Summary of natural deduction rules

Basic rules

		Conjunction		
($\wedge$I)	$\dfrac{A, B}{A \wedge B}$		$\dfrac{A, B}{B \wedge A}$	($\wedge$I)
($\wedge$E)	$\dfrac{A \wedge B}{A}$		$\dfrac{A \wedge B}{B}$	($\wedge$E)

		Disjunction		
($\vee$I)	$\dfrac{A}{A \vee B}$		$\dfrac{B}{A \vee B}$	($\vee$I)
($\vee$E)	$\dfrac{A \to C, B \to C, A \vee B}{C}$			

		Implication		
($\to$I)	If $\dfrac{\text{assumptions}, A}{B}$ then $\dfrac{\text{assumptions}}{A \to B}$ (antecedent of the "If … then" shown in a subcomputation box)		$\dfrac{A, A \to B}{B}$ (also known as modus ponens)	($\to$E)

		Negation		
($\neg$I)	$\dfrac{A \to B, A \to \neg B}{\neg A}$		$\dfrac{\neg A \to B, \neg A \to \neg B}{A}$	($\neg$E)

		Equivalence		
($\leftrightarrow$I)	$\dfrac{A \to B, B \to A}{A \leftrightarrow B}$		$\dfrac{A \leftrightarrow B}{(A \to B) \wedge (B \to A)}$	($\leftrightarrow$E)

Variant rules

		Disjunction		
($\vee$E1)	$\dfrac{A \vee B, \neg A}{B}$ (disjunctive syllogism)		$\dfrac{A \vee B, \neg B}{A}$ (disjunctive syllogism)	($\vee$E2)
(CD)	$\dfrac{A \to C, B \to D, A \vee B}{C \vee D}$ (constructive dilemma)		$\dfrac{A \to C, B \to D, \neg C \vee \neg D}{\neg A \vee \neg B}$ (destructive dilemma)	(DD)

		Implication		
($\to$I1)	$\dfrac{\neg A}{A \to B}$		$\dfrac{B}{A \to B}$	($\to$I2)
($\to$E1)	$\dfrac{A \to B}{\neg A \vee B}$			

		Negation		
($\neg$E1)	$\dfrac{\neg \neg A}{A}$		$\dfrac{\neg A \to B, A \to B}{B}$	($\neg$E2)
($\neg$I1)	$\dfrac{A}{\neg \neg A}$		$\dfrac{A \to B, \neg A \to B}{B}$	($\neg$2)