Counterparty credit risk, collateral and funding: next generation valuation models under interconnected risks

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Agenda I

1. The Classical Theory in a nutshell
2. Pre-funding subtleties and Payout risk
   - Bilateral Risk and DVA
   - DVA Hedging?
   - Risk Free Closeout or Replacement Closeout?
   - Can we neglect first to default risk?
3. Counterparty Credit Risk and Collateral Margining
   - Collateralization, Gap Risk and Re-Hypothecation
4. Adding Collateral Margining Costs and Funding rigorously
   - Risk-Neutral Modelling of Bilateral CVA with Margining
   - The recursive nature of funding adjusted prices
   - Funding Costs, CVA Desk and Bank Structure
5. Conclusions and References

Thales - Bachelier - de Finetti - Black Scholes...

Derivatives outstanding notional as of June 2011 (BIS) is estimated at **708 =** 7.08×10^{14} **trillions USD** (World GDP: 79 Trillions)

Options??? Around 580 B.C., Thales purchased options on the future use of olive presses and made a fortune when the olives crop was as abundant as he had predicted, and presses were in high demand. (Thales is considered to be the father of sciences and western philosophy... a lot to answer for). More recently...

- **Louis Bachelier** (1870 – 1946) (First to introduce Brownian motion W_t in Finance, First in the modern study of Options);
- **Bruno de Finetti** (1906 – 1985) (Father of the subjective interpret of probability; defines the risk neutral measure in a way that is very similar to current theories: first to derive no arbitrage (ante-litteram!) through inequalities constraints, discrete setting).

Modern theory follows Nobel awarded **Black, Scholes and Merton** (and then Harrison and Kreps etc) on the correct pricing of options.

Valuation of financial products, options and derivatives

An option is a contract built on an underlying asset, for example an equity stock S. Call Option: $(S_T - K)^+$.

To price this options we do this: we try to find a trading strategy in the underlying stock S and on a risk free bank account B that perfectly replicates the option at the final time T.

Replicates: Final value V of the strategy satisfies $V_T = (S_T - K)^+$.

The strategy is also self-financing: It does not require any cash injection (or allow for cash withdrawal).

The initial cost V_0 of setting up the strategy then leads to the price of the option.

This is obtained by a PDE that is derived via:
The self financing condition + Ito's formula (=The Chain rule for Differential Equations driven by Brownian noise).

Valuation of financial products, options and derivatives

Then we have a theorem (Feynman Kac) that allows to interpret the solution of the PDE as a risk neutral expectation.

Namely: the price of the option is simply an expected value of the discounted payoff $D(t, T)(S_T - K)^+$, *but under a probability measure where the local return of S is the same as the risk free bank account B.*

WE DON'T NEED TO KNOW THE LOCAL RETURN OF S TO PRICE AN OPTION ON S's RETURN!!!

This contributed to the popularity of the derivatives markets.

Derivatives outstanding notional as of June 2011 (BIS) is estimated at **708 =** 7.08×10^{14} **trillions USD** (World GDP: 79 Trillions)

Valuation of financial products, options and derivatives

However, all the above assumes a lot of things:

- Short selling is allowed
- Infinitely divisible shares
- No transaction costs
- No dividends in the stock
- No default risk of the parties in the deal
- No funding costs
- Continous time and continuous trading/hedging
- Perfect market information
- ....

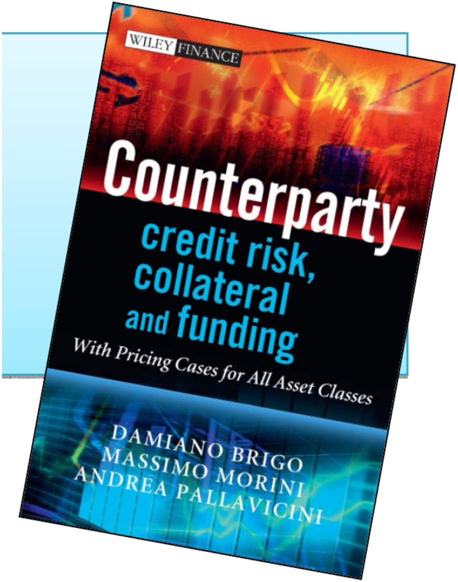
Valuation of financial products, options and derivatives

Pre-2007 the emphasis was PRICING/HEDGING COMPLEX DERIVATIVES on simple risks (pure equity risk, pure interest rate risk, etc)

Now we need to price SIMPLE DERIVATIVES such as Interest Rate Swaps under COMPLEX RISKS (credit, liquidity, funding, collateral, gap risk, multiple curves...)

This new task is much harder, not least because many of the new risks are INTERCONNECTED.

