Practice with Abstract Vector Spaces

Math 70

March 18, 2020

(1) Consider two different polynomial vector spaces $\mathbb{P}_2$ and $\mathbb{P}_4$.

(a) Is $\mathbb{P}_2$ a vector subspace of $\mathbb{P}_4$?

(b) Find a spanning (or generating) set for $\mathbb{P}_2$.

(c) Consider the linear transformation $T : \mathbb{P}_2 \to \mathbb{P}_4$ described by

$$T(a_0 + a_1 t + a_2 t^2) = (a_1 + a_2) + a_1 t + 3a_0 t^4.$$

Describe the kernel and range of T. What is a spanning set for the range?

(2) Consider the vector space of finitely supported signals $\mathbb{S}_f$, that is all bi-infinite sequences $\{y_k\}$, where only finitely many of the y_k are nonzero. Consider the map $T : \mathbb{S}_f \to \mathbb{R}$, where

$$T(\{y_k\}) = \sum_{i=-\infty}^{\infty} y_k.$$

(a) Is T a linear transformation?

(b) What is the kernel of T? Do you think it would be possible to find a spanning set for the kernel?

(3) Consider the transformation $T : \mathbb{P}_2 \to \mathbb{R}^2$ described by

$$T(p(t)) = \begin{bmatrix} p(0) \\ p'(0) \end{bmatrix}.$$

Is this transformation linear?

(4) Consider the space V of all linear transformations from $\mathbb{R}^2 \to \mathbb{R}^2$.

(a) Is V is vector space? Why or why not?

(b) Using what we know about standard matrices for linear transformations, can you think of another way to describe V?