4CCS1ELA: Tutorial list — Week 7

1. Consider the following predicate symbols with which we associate the following meanings:

- $S(x)$ represents "x is a student"
- $L(x)$ represents "x is a lecture"
- $A(x, y)$ represents "x attends y"

Provide a first-order formula encoding the following sentence:

"*At least one student attended every lecture*"

(be careful, this sentence is ambiguous. . . , provide a formula for each of the two different meanings)

2. Consider the following predicate symbols with which we associate the following meanings:

- $B(x)$ means "x is a bird"
- $W(x)$ means "x is a worm"
- $E(x, y)$ means "x eats y"

Using these predicates, represent in first-order logic each of the following statements:

(i) *Every bird eats every worm*
(ii) *Some birds do not eat some worms*
(iii) *No bird is eaten by a worm*
(iv) *Some worms do not get eaten by birds*

3. Identify which occurrences of variables in the formulas below are free and which occurrences are bound. Justify you answers.

(i) $y \geq 0 \ \wedge \forall x(N(x) \to x \geq y)$
(ii) $x \geq 0 \ \wedge \forall x(N(x) \to x \geq y)$
(i) $\forall x(N(x) \to \exists y(N(y) \wedge x \geq y))$

Here N is a unary predicate symbol, $\geq$ is a binary predicate symbol in infix notation, and $x \geq y$ is an atom in infix notation (infix notation means that the predicate symbol appears in between the terms).

4. Let ϕ be a well-formed formula (wff), i.e. a σ-formula belonging to $\mathcal{L}[\sigma]$, interpreted over the domain set D and $d \in D$. Then $\phi(x/d)$ denotes the wff obtained from ϕ by replacing all *free* occurrences of X in ϕ by d.

Compute the following substitutions and determine the meaning (the truth-values) of the resulting sentences over the set $\mathbb{N} = \{0, 1, 2, \dots\}$ of natural numbers. Here $N(x)$ denotes "x is a natural number", predicates $\geq$ and $>$ have their usual interpretation and are expressed with their usual infix notation.

(i) $\big(y \geq 0 \ \wedge \forall x(N(x) \to x \geq y)\big)(y/3)$
(ii) $\big(x \geq 0 \ \wedge \exists y(N(y) \wedge x \geq y)\big)(x/3)$
(iii) $\big(\forall x(N(x) \to \exists y(N(y) \wedge x > y))\big)(x/3)$
(iv) $\big(\forall x(N(x) \to \exists y(N(y) \wedge y > x))\big)(y/3)$