Homework 10

Graded

Student

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Total Points

18.25 / 20 pts

Question 1

Surface integral notation

1 / 1 pt

✔ **– 0 pts** Problem reasonably attempted

– 1 pt Problem unanswered

Question 2

Parametrizing surfaces

2 / 2 pts

✔ **– 0 pts** Problem reasonably attempted

– 1 pt Poor attempt

– 2 pts Problem unanswered

Question 3

Evaluating a surface integral

3.5 / 5 pts

– 0 pts Correct

✔ **– 1.5 pts** Wrong normal vector

– 1 pt Normal vector pointed in the wrong direction

– 1.5 pts Calculation error in evaluation of integral

– 1 pt Lost a negative sign

– 5 pts No meaningful attempt

the answer comes out right because you have a fortunate number of negative signs swapped

Question 4

Evaluating a surface integral given a parametrization

5 / 5 pts

✔ **− 0 pts** Correct.

− 2 pts Integration mistake.

− 0.5 pts Simple algebra mistake.

− 0.5 pts Missing description of direction of net flux.

− 3 pts Insufficient amount of work before answer or partial attempt.

− 5 pts Nearly no work or no attempt.

− 0.75 pts Incorrect sign on net flux and incorrect description.

− 2 pts Problem with setting up surface integral.

− 0.5 pts Small mistake.

Question 5

Average temperature on a surface

2 / 2 pts

✔ **− 0 pts** Problem reasonably attempted

− 2 pts Click here to replace this description.

Question 6

Computing flux across a surface

4.75 / 5 pts

− 0 pts Correct

− 1 pt Incorrect parametrization

− 0.25 pts Minor sign/algebraic/notation error

− 0.5 pts No guess given for whether or not integral will be positive, negative, or zero.

− 0.5 pts Algebraic error

− 0.5 pts Incorrect spherical bounds

− 2.5 pts Serious conceptual errors

− 1 pt Conceptual error

− 0.5 pts Incorrect orientation on surface

− 0.25 pts The educated guess is not a guess for the integral but an interpretation of the result

Calc 3 HW 10

1 a) $\iint_S f(x,y,z)\,dS = \iint_R f(x(u,v), y(u,v), z(u,v)) |r_u \times r_v| \, du\,dv$

R is the region of integration and $|r_u \times r_v|$ represents the magnitude of the cross product between $\frac{dr}{du}$ and $\frac{dr}{dv}$. dS represents the area element surface. To calculate it, it is $|r_u \times r_v| \, dA$

1 b) $\vec{n}$ is the normal vector to S.

$\iint_S \vec{F} \cdot \vec{n} \, dS = \iint_R \vec{F} \cdot (r_u \times r_v) \, du\,dv \quad dS = |r_u \times r_v| \, dA$

dS still represents the same thing, it is there cancell with $\hat{n} \cdot dS$ $(r_u \times r_v)\,dA$

2 a) $\vec{f}(u,v) = \langle u, v, u+v \rangle \quad 1 \le u \le 2, \; 1 \le v \le 3$

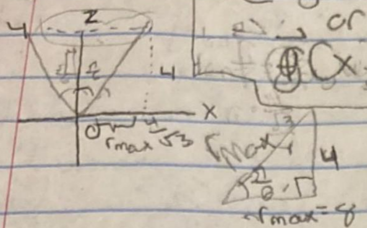
b) $\vec{f}(u,v) = \langle u, v, u^2+v^2 \rangle \quad -1 \le u^2 + v^2 \le 1$

$\vec{f}(u,v) = \langle \sqrt{v}\cos u, \sqrt{v}\sin u, v \rangle \quad 0 \le u \le 2\pi \quad 0 \le v \le 1$

c) $\vec{f}(u,v) = \langle v\cos u, v\sin u, v^2 \rangle \quad 0 \le u \le 2\pi \quad 1 \le v \le \sqrt{2}$

$\vec{f}(u,v) = \langle u, v, u^2+v^2 \rangle \quad 1 \le u^2+v^2 \le 4$

2 d)



$\vec{f}(u,v) = \langle \frac{\sqrt{3}}{3} v \cos u, \frac{\sqrt{3}}{3} v \sin u, v \rangle$ $\leftarrow$ $0 \leq u \leq 2\pi$, $0 \leq v \leq 4$

or

$\vec{g}(x,y) = \langle \frac{x}{2} \cos y, \frac{x}{2} \sin y, \frac{\sqrt{3}}{2} x \rangle$ $\leftarrow$ $0 \leq x \leq \frac{8}{\sqrt{3}}$, $0 \leq y \leq 2\pi$

3 $\iint_S \vec{F} \cdot \vec{n} \, dS = \iint_R -f z_x - g z_y + h \, dA \qquad \langle f, g, h \rangle = \vec{F}$

$z = 2x - y$

$= \iint_R -3x^2(2) + 2xy(-1) + 8 \, dx\, dy$

$= \int_0^2 \int_0^2 -6x^2 - 2xy + 8 \, dx\, dy$

$= \int_0^2 -2x^3 - x^2 y + 8x \Big|_0^2 dy$

$= \int_0^2 -16 - 4y + 16 \, dy = \int_0^2 -4y \, dy = -2y^2 \Big|_0^2$

$= \boxed{-8}$

Question assigned to the following page: 4

4 $\iint_S \vec{F}\cdot\vec{n}\,dS = \iint_R F\cdot(\vec{r}_u \times \vec{r}_v)\,dA$

