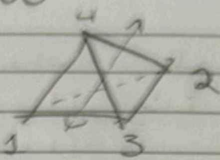
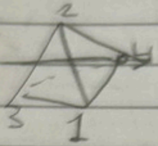
Math 145 HW

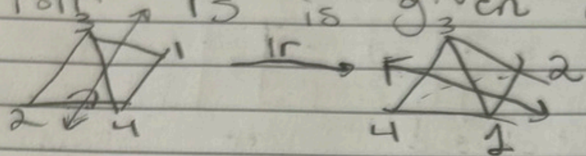
1.3) sr is given as



So srs is:



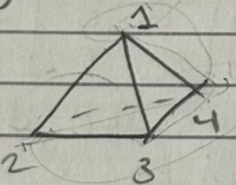
For rsrr, rs is given as:



Can see axis rotates r so
goes between 12, 34 : {3

1.4) So can think of this as holding
1 element on top, then rotating it twice
to make all symmetries. Making 4x3 total

Starting as



, we clearly
see {e, r, rr} generates those 3 associated
w/ 1.

To get 4 on top, it requires one s, and
it follows clearly that s, sr, srr generates
that list.

To push 2 on top, requires rrs as need
two rotations to get it in position. Also right
after it follows rrsr. To get the last one,
rotate again w/ rrsrr.

Lastly, need to put 3 on top. So, see
rs puts 3 on top, then rsr, rsrr are other positions

This makes the set

$$\boxed{S = \{e, r, rr, s, sr, srr, rrs, rrsr, rrsrr, rs, rsr, rsrr\}}$$

2.3)

i)

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} d & e \\ e & f \end{bmatrix} = \begin{bmatrix} ad+be & ae+bf \\ bd+ce & be+cf \end{bmatrix}$$

So not closed under group operation and not a a group

ii)

$$\begin{bmatrix} a & b \\ c & a \end{bmatrix}\begin{bmatrix} d & e \\ f & d \end{bmatrix} = \begin{bmatrix} ad+bf & ae+bd \\ cd+af & ce+ad \end{bmatrix}$$

So not closed under group operations so not a group

iii)

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}\begin{bmatrix} d & e \\ 0 & f \end{bmatrix} = \begin{bmatrix} ad & ae+bf \\ 0 & cf \end{bmatrix}$$

$adcf \neq 0$ as $ac, df \neq 0$ so closed under operation.

Matrix multiplication is associative and $\mathbb{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is in the group and

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

As $ac \neq 0$, $\det \neq 0$ so $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is invertible, and an inverse exists. Therefore, this is a group.

v) Not a group, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\det A \neq 0$, We know $A^{-1} = -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \notin$ the group

So therefore the collection isn't a group.

2.5)

Injective: Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be an isometry under some distance function d.

f is injective if $f(\vec{a}) = f(\vec{b}) \Rightarrow \vec{a} = \vec{b}$ for $\vec{a}, \vec{b} \in \mathbb{R}^2$

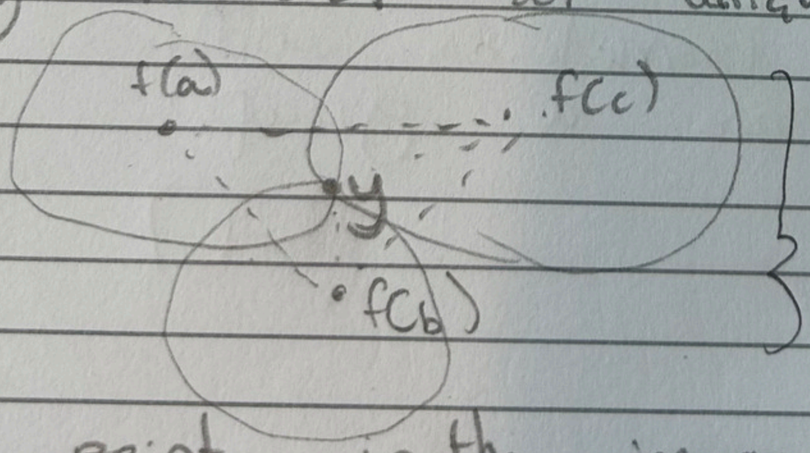
If $f(\vec{a}) = f(\vec{b})$, then $d(f(\vec{a}), f(\vec{b})) = 0$ and as f is an isometry, then $d(\vec{a}, \vec{b}) = 0$ so $\vec{a} = \vec{b}$ and f is injective.

Surjective: Show $\forall \vec{y} \in \mathbb{R}^2 \; \exists x \in \mathbb{R}^2$ s.t. $f(x) = y$.

Let $a, b, c \in \mathbb{R}^2$ and be unique.
As f is injective, $f(a) \neq f(b) \neq f(c)$

Let $y \in \mathbb{R}^2$ so let $d(f(a), \vec{y}) = x_1$
$d(f(b), \vec{y}) = x_2$
$d(f(c), \vec{y}) = x_3$

If we draw 3 circles, of radius x_1, x_2, x_3 centered on $f(a), f(b), f(c)$ they intersect at unique point y.



Miquel's Thm.

This point in the image is unique, as if it's not, then f isn't injective.
So, get the unique x that maps to y by taking preimage and then using the preserved distances.

To show this is a group.

Isometries are closed under composition.
Let f, g be isometries

$f \circ g$ is an isometry, as for $\vec{a}, \vec{b} \in \mathbb{R}^2$

$$d(\vec{a}, \vec{b}) = c \Rightarrow d(g(\vec{a}), g(\vec{b})) = c$$
$$\Rightarrow d(f(g(\vec{a})), f(g(b))) = c$$

Function compositions are associative so the group is associative

As isometries are bijective, they are invertible and for isometry f, it's inverse f^{-1} is the function that maps any point back to it's initial point in $\mathbb{R}^2$. As $f^{-1} \circ f =$

The identity is $I: x \to x$ as $f \circ I = f$ and $I \circ f = f$ as I maps every point to itself.

Therefore, the isometries under composition are a group

$\square$