Monday, March 13

Saturday, March 11, 2023 15:26

Student Hours :

11:00 AM -12:00 noon on Mondays
2:00-3:00 PM on Thursdays
1:30-2:30 PM on Fridays in the Math Department Lounge
+ just ask if you want to talk at other times

No student hours this Friday as I'll be on my way to Amsterdam!

Note challenge problems on HW 6.

Group Work on Wednesday

Next week we will read parts of Chapter 8 on Marsden-Hoffman, which is on Canvas.

Eric Grinberg Talk Thursday, 3:00-4:00 Anderson 206!

Colloquium

Why Reprove It?
(or Theorems Which Keep on Giving)

Eric Grinberg
Professor of Mathematics
U Mass Boston

(note unusual day, time, and place)

Day: THURSDAY, March 16, 2023
Time: 3:00 PM—4:00 PM
Place: Anderson 206
Reception Following the Lecture: Joyce Cummings Center 501

Abstract:

Some theorems are discovered, proved and, one hopes, remembered, invoked and cited. Others are proved and reproved, again and again.
The latter type is getting increasing attention. We'll explore this phenomenon through 2+1 theorems in geometry and linear algebra, and ponder reasons to reprove.

The talk will include audience participation. Bring a sheet of paper and prepare to draw a large triangle and an angle or two (protractor optional). Save the backside for row reducing a matrix, or aim your device to SageMath. This talk is suitable for the full spectrum from undergrads to graduate students.

Thm 18.16 Let $\mathbb{I}$ be a gen rect, $f: \mathbb{I} \to \mathbb{R}$ cont. Then f is integrable.

Pf $f: \mathbb{I} \to \mathbb{R}$ cont. $\therefore$ f is uniformly cont.

Let $\varepsilon > 0$ Let $\delta > 0$ st.

if $\bar{x}$ and $\bar{y}$ are in $\mathbb{I}$ and $\|\bar{x} - \bar{y}\| < \delta$,

then $|f(\bar{x}) - f(\bar{y})| < \frac{\varepsilon}{\mathrm{vol}(\mathbb{I})}$

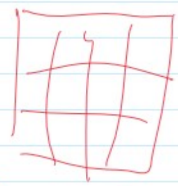
[Def ... $\mathbb{R}^n$ $A \neq \emptyset$

then $|f(\bar{x}) - f(\bar{y})| < \frac{\varepsilon}{\mathrm{vol}(\mathbb{I})}$

Def if A is bndd subset of $\mathbb{R}^n$ $A \neq \phi$
then define $\mathrm{diam}(A) = \sup_{\substack{\bar{x} \in A \\ \bar{y} \in A}} \|x - y\|$

if $\mathbb{P}$ is a part. of gen rect $\mathbb{I}$
then $\mathrm{gap}\ \mathbb{P} = \max_{J \text{ in } \mathbb{P}} \mathrm{diam}\, J$

$\mathbb{R}^2$



$\mathbb{R}$

Let $\mathbb{P}$ be any part of $\mathbb{I}$ with $\mathrm{gap}(\mathbb{P}) < \delta$
ie $\forall J$ in $\mathbb{P}$ $\bar{x}, \bar{y}$ in J $\|x - y\| < \delta$.

To calculate $U(f, \mathbb{P}) - L(f, \mathbb{P}) < \varepsilon$ (show this)

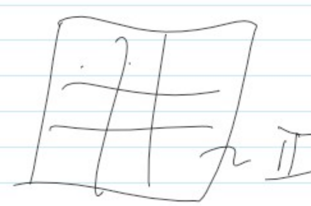
$$\sum_{J \text{ in } \mathbb{P}} \left(M(f, J) - m(f, J) \right) \mathrm{vol}(J).$$

f is cont on J. J seq cpt so
by EVT $\exists$ x^J_m x^J_M in J
st. $f(x^J_m) = m(f, J)$ $f(x^J_M) = M(f, J)$

$$\sum_{J \text{ in } \mathbb{P}} \left(f(x^J_M) - f(x^J_m) \right) \mathrm{vol}(J)$$

$$< \sum_{J \text{ in } \mathbb{P}} \frac{\varepsilon}{\mathrm{vol}\,\mathbb{I}} \mathrm{vol}\,J = \frac{\varepsilon}{\mathrm{vol}\,\mathbb{I}} \sum_{J \text{ in } \mathbb{P}} \mathrm{vol}\,J$$

$$= \frac{\varepsilon}{\mathrm{vol}\,\mathbb{I}} \mathrm{vol}\,\mathbb{I} = \varepsilon$$



