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Math 87 HW 8
Due 3/29/2022

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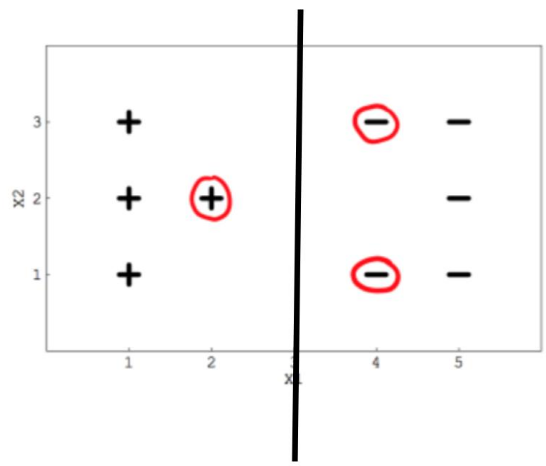
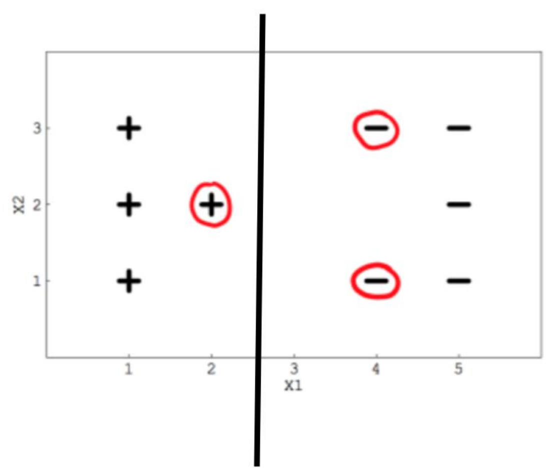
1. Comprehending SVM. Suppose we have data drawn from two different populations, shown in the figure below as (+) and () (From pdf). Support vector machines are used to linearly separate such classes of data.

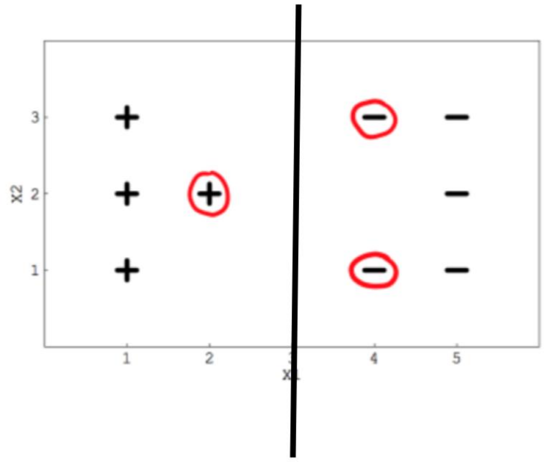
   (a) Draw the separating line SVM would generate to separate these two classes of data.
   The black line is the separator. I think this makes sense because the data is equally distributed across both sides.

   (b) Suppose that the (+) in the red circle was deleted from the data set. Would the supporting line change? If so, draw it.
   The black line is the separator , since the plus sign is removed, the classification line should move to the left, as the (+) is concentrated further to the left.

   (c) Suppose that all red circled data points were deleted. Would the supporting line change? If so, draw it.
   With all the red points deleted, since the data is symmetrically distributed around $x_1 = 3$, it makes sense that this would be the separating line.



2. Computing kernels

   (a) Let $x = (x_1, x_2), y = (y_1, y_2) \epsilon R^2$. Suppose we use the kernel K(x, y) = (x·y+ c)^2, where x · y is the dot product between x and y. Compute the higher dimensional embedding (i.e. the feature map) of **x** corresponding to this kernel.

   We can start by distributing out K(x, y) and getting that:
   $K(x, y) = (x_1 * y_1 + x_2 * y_2 + c)^2$
   $= (x_1 * y_1)^2 + 2x_1y_1 * x_2y_2 + 2cx_1y_1 + 2cx_2y_2 + (x_2y_2)^2 + c^2$
   $= (x_1^2, x_2^2, \sqrt{2}x_1x_2, \sqrt{2c} * x_1, \sqrt{2c} * x_2, c) \cdot (y_1^2, y_2^2, \sqrt{2}y_1y_2, \sqrt{2c} * y_1, \sqrt{2c} * y_2, c)$
   The following 6 dimensional vectors are the feature mapping:
   $v(x) = (x_1^2, x_2^2, \sqrt{2}x_1x_2, \sqrt{2c} * x_1, \sqrt{2c} * x_2, c)$
   $v(y) = (y_1^2, y_2^2, \sqrt{2}y_1y_2, \sqrt{2c} * y_1, \sqrt{2c} * y_2, c)$

   To prove that this is correct, we can do $v(x) \cdot v(y)$, which goes in reverse of the steps above, and ends with K(x, y).