Chapter two

Relations

2-1 Introduction :-

(a , b) = (b , a) unless a = b

Further more , (a , b) = (c , d) if and only if a = c , b = d

2-2 product sets :-

A x B read A cross B

A x B = { (a , b) : a ϵ A and b ϵ B }

A^2 instead of A x A

EX :- A = (1 , 2) & B = (a , b , c)

A x B = { (1 , a) , (1 , b) , (1 , c) , (2 , a) , (2 , b) , (2 , c) }

B x A = { (a , 1) , (a , 2) , (b , 1) , (b , 2) , (c , 1) , (c , 2) }

A x A = { (1 , 1) , (1 , 2) , (2 , 1) , (2 , 2) }

There are two things worth noting in the above example : -

First of all :- A x B ≠ B x A

Secondary , n (A x B) = 6 = 2 – 3 = n (A) n (B)

The idea of a product of sets

A1 X A2 ………… An or πAi i = 1

2-3 Relations :-

If (x , y) ϵ R x R y

If (x , y) ϵ R $x \not{R} y$

The domain of a relation R is the set of all first element of the ordered pairs which belong to R , and the range of R is the set of second elements .

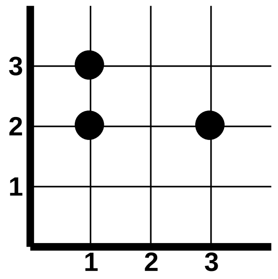
EX :- Let A = { 1 , 2 , 3 } , and R = { (1 , 2) , (1 , 3) , (3 , 2) }

Then R is Relation on A since it is a subset of A x A .

1 R2 , 1 R3 , 3 R2 , but $1 \not R 1$, $2 \not R 1$, $2 \not R 2$, $2 \not R 3$, $3 \not R 1$, $3 \not R 3$

The domain of R is { 1 , 3 } and the range of R is { 2 , 3 }

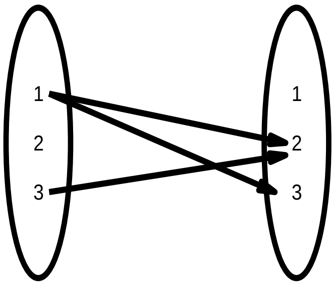
2-4 :- pictorial Representations of Relations :-



(i)

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{array}$$

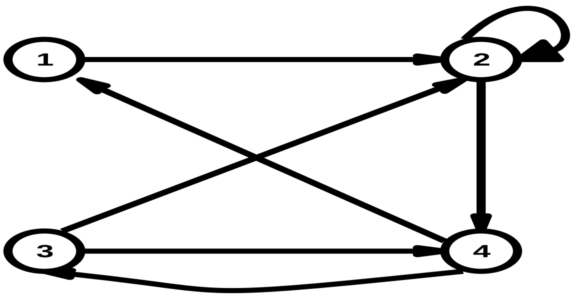
(ii)



(iii)

Coordinate diagram of A x A (matrix of the relation) (arrow diagram of the relation)

R = { (1 , 2) , (1 , 3) , (3 , 2) }



Directed graph of the relation

R = { (1 , 2) , (2 , 2) , (2 , 4) , (3 , 2) , (3 , 4) , (4 , 1) , (4 , 3) }

<u>EX(1) :-</u> Let A = { eggs , milk , corn } and B = { cows , goats , hens }

We can define a relation R form A to B by (a , b) ϵ R if a is produced by b .

Solution :-

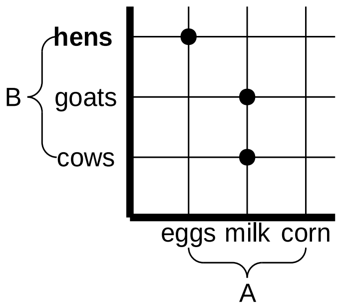
R = { (eggs , hens) , (milk , cows) , (milk , goats) }

With respect to this relation

Eggs R hens , milk R cows , milk R goats

domain of R = { eggs , milk }

the range of R = { hens , cows , goats }



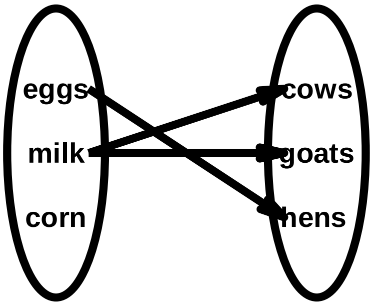
(i)

(ordinate diagram of A x B)

$$\begin{array}{c|ccc} & \overbrace{\text{cows} \quad \text{goats} \quad \text{hens}}^{B} & & \\ \text{eggs} & 0 & 0 & 1 \\ A \quad \text{milk} & 1 & 1 & 0 \\ \text{corn} & 0 & 0 & 0 \end{array}$$

(ii)

(matrix of the relation)



(iii)

(arrow diagram of the relation)

<u>EX(2):-</u> suppose we say that two countries are adjacent if they have some part of their boundaries in common . Then " is adjacent to" is a relation R on the countries of the earth .

