

- Question 1.** (a) Check that rs and r^2s together generate D_n .
- (b) Find all subgroups of D_4 and D_5 .

Question 2. Draw a diagonal in a regular hexagon. List those plane symmetries of the hexagon which leave the diagonal fixed, and those which send the diagonal to itself. Show that both collections of symmetries are subgroups of the group of all plane symmetries of the hexagon.

Question 3. Let f be an isometry from the real line to itself which sends the integers to integers.

- (a) Show that when f has no fixed points, f is a translation by an integer distance.
- (b) Show that when f leaves exactly one point fixed, this point is either an integer or lies midway between two integers and f is a reflection across this fixed point.
- (c) Show that when f fixes more than one point that it is the identity map.

Question 4. Express each of the following elements of S_6 as a product of disjoint cyclic permutations, and as a product of transpositions.

(a)

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 5 & 1 & 6 & 4 \end{bmatrix}$$

(b) $(1235)(2134)$

(c) $(135)(234)(246)(345)$

Question 5. Show that the elements of S_5 which send the numbers 1, 3, 5 among themselves form a subgroup of S_5 . What is the order of this subgroup? Then do the same for the elements of S_5 sending 2, 4 among themselves.

Question 6 (Challenging). Show that:

- (a) When n is odd (123) and $(12 \cdots n)$ generate A_n .
- (b) When n is even (123) and $(23 \cdots n)$ generate A_n .

Question 7. Label the vertices of the tetrahedron $\{1, 2, 3, 4\}$. Check that each of the following are subgroups of the rotational symmetry group of the tetrahedron, and write as

- (a) All rotational symmetries fixing the vertex 1 and the midpoint of vertices 2, 3, 4.
- (b) All rotational symmetries fixing the midpoint of vertices 1, 2 and the midpoint of vertices 3, 4.
- (c) All rotational symmetries fixing vertex 4 and the midpoint of vertices 2, 3.

Show that each of these collections form a subgroup of the group of rotational symmetries of the tetrahedron.