

4CCS1ELA—ELEMENTARY LOGIC WITH APPLICATIONS

3—IMPORTANT SEMANTICAL NOTIONS

3.2—QUINE'S METHOD AND SATISFIABILITY

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Outline

1. Quine's Method

2. Satisfiability

QUINE'S METHOD

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Quine's Method

For any formula W and propositional variable P :

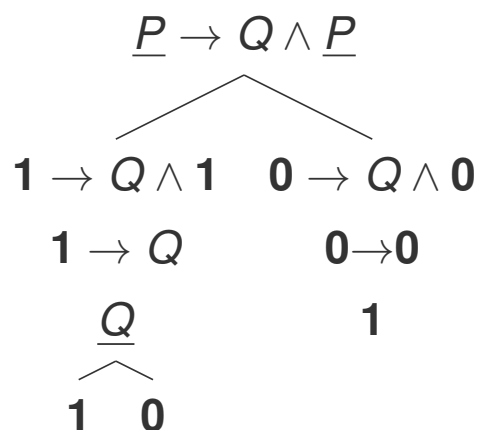
- W is a tautology if and only if $W(P/0)$ and $W(P/1)$ are tautologies.
- W is a contradiction if and only if $W(P/0)$ and $W(P/1)$ are contradictions.

Quine's method can be described graphically with a binary tree (Hein, Section 6.2).

- When no propositional symbols remain:

- W is a tautology if all of the leaves in the tree are true (i.e., **1**)
- W is a contradiction if all leaves in the tree are false (i.e., **0**)
- Otherwise, W is a *contingency* (i.e., sometimes true, sometimes false)

Example



Conclusion. The formula $P \rightarrow Q \wedge P$ is a contingency, because neither all leaves in the tree are true, nor all of them are false.

SATISFIABILITY

Satisfiability

Satisfiability

A formula F is **satisfiable** if there is an interpretation v that makes the formula F true. In this case, we say that v **satisfies** F .

A set $\mathcal{S} = \{A_1, \dots, A_n\}$ of propositional formulae is **satisfiable** (**consistent**) if there is an interpretation v satisfying *every* formula in \mathcal{S} .

	p_1	\dots	p_m	A_1	\dots	A_n
v	e_1	\dots	e_m	1	\dots	1

p_1, \dots, p_m are all the propositional symbols appearing in \mathcal{S} .

The set of formulae $\{A_1, \dots, A_n\}$ is satisfiable if, and only if, the conjunction $A_1 \wedge \dots \wedge A_n$ is satisfiable

Example

Let $A_1 = P \rightarrow Q$, $A_2 = Q \rightarrow R$ and $A_3 = R \rightarrow P$ and \mathcal{S} be the set of formulae $\{A_1, A_2, A_3\}$. The combined truth-table for \mathcal{S} is:

	p_1	p_2	p_3	A_1	A_2	A_3
	P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$R \rightarrow P$
v_0	0	0	0	1	1	1
v_1	0	0	1	1	1	0
v_2	0	1	0	1	0	1
v_3	0	1	1	1	1	0
v_4	1	0	0	0	1	1
v_5	1	0	1	0	1	1
v_6	1	1	0	1	0	1
v_7	1	1	1	1	1	1

Thus, \mathcal{S} is satisfiable, since v_0 and v_7 satisfy every formula in \mathcal{S} (we can also say that \mathcal{S} is *consistent*).

Models

A *model* is an interpretation that makes a formula (or set of formulae) true.

We denote the fact that v is a model of A by $v \models A$.

The set of all models of a formula A is denoted by $\text{mod}(A)$. We use the same symbol for a set of formulae \mathcal{S} , as before.

Thus, in the Example of Slide 3.2.8 we have that $\text{mod}(\mathcal{S}) = \{v_0, v_7\}$.

Example

Let \mathcal{S} be the set of formulae $\{P \leftrightarrow \neg Q, Q \leftrightarrow R, R \leftrightarrow P\}$. The truth-table for its formulae is:

	P	Q	R	$P \leftrightarrow \neg Q$	$Q \leftrightarrow R$	$R \leftrightarrow P$
v_0	0	0	0	0	1	1
v_1	0	0	1	0	0	0
v_2	0	1	0	1	0	1
v_3	0	1	1	1	1	0
v_4	1	0	0	1	1	0
v_5	1	0	1	1	0	1
v_6	1	1	0	0	0	0
v_7	1	1	1	0	1	1

\mathcal{S} is **not** satisfiable (i.e., \mathcal{S} is inconsistent) and thus, $\text{mod}(\mathcal{S}) = \emptyset$.

To know more...

- Quine's method is shown in Section 6.2 of Hein's "Discrete Structures, Logic, and Computability", 4th edition.
- Satisfiability can be found in Sections 1.2 and 1.3 of Gabbay and Rodrigues' "Elementary Logic with Applications, 1st edition.