Quiz 7, Monday October 25.

Question. (a) *Define what it means for a relation $\sim$ to be an equivalence relation.*

A relation is an equivalence relation if it is reflexive, symmetric, and transitive.

(b) *Consider the following relation in the set of integers $a \sim b$ if and only if, there exists an integer k such that $a - b = 4k$. Show that $\sim$ is an equivalence relation.*

If the relation is reflexive, then $\forall a \in A$, $a \sim a$,

Meaning $a - a = 4k$, $0 = 4k$, $k = 0$

If relation is symmetric then if $a \sim b$ then $b \sim a$.

If $a \sim b$, there is a k such that $a - b = 4k$, To show $b \sim a$, then $-1(a - b = 4k) \to b - a = -4k \to b - a = 4(-k)$,

Meaning $\frac{a-b}{4} \in Z$, and $-k$ still $\in Z$

To show transitive. If $a \sim b$ and $b \sim c$, then $a \sim c$, $a, b, c \in Z$.

$a - b = 4k$ $\quad b - c = 4k$. If $a \sim c$, then $a - c = 4k$

$a - b = 4k$

$-c + b = 4k$

$a - c = 8k \to a - c = 4(2k)$, $2k \in Z$ meaning the relation is transitive

(c) *Describe the equivalence classes or cosets of* $\mathbb{Z}$ *by this equivalence.*

If $a-b=4k$, then $\frac{a-b}{4}=k$.

$[a] = \{x \in A \mid x \sim a\}$

$[0] = \{4k$, where $k \in \mathbb{Z}\}$

$[1] = \{4k+1,\ k \in \mathbb{Z}\}$

$[2] = \{4k+2,\ k \in \mathbb{Z}\}$

$[3] = \{4k+3,\ k \in \mathbb{Z}\}$

4 total classes / cosets

~~$[4] = \{4k+4, k \in \mathbb{Z}\}$~~, but $4k+4$ is same class as $4k$, so repeat in a cycle