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Motivation

Bayesian
search
strategy

Summary

Bayesian estimation

Bayesian search strategy

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Summary

- Suppose we are trying to find a lost object.
- We have some ideas of regions where to search.
- There is no certainty that we will find the object, even if we search the region in which it is located.
- For example, searches of the ocean floor for lost objects.

- $A_j :=$ Event missing object is in region r_j
- $B_j :=$ Event object would be found if it was in r_j, and r_j was searched
- $P(A_j \mid B_j^C)$ is probability that the item is in r_j, given that r_j was just searched unsuccessfully.
- Start with *prior* estimates for the probabilities $\{P(A_j)\}_{j=1}^n$. These could be as simple as the uniform distribution

$$\forall j \in \{1, \ldots, n\} : \ P(A_j) = \frac{1}{n}.$$

- Suppose region r_j is searched, and nothing is found there.

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- Apply Bayes' Theorem, and note that $P\left(B_j^C \mid A_j^C\right) = 1$

$$P\left(A_j \mid B_j^C\right) = \frac{P\left(B_j^C \mid A_j\right) P(A_j)}{P\left(B_j^C \mid A_j\right) P(A_j) + P\left(B_j^C \mid A_j^C\right) P(A_j^C)}$$

$$= \frac{[1 - P\left(B_j \mid A_j\right)] P(A_j)}{[1 - P\left(B_j \mid A_j\right)] P(A_j) + [1 - P(A_j)]} = \left[\frac{1 - P\left(B_j \mid A_j\right)}{1 - P\left(B_j \mid A_j\right) P(A_j)}\right] P(A_j)$$

- Hence, after unsuccessful search in r_j, prior for $P(A_j)$ is updated to obtain

$$\boxed{P^*(A_j) = P\left(A_j \mid B_j^C\right) = \left[\frac{1 - P\left(B_j \mid A_j\right)}{1 - P\left(B_j \mid A_j\right) P(A_j)}\right] P(A_j) < P(A_j).}$$

- Rescale all the other priors, $P(A_k)$ where $k \neq j$,

$$P^*(A_k) = P\left(A_k \mid B_j^C\right) = \alpha_j P(A_k)$$

- Demand normalization $1 = \sum_{m=1}^{n} P^*(A_m)$ to find

$$\alpha_j = \frac{1}{1 - P\left(B_j \mid A_j\right) P(A_j)}$$

- So the other priors update as follows

$$P^*(A_k) = P\left(A_k \mid B_j^C\right) = \left[\frac{1}{1 - P\left(B_j \mid A_j\right) P(A_j)}\right] P(A_k) > P(A_k).$$

- For all $m = 1, \ldots, n$:
    - Determine the probabilities that a search in r_m will find the object if it is in r_m.
    - Guess the priors $P(A_m)$.
- Now repeat until the object is found:
    - Suppose value of m for which $P(A_m)$ is highest is $m = j$.
    - Conduct a search in region r_j
    - If the search is unsuccessful, update the priors and repeat.

- We have studied Bayesian search strategy
- We have presented an algorithm for Bayesian search