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Global Visco-Acoustic Full Waveform Inversion

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Summary

Full Waveform Inversion aims to determine parameters of the subsurface by minimising the misfit between the simulated and recorded seismic data. The quality of such fit depends on many different aspects, as for example, the inversion algorithm and the accuracy of the constitutive laws. The latter is particularly important as if there are factors that are not taken into account in the seismic simulation then the inversion algorithm will compensate for their existence in the parameter(s) being estimated. One of such factors is attenuation. Here we introduce an approach that jointly estimates velocity and attenuation using a combination of Quantum Particle Swarm Optimisation with the conventional gradient descent method. This hybrid approach takes advantage of the fact that it is sufficient to estimate smooth models of Q and for this reason these can be represented with a sparse support, thus decreasing substantially the number of weights of the basis functions that have to be estimated and making the use of global algorithms practical. We demonstrate that the proposed method mitigates cross-talk between velocity and attenuation, while allowing the convergence towards accurate models of attenuation and velocity, thus being an effective method for velocity model building and consequently for seismic imaging.

Introduction

Full Waveform Inversion (FWI) has been applied successfully to a wide collection of data sets. The vast majority of examples found in the literature focuses mainly on using constitutive laws that account only for media that are perfectly elastic. This is, the conversion of potential to kinetic energy, and vice-versa is instantaneous. Nonetheless, it is observed that geological materials exhibit a dependency on the temporal-scale of the source of energy and for this reason seismic energy is dissipated and dispersed as a consequence of attenuation. Thus, when inverting data, and especially in the context of fitting the entire seismic record, as in the case of FWI, the constitutive law should include this important effect as the waveforms are attenuated and distorted as a consequence of attenuation. Otherwise the estimates of the velocity model incorporate the unaccounted effects of attenuation introducing an error in the inverted velocity.
FWI requires the solution of the wave equation for a given rheology. This can be obtained in the frequency or in the time-domain. In the case of media with attenuation, the frequency domain has a natural advantage as the attenuation, given by the quality factor Q, is represented by an imaginary component of the velocity. However, frequency-domain methods require the solution of a linear system, that can be solved by a factorization method, thus scaling poorly with the size of the problem, or they require the implementation of specific pre-conditioners for the physics of the wave equation being solved. Thus time-domain approaches are preferred. In the time-domain, visco-acoustic (or visco-elastic) media are generally described by a relaxation mechanism. The Standard Linear Solid (SLS) or Zener's body is the most widely used mechanism. It is observed that Q is constant over the frequency range of interest in seismic experiments. The invariance of Q over the frequency range is obtained combining several of these relaxation mechanisms (Blanch et al., 1995). This has an additional computational cost as it leads to a larger number of equations that need to be time-stepped numerically.
Operto et al., (2015), and Plessix et al. (2016) presented examples of the application of FWI to data sets affected by attenuation. The cited approaches focus on the minimisation of the objective function by updating the model parameters iteratively with the gradient of the objective function. FWI is a high-dimensional inverse problem and for this reason the use of first order local optimisation methods (e.g. steepest descent) is generally preferred. Nonetheless, this type of approach leads generally to cross-talk when several classes of parameters are being estimated. This phenomenon is also observed when jointly estimating velocity and attenuation (Hak and Mulder, 2010; Plessix et al., 2016).
Attenuation is generally a parameter that varies smoothly in space, thus the reconstruction of the long wavelengths of this parameter is sufficient in order to predict both the kinematics and dynamics of wave propagation. Because of this property (low spatial variation), this parameter can be represented in a sparse basis, leading to a small support (the space of the basis functions) hence reducing dramatically the dimension of the parameter space for Q. This makes the use of global optimisation methods practical in this scope.
Here we present a hybrid algorithm that allows to jointly estimate velocity and attenuation combining Quantum Particle Swarm Optimisation (QPSO) (Sun et al., 2004) and the conventional FWI algorithm using a steepest descent for the local updates of velocity (Debens et al., 2015).
Here we present a synthetic example comparing the proposed approach with local optimisation methods for the joint estimation of velocity and attenuation, demonstrating that the outlined hybrid scheme does not suffer from cross-talk and estimates the true model with a very reasonable accuracy, leading to the an estimate of velocity that is comparable to the one if the true attenuation was known *a priori*.

Forward modelling approach

The propagation of pressure waves in a visco-acoustic medium is described by (Carcione et al., 1988):

$$\frac{\partial^2 p}{\partial t^2} = M_r \dot{G}(t) * \left(\nabla \cdot \left(\frac{1}{\rho} \nabla p \right) + s(t) \right) \tag{1}$$

where the time dependency of the constitutive law is given by,

$$G(t) = \left[1 - \frac{1}{L}\sum_{l=1}^{L}\left(1 - \frac{\tau_{\varepsilon,l}}{\tau_{\sigma,l}}\right) e^{-t/\tau_{\sigma,l}}\right] H(t) \tag{2}$$

where, M_R is the relaxed bulk modulus, $\tau_{\varepsilon,l}$ is the strain relaxation time for the l-th relaxation body, $\tau_{\sigma,l}$ is the stress relaxation time for the l-th relaxation body and $H(t)$ is the Heaviside function. The method of memory-variables (Carcione et al., 1988) is used for time-stepping the wave equation 1.

