very small homework #5

Graded

Student

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Total Points

7.5 / 10 pts

Question 1

7.8 — **5** / 5 pts

✔ **– 0 pts** Correct

– 1 pt Did not show that the function is a bijection

– 2 pts Did not show the function is an isomorphism

Question 2

7.9 — **2.5** / 5 pts

– 0 pts Correct

– 0.5 pts Stated functions $\mathbb{Z}_{12} \to \mathbb{Z}_{12}$ are not isomorphisms

✔ **– 0.5 pts** That $\phi(x)$ generates G requires more explanation

– 0.5 pts $\mathbb{Z}_{12}$ has more than one generator

– 0.5 pts $\mathbb{Z}$ has more than one generator

✔ **– 2 pts** That ϕ is completely determined by $\phi(x)$ requires more explanation

Invoking more general ideas of how isomorphisms interact with generators is (1) circular because that is exactly what we are asking you to explain and (2) slightly inaccurate because these tricks don't work for non-cyclic groups. Namely that although isomorphisms are determined by where they send generators, functions sending generators to generators are not always isomorphisms.

Question assigned to the following page: 1

7.8) Consider $(\mathbb{Z},+)$ and the subgroup set $(2\mathbb{Z},+)$ where $2\mathbb{Z}$ denotes the even integers

$2\mathbb{Z}$ is a subgroup of $\mathbb{Z}$, as $0 \in 2\mathbb{Z}$, for $a \in 2\mathbb{Z}$, $-a \in 2\mathbb{Z}$ and $a-a=0$, and $a,b \in 2\mathbb{Z}$ $\exists m,n$ s.t. $2m=a$, $2n=b$ so $a+b=2m+2n=2(m+n) \in 2\mathbb{Z}$

To show $2\mathbb{Z}$ is isomorphic to $\mathbb{Z}$ we define $\phi: \mathbb{Z} \to 2\mathbb{Z}$
$x \mapsto 2x$

ϕ is injective. Let $x_1, x_2 \in \mathbb{Z}$
$f(x_1)=f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

ϕ is surjective. Let $y \in 2\mathbb{Z}$, then by def. of even numbers $\exists k \in \mathbb{Z}$ s.t. $y = 2k$, meaning $\phi(k)=y$

So ϕ is bijective. To show, ~~presurph~~ ... let $x,y \in \mathbb{Z}$

$$\phi(x+y)=2(x+y)=2x+2y=\phi(x)+\phi(y)$$

Therefore, $(\mathbb{Z},+)$ is isomorphic to $(2\mathbb{Z},+)$ and this is proper subgroup isomorphic to group $\square$

Question assigned to the following page: 2

7.9)

So isomorphism preserves generators,
So as x is a generator of G,
then $\varphi(x)$ is a generator of G as well.

As G is cyclic and $\varphi(x)$ is generator,
$\varphi(x)$ is completely determined by $\varphi(x)$
as $\varphi: G \to G$.

So isomorphism $\phi: \mathbb{Z} \to \mathbb{Z}$ come from
generators which are $-1, 1$ so
$\phi(x) = x$ and $\phi(x) = -x$ are isomorphisms
from $\mathbb{Z}$ to itself.

Same logic to get isomorphisms $\phi: \mathbb{Z}_{12} \to \mathbb{Z}_{12}$
Generators of $\mathbb{Z}_{12}$ are $\{1, 5, 7, 11\}$ as are
coprime with 12.
So, that gives the ~~bis~~ isomorphisms:

$$\phi(x) = x \qquad \phi(x) = x+6$$
$$\phi(x) = x+4 \qquad \phi(x) = x+10$$

} maps 1 to other generators.

* I don't think we fully proved, but should follow
from idea that for a, b coprime $LCM(a,b) = ab$
so will get each $a \bmod b$ between 0 and $b-1$.
Also can just calc and see these are
generators.