Problem

Let $f_n(x) = ae^{-nax} - be^{-bnx}$ for $x > 0$ and $0 < a < b$.

(a) Prove that $\sum_{n=1}^{\infty} \int_0^{\infty} |f_n(x)| dx = \infty$.

(b) Prove that $\sum_{n=1}^{\infty} \int_0^{\infty} f_n(x) dx = 0$.

(c) Prove that $\sum_{n=1}^{\infty} f_n(x) \in L^1([0, \infty))$, and $\int_0^{\infty} \sum_{n=1}^{\infty} f_n(x) dx = \ln(b/a)$.