A mathematical model of the environmental effects on long jump performance of world class athletes

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A Mathematical Model of the Environmental Effects on Long Jump Performance of World Class Athletes

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Abstract

A model is presented that combines experimental data on the relationship between approach velocity and takeoff angle with a five-parameter model to approximate environmental effects on approach velocity and in-flight travel. Results indicate that wind speed provides the greatest influence on jump distance, followed by air density which itself is a product of altitude, temperature, air pressure, and humidity. Local fluctuations in the Earth's surface gravitational field strength are shown to have a slight effect on performance. Previously, analysis attributed the majority of performance increase to faster approach speed and takeoff velocity. These new results suggest a diminishing return to performance from an increase in approach speed.

KEYWORDS: DRAG REDUCTION, WIND AND ALTITUDE ASSISTANCE, DENSITY ALTITUDE, MATHEMATICAL MODELING OF ATHLETIC PERFORMANCES

Introduction

Athletic performances in track and field can be manipulated both physiologically and environmentally. The testing for performance enhancing drugs is an attempt to normalize performances that were manipulated with drugs. Environmental factors are normalized by declaring performances with an excessive tailwind "illegal." The standard in sprints and long jump is to classify performances which benefit from a tailwind of greater than 2.0 m/s as ineligible for recognition as world records. The crude nature of this standard has led to studies that focus on instead adjusting performances with consideration to the effects of atmospheric drag. (see Ward-Smith (1984, 1986), Linthorne (1994b), Mureika (2001, 2003, 2006), Frolich (1984)) Most models examine sprint performances where the longer time interval allows atmospheric drag to provide greater influence than in jumps. However, historic data indicates that both the jumps and sprints may be measurably affected by atmospheric conditions. The 1968 Olympics – which took play in the high altitude of Mexico City -- saw records in both sprints and jumps broken by large margins. The most remarkable performance was recorded in the long jump. Bob Beamon of the United States jumped 8.90 meters, breaking the previous record by more than 0.5 meters. This jump was analyzed by Ward-Smith (1986), who examined primarily the effect of wind and altitude on the speed of the approach before the the jump rather than the flight through the air. Modeling the approach as a sprint provides inaccuracies because instead of approaching at maximum pace, long jumpers use a slower sprint to mitigate biomechanical difficulties of taking off at maximum pace (Seyfarth *et al.*, 2000).

We provide an accurate model for both the in-flight phase and slow sprint approach to the long jump. Analysis concerns differences in performance with respect to relative gravity, physical altitude, density altitude, and tailwind speed. This model can be used to normalize performance to an environmentally neutral locations and to provide information concerning takeoff velocity and the in-flight path of a jumper. The model is split into an aerial phase and an approach phase which provides an approximation for the effect of the environment on takeoff speed and angle.

The Aerial Phase

To accurately simulate atmospheric effects on the long jump, a model should incorporate and allow variation of all parameters of interest. The greatest environmental effect on the long jump is wind speed, which can decrease or increase the force of drag on an athlete. In the aerial portion of the model we will approximate the body of the long jumper as a rigid projectile, as shown in Figure 1.

