Probabilistic
Graphical
Models

Representation

Independencies

# Preliminaries

独立事件基础

satisfy

independent

# Independence

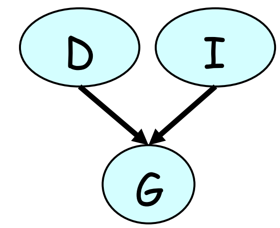
- For events $\alpha$, $\beta$, $P \models \alpha \perp \beta$ if: satisfies independence
  - $P(\alpha \cap \beta) = P(\alpha) \cdot P(\beta)$
  - ~~$P(\alpha|\beta) = P(\alpha)$~~
  - $P(\beta|\alpha) = P(\beta)$
- For random variables $X,Y$, $P \models X \perp Y$ if:
  - $P(X, Y) = P(X)\,P(Y)$
  - $P(X|Y) = P(X)$ universal
  - $P(Y|X) = P(Y)$ $\forall x,y \quad P(x,y) = P(x) \cdot P(y)$

Daphne Koller

# Independence

| I | D | G | Prob. |
|---|---|---|---|
| $i^0$ | $d^0$ | $g^1$ | 0.126 |
| $i^0$ | $d^0$ | $g^2$ | 0.168 |
| $i^0$ | $d^0$ | $g^3$ | 0.126 |
| $i^0$ | $d^1$ | $g^1$ | 0.009 |
| $i^0$ | $d^1$ | $g^2$ | 0.045 |
| $i^0$ | $d^1$ | $g^3$ | 0.126 |
| $i^1$ | $d^0$ | $g^1$ | 0.252 |
| $i^1$ | $d^0$ | $g^2$ | 0.0224 |
| $i^1$ | $d^0$ | $g^3$ | 0.0056 |
| $i^1$ | $d^1$ | $g^1$ | 0.06 |
| $i^1$ | $d^1$ | $g^2$ | 0.036 |
| $i^1$ | $d^1$ | $g^3$ | 0.024 |

这个就是概率图模型




P(I,D) = ×

| I | D | Prob |
|---|---|---|
| $i^0$ | $d^0$ | 0.42 |
| $i^0$ | $d^1$ | 0.18 |
| $i^1$ | $d^0$ | 0.28 |
| $i^1$ | $d^1$ | 0.12 |

P(I)

| I | Prob |
|---|---|
| $i^0$ | 0.6 |
| $i^1$ | 0.4 |

P(D)

| D | Prob |
|---|---|
| $d^0$ | 0.7 |
| $d^1$ | 0.3 |

# Conditional Independence

- For (sets of) random variables **X**,**Y**,**Z**

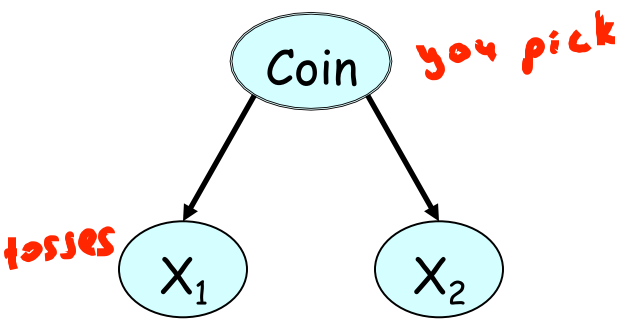
  satisfies indep conditioning

  $P \models (\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z})$ if:

  given

  - $P(\mathbf{X}, \mathbf{Y} \mid \mathbf{Z}) = P(\mathbf{X} \mid \mathbf{Z})\, P(\mathbf{Y} \mid \mathbf{Z})$
  - $P(\mathbf{X} \mid \mathbf{Y}, \mathbf{Z}) = P(\mathbf{X} \mid \mathbf{Z})$
  - $P(\mathbf{Y} \mid \mathbf{X}, \mathbf{Z}) = P(\mathbf{X} \mid \mathbf{Z})$
  - $\boxed{P(\mathbf{X}, \mathbf{Y}, \mathbf{Z}) \propto \phi_1(\mathbf{X}, \mathbf{Z})\, \phi_2(\mathbf{Y}, \mathbf{Z})}$

# Conditional Independence




$P \not\models X_1 \perp X_2$

$P \models (X_1 \perp X_2 \mid C)$

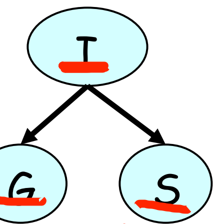
# Conditional Independence

$P(I, S, G)$

| I | S | G | Prob. |
|---|---|---|---|
| $i^0$ | $s^0$ | $g^1$ | 0.114 |
| $i^0$ | $s^0$ | $g^2$ | 0.1938 |
| $i^0$ | $s^0$ | $g^3$ | 0.2622 |
| $i^0$ | $s^1$ | $g^1$ | 0.006 |
| $i^0$ | $s^1$ | $g^2$ | 0.0102 |
| $i^0$ | $s^1$ | $g^3$ | 0.0138 |
| $i^1$ | $s^0$ | $g^1$ | 0.252 |
| $i^1$ | $s^0$ | $g^2$ | 0.0224 |
| $i^1$ | $s^0$ | $g^3$ | 0.0056 |
| $i^1$ | $s^1$ | $g^1$ | 0.108 |
| $i^1$ | $s^1$ | $g^2$ | 0.0096 |
| $i^1$ | $s^1$ | $g^3$ | 0.0024 |

$P(S,G \mid i^0)$

| S | G | Prob. |
|---|---|---|
| $s^0$ | $g^1$ | 0.19 |
| $s^0$ | $g^2$ | 0.323 |
| $s^0$ | $g^3$ | 0.437 |
| $s^1$ | $g^1$ | 0.01 |
| $s^1$ | $g^2$ | 0.017 |
| $s^1$ | $g^3$ | 0.023 |



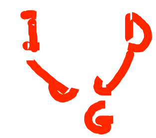

$P(S \mid i^0)$

| S | Prob |
|---|---|
| $s^0$ | 0.95 |
| $s^1$ | 0.05 |

$P(G \mid i^0)$

| G | Prob. |
|---|---|
| $g^1$ | 0.2 |
| $g^2$ | 0.34 |
| $g^3$ | 0.46 |

# Conditioning can Lose Independences




| I | D | G | Prob. |
|---|---|---|---|
| $i^0$ | $d^0$ | $g^1$ | 0.126 |
| $i^0$ | $d^0$ | $g^2$ | 0.168 |
| $i^0$ | $d^0$ | $g^3$ | 0.126 |
| $i^0$ | $d^1$ | $g^1$ | 0.009 |
| $i^0$ | $d^1$ | $g^2$ | 0.045 |
| $i^0$ | $d^1$ | $g^3$ | 0.126 |
| $i^1$ | $d^0$ | $g^1$ | 0.252 |
| $i^1$ | $d^0$ | $g^2$ | 0.0224 |
| $i^1$ | $d^0$ | $g^3$ | 0.0056 |
| $i^1$ | $d^1$ | $g^1$ | 0.06 |
| $i^1$ | $d^1$ | $g^2$ | 0.036 |
| $i^1$ | $d^1$ | $g^3$ | 0.024 |

$P(I, D \mid g^1)$

| I | D | Prob. |
|---|---|---|
| $i^0$ | $d^0$ | 0.282 |
| $i^0$ | $d^1$ | 0.02 |
| $i^1$ | $d^0$ | 0.564 |
| $i^1$ | $d^1$ | 0.134 |

条件改变，事件间可能就不独立了