Homework 1

● Graded

Student

Scott A. Fullenbaum

Total Points

19.5 / 20 pts

Question 1

(no title) **2.5** / 2.5 pts

✔ **– 0 pts** Completion

– 2.5 pts Problem not answered

– 2 pts Late assignment

Question 2

(no title) **5** / 5 pts

✔ **+ 2 pts** Completed the square for each term.

✔ **+ 1 pt** Re-wrote equation in the general form of squares of terms on one side and constants on the other side.

✔ **+ 1 pt** Identified that solutions either form sphere, single point, or do not exist.

✔ **+ 1 pt** Identified these classifications of solutions with the values of the constant a.

+ 0 pts Not answered

Question 3

(no title) **5** / 5 pts

+ 2.5 pts Found $x = proj_v(u)$

+ 2.5 pts Found corresponding y

✔ **+ 5 pts** Set up a system of equations using dot and/or cross products

+ 3 pts Some computational errors

+ 0 pts No attempt

Question 4

(no title) **2.5** / 2.5 pts

✔ **– 0 pts** Completed

Question 5

(no title)

4.5 / 5 pts

- **− 0 pts** Correct
- **− 2 pts** Incorrect cross product or determinant definition
- ✔ **− 0.5 pts** Absolute value misplaced/forgotten
- **− 2 pts** Incorrect formula/process
- **− 2 pts** Volume cannot be negative!
- **− 1 pt** Arithmetic error
- **− 1 pt** Legibility

Questions assigned to the following page: 1 and 2

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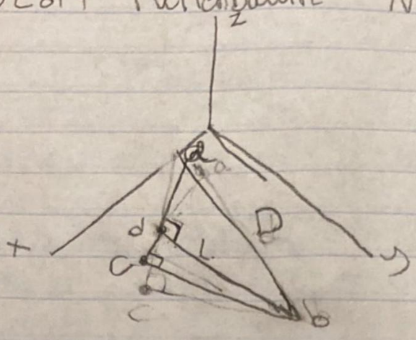
1)



a: (a_1, a_2, a_3)
b: (b_1, b_2, b_3)

c: (
d: (

$\overline{bc} = |a_2 - b_2|$
$\overline{ad} = |a_3 - b_3|$
$cd = |a_1 - b_1|$

$$L^2 = (\overline{cd})^2 + (\overline{bc})^2$$
$$L^2 = (a_1 - b_1)^2 + (a_2 - b_2)^2$$

$$D^2 = (\overline{ad})^2 + L^2$$
$$D^2 = (a_3 - b_3)^2 + (a_1 - b_1)^2 + (a_2 - b_2)^2$$

$$D = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

2 $x^2 + y^2 + z^2 - 4x + 8y - 10z = a$

$$x^2 - 4x + 4 + y^2 + 8y + 16 + z^2 - 10z + 25 = a + 4 + 16 + 25$$
$$(x-2)^2 + (y+4)^2 + (z-5)^2 = a + 45$$

if $a < -45$, no solution exists
if $a = -45$, 1 solution exists at $(2, -4, 5)$
if $a > -45$, the equation forms a sphere centered at $(2, -4, 5)$ with a radius of $\sqrt{a+45}$

Questions assigned to the following page: 3 and 4

3 $\vec{w} = \langle 3, 3, 4 \rangle$ $\vec{v} = \langle 2, 0, 1 \rangle$

$\vec{x} \parallel \vec{v}$, so $\frac{\vec{x}}{|\vec{x}|} = \frac{\vec{v}}{|\vec{v}|} = \langle \frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}} \rangle$

$\vec{x} = \langle \frac{2a}{\sqrt{5}}, 0, \frac{a}{\sqrt{5}} \rangle$ $y = \langle x_1, x_2, x_3$

since $\vec{y} \perp \vec{v}$, $\vec{y} \cdot \vec{v} = 0$, and if $\vec{y} = \langle y_1, y_2, y_3 \rangle$,
then $\vec{y} \cdot \vec{v} = 2y_1 + y_3 = 0 \rightarrow -2y_1 = y_3$

