Tuesday, November 16

1. Let's think about a slightly more elaborate model of two competing species:

$$\frac{dx_1}{dt} = x_1\left(1-\frac{x_1}{2}\right)(1-x_2)$$
$$\frac{dx_2}{dt} = x_2\left(1-\frac{x_2}{3}\right)(1-x_1)$$

   (a) Discuss what might motivate these equations.

   (b) Find all fixed points. (Hint: there are five of them.)

   (c) What can you say about the fixed points based on the eigenvalues of the Jacobi matrices?

   (d) Is stable coexistence possible here?

2. Suppose that J is an $n \times n$ matrix with n eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, and associated eigenvectors $v_1, v_2, \ldots, v_n$. Assume $v_1, v_2, \ldots, v_n$ are linearly independent. Explain: If $\lambda_1, \ldots, \lambda_n$ are negative real numbers, then all solutions of

$$\frac{du}{dt} = Ju$$

   converge to zero. So while a *single* eigenvalue being negative does not tell you that the fixed point $u = 0$ fo $\frac{du}{dt} = Ju$ is stable, if *all* eigenvalues are negative, then indeed it is stable.

3. Explain: If a, b are real numbers, and $\lambda = a + ib$ (with $i = \sqrt{-1}$), then $e^{\lambda t}$ converges to zero as $t \to \infty$ if and only if $a < 0$. If $a > 0$, then $|e^{\lambda t}|$ converges to ∞ as $t \to \infty$. (This is needed to understand how the analysis we gave in class extends to the case of complex eigenvalues.)