Tuesday October 26

Think about the function

$$F(x) = cx(1-x),$$

where c is a constant.

1. Prove that F is a function from $[0,1]$ into $[0,1]$ if $0 \le c \le 4$, but not otherwise. From here on, we will therefore assume

$$0 \le c \le 4.$$

2. Explain: $x = 0$ is always a fixed point, and if $c < 1$, then fixed point iteration converges to $x = 0$ no matter how you choose $x^{(0)} \in [0,1]$.

3. Explain: If $c > 1$, the fixed point $x = 0$ is repelling.

4. Suppose $c = 1$, so $F(x) = x(1-x)$. Is it true that fixed point iteration converges to $x = 0$ no matter how you choose $x^{(0)} \in [0,1]$?

5. Explain: There is a non-zero fixed point of F in $[0,1]$ if and only if $1 < c \le 4$.

6. Explain: The non-zero fixed point of F in $0,1]$ is attracting if $1 < c < 3$, and repelling if $3 < c \le 4$. So the interesting window is $3 < c \le 4$, since then there are two fixed points, but both are repelling.

7. Let $G(x) = F(F(x))$. Explain why fixed points of F are fixed points of G

8. Explain: If x_* is a fixed point of F, and $|F'(x_*)| < 1$, then $|G'(x_*)| < 1$. Similarly, if $|F'(x_*)| > 1$, then $|G'(x_*)| > 1$.

9. For the fixed point iteration $x^{(k)} = F(x^{(k-1)}$, what is the significance of fixed points of G that are not fixed points of F?