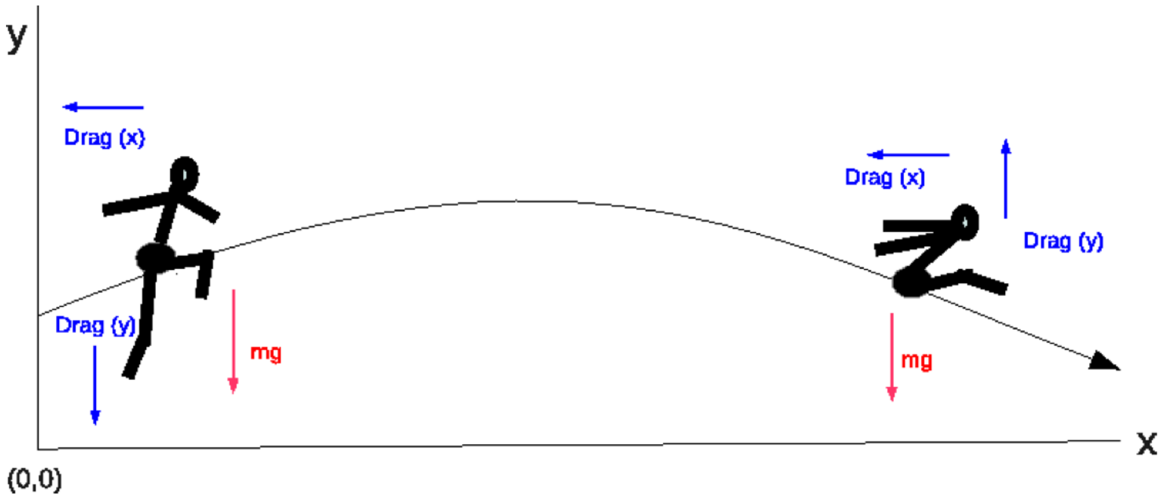

Figure 1. The free body diagram depicts the in-flight forces on the athlete. Specifically, the force of drag parameter (which is resistive in the horizontal direction but contributive in the vertical direction after the peak altitude has been reached) and the force of gravity which always acts in the negative vertical direction.

It is an accurate approximation to assume drag forces are proportional to the squared velocity of the jumper. Analyzing the forces present on the athlete provide the most accurate model, because they can be decomposed into a horizontal and vertical components. It is understood from mechanics that for low wind speed, that only forces that act an in-flight athlete are gravitational acceleration (g) that acts only in the negative vertical direction, and the drag force, which acts in the direction opposing movement. We assume that wind (w) is present in only in the horizontal direction. These assumptions are consistent with the fact that modern athletic stadiums are relatively enclosed, which prevents both extreme horizontal wind and measurable wind in the vertical direction. We can write these equations as a system of force equations for each direction:

$$\frac{d^2x}{dt^2} = -k_x\left(\frac{dx}{dt} - w\right)^2 \qquad \frac{d^2y}{dt^2} = -g - k_y\left(\frac{dy}{dt}\right)\left|\frac{dy}{dt}\right| \tag{1}$$

where $g = 9.8$ m/s^2, w is the wind speed in the horizontal direction (vertical winds are ignored), and k represents the drag parameters in each direction as follows:

$$k_x = \frac{1}{2}\frac{C_d A_x \rho}{m} \qquad k_y = \frac{1}{2}\frac{C_d A_y \rho}{m} \tag{2}$$

where m is the average mass of a long jumper (75 kg), ρ is air density, and C_d is the dimensionless experimentally-determined drag coefficient of 0.6 (Brownlie *et al.* 2004), which is further described in Figure 2. Total drag is greatest when the air flow is neither laminar nor completely turbulent and both forms of drag are present. Material science in sport has provided different methods to reduce the time spent in this partially turbulent state. This can be accomplished by either quickly transitioning to turbulent flow (*e.g.* dimples on a golf ball) or by remaining in laminar flow (*e.g.* the smooth material of a speed skaters suit). Calculations on the material used by elite sprinters resulted in an effective result to the non-dimensional drag coefficient (C) of 0.40%, from 1.0 to 0.6.

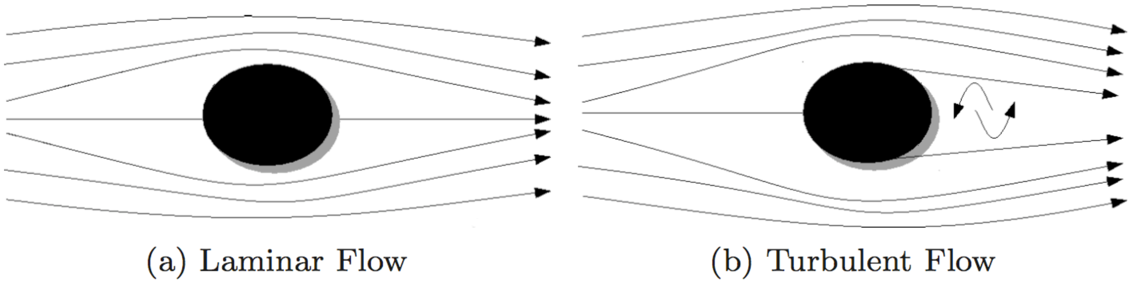

Figure 2. At low velocities (a) friction drag reduces an athletes ability to move throw the atmosphere. As velocity increases the point at which the boundary fluid layer separated farther from the front of the object resulting in a pressure differential which causes form drag. During turbulent flow (b) friction drag is reduced and form drag dominates.

