Homework 1

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Due: **Friday 9/24** on Canvas. Prepare your answers as a **single PDF file**.

Propositional Logic questions

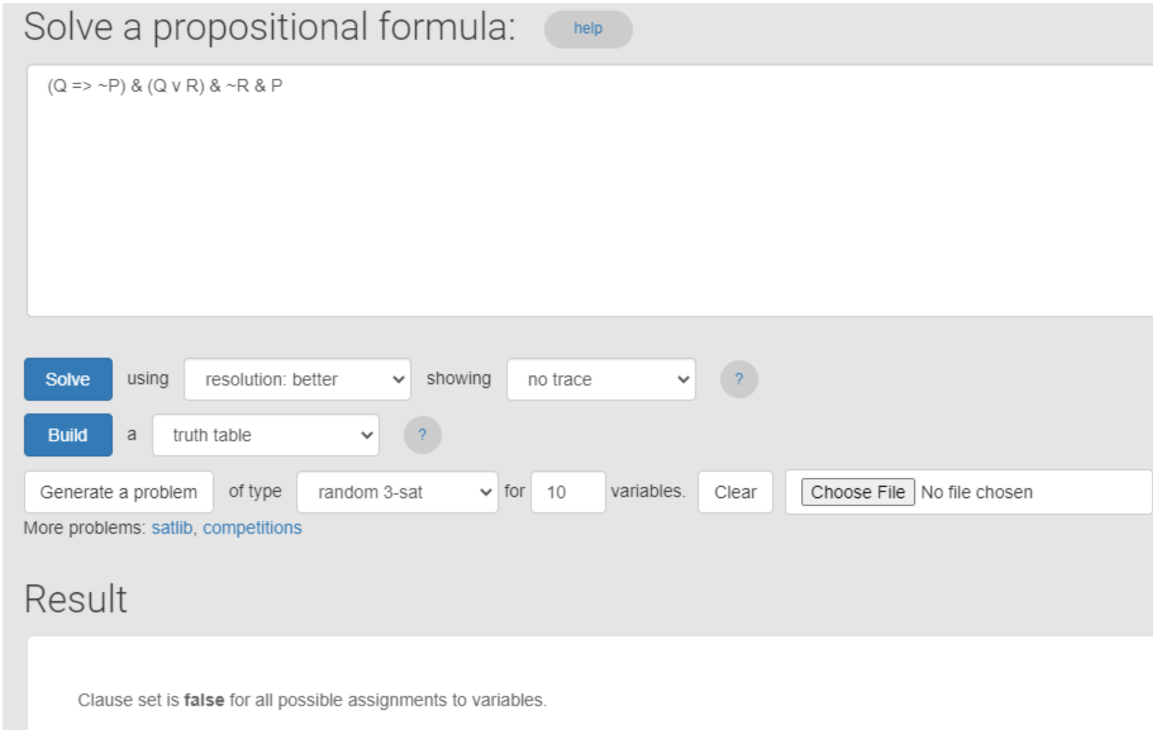
1. Determine whether the following statements are well-formed formulae in Propositional Logic. (Answer yes or no)
   a. $P \rightarrow Q \vee (R \wedge S)$ No, needs extra parenthesis
   b. $P \leftarrow \rightarrow Q$ No, invalid operators, unknown symbol
   c. $P \rightarrow \sim Q \vee Q$ No, needs parenthesis

2. Determine whether the following Propositional Logic statements are valid or invalid arguments. You may use a truth table, proofs using rules of inference, or resolution (specify which method you are using).
   a. Premises: $Q \rightarrow \sim P$, $Q \vee R$, $\sim R$; Conclusion: $\sim P$
      i. Prove using a truth table

P	Q	R	~P	Q→~P *	QVR *	~R *	~P Concl
T	T	T	F	F	T	F	F
T	T	F	F	F	T	T	F
T	F	T	F	T	T	F	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	F	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	F	T
F	F	F	T	T	F	T	T

Yes, these statements are valid arguments because the conclusion is true whenever the premises are true

      ii. Verify the answer using any of the solvers at https://logictools.org/prop.html. Include a screenshot of the webpage with the output.
         - Hint: To prove "KB entails **α**", you show that "KB ∧ ~**α**" is unsatisfiable. Write your sentences in the ASCII format described at Propositional logic syntax. Use parentheses to remove ambiguity. For example, this problem can be checked with the formula: `(Q => ~P) & (Q v R) & ~R & P`



