spring 2022

MA 166: Statistics

Homework 7 (v1.1) [1]

Assigned Monday 7 March 2022
Due Monday 14 March 2022 at 11:59 pm EDT.

1. **Larsen & Marx, Section 7.4, Problem 7.4.7, page 393: Cell phones emit radio frequency energy that is absorbed by the body when the phone is next to the ear and may be harmful. The table in the next column gives the absorption rate for a sample of twenty high-radiation cell phones. (The Federal Communication Commission sets a maximum of 1.6 watts per kilogram for the absorption rate of such energy.) Construct a 90% confidence interval for the true average cell phone absorption rate.**

1.54	**1.41**
1.54	**1.40**
1.49	**1.40**
1.49	**1.39**
1.48	**1.39**
1.45	**1.39**
1.44	**1.38**
1.42	**1.38**
1.41	**1.37**
1.41	**1.33**

We have $n = 20$, and we can compute $\overline{y} = 1.4255$, $s = 0.05643$ and $s^2 = 0.003184$. If our desired level of confidence is $\alpha = 0.1$, we need to know

$$t_{\alpha/2,n-1} = t_{0.05,19} = 1.7291.$$

The 90% confidence interval is then

$$\boxed{\left(\overline{y} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \overline{y} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right) = (1.4037, 1.4473)}$$

2. **Larsen & Marx, Section 7.4, Problem 7.4.11, page 394: In a nongeriatric population, platelet counts ranging from 140 to 440 (thousands per mm3 of blood) are considered "normal." The following are the platelet counts recorded for twenty-four female nursing home residents (180).**

Subject	Count	Subject	Count
1	125	13	180
2	170	14	180
3	250	15	280
4	270	16	240
5	144	17	270
6	184	18	220
7	176	19	110
8	100	20	176
9	220	21	280
10	200	22	176
11	170	23	188
12	160	24	176

Use the following sums:

$$\sum_{i=1}^{24} \boldsymbol{y_i} = \mathbf{4645} \qquad \textbf{and} \qquad \sum_{i=1}^{24} \boldsymbol{y_i^2} = \mathbf{959,265.}$$

How does the definition of "normal" above compare with the 90% confidence interval?

We have $n = 24$, and we can compute $\overline{y} = 1.4255$. Since the definition of "normal" given is two sided, we choose the null hypothesis $H_0 : \overline{y} = 1.4255$ versus the alternative hypothesis $H_0 : \overline{y} \neq 1.4255$. Since we are looking for confidence $\alpha = 0.1$, we look up the value

$$t_{\alpha/2,n-1} = t_{0.05,23} = 1.71387.$$

To calculate s with the information given, note that

$$\begin{aligned}
s^2 &= \left(\frac{n}{n-1}\right)\sigma^2 \\
&= \left(\frac{n}{n-1}\right)\left[\frac{1}{n}\sum_{i=1}^{n} y_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)^2\right] \\
&= \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2\right) - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n} y_i\right)^2 \\
&= \frac{1}{23}(959,265) - \frac{1}{24 \times 23}(4645)^2 = 2620.17,
\end{aligned}$$

so that $s = \sqrt{2620.17} = 51.1876$.

The 90% confidence interval is then

$$\boxed{\left(\overline{y} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \overline{y} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right) = (175.6, 211.4)}$$

This is not even close to the given definition of "normal," as is commonly used for nursing home residents.

3. **Larsen & Marx, Section 7.5, Problem 7.5.17, page 412: A stock analyst claims to have devised a mathematical technique for selecting high-quality mutual funds and promises that a client's portfolio will have higher average ten-year annualized returns and lower volatility; that is, a smaller standard deviation. After ten years, one of the analyst's twenty-four-stock portfolios showed an average ten-year annualized return of 11.50% and a standard deviation of 10.17%. The benchmarks for the type of funds considered are a mean of 10.10% and a standard deviation of 15.67%.**

   (a) **Let $\boldsymbol{\mu}$ be the mean for a twenty-four-stock portfolio selected by the analyst's method. Test at the 0.05 level that the portfolio beat the benchmark; that is, test $\boldsymbol{H_0 : \mu = 10.1}$ versus $\boldsymbol{H_1 : \mu > 10.1}$.**

   We have $n = 24$, $\mu_0 = 10.1$, $s = 10.17$, and $\overline{y} = 11.5$. It follows that

   $$\frac{\overline{y} - \mu_0}{s/\sqrt{n}} = 0.6744.$$

   Since this is a one-sided test (we want to see if the analyst's method will *beat* the benchmark), and since the confidence level is $\alpha = 0.05$, we look up the quantity

   $$t_{\alpha,n-1} = t_{0.05,23} = 1.71387.$$

   Since $\frac{\overline{y}-\mu_0}{s/\sqrt{n}} < t_{\alpha,n-1}$, we conclude that we should *not* reject the null hypothesis in this situation. We can not conclude that the analyst's methodology is better than the baseline.

   (b) **Let $\boldsymbol{\sigma}$ be the standard deviation for a twenty-four-stock portfolio selected by the analyst's method. Test at the 0.05 level that the portfolio beat the benchmark; that is, test $\boldsymbol{H_0 : \sigma = 15.67}$ versus $\boldsymbol{H_1 : \sigma < 15.67}$.**

   Again, we have $n = 24$. This time we are testing $H_0 : \sigma^2 = \sigma_0^2$ versus $H_1 : \sigma^2 < \sigma_0^2$ at the $\alpha = 0.05$ level of confidence.

   We reject H_0 if the test statistic,

   $$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(23)(10.17)^2}{(15.67)^2} = 9.688$$

   is less than or equal to

   $$\chi^2_{\alpha,n-1} = \chi^2_{0.05,23} = 13.091.$$

   It is, so, in this case, we reject the null hypothesis. The analyst's method does seem to result in less volatility.