MATH 42 HOMEWORK 10

This homework is due at 11:59 p.m. (Eastern Time) on Friday, December 4. Scan the completed homework and upload it **as one pdf file** to Gradescope. The Canvas module "Written Assignments" has instructions for how to upload the assignment to Gradescope. This assignment covers §17.4−17.6.

Be sure to show work (integration by parts, substitutions, etc.) when calculating integrals. Unless stated in the problem, it is insufficient to simply respond with a numerical evaluation of definite integrals or an antiderivative of a non-standard integrand.

(1) Let S be a surface parameterized by $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle, (u,v) \in D$ for some domain D in the two variables u, v.

  (a) If $f(x,y,z)$ is a (scalar) function, express $\iint_S f(x,y,z)\,dS$ as an integral in u,v. What does the dS mean here? How do you compute it?

  (b) If $\vec{F}$, is a vector field, write $\iint_S \vec{F}\cdot\vec{n}\,dS$ as an integral in u,v. What is $\vec{n}$? Is the dS the same as before? How do we compute it in terms of u,v?

(2) Can you parametrize the following surfaces? Can you give more than one answer to each?

  (a) The surface $z = x + y$ for $1 \le x \le 2, 1 \le y \le 3$

  (b) The portion of the paraboloid $z = x^2 + y^2$ that lies over the unit disk.

  (c) The portion of the paraboloid $z = x^2 + y^2$ that lies over the annulus centered at the origin with radius ranging from 1 to 2.

  (d) A cone with vertex at the origin, opening up around the z-axis up to $z = 4$, with angle between the z-axis and the cone $\pi/6$.

(3) Evaluate the surface integral $\iint_S \vec{F}\cdot\vec{n}\,dS$ where $\vec{F} = 3x^2\vec{i} - 2xy\vec{j} + 8\vec{k}$ and S is the graph of the function $f(x,y) = 2x - y$ for $0 \le x \le 2$ and $0 \le y \le 2$, oriented so that the normal vectors point upward (i.e., in the positive z direction).

(4) Compute $\iint_S \vec{F}\cdot\vec{n}\,dS$ where $\vec{F} = \langle y, -x, z\rangle$ and the surface S is given by the parametrization $\vec{r}(u,v) = \langle 2u, 2v, 5 - u^2 - v^2\rangle$, with $u^2 + v^2 \le 1$. (Note that the parametrization allows us to orient the surface S, since the unit normals to S are proportional to the cross products $\vec{r}_u \times \vec{r}_v$.) Describe the direction of the net flux of $\vec{F}$ across the surface.

(5) Find the average value of the temperature function $T(x,y,z) = 80 - 24z^2$ on the surface S defined by the cone: $z^2 = x^2 + y^2$ for $0 \le z \le 3$. The average temperature is found by adding up (integrating) all of the "temperature" and dividing by the surface area.

(6) Compute the flux $\iint_S \vec{F}\cdot\vec{n}\,dS$ over the hemisphere S given by $x^2+y^2+z^2 = 9$, $x^2+z^2 \le 9$, and $y \ge 0$, where $\vec{F} = x\vec{i} + y^4\vec{j} + z\vec{k}$. Here S is oriented so that the normal vectors point rightward (i.e., in the positive y direction). Before evaluation, make an educated guess as to whether or not this integral will be positive, zero, or negative, based on the vector field and the geometry of the region.