Math 165 HW 4

I worked w/ David on the HW

1 a) num of heads = X

X	P(X)
0	1/8
1	3/8
2	3/8
3	1/8

b) num of heads = X, want $|X-1|$ outcomes of $|X-1|$ are 0, 1, 2.

$\lvert X-1 \rvert$	$P(\lvert X-1 \rvert)$
0	3/8
1	4/8
2	1/8

2 a)

X_1 \ X_2	1	2	3	4	5	6
6	1/36					1/36
5						
4			1/36	1/36		
3			1/36	1/36		
2						
1	1/36					1/36

$P(X_1, X_2)$ for all $(X_1, X_2) = 1/36$

b)

Y_1 (min) \ Y_2 (max)	1	2	3	4	5	6
6	0	0	0	0	0	1/36
5	0	0	0	0	1/36	2/36
4	0	0	0	1/36	2/36	2/36
3	0	0	1/36	2/36	2/36	2/36
2	0	1/36	2/36	2/36	2/36	2/36
1	1/36	2/36	2/36	2/36	2/36	2/36

$Y_1 = \min(X_1, X_2)$
$Y_2 = \max(X_1, X_2)$

When $X_1 = X_2$ one outcome for min and max. For the other possible combos 2/36 as 2 outcomes

2

3

number of draws	2	3	4	5
P(X)	$1/7$	$2/7$	$8/35$	$12/35$

↑ Use geometric distribution.

$P(X=3) = \frac{6}{7} \cdot \frac{2}{6} = \frac{2}{7}$

$P(X=4) = \frac{6}{7} \cdot \frac{4}{6} \cdot \frac{2}{5} = \frac{8}{35}$

$P(X=5) = \frac{6}{7} \cdot \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{4}{4} = \frac{12}{35}$

4 a) $g=4$: p^4 $\quad$ $g=6$: $\binom{5}{3} p^4 q^2$

$g=5$: $\binom{4}{3} p^4 q$ $\quad$ $g=7$: $\binom{6}{3} p^4 q^3$

To find coefficient, need 3 wins in $n-1$ games as last game is win

$F_g(n) = \binom{n-1}{3} p^4 q^{n-4}$, $4 \le n \le 7$

b) $P(A \text{ wins}) = \sum_{n=4}^{7} \binom{n-1}{3} p^4 q^{n-4}$

$= p^4 + \binom{4}{3} p^4 q + \binom{5}{3} p^4 q^2 + \binom{6}{3} p^4 q^3$

c) For $p = 2/3 \rightarrow \left(\frac{2}{3}\right)^4 \left(1 + \binom{4}{3} \cdot \frac{1}{3} + \binom{5}{3} \cdot \frac{1}{9} + \binom{6}{3}\left(\frac{1}{27}\right)\right)$

$= \boxed{0.827}$

d) $P(A \text{ wins}) \ge P(X \ge 4)$ as it is impossible for team A to win in under 4 games, and will take them at least 4. If $P(A \text{ wins}) = \sum_{n=4}^{7} \binom{n-1}{3} p^4 q^{n-4} = P(X \ge 4)$ as it means chance A will win in up to n games, w $4 \le n \le 7$, nor $P(X \ge 4)$, as 7 is a bound of the problem.

e)

G	1	2	3	4	5	6	7
P(G)	1	1	1	1	$1-p^4$	$1-\binom{4}{3}p^4 q$	$1-\binom{5}{3}p^4 q^2$

$-p^4$ Dependent on info Series ends in previous game

G is independent of series winner for $p = 0.5$

Math 165 HW 4

5a) $P(X \leq 3 \cap Y > 4) = 0.39$

b) $P(X \leq 3 \cup Y > 4) = P(X \leq 3) + P(Y > 4) - P(X \leq 3 \text{ and } Y > 4)$

$P(X \leq 3 \cup Y \geq 4) = (.09 + .28 + .52) + .41 - 0.39$

$= \boxed{0.91}$

c) $P(X = Y) = P((1,1)) + P((3,3))$

$= .07 + .03 = \boxed{0.1}$

d)

X	P(X)
1	0.52
2	0.28
3	0.09
4	0.11
5	

Y	P(Y)
1	0.23
3	0.36
5	0.41

e) $P(X \leq 3) = 1 - P(X=4) = 1 - 0.11 = \boxed{0.89}$

6

X \ Y	4	5	6	
0	0.2	0.24	0.16	0.6
1	$.08\bar{3}$	0.1	$.0\bar{6}$	0.25
2	0.05	.06	.04	0.15
	.3	0.4	$.2\bar{6}$	

$P(X \cap Y) = P(X)\,P(Y)$

$\frac{2}{5}$ $\quad P(X=5 \cap Y=1) = P(X=5)\,p(y=1)$

$0.1 = .4\,p(y=1)$

$.25 = p(y=1)$

$P(Y=6 \cap X=0) = P(Y=6)\,P(X=0)$

$.16 = .6\,P(X=6)$

$/ .6$

$P(X=6) = .2\bar{6}$

.1212
.1818
.0303
.3393

.0666
.0606
+.0100
.4372 =

7)

y \ x	0	1	2	3	
4	1/495	0	0	0	1/495
3	4/99	12/495	0	0	32/495
2	12/99	18/99	18/495	0	168/495
1	8/99	24/99	12/99	4/495	224/495
0	1/99	6/99	6/99	1/99	70/495
	126/495	252/495	108/495	9/495	1 → total P(x)

$\frac{126}{495}\ \frac{252}{495}\ \frac{108}{495}\ \frac{9}{495} = 1 \leftarrow$ total P(y)

if $X+Y>4$ $P(X+Y)=0$

To solve use hypergeometric distribution:

$$P = \frac{\binom{3}{x}\binom{4}{y}\binom{5}{4-x-y}}{\binom{12}{4}}$$ where $x+y\leq 4$, $x,y\geq 0$

Since my answers are in the form of repeated decimals, the addition doesn't add up perfectly, the $\sum p(x)=1$ and $\sum p(y)=1$

8 a) range of X is $[r, \infty)$ as must go at least r attempts, and no hardstop is set so could go on forever

b) In 1st $n-1$ attempts are $r-1$ successes and $(n-1)-(r-1) = n-r$ failures, and any $\binom{n-1}{r-1}$ combinations

$P(X=n) = \binom{n-1}{r-1} p^{r-1}(1-p)^{n-r} \cdot p$

$P(X=n) = \binom{n-1}{r-1} p^{r}(1-p)^{n-r}$

c) When $r=1$, $P(X=n) = \binom{n-1}{0} p(1-p)^{n-1}$

$\binom{n-1}{0} = 1$, $P(X=n) = p(1-p)^{n-1}$, which is the geometric formula