Chapter three

Function

<u>3.1 definition :</u> A function from A to B is a triple of objects < f , A , B > , where A and B are classes and f is a sub class of A x B with the following properties .

F1 . $X \in A$, $y \in B$ such that $(x, y) \in f$

F2 . If $(x, y1) \in f$ and $(x, y2) \in f$, then $y1 = y2$

It is customary to write $f : A \to B$ instead of $< f , A , B >$

Thus , F1 state that

Every element $X \in A$ has an image $y \in B$ F2 states that if $X \in A$, then

The image of X is unique :-

For if $(x, y) \in f$ and $(x, y2)$ that is , if y1 and y2 are both images of X , then F2 dicates that

$y1 = y2$, It follows that F1 and F2 combined state that .

Every element $X \in A$ has a uniquely determined image $y \in B$

<u>Theorem (1) :-</u> let A & B be classes and let f be a graph . then $f : A \to B$ is a function if and only if

(i) F2 holds (ii) dom F = A , and (iii) ran f $\subseteq$ B

<u>Theorem (2) :-</u> let $f : A \to B$ and $g : A \to B$ be function , Then $f = g$ if and only if $f(x) = g(x)$, $\forall$ $X \in A$

Proof :- first , let us assume that f = g , then , for arbitrary

$X \in A$, $y = f(x) \leftrightarrow (x, y) \in f$

$\leftrightarrow (x, y) \in g$

$\leftrightarrow y = g(x)$: thus , $f(x) = g(x)$

Conversely , assume that $f(x) = g(x)$, $X \in A$, Then

$(x, y) \in f \leftrightarrow y = f(x)$

$\leftrightarrow y = g(x)$

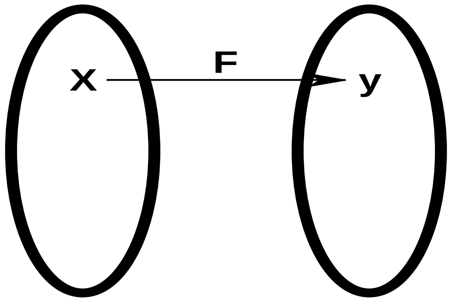
$\leftrightarrow$ (x , y) $\in$ g thus , f = g .

Let f : A → B be a function if (x , y) $\in$ f , we say that y is the image of X (with respect to f ;

we also say that X is the pre-image of y (with respect to f) ; we also say that f maps X on to y

, and symbolize this statement by $x \xrightarrow{F} y$

(the reader may , if he wishes , picture these state as in Fig 1)



F maps X on to y

y is the image of X

Fig (i)

X is the : pre-image of y

<u>Injective , sujective and Bi jective function :-</u>

<u>Definition :-</u> A function f : A → B is said to be injective if it has the following property .

INJ . If (X1 , y) $\in$ f and (X2 , y) $\in$ then X1 = X2

$$X_1 \rightarrow y \leftarrow X_2 \qquad\qquad X_1 \rightarrow f(x) = f(X_2) \leftarrow X_2$$

IN J :- If f (X1) = f (X2) , then X1 = X2

IN J :- y has no more than one pre-image .

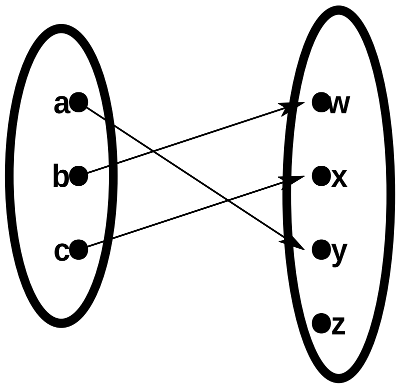
Definition :- A function f : A → B is said to be surjective if it has the following property : -

SUR J :- ∀ y ∊ B , ∃ ∊ A ∍ y = f (x)

F : A → B is surjective if and only if ran f = B

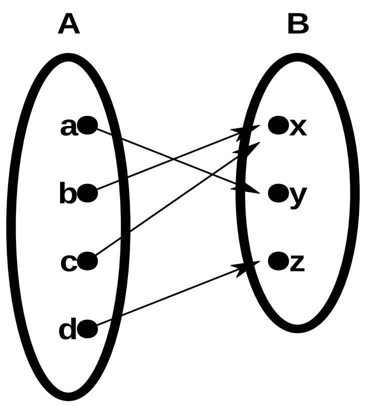
Every element of B is the image

Of at least one element of A .



