Discrete mathematics

Syllabus

- Chapter One: - “classes & sets “

- Chapter Two: - “Relation “

- Chapter Three: - “functions “

- Chapter Four: - “vectors & matrices “

- Chapter five: - “Graph theory”

- Chapter six: - “planar Graphs”

- Chapter Seven: - “Directed Graphs “

- Chapter Eight: - “combinatorial Analysis “

Chapter one

Classes & sets

Definition (1) :- let A & B be classes :- we define A = B to mean that every element of A is an element of B and vice versa

In symbols:-

$$A = B \text{ if } X \in A \rightarrow X \in B \text{ and } X \in B \rightarrow X \in A$$

Definition (2):- let A & B be classes:- we define $A \subseteq B$ to mean that every element of A is an element of B

In symbols

$$A \subseteq B \quad \text{if} \quad X \in A \rightarrow X \in B$$

Theorem (1) :- for all classes A , B & C , the following hold :-

(i) $A = A$

(ii) $A = B \rightarrow B = A$

(iii) $A = B \;\&\; B = C \rightarrow A = C$

(iv) $A \subseteq B \;\&\; B \subseteq A \rightarrow A = B$

(v) $A \subseteq B \;\&\; B \subseteq C \rightarrow A \subseteq C$

Definition :-

The union :-

$$A \cup B = \{ X : X \in A \text{ or } X \in B \}$$

The intersection :-

$$A \cap B = \{ X : X \in A \text{ and } X \in B \}$$

Theorem (2) : for every class A , the following hold

(i) $\emptyset \subseteq A$ — U = The Universal set

(ii) $A \subseteq U$ — $\emptyset$ = The empty set

Theorem (3) : If A & B are any classes , then

(i) $A \subseteq A \cup B$ and $B \subseteq A \cup B$

(ii) $A \cap B \subseteq A$ and $A \cap B \subseteq B$

Theorem (4) :- If A & B are classes , then

(i) $A \subseteq B$ if and only if $A \cup B = B$

(ii) $A \subseteq B$ if and only if $A \cap B = A$

Theorem (5) :- (Absorption laws) for all classes A and B

(i) $A \cup (A \cap B) = A$

(ii) $A \cap (A \cup B) = A$

Theorem (6) :- for every class A ,

$$(A')' = A$$

Proof : - $X \in (A')' \rightarrow X \notin A' \rightarrow X \in A$

$X \in A \rightarrow X \notin A' \rightarrow X \in (A')'$

Theorem (7) :-

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof :-

First , $X \in (A \cup B)' \rightarrow X \notin A \cup B$

$\rightarrow X \notin A$ and $X \notin B$

$\rightarrow X \in A'$ and $X \in B'$

$\rightarrow X \in (A' \cap B')$

Next , $X \in (A' \cap B') \rightarrow X \in A'$ and $X \in B'$

$\rightarrow X \notin A$ and $X \notin B$

$\rightarrow X \notin A \cup B$

$\rightarrow X \in (A \cup B)'$

<u>Theorem (8)</u>:- For all classes A, B and C, the following are there

(1) Commutative laws :- (i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

(2) Idempoteat laws :- (iii) $A \cup A = A$

(iv) $A \cap A = A$

(3) Associative laws :- (v) $A \cup (B \cup C) = (A \cup B) \cup C$

(vi) $A \cap (B \cap C) = (A \cap B) \cap C$

(4) Distributive laws :- (vii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(viii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

<u>Theorem (9)</u>:- for every class A

(i) $A \cup \emptyset = A$ (ii) $A \cap \emptyset = \emptyset$

(iii) $A \cup u = u$ (iv) $A \cap u = A$

(v) $u' = \emptyset$ (vi) $\emptyset' = u$

(vii) $A \cup A' = u$ (viii) $A \cap A' = \emptyset$

<u>EX (1) :-</u> prove that $A \cap (A' \cup B) = A \cap B$

Proof :- $A \cap (A' \cup B) = (A \cap A') \cup (A \cap B)$ (8 - vii)

$= \emptyset \cup (A \cap B)$ (9 - viii)

$= A \cap B$ (9 - i)