Using resolution shows that adding $\sim(\sim P)$ means P to the KB contradicted. Therefore, the conclusion is true which confirms the results found in part(i).

b. Premises: $\sim P \vee Q$, $P \rightarrow (R \wedge S)$, $S \rightarrow Q$; Conclusion: $Q \vee R$

i. Prove using resolution refutation

$P \rightarrow (R \wedge S) \equiv \sim P \vee (R \wedge S) \equiv (\sim P \vee R) \wedge (\sim P \vee S)$

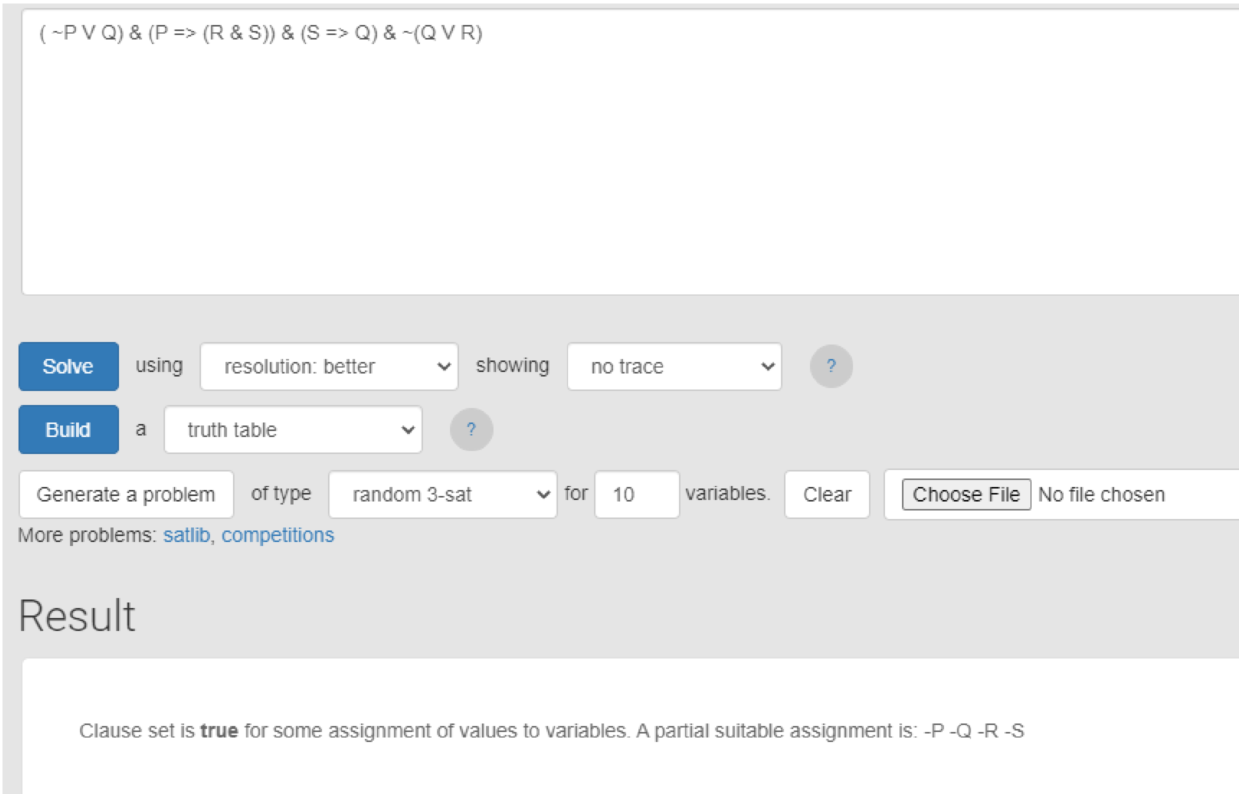
$S \rightarrow Q \equiv \sim S \vee Q$

$\sim(Q \vee R) \equiv \sim Q \wedge \sim R$

1. $\sim P \vee Q$
2. $\sim P \vee R$
3. $\sim P \vee S$
4. $\sim S \vee Q$
5. $\sim Q$
6. $\sim R$
7. $\sim S$ (Resolution on sentences 4 and 5)
8. $\sim P$ (Resolution on sentences 2 and 6)

We are stuck. Since the conclusion cannot be proven, it is an invalid conclusion.

ii. Verify the answer using any of the solvers at https://logictools.org/prop.html. Give the formula in ASCII format for testing satisfiability and a screenshot of the webpage with the output.



