Homework 2

● Graded

Student

Scott A. Fullenbaum

Total Points

20 / 20 pts

Question 1

Vector Operations

5 / 5 pts

✔ **+ 1 pt** A Correct

✔ **+ 1 pt** B Correct

✔ **+ 1 pt** C Correct

✔ **+ 1 pt** D Correct

✔ **+ 1 pt** E Correct

Question 2

Curve Sketching

2.5 / 2.5 pts

✔ **– 0 pts** Completion

– 2.5 pts Not completed

Question 3

Velocity & Acceleration of Curves

2.5 / 2.5 pts

✔ **– 0 pts** Completion

– 2.5 pts No Completion

Question 4

Integration

5 / 5 pts

- ✔ **– 0 pts** Correct.

– 0.75 pts Evaluate the integrals and show the general result before calculating the antiderivative at the limits of integration.

– 0.5 pts Incorrect solution for part (a).

– 0.5 pts Incorrect solution for part (b).

– 0.5 pts Incorrect solution for part (c).

– 0 pts Incorrect solution for part (d) but the mistake originates from the typo on the first version of assignment 2.

– 0.5 pts Missing constants of integration in part (c).

– 0.5 pts Incorrect integration of tan(t) in part (c).

– 0.5 pts The x-component of the integral in part (d) is incorrect.

– 0.25 pts The constants of integration in part (c) is a vector as all the constants could be different.

– 2.5 pts Final answers should be vectors; each integration over a component remains independent.

– 0.75 pts Keep vector notation via angle brackets or unit vectors throughout the calculations.

– 0.5 pts Incorrect solution for part (d).

Question 5

Finding position vectors

5 / 5 pts

- ✔ **+ 1 pt** Correct velocity vector
- ✔ **+ 1 pt** Correct initial condition for velocity
- ✔ **+ 1 pt** Correct position vector
- ✔ **+ 1 pt** Correct initial condition for position
- ✔ **+ 1 pt** Sufficient work shown (show trig integrals!)

+ 0.5 pts Sufficient work shown but small error(s) present

Question assigned to the following page: 1

Scott Fullasbaum Math 42 HW2

1a) $(\vec{c}\cdot\vec{b})\cdot(\vec{a}+\vec{b}+\vec{c})=$

$10(\vec{a}+\vec{b}+\vec{c})=10\langle 9,4,4\rangle=\langle 90,40,40\rangle$

b) $(\vec{b}\times\vec{c})\cdot\vec{b}$

$\vec{b}\times\vec{c}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\2&2&2\\4&0&1\end{vmatrix}=\begin{vmatrix}2&2\\0&1\end{vmatrix}\vec{i}-\begin{vmatrix}2&2\\4&1\end{vmatrix}\vec{j}+\begin{vmatrix}2&2\\4&0\end{vmatrix}\vec{k}$

$=\langle 2,6,-8\rangle.$

$\langle 2,6,-8\rangle\cdot\langle 2,2,2\rangle=0$

c) $\vec{a}+(\vec{b}\times\vec{a})$

$\vec{b}\times\vec{a}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\2&2&2\\3&2&1\end{vmatrix}=\begin{vmatrix}2&2\\2&1\end{vmatrix}\vec{i}-\begin{vmatrix}2&2\\3&1\end{vmatrix}\vec{j}+\begin{vmatrix}2&2\\3&2\end{vmatrix}\vec{k}$

$=\langle -2,4,-2\rangle$

$\langle -2,4,-2\rangle+\langle 3,2,1\rangle=\langle 1,6,-1\rangle$

d) $\vec{b}\cdot(\vec{a}\times\vec{c})$

$\vec{a}\times\vec{c}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\3&2&1\\4&0&1\end{vmatrix}=\begin{vmatrix}2&1\\0&1\end{vmatrix}\vec{i}-\begin{vmatrix}3&1\\4&1\end{vmatrix}\vec{j}+\begin{vmatrix}3&2\\4&0\end{vmatrix}\vec{k}$

$=\langle 2,1,-8\rangle$

$\langle 2,1,-8\rangle\cdot\langle 2,2,2\rangle=-10$

e) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{b})$

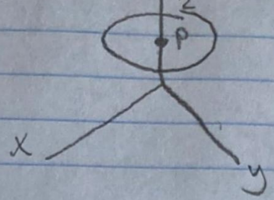
From part c, $\vec{b}\times\vec{a}=\langle -2,4,-2\rangle$ so, $\vec{a}\times\vec{b}=\langle 2,-4,2\rangle$

From part b, $\vec{b}\times\vec{c}=\langle 2,6,-8\rangle$, so $\vec{c}\times\vec{b}=\langle -2,-6,8\rangle$

$\langle 2,-4,2\rangle\cdot\langle -2,-6,8\rangle=-4+24+16=36$

Question assigned to the following page: 2

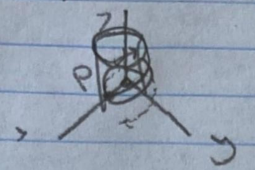
2 a) $\vec{r}(t) = \langle 2\cos(t), 2\sin(t), 5 \rangle$



