Homework 5

Graded

Student

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Total Points

19 / 20 pts

Question 1

Maxmin

4 / 5 pts

– **0 pts** Correct

✔ – **0.5 pts** (a) Boundary not accounted for correctly

– **0.5 pts** (a) Interior not accounted for correctly

– **0.5 pts** (b) Interior not accounted for correctly (grad(f) = 0)

– **0.5 pts** (b) Edges not accounted for correctly

– **0.5 pts** (b) Corners not checked

✔ – **0.5 pts** (c) Boundary not accounted for correctly/sufficiently

– **0.5 pts** (c) Interior not accounted for correctly (grad(f)= 0)

– **1 pt** Region or surface misinterpreted

– **0.5 pts** Algebraic error

– **1 pt** Multiple algebraic errors

– **0.5 pts** Conceptual issues present

– **1 pt** Serious conceptual issues present

(a) The border points given do not constitute the entire boundary.

(c) similar to (a) -- see solutions

Question 2

Lagrange Multipliers 1

2 / 2 pts

✔ – **0 pts** Completion

– **2 pts** No attempt

Question 3

Langrange Multipliers 2

5 / 5 pts

✔ **− 0 pts** Correct

− 1 pt No case analysis

− 1 pt Calculation errors in case 1

− 1 pt Calculation errors in case 2

− 1.5 pts No work shown in case 1

− 1.5 pts No work shown in case 2

− 1 pt Extremal values for f not given

− 1.5 pts In correct extremal value(s)

Question 4

Lagrange Multipliers 3

5 / 5 pts

✔ **− 0 pts** Correct.

− 0.5 pts Incorrect function for area of inscribed rectangle.

− 0.75 pts Incorrect derivatives for gradient of F.

− 0.5 pts Incorrect value for Lagrange multiplier in part b.

− 0.5 pts Incorrect or missing answer for largest area.

− 0.25 pts Incorrectly drawn figure/intersection of ellipse and axes unlabeled/wrong. Or missing figure.

− 0.5 pts Did not show that Lagrange multiplier is 2ab in part b.

− 0.5 pts Incorrect, unclear, or tautological interpretation of Lagrange multiplier in final part.

− 1.5 pts Incorrect solving of points yielding maximum area.

− 0.5 pts Area cannot be negative.

Question 5

Integrals

3 / 3 pts

✔ **− 0 pts** Complete, on time

− 1 pt No attempt

− 2 pts late

− 3 pts No attempt, late

Question assigned to the following page: 1

Math 42 HW 5

1 a) $= f(x,y) = x^2 - y^2 + 1$

$f_x = 2x \quad 0 = 2x$
$f_y = -2y \quad 0 = -2y$] at $(0,0)$

$f(0,0) = 1$

Border points of domain are $(1,0)$, $(-1,0)$, $(0,1)$, and $(0,-1)$

$f(1,0) = 2 \quad f(0,1) = 0$

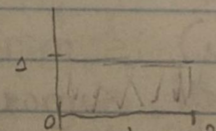
$f(-1,0) = 2 \quad f(0,-1) = 0$

f is minimum of 0 at $(0,1)$ and $(0,-1)$. f is maximum at $(1,0)$ and $(-1,0)$ and the value is 2.

1 b) $f(x,y) = x^2 + 2y^2 - 3xy$

$f_x = 2x - 3y = 0$
$f_y = 4y - 3x = 0$] at $(0,0)$

$R = \{(x,y) : 0 \leq x \leq 2, 0 \leq y \leq 1\}$



so, have to evaluate $f(x,y)$ on border lines $x=0$, $x=2$, $y=0$, and $y=1$

When $x=0$, $f(0,y) = 2y^2$, $f'(0,y) = 4y$, $f(0,y)$ has a minimum at $y=0$, as $f'(0,y)$ switches from negative to positive. $f(0,0) = 0$

If $x=2$, $f(2,y) = 2y^2 - 6y + 4$, $f'(2,y) = 4y - 6$, $f(2,y)$ has a minimum at $y = \frac{3}{2}$. However, $y = 3/2$ outside domain

If $y=0$, $f(x,0) = 2x^2$, $f'(x,0) = 4x$. $f(x,0)$ has a minimum at $x=0$. $f(0,0) = 0$

If $y=1$, $f(x,1) = x^2 - 3x + 2$, $f'(x,1) = 2x - 3$, $f(x,1)$ has a relative min at $x = 3/2$. $f(\frac{3}{2}, 1) = -\frac{1}{4}$

Question assigned to the following page: 1

1 b) Must additionally evaluate corners

$f(0,0)=0$ $f(0,1)=2$

$f(2,0)=4$ $f(2,1)=0$

$f(x,y)$ has a minimum of $-\frac{1}{4}$ at $(\frac{3}{2}, \frac{3}{2})$ and a maximum of 4 at $(2,0)$ over the given interval

1 c) $f(x,y)=x^2+y^2-2y+1$

$f_x=2x=0$
$f_y=2y-2=0$ at $(0,1)$

Border points that need evaluation are $(0,4)$, $(0,-4)$, $(4,0)$ and $(-4,0)$.

