Thursday, Octoberr 21

1. **Motion on a circle.** A point $(x,y) \in \mathbb{R}^2$ lies on the unit circle (the circle centered at $(0,0)$ with radius 1) if and only if (x,y) can be written in the form $(\cos\theta, \sin\theta)$ for some $\theta \in \mathbb{R}$. Of course, if you add an integer multiple of 2π to θ, then the point (x,y) doesn't change.

   Motion on the unit circle can be described by an equation of the forrm

   $$\frac{d\theta}{dt} = f(\theta). \tag{1}$$

   However, the function f must be 2π-periodic for (1) to describe the motion of a point on the unit circle. Explain why.

   So
   $$\frac{d\theta}{dt} = 3 + \cos\theta$$
   describes motion on the unit circle, butt
   $$\frac{d\theta}{dt} = 3 + e^{\theta}$$
   does not.

2. **Oscillators described by a single differential equation.** We saw earlier that x cannot oscillate if
   $$\frac{dx}{dt} = f(x).$$
   However, suppose we have a differential equation like (1) describing motion on the unit circle. Then
   $$x(t) = \cos\theta(t) \quad \text{and} \quad y(t) = \sin\theta(t)$$
   may be functions that perpetually oscillate. What do you have to assume about the function f in (1) to ensure that $x(t)$ and $y(t)$) perpetually oscillate?

3. **An example.** Suppose
   $$\frac{d\theta}{dt} = 2\pi.$$
   What will the function $x(t) = \cos\theta(t)$ look like?

4. **Another example.** Suppose
   $$\frac{d\theta}{dt} = 1.01 + \cos\theta.$$
   What do you think the function $x(t) = \cos\theta(t)$ will look like? You can figure this out qualitatively by thinking about it, but you can also solve the differential equation using the midpoint method, then plot $\cos\theta(t)$ as a function of t.