Presentation based on the Forthcoming Book



An online colloquial survey

For an introductory dialogue on Counterparty Risk, illustrating the themes of the book, see

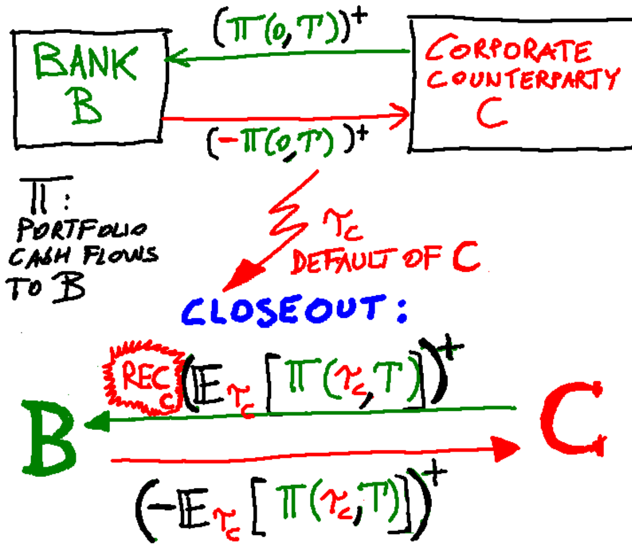
CVA Q&A

D. Brigo (2012). Counterparty Risk FAQ: Credit VaR, CVA, DVA, Closeout, Netting, Collateral, Re-hypothecation, Wrong Way Risk, Basel, Funding, and Margin Lending. SSRN.com, arXiv.org.

See also References at the end of this presentation.

Let's start by introducing COUNTERPARTY CREDIT RISK

Context



$(\Pi(0,T))^+$
BANK
B
CORPORATE
COUNTERPARTY
C
$(-\Pi(0,T))^+$
Π :
PORTFOLIO
CASH FLOWS
TO B
τ_C
DEFAULT OF C
CLOSEOUT :
$\mathrm{REC}_C\left(\mathbb{E}_{\tau_C}[\Pi(\tau_C,T)]\right)^+$
B
C
$\left(-\mathbb{E}_{\tau_C}[\Pi(\tau_C,T)]\right)^+$

The case of symmetric counterparty risk

$$\mathbb{E}_t\left\{\Pi_B^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi_B(t,T)\right\} + \mathrm{DVA}_B(t) - \mathrm{CVA}_B(t)$$

$$\mathrm{DVA}_B(t) = \mathbb{E}_t\left\{\mathrm{LGD}_B \cdot \mathbf{1}(t < \tau^{1st} = \tau_B < T) \cdot D(t,\tau_B) \cdot [-\mathrm{NPV}_B(\tau_B)]^+\right\}$$

$$\mathrm{CVA}_B(t) = \mathbb{E}_t\left\{\mathrm{LGD}_C \cdot \mathbf{1}(t < \tau^{1st} = \tau_C < T) \cdot D(t,\tau_C) \cdot [\mathrm{NPV}_B(\tau_C)]^+\right\}$$

- Obtained simplifying a first principles cash flows formula and taking expectation.
- 2nd term : adj due to scenarios $\tau_B < \tau_C$. This is positive to the investor/ Bank B and is called ”Debit Valuation Adjustment” (DVA)
- 3d term : Counterparty risk adj due to scenarios $\tau_C < \tau_B$
- Bilateral Valuation Adjustment as seen from B: $\mathrm{BVA}_B = \mathrm{DVA}_B - \mathrm{CVA}_B$.
- If computed from the opposite point of view of “C” having counterparty “B”, $\mathrm{BVA}_C = -\mathrm{BVA}_B$. Symmetry.

The case of symmetric counterparty risk

Strange consequences of the formula new mid term, i.e. DVA

- credit quality of investor WORSENS $\Rightarrow$ books POSITIVE MARK TO MKT
- credit quality of investor IMPROVES $\Rightarrow$ books NEGATIVE MARK TO MKT
- Citigroup in its press release on the first quarter revenues of 2009 reported a *positive* mark to market due to its *worsened* credit quality: “Revenues also included [...] a net 2.5$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads”

The case of symmetric counterparty risk: DVA?

`October 18, 2011, 3:59 PM ET, WSJ. Goldman Sachs Hedges Its Way to Less Volatile Earnings.`

Goldman's DVA gains in the third quarter totaled $450 million [...] $1.9 billion in DVA gains that J.P. Morgan Chase and Citigroup each recorded for the third quarter. Bank of America reported $1.7 billion of DVA gains in its investment bank [...]

Is DVA real? **DVA Hedging**. Buying back bonds? Proxying?

DVA hedge? One should sell protection on oneself, buying back bonds? Difficult.
Most times: proxying. Sell protection on a number of names highly correlated to oneself *(above WSJ interview, systemic risk problem)*
Even if DVA can be partly unreal to us because we can't hedge it, it is REAL FOR THE OTHER PARTY, since it's the other party's CVA. Price Reality becomes a matter of PERSPECTIVE.

DVA or no DVA? Accounting VS Capital Requirements

NO DVA: Basel III, page 37, July 2011 release

This CVA loss is calculated without taking into account any offsetting debit valuation adjustments which have been deducted from capital under paragraph 75. Stefan Walter spoke about *"perverse incentives"*

YES DVA: FAS 157

Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entitys credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements FAS 157 (see also IAS 39)

Funding and DVA

DVA a component of FVA?