$\vec{r}_u = \langle 2, 0, -2u \rangle$

$\vec{r}_v = \langle 0, 2, -2v \rangle$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -2u \\ 0 & 2 & -2v \end{vmatrix} = \left\langle \begin{vmatrix} 0 & -2u \\ 2 & -2v \end{vmatrix}, -\begin{vmatrix} 2 & -2u \\ 0 & -2v \end{vmatrix}, \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \right\rangle$$

$$= \langle 4u, 4v, 4 \rangle$$

$\vec{F} = \langle y, -x, z \rangle = \langle 2v, -2u, 5-u^2-v^2 \rangle$

$$\iint_D \langle 2v, -2u, 5-u^2-v^2 \rangle \cdot \langle 4u, 4v, 4 \rangle \,du\,dv \quad D: u^2+v^2 \leq 1$$

$$= \iint_D 8uv - 8uv + 20 - 4u^2 - 4v^2 \,du\,dv$$

$$= \iint_D 20 - 4(u^2+v^2)\,du\,dv$$ convert to polar, $u = r\cos\theta$, $v = r\sin\theta$

$$= \int_0^{2\pi}\int_0^1 20r - 4r^3\,dr\,d\theta$$

$$= 2\pi(10r^2 - r^4)\Big|_0^1$$

$$= 2\pi(10-1) = \boxed{18\pi}$$, and since flux is positive, it is pointing outwards, as $\vec{n}$ is outw

Question assigned to the following page: 5

5 $A_T = \dfrac{\iint_S T(x,y,z)\,dS}{\iint_S dS}$

$z^2 = x^2+y^2$

$z = r$ $\quad 0 \le u \le 2\pi,\ 0 \le v \le 3$

$\vec{r}(u,v) = \langle v\cos u, v\sin u, v \rangle$

$\vec{r}_u = \langle -v\sin u, v\cos u, 0 \rangle$

$\vec{r}_v = \langle \cos u, \sin u, 1 \rangle$

$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -v\sin u & v\cos u & 0 \\ \cos u & \sin u & 1 \end{vmatrix} = \left\langle \begin{vmatrix} v\cos u & 0 \\ \sin u & 1 \end{vmatrix}, -\begin{vmatrix} -v\sin u & 0 \\ \cos u & 1 \end{vmatrix}, \begin{vmatrix} -v\sin u & v\cos u \\ \cos u & \sin u \end{vmatrix} \right\rangle$

$= \langle v\cos u, v\sin u, -v \rangle$

$|\vec{r}_u \times \vec{r}_v| = \sqrt{v^2\cos^2 u + v^2\sin^2 u + v^2}$

$= v\sqrt{2} = |\vec{r}_u \times \vec{r}_v|$

$A_T = \dfrac{\int_0^{2\pi}\int_0^3 (80-24v^2)(v\sqrt{2})\,dv\,du}{\int_0^{2\pi}\int_0^3 v\sqrt{2}\,dv\,du}$

$A_T = \dfrac{2\pi\int_0^3 80\sqrt{2}\,v - 24\sqrt{2}\,v^3\,dv}{2\pi\int_0^3 v\sqrt{2}\,dv}$

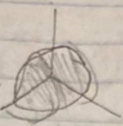
$A_T = \dfrac{40\sqrt{2}\,v^2 - 6\sqrt{2}\,v^4 \Big|_0^3}{\frac{\sqrt{2}}{2}v^2 \Big|_0^3}$

$A_T = \dfrac{360\sqrt{2} - 486\sqrt{2}}{\frac{9\sqrt{2}}{2}}$

$\boxed{A_T = -28}$

Question assigned to the following page: 6

6 $\iint_S \vec{F}\cdot\vec{n}\,dS =$ $z = \pm\sqrt{9-x^2-y^2}$



$= \vec{n} = \langle -z_x, -z_y, 1\rangle = \langle \frac{x}{z}, \frac{y}{z}, 1\rangle$

$\iint_{S_1} \vec{F}\cdot\vec{n}\,dS + \iint_{S_2} \vec{F}\cdot\vec{n}\,dS = \iiint_E \operatorname{div}\vec{F}\,dV$

$S_1: \langle 3\cos t, 0, 3\sin t\rangle$

$S_2: x^2+y^2+z^2 = 9$ for $y \geq 0$

$\iint_{S_1} \langle 3\cos t, 0, 3\sin t\rangle \cdot \langle 0, 1, 0\rangle + \iint_{S_1} \vec{F}\cdot\vec{n}\,dS = \iiint_E \operatorname{div} F\,dV$

$= \cancel{\iint_{S_1} 0\,dS} + \iint_{S_1} \vec{F}\cdot\vec{n}\,dS = \iiint_E \operatorname{div} F\,dV$

$0 \leq u \leq \pi$

$0 \leq v \leq 2\pi$

$= \iint_{S_1} \vec{F}\cdot\vec{n}\,dS = \iiint_E \operatorname{div} F\,dV$

$\iiint_E \operatorname{div} F\,dV = \iiint_E 1 + 4y^3 + 1\,dV = \iiint_E 2 + 4y^3\,dV$

$\iiint_E 2+4y^3\,dV = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-z^2}} 2+4y^3\,dy\,dz\,dx$

$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 2\sqrt{9-x^2-z^2} + (9-x^2-z^2)^2\,dz\,dx$

Question assigned to the following page: 6

If $z=y$

6) $$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 2\sqrt{9-x^2-y^2} + (9-x^2-y^2)^2 \, dz\, dx$$

$$= \int_{0}^{2\pi}\int_{0}^{3} 2r\sqrt{9-r^2} + r(9-r^2)^2 \, dr\, d\theta$$

$$= 2\pi\left(-\frac{2}{3}(9-r^2)^{3/2} - \frac{(9-r^2)^3}{6}\right)\Big|_0^3$$

$$= 2\pi\left(-(-18 - \frac{729}{6})\right) = 279\pi$$

The Flux is 279π. An educated guess would be that the flux is positive because the flux is in the direction of the normal vector.