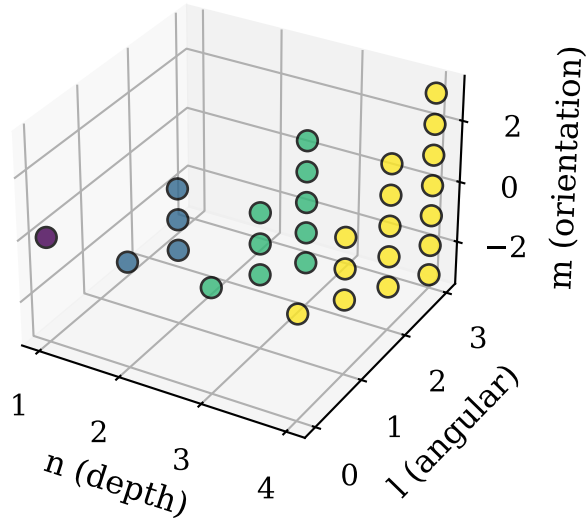


# Figure 3: Partition Geometry → Spatial Coordinates

## A. Partition Coordinates (n, l, m, s)



## B. Geometric Constraints

**PARTITION CONSTRAINTS**

$n \in \mathbb{Z}^+$  (depth  $\geq 1$ )

$0 \leq l \leq n-1$  (angular limit)

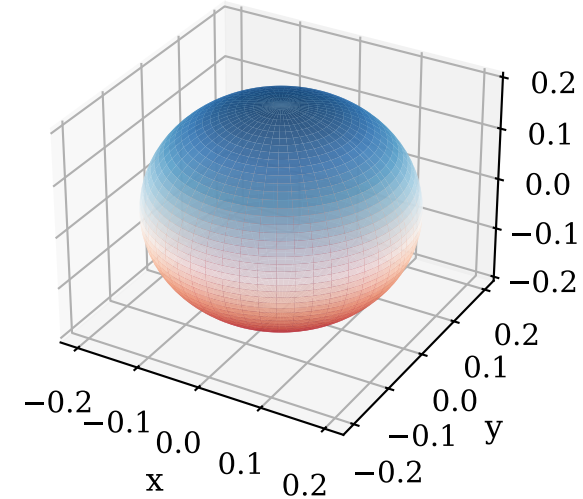
$-l \leq m \leq +l$  (orientation range)

$s = \pm \frac{1}{2}$  (chirality/spin)

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Capacity:  $2n^2$  states per shell

## C. Angular Structure $Y_2^1(\theta, \varphi) \rightarrow 3D$ Space



## D. Mapping to Space

$l \in \{0, 1, \dots, n-1\}$  SO(3) representations

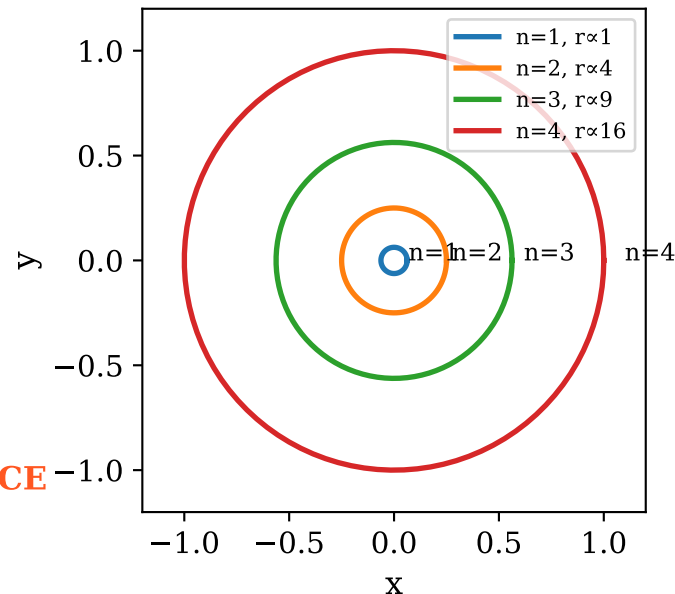
$m \in \{-l, \dots, +l\}$  (2l+1) orientations

(l, m) together Spherical harmonics

n (radial) →  $r \propto n^2$  extension

**RESULT:** 3D EUCLIDEAN SPACE

## E. Radial Extension $r \propto n^2$ (Bohr-like)



## F. Dimensionality Theorem

WHY D = 3?

The constraint structure:

- $l \in \{0, 1, \dots, n-1\}$
- $m \in \{-l, \dots, +l\}$

Has exactly 2 angular quantum numbers (l, m).

This is the UNIQUE signature of SO(3).

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D = 3 is DERIVED, not assumed!