

Exercise 1: asymptotic notation

$10n$ is: $O(n)$ $O(n^2)$ $O(\log n)$ $\Theta(n^2)$ $\Theta(n)$

$n/2 + 5 \log n$ is: $O(n)$ $O(n^2)$ $\Theta(n \log n)$ $\Theta(n^2)$ $\Theta(n)$

$7n^3 - 9n^2$ is: $O(n)$ $O(n^2)$ $\Omega(n^2)$ $O(\log n)$ $\Theta(n^2)$ $\Theta(n^3)$

Circle all correct answers.

Exercise 2

The following Java method determines whether the elements in a given range of array `arr` are all unique.

```
public static boolean isUniqueLoop(int[] arr, int start, int end) {  
    for ( int i = start; i < end; i++ )  
        for ( int j = i+1; j <= end; j++ )  
            if ( arr[i] == arr[j] )  
                return false; // the same element at locations i and j  
    // all elements are unique  
    return true;  
}
```

What is the worst-case running time of this method, in terms of the number n of elements under consideration ($n = \text{end} - \text{start} + 1$) ?

Is there a better (faster) way to find out if all elements are unique?

```
public static boolean isUniqueLoop(int[] arr, int start, int end) {  
    for ( int i = start; i < end; i++ )  
        for ( int j = i+1; j <= end; j++ )  
            if ( arr[i] == arr[j] )  
                return false; // the same element at locations i and j  
    // all elements are unique  
    return true;  
}
```

operations
3
 $2n+1$
 $2n+1$
 $2n+2n+1$
1
1
total:

$$\begin{aligned} & 5 + (2n+1)(2n+1)(2n+2n+1) \\ &= (4n^2+4n+1)(4n+1)+2 \\ &= 16n^3+4n^2+16n^2+4n+4n+1+2 \\ &= 16n^3+20n^2+8n+8 \end{aligned}$$

Better solution: Put all elements in an array in a HashSet and compare the length. If they are truly all unique, then the HashSet and array length will be the same value.

Exercise 3

The following Java method determines whether the three sets of integers, given in arrays a, b and c, have a common element.

```
public static boolean haveSameElement(int[] a, int[] b, int[] c) {  
    for ( int i=0; i < a.length; i++ )  
        for ( int j=0; j < b.length; j++ )  
            for ( int k=0; k < c.length; k++ )  
                if ( (a[i] == b[j]) && (b[j] == c[k]) )  
                    return true; // a common element found  
    // no common element  
    return false;  
}
```

What is the worst-case running time of this method, if each array is of size n ?

Is there a better (faster) way to find out if the arrays have a common element?

#operations

3
 $2n+1$
 $2n+1$
 $2n+1$
7
1
1

$$(2n+1)^3 + 7 + 5$$

$$=(4n^2 + 4n + 1)(2n + 1) + 12$$

$$= 8n^3 + 4n^2 + 8n^2 + 4n + 8n + 1 + 12$$

$$= 8n^3 + 12n^2 + 12n + 13$$

Better solution:

Put the arrays in HashSets.

Put array a and array b together in one HashSet

and then array b and array c in one HashSet.

Compare the lengths of array a and array b to the length of HashSet a and b.

Likewise with array b and array c with HashSet b and c.

If they differ in length, put HashSet a and b and HashSet b and c in a new HashSet.

If the length of HashSet a and b and HashSet b and c differ to the new HashSet, there is a common element.

Exercise 4

Design the following algorithm and implement it as a Java method

Algorithm *countOnes*(*A*, *n*)

Input two-dimensional $n \times n$ binary array *A* (each entry is either 0 or 1)

Output two-dimensional $n \times n$ array *S*, where *S*[*i*][*j*] is the number of 1's in the subarray with the top-left corner at (0,0) and the bottom-right corner at (*i*,*j*).

Example. Input:

	0			<i>j</i>	
	1	0	1	1	0
	0	1	1	1	0
<i>i</i>	1	0	1	0	1
	1	1	0	0	1
	0	0	1	1	0

Output:

	1	1	2	3	3
	1	2	4	6	6
<i>i</i>	2	3	6	8	9
	3	5	8	10	12
	3	5	9	12	14

What is the running time of your algorithm in terms of n ?

Try to obtain the running time as low as you can.

The target running time is $\Theta(n^2)$.

```
1 import java.util.Arrays;
2 /**
3  * Write a description of class Test here.
4  *
5  * @author (your name)
6  * @version (a version number or a date)
7  */
8 public class HelloWorld
9 {
10     public int[][] countOnes()
11     {
12         int[][] array = {
13             {1, 0, 1, 1, 0},
14             {0, 1, 0, 1, 0},
15             {1, 1, 1, 0, 1},
16             {1, 1, 0, 0, 1},
17             {0, 0, 1, 1, 0}
18         };
19         int count = 0;
20         int[][] oneCount = array;
21         for (int i = 0; i < array.length; i++) {
22             for (int j = 0; j < array[i].length; j++) {
23                 if (array[i][j] == 1) {
24                     count++;
25                 }
26                 oneCount[i][j] = count;
27             }
28         }
29         System.out.println(Arrays.deepToString(array));
30         return oneCount;
31     }
32 }
33
34
35
36
37
38
```