DVA is related to funding costs when the payout is uni-directional, eg shorting/issuing a bond, borrowing in a loan, or going short a call option.

Indeed, if we are short simple products that are uni-directional, we are basically borrowing.

As we shorted a bond or option, for example, we received cash V_0 in the beginning, and we will have to pay the product payout in the end.

This cash can be used by us to fund other activities, and allows us to spare the costs of fuding this cash V_0 from our treasury.

Funding and DVA

Our treasury usually funds in the market, and the market charges our treasury a cost of funding that is related to the borrowed amount V_0, to the period T and to our own bank credit risk $\tau_B < T$.

In this sense the funding cost we are sparing when we avoid borrowing looks similar to DVA: it is related to the price of the object we are shorting and to our own credit risk.

However quite a number of assumptions is needed to identify DVA with a *pure funding benefit*, as we will see below.

Closeout: Replacement (ISDA?) VS Risk Free

NPV_B in the CVA/DVA formulas we have seen before, is the **(credit-) Risk Free** Net Present Value (residual value) at the first default. Risk free means that this residual value is computed without taking into account any residual credit quality. This is called RISK FREE CLOSEOUT

In other terms, if we replace the defaulted deal with a new, equivalent deal with a *new counterparty*, we are not accounting for the CVA of the new counterparty. And what if that counterparty defaults too before the end of the deal? *We could enter into a recursive/infinitely regressing boundary condition here.*

Closeout: Replacement (ISDA?) VS Risk Free

We are not accounting for our DVA either when pricing the NPV at the default of the counterparty (assuming this happens first).

But the very fact that we have been using a formula with CVA and DVA previously at time $t < \tau^1$, makes our valuation AT default inconsistent, or at least **discontinuous**. I was including my DVA before. The counterparty defaults, I take away my DVA all of a sudden.

If we DO inlcude residual DVA (and possibly CVA), we have a REPLACEMENT CLOSEOUT, that is the cost of replacing the defaulted deal with an equivalent one in the market.

On the other hand, if we are closing the position and liquidating it NOW, why should we account for any residual credit risk?

ISDA is giving some "soft" suggestions on this issue, in favour of the REPLACEMENT closeout. But this is a nightmare computationally.

Closeout: Replication (ISDA?) VS Risk Free

We can study RISK FREE vs REPLICATION CLOSEOUT on CVA/DVA for a LOAN. There should be **NO Impact** of an early default of the **Lender** on the loan price adjustment. Instead:

Statistical Dependence $(\tau_B, \tau_C) \rightarrow$ Closeout$\downarrow$	independence	co-monotonicity
Risk Free	Negatively Impacts Borrower	**No Impact**
Replication	**No Impact**	Further Negatively Impacts Lender

For a numerical case study and more details see Brigo and Morini (2010, 2011).

A simplified formula without τ^{1st} for bilateral VA

There is another source of confusion on CVA/DVA. Some market players take τ^1 out.

- The simplified formula is only a simplified representation of bilateral risk and ignores that upon the first default closeout proceedings are started, thus involving a degree of double counting
- It is attractive because it allows for the construction of a bilateral counterparty risk pricing system based only on a unilateral one.
- The correct formula involves default dependence between the two parties through τ^{1st} and allows no such incremental construction

A simplified formula without τ^{1st} for bilateral VA

One can show easily that the difference between the full correct formula and the simplified formula is

$$
\begin{aligned}
D^{BC} &= E_0[1_{\{\tau_B<\tau_C<T\}} LGD_C D(0,\tau_C)(E_{\tau_C}(\Pi(\tau_C,T)))^+] \\
&- E_0[1_{\{\tau_C<\tau_B<T\}} LGD_B D(0,\tau_B)(-E_{\tau_B}(\Pi(\tau_B,T)))^+].
\end{aligned} \tag{1}
$$

For an equity forward, we compute the difference D^{BC} between the correct bilateral risk free closeout formula and the simplified one.

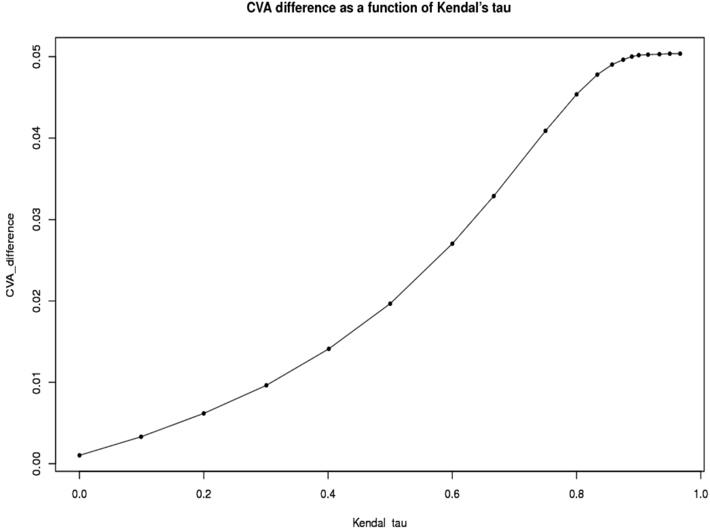


Figure: D^{BC} plotted against Kendall's tau between τ_B and τ_C, all other quantities being equal: $S_0 = 1$, $T = 5$, $\sigma = 0.4$, $K = 1$, $\lambda_B = 0.1$, $\lambda_C = 0.05$.

