## Quiz 6, October 18

**Question.** (a) *Give an algebraic proof of the expression*

$$2^n = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{k} + \cdots + \binom{n}{n-1} + \binom{n}{n}$$

*Hint:* $2 = 1 + 1$.

$$2^n = (1+1)^n = \sum_{k=0}^{n} \binom{n}{k} 1^n 1^{n-k}$$

$$2^n = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{k} + \binom{n}{n-1} + \binom{n}{n}$$

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(b) *Give a combinatorial proof of the expression*

$$2^n = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{k} + \cdots + \binom{n}{n-1} + \binom{n}{n}$$

*Hint: $2^n$ is the number of subsets of a set with $n$ elements.*

If have some set A with the cardinality n, it has $2^n$ subsets. When forming a subset from A, the number of subsets of size k, where $k \leq n$ is $C(^n_k)$ as order of a subset doesn't matter. Therefore, the total number of subsets of K is the sum of them all, which is $C(^n_0) + C(^n_1) + \ldots + C(^n_k) + \ldots + C(^n_{n-1}) + C(^n_n)$. We know a # of subsets is $2^n$. So the sum above is also $2^n$.