

The force, power and energy of the 100 meter sprint

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Abstract

At the 2008 Summer Olympics in Beijing, Usain Bolt broke the world record for the 100 m sprint. Just one year later, at the 2009 World Championships in Athletics in Berlin he broke it again. A few months after Beijing, Eriksen et al. studied Bolt's performance and predicted that Bolt could have run about one-tenth of a second faster, which was confirmed in Berlin. In this paper we extend the analysis of Eriksen et al. to model Bolt's velocity time-dependence for the Beijing 2008 and Berlin 2009 records. We deduce the maximum force, the maximum power, and the total mechanical energy produced by Bolt in both races. Surprisingly, we conclude that all of these values were smaller in 2009 than in 2008.

I. INTRODUCTION

Eriksen, Kristiansen, Langangen, and Wehus recently analyzed Usain Bolt's 100 m sprint at the 2008 Summer Olympics in Beijing and speculated what the world record would have been had he not slowed down in celebration near the end of his race.¹ They analyzed two scenarios for the last two seconds – the same acceleration as the second place runner (Richard Thompson) or Bolt's acceleration was 0.5 m/s^2 greater than Thompson's. They found race times of $9.61 \pm 0.04 \text{ s}$ and $9.55 \pm 0.04 \text{ s}$ for the first and second scenarios, respectively. In comparison, Bolt's actual race time in Beijing was 9.69 s . The predictions in Ref. 1 were apparently confirmed at the 2009 World Championships in Athletics in Berlin (race time 9.58 s), suggesting that Bolt had improved his performance. However, the runners in Berlin benefited from a 0.9 m/s tailwind that, as we will show, cannot be neglected.

In this paper we analyze Bolt's records in Beijing and Berlin using a function proposed by Tibshirani^{2,3} to describe the time dependence of the velocity in both races. This paper is organized as follows. As a first step, we fit an empirical function, $v(t)$, to the data available from the International Association of Athletics Federation.⁴ From this fit we can deduce the maximum acceleration, the maximum power, and the total mechanical energy produced by the runner. We then compare Bolt's performances in Berlin 2009 and Beijing 2008, concluding that all of these values were smaller in 2009 than in 2008.

II. MODELING THE TIME DEPENDENCE OF THE VELOCITY

In a seminal paper, Keller⁵ analyzed the problem faced by runners – how should they vary their velocity to minimize the time to run a certain distance? In Ref. 5 the resistance of the air was assumed to be proportional to the velocity and, for the 100 m sprint, it was assumed that the best strategy is to apply as much constant force, F , as possible. The equation of motion was written as

$$m \frac{dv}{dt} = F - \alpha v, \quad (1)$$

where v is the speed of the runner. The solution of Eq. (1) is $v(t) = (F/\alpha)(1 - e^{-\alpha t/m})$. Although this solution seems to answer the proposed question, it has some limitations. One problem is that the air flow is turbulent, not laminar, and thus the drag force is proportional to v^2 , rather than v . Another problem is that runners must produce energy (and thus a

force) to replace the energy expended by the vertical movement of the runners' center of mass and the loss of energy due to the movement of their legs and feet. (See Ref. 6 for an analysis of the energy cost of the vertical movement in a horse race.) Also, the runners' velocity declines slightly close to the end of the 100 m race, which is not taken into account in Eq. (1).

Tibshirani^{2,3} proposed that a runner's force decreases with time according to $f(t) = F - \beta t$, so that the solution of Newton's second law is

$$v(t) = \left(\frac{F}{\alpha} + \frac{m\beta}{\alpha^2} \right) (1 - e^{-\alpha t/m}) - \frac{\beta t}{\alpha}. \quad (2)$$

This solution solves some of the limitations discussed in the previous paragraph, but it is still incomplete. As we know, the air resistance is proportional to v^2 , but in the above models the air resistance was supposed to be proportional only to v , probably to simplify the dynamical equations such that an analytical solution could be obtained.