$f(0,1)=0$ $f(0,-4)=25$ $f(-4,0)=17$

$f(0,4)=9$ $f(4,0)=17$

$f(x,y)$ is minimum at $(0,1)$ and has value of 0.
$f(x,y)$ is maximum at $(0,-4)$ and has value of 25

Question assigned to the following page: 2

2 $V = 500 = xyz$,
$A = xy + 2xz + 2yz$

So, minimizing $A(x,y,z)$ w/ constraint $g(x,y,z) = 500 - xyz = 0$

$F(x,y,z,\lambda) = xy + 2xz + 2yz - \lambda(500 - xyz)$

$\nabla F(x,y,z,\lambda) = \langle y + 2z + \lambda yz,\ x + 2z + \lambda xz,\ 2x + 2y + \lambda xy, -g \rangle$

$y + 2z + \lambda yz = 0$ $\qquad \lambda = \frac{y+2z}{-yz} = -\frac{1}{2} - \frac{2}{y}$

$x + 2z + \lambda xz = 0$ $\qquad \lambda = \frac{x+2z}{-xz} = -\frac{1}{2} + \frac{-2}{x}$

$2x + 2y + \lambda xy = 0$ $\qquad \lambda = \frac{2x+2y}{-xy} = -\frac{2}{y} - \frac{2}{x}$

$xyz - 500 = 0$

$-\frac{1}{2} - \frac{2}{y} = -\frac{1}{2} - \frac{2}{x}$ $\qquad -\frac{1}{2} - \frac{2}{x} = -\frac{2}{y} - \frac{2}{x}$

$\frac{2}{y} = \frac{2}{x}$, $y = x$ $\qquad -\frac{1}{2} = -\frac{2}{y}$, $y = 2z$

$xyz = 500$, $y \cdot y \cdot \frac{y}{2} = 500$, $\sqrt[3]{y^3} = 1000$, $y = 10$, meaning, $x = 10$ and $z = 5$.

The dimensions of the box are $10 \times 10 \times 5$.
This method is different than the exam as it doesn't explicitly rely on us solving for the minimums of the function.

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Question assigned to the following page: 3

3 $f(x,y,z) = x^2+y^2+z^2$

$G(x,y,z) = x^2+y^2+z^2-4xy-1=0$

$\nabla f(x,y,z) = \langle 2x, 2y, 2z \rangle$

$\nabla G(x,y,z) = \langle 2x-4y, 2y-4x, 2z \rangle$

$\nabla f = \lambda \nabla g$

$2x = \lambda(2x-4y)$

$2y = \lambda(2y-4x)$

$2z = 2\lambda z$, this leads to 2 scenarios where either $\lambda = 1$ or $z=0$

Case 1: $\lambda = 1$

If $\lambda = 1$, $2x = 2x-4y$, $0=-4y$, meaning $y=0$

$2y = 2y-4x$, $0=-4x$, meaning $x=0$.

$G(0,0,z) = z^2-1=0 \quad z=\pm 1$

$f(0,0,1) = 1, \quad f(0,0,-1) = 1$

Case 2: $z=0$

if $z=0$, $\dfrac{2x}{2x-4y} = \lambda = \dfrac{2y}{2y-4x}$

$$\frac{x}{x-2y} = \frac{y}{y-2x}$$

$$x(y-2x) = y(x-2y)$$

$$xy-2x^2 = xy-2y^2$$

$$\pm x = y$$

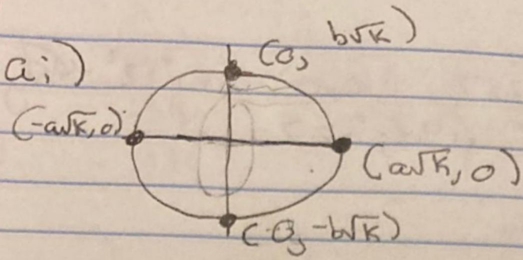
$G(x,x,0) = -2x^2-1=0$, no real solutions.

$G(x,-x,0) = 2x^2+4x^2-1=0$, $6x^2=1$, $x=\pm\frac{1}{\sqrt{6}}$. Whether x is positive or negative does not matter as x and y are both squared in f.