Collateral Management and Gap Risk

Collateral (CSA) is considered to be the solution to counterparty risk.

Periodically, the position is re-valued (”marked to market”) and a quantity related to the change in value is posted on the collateral account from the party who is penalized by the change in value.

This way, the collateral account, at the periodic dates, contains an amount that is close to the actual value of the portfolio and if one counterparty were to default, the amount would be used by the surviving party as a guarantee (and viceversa).

Gap Risk is the residual risk that is left due to the fact that the realingment is only periodical. If the market were to move a lot between two realigning (”margining”) dates, a significant loss would still be faced.
Folklore: Collateral completely kills CVA and gap risk is negligible.

Collateral Management and Gap Risk I

Folklore: Collateral completely kills CVA and gap risk is negligible.

We are going to show that there are cases where this is not the case at all (B. Capponi and Pallavicini 2012, Mathematical Finance)

- Risk-neutral evaluation of counterparty risk in presence of collateral management can be a difficult task, due to the complexity of clauses.
- Only few papers in the literature deal with it. Among them we cite Cherubini (2005), Alavian *et al.* (2008), Yi (2009), Assefa *et al.* (2009), Brigo et al (2011) and citations therein.
- Example: Collateralized bilateral CVA for a netted portfolio of IRS with 10y maturity and 1y coupon tenor for different default-time correlations with (and without) collateral re-hypothecation. See B, Capponi, Pallavicini and Papatheodorou (2011)

Collateral Management and Gap Risk II

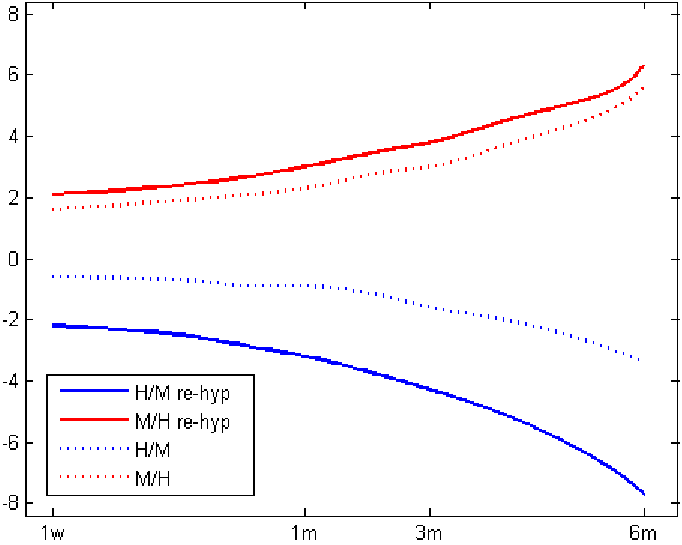


Figure explanation

Bilateral valuation adjustment, margining and rehypotecation

The figure shows the BVA(DVA-CVA) for a ten-year IRS under collateralization through margining as a function of the update frequency δ with zero correlation between rates and counterparty spread, zero correlation between rates and investor spread, and zero correlation between the counterparty and the investor defaults. The model allows for nonzero correlations as well.
Continuous lines represent the re-hypothecation case, while **dotted lines** represent the opposite case. The *red line* represents an investor riskier than the counterparty, while the *blue line* represents an investor less risky than the counterparty. All values are in basis points.

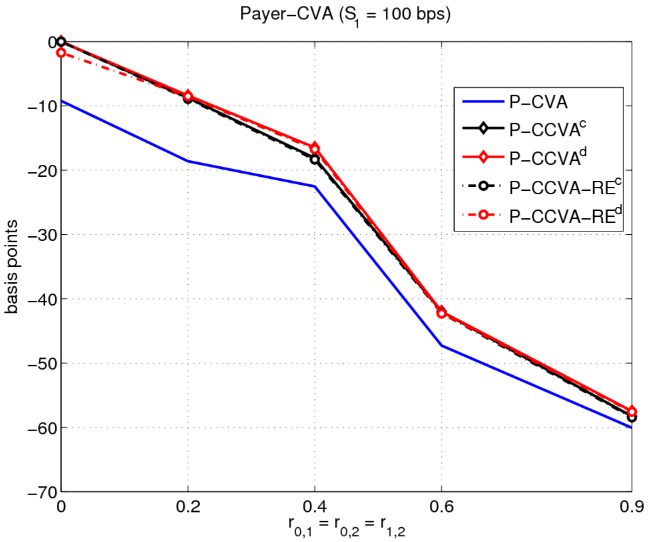
See the full paper by Brigo, Capponi, Pallavicini and Papatheodorou 'Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-Hypotecation and Netting" available at `http://arxiv.org/abs/1101.3926`