To describe the cross sectional area of an athlete A_x, we first remember that the cross sectional area of a long jumper is time dependent in the horizontal direction. The jumper starts out in an outstretched position but finishes the jump in a crouch. This shrinking of cross sectional area will be approximated with the function $A = (t+2)^{-1}$, where the time (t) varies from zero to about one second, as A varies from 0.5 m^2 to 0.33 m^2. A_y is the horizontal cross sectional area of the athletes body and is approximated as 0.1 square meters.

The parameter most often studied in physical models is wind speed. Mathematically, the effect of the wind speed on the model is the simplest, but also makes the most difference in jump performance. In fact, the wind speed is the only aerodynamically-influential quantity measured at the Olympic Games. An analysis of the effect of wind speed will allow us to understand just how great a difference wind speed can have on an athlete's performance.

Positive wind speeds are tailwinds that decrease the ambient drag force acting on the sprinter. Conversely, negative wind speeds are headwinds into which the athlete runs, increasing the aerodynamic drag. It is important to remember that moderate wind speeds do not aid an athlete by providing a force to "push" the athlete through the air, but acts to counteract drag.

The parameter that is closest to being constant is the value of gravitational acceleration. The value of gravitational acceleration varies due to the distance between the earth's center and the athlete's center of mass. This distance varies with respect to physical altitude and latitude (because of the oblate shape of the earth). Differences in gravitational acceleration are not considered in most models because variation between most track and field venues is negligible. However, that the because the most famous long jump was performed at high altitude and close to the equator in Mexico City justifies the scrutiny of gravity. An approximation for measured gravity can be built up by combining the law of universal gravitation and the international gravity formula (Ceasure, 1987). When both are combined, the following approximation for measured gravity is obtained:

$$g = \frac{9.7803\left(\frac{(1+0.001913)\sin^2\lambda}{\sqrt{(1-0.006694)\sin^2\lambda}}\right)}{1+\frac{h^2}{R^2}} \tag{3}$$

where h is the physical elevation in meters, R is the Earth's average radius, and λ is the latitude. This equation should allow us to approximate the variation in measured gravity with respect to latitude λ and altitude h. To observe whether stadium conditions vary enough to affect the value of g, the measured gravity was calculated with accurate latitude and altitude for different stadiums. When six different major cities are used from around the world the data showed a maximum change in the gravitational acceleration of about .03 ms^{-2}, which is significant enough to affect an athlete's performance (at the centimetre scale).

The differences in measured gravity at six different track stadiums around can be observed in Table 1. The variations in g at different stadiums is significant enough to effect long jump performance, albeit to a small degree. However, to examine long jump performances differences while only varying the gravity is not necessary or realistic, because the factors that effect measured gravity, physical altitude and geographic location also cause variation air density which significantly effect the drag parameter k .

Table 1. Approximate value of the local gravitational acceleration [ms^{-2}] in different world class stadium locations (from Equation 3).

Venue	g [ms^{-2}]
Mexico City, Mexico	9.782
Colorado Springs, USA	9.790
Sydney, Australia	9.795
Los Angeles, USA	9.795
Oslo, Norway	9.801
Hammerfest, Norway	9.811

In past models air density (ρ) is often approximated as only a function of physical altitude. Physical altitude however does not provide as accurate a description of the air density as density altitude (H_ρ) that represents the effective altitude when considerations are made to barometric pressure, temperature and humidity. Mureika (2006c) has shown the following equation to be an accurate representation of density altitude,

$$H_\rho = \frac{T_0}{\Lambda}\left(1 - \left(\frac{RT_o\rho(H)}{\mu P_o}\right)^{\frac{\Lambda R}{g\mu} - \Lambda R}\right) \tag{4}$$

where the parameters are defined as

$\Lambda = 6.5 \times 10^{-3}\,\mathrm{K \cdot m^{-1}}, \quad R = 8.314\ \mathrm{J \cdot K \cdot mol^{-1}}$
$T_o = 288.15\ K, P_o = 101.325\ kPa, \quad g = 9.80\ \mathrm{m \cdot s^{-2}}, \quad \mu = 2.89x10^{-2}\mathrm{kg \cdot mol^{-1}}$