$P = \langle 0, 0, 5 \rangle$

$\vec{r}(t)$ is a circle with radius 2 centered at $\langle 0, 0, 5 \rangle$

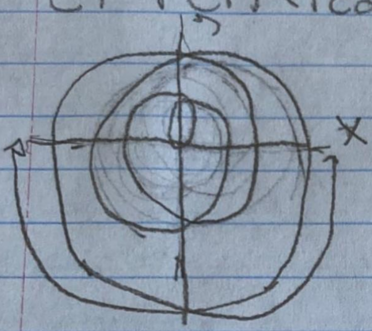
b) $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$



$P = \langle 1, 0, 0 \rangle$

$\vec{r}(t)$ is a cylinder, with ~~radius~~ constant difference from the center of 1

c) $\vec{r}(t) = \langle t\cos t, t\sin(t) \rangle$

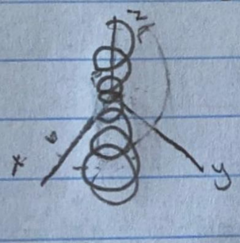


$\frac{y}{x} = \tan t \quad x^2 + y^2 = t^2 \quad \sqrt{x^2+y^2} = t$

$y = x\tan t$

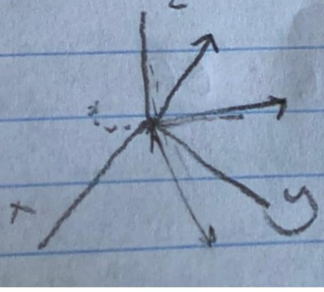
$\vec{r}(t)$ is a parameterization of the equation $y = x\tan\sqrt{x^2+y^2}$. It is also a spiral in 2D

d) $\vec{r}(t) = \langle t\cos t, t\sin t, t \rangle$



$\vec{r}(t)$ is a spiral with radius increasing further away you get from the origin.

e) $\vec{r}(t) = \langle 3t^2, 4t^2, 5t^2 \rangle$



$\vec{r}(t)$ is a line that is increasing in the x, y, and z planes

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3 a) $\vec{r}(t) = \langle \frac{2}{t}, \frac{3}{t+1} \rangle$ $\quad \vec{r}'(t) = \vec{v}(t)$

$\vec{v}(t) = \langle -\frac{2}{t^2}, -\frac{3}{(t+1)^2} \rangle$ $\quad \vec{v}'(t) = \vec{a}(t)$

$\vec{a}(t) = \langle \frac{4}{t^3}, \frac{6}{(t+1)^3} \rangle$

$\vec{v}(2) = \langle -\frac{1}{2}, -\frac{1}{3} \rangle$

$\vec{a}(2) = \langle \frac{1}{2}, \frac{2}{9} \rangle$

speed at $t=2$ is $\sqrt{\frac{1}{4} + \frac{1}{9}} = \sqrt{\frac{13}{36}} = \frac{\sqrt{13}}{6}$

b) $\vec{r}(t) = \langle e^{2t}, e^{-t} \rangle$

$\vec{v}(t) = \langle 2e^{2t}, -e^{-t} \rangle$

$\vec{a}(t) = \langle 4e^{2t}, e^{-t} \rangle$

$\vec{v}(4) = \langle 2e^{8}, -e^{-4} \rangle$

$\vec{a}(4) = \langle 4e^{8}, e^{-4} \rangle$

speed at $t=4$ is $\sqrt{4e^{16} + e^{-8}} = e^{-4}\sqrt{4e^{24}+1}$

c) $\vec{r}(t) = \langle e^t \cos t, e^t \sin t, e^t \rangle$

$\vec{v}(t) = \langle e^t \cos t - e^t \sin t, e^t \sin t + e^t \cos t, e^t \rangle$

$\vec{a}(t) = \langle e^t \cos t - e^t \sin t - e^t \sin t - e^t \cos t, e^t \sin t + e^t \cos t + e^t \cos t - e^t \sin t, e^t \rangle$

$\vec{a}(t) = \langle -2e^t \sin t, 2e^t \cos t, e^t \rangle$

$\vec{v}(\frac{\pi}{2}) = \langle -e^{\frac{\pi}{2}}, e^{\frac{\pi}{2}}, e^{\frac{\pi}{2}} \rangle$

$\vec{a}(\frac{\pi}{2}) = \langle -2e^{\frac{\pi}{2}}, 0, e^{\frac{\pi}{2}} \rangle$

speed at $t = \frac{\pi}{2}$ is $\sqrt{e^{\pi} + e^{\pi} + e^{\pi}} = \sqrt{3e^{\pi}}$

d) $\vec{r}(t) = \langle 1+t, 2t, 2+3t \rangle$

$\vec{v}(t) = \langle 1, 2, 3 \rangle$

$\vec{a}(t) = \langle 0, 0, 0 \rangle$

$\vec{v}(2) = \langle 1, 2, 3 \rangle$

$\vec{a}(2) = \langle 0, 0, 0 \rangle$

speed at $t=2$ is $\sqrt{14}$

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3 e) $\vec{r}(t) = \langle 2t, e^{-t^2} \rangle$