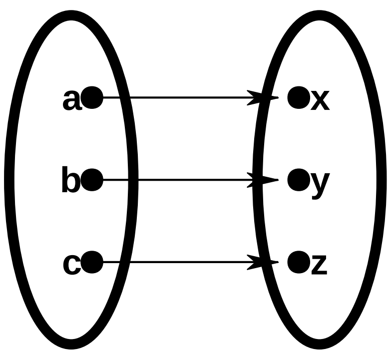
F is injective

F i g (2)



F is surjective

F i g (3)



F is Bijective

F ig (4)

<u>Definition :-</u> A function $f = A \rightarrow B$ is said to be bijective if it is both injective and surjective .

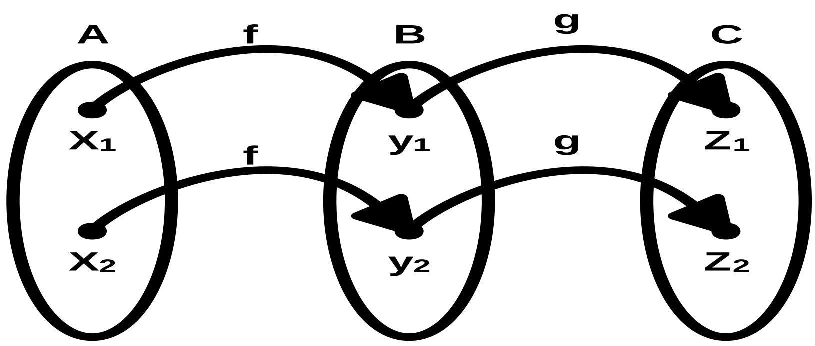
To say that $f : A \rightarrow B$ is injective is to say that every element of B is the image of on more than one element of A : to say that f is surjective is to say that every element of B is the image of at least one element of A ; thus , to say that f is bijective is to say that every element of B is the image of exactly one element of A (Fig . 4)

In other words if $f : A \rightarrow B$ is a bijective function , every element of A has exactly one image in B and every element of B has exactly one pre-image in A ; thus all the element of A and all the element s of B are . associated in pairs ; for this reason if f is bijective , it is some times called a one-to-one correspond dence between A & B .

<u>Properties of composite function and inverse function :-</u>

Theorem :- If $f : A \rightarrow B$ and $g : B \rightarrow C$ are function , then $g \circ f : A \rightarrow C$ is function .

$$[g \circ f](x) = g[f(x)]$$

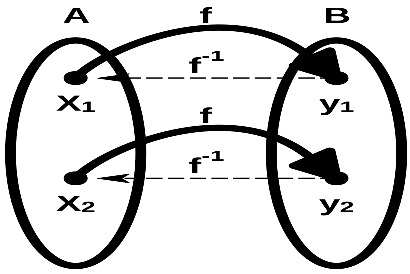


$$Z1 = (g \circ f)(X1) = g(f(X1))$$

$$Z2 = (g \circ f)(X2) = g(f(X2))$$

<u>Definition :-</u> A function $f : A \rightarrow B$ is said to be invertible if $f^{-1} : B \rightarrow A$ is a function

$Y = f(x)$ If and only if $X = f^{-1}(y)$



<u>Theorem :-</u> If f : A → B is a bijective function then f^{-1} = B → A is a bijective function .

<u>EX :-</u> let the function f and g be defined by f(x) = 2x + 1 and g (x) = $X^2 - 2$, find the formula defining the composition function g o f .

Compute g o f as follows : (g o f) (x) = g (f (x))

$$= g(2x + 1) = (2x + 1)^2 - 2$$

$$= 4X^2 + 4X - 1$$

Observe that the same answer can be found by writing y = f (x) = 2X + 1 and Z = g (y) = $y^2 - 2$

And the eliminating y from both equation .

$$Z = y^2 - 2 = (2x + 1)^2 - 2 = 4X^2 + 4X - 1$$