Using resolution shows that adding ~(Q ∨ R) to the KB gave a true statement. Adding a negated conclusion should have given a false statement. Therefore, this an invalid conclusion which confirms the results found in part(i).

c. Premises: P → (Q→R), Q; Conclusion: P → R

i. Prove using resolution refutation

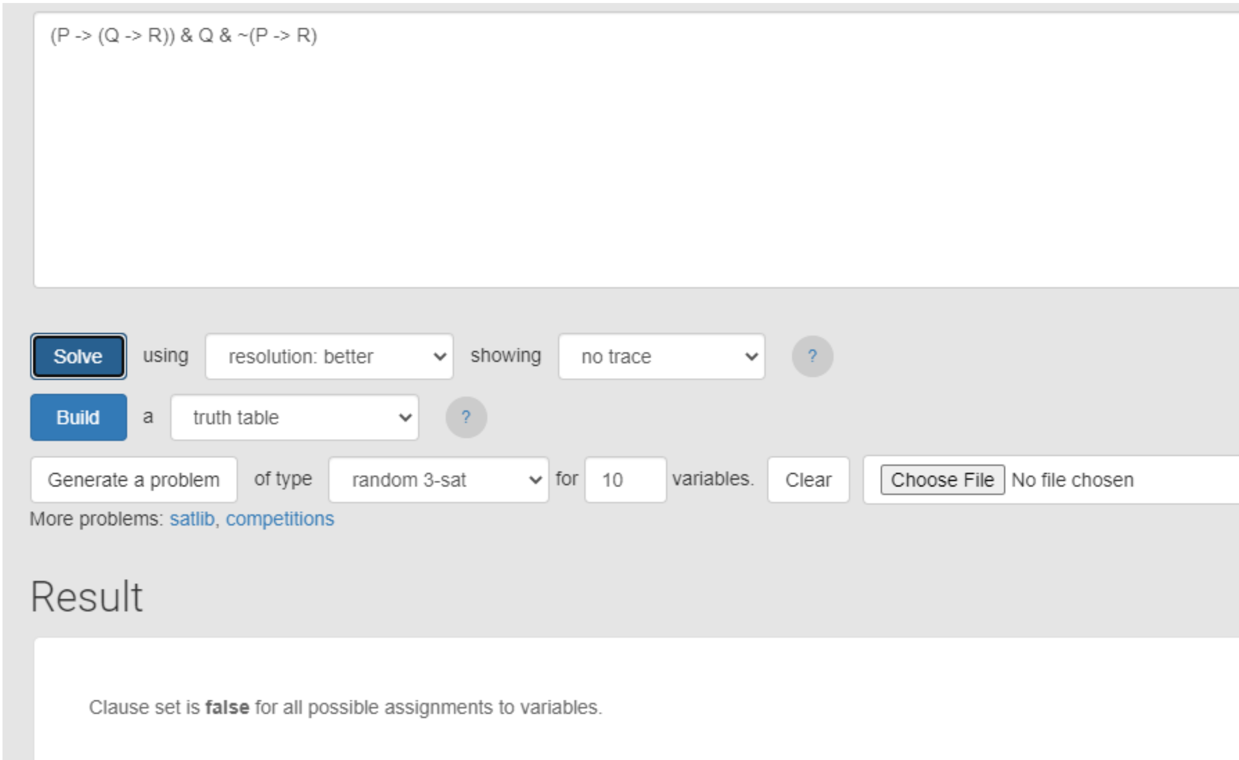
P → (~Q V R) ≡ ~P v (~Q V R)

P → R ≡ ~ (~P V R) ≡ P ∧ ~R

1. ~P v ~Q V R
2. Q
3. P
4. ~R
5. ~P V R (Resolution on sentences 1 and 2)
6. ~P V ~Q (resolution of 4 and 1)
7. ~Q V R (resolution of 3 and 1)
8. ~P (resolution of 2 and 6)
9. R (resolution of 2 and 7)
10. Resolutions on 3 and 8 would get us an empty set and that is FALSE

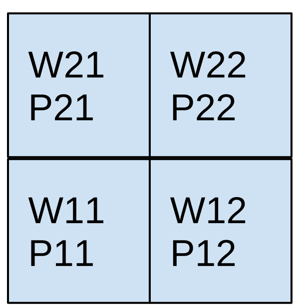
Therefore, P → R follows. Yes, this is a valid conclusion.

ii. Verify the answer using any of the solvers at https://logictools.org/prop.html. Give the formula in ASCII format for testing satisfiability and a screenshot of the webpage with the output.



