Instituto Tecnológico y de Estudios Superiores de Monterrey
Campus Monterrey

MA3014
Estancia de Investigación Control de Qbits para Cómputo Cuántico
(Grupo 577)

Actividad: Qubit Control 1

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Follow the instructions

Quantum objects must be controlled in order to implement some operations on them. The purpose of this activity is to model the dynamics of a qubit interacting with a uniformed magnetic field. This constitutes a first step in the quantum control to implement quantum gates.

The energy of a qubit is electric field can be modeled using the following operator

$$H = \frac{\Delta}{2} Z + V(t) X_+ + \overline{V}(t) X_-$$

Where the operators

$X_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $X_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

The constant Δ might represent the magnitude a constant field in the z-direction and the complex time-dependent function V is known as the control field that will be useful to manipulate the dynamics.

In order to find the time-evolution of the qubit the Schrödinger equation

$$i \frac{d}{dt} |\psi\rangle = H |\psi\rangle$$

(Note that we are in a free-units system) This is second order matrix differential equation. This means that given an initial state

$|\psi(0)\rangle = a_0 |0\rangle + b_0 |1\rangle$, where $a_0 = \cos\theta_0, \quad b_0 = e^{i\varphi_0} \sin\theta_0$ (0)

The solution of equation () will provide the state of the system at any time, that is to say,

$$|\psi(t)\rangle = a(t)|0\rangle + b(t)|1\rangle$$

Where the coefficients as time-dependent functions provide a curve in the Bloch sphere.

The Schrödinger equation is not exactly solvable for any control field V. Thus, in this activity we provide a solution for the simple case $V(t) = 0$, that is to say,

$$H = \frac{\Delta}{2} Z \tag{1}$$

The correspondent time-dependent qubit can be computed as

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle$$

Where U is the time-evolution operator and reads

$$U(t) = \exp(-iHt) \tag{2}$$

A) Using the Taylor series of the exponential function show that U can be written as

$$U(t) = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{(2k)!} H^{2k} - i \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)!} H^{2k+1} \tag{3}$$

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Control de Qbits 1

A) Using the Taylor series of the exponential function show that U can be written as:

$$U(t) = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{(2k)!} H^{2k} - i \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)!} H^{2k+1} \quad (3)$$

Note:
U is the time-evolution operator and its defined as

$$U(t) = \exp(-iHt) = e^{-iHt}$$

Usaremos la serie de Taylor ó MacLaurin fuction para funciones exponenciales de la siguiente manera

$$e^x = \cancel{\frac{x^0}{0!}}\, 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \quad \text{desde } [0, \infty)$$

$x = -iHt$

$$e^{-iHt} = \frac{(-iHt)^0}{0!} + \frac{(-iHt)}{1!} + \frac{(-iHt)^2}{2!} + \frac{(-iHt)^3}{3!} + \dots$$

Para facilitar la estructura de la solución separaremos la operación en la suma de la agrupación de todos los pares y de los impares.

$$= \left(\frac{(-iHt)^0}{0!} + \frac{(-iHt)^2}{2!} + \frac{(-iHt)^4}{4!} \right) + \left(\frac{(-iHt)}{1!} + \frac{(-iHt)^3}{3!} + \frac{(-iHt)^5}{5!} \right)$$

$$= \left((-1)^0 + \frac{(-1)^2 H^2 t^2}{2!} + \frac{(-1)^4 H^4 t^4}{4!} \right) + i \left(\frac{(-1) Ht}{1!} + \frac{(-1)^3 H^3 t^3}{3!} + \frac{(-1)^5 H^5 t^5}{5!} \right)$$

Igualamos valores de (-1) para ambos conjuntos

$$= \left((-1)^0 + \frac{(-1)^2 H^2 t^2}{2!} + \frac{(-1)^4 H^4 t^4}{4!} \right) + i(-1) \left(\frac{(-1)^0 Ht}{1!} + \frac{(-1)^2 H^3 t^3}{3!} + \frac{(-1)^4 H^5 t^5}{5!} \right)$$

$$= \left((-1)^0 + \frac{(-1)^2 H^2 t^2}{2!} + \frac{(-1)^4 H^4 t^4}{4!} \right) - i \left(\frac{(-1)^0 Ht}{1!} + \frac{(-1)^2 H^3 t^3}{3!} + \frac{(-1)^4 H^5 t^5}{5!} \right)$$

