MATH 70
Problem Session Worksheet 2
2/18/2021

1 Suppose C is an $n \times n$ matrix and there is some $\mathbf{b} \in \mathbb{R}^n$ such that $C\mathbf{x} = \mathbf{b}$ has infinitely many solutions. Do the columns of C span all of $\mathbb{R}^n$? Justify your answer.

2

(a) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^n$ and let $A = \begin{bmatrix} \mathbf{a_1} & \mathbf{a_2} & \dots & \mathbf{a_n} \end{bmatrix}$ be an $m \times n$ matrix. Using the definition of matrix-vector multiplication, prove that

$$A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}.$$

(b) Suppose $\mathbf{u}$ and $\mathbf{v}$ are both solutions to the matrix-vector equation $A\mathbf{x} = \mathbf{0}$. Show that $\mathbf{u} + \mathbf{v}$ is also a solution.

3 Let

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

and let $b = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

(a) Check that $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is a solution to $A\mathbf{x} = \mathbf{b}$.

(b) Check that $\mathbf{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ is also a solution to $A\mathbf{x} = \mathbf{b}$.

(c) Is $\mathbf{u} + \mathbf{v}$ a solution to $A\mathbf{x} = \mathbf{b}$?

With the least amount of work possible, decide which of the following sets of vectors are linearly independent, and give a reason for each answer.

(a)

$$\left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 6 \\ 4 \end{bmatrix} \right\}$$

(b) The columns of

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 7 & 8 & 0 \\ 9 & 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 & 8 \end{bmatrix}$$

(c)

$$\left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

(d)

$$\left\{ \begin{bmatrix} 8 \\ 2 \\ 1 \\ 4 \end{bmatrix} \right\}$$