The problem of determining the best form of the function to describe the time dependence of the velocity of a runner in the 100 m sprint was addressed in Refs. 7,8, but is still open. We fitted Bolt's velocity in Beijing 2008 and Berlin 2009 using an equation with the same form as Eq. (2), but with three parameters to be fitted to his position as a function of time. We assume that the velocity is given by

$$v(t) = a(1 - e^{-ct}) - bt, \quad (3)$$

and the corresponding position is

$$x(t) = at - \frac{bt^2}{2} - \frac{a}{c}(1 - e^{-ct}). \quad (4)$$

III. EXPERIMENTAL DATA AND RESULTS

The parameters a , b , and c in Eq. (4) were fitted using the least-squares method constrained such that $x(t_f) = 100$ m, where t_f is the running time minus the 'reaction time' (the elapsed time between the sound of the starting pistol and the initiation of the run). The split times for each 10 m of Bolt's race are reproduced in Table I. In Table I we also show the split times calculated using the parameters given in Table II.

The data variances were estimated as follows. First, we assumed that the uncertainties of the split times are all equal. Then, we estimated the variances by the sum of the squares

of the differences between the observed split times and the calculated values divided by the number of degrees of freedom (the number of data points minus the number of fitted parameters), ν , considering both the Beijing and Berlin data:

$$\sigma^2 = \frac{1}{\nu} \left(\sum_i^{\text{Beijing data}} (t_i - t_i^{\text{fitted}})^2 + \sum_i^{\text{Berlin data}} (t_i - t_i^{\text{fitted}})^2 \right) \quad (5)$$

The obtained result was $\sigma = 0.02$ s. Then, we generated 100 sets of split times by summing normally distributed random numbers centered at zero with standard deviation equal to 0.02 s to the observed split times.

For each simulation we fitted new values of the parameters. Thus, we obtained 100 values for each parameter, both to the Beijing and Berlin competitions. The uncertainties of the fitted parameters were estimated by the standard deviation of the 100 fitted parameters. The uncertainties calculated by this procedure reflect the uncertainties of the measured split times, the rounding of the published data, and the inadequacy of the model function.

Bolt's velocities in Beijing and Berlin are shown in Fig. 1. In Berlin we observe a smaller initial acceleration and a smaller velocity decrease at the end of the race compared to his performance in Beijing. The mechanical power developed by Bolt is given by

$$P(t) = m \frac{dv}{dt} v + \kappa (v - v_{\text{wind}})^2 v + 200 \text{ W}, \quad (6)$$

where the parameter $\kappa = C_d \rho A / 2$ corresponds to the dissipated power due to drag and v_{wind} is the tailwind speed, which was zero in Beijing and equal to 0.9 m/s in Berlin. For $C_d = 0.5$, a typical value of the drag coefficient,^{5,8} $\rho = 1.2 \text{ kg/m}^3$ for the air density, and a cross section of $A \approx 1 \text{ m}^2$, we obtain $\kappa = 0.3 \text{ W/(m/s)}^3$. A constant power of 200 W is estimated for the power expended due to the vertical movement of the Bolt's center of mass.⁹ We used $m = 86 \text{ kg}$ for his mass.¹⁰ The mechanical power generated by Bolt is shown in Fig. 2. We see that the maximum power generated by Bolt in Berlin is smaller than the power he generated in Beijing.

Table III shows Bolt's maximum acceleration and power at Beijing 2008 and at Berlin 2009 calculated using the fitted function $v(t)$. We also show the total mechanical energy produced in both competitions, calculated from the time integral of $P(t)$. Within the limitations of the model, Bolt's performance in Beijing (maximum force and power and the total energy) is higher than in Berlin. Also the predictions in Ref. 1 seem to be vindicated.

IV. DISCUSSION

From Eq. (3) we see that the velocity would become negative at times after the end of the race: $t = 2.5$ and 6.5 minutes, for Beijing and Berlin, respectively. Thus, the model is applicable to competitions of short duration, as can be observed by comparing the differences between the measured and fitted split times. The differences are very small and even in the worst cases (10 m and 90 m in the Beijing Olympic Games) the corresponding differences in the actual and calculated distances are not greater than 0.6 m. The quality of the model function can also be observed by comparing the reported maximum velocity of 12.27 m/s at 65.0 m in Berlin with the result obtained from the fit equal to 11.92 m/s at 57.6 m.