Figure explanation

From the fig, we see that the case of an investor riskier than the counterparty (M/H) leads to positive value for DVA-CVA, while the case of an investor less risky than the counterparty has the opposite behaviour. If we inspect the DVA and CVA terms as in the paper we see that when the investor is riskier the DVA part of the correction dominates, while when the investor is less risky the counterparty has the opposite behaviour.
Re-hypothecation enhances the absolute size of the correction, a reasonable behaviour, since, in such case, each party has a greater risk because of being unsecured on the collateral amount posted to the other party in case of default.

Let us now look at a case with more contagion: a CDS.

Collate


Payer–CVA (S_1 = 100 bps)
basis points
0
−10
−20
−30
−40
−50
−60
−70
0
0.2
0.4
0.6
0.9
$r_{0,1} = r_{0,2} = r_{1,2}$
P–CVA
P–CCVA^c
P–CCVA^d
P–CCVA–RE^c
P–CCVA–RE^d

Collateral Management and Gap Risk II

The figure refers to a payer CDS contract as underlying. See the full paper by Brigo, Capponi and Pallavicini (2011) for more cases.

If the investor holds a payer CDS, he is buying protection from the counterparty, i.e. he is a protection buyer.

We assume that the spread in the fixed leg of the CDS is 100 while the initial equilibrium spread is about 250.

Given that the payer CDS will be positive in most scenarios, when the investor defaults it is quite unlikely that the net present value be in favor of the counterparty.

We then expect the CVA term to be relevant, given that the related option will be mostly in the money. This is confirmed by our outputs.

Collateral Management and Gap Risk III

We see in the figure a relevant CVA component (part of the bilateral DVA - CVA) starting at 10 and ending up at 60 bps when under high correlation.

We also see that, for zero correlation, collateralization succeeds in completely removing CVA, which goes from 10 to 0 basis points.

However, collateralization seems to become less effective as default dependence grows, in that collateralized and uncollateralized CVA become closer and closer, and for high correlations we still get 60 basis points of CVA, even under collateralization.

The reason for this is the instantaneous default contagion that, under positive dependency, pushes up the intensity of the survived entities, as soon as there is a default of the counterparty.

Collateral Management and Gap Risk IV

Indeed, the term structure of the on-default survival probabilities (see paper) lies significantly below the one of the pre-default survival probabilities conditioned on $\mathcal{G}_{\tau-}$, especially for large default correlation.

The result is that the default leg of the CDS will increase in value due to contagion, and instantaneously the Payer CDS will be worth more. This will instantly increase the loss to the investor, and most of the CVA value will come from this jump.

Given the instantaneous nature of the jump, the value at default will be quite different from the value at the last date of collateral posting, before the jump, and this explains the limited effectiveness of collateral under significantly positive default dependence.

Basic Payout plus Credit and Collateral: Cash Flows I

- We calculate prices by discounting cash-flows under the pricing measure. Collateral and funding are modeled as additional cashflows (as for CVA and DVA)
- We start from derivative's cash flows.

$$\bar{V}_t(C;F) \quad := \quad \mathbb{E}_t[\,\Pi(t, T \wedge \tau) + \dots\,]$$

where

$\longrightarrow$ $\tau := \tau_C \wedge \tau_I$ is the first default time, and

$\longrightarrow$ $\Pi(t,u)$ is the sum of all discounted payoff terms up from t to u,

Basic Payout plus Credit and Collateral: Cash Flows II

- As second contribution we consider the collateralization procedure and we add its cash flows.

$$\begin{aligned}\bar{V}_t(C;F) &:= \mathbb{E}_t[\,\Pi(t, T \wedge \tau)\,] \\ &+ \mathbb{E}_t\Big[\,\gamma(t, T \wedge \tau; C) + 1_{\{\tau < T\}} D(t,\tau) C_{\tau^-} + \ldots\Big]\end{aligned}$$

where

$\longrightarrow$ C_t is the collateral account defined by the CSA,

$\longrightarrow$ C_{τ^-} is the pre-default value of the collateral account, and

$\longrightarrow$ $\gamma(t, u; C)$ are the collateral margining costs up to time u.

- Notice that when applying close-out netting rules, first we will net the exposure against C_{τ^-}, then we will treat any remaining collateral as an unsecured claim.

