Live Q&A 3 Transcript

[pardon my handwriting!]

Preliminaries

- If you missed last week, please attend the replacement session if you can!
- Thanks to everyone that submitted the feedback form on KEATS. The poll closed yesterday night but I promise that by next week we will discuss it.
- This week you are asked again to give feedback on the SGTs

Exercises

3.3 For each of the following functions, determine whether…

(a) one to one?

$\forall x, y \quad x \neq y \rightarrow x+1 \neq y+1$ We need a general argument

$\forall x+1 = y+1$

$x = y$

not onto as it doesn't cover all Natural numbers

0 is not covered.

(b) not one to one e.g. two different pairs

e.g: (3,4) → 7

(2,5) → 7

onto? i.e: every natural number can be obtained by the sum of two natural numbers.

e.g. $0 + 0 = 0$

3.4 Give an example…

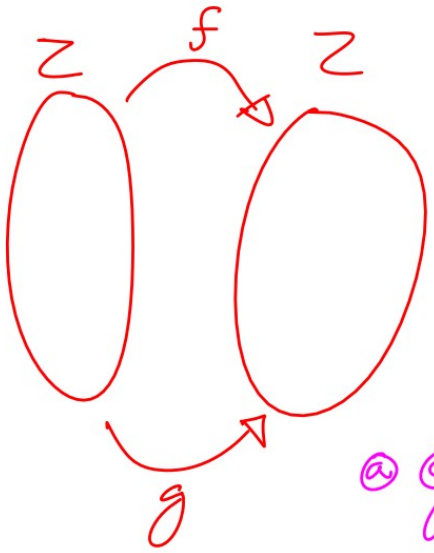
(a) $f(n) = 2n$ ⇒ no odd numbers

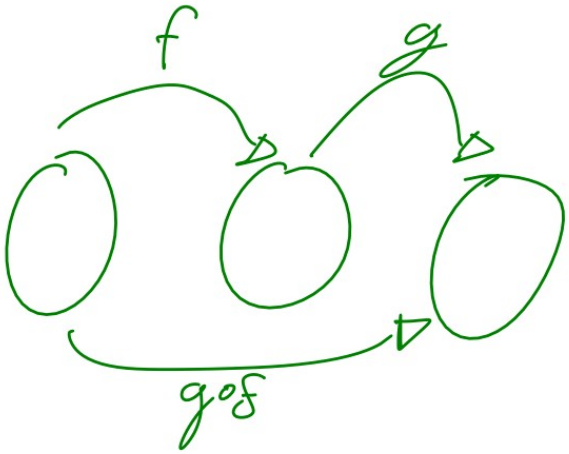
(b) $$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{(n-1)}{2} & \text{if } n \text{ is odd} \end{cases}$$ ⇒ Multiple values in domain mapped in codomain.

(c) $$f(n) = \begin{cases} n & \text{if } n \text{ is odd} \\ n+2 & \text{if } n \text{ is divisible by 4} \\ n-2 & \text{if } n \text{ is even but not divisible by 4.} \end{cases}$$ ⇒ all numbers are mapped and have a 1 to 1 relation

$f(n) = 42$ ⇒ everything is mapped to 42.

3.5 Let *f* and *g* be…





ⓐ $(g \circ f)(k) = g(f(k)) = \begin{cases} g(k-3) & \text{if } k > 10 \\ g(k^2) & \text{if } k \leq 10 \end{cases}$

ⓑ $(g \circ f)(-8) = g(f(-8)) = 64$

$(g \circ f)(11) = 11 - 3 = 8$

$(g \circ f)(14) = 14 - 3 + 1 = 12$ ⇒ Aji's example

3.7 Prove that Z is a countable set by...

Every negative number can be mapped to an odd number, whilst positive numbers can be mapped to even numbers.

$$f(n) = \begin{cases} 2n & \text{if } n \geq 0 \\ -2n-1 & \text{if } n < 0 \end{cases}$$