Although long jump performance could be effectively modeled by using density altitude as a parameter, the derivation of the drag parameter requires the air density (ρ) itself to be examined. Assuming ideal gases, air density can be described as

$$\rho = \frac{P - P_v}{R_a T} + \frac{P_v}{R_v T} \quad , \tag{5}$$

where P is the total air pressure, the gas constants are R_a = 287.05, and R_v = 461.50, and the pressure of water vapor P_v is described by the Magnus-Teton equation (Murray, 1967)

$$P_v \approx \left(\frac{H_r}{100}\right) \cdot 10^{7.5T/(237.7+T)} \quad , \tag{6}$$

where H_r is the relative humidity and T is the temperature in degrees Celsius.

This equation for the air density is crucial to our analysis, as it includes three of the parameters often overlooked in environmental analysis of athletic performance, barometric pressure, temperature and humidity. It is important to remember that the barometric pressure that is usually reported in weather forecasts is the corrected sea level pressure, similar to the altitude corrected for air density formula just derived. However, this sea level corrected barometric pressure (usually between 100-102 kPa) is not what we are examining for in the model. Thus, for forecast values, the correction for sea level barometric pressure is necessary, (Lawrence Livermore National Laboratory Meteorological Website, 1997)

$$P_{\text{corrected}} = P_{\text{SL}}\left(\frac{288 - 0.0065h}{288}\right)^{5.2561} . \tag{7}$$

This correction is used in the model to allow inputs to be sea level corrected barometric pressure. This correction allows reported values to be used for the inputs to the third and final parameter in the model air density. As is expected, differences in air density provide a greater effect on performance than gravity but less of an effect than wind speed.

The Approach Phase

The approximation of athletes as rigid bodies in motion during the aerial phase is a predictively-accurate but an incomplete way to model the environmental effects of the long jump. Although, aerial phase mechanics and the environment both affect athlete performance, the biggest influence on jump distance is the takeoff velocity and angle.

The environmental effect on take off velocity requires the approach sprint to be modeled with respect to wind speed and air density. Although sprint models are ubiquitous, modeling this phase in the same manner as a sprint model is inaccurate, because long jumpers do not take off at maximal sprint velocities. By modifying a model originally designed to simulate the 100 m sprint, the long jump preparatory phase can be obtained. Mureika (2001) discusses a quasi-physical model for the 100 m sprint using the following system of differential equations

$$\begin{aligned} \dot{x} &= v(t) \\ \dot{v} &= f_s + f_m - f_v - f_d \end{aligned} \tag{8}$$

Here, (f_s) is the drive term, (f_m) is the maintenance term, (f_v) is the velocity limiting term, and (f_d) is the force of drag. Each term is defined as follows:

$f_s = f_0 \exp(-\sigma t^2)$, This term describes the drive force of the runner. It falls off very quickly with time to mimic the explosive start of a sprinter out of the blocks. This term is left out of the long jump model because long jumpers do not start from blocks, but rather from a "standing" position.

$f_m = f_1 \exp(-c\,t)$. This term describes the maintenance of force for the runner. The exponential being raised to a negative time coefficient means that this term will fall off as time passes (modeling fatigue).

$f_v = \alpha v(t)$ This term serves to limit top speed or leg turnover rate.

$f_d = \frac{1}{2}\rho C_d \frac{A}{m}\left(v(t) - w\right)^2$. This term represents drag. It is decreased by wind speed and lower air density.

This model can be used to simulate the approach or preparatory phase of the long jump if the drive term f_s removed (alternatively setting $f_0 = \sigma = 0$). It is assumed here that the main difference in the long jump run from the sprints is the lack of the driving out of the blocks. If this system is integrated numerically over a 30 meter distance (preparatory phase) it will provide initial velocity which can then be used along with take off angle as the inputs for the aerial phase. Following (Mureika 2001), the following values of the parameters are adopted: $f_1 = 5.15$, $\alpha = 0.323$, $c = 0.0385$, $m = 75$ kg, $C_d = 0.6$, and $A = 0.45$ m^2.