Basic Payout plus Credit and Collateral: Cash Flows III

- The cash flows due to the margining procedure on the time grid $\{t_k\}$ are equal to

$$\gamma(t,u;C) := \sum_{k=1}^{n-1} 1_{\{t \le t_k < u\}} D(t,t_k) C_{t_k} \left(1 - P_{t_k}(t_{k+1})(1 + \alpha_k \tilde{c}_{t_k}(t_{k+1}))\right)$$

  where $\alpha_k = t_{k+1} - t_k$ and the collateral accrual rates are given by

$$\tilde{c}_t := c_t^+ 1_{\{C_t > 0\}} + c_t^- 1_{\{C_t < 0\}}$$

- Then, according to CSA, we introduce the pre-default value of the collateral account C_{τ^-} as

$$C_{\tau^-} := \sum_{k=1}^{n-1} 1_{\{t_k < \tau < t_{k+1}\}} C_{t_k} \frac{P_\tau(t_{k+1})}{P_{t_k}^{\tilde{c}}(t_{k+1})}$$

Close-Out: Trading-CVA/DVA under Collateral – I

- As third contribution we consider the cash flow happening at 1st default, and we have

$$
\begin{aligned}
\bar{V}_t(C;F) &:= \mathbb{E}_t[\,\Pi(t,T\wedge\tau)\,] \\
&+ \mathbb{E}_t\big[\,\gamma(t,T\wedge\tau;C) + 1_{\{\tau<T\}}D(t,\tau)C_{\tau^-}\,\big] \\
&+ \mathbb{E}_t\big[\,1_{\{\tau<T\}}D(t,\tau)\,(\theta_\tau(C,\varepsilon) - C_{\tau^-}\,) + \dots\big]
\end{aligned}
$$

where

$\longrightarrow$ ε_τ is the amount of losses or costs the surviving party would incur on default event (close-out amount), and

$\longrightarrow$ $\theta_\tau(C,\varepsilon)$ is the on-default cash flow.

- θ_τ will contain collateral adjusted CVA and DVA payouts for the instument cash flows
- We define θ_τ including the pre-default value of the collateral account since it is used by the close-out netting rule to reduce exposure

Close-Out: Trading-CVA/DVA under Collateral – II

- The close-out amount is not a symmetric quantity w.r.t. the exchange of the role of two parties, since it is valued by one party after the default of the other one.

$$\varepsilon_\tau := 1_{\{\tau=\tau_C\}}\varepsilon_{I,\tau} + 1_{\{\tau=\tau_I\}}\varepsilon_{C,\tau}$$

- Without entering into the detail of close-out valuation we can assume a close-out amount equal to the risk-free price of remaining cash flows inclusive of collateralization and funding costs. More details in the examples.
  - $\longrightarrow$ See ISDA document "Market Review of OTC Derivative Bilateral Collateralization Practices" (2010).
  - $\longrightarrow$ See, for detailed examples, Parker and McGarry (2009) or Weeber and Robson (2009)
  - $\longrightarrow$ See, for a review, Brigo, Morini, Pallavicini (2013).

Close-Out: Trading-CVA/DVA under Collateral – III

- At transaction maturity, or after applying close-out netting, the originating party expects to get back the remaining collateral.
- Yet, prevailing legislation's may give to the Collateral Taker some rights on the collateral itself.
  - ⟶ In presence of re-hypothecation the collateral account may be used for funding, so that cash requirements are reduced, but counterparty risk may increase.
  - ⟶ See Brigo, Capponi, Pallavicini and Papatheodorou (2011).
- In case of collateral re-hypothecation the surviving party must consider the possibility to recover only a fraction of his collateral.
  - ⟶ We name such recovery rate REC'_I, if the investor is the Collateral Taker, or REC'_C in the other case.
  - ⟶ In the worst case the surviving party has no precedence on other creditors to get back his collateral, so that

$$\text{REC}_I \leq \text{REC}'_I \leq 1 \;, \quad \text{REC}_C \leq \text{REC}'_C \leq 1$$

Close-Out: Trading-CVA/DVA under Collateral – IV

- The on-default cash flow $\theta_\tau(C,\varepsilon)$ can be calculated by following ISDA documentation. We obtain

$$\begin{aligned}\theta_\tau(C,\varepsilon) &:= 1_{\{\tau=\tau_C<\tau_I\}}\left(\varepsilon_{I,\tau} - \mathrm{L{\scriptstyle GD}}_C(\varepsilon^+_{I,\tau} - C^+_{\tau-})^+ - \mathrm{L{\scriptstyle GD}}'_C(\varepsilon^-_{I,\tau} - C^-_{\tau-})^+\right)\\ &+ 1_{\{\tau=\tau_I<\tau_C\}}\left(\varepsilon_{C,\tau} - \mathrm{L{\scriptstyle GD}}_I(\varepsilon^-_{C,\tau} - C^-_{\tau-})^- - \mathrm{L{\scriptstyle GD}}'_I(\varepsilon^+_{C,\tau} - C^+_{\tau-})^-\right)\end{aligned}$$

  where loss-given-defaults are defined as $\mathrm{L{\scriptstyle GD}}_C := 1 - \mathrm{R{\scriptstyle EC}}_C$, and so on.