Los números pares podemos expresarlos como $\sum_{k=0}^{\infty} = 2k$
y los impares como $\sum_{k=0}^{\infty} = 2k+1$
Por lo que podemos definir:

$$\boxed{= \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{(2k)!} H^{2k} - i \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)!} H^{2k+1}}$$

B) Write the operator $\boldsymbol{H}$ in its matrix form and show

$$H^{2k} = \begin{pmatrix} \Delta^{2k}/2^{2k} & 0 \\ 0 & \Delta^{2k}/2^{2k} \end{pmatrix}$$

$$H^{2k+1} = \begin{pmatrix} \Delta^{2k+1}/2^{2k+1} & 0 \\ 0 & -\Delta^{2k+1}/2^{2k+1} \end{pmatrix}$$

B) Write the operator H in its matrix form and show

$$H^{2k} = \begin{pmatrix} \frac{\Delta^{2k}}{2^{2k}} & 0 \\ 0 & \frac{\Delta^{2k}}{2^{2k}} \end{pmatrix}$$

$$H^{2k+1} = \begin{pmatrix} \frac{\Delta^{2k+1}}{2^{2k+1}} & 0 \\ 0 & \frac{\Delta^{2k+1}}{2^{2k+1}} \end{pmatrix}$$

Considering that

$V(t) = 0$

$$H = \frac{\Delta}{2}\cdot Z + \cancel{V(t)X_+}^{\,0} + \cancel{V^*(t)X_-}^{\,0}$$

$$H = \frac{\Delta}{2}\cdot Z$$

← Producto con Matriz de Pauli

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H = \frac{\Delta}{2}\cdot\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \frac{\Delta}{2} & 0 \\ 0 & -\frac{\Delta}{2} \end{pmatrix}$$

Tomando en cuenta la propiedad de exponentes aplicados a matrices con diagonal:

$$H^x = \begin{pmatrix} a^x & 0 \\ 0 & b^x \end{pmatrix}$$

entonces:

$$H^{2k} = \begin{pmatrix} \left(\frac{\Delta}{2}\right)^{2k} & 0 \\ 0 & \left(-\frac{\Delta}{2}\right)^{2k} \end{pmatrix} \qquad H^{2k+1} = \begin{pmatrix} \left(\frac{\Delta}{2}\right)^{2k+1} & 0 \\ 0 & \left(-\frac{\Delta}{2}\right)^{2k+1} \end{pmatrix}$$

Realizamos expansión de los exponentes (exp 2k cambia signo debido a la doble negación)

$$H^{2k} = \begin{pmatrix} \frac{\Delta^{2k}}{2^{2k}} & 0 \\ 0 & \frac{\Delta^{2k}}{2^{2k}} \end{pmatrix} \qquad H^{2k+1} = \begin{pmatrix} \frac{\Delta^{2k+1}}{2^{2k+1}} & 0 \\ 0 & -\frac{\Delta^{2k+1}}{2^{2k+1}} \end{pmatrix}$$

C) Substitute the former expressions into (3) and show that the time-evolution operator can be written as

$$U(t) = \begin{pmatrix} \cos(\Delta t/2) + i\sin(\Delta t/2) & 0 \\ 0 & \cos(\Delta t/2) - i\sin(\Delta t/2) \end{pmatrix}$$

(You should remember which the infinite series of functions cos and sin)

c) Substitute the former expressions into (3) and show that the time-evolution operator can be written as:

$$U(t) = \begin{pmatrix} \cos(\Delta t/2) + i\sin(\Delta t/2) & 0 \\ 0 & \cos(\Delta t/2) - i\sin(\Delta t/2) \end{pmatrix}$$

Sustitute expressions from B) into equation from A)

$$U(t) = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{(2k)!} H^{2k} - i \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)} H^{2k+1}$$

$$H^{2k} = \begin{pmatrix} \frac{\Delta^{2k}}{2^{2k}} & 0 \\ 0 & \frac{\Delta^{2k}}{2^{2k}} \end{pmatrix} \qquad H^{2k+1} = \begin{pmatrix} \frac{\Delta^{2k+1}}{2^{2k+1}} & 0 \\ 0 & \frac{\Delta^{2k+1}}{2^{2k+1}} \end{pmatrix}$$