In a purely physical model it would seem advantageous for an athlete to jump at a 45 degree angle while maximizing velocity, since this angle would allow the athlete to maximize the horizontal the velocity necessary to travel forward as well as the vertical velocity necessary to allow the athlete time in the air before landing. However, an athlete approaching with a high velocity will be able to take off with a large horizontal take off velocity, but will not have enough ground contact time to create a large vertical force to allow hang-time. Athletes attempt to provide an increase the vertical force by planting the foot ahead of the body to allow more time to create the force. Unfortunately, the longer the athlete takes to create a vertical force the more horizontal breaking that that will take place. Thus, biomechanical compromises between vertical force and horizontal breaking force elite athletes to favor a take off angle between 20 and 25 degrees. Seyfarth *et al.* (2001) has experimentally determined the take off angle of elite long jumpers to be close to 21 degrees with a take off velocity of slightly less than 10 m/s.

More recently, Bridgett and Linthorne (2006) have conducted experimental research to determine how elite long jumpers take off angle changes with respect to initial velocity. Like Seyfarth *et al.* (2000), the authors found that elite long jumpers jump with take off angles close to 21 degrees, but that angle varies with respect to takeoff velocity. A linear fit experimental data that shows that as takeoff velocity increases takeoff angle decreases. Incorporating this relationship into the approach phase will provide an experimentally verified relationship between the environmentally affected takeoff velocities and take off angle.

Data and Analysis

The described model can generate data that yields information about long jump performance increase in both the aerial and approach phases with respect to the variation of the following inputs: wind speed, altitude, latitude, humidity, temperature, and air pressure. With the exception of wind speed all other inputs are heavily correlated to geographic location or in the case of elite athletes, specific stadiums during the summer and fall months. For this reason analysis will concern the effect of the stadiums most often implicated in changing jump performance. Three different locations with extremal realistic parameters [temperature (ºC), station pressure (kPa), relative humidity (%)] will be considered: Los Angeles, CA [25 ºC, 101.3 kPa, 10 %], Mexico City [35 ºC, 100.0 kPa, 60 %], and Hammerfest, Norway [15 ºC, 102.0 kPa, 2 %]. The cold and dry conditions in the northern location of Hammerfest should produce the greatest performance decreases, while the hot, humid conditions in Mexico should produce the highest performance increases. Los Angeles (a typical sea-level venue, similar to European arenas) will provide the baseline performance for different wind speeds, although it should be noted that aside from the lower relative humidity, these conditions are reflective of many sea level competition stadia.

Table 2. Effect on Performance (cm) for different venues as compared to Los Angeles at the same wind speed (m/s) at 25°C, 101.5 kPa, and 10% humidity. Values in parenthesis are temperature, barometric pressure, and relative humidity of indicated venues (°C, kPa, %).

	Mexico City (35,100,60)			Hammerfest (15,102,2)		
Wind (ms^{-1})	Total (cm)	Flight (cm)	Approach (cm)	Total (cm)	Flight (cm)	Approach (cm)
-4	17	4.3	12.7	-2	-0.4	-1.6
-2	15	5.6	9.4	-2	-0.5	-1.5
0	9.1	2.4	6.7	-1.9	-0.8	-1.1
2	6	1.6	4.4	-0.01	0.79	-0.8
4	6	3.2	2.8	-0.07	0.63	-0.7

The data generated by this model serves to illustrate two important characteristics of the environmental effects on long jump performance. The first effect to note is the actual performance increase without wind. Comparing performances without wind is important because all modern events have wind gauge readings reported, but rarely consider the air pressure or humidity during the event. The model suggests that without wind an athlete in Mexico City can expect about 9 cm increase in performance while the same athlete jumping in Norway should only see a performance loss of 2 cm. This suggests that while the performance increase in Mexico is real, the drop in performance often attributed to cold conditions may be the result of physiological difficultly rather than the environment.

