March 4th - Exam Review/Practice

Math 70

March 4th, 2021

Note: I have NOT seen the exam, and this set of problems may or may not be representative of what is on the exam.

(1) Determine if the following statements are true or false. If false, explain why.

(a) Let $\mathbf{u}$ and $\mathbf{v}$ be nonzero vectors in $\mathbb{R}^n$. The zero vector $\mathbf{0}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

(b) A vector $\mathbf{b}$ is a linear combination of the columns of a matrix A if and only if the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution.

(c) If S is a linear dependent set, then each vector in S is a linear combination of the others.

(d) The equation $A\mathbf{x} = \mathbf{b}$ is consistent if the augmented matrix $[A \quad \mathbf{b}]$ has a pivot position in every row.

(e) A homogeneous linear system is always consistent.

(f) The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A.

(g) The set $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ can always be thought of as a plane through the origin.

(h) If A is a 3×2 matrix, then the tranformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot be one-to-one.

(2) Using ideas from all across Chapter 1 (span, linear systems, homogeneous systems, linear (in)dependence, linear transformations, etc.), come up with as many equivalent statements as possible (at least 3-4) to: the linear transformation $T(\mathbf{x}) = A\mathbf{x}$, for an $m \times n$ matrix A is one-to-one. Additionally, what can you say about m and n?

(3) Using ideas from all across Chapter 1 (span, linear systems, homogeneous systems, linear (in)dependence, linear transformations, etc.), come up with as many equivalent statements as possible (at least 3-4) to: the linear transformation $T(\mathbf{x}) = A\mathbf{x}$, for an $m \times n$ matrix A is onto. Additionally, what can you say about m and n?

(4) (Taken from Exam 1 Spring 2020) Consider the linear system

$$\begin{array}{ccccccc} x_1 & + & x_2 & + & x_3 & = & k \\ 2x_1 & + & hx_2 & + & 2x_3 & = & 3. \end{array}$$

(a) Write the augmented matrix of the linear system and reduce it to row echelon form.

(b) Find all values of h and k (if there are any) so that the linear system has (i) no solutions; (ii) only one solution; (iii) infinitely many solutions.