$\vec{v}(t) = \langle 2, -2te^{-t^2} \rangle$

$\vec{a}(t) = \langle 0, -2e^{-t^2} + 4t^2e^{-t^2} \rangle$

$\vec{v}(1) = \langle 2, -2e^{-1} \rangle$

$\vec{a}(1) = \langle 0, \frac{2}{e} \rangle$

Speed at $t = 1$ is $\sqrt{4 + \frac{4}{e^2}}$

4 a)

$$\int_0^2 6t^2 i - 4tj + 3k\,dt = \langle 2t^3, -2t^2, 3t \rangle \Big|_0^2$$
$$= \langle 16, -8, 6 \rangle - \langle 0, 0, 0 \rangle$$
$$= \langle 16, -8, 6 \rangle$$

b)
$$\int_{-1}^1 -5t\,i + 8t^3 j - 3t^2 k\,dt = \langle -\frac{5t^2}{2}, 2t^4, -t^3 \rangle \Big|_{-1}^1$$
$$= \langle -\frac{5}{2}, 2, -1 \rangle - \langle -\frac{5}{2}, 2, 1 \rangle$$
$$= \langle 0, 0, -2 \rangle$$

c)
$$\int \sin(t)\vec{i} + \cos(t)j + \tan(t)k\,dt$$
$$= \langle -\cos t, \sin t, \ln|\sec t| \rangle + C$$

d)
$$\int_0^1 te^{t^2}\vec{i} + \sqrt{t}\,\vec{j} + \frac{1}{1+t^2}\vec{k}\,dt = \langle \frac{1}{2}e^{t^2}, \frac{2}{3}t^{3/2}, \tan^{-1}(t) \rangle \Big|_0^1$$
$$= \langle \frac{e}{2}, \frac{2}{3}, \frac{\pi}{4} \rangle - \langle \frac{1}{2}, 0, 0 \rangle$$
$$= \langle \frac{e-1}{2}, \frac{2}{3}, \frac{\pi}{4} \rangle$$

5 $\vec{a}(t) = \langle e^t, t^2+t, \sin t \cos t \rangle$

$\vec{v}(t) = \int \vec{a}(t)\, dt$

$\vec{v}(t) = \int e^t i + (t^2+t) j + \sin t \cos t\, k\, dt$

$\vec{v}(t) = \langle e^t + C_1, \frac{t^3}{3} + \frac{t^2}{2} + C_2, \frac{\sin^2 t}{2} + C_3 \rangle$

$\vec{v}(0) = \langle 1, 0, 2 \rangle$

$\langle 1, 0, 2 \rangle = \langle 1 + C_1, C_2, C_3 \rangle$ $C_1 = 0$

$C_2 = 0$

$C_3 = 2$

$\vec{v}(t) = \langle e^t, \frac{t^3}{3} + \frac{t^2}{2}, \frac{\sin^2 t}{2} + 2 \rangle$

$\vec{r}(t) = \int \vec{v}(t)\, dt$

$\vec{r}(t) = \int e^t i + (\frac{t^3}{3} + \frac{t^2}{2}) j + (\frac{\sin^2 t}{2} + 2) k\, dt$

$= \langle e^t + C_1, \frac{t^4}{12} + \frac{t^3}{6} + C_2, \int \frac{\sin^2 t}{2} + 2\, dt \rangle$

$\int \frac{\sin^2 t}{2} + 2\, dt = 2t + \int \frac{\sin^2 t}{2}\, dt$

$\int \frac{\sin^2 t}{2}\, dt = \int \frac{1 - \cos 2t}{4}\, dt = \frac{t}{4} - \frac{\sin 2t}{8} + 2t + C_3 = \frac{9t}{4} - \frac{\sin 2t}{8} + C_3$

$\vec{r}(t) = \langle e^t + C_1, \frac{t^4}{12} + \frac{t^3}{6} + C_2, \frac{9t}{4} - \frac{\sin 2t}{2} + C_3 \rangle$

$\vec{r}(0) = \langle 0, 0, 0 \rangle = \langle 1 + C_1, C_2, C_3 \rangle$ $C_1 = -1$

$C_2 = 0$

$C_3 = 0$

$\vec{r}(t) = \langle e^t - 1, \frac{t^4}{12} + \frac{t^3}{6}, \frac{9t}{4} - \frac{\sin(2t)}{8} \rangle$

$\vec{v}(t)$

$\vec{v}(t) = \langle e^t, \frac{t^3}{3} + \frac{t^2}{2}, \frac{\sin^2 t}{2} + 2 \rangle$

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