Wednesday, January 25

Monday, January 23, 2023 12:24

TA Help session 10:30 Fridays, Math library, JCC 574
Student hours with Todd 1:30-3:00 my office JCC 575 (end of hall)

GROUP WORK Today!

MATHEMATICAL CONTEST IN MODELING: February 16-20, 2023. TEAMS OF THREE UNDERGRADS

https://www.contest.comap.com/undergraduate/contests/

Information session: Today! Wednesday, January 25th, 6pm or 7:30pm, JCC 574

Please RSVP to either Arkadz Kirshtein (Arkadz.Kirshtein@tufts.edu) or James Adler (james.adler@tufts.edu)

DIRECTED READING PROGRAM: The Directed Reading Program will pair an undergraduate with a graduate student to work on a joint reading project during the term. The program is modeled on something started at the University of Chicago over 10 years ago. At other universities, and at Tufts in the past, the program has been a huge success at helping students interested in mathematics get exposure to beautiful topics beyond the classroom, as well as connect to peers and mentors.

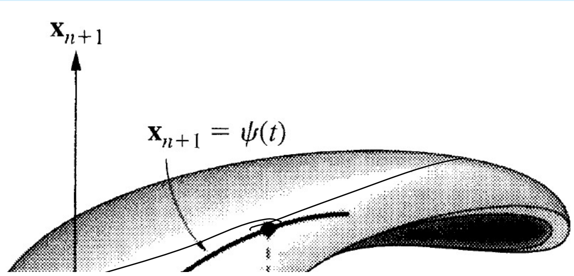
A list of projects and descriptions can be found here: https://drive.google.com/file/d/1ffyVId43yPtFP-9GiODrtHf3ZlJ2Nc2S/view?usp=sharing

f you are interested in participating, please complete the application: https://forms.gle/P46BCsEKvdnzftLo9

Save the date! AWM Panel & Lunch with Malena Espanol
Friday February 3rd at 1pm in JCC 501

Malena Espanol is an assistant professor in the school of Mathematical and Statistical Sciences at Arizona State University. She earned a Ph.D. in math from Tufts in 2009. The Tufts AWM chapter is excited to host Dr. Espanol for a Q&A over lunch! Everyone in the Tufts community is welcome to join.

Please RSVP at https://tufts.qualtrics.com/jfe/form/SV_0cR5K8g15jJQ7eC



$X_{n+1} = f(X)$

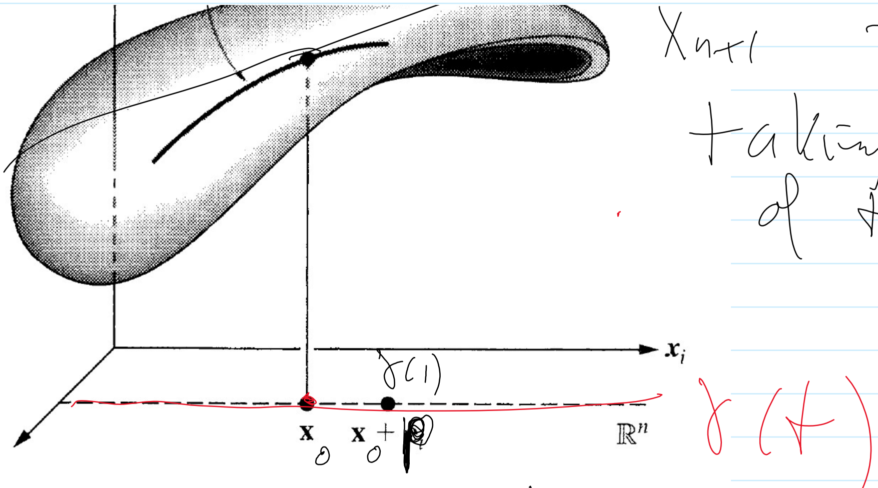


FIGURE 13.2 $\psi(t) = f(\mathbf{x}_0 + t p)$.