Do sustitution:

$$U(t) = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{(2k)!} \begin{pmatrix} \frac{\Delta^{2k}}{2^{2k}} & 0 \\ 0 & \frac{\Delta^{2k}}{2^{2k}} \end{pmatrix} - i \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)!} \begin{pmatrix} \frac{\Delta^{2k+1}}{2^{2k+1}} & 0 \\ 0 & -\frac{\Delta^{2k+1}}{2^{2k+1}} \end{pmatrix}$$

We do the corresponding operations (multiply):

$$= \begin{pmatrix} \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{(2k)!} \left(\frac{\Delta^{2k}}{2^{2k}}\right) & 0 \\ 0 & \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{(2k)!} \left(\frac{\Delta^{2k}}{2^{2k}}\right) \end{pmatrix} - i \begin{pmatrix} \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)!} \left(\frac{\Delta^{2k+1}}{2^{2k+1}}\right) & 0 \\ 0 & -\sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)!} \left(\frac{\Delta^{2k+1}}{2^{2k+1}}\right) \end{pmatrix}$$

negative

Simplifyed with Taylor series:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{3!} \qquad \& \qquad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

(paired values) (odd values)

$$= \begin{pmatrix} \cos(t\Delta/2) & 0 \\ 0 & \cos(t\Delta/2) \end{pmatrix} - i \begin{pmatrix} \sin(t\Delta/2) & 0 \\ 0 & -\sin(t\Delta/2) \end{pmatrix}$$

Simplificando valores de matrices:

$$= \begin{pmatrix} \cos(\Delta t/2) - i\sin(\Delta t/2) & \emptyset \\ \emptyset & \cos(\Delta t/2) + i\sin(\Delta t/2) \end{pmatrix}$$

Dicha respuesta es equivalente a la esperada debido a la regla trigonometrica de negative angle identities. (Y un pequeño error de dedo en la act jeje)

D) Apply the operator *U* to the state (0), and find the Bloch vector coordinates

D) Apply the operator U to the state (0), and find the Bloch vector coordinates

state (0):

$$|\psi(0)\rangle = a_0|0\rangle + b_0|1\rangle, \quad \text{where } a_0 = \cos\frac{\theta_0}{2}, \quad b_0 = e^{i\varphi_0}\sin\frac{\theta_0}{2}$$

Time dependent equation $\longrightarrow |\psi(t)\rangle = U(t)|\psi(0)\rangle$

$$U(t) = \begin{pmatrix} \cos(\Delta t/2) - i\sin(\Delta t/2) & 0 \\ 0 & \cos(\Delta t/2) + i\sin(\Delta t/2) \end{pmatrix}$$

State (0) is represented as

$$\psi(0) = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\theta}{2}_0\right) \\ e^{i\varphi}\sin\left(\frac{\theta}{2}_0\right) \end{pmatrix}$$

Con base a dichos planteamientos y con el fin de optimizar el tiempo de desarrollo de dicha aplicación de *U* se desarrolló el siguiente código en Python utilizando los puntos antes mencionados para obtener el estado del Qubit tiempo-dependiente, así como las coordenadas <X>,<Y>,y <Z> del Vector aplicado a la Esfera de Bloch.

```
#Inciso D) Apply the operator U to the state (0), and find the Bloch
vector coordinates

#Andrea Catalina Fernández Mena

import numpy as np
```

```
#Valores iniciales

time = 0
theta = np.pi/4
phi = 0
delta = 1

#Representar valores iniciales del ket

a = np.cos(theta/ 2 )
b=  np.exp(1j * phi) * np.sin(theta/2)  #1j representa valores
imaginarios

#Definir U(t)
Ut = np.array([ [(np.cos(delta*time)/ 2) - 1j*np.sin((delta*time)/ 2) ,
0] , [0 ,(np.cos(delta*time)/ 2) + 1j*np.sin((delta*time)/ 2) ] ])

#Calcula el estado time-dependent del Qbit
   #Ket puede representarse como una matriz
   #Np.dot sirve para hacer dot producto multiplicacion de matrices
psi_t = np.dot(Ut, np.array([a,b]))

print("Time dependent qubit state: ", psi_t)

#Calculate the Bloch Vector Component
nX = 2* np.real(np.conj(psi_t[0]) * psi_t[1])  #posicion 0 es A y
posicion 1 es B
nY = 2* np.imag(np.conj(psi_t[0]) * psi_t[1])  #posicion 0 es A y
posicion 1 es B
nZ = np.abs(psi_t[0])**2 - np.abs(psi_t[1])**2

print("Bloch vector coordinates")
print("Nx = ", nX)
print("Ny = ", nY)
print("Nz = ", nZ)
```