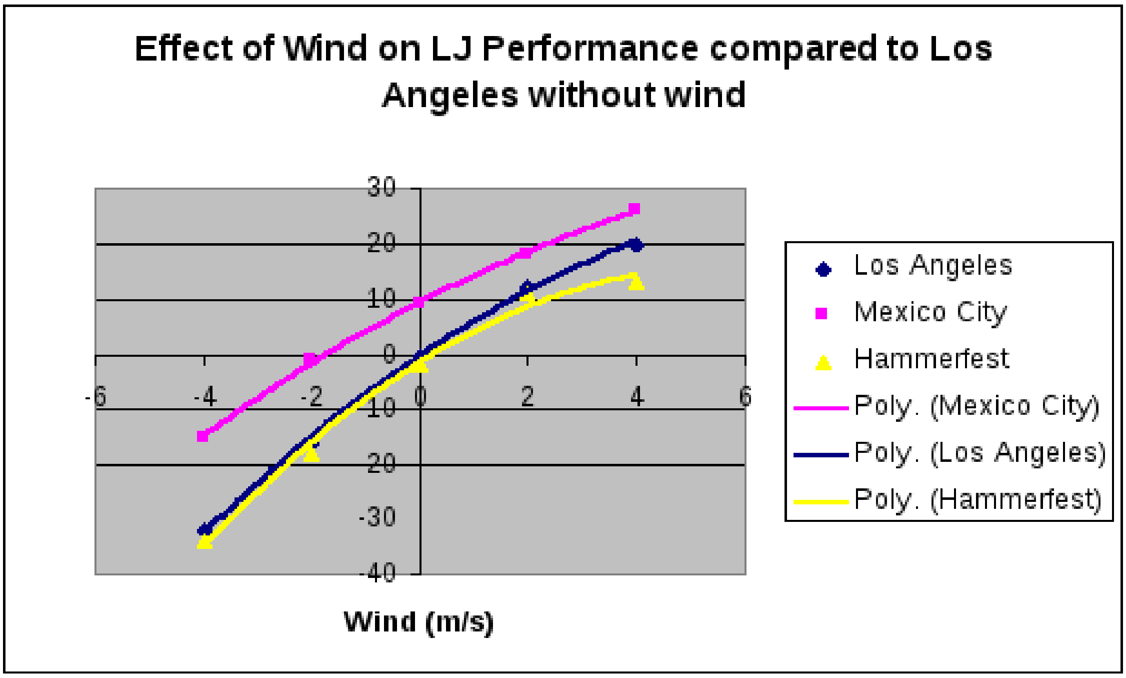

Figure 3. Altitude-based performance correction curves for various wind speeds, as compared to sea-level (Los Angeles) standard.

When the maximum legal wind in Mexico is considered (2 m/s) an elite athlete is expected to benefit from a performance increase of about 18 cm, far less than the value of 31 cm calculated by Ward-Smith (1986b) who attributed the majority of the performance increase to be an effect of the increased sprinting speed at altitude but did not take into account the different jumping angle the sprinting speed would create. Effectively, this model agrees with or is slightly higher than most aerial predictions (Ward Smith, 1986 and Frolich, 1985) but predicts significantly lower values for total performance increase because of the effect of sprint speed on take off angle. It is important to note that without the correction for take-off angle this model actually predicts a total performance of greater than 35 cm on a hot humid day in Mexico, 4 cm more than the difference found by Ward Smith. This difference is due to the effect of humidity, temperature, pressure and the locally-measured value of gravitational acceleration, all of which were not considered in the 1986 paper. It is only because of the confluence of speed and take off angle that the model's total performance increase is far less than what has been previously predicted.

Table 3. Total Performance corrections (cm) compared to performances in Los Angeles without wind.

	Mexico City		Hammerfest		Los Angeles	
Wind (ms^{-1})	**Total (cm)**	**Increase**	**Total (cm)**	**Increase**	**Total (cm)**	**Increase**
-4	-14.9		-33.9		-31.9	
-2	-1		-17.9		-15.9	
0	9.1		-1.9		0	
2	18.1		11.1		12.1	
4	26.1		13.1		20.1	

It is important that while this model suggests unfavorable conditions should not cause performance to decrease drastically, there is a very real (if overestimated) effect on long jump performance in favorable conditions that aided but did not invalidate the 8.90 meter jump of Bob Beamon. The model suggests that Beamon's advantage of 18 cm (2 m/s of wind, compared to sea level without wind in Los Angeles) were responsible for less than a third of the margin between Beamon's jump and the previous record. Even if one examines the possibility of a faulty wind reading and provides Beamon with what would have been a very unusual tailwind (4 m/s) the physical effects of drag would only have produced 26 centimeters of aid compared to jumping at sea level Los Angeles.

The second notable feature of the data generated by the model is the non-linear relationship between wind speed and increased performance. Two reasons serve to create this effect. First, the drag force depends on the square of the relative velocity of the athlete and the air which causes the performance decrease in running into a tailwind much larger than the increase in running in front of a headwind. Second, using Linthorne's linear relationship between take-off speed and angle causes the positive effect of a tailwind to be lessened the faster the athlete is able to move. Although the relationship between speed and angle is approximated to be linear the trigonometric functions used to decompose the speed into separate vectors are non-linear.

