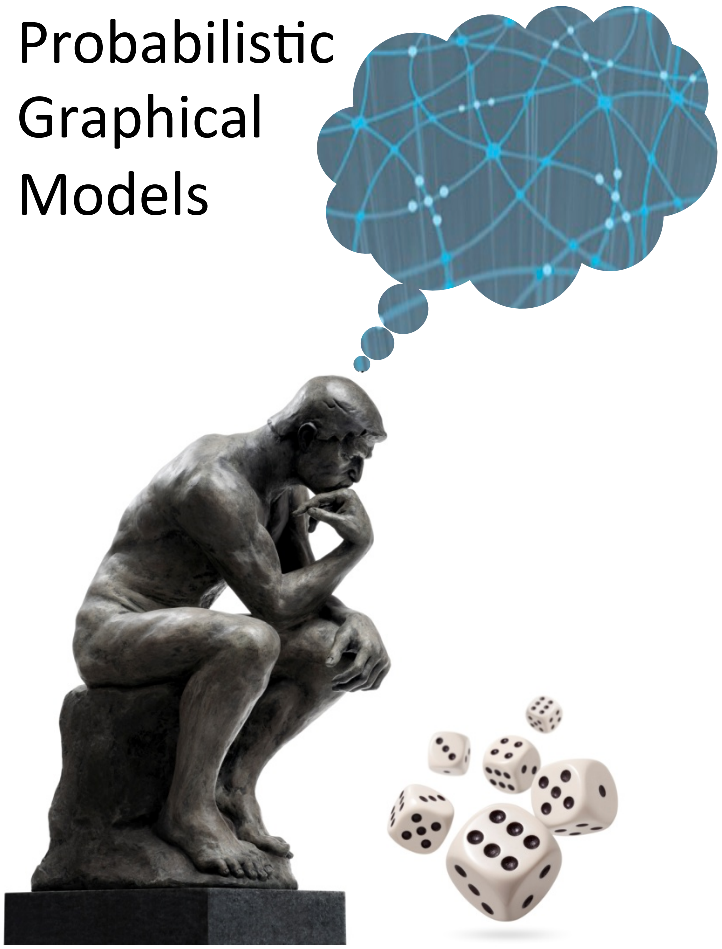
Probabilistic
Graphical
Models



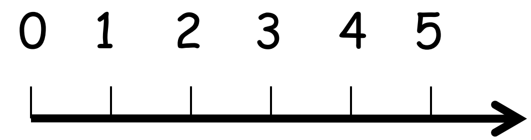
Representation

Template Models

Temporal Models

Distributions over Trajectories

discretize time



- Pick time granularity Δ sensor
- $X^{(t)}$ – variable X at time $t\Delta$ 时序模型的变量表示
- $X^{(t:t')} = \{X^{(t)}, \ldots, X^{(t')}\}$ $(t \le t')$ 区间表示法
- Want to represent $P(X^{(t:t')})$ for any t, t' =P (xt, ...,xtp)

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Markov Assumption

时序模型中，计算概率的公式是这个

$$P(\boldsymbol{X}^{(0:T)}) = P(\boldsymbol{X}^{(0)}) \prod_{t=0}^{T-1} P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(0:t)})$$

time flows forward

state at t+1

state

chain rule for probabilities

说明：你一旦知道了现在，再算下一步的时候，就不用考虑过去了

有了这个条件，上式就可以简化为下式

$$(\boldsymbol{X}^{(t+1)} \perp \boldsymbol{X}^{(0:t-1)} \mid \boldsymbol{X}^{(t)})$$

next step

past

present

forget

$$P(\boldsymbol{X}^{(0:T)}) = P(\boldsymbol{X}^{(0)}) \prod_{t=0}^{T-1} P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(t)})$$

Is this true?

X = Location of robot

$L^{t+1} \perp L^{t-1} \mid L^{t}$?

probably not

velocity

enrich state by adding v, and other variables

(adding dependencies back in time – semi. Markov)

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时不变性

Time Invariance

next time

current time

- Template probability model $P(X' \mid X)$
- For all t:

$$P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(t)}) = P(\boldsymbol{X}' \mid \boldsymbol{X})$$