$\vec{y} = \langle y_1, y_2, -2y_1 \rangle$

$\vec{w} = \vec{x} + \vec{y}$, $\langle 3, 3, 4 \rangle = \langle \frac{2a}{\sqrt{5}} + y_1, y_2, \frac{a}{\sqrt{5}} - 2y_1 \rangle$

$y_2 = 3$

$\frac{2a}{\sqrt{5}} + y_1 = 3$ $4 = \frac{a}{\sqrt{5}} - 2y_1$

$y_1 = 3 - \frac{2a}{\sqrt{5}}$, $4 = \frac{a}{\sqrt{5}} - 2(3 - \frac{2a}{\sqrt{5}})$

$4 = \frac{5a}{\sqrt{5}} - 6$, $2\sqrt{5} = a$, $y_1 = -1$

$\vec{x} = \langle 4, 0, 2 \rangle$ $\vec{y} = \langle -1, 3, 2 \rangle$

4 a) $\vec{w} = \langle -1, 4 \rangle$ $\vec{v} = \langle -4, 2 \rangle$

$\text{proj}_{\vec{v}} \vec{w} = \frac{\vec{w} \cdot \vec{v}}{|\vec{v}|^2} \vec{v} = \frac{12}{20} \langle -4, 2 \rangle = \langle -\frac{12}{5}, \frac{6}{5} \rangle$

$\text{scal}_{\vec{v}} \vec{w} = \frac{\vec{w} \cdot \vec{v}}{|\vec{v}|} = \frac{12}{\sqrt{20}}$

$\text{proj}_{\vec{v}} \vec{w} = \langle -\frac{12}{5}, \frac{6}{5} \rangle$ $\text{scal}_{\vec{v}} \vec{w} = \frac{12}{\sqrt{20}}$

4 b) $\vec{w} = \langle 3, 3, -3 \rangle$ $\vec{v} = \langle 1, -1, 2 \rangle$

$\text{proj}_{\vec{v}} \vec{w} = \frac{\vec{w} \cdot \vec{v}}{|\vec{v}|^2} \vec{v} = \frac{-6}{6} \langle 1, -1, 2 \rangle = \langle -1, 1, -2 \rangle$

$\text{scal}_{\vec{v}} \vec{w} = \frac{\vec{w} \cdot \vec{v}}{|\vec{v}|} = -\frac{6}{\sqrt{6}}$

$\text{proj}_{\vec{v}} \vec{w} = \langle -1, 1, -2 \rangle$ $\text{scal}_{\vec{v}} \vec{w} = -\frac{6}{\sqrt{6}}$

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Questions assigned to the following page: 4 and 5

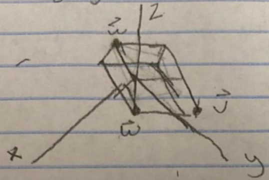
4 c) $\vec{w} = \langle 5, 1, -5 \rangle$ $\vec{v} = \langle -1, 1, -2 \rangle$

$\text{proj}_{\vec{v}}\vec{w} = \dfrac{\vec{w}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \dfrac{6}{6}\langle -1, 1, -2 \rangle = \langle -1, 1, -2 \rangle$

$\text{Scal}_{\vec{v}}\vec{w} = \dfrac{\vec{w}\cdot\vec{v}}{|\vec{v}|} = \dfrac{6}{\sqrt{6}}$

$\text{proj}_{\vec{v}}\vec{w} = \langle -1, 1, -2 \rangle$ $\text{Scal}_{\vec{v}}\vec{w} = \dfrac{6}{\sqrt{6}}$

5 $\vec{u} = \langle 1, 0, 3 \rangle$ $\vec{v} = \langle 0, 2, 1 \rangle$ $\vec{w} = \langle 1, 1, 1 \rangle$



$\text{Volume} = (\vec{v}\times\vec{w})\cdot\vec{u}$

$$\vec{v}\times\vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}\vec{i} - \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}\vec{j} + \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix}\vec{k}$$

$= \vec{i} + \vec{j} - 2\vec{k}$

$(\vec{v}\times\vec{w})\cdot\vec{u} = \langle 1, 1, -2 \rangle \cdot \langle 1, 0, 3 \rangle = 1 - 6 = |-5| = 5$

The volume of the parallelpiped is 5

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