- If both parties agree on exposure, namely $\varepsilon_{I,\tau} = \varepsilon_{C,\tau} = \varepsilon_\tau$ then

$$\begin{aligned}\theta_\tau(C,\varepsilon) &:= \varepsilon_\tau - 1_{\{\tau=\tau_C<\tau_I\}}\Pi_{\text{CVAcoll}} + 1_{\{\tau=\tau_I<\tau_C\}}\Pi_{\text{DVAcoll}}\\ \Pi_{\text{CVAcoll}} &= \mathrm{L{\scriptstyle GD}}_C(\varepsilon^+_\tau - C^+_{\tau-})^+ + \mathrm{L{\scriptstyle GD}}'_C(\varepsilon^-_\tau - C^-_{\tau-})^+\\ \Pi_{\text{DVAcoll}} &= \mathrm{L{\scriptstyle GD}}_I((-\varepsilon_\tau)^+ - (-C_{\tau-})^+)^+ + \mathrm{L{\scriptstyle GD}}'_I(C^+_{\tau-} - \varepsilon^+_\tau)^+\end{aligned}$$

Close-Out: Trading-CVA/DVA under Collateral – V

- In case of re-hypothecation, when $\mathrm{L}\text{GD}_C = \mathrm{L}\text{GD}'_C$ and $\mathrm{L}\text{GD}_I = \mathrm{L}\text{GD}'_I$, we obtain a simpler relationship

$$\begin{aligned}\theta_\tau(C,\varepsilon) \quad := \quad & \varepsilon_\tau \\ - \quad & 1_{\{\tau=\tau_C<\tau_I\}}\mathrm{L}\text{GD}_C(\varepsilon_{I,\tau} - C_{\tau^-})^+ \\ - \quad & 1_{\{\tau=\tau_I<\tau_C\}}\mathrm{L}\text{GD}_I(\varepsilon_{C,\tau} - C_{\tau^-})^-\end{aligned}$$

Funding and Hedging – I

- As fourth and last contribution we consider the funding and hedging procedures and we add their cash flows.

$$
\begin{aligned}
\bar{V}_t(C;F) \;&:=\; \mathbb{E}_t[\,\Pi(t, T\wedge\tau)\,] \\
&+\; \mathbb{E}_t\left[\,\gamma(t, T\wedge\tau; C) + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau(C,\varepsilon)\,\right] \\
&+\; \mathbb{E}_t[\,\varphi(t, T\wedge\tau; F, H)\,]
\end{aligned}
$$

where

$\longrightarrow$ F_t is the cash account needed for trading,
$\longrightarrow$ H_t is the risky-asset account implementing the hedging strategy, and
$\longrightarrow$ $\varphi(t,T;F,H)$ are the cash F and hedging H funding costs up to u.

- In classical Black Scholes on Equity, for a call option (no credit risk, no collateral, no funding costs),

$$\bar{V}_t^{\text{Call}} = \Delta_t S_t + \eta_t B_t =: H_t + F_t, \quad \tau = +\infty, \gamma = 0, \varphi = 0.$$

Funding and Hedging – II

- The cash flows due to the funding and hedging strategy on the time grid $\{t_j\}$ are equal to

$$\begin{aligned}\varphi(t,u;F,H) \quad := \quad & \sum_{j=1}^{m-1} 1_{\{t \le t_j < u\}} D(t,t_j)(F_{t_j} + H_{t_j}) \left(1 - P_{t_j}(t_{j+1})(1 + \alpha_k \tilde{f}_{t_j}(t_{j+1}))\right) \\ - \quad & \sum_{j=1}^{m-1} 1_{\{t \le t_j < u\}} D(t,t_j) H_{t_j} \left(1 - P_{t_j}(t_{j+1})(1 + \alpha_k \tilde{h}_{t_j}(t_{j+1}))\right)\end{aligned}$$

where the funding and lending rates for F and H are given by

$$\tilde{f}_t := f_t^+ 1_{\{F_t > 0\}} + f_t^- 1_{\{F_t < 0\}} \; , \quad \tilde{h}_t := h_t^+ 1_{\{H_t > 0\}} + h_t^- 1_{\{H_t < 0\}}$$

Funding and Hedging – III

Cash is borrowed $F > 0$ from the treasury at an interest f^+ (cost) or is lent $F < 0$ at a rate f^- (revenue)

Risky Hedge asset is worth H. Cash needed to buy $H > 0$ ie the risky hedge is borrowed at an interest f^+ from the treasury (cost); in this case H can be used for asset lending (Repo for example) at a rate h^+ (revenue);

On the other hand if risky hedge is worth $H < 0$, we may borrow from the repo market by posting the asset H as guarantee (rate h^-, cost), and lend the obtained cash to the treasury to be remunerated at a rate f^- (revenue).

It is possible to include the risk of default of the funder and funded, leading to CVA and DVA adjustments for the funding position, see PPB.

The Recursive Nature of Pricing Equations – I

$$(*) \quad \bar{V}_t(C;F) = \mathbb{E}_t\big[\,\Pi(t, T\wedge\tau) + \gamma(t, T\wedge\tau) + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau(C,\varepsilon)\,\big] + \mathbb{E}_t[\,\varphi(t, T\wedge\tau; F, H)\,]$$

where we recall that $\varphi(t, T\wedge\tau; F) =$ sum of all the Investor funding borrowing and lending positions costs/revenues to hedge its trading position, up to the 1st default.