Dicho código imprimiendo como resultados, los cuales representar coordenadas para A y B en el caso del estado del Qubit tiempo-dependiente, arroja las coordenadas de la esfera de Bloch en decimales con un valor equivalente a sus identidades trigonométricas correspondientes:

```
PS D:\Documentos\Septimo Semestre\Control de QBits para Cómputo Cuántico\Act2_A01197705> python -u "d:\Documentos\Septimo Semestre\Control de QBits para Cómputo Cuántico\Act2_A01197705\incisoD_A01197705.py"
Time dependent qubit state:  [0.46193977+0.j 0.19134172+0.j]
Bloch vector coordinates
Nx =  0.1767766952966369
Ny =  0.0
Nz =  0.17677669529663687
PS D:\Documentos\Septimo Semestre\Control de QBits para Cómputo Cuántico\Act2_A01197705>
```

E) Generate a code (Matlab, Pyton, Mathematica) to make an animation of the dynamics. Note that for each value of the time a different point in the Bloch sphere can be represented. The sequence of points will determine a trajectory in the sphere. To test your code you may use $\theta = \pi/4$, $\varphi = 0$ and $\Delta = 1$.

```
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#Bloch Sphere Dynamics Animation

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Circle
from matplotlib.animation import FuncAnimation

# Condiciones iniciales para esfera de Bloch (test clauses)
delta = 1
theta = np.pi / 4
phi = 0
a = np.cos(theta)
b = np.exp(1j * phi) * np.sin(theta)

# Guardar funcionalidades de Inciso D para coordenadas dentro de
funciones lambda
U = lambda t: np.array([[np.cos((delta * t) / 2) - 1j * np.sin((delta *
t) / 2), 0],[0, np.cos((delta * t) / 2) + 1j * np.sin((delta * t) /
2)]])
psi = lambda uT: np.dot(uT, np.array([a, b]))
bloch = lambda a, b: [2*np.real(np.conj(a)*b), 2*np.imag(np.conj(a)*b),
np.abs(a)**2 - np.abs(b)**2]

#Incluir funciones lamda dentro de un método funcion
def bloch_Psi_Plot(t):
  uT = U(t)
  a, b = psi(uT)
  n = bloch(a, b)
  return n

# Subplots para proyecciones de  X-Y, X-Z, & Y-Z
```
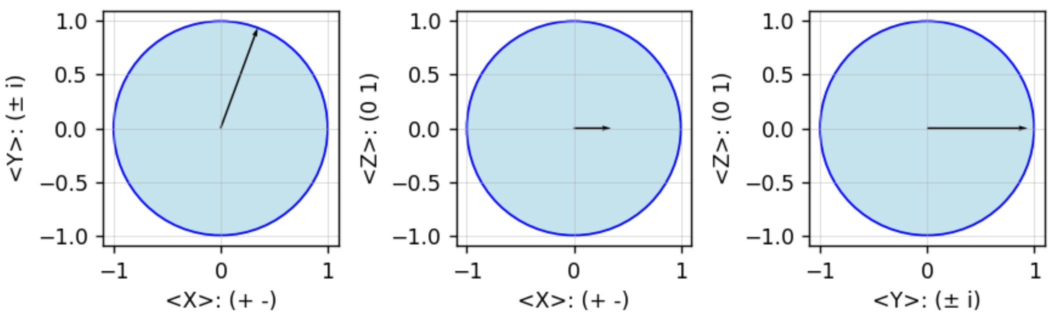

```
fig, axes = plt.subplots(1, 3, figsize=(8, 4))
plt.suptitle('Bloch Sphere Dynamics Animation - A01197705',
fontsize=13)
plt.subplots_adjust(wspace=0.5)

#Actualización de frames para posicionamiento
def update(frame):
    t = frame / ratio_frames
    n_vector = bloch_Psi_Plot(t)
    projections = [(axes[0], "<X>: (+ -)", "<Y>: (± i)", [n_vector[0],
n_vector[1]]),
                   (axes[1], "<X>: (+ -)", "<Z>: (0 1)", [n_vector[0],
n_vector[2]]),
                   (axes[2], "<Y>: (± i)", "<Z>: (0 1)", [n_vector[1],
n_vector[2]])]

    for ax, x_label, y_label, vector in projections:
        ax.clear()
        ax.add_patch(Circle((0, 0), 1, fill=True, color='lightblue',
alpha=0.7))
        ax.add_patch(Circle((0, 0), 1, fill=False, color='b', alpha=1))
        ax.set_xlim([-1.1, 1.1])
        ax.set_ylim([-1.1, 1.1])
        ax.set_xlabel(x_label)
        ax.set_ylabel(y_label)
        ax.set_aspect('equal', adjustable='box')
        ax.grid(True, linestyle='-', linewidth=0.5, alpha=0.5)
        ax.quiver(0, 0, vector[0], vector[1], angles='xy',
scale_units='xy', scale=1, color='k')

#Animacion
ratio_frames = 10  # Adjust as needed
animation_duration_s = 10
frames_total = animation_duration_s * ratio_frames
interval_frame = 1000 / ratio_frames

animation = FuncAnimation(fig, update, frames=frames_total,
interval=interval_frame)

plt.show()
```