Table 4. Top personal men's long jump performances per athlete and corrections (cm) with legal wind aid (w≤ 2 m/s). and illegal wind (w > 2.0 m/s). I = illegal wind, A = legal wind, U = unaided or negligible assitance (w < 0.5 m/s).

Rank	**Name**	**Top Dist (m)**	**Wind (ms^{-1})**	**Legal (m)**	**Wind (ms^{-1})**	**Unaided (m)**	**Wind (ms^{-1})**	**Total Aid (m)**
1 (I)	M Powell	8.99	4.4	8.95	0.3	8.95	0.3	4
2 (U)	M Powell	8.95	0.3	8.95	0.3	8.95	0.3	0
3 (A)	Lewis	8.91	3	8.79	1.9	8.72	-0.3	19
4 (A)	Beamon	8.9	2	8.9	2	8.3	0	50
5 (I)	M Powell	8.9	3.7	8.95	0.3	8.95	0.3	-5
6 (A)	Emmiyan	8.86	1.9	8.86	1.9	8.61	-0.3	25
7 (A)	Lewis	8.79	1.9	8.79	1.9	8.72	-0.2	7
8 (I)	Pedroso	8.79	3	8.7	1.6	8.66	0.3	13
9 (I)	Lewis	8.77	3.4	8.79	1.9	8.72	-0.3	5
10 (A)	Lewis	8.76	1	8.79	1.9	8.72	-0.3	4

Although this model predicts less performance increase than previous models, the experimental data generated by Linthorne is bounded by the data generated by the model. Experimentally, Linthorne found 2 m/s and 4 m/s tailwind to produce about 8 and 14 cm of aid, respectively while the model predicts 12 and 20 cm of aid. The computer model (Figure 3) also agrees with the effect of diminishing gains between take off speed and jump performance that Linthorne also observed.

The model can also be examined parameter by parameter to examine the predictive difference between past models. This model examined variations in gravitational acceleration rather than assuming a constant value, as well as the effect of temperature, humidity and pressure on air density. These considerations proved to slightly improve the environmental aid on a long jumper. However, this model also used a modified sprint model (drive term removed), parameterized the jumpers cross sectional area and used Linthorne's experimental data with respect to takeoff speed and angle. These considerations proved to drastically decrease the expected aid that the environment has on long jump distance. Perhaps most importantly in the world of track and field is the fact that such considerations show that less than half of the distance that Beamon broke the world record jump by can be attributed to Mexico City.

Finally, this model is anecdotally supported through historical data. Although the approach phase of the model was created using a pre-existing sprint model, the correction for speed and angle caused the model to predict significantly less performance aid for the long jump than the 100-meter sprint. If the top 10 performances (including those declared illegal for wind assistance) in the 100-meter sprint and the long jump are examined it is not surprising that every sprinter and eight of ten of the jumpers enjoyed greater than 1 m/s wind assistance. The top ten sprint times were bolstered by an average wind speed of 3.82 m/s which allowed the sprinters an to improve on their top performances with negligible wind speed (less than 0.5 m/s) by an average of 0.22 seconds, the same margin between the top legal 100m sprint and the 258th fastest recorded 100m sprint. In contrast, the long jump performances were achieved with an average tailwind of only 2.46 m/s which supplied the jumpers with an average boost of 12.2 cm (The model predicts 15 cm), which is approximately equivalent to the difference between the top legal long jump and the 4th farthest recorded long jump. Additionally, historical data suggests a generous tailwind provides a greater advantage in the sprints than the jumps. Five of the ten fastest men in any conditions joined the group with wind-assistance, while only one of the top ten longer jumpers of all time achieved his mark with wind assistance.

In closing, we comment on potential future applications of this research. Beyond normalization of performance, the model may be useful for training and technique implementation for different environments. For example, if the environment is very unfavorable to the aerial phase (*i.e.* High relative gravity, large headwind), the athlete may be better served to use a faster takeoff speed and a lower takeoff angle, which would reduce the length of time spent in the aerial phase but provide a faster initial horizontal velocity.

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