QUIZ 8, OCTOBER 27

Question. (a) *Consider the assignment* $f_1 : \mathbb{Z} \to \mathbb{Z}_4$ *given by* $f_1(z) = [z]_4$. *Is this a well defined function? Why or why not?*

$f_1(z) = [z]_4$

each $z \in \mathbb{Z}$ can only be defined in one equivalence class, meaning each $f_1(z)$ is mapped to only one equivalence class, making it well defined.

(b) *Consider the assignment* $f_2 : \mathbb{Z}_4 \to \mathbb{Z}$ *given by* $f_2([z]_4) = z$. *Is this a well defined function? Why or why not?*

$f_2([5]$ not well defined, as $[z]_4$ can map the same z to multiple places as the equivalence class can repeat.

Using ex.) from class, $[2]_4$ from $a-b=4k$, would contain all integers with remainder two, which is infinitely many

(c) Consider the assignment $f_3 : \mathbb{Z}_8 \to \mathbb{Z}_4$ given by $f_3([z]_8) = [z]_4$. Is this a well defined function? Why or why not?

$f_3([z]_8) = [z]_4$

No $[2]_8$ has $[2]_8$ and $[2]_4$, and $[2_4]$ only has

$[2]_4 \mathbb{Z}_8 \to \mathbb{Z}_4$

$[2]_8 \Rightarrow [2]_4$

$[z_1]_8 = [z_2]_8 \Rightarrow [z_1]_4 = [z_2]_4$

$\exists k \in \mathbb{Z}\ z_1 - z_2 = 8k \to \exists k\ z_1 - z_2 = 4k$

$z_1 - z_1 - z_2 = 4(2k)$ so $2k = k$

Therefore here, you get

$[z_1]_8 = [z_2]_8 \Rightarrow [z_1]_4 = [z_2]_4$

and f_3 is a well defined function.

(d) Consider the assignment $f_4 : \mathbb{Z}_4 \to \mathbb{Z}_8$ given by $f_4([z]_4) = [z]_8$. Is this a well defined function? Why or why not?

$f_4([1]_4) = [1]_8 = [9]_8 = [17]_8 \ldots [8k+1]_8$ $k \in \mathbb{N}$

No meaning that f_4 is not well defined as an input has multiple outputs.