Wednesday, February 1

Friday, January 27, 2023 11:21

TA Help session 10:30 Fridays, Math library, JCC 574
Student hours with Todd 1:30-3:00 my office JCC 575 (end of hall)

Student hours will start on Friday at 2:00 and we can continue to 3:30 (because of AWM panel and lunch)

Still get-to-know-you meeting slots available and when they fill up, I'll add more
https://docs.google.com/spreadsheets/d/1T8o6af3Oe3uA3aswPvv1pm0FdnmQ6oaiF5Le623wdLY/edit?usp=sharing

MATHEMATICAL CONTEST IN MODELING: February 16-20, 2023. TEAMS OF THREE UNDERGRADS
https://www.contest.comap.com/undergraduate/contests/

DIRECTED READING PROGRAM: grad student and undergrad read a math book or article and learn about it together
A list of projects and descriptions can be found here: https://drive.google.com/file/d/1ffyVId43yPtFP-9GiODrtHf3ZlJ2Nc2S/view?usp=sharing

Application: https://forms.gle/P46BCsEKvdnzftLo9

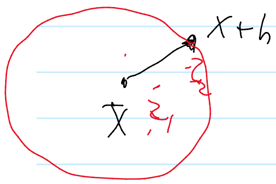
Save the date! AWM Panel & Lunch with Malena Espanol
Friday February 3rd at 1pm in JCC 501

Malena Espanol is an assistant professor in the school of Mathematical and Statistical Sciences at Arizona State University. She earned a Ph.D. in math from Tufts in 2009. The Tufts AWM chapter is excited to host Dr. Espanol for a Q&A over lunch! Everyone in the Tufts community is welcome to join.

Proposition 13.15 The Mean Value Proposition Let $\mathbf{x}$ be a point in $\mathbb{R}^n$ and let r be a positive number. Suppose that the function $f : \mathcal{B}_r(\mathbf{x}) \to \mathbb{R}$ has first-order partial derivatives. Then if the point $\mathbf{x}+\mathbf{h}$ belongs to $\mathcal{B}_r(\mathbf{x})$, there are points $\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_n$ in $\mathcal{B}_r(\mathbf{x})$ such that

$h = (h_1, \ldots, h_n)$

$$f(\mathbf{x}+\mathbf{h}) - f(\mathbf{x}) = \sum_{i=1}^{n} h_i \frac{\partial f}{\partial x_i}(\mathbf{z}_i), \qquad (13.19)$$



and

$$\|\mathbf{x} - \mathbf{z}_i\| < \|\mathbf{h}\| \qquad \text{for each index } i \text{ with } 1 \le i \le n.$$

Definition Let $\mathcal{O}$ be an open subset of $\mathbb{R}^n$ that contains the point $\mathbf{x}$ and suppose that the function $f : \mathcal{O} \to \mathbb{R}$ has first-order partial derivatives at $\mathbf{x}$. We define the *gradient* of the function $f : \mathcal{O} \to \mathbb{R}$ at the point $\mathbf{x}$, denoted by $\nabla f(\mathbf{x})$, to be the point in $\mathbb{R}^n$ given by

$$\nabla f(\mathbf{x}) = \left(\frac{\partial f}{\partial x_1}(\mathbf{x}), \frac{\partial f}{\partial x_2}(\mathbf{x}), \ldots, \frac{\partial f}{\partial x_n}(\mathbf{x}) \right)$$

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Goal: generalize MVT from $\mathbb{R}$ to $\mathbb{R}^n$

recall MVT on $\mathbb{R}$ $f : [a,b] \to \mathbb{R}$ cont diff on (a,b) Then $\exists\, c \in (a,b)$

st $f(b) - f(a) = f'(c)\,(b-a)$

generalize on $\mathbb{R}^n$

Thm directional deriv thm

$\mathcal{O}$ open in $\mathbb{R}^n$ $f \in C^1(\mathcal{O})$ (cont. diff on $\mathcal{O}$)

$\bar{x} \in \mathcal{O}$ $\bar{h} \in \mathbb{R}^n \setminus \{0\}$

$$\frac{\partial f}{\partial \bar{h}}(\bar{x}) = \langle \nabla f(\bar{x}), \bar{h} \rangle$$

eg $f(x,y) = x^2 e^{xy}$ $\bar{h} = (1,2)$

Find $\frac{\partial f}{\partial \bar{h}}(\bar{x})$ $\bar{x} = (3,4)$ as $f \in C^1(\mathcal{O})$

use dir deriv thm.

$$\frac{\partial f}{\partial \bar{h}}(\bar{x}) = \langle \nabla f(\bar{x}), \bar{h} \rangle$$

$$\frac{\partial f}{\partial \bar{h}}(3,4) = \langle \nabla f(3,4),\ (1,2) \rangle$$

$\nabla f(x,y) = (x^2 y\, e^{xy} + 2x e^{xy},\ x^3 e^{xy})$ $\nabla f(3,4) = (36 e^{12}, 27 e^{12})$

$= \langle (\cancel{36}\,e^{12}, 27 e^{12}), (1,2) \rangle$

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$\longrightarrow = \langle (\cancel{36}\,42\, e^{12}, 27e^{12}), (1,2) \rangle_2$

$\cancel{36}\,42\, e^{12} + 54e^{12} = 96e^{12}$

pf of thm use Mean Value prop and $f \in C^1$

I put a copy of the Math 136-01 notes, which are in this directory and labeled *136-02-2.1-notes from section 1.pdf* .

They continue with the proof of the Directional Derivative Theorem, so you can start there. I've noted that

I'm sorry that my computer stylus broke, and I hope to have it working on Monday.