MATH 70
Problem Session Worksheet 4
3/11/2021

1 Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ (x_1 + x_2)^3 \end{bmatrix}.$$

Is T a linear transformation? If so, prove it. If not, provide a specific counterexample.

2 Determine the standard matrix for the following linear transformations $\mathbb{R}^2 \to \mathbb{R}^2$ defined by reflection across the x_2-axis. Do the same for the linear transformation defined by reflection across the line $x_1 = x_2$.

3 Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation.Assume T is one-to-one. Show that, if $T(\mathbf{v_1}), T(\mathbf{v_2}), T(\mathbf{v_3})$ span $\mathbb{R}^3$, then $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ span $\mathbb{R}^3$.

4 IF C is a 6×6 matrix and the equation $C\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b} \in \mathbb{R}^6$, is it possible that for some $\mathbf{b}$, the equation $C\mathbf{x} = \mathbf{b}$ has more than one solution?

5 Suppose H is a 5×5 matrix and suppose there is a vector $\mathbf{v} \in \mathbb{R}^5$ which is not a linear combination of the columns of H. What can you say about the number of solutions to $H\mathbf{x} = \mathbf{0}$?