Math 165 HW 1.

I worked with David O. on HW 1.

1 a) Represent an outcome with (a,b) where $a = 1^{st}$ ticket#, $b = 2^{nd}$ ticket#. $P = \frac{\#(a,b)}{|\Omega|}$ where $(a,b) = (1,2)$
1 outcome where true w/ $|\Omega| = n \cdot n = n^2$.

$\boxed{P = \frac{1}{n^2}}$

b) Using (a,b) $a \leq n-1$ and b has only 1 value, but a has $n-1$ outcomes. $P = \frac{\#(a,b)}{|\Omega|} = \frac{(n-1) \times 1}{n^2} = \boxed{\frac{n-1}{n^2}}$

c) 1^{st} ticket has $n-1$ possibilities, 2^{nd} ticket has n possibilities, but must be $\geq 1^{st}$ ticket, so to find this take sum of 1^{st} n integers, which is $\frac{(n-1)n}{2}$ possibilities, from textbook. $P = \frac{(n-1)n}{2n^2} = \boxed{\frac{1}{2}(1-\frac{1}{n})}$

d) For a, using (a,b) $P(a=1) = \frac{1}{n}$, $P(b=2) = \frac{1}{n-1}$
So $P(a=1 \cap b=2) = \frac{1}{n(n-1)}$

For b, 1^{st} ticket, $a \leq n-1$, $b \leq n-1$, $P(a \leq n-1 = \frac{n-1}{n})$
$P(b=n) = \frac{1}{n-1}$, can multiply, giving $\boxed{\frac{1}{n} = P}$

For c, 1^{st} ticket up to $n-1$, 2^{nd} ticket has combinations of sum of 1^{st} n integers. So have $\frac{(n-1)n}{2} \cdot \frac{1}{n(n-1)} = \boxed{\frac{1}{2}}$

2 a) Let (a,b) be ordered pair combinations of:
$(a,b) = \binom{52}{2} = \boxed{2652}$

b) $\frac{4 \text{ aces}}{52 \text{ cards}} = \boxed{\frac{1}{13}}$

c) Since taking combinations order doesn't matter, meaning $\boxed{1/13 \text{ chance}}$

d) $P = \frac{\#(a,b)\ a,b = \text{Ace}}{\#\Omega} = \frac{\binom{4}{2}}{2652} = \boxed{\frac{1}{221}}$

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2e) If there's atleast 1 ace, then either 1st, 2nd, or both are aces. Subtract the both as it's counted twice, so P(at least 1 ace) =
$\frac{1}{13}+\frac{1}{13}-\frac{1}{221}=\boxed{\frac{33}{221}}$

3 a) (a,b) unordered pair a = 1st die roll, b = 2nd die roll
$a\le 2$, $b\le 2$, Meaning roll is (1,1) (1,2), (2,1), (2,2)
$P=\frac{\#(a,b)_{max\le 2}}{\#\Omega}\quad \frac{4}{36}=\boxed{\frac{1}{9}}$

b) $\max(a,b)\le x$, $x\in[1,6]$ has x^2 outcomes since for a and b, are x rolls where $\max\le x$, so multiplying these, $x\cdot x=x^2$. For $x=3$, $P=\frac{9}{36}=\boxed{\frac{1}{4}}$

c) For $\max(a,b)=x$, are $2x-1$ combinations since:
$\underbrace{(1,x)\ldots\ldots(x,x)}_{x \text{ combinations}},\ \underbrace{(x,x-1)\ (x,x\ \ (x,1)}_{x-1 \text{ combinations}}$ $\quad x+x-1=2x-1$

$P(\max=x)=\frac{2x-1}{\#\Omega}=\frac{2x-1}{36}$ for max = 3, $\boxed{P=\frac{5}{36}}$

d) Using formulas from b and c for max of x

For b) $\max\le x$	For c) $\max=x$
$x\le 1\to P=\frac{1}{36}$	$x=1\to\frac{1}{36}$
$x\le 2\to P=\frac{4}{36}$	$x=2\to 3/36$
$x\le 3\to P=\frac{9}{36}$	$x=3\to 5/36$
$x\le 4\to P=\frac{16}{36}$	$x=4\to 7/36$
$x\le 5\to P=\frac{25}{36}$	$x=5\to 9/36$
$x\le 6\to P=\frac{36}{36}$	$x=6\to 11/36$

e) Sum of P's $\sum_{x=1}^{6}P(x)=1$, as max must be between 1 and 6. They do add to 1, showing this.

