## Tufts University Department of Mathematics Groupwork #7: Due on $11/3^1$

Fall, 2023

## ${\bf Problem}$

Let 
$$f_n(x) = ae^{-nax} - be^{-bnx}$$
 for  $x > 0$  and  $0 < a < b$ .

- (a) Prove that  $\sum_{n=1}^{\infty} \int_{0}^{\infty} |f_n(x)| dx = \infty$ .
- (b) Prove that  $\sum_{n=1}^{\infty} \int_{0}^{\infty} f_n(x) dx = 0$ .
- (c) Prove that  $\sum_{n=1}^{\infty} f_n(x) \in L^1([0,\infty))$ , and  $\int_0^{\infty} \sum_{n=1}^{\infty} f_n(x) dx = \ln(b/a)$ .

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