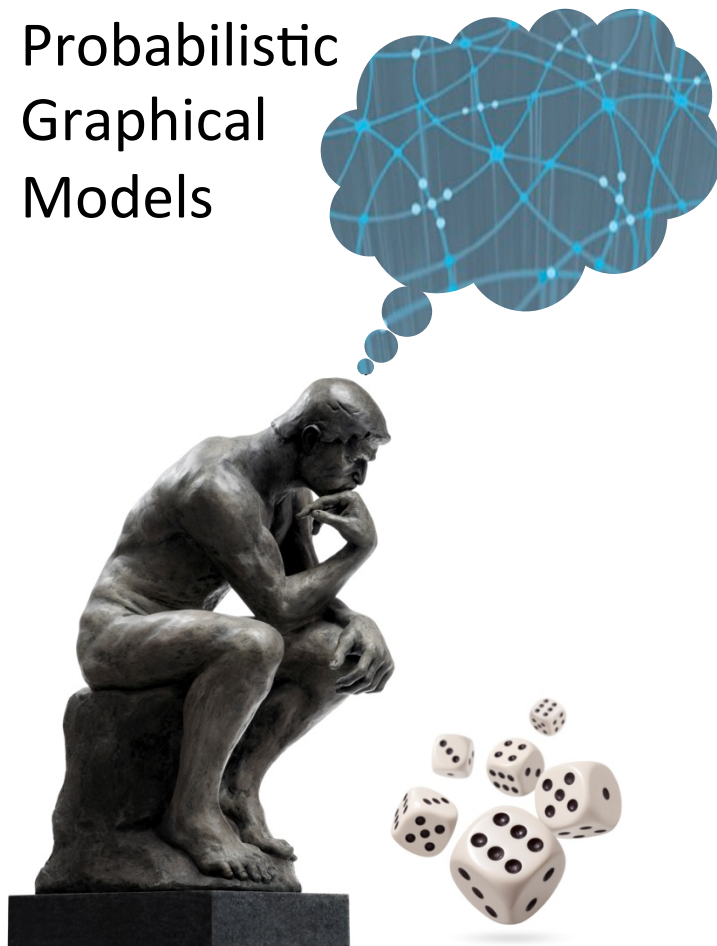


Probabilistic  
Graphical  
Models



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Introduction

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# Preliminaries: Distributions

关于 概率分布的基础

联合分布

# Joint Distribution $P(I, D, G)$

- Intelligence (I)  $\leftarrow 2$ 
    - $i^0$  (low),  $i^1$  (high),
  - Difficulty (D)  $\leftarrow 2$ 
    - $d^0$  (easy),  $d^1$  (hard)
  - Grade (G)  $\leftarrow 3$ 
    - $g^1$  (A),  $g^2$  (B),  $g^3$  (C)
- parameters  
 $2 \times 2 \times 3 = 12$
- independent params  
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I	D	G	Prob.
$i^0$	$d^0$	$g^1$	0.126
$i^0$	$d^0$	$g^2$	0.168
$i^0$	$d^0$	$g^3$	0.126
$i^0$	$d^1$	$g^1$	0.009
$i^0$	$d^1$	$g^2$	0.045
$i^0$	$d^1$	$g^3$	0.126
$i^1$	$d^0$	$g^1$	0.252
$i^1$	$d^0$	$g^2$	0.0224
$i^1$	$d^0$	$g^3$	0.0056
$i^1$	$d^1$	$g^1$	0.06
$i^1$	$d^1$	$g^2$	0.036
$i^1$	$d^1$	$g^3$	0.024

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# Conditioning

condition on  $g^1$

I	D	G	Prob.
$i^0$	$d^0$	$g^1$	0.126
<del><math>i^0</math></del>	<del><math>d^0</math></del>	<del><math>g^2</math></del>	<del>0.168</del>
<del><math>i^0</math></del>	<del><math>d^0</math></del>	<del><math>g^3</math></del>	<del>0.126</del>
$i^0$	$d^1$	$g^1$	0.009
<del><math>i^0</math></del>	<del><math>d^1</math></del>	<del><math>g^2</math></del>	<del>0.045</del>
<del><math>i^0</math></del>	<del><math>d^1</math></del>	<del><math>g^3</math></del>	<del>0.126</del>
$i^1$	$d^0$	$g^1$	0.252
<del><math>i^1</math></del>	<del><math>d^0</math></del>	<del><math>g^2</math></del>	<del>0.0224</del>
<del><math>i^1</math></del>	<del><math>d^0</math></del>	<del><math>g^3</math></del>	<del>0.0056</del>
$i^1$	$d^1$	$g^1$	0.06
<del><math>i^1</math></del>	<del><math>d^1</math></del>	<del><math>g^2</math></del>	<del>0.036</del>
<del><math>i^1</math></del>	<del><math>d^1</math></del>	<del><math>g^3</math></del>	<del>0.024</del>

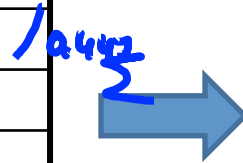
# Conditioning: Reduction

I	D	G	Prob.
$i^0$	$d^0$	$g^1$	0.126
$i^0$	$d^1$	$g^1$	0.009
$i^1$	$d^0$	$g^1$	0.252
$i^1$	$d^1$	$g^1$	0.06

# Conditioning: Renormalization

让reduction后的概率之和仍是1

I	D	G	Prob.
$i^0$	$d^0$	$g^1$	0.126
$i^0$	$d^1$	$g^1$	0.009
$i^1$	$d^0$	$g^1$	0.252
$i^1$	$d^1$	$g^1$	0.06



$P(I, D, g^1)$   
unnormalized measure 0.447

I	D	Prob.
$i^0$	$d^0$	0.282
$i^0$	$d^1$	0.02
$i^1$	$d^0$	0.564
$i^1$	$d^1$	0.134

$P(I, D | g^1)$

# Marginalization

$P(I, D)$

Marginalize I

I	D	Prob.
$i^0$	$d^0$	0.282
$i^0$	$d^1$	0.02
$i^1$	$d^0$	0.564
$i^1$	$d^1$	0.134

D	Prob.
$d^0$	0.846
$d^1$	0.154