MATH 42 HOMEWORK 3

Topics covered: planes, surfaces, curve length from §13.5, 13.6, 14.4, and 15.1

This homework is due at 11:59 p.m. (Eastern Time) on Wednesday, September 30. Scan the completed homework and upload it **as one pdf file** to Gradescope. The Canvas module "Written Assignments" has instructions for how to upload the assignment to Gradescope.

(1) Find the length of the parameterized curve over the specified time interval:
   (a) $\vec{r}(t) = \langle \sin(t), \cos(t), t\sqrt{3} \rangle$ for $t \in [0, 5]$
   (b) $\vec{r}(t) = \langle \frac{1}{2}t^2, \frac{2\sqrt{2}}{3}\pi^{1/4}t^{3/2}, t\sqrt{\pi} \rangle$ for $t \in [0, \pi]$

(2) Consider the plane passing through $A(-2, 4, 3)$, $B(1, 0, -3)$, and $C(3, 2, -1)$.
   (a) Identify the unit normal vector of the plane.
   (b) Express the equation of the plane in the form $\alpha x + \beta y + \gamma z + \delta = 0$ where α, β, γ, and δ are integers with greatest common denominator of 1. Hint: $|\delta| = 19$ in this form.

(3) Consider the plane P with unit normal vector $\hat{n} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$ that passes through the origin, $O(0, 0, 0)$.
   (a) The *dihedral angle* is the angle between two planes that intersect. Find the dihedral angle between the plane in problem (2) and the plane P.
   (b) The intersection between the plane in problem (2) and the plane P is a line. In what direction does this line point? Express the answer as a unit vector.

   Hint: Use knowledge of the unit normal vector of each plane.

(4) For any point D selected in three-dimensions, can a plane be found that passes through D and is orthogonal to the plane in problem (2)? If one such plane exists, do others and if so, how many? Explain your reasoning.

(5) Sketch the traces and at least one cross section (for a non-zero value of x, y, or z) of the surfaces:
   (a) $y = \frac{1}{4}x^2 - 2z^2$
   (b) $0 = 2 - x^2 + 2y^2 + 4z^2$
   (c) $x = y^2 + z^2$

(6) Consider the equation $f(x, y, z) = \sqrt{(x-3)^2 + (y+4)^2 + z^2}$ and its level surfaces at $f(x, y, z) = 1, 2, 3, \ldots$ Qualitatively describe the separation of these level surfaces.