Homework 8

Graded

Student

Scott A. Fullenbaum

Total Points

20 / 20 pts

Question 1

Sketching

5 / 5 pts

- ✔ **– 0 pts** Reasonably attempted

Question 2

Line Integrals

5 / 5 pts

- ✔ **– 0 pts** Correct

- **– 0.5 pts** (b) orientation of path is not consistent around the entire curve (e.g. $\langle t, 0\rangle$ and $\langle 0, t\rangle$ are both used, even though they have opposite orientation along the entire boundary of the unit square)
- **– 0.5 pts** Algebraic error
- **– 1 pt** Multiple algebraic errors
- **– 0.5 pts** Problem has been incorrectly rewritten: in (c) $\vec{F} = \langle 5z^2, 2x, x + 2y\rangle$ and $\langle t, t^2, t^2\rangle$.
- **– 0.5 pts** Incorrect use of inequalities, e.g.: $1 \leq t \leq 0$.
- **– 1 pt** Parametrization does not match given curve (not just in orientation but in topology of the curve)
- **– 1 pt** (a) Incorrectly set up line integral
- **– 1 pt** (c) Incorrectly set up line integral
- **– 0.1 pts** If using Green's Theorem, be sure to say it!
- **– 1 pt** Conceptual errors present
- **– 0.5 pts** $d\vec{r} = \vec{r}'(t)dt$
- **– 0.25 pts** Notational errors
- **– 1 pt** (b) Incorrectly set up line integral

Question 3

Conservative Vector Fields

5 / 5 pts

✔ **– 0 pts** All correct

– 1 pt A incorrect

– 1 pt B incorrect or no potential given

– 1 pt C incorrect or no potential given

– 1 pt D incorrect or no potential given

– 1 pt E incorrect or no potential given

– 1 pt Some moderate calculation errors

Question 4

Path dependence

5 / 5 pts

✔ **– 0 pts** Correct

– 0.5 pts Once, took the same derivative instead of the mixed derivative.

– 1.5 pts Multiple times: Took the same/wrong derivative instead of the mixed derivative.

– 5 pts No work.

– 0.5 pts No work explaining that the equations given are the derivatives of the functions.

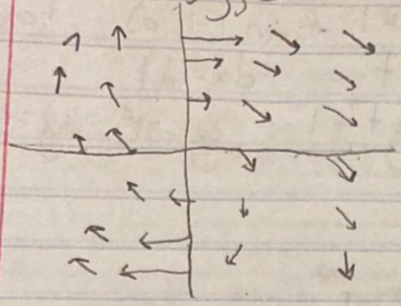
– 0.5 pts Algebra or differentiation mistake.

Question assigned to the following page: 1

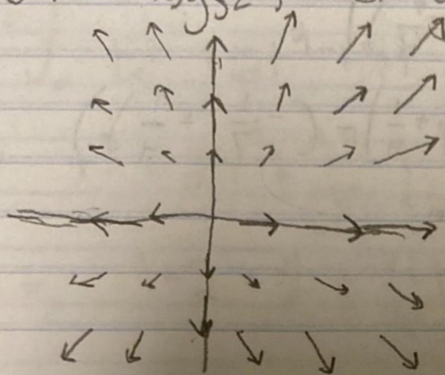
Math 42 HW 8

$y=0, \langle x, -x \rangle$

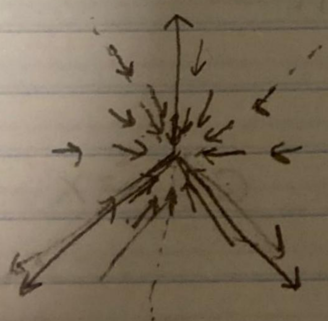
1 a) $F(x,y,z) = \langle x+y, -x \rangle$