Bloch Sphere Dynamics Animation - A01197705



F) The commutator between two operators A and B is defined as

$$[A, B] = AB - BA$$

Verify the following relations

$$[X_+, X_-] = Z$$

$$[Z, X_\pm] = \pm 2X_\pm$$

Ver procedimientos en siguiente página

f) The commutator between two operators A and B is defined as

$[A, B] = AB - BA$

Verify the following relations:

$$[X_+, X_-] = Z$$

$$[Z, X_\pm] = \pm 2X_\pm$$

Considerando que:

$$X_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad X_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Primer caso

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = Z$$

$0 \cdot 0 + 1 \cdot 1 = 1$

$0 \cdot 0 + 1 \cdot 0 = 0$

$0 \cdot 0 + 0 \cdot 1 = 0$

$0 \cdot 0 + 0 \cdot 0 = 0$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} -$$

$0 \cdot 0 + 0 \cdot 0 = 0$

$0 \cdot 1 + 0 \cdot 0 = 0$

$1 \cdot 0 + 0 \cdot 0 = 0$

$1 \cdot 1 + 0 \cdot 0 = 1$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = Z$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Por lo que

$$[X_+, X_-] = Z$$

Caso dos:

$$[Z, X_\pm] = \pm 2X_\pm$$

Considerando que:

$$X_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad X_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad [X_+, X_-] = Z$$

Se evaluara Z para ambos casos con X_+ + X_-

$$[A,B] = AB - BA$$

$$[Z, X_+] = +2X_+$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = +2X_+$$

$1\cdot0 + 0\cdot0 = 0$
$1\cdot1 + 0\cdot0 = 1$
$0\cdot0 + -1\cdot0 = 0$
$0\cdot1 + -1\cdot0 = 0$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$$

$0\cdot1 + 1\cdot0 = 0$
$0\cdot0 + 1\cdot-1 = -1$
$0\cdot1 + 0\cdot0 = 0$
$0\cdot0 + 0\cdot-1 = 0$

$$\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} = +2X_+$$

Si sacamos el 2 de la matriz veremos que

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = X_+ \quad = \quad \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} = +2X_+$$

Para evaluar a $[Z, X_-] = -2X_-$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -2X_-$$

$1\cdot0 + 0\cdot1 = 0$
$1\cdot0 + 0\cdot0 = 0$
$0\cdot0 + -1\cdot1 = -1$
$0\cdot0 + -1\cdot0 = 0$

$$\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = -2X_-$$

$0\cdot1 + 0\cdot0 = 0$
$0\cdot0 + 0\cdot-1 = 0$
$1\cdot1 + 0\cdot0 = 1$
$1\cdot0 + 0\cdot-1 = 0$

$$\begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix} = -2X_-$$

Si sacamos el -2 de la matriz tenemos que

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = X_- \longrightarrow \begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix} = -2X_-$$

Y simplificado podemos decir que

$$[Z, X_\pm] = \pm 2X_\pm$$