Hybrid Inversion Algorithm

The assumption being made here is that, the most likely model of Q is the one that leads to the largest decrease of the functional when optimising for velocity. Thus, the quality of each attenuation model is quantified by the amount of decrease of the misfit function. The velocity of pressure waves is updated with the steepest descent method, where the gradient is computed with the adjoint-state method. As the attenuation is estimated in a sparse support, the global update of the attenuation is carried out only in the low frequency region of the spectrum. Once this estimate is completed, FWI is carried out for velocity only up to the highest frequency in the source wavelet, keeping the quality factor constant throughout the successive iterations. The hybrid inversion algorithm is described as follows:

1. Generate a random population of attenuation models
2. Choose a starting velocity model for FWI
3. Estimate the goodness of each one of the attenuation models iterating locally over velocity with FWI.
4. Update the information of the population of Q models with QPSO (Shu et al., 2004)
5. Update the velocity model that will be used by each particle in the subsequent iteration.
6. Repeat steps 2 and 3 until the stopping criteria is met (number of iterations, level of misfit,…)
7. Update the velocity model with local FWI using the best velocity model determined at step 5, and the Q model estimated in steps 1 to 6.

Example

In this example it is demonstrated an application of the outlined method. Figure 1a shows the Marmousi model (true velocity model) and figure 2a shows the true Q model. One can observe that the two models do not have a spatial correlation thus making this example very suitable for testing the ambiguity of the inverse problem for multi-parameters. Figure 1b shows the starting velocity model.
The synthetic data is generated with the wave equation 1, for 345 sources spaced 24 m along the horizontal direction and at 6 m of depth. The temporal history of the source function is given by a Ricker wavelet with a peak frequency of 8 Hz. The receiver geometry is fixed with receivers placed every 12 m along the horizontal direction and at 120 m of depth, resembling an Ocean Bottom Cable (OBC) configuration. A free-surface boundary condition is imposed at the top of the model, and absorbing boundaries are used at the lateral and bottom boundaries.
The inversion for velocity and attenuation is carried out with different approaches in order to demonstrate the relevance of the hybrid scheme. The hybrid inversion is carried out at 3Hz using a swarm of 20 particles and 21 iterations. The local iterations with conventional FWI are carried out starting at 3 Hz and widening the frequency band 1 Hz, at each 6 local iterations, up to 16 Hz, thus representing a total of 84 iterations.
Figure 1c, shows the inverted velocity model, when it is assumed that the true Q model is known, demonstrating the convergence towards a model that is quantitatively and qualitatively correct. Figure 1d shows the inverted velocity model, and figure 2b) shows the inverted attenuation model, when the velocity and the attenuation are jointly updated with a local scheme. The starting model of Q is homogeneous with $Q = 10^5$. The inverted Q model converges to a model that is qualitatively close to the true model as its background is close to the true model. Nonetheless, one can observe that some of the sharp features from the velocity model are mapped into the inverted model of attenuation. In addition the inverted velocity model shows high-frequency artefacts and qualitatively one can observe that the resulting model is blurred in comparison to the model in figure 1c. This is a clear example when cross-talk between the parameters occurs. In this case, the sharp events in Q can also generate

reflections and diffractions, and the inversion scheme build these events in the Q model in order to fit events in the data that were originally created by features in the velocity model. When starting from Q obtained from the hybrid scheme, the model of Q inverted locally, also shows strong evidences of cross-talk (figure 2c). Thus the use of a better approximation for the starting model does not mitigate the cross-talk when jointly inverting for velocity and attenuation with a first order method.
Figure 1e shows the inverted velocity model when it is assumed that there is no attenuation affecting the data, this is, the inversion is carried out locally for velocity only and without using a model of

