Math 70 Wrksheet 5

1. By the definition of invertibility, and assuming $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$, by the associativity law of inverse matrix then:

$$ABC(C^{-1}B^{-1}A^{-1}) = C^{-1}B^{-1}A^{-1}\cdot(AB) = I_n$$

We can rearrange the second equation as $C^{-1}B^{-1}(A^{-1}A)BC = I_n$, $A^{-1}A = I_n$, making the equation $C^{-1}B^{-1}BC = I_n$, repeating this process w/ $B^{-1}B$ $C^{-1}(B^{-1}B)C = I_n$, $C^{-1}C = I_n$, $I_n = I_n$, which is true, meaning that $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$ QED

2. By Theorem 3 of Chapter 3, when one row of $A_{n\times n}$ is multiplied by a constant k to create $B_{n\times n}$, $\det B = k \det A$

If $A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & & a_{nn} \end{bmatrix}$, then $cA = \begin{bmatrix} ca_{11} & \cdots & ca_{1n} \\ \vdots & & \vdots \\ ca_{n1} & \cdots & ca_{nn} \end{bmatrix}$

We are multiplying n rows by c, meaning by theorem 3, $\det(cA) = \underbrace{c\cdot c\cdot c\cdots c\cdots}_{n\text{ times}}\det(A) = c^n\det(A)$ QED

To prove $\det(A^{-1}) = (\det(A))^{-1} = \frac{1}{\det(A)}$, first, we know from theorem 6 that $\det(AB) = \det A\cdot\det B$. We also know that $A\cdot A^{-1} = I_n$ where A are of size $n\times n$.

$$\det(A\cdot A^{-1}) = \det(I_n) = \det(A)\cdot\det(A^{-1})$$

$\det(I_n)$, $I_n = [\vec{e}_1\ \vec{e}_2 \ldots \vec{e}_n]$. The determinant of a diagonal matrix is the product of it's diagonal entries. I_n is a diagonal matrix where all diagonal entries equal 1. Therefore, $\det(I_n) = 1$, meaning that $1 = \det(A)\cdot\det(A^{-1})$, $\frac{1}{\det(A)} = \det(A^{-1})$ QED

3 For W to be a subspace of $\mathbb{P}_3$, 4 conditions must be fulfilled, let f and g be functions in W.

1.) W is a subset of $\mathbb{P}_3$, which is true

$p(0)$

2.) W must contain the zero vector of $\mathbb{P}_3$,
$P(0) = 0 = P(1)$, meaning that W contains the zero vector

3.) W must be closed under addition meaning that $(f+g) \in W$
- $(f+g)(x) = f(x) + g(x)$

$(f+g)(0) = (f+g)(1)$, $f(0)+g(0) = f(1)+g(1)$, which is true, meaning W is closed under addition

4.) W must be closed under scalar multiplication, meaning that $cf(x) \in W$.
- $cf(0) = cf(1)$, which is still true

Thus W is a subset of $\mathbb{P}_3$

QED

4 If $W = \{A \in M_{2\times2} \mid \det(A) = 0\}$, then $A = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ or $A = \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix}$. One column must be $\vec{0}$.

To see if W is a subspace of $M_{2\times2}$, let's check the 4 conditions.

1) W is a subset of $M_{2\times2}$

2) The $\vec{0}$ of $M_{2\times2}$ is in W, as A must have a column equivalent to $\vec{0}$

3) To prove closed under addition, let $B \in W$

4 3) To prove ~~closed~~ closed under addition, let $B \in W$

$B = \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}$, $A = \begin{bmatrix} 0 & c \\ 0 & d \end{bmatrix}$, or more specifically

where a, or b $\neq 0$, c or d $\neq 0$

$B = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $A+B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$\det(A+B) = 1 - 2 = -1 \neq 0$, meaning W is not closed under addition and W is not a subspace of $M_{2\times2}$

5 a) For $c \in \mathbb{R}$, $c\vec{0} = c(\vec{0} + \vec{0})$, as $\vec{0} + \vec{0} = \vec{0}$

$c\vec{0} = c\vec{0} + c\vec{0}$, $\vec{0} = c\vec{0}$ QED

$-c\vec{0} \quad -c\vec{0}$

b) If $c\vec{v} = \vec{0}$, then $c(\vec{v} + \vec{0}) = \vec{0}$, as $\vec{v} + \vec{0} = \vec{v}$

By distributive property, $c\vec{v} + c\vec{0} = \vec{0}$, $c\vec{v} = -c\vec{0}$,

$-c\vec{0} \quad -c\vec{0}$

C's cancel out, as $c \neq 0$, $\vec{v} = -\vec{0}$,

$-\vec{0} = \vec{0}$ by definition of zero vector, so $\vec{v} = \vec{0}$. QED

c) $\vec{u} = \vec{u} + \vec{0}$, since $\vec{x} + \vec{v} = \vec{0}$, we can substitute as $\vec{u} = \vec{u} + (\vec{x} + \vec{v})$, $\vec{u} = (\vec{u} + \vec{x}) + \vec{v}$, since $\vec{x} + \vec{u} = \vec{0}$, by commutative property, $\vec{u} + \vec{x} = \vec{0}$,

$\vec{u} = \vec{0} + \vec{v}$, $\vec{u} = \vec{v}$, Meaning the additive inverse of $\vec{x}$ is unique

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