Math 65 HW 1

1.1a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

b) $A \cap B = \{6\}$

c) $A - B = \{1, 2, 3, 4, 5\}$

d) $B - A = \{7\}$

e) $A \times B = \{(1,6), (2,6)\ (3,6)\ (4,6)\ (5,6), (6,6), (1,7), (2,7), (3,7), (4,7), (5,7), (6,7)\}$

f) $P(B) = \{\{\phi\}, \{6\}, \{7\}, \{6, 7\}\}$

1.2 For $S = T$, $S \subseteq T$ and $T \subseteq S$. Looking at S, $\{x \in R \mid x^2 < x\}$, if we perform: $\sqrt{x^2 < x} \rightarrow |x| < 1$. If $x \leq 0$, then $x^2 < x$ doesn't work as x^2 is greater than 0 for $x \in \mathbb{R}$. This means $0 < x < 1$, which is T. For $T = \{x \in \mathbb{R} \mid 0 < x < 1\}$ multiply the inequality by x, giving us $0 < x^2 < x$. As we showed earlier, x must be greater than 0, so we can rewrite it as $x^2 < x$, and that $T \subseteq S$, and $S \subseteq T$. So $S = T$.

1.3 a) $\lim_{n \to \infty} \frac{1}{n} = 0$, meaning that for $x \in \mathbb{R}$, it is contained within smaller and smaller sets for $n \in N$ except 0. Then, for $\cup_{n \in N - \{0\}} A_N$, we can use $n = 1$ as a solution, meaning that $\cup_{n \in N - \{0\}} A_n = \{-1, 1\}$. For $\cap_n A_n$, as n increases, it goes to 0. This means the only overlap is at 0 and $\cap_n A_n = \{0\}$

b) $\cup_n B_n$ where $n \in N$ for $n \geq 1$ starting at $\frac{1}{2} < x < \frac{3}{2}$, since n increases by 1, and $(n + \frac{1}{2}) - (n - \frac{1}{2}) = 1$, then $\cup_n B_n$ will contain all $x \in \mathbb{R}$, for $x > \frac{1}{2}$, with the exception of [illegible] $n \in N$. For $\cap_n B_n$ there is no overlap

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1.4) To attempt to prove, let us assume $A=B$. Thus $A\cup B = A\cap B$, WLOG can be written as $A\cup A = A\cap A$. This is true since if $A=\{a_1 \dots a_n\}$, then $\{a_1 \dots a_n\}\cup\{a_1 \dots a_n\} = \{a_1 \dots a_n\}$ where a_n is an element. $\{a_1 \dots a_n\}\cap\{a_1 \dots a_n\} = \{a_1 \dots a_n\}$ which are identical. Now, let us assume A and B are different and $B=\{b_1 \dots b_k\}$, meaning at least 1 element in B is not in A. $\{a_1 \dots a_n\}\cup\{b_1 \dots b_n\}$ contains all unique elements in A and B. However, for $A\cap B$, the set contains all elements in A and B. Since B has at least 1 unique element from A, then $A\cap B$ is missing at least 1 element in $A\cup B$, thus $A\cup B \neq A\cap B$ if $A\neq B$

1.5 1) $A=\{x\in\mathbb{R} \mid x(x-2)(x+2)\geq 0\}$

sign chart – + – + (−2, 0, 2) meaning $\boxed{A=[-2,0]\cup[2,\infty)}$

2) $B=\{x\in\mathbb{R} \mid x^3-x<0\} \rightarrow \{x\in\mathbb{R} \mid x(x-1)(x+1)<0\}$

signchart – + – + (−1, 0, 1) meaning that B

$\boxed{B=(-\infty,-1)\cup(0,1)}$

3) $\boxed{A\cap B=[-2,-1)}$ $\cup$ $[2,\infty)$ s

4) $\boxed{A\cup B=(-\infty,1)\cup[-2,\infty)}$ $[2,\infty)$

6a) $A=\{a_1 \dots a_n\}$ $B=\{b_1 \dots b_k\}$ $n,k\in N$

$P(A\cup B)$ contains subsets with elements from A and B. However, $P(A)\cup P(B)$ each subset is restricted to terms of A or B, and meaning unless $A=B$ we cannot form a subset w/ elements of A and B.

ex. $A=\{1\}$, $B=\{2\}$ $P(A\cup B)=\{\{\emptyset\},\{1\},\{2\},\{1,2\}\}$

$\{\emptyset,1\}$ $P(A)\cup P(B)=\{\{\emptyset\},\{1\},\{2\}\}$

1.6b $A=\{1,2\}$ $B=\{1,2\}$
$P(A\cup B)=P(\{1,2,3\})=\{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

$P(A)=\{\{\emptyset\}, \{1\}, \{2\}, \{1,2\}\}=P(B)$
so $P(A)\cup P(B)=P(A)$ since $A=B$, and $P(A\cup B)=P(A)\cup P(B)$

1.6c) The condition on which $P(A\cup B)=P(A)\cup P(B)$ is when $A=B$. Let $A=\{a_1 \dots a_n\}=B$.
$A\cup B=\{a_1 \dots a_n\}\cup\{a_1 \dots a_n\}=A$, thus $P(A\cup B)=P(A)$
The equation is $P(A)=P(A)\cup P(A)$, $P(A)\cup P(A)=P(A)$ as the sets are identical.

To show $A=B$ is the only condition let us assume $A\neq B$, $A=\{a_1 \dots a_n\}$, $B=\{b_1 \dots b_k\}$, $n,k\in\mathbb{N}$
This means B has at least 1 element not in A or vice versa or both.
$P(A\cup B))=P(\{a_1 \dots a_n, b_1 \dots b_k\})$ → Assume all $b_1 \dots b_n$ are unique of $a_1 \dots a_n$, but this doesn't change result, as irrelevant to proof

$P(A\cup B)$ will have 2^{n+k} elements, and will consist of subsets containing elements from A and B due to definition of a power set.
$P(A)$ will, by def of power set contain only elements in A, and the same could be said for $P(B)$
$P(A)\cup P(B)$ will then not have a single subset consisting of elements from A and B, $P(A\cup B)\not\subseteq P(A)\cup P(B)$ and $P(A\cup B)\neq P(A)\cup P(B)$. While assumed B and A had no common elements, as long as B has at least 1 unique element it works, as $P(A\cup B)$ will have a subset of $\{a_1 \dots a_n, b_k\}$, which cannot exist in $P(A)\cup P(B)$, as $P(A)$ at most will have a set $\{a_1 \dots a_n\}$, w/ no way of adding an additional unique element from b to the set