Math 70 worksheet 4

1 A counter example that proves $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is not linear is $T\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. If T is linear, then $T(c\vec{u}) = cT\vec{u}$.
For this counter example, let $c = 2$.
$T(2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}) = T(\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}) = \begin{bmatrix} 8 \\ 8 \end{bmatrix}$, $2T\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$
$\begin{bmatrix} 8 \\ 8 \end{bmatrix} \neq \begin{bmatrix} 4 \\ 8 \end{bmatrix}$, meaning that T is not linear,

2 If $T(v_1)$, $T(v_2)$, $T(v_3)$ are linearly independent in $\mathbb{R}^m$, then $c_1T(\vec{v}_1) + c_2T(\vec{v}_2) + c_3T(\vec{v}_3) = \vec{0}$ has only the trivial solution of $c_1, c_2, c_3 = 0$
By the properties of linear transformations, $c_1T(\vec{v}_1) = T(c_1\vec{v}_1)$, the equation above is rewritten as:
$T(c_1\vec{v}_1) + T(c_2\vec{v}_2) + T(c_3\vec{v}_3) = \vec{0}$. By the property of linear transformations $T(\vec{x} + \vec{y}) = T(\vec{x}) + T(\vec{y})$, We can rewrite our equation as
$T(c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3) = 0$, by the properties of linear transformations, $T(0) = 0$, meaning $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = 0$. As mentioned earlier, $c_1, c_2, c_3 = 0$, meaning that $\vec{v}_1$, $\vec{v}_2$, and v_3 are linearly independent,

3a) $\mathbb{R}^n$

3b) Column

3c) to span $\mathbb{R}^n$, needs a pivot in every row.

3d) $M = n$,

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4 a) $\begin{bmatrix} 3 & -2 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 3-2 & 6-6 \\ -1+1 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

$3\times 3,\ 2\times 3$

4 b) $\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -2 & 0 \\ -1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3-2 & -2+2 & 0 \\ 3-3 & -2+3 & 0 \\ 6-5 & -4+5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \neq I_3$

4 c) The invertible matrix theorem only applies when A and A^{-1} are square matrices, A and C are not of size $n \times n$.

4 d) $\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = I_3$

$\begin{bmatrix} a_{11} + 2a_{21} & a_{12} + 2a_{22} & a_{13} + 2a_{23} \\ a_{11} + 3a_{21} & a_{12} + 3a_{22} & a_{13} + 3a_{23} \\ 2a_{11} + 5a_{21} & 2a_{12} + 5a_{22} & 2a_{13} + 5a_{23} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since in terms of solving columns are independent, let's break this into 3 equations

$\begin{bmatrix} a_{11} + 2a_{21} \\ a_{11} + 3a_{21} \\ 2a_{11} + 5a_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} a_{12} + 2a_{22} \\ a_{12} + 3a_{22} \\ 2a_{12} + 5a_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} a_{13} + 2a_{23} \\ a_{13} + 3a_{23} \\ 2a_{13} + 5a_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

4 d) Starting with the first equation,

$$\begin{bmatrix} a_{11}+2a_{21} \\ a_{11}+3a_{21} \\ 2a_{11}+5a_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \sim \left[\begin{array}{cc:c} 1 & 2 & 1 \\ 1 & 3 & 0 \\ 2 & 5 & 0 \end{array}\right], \begin{array}{l} r_2 = r_2 - r_1 \\ r_3 = r_3 - 2r_1 \end{array}$$

$$\sim \left[\begin{array}{cc:c} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -2 \end{array}\right]$$

This means $a_{21} = -1$ and -2. This is impossible meaning the system is inconsistent.

Since the first column of A's right inverse cannot equal the first column of I_3, A does not have a right inverse