Using resolution shows that adding $\sim(P \rightarrow R)$ to the KB contradicted. Therefore, this is a valid conclusion which confirms the results found in part(i).

3. Encode the rules of a simplified version of the Wumpus world, defined on a 2x2 grid and exactly one Wumpus (and no pits). In the squares adjacent to the wumpus, you will get a stench. Use only the propositions W11, W12, W21, W22 (to represent the presence of the Wumpus) and S11, S12, S21, S22 (to represent the stench).



Initially, the agent is in square (1,1) (and so there is no Wumpus there), and there is also no stench in (1,1).

a. Give the propositional logic sentences to represent that the two squares adjacent to the wumpus will have a stench.

(S11 <=> (W21 v W12)) &

(S12 <=> (W11 v W22)) &

(S21 <=> (W11 v W22)) &

(S22 <=> (W21 v W12))

b. Give the sentences to represent that there is exactly one Wumpus

W11 V W12 V W21 V W22

~W11 V ~W12

~W11 V ~W21

~W11 V ~W22

~W12 V ~W21

~W12 V ~W22

~W21 V ~W22

c. Give the sentence to represent that there is no Wumpus in (1,1)

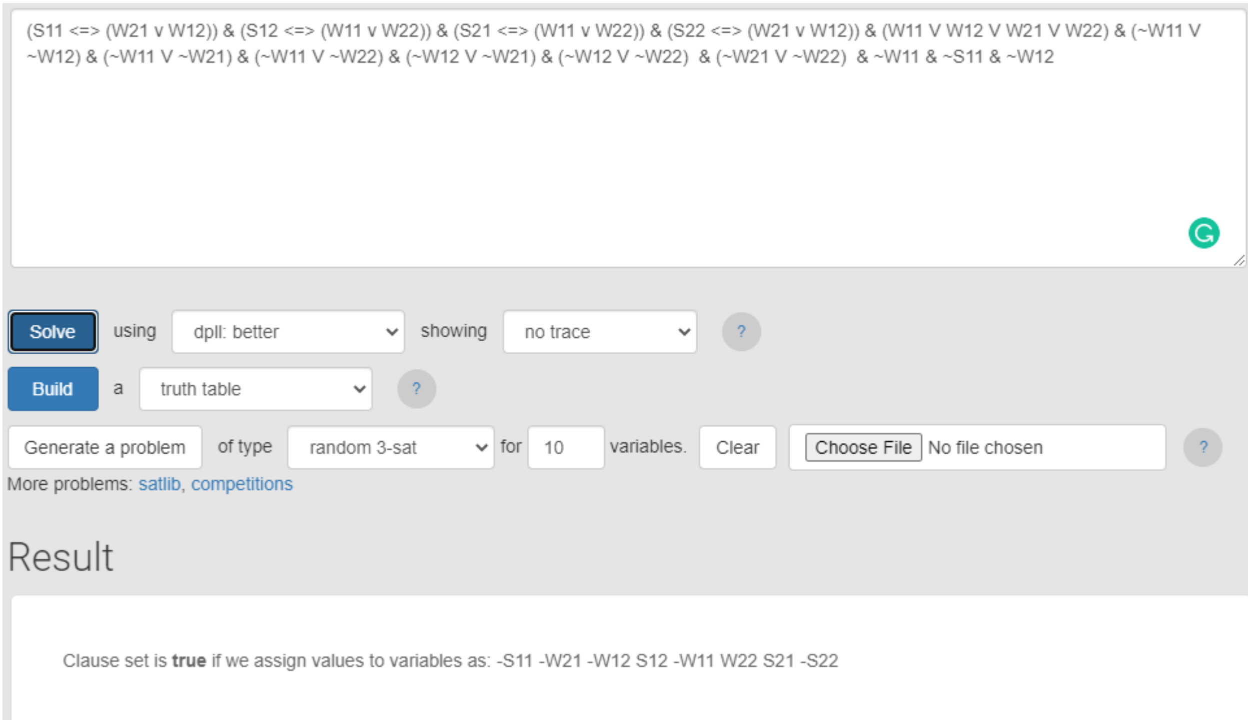
~W11

d. Give the sentence to represent that there is no stench in (1,1)