1 b) $G(x,y,z) = (x^2+y^2)^{\frac{1}{2}} \langle x,y \rangle$



1 c) $H(x,y,z) = \langle -x,-y,-z \rangle$



All vectors point to the origin

2 a) $\int_C 6x^2y\,dx + xy\,dy$ $\quad C = x^3+1$ from $(-1,0)$ to $(1,2)$

$x = t \qquad dx = dt$

$y = t^3+1 \qquad \frac{dy}{dt} = 3t^2, \; dy = 3t^2dt$

$$= \int_{-1}^{1} 6t^2(t^3+1)dt + t(t^3+1)3t^2dt$$

$$= \int_{-1}^{1} 6t^5+6t^2\,dt + \int_{-1}^{1} 3t^6 + 3t^3\,dt$$

$$= t^6+2t^3\Big|_{-1}^{1} + 3\left(\frac{t^7}{7}+\frac{t^4}{4}\right)\Big|_{-1}^{1}$$

$$= (1+2)-(1-2)+3\left(\left(\frac{1}{7}+\frac{1}{4}\right)-\left(-\frac{1}{7}+\frac{1}{4}\right)\right)$$

$$= 4+3\left(\frac{2}{7}\right)$$

$$= 4+\frac{6}{7} = \frac{34}{7}$$

2 b) $\int_C \langle y, x^2-x\rangle \cdot dr$ where C is unit square in 1st quadrant

C is simply connected

$\oint_C \langle y, x^2-x\rangle \cdot d\vec{r} \qquad P = y \qquad Q = x^2-x$

by Green's theorem

$$= \oint_C (2x-1)-1\,dA = \oint_C 2x-2\,dA = 2\oint_C x-1\,dA$$

$$= 2\int_0^1\int_0^1 x-1\,dx\,dy = 2\int_0^1 \frac{x^2}{2}-x\Big|_0^1 dy = 2\int_0^1 -\frac{1}{2}dy = -1$$

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Question assigned to the following page: 2

2 c) $\int_C 5z^2\,dx + 2x\,dy + (x+2y)\,dz \qquad C = \langle t, t^2, t^2 \rangle \quad t \in [0,1]$

$x = t \qquad dx = dt$

$y = t^2 \qquad dy = 2t\,dt$

$z = t^2 \qquad z = 2t\,dt$

$= \int_0^1 5(t^2)^2\,dt + 2t(2t)\,dt + (t+2t^2)2t\,dt$

$= \int_0^1 5t^4 + 4t^2 + 2t^2 + 4t^3\,dt$

$= \int_0^1 5t^4 + 6t^2 + 4t^3\,dt$

$= t^5 + 2t^3 + \frac{t^4}{2} \Big|_0^1$

$= 1 + 2 + \frac{1}{2} = \frac{7}{2}$

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Question assigned to the following page: 3

P Q R

3 a) $F(x,y,z) = \langle y, -x, 0 \rangle$

$P = y, \; Q = -x, \; Z = 0$

$\frac{\partial P}{\partial y} = 1 \neq \frac{\partial Q}{\partial x} = -1$

$F(x,y,z)$ is not conservative

3 b) $G(x,y) = \langle -2y^3 \sin x, \; 6y^2 \cos x + 5 \rangle$

$\frac{\partial P}{\partial y} = -6y^2 \cos x = \frac{\partial Q}{\partial x} = -6y^2 \cos x$

$G = \int g_x \, dx = \int -2y^3 \sin x \, dx = 2y^3 \cos(x) + C(y)$

$g_y = 6y^2 \cos x + 5$

$\frac{\partial}{\partial y}\left(G = 2y^3 \cos x + C(y)\right) = g_y = 6y^2 \cos x + \frac{\partial}{\partial y} C(y)$

$\frac{\partial}{\partial y} C(y) = 5, \quad C(y) = 5y$

The potential function is $g(x,y) = 2y^3 \cos x + 5y + C$

3 c) $H(x,y,z) = \langle y^2 \cos x, \; 2y \sin x + e^{2z}, \; 2y e^{2z} \rangle$

$P(x,y,z) = y^2 \cos x$

$Q(x,y,z) = 2y \sin x + e^{2z}$

$R(x,y,z) = 2y e^{2z}$

$\frac{\partial P}{\partial y} = 2y \cos x = \frac{\partial Q}{\partial x}, \quad \frac{\partial P}{\partial z} = 0 = \frac{\partial R}{\partial x}, \quad \frac{\partial Q}{\partial z} = 2e^{2z} = \frac{\partial R}{\partial y}$