<u>Definition :-</u> $A - B = A \cap B'$

<u>EX (2) :-</u> prove that $A - B = B' - A'$

Proof :- $A - B = A \cap B'$ definition

$= B' \cap A$ (8 – ii)

$= B' \cap (A')'$ (6)

$= B' - A'$ Definition of $B' - A'$

Ordered pairs Cartesian products :-

Theorem (10) :- IF $\{ X , y \} = \{ u , V \}$, then

$[X = u \text{ and } y = V]$ or $[X = V \text{ and } y = u]$

Theorem (11) :- If $(a , b) = (c , d)$ Then

$a = c$ and $b = d$

Theorem (12) :- for all classes A , B and C

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(iii) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Proof :-

(ii) $(X , y) \in A \times (B \cap C) \leftrightarrow X \in A$ and $y \in B \cap C$

$\leftrightarrow X \in A$ and $y \in B$ and $y \in C$

$\leftrightarrow (x, y) \in A \times B$ and $(X , y) \in A \times C$

$\leftrightarrow (x , y) \in (A \times B) \cap (A \times C)$

(i) $(x , y) \in (A \times B) \cap (C \times D)$

$\leftrightarrow (x , y) \in A \times B$ and $(x , y) \in C \times D$

$\leftrightarrow X \in A$ and $y \in B$ and $X \in c$ and $y \in D$

$\leftrightarrow X \in A \cap C$ and $y \in B \cap D$

$\leftrightarrow (x, y) \in (A \cap C) \times (B \cap D)$

Graphs : -

A class of ordered pairs is called a graph

Def (1):- If G is a graph , then G^{-1} is the graph defined by :-

$$G^{-1} = \{ (x , y) \mid (y , x) \in G \}$$

Def (2) :- If G and H are graphs , then G o H is the graph defined as follows :-

$$G \circ H = \{ (x , y) \mid \exists Z \ni (x , y) \in H \text{ and } (z , y) \in G \}$$

Theorem (13) :- If G , H and J are graphs , then the following statements hold

- **(i)** $(G \circ H) \circ J = G \circ (H \circ J)$
- **(ii)** $(G^{-})^{-1} = G$
- **(iii)** $(G \circ H)^{-1} = H^{-1} \circ G^{-1}$

Proof :-

(i) $(x , y) \in (G \circ H) \circ J \leftrightarrow \exists z \ni (x , z) \in J \text{ and } (z , y) \in G \circ H$

$\leftrightarrow \exists Z \ni (x, z) \in J \text{ and } (z , w) \in H \text{ and } (w, y) \in G$

$\leftrightarrow w \ni (x , w) \in H \circ J \text{ and } (w , y) \in G$

$\leftrightarrow (x , y) \in G \circ (H \circ J)$

(ii) $(x , y) \in (G^{-1})^{-1} \leftrightarrow (y , x) \in G^{-1}$

$\leftrightarrow (x , y) \in G$

(iii) $(x , y) \in (G \circ H)^{-1} \leftrightarrow (y , x) \in G \circ H$

$\leftrightarrow \exists Z \ni (y , z) \in H \text{ and } (Z , x) \in G$

$\leftrightarrow \exists Z \ni (x , z) \in G^{-1} \text{ and } (z , y) \in H^{-1}$

$\leftrightarrow (x , y) \in H^{-1} \circ G^{-1}$

Theorem (14) :- If G and H are graphs , then

- **(i)** dom G $=$ ran G^{-1}
- **(ii)** ran G $=$ dom G^{-1}
- **(iii)** dom $(G \circ H) \subseteq$ dom H
- **(iv)** ran $(G \circ H) \subseteq$ ran G

proof :-

(i) $X \in \text{dom}\, G \leftrightarrow \exists\, y \ni (x, y) \in G$

$\leftrightarrow \exists\, y \ni (y, x) \in G^{-1}$

$\leftrightarrow X \in \text{ran}\, G^{-1}$

(ii) $X \in \text{dom}\,(G \circ H) \rightarrow \exists\, y \ni (x, y) \in (G \circ H)$

$\rightarrow \exists\, Z \ni (x, z) \in H$ and $(z, y) \in$

$\rightarrow X \in \text{dom}\, H$