~S11

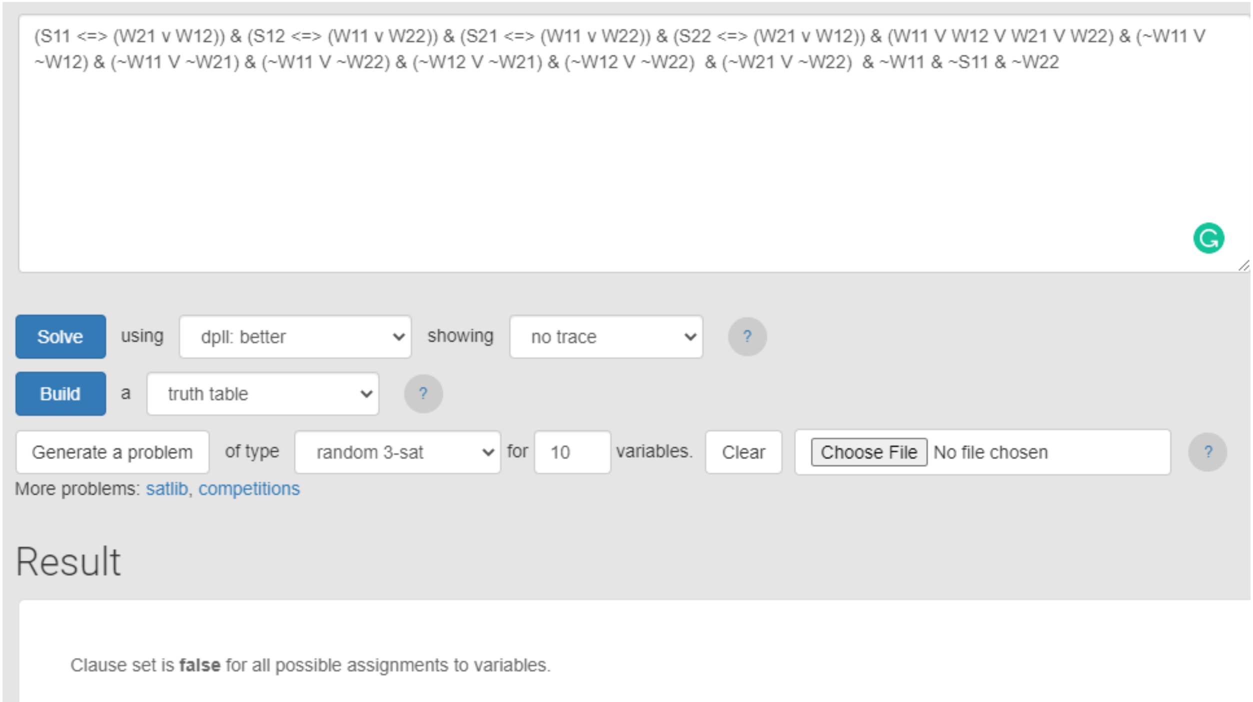
e. Is there a Wumpus in (1,2)? Check using the solver at https://logictools.org/prop.html. Hint: encode all the above sentences, the KB for this problem, in the ASCII format. Give the ASCII formula for testing satisfiability and a screenshot of the webpage with the output.

No, there is no Wumpus in (1, 2). Using resolution below shows that adding ~W12 to the KB gave a true statement. Adding a negated conclusion should have given a false statement. So, having a Wumpus in (1, 2) is an invalid conclusion.



f. Is there a Wumpus in (2,2)? Check using the solver at https://logictools.org/prop.html. Give the ASCII formula for testing satisfiability and a screenshot of the webpage with the output.

Yes, there is a Wumpus in (2, 2). Using resolution below shows that adding ~W22 to the KB gave a false statement. Adding a negated conclusion should have given a false statement. So, having a Wumpus in (2, 2) is a valid conclusion.



Predicate Logic questions

4. Determine whether the following statements are well-formed formulae in Predicate Logic.

(a) $P \rightarrow Q \vee R$	No, needs parenthesis
(b) $\forall x\, P(x) \rightarrow \exists y$	No, missing a predicate function in terms of y
(c) $\exists x\, P(x) \rightarrow \forall x P(x)$	Yes

5. Use the following predicates to write first-order Predicate Logic formulas:

P(x) = x is a programmer

S(x) = x is smart

(a) There are programmers.