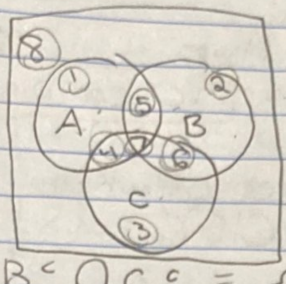
4 a) $\Omega = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,H),(T,T)\}$

b) $A = \{(T,T)\}$

$B = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\}$

$C = \{(H,2),(H,4),(H,6)\}$

5 a)



$(A \cup B \cup C)^c = \{8\}$

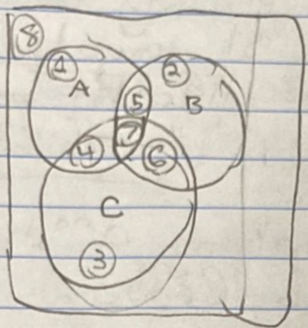
$A^c = \{2, 3, 6, 8\}$

$B^c = \{1, 3, 4, 8\}$

$C^c = \{1, 2, 5, 8\}$

$A^c \cap B^c \cap C^c = \{8\} = (A \cup B \cup C)^c$

b)



$(A \cap B \cap C) = \{7\}$

$(A \cap B \cap C)^c = \{1, 2, 3, 4, 5, 6, 8\}$

$A^c = \{2, 3, 6, 8\}$

$B^c = \{1, 3, 4, 8\}$

$C^c = \{1, 2, 5, 8\}$

$A^c \cup B^c \cup C^c = \{1, 2, 3, 4, 5, 6, 8\} = (A \cap B \cap C)^c$

6 a) By inclusion exclusion principle

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ | $1 - P(B^c) = .7 = P(B)$

$= .5 + .7 - .8 = \boxed{0.4}$

b) $P(A^c \cup B) = P(A^c) + P(B) - P(A^c \cap B)$

$P(B) = P(A^c \cap B) + P(A \cap B) \rightarrow .7 = P(A^c \cap B) + .4$

$P(A^c \cap B) = .3$

$P(A^c \cup B) = 0.5 + 0.7 - 0.3 = \boxed{0.9}$

c) $P(A) = P(A \cap B) + P(A \cap B^c)$

$0.5 = 0.4 + P(A \cap B^c)$

$= \boxed{0.1}$

7 a) There are $\binom{6}{4}$ combinations. $\binom{6}{4} = \boxed{15}$ possible Committees

b) Have 6 possible leaders and for each leader, there are $\binom{5}{3}$ ways to fill remaining commitee, so total = $6\binom{5}{3}$ = $\boxed{60 \text{ different committees}}$

8 a) With 2 empty seats, there is $\binom{20}{2}$ combinations as order is irrelevant. $\binom{20}{2}$ = $\boxed{190 \text{ combinations}}$

b) Need permutations, as order of where people sit matters, total permutations = $(18)_{18} = 18!$

c) We have 190 combinations of empty seats, and for each set of empty seats, 18! ways to sit the students. This means there is $\boxed{190 \times 18! \text{ ways to sit the students}}$

9 a) 1st person has 30 options, 2nd has 29, and so on. Can write this as $(30)_8$ as order matters and need to use permutations. $(30)_8 = \frac{30!}{22!}$

b) 1st person has 15 pairs to choose from and 2 options for each pair, 2nd has 14 pairs and 2 options for each pair. repeating this gives us:

$((15\times2) \times (14\times2) \times \ldots (8\times2)) = 2^8(15\times14\times \ldots \times9\times8)$.
$= 2^8(15)_8$

$\boxed{\text{There are } 2^8 \cdot (15)_8 \text{ total combinations, } (15)_8 = \frac{15!}{7!}}$

10 a) Each ball has 7 options, meaning ignoring color, there are 7^5 combinations. However, since 5 unique balls, there are 5! ways to place the balls, meaning there is a total # of ways of $\boxed{5! \times 7^5}$