$\therefore$ f satisfies Riem cond

Prop 18.17 let $\mathbb{I}$ be gen rect $f : \mathbb{I} \to \mathbb{R}$ bndd
If f is cont on $\mathrm{int}(\mathbb{I})$ then

If f is cont on int$(\overline{I})$ then f is int ← interior of $\overline{I}$

pf is partly CW rest this similar to pf in $\mathbb{R}$

Jordan Content Zero.

idea; characterize small sets in $\mathbb{R}^n$

MML $\mathbb{R}$

is $\mathbb{R}$ small?

in $\mathbb{R}^2$

cover this with small rect

Cover S with rects of small vol.

arbitrary

Defn let S be bdd set

S has Jordan content 0 JC0.

if $\forall \varepsilon > 0$ $\exists$ finite # of gen rects

$$J_1, J_2, \ldots J_n$$

st ① $S \subset J_1 \cup \cdots \cup J_M$ rect cover S

② $\sum_{k=1}^{M} \mathrm{vol}(J_k) < \varepsilon$ sum of vols small

eg $\overline{x}_0 \in \mathbb{R}^n$ is $\{\overline{x}_0\} = S$ of JC0

yup!

$\overline{x}_0 = (x_1, x_2, \ldots x_n)$

let $\varepsilon > 0$ cover S with finit # rects let $\varepsilon > 0$ cover S with finit # rects of arb small vol.

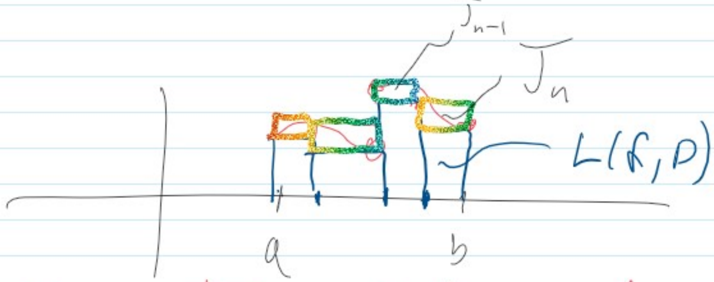
J_ε = rect containing $\bar{x}_0$ $\mathrm{vol}(J_\varepsilon) < \varepsilon$

side len $\left(\frac{\varepsilon}{2}\right)^{\frac{1}{n}}$

$J_\varepsilon = [x_1, x_1 + (\frac{\varepsilon}{2})^{\frac{1}{n}}] \times [x_2, x_2 + (\frac{\varepsilon}{2})^{\frac{1}{n}}] \times \cdots \times [x_n, x_n + (\frac{\varepsilon}{2})^{\frac{1}{n}}]$

$\bar{x}_0 \in J_\varepsilon$ $\mathrm{vol}\, J_\varepsilon = \left(\left(\frac{\varepsilon}{2}\right)^{\frac{1}{n}}\right)^n = \frac{\varepsilon}{2} < \varepsilon$

eg $f : [a,b] \to \mathbb{R}$ int.
is graph of f a JCO set in $\mathbb{R}^2$?



let $\varepsilon > 0$ let P be part of $[a,b]$
st $U(f,P) - L(f,P) < \varepsilon$

the rects J_j $[x_{j-1}, x_j] \times [\inf_{[x_{j-1},x_j]} f, \sup_{[x_{j-1},x_j]} f]$

cover graph of f

$\sum \text{area } J_j = U(f,P) - L(f,P) < \varepsilon$

Note Cantor middle thirds set has JCO.

