Extra exercise

1. Find the degree 2 interpolating polynomial $P_2(x)$ that passes through $(0,0)$ $(\pi/2, 1)$ and $(\pi, 0)$.

solution

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-\pi/2)(x-\pi)}{-\pi/2 \cdot -\pi} = \frac{x^2 - \frac{3\pi}{2}x + \frac{\pi^2}{2}}{\pi^2/2}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{x\cdot(x-\pi)}{\pi/2\cdot(\pi/2-\pi)} = \frac{x^2-\pi x}{-\pi^2/4}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{x\cdot(x-\pi/2)}{\pi\cdot(\pi/2)} = \frac{x^2-\frac{\pi}{2}x}{\pi^2/2}$$

$$P_2(x) = \frac{x^2-\pi x}{-\pi^2/4} = -\frac{4}{\pi^2}x^2 + \frac{4}{\pi}x$$

2. Using your result in (1), calculate $P_2(\pi/4)$ an approximation for $\sin(\pi/4)$

solution

$$P_2\left(\frac{\pi}{4}\right) = -\frac{4}{\pi^2}\cdot\frac{\pi^2}{16} + \frac{4}{\pi}\cdot\frac{\pi}{4}$$

$$= -\frac{1}{4} + 1 = \frac{3}{4}$$

3. Use the interpolation error formula to give an error bound for the approximation in (b).

solution

$$\left|\sin\left(\frac{\pi}{4}\right) - P_2(\pi/4)\right| \le \frac{1}{3!}\left|\sin'''(\xi)\right| \prod_{i=0}^{2}\left|(\pi/4 - x_i)\right|$$

$$\le \frac{1}{6}\cdot\frac{\pi}{4}\cdot\frac{\pi}{4}\cdot\frac{3\pi}{4} = \frac{\pi^3}{128}$$