QUIZ 9, MONDAY NOVEMBER 1.

Question 1. Let $(X, \preceq)$ be a poset

- *Define what it means for an element $x \in X$ to be minimal.*

$x \in X$ is minimal if $y \in X$ $y \preceq x \Rightarrow x = y$

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$x \in X$ is minimum if $\forall y \in X$, $x \preceq y$

- *Define what it means for an element $x \in X$ to be maximal.*

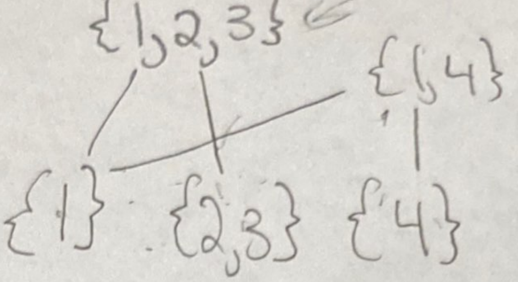
$x \in X$ is said to be maximal $\forall y \in X$, $x \preceq y \Rightarrow x = y$

Prove or disprove each of the following statements

(a) *If a poset has a minimum element, then it has to be unique.*

Assume b_1, b_2 are both minimums s.t. $b_1 \preceq b_2$ because b_2 is a minimum and $b_2 \preceq b_1$ as b_1 is a minimum. Since the relation is antisymmetric, $b_1 = b_2$ meaning it is unique

(b) *If a poset has a minimal element, then it has to be unique.*



A = {1, 2, 3, 4}

A = { {1}, {2,3}, {4}, {1,4}, {1,2,3} }

the relation is the inclusion

{1}, {2,3}, and {4} are all minimal, but aren't idential meaning the minimal element doesn't have to be unique.

(c) *If a finite poset has only one minimal element, then it is a minimum.*

If A is not minimum, A → $\exists b$, $b \preceq a$, $b \neq a$. However, a is the only minimal meaning $\exists c \in A$, $c \preceq b$, $c \neq b$, and so on forever, as none of these elements are the minimal, creating an infinite poset. However we are told the poset is finite, meaning this is a contradiction and that A actually must be the minimum and the only minimal element.

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