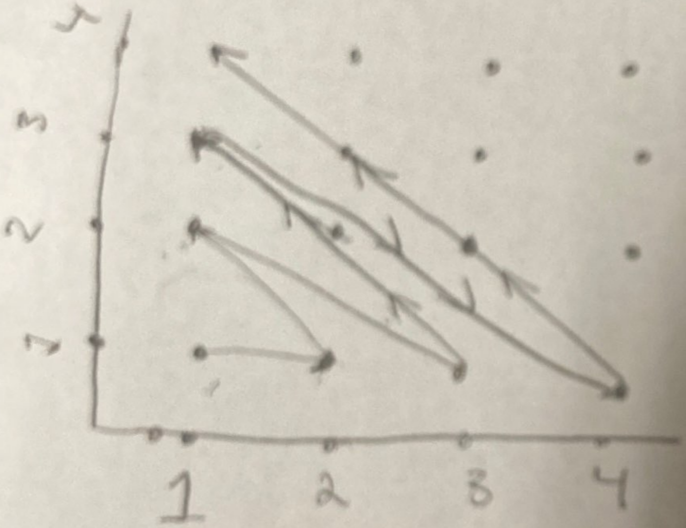
QUIZ 5, MONDAY OCTOBER 4.

Question 1. (a) *In class, we showed that the set of pairs of positive integers is countable. Give a different way of counting the pairs of positive integers Describe your counting strategy both with a picture and algebraically.*

Our method is for (a,b) $a,b \in \mathbb{N} - \{0\}$,

$$(a,b) \leq (c,d) \text{ if } \begin{cases} a+b < c+d \\ a+b = c+d \quad d \geq b \end{cases}$$



(b) *We know that the set of positive rational numbers is countable. Show that the set of all rational numbers is countable. Please be very precise and show how things follow from the definitions*

Let A_n be the set of all positive rational numbers w/ n elements. A_n is countable, and forms a biviection with $\mathbb{N}$. Let $B_n = -A_n$, and B_n is the set of all negative rational numbers. Since A is countable, then it follows that B is countable.

We can map the positive rationals to even numbers and negative rationals to odd numbers.

$Q \geq 0 \rightarrow$ even #s

$\frac{a}{b} \xrightarrow{\cdot 2b} 2a$

even numbers $\rightarrow$ pos. rationals

$n \xrightarrow{\div \frac{n}{2b}} \frac{n}{2b}$

negative rationals $\rightarrow$ odd numbers

$\frac{a}{b} \leq 0 \quad \frac{a}{b} \xrightarrow{(1-b)} -a+1$

oddnum $\rightarrow$ negative rational nums

$k \xrightarrow{-(\frac{1}{b})-1} -\frac{k}{b} - 1$

Since the 4 functions above are in bijection w/ N, the positive and negative rational numbers are in bijection w/ N and thus the set Q of all rational numbers is countable