

## 4CCS1ELA: Tutorial list 3

1. Consider the connective  $\downarrow$  (called the Pierce arrow or NOR), and defined by the truth-table below:

$P$	$Q$	$P \downarrow Q$
1	1	0
1	0	0
0	1	0
0	0	1

As you can see, the formula  $P \downarrow Q$  is true when both  $P$  and  $Q$  are false, and it is false otherwise.

- (i) Show that  $P \downarrow Q$  is logically equivalent to  $\neg(P \vee Q)$ .
- (ii) Show that  $P \downarrow P$  is logically equivalent to  $\neg P$ .
- (iii) Show that the connective  $\downarrow$  is adequate for propositional logic (in other words, show that the set of logical connectives consisting only of the connective  $\downarrow$  is functionally complete).

2. Which of the following propositional formulas are substitution instances of the formula below?

$$P \rightarrow (Q \rightarrow P)$$

If a formula is indeed a substitution instance, give the formulas substituted for  $P, Q$ .

- (i)  $\neg R \rightarrow (R \rightarrow \neg R)$
- (ii)  $\neg R \rightarrow (\neg R \rightarrow \neg R)$
- (iii)  $\neg R \rightarrow (\neg R \rightarrow R)$
- (iv)  $(P \wedge Q \rightarrow P) \rightarrow ((Q \rightarrow P) \rightarrow (P \wedge Q \rightarrow P))$
- (v)  $((P \rightarrow P) \rightarrow P) \rightarrow ((P \rightarrow (P \rightarrow (P \rightarrow P)))) ?$

3. Remember the formula  $(P \vee \neg R) \rightarrow \neg(\neg Q \vee R)$  seen in Tutorial list 1. Find a disjunctive normal form for this formula using Quine's tree.

4. Remember the following system specification used in Tutorial list 1.

*Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded.*

Determine whether this specification is consistent.