Math 65 HW 6

1a) If we have sequence of length n with k ones, there is $\binom{n}{k}$ combinations

b) $\binom{n}{k}$ is all possible ones, each remaining slot has 2 choices, 0 or 2 so 2^{n-k} remaining options. Total options $= \boxed{\binom{n}{k}2^{n-k}}$

2a) Fd $\binom{n}{4}$ except for $n=8$ where it's $\frac{\binom{8}{4}}{2}$
This is case for $n=8$ as for $n=8$ each combination of 4 elements has another comb. that is identical, meaning we must divide by 2.

3a) $\binom{n}{3} = \binom{n-1}{2} + \binom{n-1}{3}$

$\binom{n-1}{3} = \binom{n-2}{2} + \binom{n-2}{3}$

$\binom{n-k}{3} = \binom{n-k-1}{2} + \binom{n-k-1}{3}$ this repeats until $n-k-1=2$ as $\binom{2}{3}=0$

Gives sum:
$\binom{n}{3} = \binom{n-1}{2} + \binom{n-2}{2} + \ldots \binom{4}{2} + \binom{3}{2} + \binom{2}{2}$

b) For a set $\{1,2 \ldots n\}$ pick the highest element of the group of 3 to be n. There are $\binom{n-1}{2}$ remaing combinations. This can be repeated for all possible groups and sum this to get all combinations.
The sum is $\binom{n-1}{2} + \binom{n-2}{2} + \ldots \binom{4}{2} + \binom{3}{2} + \binom{2}{2}$

4) $x+y+z=85$, $x \geq 5$ $z \leq 10$
total = combinations $x \geq 5$ − comb $z \geq 11$
total $= \binom{80+3-1}{3-1} - \binom{69+3-1}{3-1} = \boxed{\binom{82}{2} - \binom{71}{2}}$

Scanned with CamScanner

5 Algebraically

$$\left(\!\binom{n}{k}\!\right) = \left(\!\binom{k+1}{n-1}\!\right)$$

$$\left(\!\binom{n}{k}\!\right) = \binom{k+n-1}{n-1} = \binom{n+k-1}{k}$$ equal

$$\left(\!\binom{k+1}{n-1}\!\right) = \binom{n-1+k+1-1}{k+1-1} = \binom{n+k-1}{k}$$

Combinatorically:

$\left(\!\binom{n}{k}\!\right)$ has $n-1$ seperators and k varieties.

k → n objects

If we were to invert it so that seperators became the varieties, it becomes $k+1$ seperators as need one edge, with now $n-1$ varieties as one less variety. So $\left(\!\binom{k+1}{n-1}\!\right)$

6 a) $\left(\!\binom{12}{3}\!\right) = \binom{9+4-1}{4-1} = \boxed{\binom{12}{3}}$

b) $\binom{12+4-1}{4-1} - \binom{12}{3}$

c) $\binom{12}{3} - \binom{6+4-1}{4-1} = \boxed{\binom{12}{3} - \binom{9}{3}}$

d) $5\binom{12}{3} + \left(\binom{15}{3} - \binom{12}{3}\right) - \left(\binom{12}{3} - \binom{9}{3}\right) = \boxed{\binom{15}{3} - \binom{12}{3} + \binom{9}{3}}$

7 Consider a lineup of $2n$ people assign person 1 w/ person 2, and so on. There are different ways to line people up as can swap pairs, and 2^n pairs to swap. Since there is n pairs, they can also be reordered $n!$ ways, meaning from $(2n)!$ possible line ups, there is $\frac{(2n)!}{2^n n!}$ unique pairs

and $\frac{n!}{2^n}\binom{2n}{n} = \frac{(2n)!}{n!\,2^n}$

Scanned with CamScanner