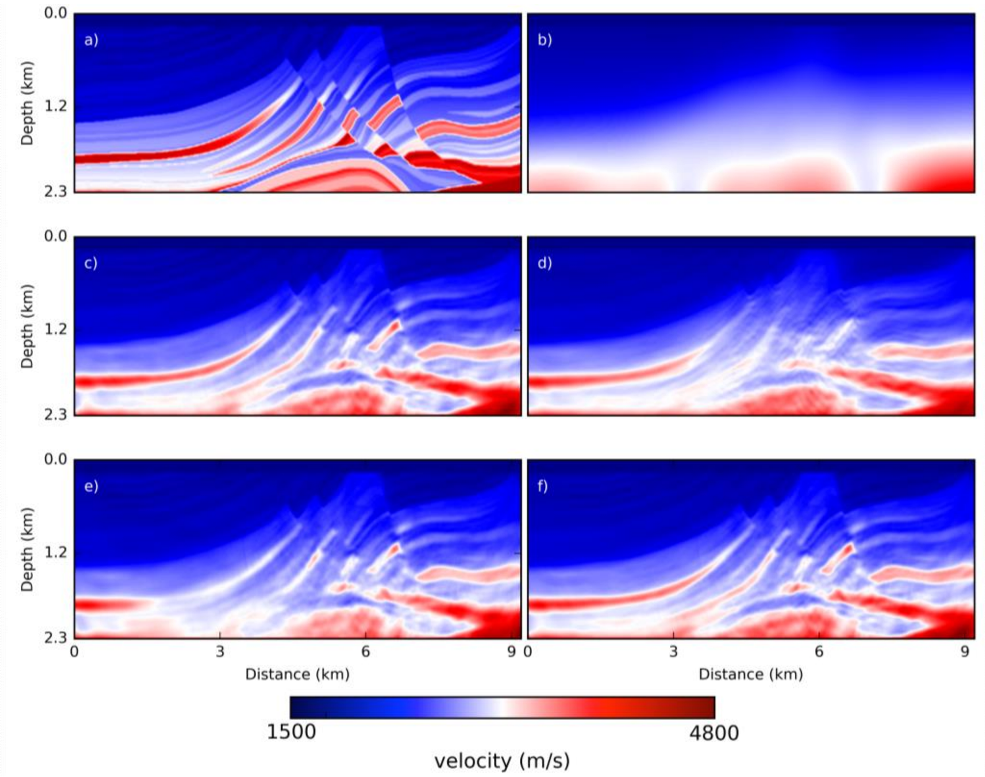

Figure 1 *a) Marmousi velocity model, b) starting velocity model, c) inverted velocity model using the true Q model (figure 1a), d) inverted velocity model inverting for both velocity and attenuation (inverted Q model is in figure 1b) using the steepest descent method, e) inverted velocity model using a constant Q model and keeping it fixed throughout the iterations, f) inverted velocity model using the inverted Q model from the hybrid inversion scheme (figure 1c).*

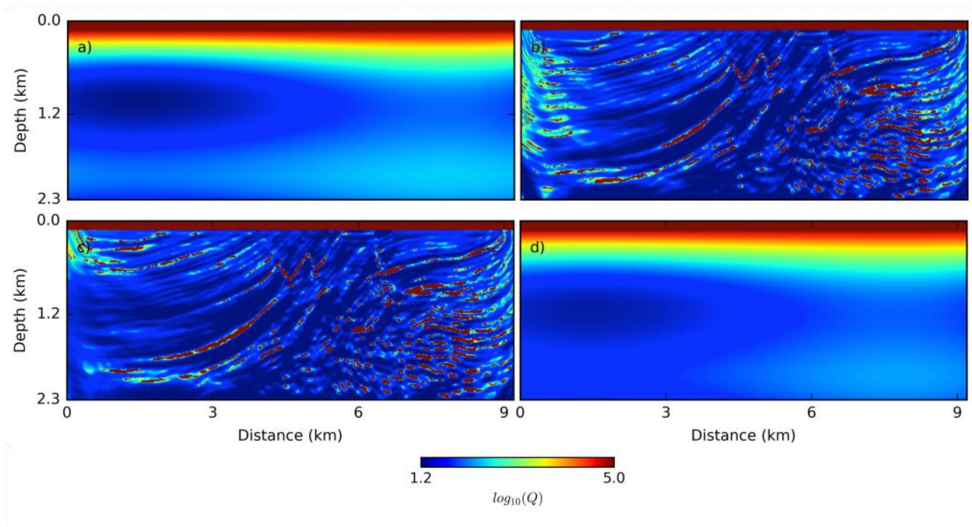

Figure 2 *a) True model of Q, b) inverted model of Q using the steepest descent method, c) inverted model of Q using the outlined hybrid scheme.*

attenuation for simulating the data. In this case, the inversion still converged to a model that keeps some of the main features of the true model. Nonetheless, the lower left region of the model is slower than the true model, and consequently some of the higher frequency anomalies are smeared. This is because in the presence of attenuation, the energy propagates slower. Thus, when inverting data that is affected by attenuation, and if the attenuation is not compensated in the inversion scheme, the kinematics of the estimated velocity model is affected by the slower events in the recorded data, also leading to slower velocity where the attenuation is higher.
The model of Q inverted with the hybrid scheme (figure 2d) demonstrates an excellent agreement with the true model, with respect to the reconstruction of the anomalies quantitatively and qualitatively. In addition there is no evidence of cross-talk of the velocity model into the reconstructed attenuation model. The model of attenuation in figure 2d is then used to carry out the inversion for velocity only (figure 1f) using conventional FWI, while keeping this model of attenuation fixed. Even though the inverted Q model (using the hybrid scheme) has some differences in comparison with the true model, the inverted velocity model has no noticeable differences in comparison with the inverted velocity model when the true model of attenuation is known (figure 2c).

Conclusions and discussion

We introduced a hybrid scheme for joint full waveform inversion of velocity and attenuation. The outlined approach allows estimating accurately the long wavelengths of attenuation. Unlike conventional joint updates, where both velocity and attenuation are updated locally, the introduced hybrid scheme is robust to cross-talk. This approach can thus be extended to any other types of rheology as for instance when the effect of anisotropy exists in the data.

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