$\exists x\, P(x)$

(b) Everyone is a programmer.

$\forall x\, P(x)$

(c) Everyone is not a programmer (i.e., every person is not a programmer)

$\forall x \sim P(x)$

(d) Not everyone is a programmer.

$\sim\forall x\, P(x)$

(e) Someone is not a programmer.

$\sim\exists x\, P(x)$

(f) All programmers are smart.

$\forall x\, P(x) \rightarrow S(x)$

(g) There are some programmers who are smart.

$\exists x\, P(x) \wedge \mathrm{S(x)}$

(h) No programmer is smart.
$\sim\exists x\ P(x)\ \land\ \mathrm{S(x)}$
(i) Some programmers are not smart.
$\exists x\ P(x)\ \land\ \sim\mathrm{S(x)}$

6. What is the result of unifying the following pairs of predicate logic expressions? Words starting with an uppercase letter are constants. (Determine if unification fails, or if it succeeds give the substitution). Note: variables start with a small letter, Constants start with a Capital letter.
   a) Mother(x) and Parent(x)
   Unification fails because we cannot change predicates.

   b) Parent (Mary, John) and Parent (x, y)
   Unification succeeds. Substitute x⇐ Mary, y⇐ John, we get Parent (Mary, John)

   c) Parent (Mary, z) and Parent (y, John)
   Unification succeeds. Substitute y⇐ Mary, z⇐ John, we get Parent (Mary, John)

7. Prove the following Predicate Logic theorem using resolution refutation
   - Premise
     1. For all persons, a person's mother is that person's parent
     2. For all persons, if the person's parent is alive then the parent is older than the person
     3. Mary is the mother of John
     4. Mary is alive
   - Conclusion
     1. Mary is older than John

   Use the following predicates
     - Mother(x,y): x is a mother of y
     - Parent(x,y): x is a parent of y
     - Older(x,y): x is older than y
     - Alive(x): x is alive

   Give:

   1. Premise 1 in Predicate logic:
      $\forall x\ \forall y\ Mother(x,y) \rightarrow Parent(x,y)$
   2. Premise 1 in CNF:
      $\forall x\ \forall y\ \sim Mother(x,y)\ \mathrm{V}\ Parent(x,y)$
   3. Premise 2 in Predicate logic:
      $\forall x\ \forall y\ Parent(x,y) \land \mathrm{Alive(x)} \rightarrow Older(x,y)$
   4. Premise 2 in CNF:
      $\forall x\ \forall y\ \sim(Parent(x,y) \land \mathrm{Alive(x)})\ V\ Older(x,y)$
      $\forall x\ \forall y\ \sim Parent(x,y)\ \mathrm{V} \sim\mathrm{Alive(x)}\ V\ Older(x,y)$
   5. Premise 3 in Predicate logic:
      $Mother(Mary, John)$

6. Premise 3 in CNF:
   $Mother(Mary, John)$
7. Premise 4 in Predicate logic:
   $Alive(Mary)$
8. Premise 4 in CNF:
   $Alive(Mary)$
9. Conclusion in Predicate logic:
   $Older(Mary, John)$
10. Negated conclusion in Predicate logic:
    $\sim Older(Mary, John)$
11. Negated conclusion in CNF:
    $\sim Older(Mary, John)$
12. Steps in the resolution proof

Resolution refutation:

1. $\sim Mother(x_1, x_2)$ V $Parent(x_1, x_2)$
2. $\sim Parent(x_3, x_4)$ V ~Alive(x_3) V $Older(x_3, x_4)$
3. $Mother(Mary, John)$
4. $Alive(Mary)$
5. $\sim Older(Mary, John)$
6. $Parent(Mary, John)$
   Resolution on lines 1 and 3 after unification of $Mother(x_1, x_2)$ and $Mother(Mary, John)$ with x_1⇐Mary and x_2⇐John
7. $\sim Parent(Mary, x_4)$ V $Older(Mary, x_4)$
   Resolution on lines 2 and 4 after unification of Alive(x_3) and $Alive(Mary)$ with x_3⇐Mary
8. $Older(Mary, John)$
   Resolution on lines 6 and 7 after unification of $Parent(Mary, x_4)$ and $Parent(Mary, John)$ with x_4⇐John
9. False
   Resolution on lines 5 and 8 (no unification needed)

Therefore, by using contradiction we proved the conclusion “Mary is older than John”.