<u>Sol .</u>

(Italy , Switzerland) ϵ R but (Canada , Mexico) ∉ R

2-5 Inverse Relation :-

Let R be relation from A to B . Then inverse of R denoted by R^{-1} , is the relation from A to B which consists R those ordered pairs which when reversed belong to R :-

$R^{-1} = \{ (b , a) : (a , b) \in R \}$

In the other words , b R^{-1} a if and only if a R b

<u>EX(1) :-</u> A = { 1 , 2 , 3 }

R = { (1 , 2) , (1 , 3) , (2 , 3) }

R^{-1} = { (2 , 1) , (3 , 1) , (3 , 2) }

<u>EX(2):-</u> Then inverse of the relation defined by

“ X is taller than Y “

“ X is shorter than Y “

<u>Solu:-</u>

R^{-1}

$$M_R = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \& \quad M R^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} = MR^T$$

<u>2-6 :- Composition of Relation :-</u>

IF R C A x B

& S C B x C

R & S a relation from A to C denoted by R o S

Defined by :-

A (R o S) C if for some b ϵ B we have a R b & b S c

R o S = { (a , c) : There exists b ϵ B for which (a , b) ϵ R and (b , c) ϵ S }

The relation R o S is called the composition of R and S

<u>EX:-</u> Let A = { 1 , 2 , 3 , 4 } , B = { a , b , c , d } , C = { x , y , z }

and let R = { (1 , a) , (2 , d) , (3 , a) , (3 , b) , (3 , d) }

S = { (b , x) , (d , z) , (c , y) , (d , z) }

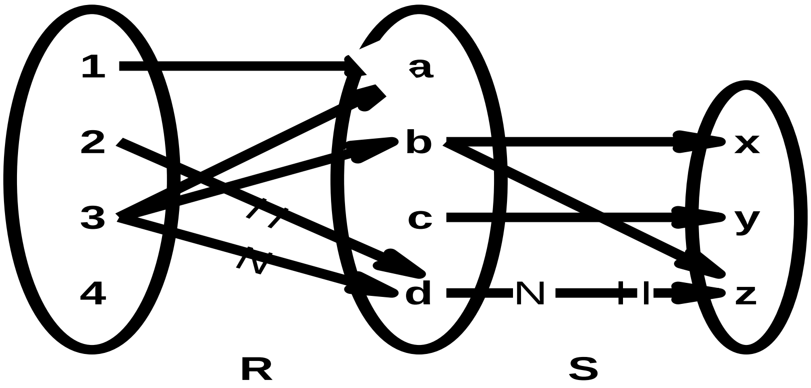


Fig (arrow diagram of R and S)

We can view two arrows as “ a path “

Similarly 2 (R o S) Z since 2Rd and dSz

3 (R o S) X

3 (R o S) z

No other element of A is connected to an element of C . Accordingly

R o S = { (2 , z) , (3 , x) , (3 , z) }

MR & Ms denote respectively the matrices of the relation R & S , Then .

$$M_R = \begin{array}{c|cccc} & a & b & c & d \\ \hline 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array} \quad \& \quad M_s = \begin{array}{c|ccc} & x & y & z \\ \hline a & 0 & 0 & 0 \\ b & 1 & 0 & 1 \\ c & 0 & 1 & 0 \\ d & 0 & 0 & 1 \end{array}$$

$$M = M_R \, M_s = \begin{array}{c|ccc} & x & y & z \\ \hline 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 3 & 1 & 0 & 2 \\ 4 & 0 & 0 & 0 \end{array}$$

M = MR Ms = MR o S

<u>Theorem 2.1 :-</u> let A , B , C and D be sets , suppose R is a relation from A to B , S is a relation from B to C and T is a relation from C to D Then (R o S) o T = R o (S o T)

2-7 properties of relation :-

(1) R is reflexive if a R b for every a in A
(2) R is symmetric if a R b implies b R a
(3) R is anti – symmetric if a R b and b R a implies a = b
(4) R is transitive if a R b and b R c implies a R c

<u>EX(1) :-</u> consider the relation C of set inclusion on any collection C of sets . note that :-

(1) A C A for any set A in C , S o C is reflexive .
(2) A C B does not imply B C A , S o C is not symmetric .
(3) If A C B and B C A then A = B , S o C is anti – symmetric
(4) If A C B and B C C then A C C , S o C is transitive .

(we assume that C has more the one set)

<u>EX (2) :-</u> consider the relation = of quality on any set A . Note that = satisfies all four of the above properties a ϵ A .

(1) a = a for any element a ϵ A
(2) if a = b then b = a
(3) if a = b and b = a then a = b
(4) if a = b and b = c then a = c

<u>EX (3) :-</u> consider the relation R = { (1 , 1) , (1 , 2) , (2 , 1) , (2 , 3) } on A = { 1 , 2 , 3 } , Then .

(1) 2 is in A but 2 R 2 is not reflexive .
(2) 2 R 3 but 3 R 2 but is not symmetric
(3) 1 R 2 and 2 R 1 but 1 ≠ 2 is not anti symmetric .
(4) 1 R 2 and 2 R 3 but 1 R 3 S o R is not transitive .

<u>Partition :-</u> let S be any nonempty set , A partition of S is a subdivision of S into non overlapping , non empty subset precisely , a partition of S a collection [Ai] of non empty subset of S such that :-

(i) Each a in S belongs to one of the A .
(ii) The sets of [Ai] are mutually disjoint .

The subset in a partition are called cells .

<u>EX :-</u> consider the following collection of subsets of S = { 1 , 2 , ... 8 , 9 }

(i) [{ 1 , 3 , 5 } , { 2 , 6 } , { 4 , 8 , 9 }]
(ii) [{ 1 , 3 , 5 } , { 2 , 4 , 6 , 8 } , { 5 , 7 , 9 }]

(iii) [{ 1 , 3 , 5 } , { 2 , 4 , 6 , 8 } , { 7 , 9 }]

Then :-

- **(i) is not a partition of S since 7 in S does not belong of any of the subsets , furthermore .**
- **(ii) Is not a partition of S since { 1 , 3 , 5 } and { 5 , 7 , 9 } are not disjoint on the other hand .**
- **(iii) Is a partition of S .**