对所有 t

traffic: time of day, day of week, football

enrich model by including

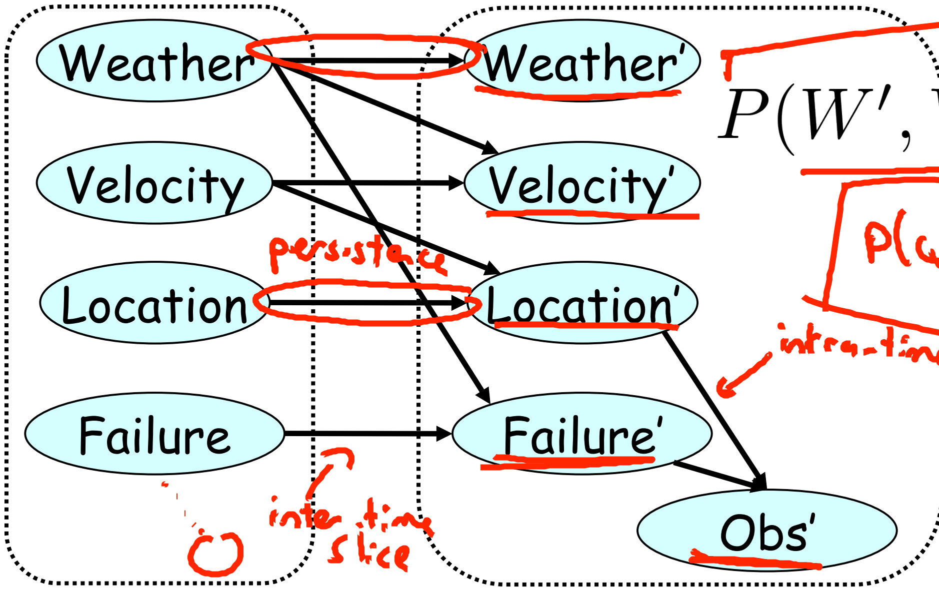
$P(X^{(t+1)}|X^{(t)})$都相同

则该过程可用转移模型来表示

模板转移模型

Template Transition Model

network fragment



Time slice *t* Time slice *t+1*

intra-time-slice

cond dist t+1 given t

$$P(W', V', L', F', O' \mid W, V, L, F) =$$

$$P(w'|w) \cdot P(V'|W,V) \cdot P(L'|L,V) \cdot \underbrace{P(F'|F,W)}_{t \text{ to } t+1} \cdot \underbrace{P(O'|L',F')}_{t+1}$$

rapidly-acting

$$\cancel{P(W, V, L, F)}$$

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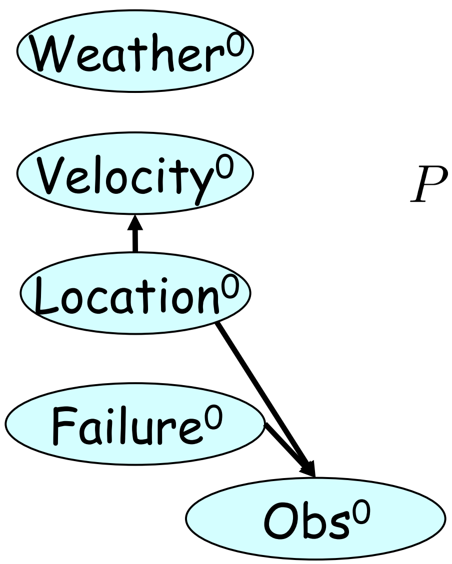
Initial State Distribution