Recursive pricing algorithm (see full PPB (2011) paper for details)

We obtain a *recursive* equation: the product price $\bar{V}_t(C, F)$ in $(*)$ depends on the funding strategy $F((t, T])$ after t via φ, and the funding $F = \bar{V} - (C-)H$ after t depends on the future product price $\bar{V}((t, T])$.

- This recursive equation can be solved iteratively via LS MC techniques as in standard CVA calculations $\rightarrow$ See PPB (2011)

The Recursive Nature of Pricing Equations – II

- $\longrightarrow$ Numerical solutions based on BSDE techniques are required to solve the general problem.
- $\longrightarrow$ See Pallavicini, Perini, Brigo (2011) for a discrete time solution.
- $\longrightarrow$ See Crépey et al. (2012a) for further examples.

- The recursive feature of pricing equations is hidden in simplified approaches starting either from spreading the discount curve, or from adding simplistic extra pricing terms (FVA?).
- A different approach, leading to similar results, is followed by Crépey et al. (2011) or Burgard and Kjaer (2010,2011) where the usual risk-neutral evaluation framework is extended to include many cash accounts accruing at different rates.

Explain Funding Rates: Trading vs. Funding DVA – I

- The funding rate $\tilde{f}_t$ is determined by the party managing the funding account for the investor, usually the bank's treasury according to its liquidity policy:
  - $\longrightarrow$ trading positions may be netted before searching for funds on the market;
  - $\longrightarrow$ a Funds Transfer Pricing (FTP) process may be implemented to gauge the performances of different business units;
  - $\longrightarrow$ a maturity transformation rule can be used to link portfolios to effective maturity dates;
  - $\longrightarrow$ many source of funding can be mixed to obtain the internal funding curve; etc...
- In the literature the role of the treasury is usually neglected, leading to some controversial results particularly when the funding positions are not distinguished from the trading positions.

Explain Funding Rates: Trading vs. Funding DVA – II

- In particular, the false claim “funding costs are the DVA”, or even “there are no funding costs at all”, are often cited in the practitioners’ literature.

$\longrightarrow$ See the querelle following Hull and White (2012), ”FVA =0” (???)

DealPrice = RiskFreePrice - CVA + DVA $\pm$ FVA?

Can we simply add a new term called FVA to account for funding costs, ”funding valuation adjustment”?

We have seen that when including funding we obtain a recursive nonlinear problem on a specific portfolio (netting set? Aggregation level? Treasury decision?).

Not additive with CVA and DVA as these cash flows feed each other in a nonlinear and overlapping way. These risk interact and we can only compute a total adjustment.

Funding structures inside a bank?

Funding implications on a Bank structure

Including funding costs into valuation, even via a simplistic FVA, involves methodological, organisational, and structural challenges.

Many difficulties are similar to CVA's and DVA's, so Funding can be integrated in the CVA effort typically.

- Reboot IT functions, analytics, methodology, by adopting a consistent global methodology including a consistent credit-debit-collateral-funding adjustment
- Very strong investment, discontinuity, and against the "internal competition" culture
- OR include separate and inconsistent CVA and FVA adjustments, accepting simplifications and double counting.
- It can be important to analyze the global funding implications of the whole trading activity of the bank.

Conclusions on funding

The law of one price

FVA cannot be charged to the counterparty, differently from CVA, and cannot be bilateral, since we do not know the funding policy of our counterparties. So even if DVA was giving us some hope to realign symmetry of prices, funding finally destroys the law of one price and makes prices a matter of perspective. bid ask?

Is the funding inclusive ”price” a real price?

Each entity computes a different funding adjusted price for the same product. The funding adjusted ”price” is not a price in the conventional term. We may use it to book the deal in our system or to pay our treasury but not to charge a client. It is more a ”value” than a ”price”.

Conclusions I

- Counterparty Risk adds exotic optionality even to vanilla portfolios.
- Highly specialized hybrid modeling framework. MODEL RISK
- Bilateral CVA brings in symmetry but also paradoxical statements
- The DVA term in bilateral CVA is hard to hedge, especially in the jump-to-default risk component.
- Bilateral CVA requires a choice of closeout? First to default risk? PAYOUT RISK.
- Gap risk in collateralization remains relevant in presence of strong contagion
- Funding costs can be included consistently but they break the law of one price
- Credit Debit and Funding costs are NOT separable...

Conclusions II

- ... can be included implicitly in term structure models to explain multiple curve LIBOR vs OIS (in progress, BP 2013)
- ... and can alter the structure of the bank organization and are politically sensitive
- Basel III will make CVA (FVA?) capital requirements rather severe
- Proper valuation and management of CVA/DVA/FVA requires a *Consistent Global Valuation approach* because these risks are all INTERCONNECTED.
- This is the real Mathematical Finance challenge of our times

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