10 b) If container 1 is empty, then ball has 6 options w/ number of orders for color being the same as a), so total = $\boxed{5! \times 6^5 \text{ ways}}$

c) If no 2 golf balls go in same container, then there are $(7)_5$ permutations as order matters. $(7)_5 = \boxed{2520 \text{ ways}}$

d) ball 1 will be unique, then ball 2 has 6/7 options where it can be unique, this repeats for balls 3 → 5, and probability decreasing by 1/7

$$P = 1 \times \frac{6}{7} \times \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{360}{7^4} = \boxed{0.15}$$

11 a) Ω consists of all combinations of good and bad light bulbs, meaning there is 0, 1, or 2 good lightbulbs, w/ rest being bad. $|\Omega| = {}_{13}C_2 = 78$

b) If 0 are good, then both are bad.

$$P = \frac{\#2\text{ bad}}{\#\Omega} = \frac{{}_3C_2}{{}_{13}C_2} = \boxed{\frac{3}{78}}$$

→ number of outcomes

c) If 1 is good, 1 is bad. Use multiplication rule to solve

$$P = \frac{\#1\text{ good} \cap \#1\text{ bad}}{\#\Omega} = \frac{{}_{10}C_1 \cdot {}_3C_1}{{}_{13}C_2} = \boxed{\frac{30}{78}}$$

d) 2 good, then $P = \frac{\#2\text{ good}}{\#\Omega} = \frac{{}_{10}C_2}{{}_{13}C_2} = \boxed{\frac{45}{78}}$

12 a) $P(3\text{ choc. chip}) = \frac{\#\text{ w/ 3 choc chip}}{\#\Omega} = \frac{{}_{10}C_3}{{}_{50}C_3} = \boxed{.006}$

$\# \rightarrow$ number of combinations w/

b) $P(2\text{ choc. chip}, 1\text{ vanilla}) = \frac{{}_{10}C_2 \cdot {}_{10}C_1}{{}_{50}C_3} = \boxed{.023}$

c) $P(1\text{ choc.}, 1\text{ van.}, 1\text{ oatmeal}) = \frac{{}_{10}C_1 \cdot {}_{10}C_1 \cdot {}_{10}C_1}{{}_{50}C_3} = \boxed{0.051}$

d) $P(0\text{ choc. chip}) = \frac{{}_{40}C_3}{{}_{50}C_3} = \boxed{.504}$

e) P(all unique), 1st unique, then 2nd has $\frac{40}{49}$ as can't be same as 9 cookies that share missing type.

Same logic for third, so has $\frac{30}{48}$ chance of having unique

$P(\text{all 3 unique}) = 1 \times \frac{40}{49} \times \frac{30}{48} = \boxed{0.51}$

13 a) $\#\Omega = \binom{25}{5}\binom{20}{4}$ For 13 H=x, Hillary has x dented,

C=y, Clyde has y dented.

where H=1 and C=2 is $\binom{10}{1}\binom{15}{4} \cdot \binom{9}{2}\binom{11}{2}$ as

must get precise amount of dented and undented

$$P = \frac{\binom{10}{1}\binom{15}{4} \cdot \binom{9}{2}\binom{11}{2}}{\binom{25}{5}\binom{20}{4}} = \boxed{.105}$$

b) # outcomes w/ H=1 are $\binom{10}{1}\binom{15}{4}$, $|\Omega| = \binom{25}{5}$

$$P = \frac{\binom{10}{1}\binom{15}{4}}{\binom{25}{5}} = \boxed{0.257}$$

c) # outcomes H=2 is $\binom{10}{2}\binom{15}{3}$ $P(C \geq 2) = 1 - (P(C=0) + P(C=1))$

$P(C=0) = \frac{\binom{8}{0}\binom{12}{4}}{\binom{20}{4}}$ $P(C=1) = \frac{\binom{8}{1}\binom{12}{3}}{\binom{20}{4}} = .363$

$.102$

$P = P(H=1) \cdot (1 - P(C=0) - P(C=1))$

$= .385(1 - .102 - .363) = \boxed{.206}$