4.1 Let X and Y be sets such that $|X| = m$ and $|Y| = n$ for some $m, n \in \mathbf{N}^+$. How many one-to-one functions are there from X to Y?

4.2 We survey 205 students to see whether they are taking courses in three areas: computer science, mathematics and physics. The results show that there are 5 students who don't take any of these, 90 take computer science, 110 take mathematics, and 60 take physics. Further, 20 students take both computer science and mathematics, 20 take computer science and physics, and 30 take mathematics and physics. How many students take courses in all three areas?

4.3 A drawer contains 12 brown socks and 12 black socks, all unmatched. A man takes socks out at random in the dark.

(a) How many socks must he take out to be sure that he has at least two socks of the same colour?

(b) How many socks must he take out to be sure that he has at least two black socks?

4.4 How many ways are there to choose a bag of 6 donuts from 8 varieties,

(a) if there are no two donuts in the bag of the same variety?

(b) if all donuts in the bag are of the same variety?

(c) if there are no restrictions on the contents of the bag?

(d) if there must be at least two varieties in the bag?

(e) if there must be at least three blueberry-filled donuts in the bag ?

(f) if there can be no more than two blueberry-filled donuts in the bag ?

4.5 A group contains 5 men and 5 women. How many ways are there to arrange these people in a row where the men and women alternate?

4.6 How many different words can be made by rearranging the letters in the word $BANANA$?

4.7 Prove Pascal's identity using the defining formula of $\binom{n}{k}$ (see lecture slides 129 and 127).