$X_{n+1} = f(X)$

taking a deriv of the fn $f \circ \gamma(t)$ @ $t=0$

$\|p\| = 1$

Definition Let A be a subset of $\mathbb{R}^n$ and let $\mathbf{x}_*$ be a limit point of A. Given a function $f : A \to \mathbb{R}$ and a real number ℓ, we write

$$\lim_{\mathbf{x} \to \mathbf{x}_*} f(x) = \ell \tag{13.2}$$

provided that whenever $\{\mathbf{x}_k\}$ is a sequence in $A \backslash \{\mathbf{x}_*\}$ that converges to $\mathbf{x}_*$, the image sequence $\{f(\mathbf{x}_k)\}$ converges to ℓ.

eg Find $\lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2}-1}{x^2+y^2}$ or show it DNE

Let $\{(x_k, y_k)\}$ be seq in $\mathbb{R}^2 \setminus \{(0,0)\}$

$(x_k, y_k) \to (0,0)$ $\quad (x_k, y_k) \neq \vec{0}$

consider $f(x_k, y_k) = \frac{e^{x_k^2+y_k^2}-1}{x_k^2+y_k^2}$

let $t_k = x_k^2 + y_k^2$

$t_k \to 0$ $\quad \frac{e^{t_k}-1}{t_k} \to ?$

$$\lim_{k\to\infty} \frac{e^{t_k}-\overset{e^0}{1}}{t_k - 0} = \frac{d}{dt} e^t \Big|_{t=0} = e^t \Big|_{t=0} = 1$$

$f'(0) = \lim_{t\to 0} \frac{f(0+t)-f(0)}{t}$

$$f'(0) = \lim_{t\to 0} \frac{f(0+t) - f(0)}{t}$$

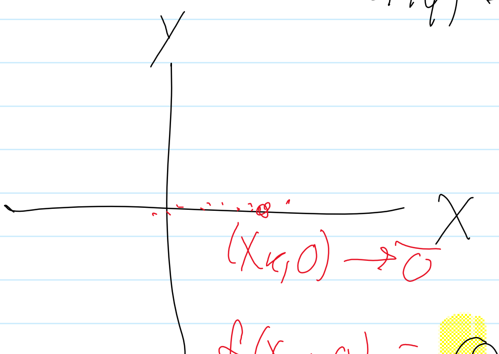
convert to seq.

$f(t) = e^t$ eval. deriv. at 0

Ex

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4+y^2} & (x,y) \neq 0 \\ 14 & (x,y) = 0. \end{cases}$$

See if $\lim_{(x,y)\to(0,0)} f(x,y)$ exists. If so find it. If not explain.



(x_n, y_n) seq in $\mathbb{R}^2 \setminus \{\vec{0}\}$

$(x_n, y_n) \to 0$

$$f(x_n,y_n) = \frac{x_n^2\, y_n}{x_n^4 + y_n^2}$$

$f(x_n,0) = 0 \xrightarrow[n\to\infty]{} 0$

let $y_n = x_n^2$

$$f(x_n, x_n^2) = \frac{x_n^2\, x_n^2}{x_n^4 + x_n^4} = \frac{x_n^4}{2x_n^4} = \frac{1}{2}$$

$f(x_n, x_n^2) \to \frac{1}{2}$ 2 diff lim 2 seq limit dne

Thm $A \subset \mathbb{R}^n$ x_0 lim pt of A $f: A \to \mathbb{R}$ $g: A \to \mathbb{R}$

assume $\lim_{x\to x_0} f(x) = l$ $\lim_{x\to x_0} g(x) = m$

then $\lim_{x\to x_0} (f+g)(x) = l + m$

$$\lim_{x \to x_0} f(x)g(x) = lm$$

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{l}{m} \quad \text{if } m \neq 0$$

pf use analogous thms for limits of seq as limit of fns are defined in term of limits of seq.

Thm $A \subset \mathbb{R}^n$ x_0 lim pt of A

$l = (l_1, l_2, \dots l_m) \in \mathbb{R}^m$

$F : A \to \mathbb{R}^m$ $\quad F(x) = (\underbrace{F_1(x), F_2(x), \dots F_m(x)}_{\text{component fns}})$

$F_j(x) : A \to \mathbb{R}$

$\lim_{x \to x_0} F(x) = l$ iff

$\lim_{x \to x_0} F_j(x) = l_j \quad j = 1 \dots m$

limit of F as $x \to x_0$ is l iff limit of each coord fn is the corresp component of l.

pf uses componentwise conv thm.