Thm 18.20 Jordan integrability thm

let $\bar{I}$ be gen rect $f : \bar{I} \to \mathbb{R}$ bndd

Let $D(f, \bar{I}) = \{\bar{x} \in \bar{I} \mid f \text{ is discontinuous at } \bar{x}\}$

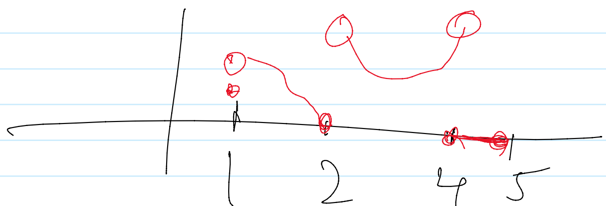
Let I be gen rect $f : I \to \mathbb{R}$ bdd

Let $D(f, I) = \{\bar{x} \in I \mid f$ is discontinuous at $\bar{x}\}$.

If $D(f,I)$ is JCO then f is integrable on I

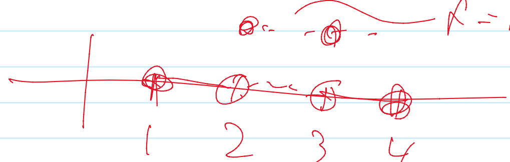
Eg Can we use Jordan int Thm to sh these fns are integrable

(a)



$D(f,[1,5]) = \{1,2,4\}$ has JCO

(b)



$f = 1$ if $x \in [2,3] \cap \mathbb{Q}$, 0 o.w.

$D(f,[1,4]) = [2,3]$

Soon you'll be able to prove: non trivial intervals don't have JCO.

Thm Let S_1, S_2 have JCO $S_1 \cup S_2$ is JCO

by induction finite union of JCO sets are JCO.

Thm If S is JCO $T \subset S$ then T is JCO (as any cover of S with small total vol also covers T)

Eg $f : [0,1]^2 \to \mathbb{R}$ $D(f, [0,1]^2) =$

then f is int, i.e. $D(f,I)$ is graph of int fn.

Pf of Jordan decomp Thm

$I = [a_1,b_1] \times \cdots \times [a_n,b_n]$ gen rect

$I = [a_1, b_1] \times \cdots [a_n, b_n]$ gen rect

$f : I \to \mathbb{R}$ bndd

$S = D(f, I)$ has JC 0.

let $\varepsilon > 0$. $M > 0$ st $\forall x \in I$ $-M \le f(x) \le M$

goal: find a part of I P^* st

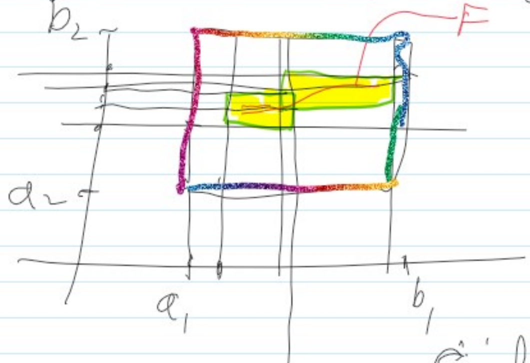
$U(f, P^*) - L(f, P^*) < \varepsilon$ then by Riem cond f will be int.

As S has JC 0 $\exists$ gen rect

$\tilde{J}_1, \tilde{J}_2, \ldots \tilde{J}_p$ st

$S \subset \tilde{J}_1 \cup \cdots \cup \tilde{J}_p$ and $\sum \text{vol}(\tilde{J}_p) < \frac{\varepsilon}{4M}$

WLOG assume $\tilde{J}_1 \cdots \tilde{J}_p$ are cont in I



make a part. of I using vertices of the $\tilde{J}_i$

let $F = \tilde{J}_1 \cup \cdots \cup \tilde{J}_p$

call this part.

$P = (P_1, P_2, \ldots P_n)$

each rect. in P is either in F

or disjoint from int F.

let $J_1', \ldots J_e'$ be the rect in P

that are contd in F

$\therefore \sum_{i=1}^{e} \text{vol}(J_i') < \frac{\varepsilon}{4M}$

let $J_1, J_2 \cdots J_m$ be the rects

in P that are disjoint from F