An important question related to short races is the effect of the air resistance on the running time.^{6,11–16} In our model the power produced by the runner [see Eq. (6)] is used to accelerate the runner’s body and to overcome air resistance (there also is a constant power of 200 W, a kind of “parasitic power loss” dominated by the up-down movement of the runner’s center of mass). If the runner is assisted by a tailwind, we assume that all the power saved to overcome air resistance will be used to accelerate the runner’s body. Given this assumption, we ask what would the Beijing record have been for a 0.9 m/s tailwind (the wind measurement on the day of the Berlin race)? We use Eq. (6) and the same initial acceleration for Bolt in Beijing to deduce a new velocity profile in Berlin. We find that the record at the Beijing Olympic Games would have been reduced by 0.16 s to 9.53 s. Also, without the tailwind in Berlin and with the same power profile developed by the runner in this race, the time would increase by 0.16 s.

The currently accepted theoretical and experimental results for the wind effect on 100 m sprint times,^{6,11–16} gives a typical reduction of about 0.05 s in the race time for a tailwind of 1 m/s. A possible cause of our overestimation of the wind effect is the value of the drag coefficient κ that we used. A smaller value of κ implies that the wind effect is less important and, thus, the power saved in the presence of a tailwind would imply a smaller gain of velocity and, consequently, time. For instance, if we used $\kappa = 0.1 \text{ W}/(\text{m/s})^3$ the time reduction would be ≈ 0.06 s in the 100 m sprint for a 0.9 m/s tailwind – close to the accepted value. Nevertheless, we chose $\kappa = 0.3 \text{ W}/(\text{m/s})^3$ (see the paragraph after Eq. (6)), which agrees with the recommended κ values^{8,13,14} to estimate the energy and power produced by runners in the 100 m sprint.

Another possible origin for the discrepancy between our result for the wind effect and the literature is the force applied by the runner. We considered that the energy and the ground force applied by the runner can be deduced from the fitted position position versus time function. However, some workers prefer to formulate a model based on the applied forces (see, for instance, Refs. 8,11,12,14). These authors usually obtain better agreement for the effects of wind as well as altitude in short races.

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Tables

	Split times (s)			
Distances (m)	Beijing 2008	Fitted	Berlin 2009	Fitted
10	1.85	1.91	1.89	1.91
20	2.87	2.88	2.88	2.89
30	3.78	3.77	3.78	3.78
40	4.65	4.63	4.64	4.63
50	5.50	5.47	5.47	5.46
60	6.32	6.31	6.29	6.29
70	7.14	7.15	7.10	7.11
80	7.96	7.95	7.92	7.93
90	8.79	8.84	8.75	8.76
100	9.69	9.69	9.58	9.58
Reaction time	0.165		0.146	

TABLE I: Bolt’s split times at 10 m intervals in Beijing 2008 and Berlin 2009. The estimated standard deviations of the data are 0.02 s (see text). The last line shows the reaction times, which were subtracted from the split times before the fit. We also show the split times calculated using the fitted parameters shown in Table II. The data is from Ref. 4.

Parameters	2008	2009
a	12.49(3) m/s	12.43(3) m/s
b	0.081(4) m/s ⁻²	0.032(3) m/s ⁻²
c	0.814(4) s ⁻¹	0.783(3) s ⁻¹

TABLE II: Fitted parameters of Eqs. (3) and (4). The estimation of the parameter uncertainties is described in the text.

	Beijing 2008	Berlin 2009
Maximum acceleration	10.09(3) m/s ²	9.70(3) m/s ²
Maximum power	2934(3) W	2827(3) W
Total energy	11611(7) J	11531(6) J

TABLE III: Maximum acceleration, maximum power, and total mechanical energy produced by Bolt in Beijing and Berlin. The standard deviations were calculated using the parameter uncertainties given in Table II.

Figure captions

FIG. 1: Fitted $v(t)$ of Usain Bolt in Beijing 2008 (solid line) and in Berlin 2009 (dashed line).

FIG. 2: Total mechanical power of Usain Bolt in Beijing 2008 (solid line) and in Berlin 2009 (dashed line).



