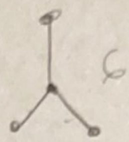
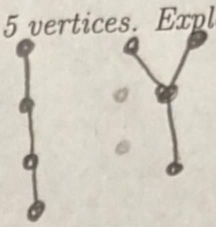


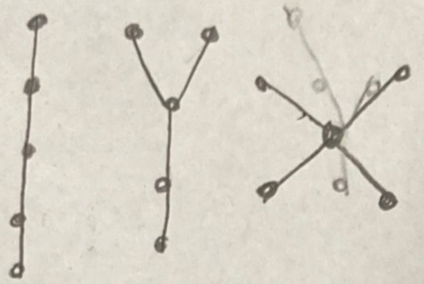
Quiz December 1, Trees

Question. (a) *If T is a tree with $n \geq 2$ vertices and $d_1 \leq d_2 \leq \cdots \leq d_n$ are the degrees of the vertices, show that $d_1 = d_2 = 1$ and $\sum_{i=1}^{n} d_i = 2n-2$.*

Any tree has at least 2 leafs, meaning $d_1 = d_2 = 1$.
For a tree with n vertices, has n-1 edges, so it's degrees add up to $2(n-1) = 2n-2$

(b) *Find all possible trees (up to isomorphism) with 2, 3, 4 and 5 vertices. Explain why there aren't any more.*

n=2  n=3  n=4 

n=5 

Any tree T ~~with~~ n vertices can't have more than n-1 leafs, since if there were n leaves, all vertices would be of degree 1, which means $\sum d_i = n$ which isn't the case. There's one of each number of leaves below as long as there is at least 2 leaves, accounting for isomorphisms.

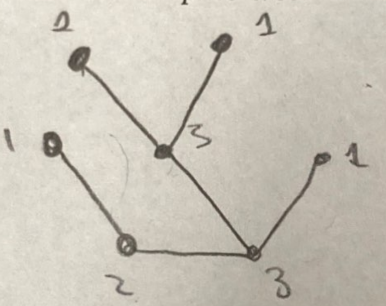
(c) *Assume that $d_1, d_2, \ldots, d_n$. are integers, $d_i \geq 1$ and $\sum_{i=1}^{n} d_i = 2n - 2$. Show that there exists a tree whose vertices have these degrees.*

Proof by induction for $n \geq 2$,

$n = 2$ $d_1, d_2 \geq 1$ and $\sum d_i = 2(2) - 2 = 2$

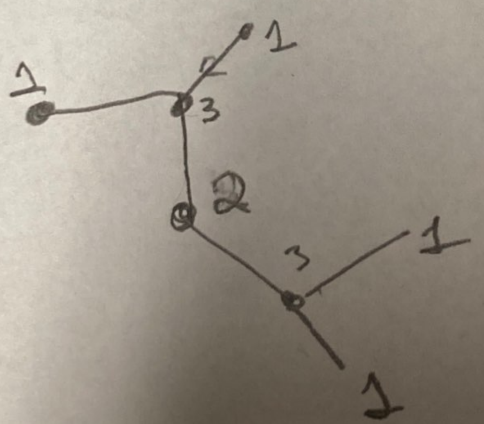
So it's true for n vertices, have to show for $n+1$. If we have a tree with n vertices, it has at least 2 leafs. We can add a vertice V_{n+1} connected only to a leaf by e_{n+1}, the sum of degrees $\sum_{i=1}^{n+1} d_{i_{n+1}} = 2n-2+2 = 2n$, and $d_{n+1} = 1$, Meaning it fulfills properties and still is tree

(d) *Give an example of a collection of integers $d_1, d_2, \ldots, d_n$, $d_i \geq 1$ and $\sum_{i=1}^{n} d_i = 2n - 2$. such that there are two non-isomorphic trees whose vertices have these degrees.*



1, 1, 1, 1, 2, 3, 3

collection of integers
1, 1, 1, 1, 2, 3 3



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