Tufts University
Department of Mathematics

Math 136 **Class work for the Week of January 30, 2023** [1] **Spring, 2023**

Due date: 11:59 pm, five days after we hand it out. So, if we hand it out on Wednesday, February 1, it will be due Monday, February 6, and if we hand it out on Monday, February 6, it will be due Saturday, February 11 on Gradescope. Please enter the names of all group members into Gradescope when submitting.

Every week we plan to work in small groups of at most four students to learn to write proofs and to solve problems. I will grade each group's work.

- *Scribe:* each week, someone in the group will volunteer to submit to Gradescope the group's answer along with group member names. This role should rotate through the group.
- *Respect:* when discussing problems, please make sure that everyone feels comfortable speaking and that all feedback is supportive and encouraging.
- Please keep your group name so you can rejoin the same group each week.

1. Consider the following definition.

Definition 1 *Let* $\mathcal{O} \subset \mathbb{R}^n$ *and let* $f : \mathcal{O} \to \mathbb{R}$ *Let* $\mathbf{x}_0 \in \mathcal{O}$*. Then,* f is differentiable at $\mathbf{x}_0$ *if there is a vector* $\mathbf{b} \in \mathbb{R}^n$ *such that*

$$\lim_{\mathbf{h}\to\bar{\mathbf{0}}} \frac{|f(\mathbf{x}_0+\mathbf{h}) - [f(\mathbf{x}_0) + \langle \mathbf{b}, \mathbf{h}\rangle]|}{\|\mathbf{h}\|} = 0 \tag{1}$$

.

You will show on homework that if f is differentiable at $\mathbf{x}_0$ then f has first order partial derivatives at $\mathbf{x}_0$ and $\mathbf{b} = \nabla f(\mathbf{x}_0)$. This shows that the vector $\mathbf{b}$ in (1) is unique. This vector is called the *derivative vector of* f *at* $\mathbf{x}_0$, and is denoted $Df(\mathbf{x}_0)$.

(a) Let $f(x,y) = 3 + 4x + 5y$ and let $(x_0, y_0) \in \mathbb{R}^2$. Use the information in this problem to guess a candidate for $\mathbf{b} = Df(\mathbf{x}_0)$ in the definition of derivative.

(b) Use the limit in (1) to prove that the function f from part (1a) is differentiable at (x_0, y_0) (i.e., plug in your vector $\mathbf{b}$ from (1a) in to (1) and take the limit).

(c) Are continuously differentiable functions (a.k.a. C^1 functions) differentiable? Why or why not?