$f(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, 0) = \frac{1}{3}$

Question assigned to the following page: 4

4 ai)



aii) if $g(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - k = 0$ the point where the rectangle and the ellipse meet is $(x, \sqrt{b^2(k - \frac{x^2}{a^2})})$, the area of $f(x,y)$ is $4xy$

aiii)

$$\nabla F(x,y,\lambda) = \langle 4y - \frac{2\lambda x}{a^2}, 4x - \frac{2\lambda y}{b^2}, -g \rangle = \vec{0}$$

$$x\left(4y - \frac{2\lambda x}{a^2} = 0\right)$$
$$+\, y\left(4x - \frac{2\lambda y}{b^2} = 0\right)$$

$$8xy - 2\lambda\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 0 \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = k, \text{ from } g(x,y)$$

$$8xy = 2\lambda k$$
$$4xy = \lambda k$$

$$4y - 2\left(\frac{4xy}{k}\right)\left(\frac{x}{a^2}\right) = 0$$
$$4y = \frac{8x^2 y}{ka^2}$$
$$\frac{ka^2}{2} = x^2, \quad x = a\sqrt{\frac{k}{2}}$$

$$4x - 2\left(\frac{4xy}{k}\right)\left(\frac{y}{b^2}\right) = 0, \quad 1 = \frac{8y^2}{kb^2}, \quad y = b\sqrt{\frac{k}{2}}, \quad x = a\sqrt{\frac{k}{2}}, \quad \lambda = \frac{4xy}{k}$$

iv) $A = 4xy$, $A = 4\left(a\sqrt{\frac{k}{8}}\right)\left(b\sqrt{\frac{k}{8}}\right)$, $A = \frac{4abk}{8}$

$$A = \frac{2abk}{2}$$

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Question assigned to the following page: 3

Using $G(x,y,z)$, we can rewrite $f(x,y,z)$ as $1+4xy=x^2+y^2+z^2$. There are infinitely many solutions to this equation as there are no restrictions on x, y, or z beyond the constraint curve. ~~As~~ a result there is no max of $f(x,y,z)$ on the given constraint.

$\boxed{\text{The minimum of } f(x,y,z) \text{ is } \frac{1}{3} \text{ and } f(x,y,z) \text{ has no max on the given constraint curve.}}$

Questions assigned to the following page: 4 and 5

4 b) $H(K) = 2abK$, $\lambda = 2ab$

$\frac{dH}{dK} = 2ab = \lambda$, so $\frac{dH}{dK} = \lambda$

4 c) The general relation $\frac{dH(K)}{dK} = \lambda$ shows the Lagrange multiplier measures the rate of change of max area w/ respect to K.

5 a) $\int_0^1\int_0^1 x^3+y^2+2xy\,dy\,dx = \int_0^1 x^3y + \frac{y^3}{3} + xy^2 \Big|_0^1 dx = \int_0^1 x^3 + \frac{1}{3} + x\,dx$

$\int_0^1 x^3 + x + \frac{1}{3}\,dx = \frac{x^4}{4} + \frac{x^2}{2} + \frac{x}{3}\Big|_0^1 = \frac{1}{4} + \frac{1}{2} + \frac{1}{3} = \frac{13}{12}$

b) $\int_1^{\ln 3}\int_0^{\ln 2} e^{x+2y}\,dx\,dy = \int_1^{\ln 3} e^{x+2y}\Big|_0^{\ln 2} dy = \int_1^{\ln 3} e^{\ln 2+2y} - e^{2y}\,dy$

$\int_1^{\ln 3} 2e^{2y} - e^{2y}\,dy = e^{2y} - \frac{e^{2y}}{2}\Big|_1^{\ln 3} = \left(e^{\ln 9} - \frac{e^{\ln 9}}{2}\right) - \left(e^2 - \frac{e^2}{2}\right)$

$9 - \frac{9}{2} - \left(\frac{e^2}{2}\right)$

$\frac{9-e^2}{2}$

c) $\int_{-1}^1\int_0^1 x^{10} - 2x^5y^5 + y^{10}\,dx\,dy = \int_{-1}^1 \frac{x^{11}}{11} - \frac{2y^5x^6}{6} + xy^{10}\Big|_0^1 dy$

$\int_{-1}^1 \frac{1}{11} - \frac{2y^5}{6} + y^{10}\,dy = \frac{y}{11} - \frac{2y^6}{36} + \frac{y^{11}}{11}\Big|_{-1}^1 = \frac{4}{11}$

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Question assigned to the following page: 5

5 d) $\int_0^1\int_0^{\frac{\pi}{2}} x^5\cos(x^3y)\,dy\,dx = \int_0^1 \frac{x^5\sin(x^3y)}{x^3}\Big|_0^{\frac{\pi}{2}} dx$

$\int_0^1 x^2\sin(x^3y)\Big|_0^{\frac{\pi}{2}} dx = \int_0^1 x^2\sin\left(\frac{\pi x^3}{2}\right)dx = \cos(x^3)\Big|_0^1$

$= -\frac{1}{3}\cos(1) + \frac{1}{3}$

$\int_0^1 x^2\sin\left(\frac{\pi x^3}{2}\right)dx \qquad t = \frac{\pi x^3}{2}$

$dt = \frac{3x^2\pi}{2}$

$\frac{2}{3\pi}\int_0^{\frac{\pi}{2}} \sin t\,dt = \frac{2}{3\pi}$