ASSIGNMENTS !45

1. Part I

- From the book: 1.3, 1.4 2.3 and 2.5
- From the book: 2.8,3.1,3.6,3.9,4.1,4.2 Due Sept 18 at 11:59 pm (so you have time to ask questions in class)
- homework 3: 5.4, 5.5, 5.7, 5.8, 6.2, 6.3 Due Monday, Sept 25
- Homework 4: 6.7, 6.5, 6.6, 6.12, 7.5, 7.7 Due Monday October 2
- Homework 5: 7.8, 7.9 due Wednesday October 4
- Test: Friday October 6 Review sheet (not to turn in)

2. Part II

- Due Monday, October 16:
  Using Cayley's theorem, find a subgroup of S_5 that is isomorphic to $\mathbb{Z}_5$.
  From the book: 8.3,8.6,8.11
- Due Monday, October 23: Products and LaGrange's Theorem
  10.2 10.5 10.7, 11.2 11.9
- Due Monday, October 30: Partitions and Cauchy's theorem
  12.1, 12.8, 12.10 13.4, 13.7, 13.8
- Due Monday, November 6: Catch-up and Conjugacy: 1) 14.2, 2) 14.3, 3) 14. 4

  -If every element of a group has order 2, is the group abelian? Prove your answer.

  -Use the following facts to prove that R_p, (the multiplicative group on non-zero elements of Z_p) is cyclic:
  (1) Given a polynomial whose coefficients are elements of Z_n, the number of roots (each root considered mod n) is at most the degree of the polynomial (and in particular, $x^k - 1 = 0$ has at most k roots mod n)
  (2) Let G be a finite abelian group, and m be the least common multiple of the orders of elements of G. Then G contains an element of order m.

  1) you made take as given and 2) is on a previous homework
- Due Monday November 13: From the book: 15.2 15.3, 15.14, 15.15

  -A *homomorphism* from a group G to a group H is a function $\phi : G \to H$ such that $\phi(x)\phi(y) = \phi(xy)$ for all $x, y \in G$. (The left-hand multiplication in the H, the right in G.) Show that the kernel of a homomorphism $\phi : G \to H$ is a normal subgroup of G.

  -What are all the Quotient group obtained from $\mathbb{Z}$? That is, describe all the groups $\mathbb{Z}/K$, where K is a normal subgroup of $\mathbb{Z}$.

3. Part III

- Due *Tuesday* Nov 28 -one time thing because of last posting/thanksgiving/etc
  1) 17.2
  2) 17.4
  3) 17.9
  4) Give an example of groups G, H and a surjective homomorphism $\phi : G \to H$ such that G does not have a subgroup isomorphic to H. Prove your answer, and conclude that G is not isomorphic to $H \times K$ for any group K. Explain why this follows.
- Due *Thursday* Dec 7. This is your last real homework :(
  1) 18.1
  2) 18.5
  3) 20.1
  4) 20.4
  5) 20.11
  6) 21.2
  7) 21.6