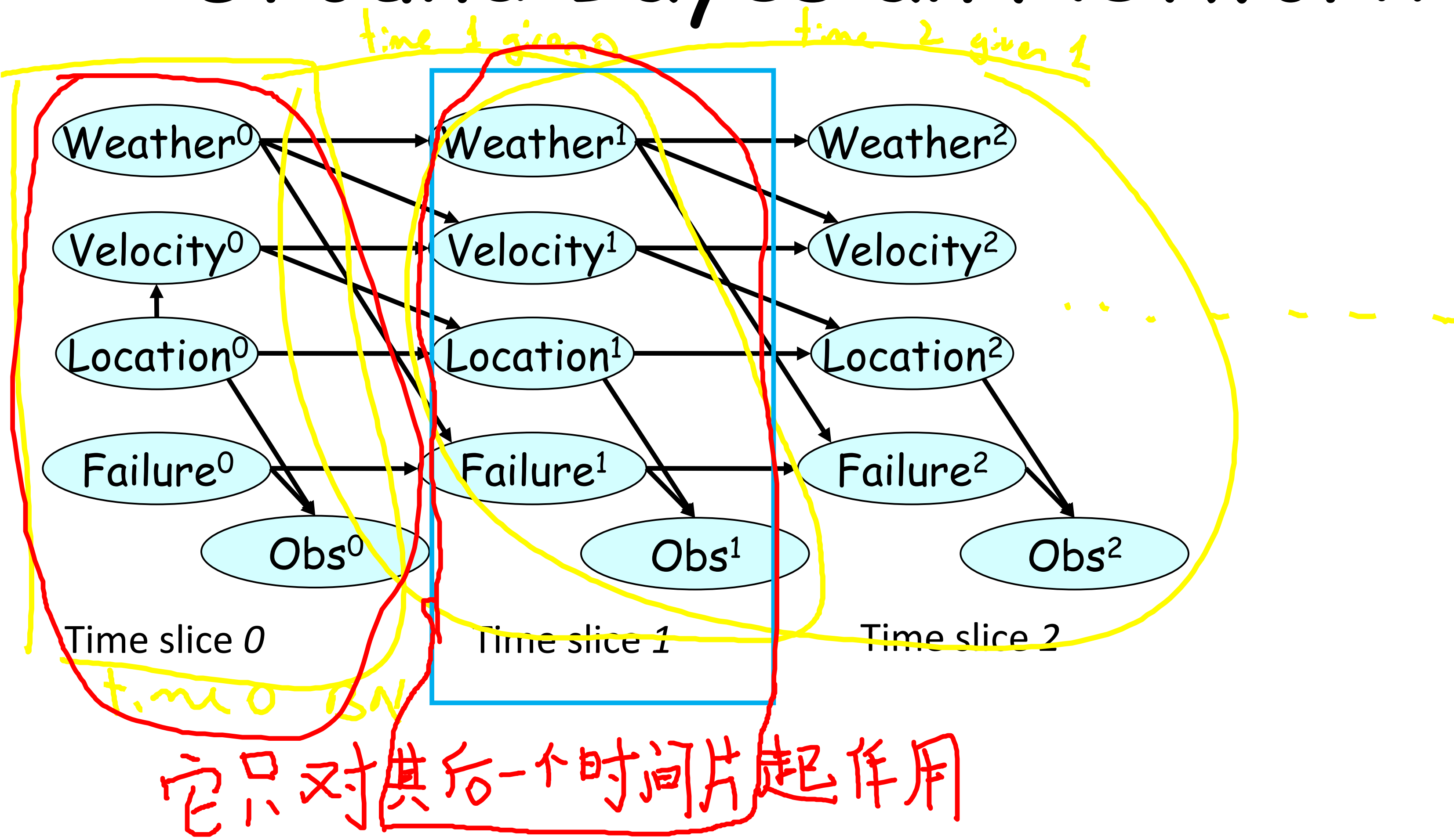
Time slice *0*

$$P(W^{(0)}, V^{(0)}, L^{(0)}, F^{(0)}, O^{(0)}) =$$

$$P(W^{(0)})P(V^{(0)} \mid L^{(0)})P(L^{(0)})P(F^{(0)})P(O^{(0)} \mid F^{(0)}, L^{(0)})$$

chain rule

Ground Bayesian Network



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2-时间片段贝叶斯网（2TBN）

2-time-slice Bayesian Network

- A transition model (2TBN) over $X_1,\dots,X_n$ is specified as a BN fragment such that:
  - The nodes include $X_1',\dots,X_n'$ and a subset of $X_1,\dots,X_n$
  - Only the nodes $X_1',\dots,X_n'$ have parents and a CPD
- The 2TBN defines a conditional distribution

template

the time t var that directly state at t+1

CPD=条件概率分布

$$P(\boldsymbol{X}' \mid \boldsymbol{X}) = \prod_{i=1}^{n} P(X_i' \mid \mathbf{Pa}_{X_i'})$$

chain rule

满足该条件的，就是2TBN

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Dynamic Bayesian Network

- A dynamic Bayesian network (DBN) over $X_1,\dots,X_n$ is defined by a
  - $\underbrace{\text{2TBN } BN_{\rightarrow}}$ over $X_1,\dots,X_n$ dynamics
  - time 0 – a Bayesian network $BN^{(0)}$ over $X_1^{(0)},\dots,X_n^{(0)}$

Ground Network

- For a trajectory over 0,…,T we define a ground (unrolled network) such that

time 0
  - The dependency model for $X_1^{(0)}, \ldots, X_n^{(0)}$ is copied from $BN^{(0)}$

transit.
  - The dependency model for $X_1^{(t)}, \ldots, X_n^{(t)}$ for all $t > 0$ is copied from $BN_{\rightarrow}$