Question assigned to the following page: 3

3 c) $H(x,y,z)$ is conservative $H = \nabla h$

$H(x,y,z) = \int h_x\,dx = \int y^2\cos x\,dx = y^2\sin x + C(y,z)$

$h_y = 2y\sin x + e^{2z}$

$H_y(x,y,z) = 2y\sin x + \frac{dC}{dy} = 2y\sin x + e^{2z}$

$\frac{dC}{dy} = e^{2z}$

$\int dC = \int e^{2z}\,dy$

$C(y,z) = ye^{2z} + C(z)$

$H(x,y,z) = y^2\sin x + ye^{2z} + C(z)$

$H_z = 2ye^{2z} + \frac{dC}{dz} = 2ye^{2z}$ $\quad \frac{\partial C}{\partial z} = 0,\ C = c$

The potential function of $H(x,y,z)$ is $H(x,y,z) = y^2\sin x + ye^{2z} + C$, where c is a constant. H is conservative

3 d) $I(x,y) = \langle 5y^3 + 4y^3\sec^2 x,\ 15xy^2 + 12y^2\tan x \rangle$

$P(x,y) = 5y^3 + 4y^3\sec^2 x,\ Q(x,y) = 15xy^2 + 12y^2\tan x$

$\frac{\partial P}{\partial y} = 15y^2 + 12y^2\sec^2 x = \frac{\partial Q}{\partial x}$, $I(x,y)$ is conservative

$I = \int I_x\,dx = \int 5y^3 + 4y^3\sec^2 x\,dx = 5xy^3 + 4y^3\tan x + C(y)$

$I_y = 15xy^2 + 12y^2\tan x + \frac{dC}{dy} = 15xy^2 + 12y^2\tan x$

$\frac{dC}{dy} = 0,\ C(y) = c$

The potential function of I is $I(x,y) = 5xy^3 + 4y^3\tan x + c$, where c is a constant.

Question assigned to the following page: 3

3 c) $J(x,y,z) = \langle y\sec^2 x - ze^x, \tan x, -e^x \rangle$

$P = y\sec^2 x - ze^x$

$Q = \tan x$

$R = -e^x$

$\frac{\partial P}{\partial y} = \sec^2 x = \frac{\partial Q}{\partial x}$, $\frac{\partial P}{\partial z} = -e^x = \frac{\partial R}{\partial x}$, $\frac{\partial Q}{\partial z} = 0 = \frac{\partial R}{\partial y}$

$J(x,y,z)$ is conservative

$J(x,y,z) = \int J_x\,dx = \int y\sec^2 x - ze^x\,dx = y\tan x - ze^x + C(y,z)$

$J_y(x,y,z) = \tan x + \frac{\partial C}{\partial y} = \tan x$

$\frac{\partial C}{\partial y} = 0$

$C(y,z) = C(z)$ where c is a constant.

$J(x,y,z) = y\tan x - ze^x + C(z)$

$J_z(x,y,z) = -e^x + \frac{\partial C}{\partial z} = -e^x$ $\quad ze^x - e^x$

$\frac{\partial C}{\partial z} = 0$

$C(z) = c$

The potential function for $J(x,y,z)$ is $J(x,y,z) = y\tan x - ze^x + c$, where c is a constant.

Question assigned to the following page: 4

4 a) $F(x,y) = 4xy^3 \vec{i} + 2xy^3 \vec{j}$ $\quad P = 4xy^3$

$Q = 2xy^3$

$\frac{\partial P}{\partial y} = 12xy^2 \neq \frac{\partial Q}{\partial x} = 6xy^2$

F(x,y) is not conservative and therefore it's line integral not independent of path.

4 b) $G(x,y) = e^x \vec{i} + (3 - e^x \sin y) \vec{j}$

$\frac{\partial P}{\partial y} = 0 \neq \frac{\partial Q}{\partial x} = -e^x \sin y$

G(x,y) is not conservative and therefore it's line integral is not independent of path

4 c) $F(x,y,z) = \langle \overset{P}{e^y \cos z}, \overset{Q}{xe^y \cos z}, \overset{R}{xe^y \sin z} \rangle$

$\frac{\partial P}{\partial y} = e^y \cos z = \frac{\partial Q}{\partial x} = e^y \cos z$

$\frac{\partial P}{\partial z} = -e^y \sin z \neq \frac{\partial R}{\partial x} = e^y \sin z$

F(x,y,z) is not conservative, therefore

$\int_c e^y \cos(z)\,dx + xe^y \cos(z)\,dy + xe^y \sin z\,dz$ is not independent of path