Math 65 HW 5

1 a) If A is one-to-one, then for $a \in A$
$f(a_1) = f(a_2)$, $a_1 = a_2$. Each $a \in A$ maps to a unique $f(a) \in B$. Therefore, $|B| \geq |A| + c$, where c = number of elements in B that cannot be mapped to for a. Since $c \geq 0$, as f is not onto, $|B| \geq |A|$

b) If f is onto, for $a \in A$, $b \in b$ $\forall b$ $\exists a$ such that $f(a) = b$. Since $f(a) = b$, then $|B| \leq |A|$, as since f is not one-to-one $f(a_1) = f(a_2)$, but $a_1 \neq a_2$

2 a) $\Rightarrow$ If $f: A \to B$ is injective, it is surjective.
$f(a_1) = f(a_2)$ $a_1 = a_2$, $a_1, a_2 \in A$. Since A and B have n elements, $\forall b \in B$ $\exists a$ $f(a) = b$, meaning f is surjective.
$\Leftarrow$ If $f: A \to B$ is surjective, it is injective.
$\forall b \in B$, $\exists a \in A$ $f(a) = b$. Since at least one a for each b, but $|A| = |B| = n$ So only can have one a for every b, meaning f is injective

3 a) $(a,b) \to (c,d)$

$x \to mx + n$

$ma + n = c$ $\quad m(a-b) = c-d$, $m = \frac{c-d}{a-b}$

$mb + n = d$

$n = c - ma$

$n = c - \left(\frac{c-d}{a-b}\right) a \quad \frac{a-b}{a-b} c - \left(\frac{ac-ad}{a-b}\right) = \frac{c(a-b) - ac + ad}{a-b}$

$n = \frac{ad-bc}{a-b} \leftarrow = \frac{ca - cb - ac + ad}{a-b}$

$f: (a,b) \to (c,d)$ $\frac{c-d}{a-b} x + \frac{ad-bc}{a-b}$

To show a bijection

f is one-to-one: $z \in \mathbb{R}$

$f(z_1) = f(z_2)$ $\frac{c-d}{a-b} z_1 + \frac{ad-bc}{a-b} = \frac{c-d}{a-b} z_2 + \frac{ad-bc}{a-b}$ $z_1 = z_2$

f is onto

$f(x) = z$, $\forall z \in \mathbb{R}$ $\exists x$ such that $f(x) = z$

$z = \frac{c-d}{a-b} x + \frac{ad-bc}{a-b}$ $\quad \frac{z(a-b) - ad + bc}{c-d} = x$, as $c \neq d$, f is onto

f is onto and one-to-one, therefore $\frac{c-d}{a-b} x + \frac{ad-bc}{a-b}$ forms a bijection

3b) let $f: (a,b) \to \mathbb{R}$ be $\tan x$ $(a = -\frac{\pi}{2}, b = \frac{\pi}{2})$
$(-\frac{\pi}{2}, \frac{\pi}{2}) \xrightarrow[\tan x]{} \mathbb{R}$.

To show bijection, we know arctanx exists
tan(arctanx) = X , arctan(tanx) = X, Since an inverse of tanx, arctanx exists, then tanx on $(-\frac{\pi}{2}, \frac{\pi}{2})$ forms a bijection with

4a) S_1, S_2 are disjoint sets, meaning they both have unique elements. Also S_1, S_2 are countable.
Let us do the following map
$$\begin{cases} 2S_1 = S_1' \\ 2S_2 + 1 = S_2' \end{cases}$$

S_1' is a set of odd numbers, S_2' is a set of even numbers. So $T = S_1' \cup S_2'$ which is a subset of N and forms a bijection with it, meaning T is countable.

b) $I = R - Q$
R is uncountable, but Q is countable. When we remove any countable set from an uncountable set, it remains uncountable, meaning, I is uncountable.

5 We can order all finite sets and number them

1 ATCGC ..... T
2 GCTA ...... G
3 TTCC ...... A

$\Bigg\}$ 3! combinations of size n

Can repeat this for size 1... n. and order them all, as n is finite. This means that since we can order them all, the total number of combinations is finite, and therefore countable.

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6 a) A has n elements, B has m elements and $A \cap B = \emptyset$ meaning all elements in A and B are unique.

$|A| = n$ $|B| = m$ $A \cup B = \{\underbrace{A_1 \ldots A_n}, \underbrace{B_1 \ldots B_m}\}$

$$|A \cup B| = n + m = \overset{n}{|A|} + \overset{m}{|B|}$$

b) None of the sets overlap and all have unique elements.

Let $|A| = m$, $|B| = n$, $|C| = p$

$$A \cup B \cup C = \{\underbrace{a_1 \ldots a_m}_{m}, \underbrace{b_1 \ldots b_n}_{n}, \underbrace{c_1 \ldots c_p}_{p}\}$$

$$|A \cup B \cup C| = m + n + p = |A| + |B| + |C|$$