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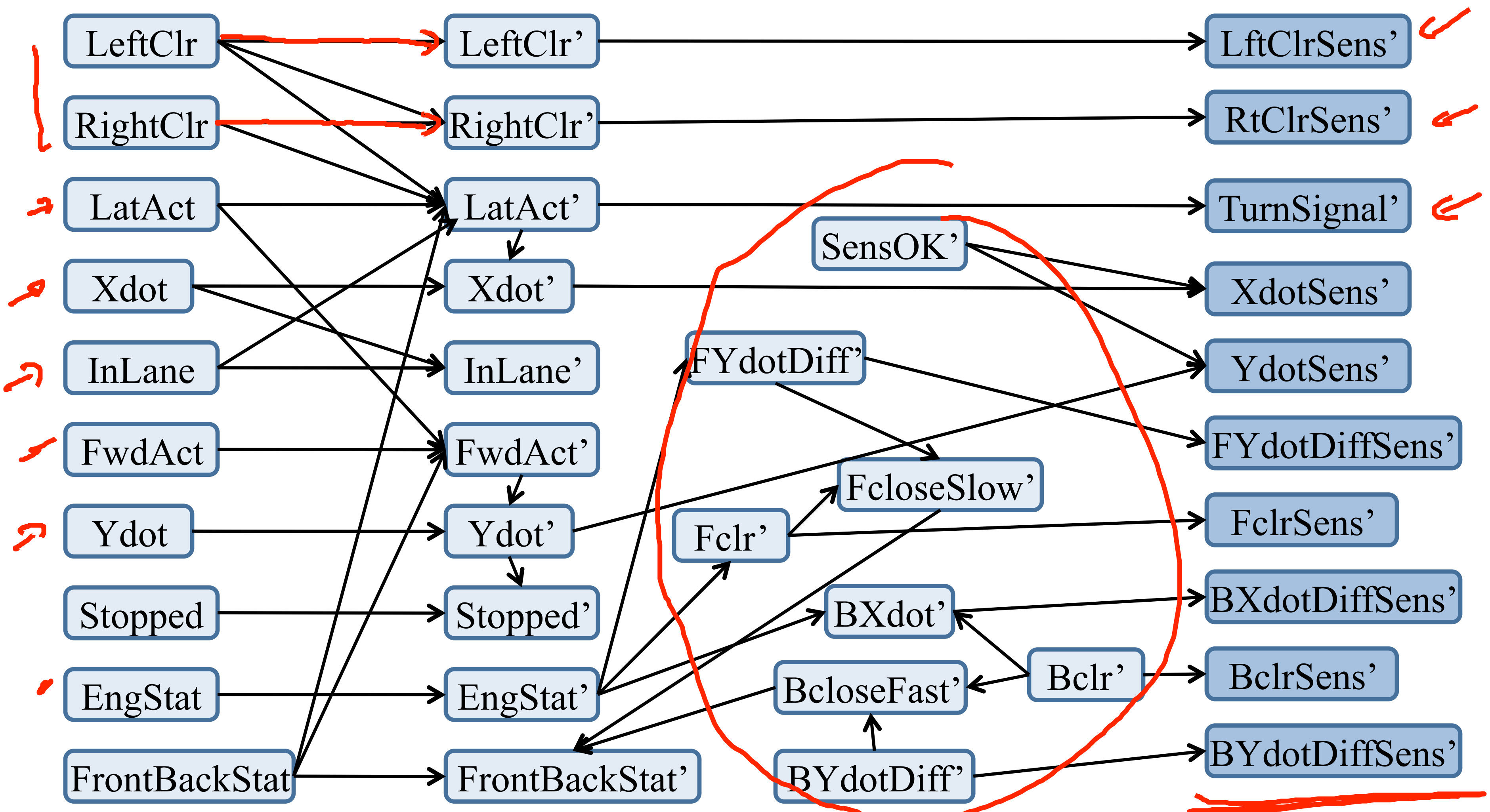

Huang, Koller, Malik, Ogasawara, Rao, Russell, Weber, AAAI 94

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Summary

- DBNS are a compact representation for encoding structured distributions over arbitrarily long temporal trajectories
- They make assumptions that may require appropriate model (re)design:
  - Markov assumption
  - Time invariance