Thm ε-δ cond for limits

$F: A \to \mathbb{R}^m$ x_0 limit pt of A

$\lim_{x \to x_0} F(x) = l$ iff

ε-δ cond: $\forall \varepsilon > 0 \ \exists \delta > 0$ st

$\forall x \in A$ and $0 < \|x - x_0\| < \delta$

then $\|F(x) - l\| < \varepsilon$

pf is as pf of ε-δ cond for continuity

Derivatives: O open in $\mathbb{R}^n$

$\bar{x}_0 \in O$ $f: O \to \mathbb{R}$

NNL $f'(\bar{x}_0) = \lim_{\bar{x} \to \bar{x}_0} \dfrac{f(\bar{x}) - f(\bar{x}_0)}{\bar{x} - \bar{x}_0}$

Can't divide by vector!

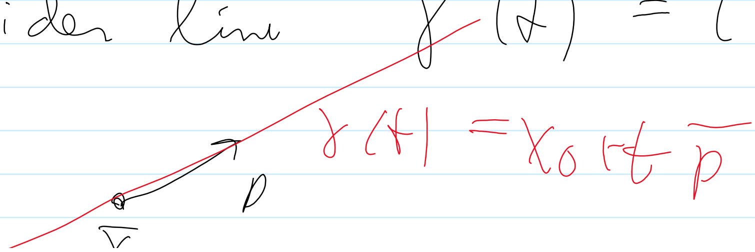
approach derivs by reduction to fns on $\mathbb{R}$

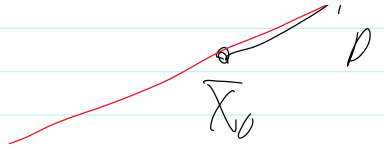
Directional derivatives

$\bar{p} \in \mathbb{R}^n \setminus \{0\}$ O open set

$\bar{x}_0 \in O$ $f: O \to \mathbb{R}$

consider line $\gamma(t) = (\bar{x}_0 + t\bar{p})$





take deriv in t @ $t=0$
of $f \circ \gamma(t) = f(\overline{x}_0 + t\overline{p})$

Defn $\mathcal{O}$ open, $\mathcal{O} \subset \mathbb{R}^n$, $\overline{x}_0 \in \mathcal{O}$, $\overline{p} \in \mathbb{R}^n \setminus 0$

$f : \mathcal{O} \to \mathbb{R}$

we define directional deriv of f at $\overline{x}_0$ in direction $\overline{p}$ as

$$\frac{df}{d\overline{p}}(x_0) = D_{\overline{p}} f(x_0) = \lim_{t \to 0} \frac{f(\gamma(t)) - f(x_0)}{t}$$

$$\gamma(t) = \overline{x}_0 + t\overline{p}$$

$$= \lim \frac{f(\overline{x}_0 + t\overline{p}) - f(\overline{x}_0)}{t}$$

$$= \frac{d}{dt} f(\overline{x}_0 + t\overline{p}) \Big|_{t=0}$$

if it exists.

recall std basis of $\mathbb{R}^n$

$$\overline{e}_1 = (1, 0, \ldots, 0)$$
$$\overline{e}_2 = (0, 1, \ldots, 0)$$
$$\overline{e}_n = (0, \ldots, 0, 1)$$

We define $\frac{\partial f}{\partial x_j}(\overline{x}_0) = D_{e_j} f(x_0)$

$= D_{e_j}$ $f(x_1, x_2, \ldots, x_j + t, \ldots x_n) - f(\overline{x}_0)$

$$= \lim_{t \to 0} f\,\frac{(x_1, x_2, \ldots x_j + t, x_{j+1}, \ldots x_n) - f(\bar{x}_0)}{t}$$

$$\gamma(t) = \bar{x}_0 + t\bar{e}_j$$

$$= (x_1, x_2, \ldots, x_j, \ldots x_n) + t\,(0, 0, \ldots, 1, 0 \ldots 0)$$

j coord

So

deriv of f in x_j @ $\bar{x}_0$ when all other coords are fixed.