Decoherence of Primordial Perturbations in the View of a Local Observer

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Seminar talk
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Based on
2504.10472 with Junsei Tokuda (McGill University)

Outline

- ❑ **Introduction**
- ❑ **Decoherence in cosmology**
  - ➢ Wavefunction formalism
  - ➢ Decoherence rate and divergences
- ❑ **IR divergence: local observer effect**
- ❑ **UV divergence: time-averaged observables**

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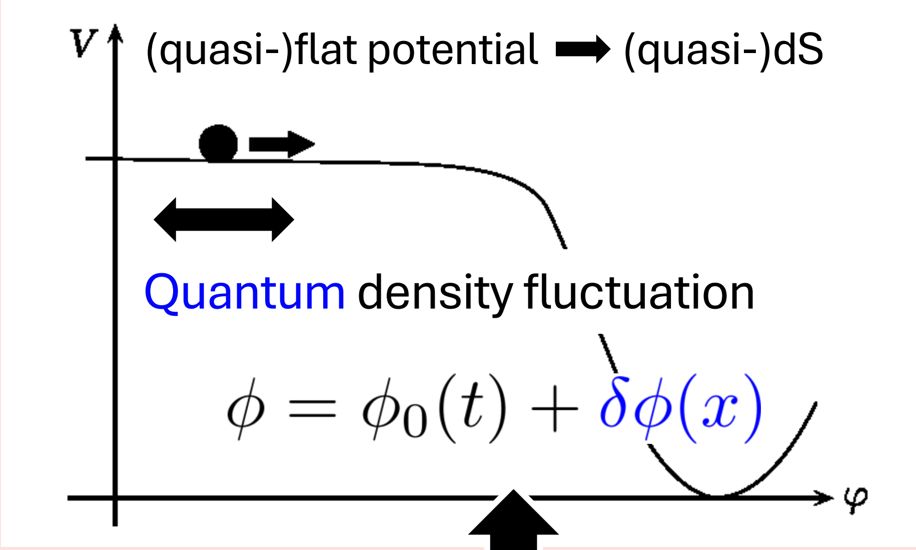
- ❑ **IR divergence: local observer effect**

- ❑ **UV divergence: time-averaged observables**

Inflation as a source for cosmological perturbations



(quasi-)flat potential ➡ (quasi-)dS

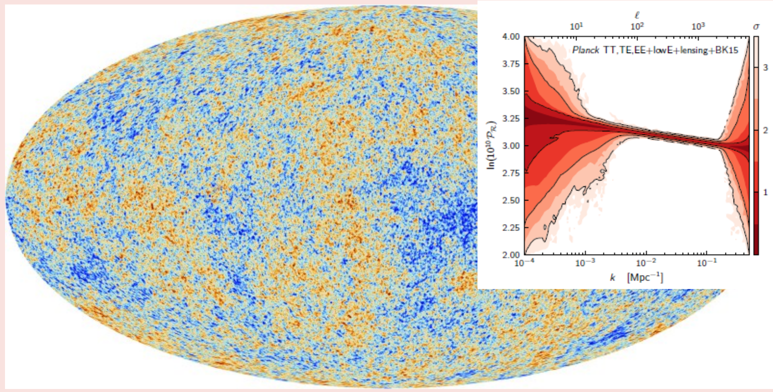
Quantum density fluctuation

$$\phi = \phi_0(t) + \delta\phi(x)$$

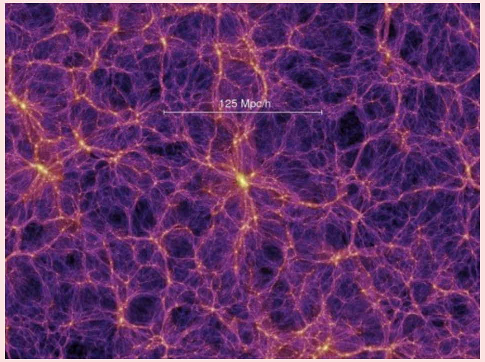
Quantum curvature perturbation

$$h_{ij} = (e^{\zeta(x)} a(t))^2 (\delta_{ij} + \gamma_{ij})$$

How (fast) classicalized?



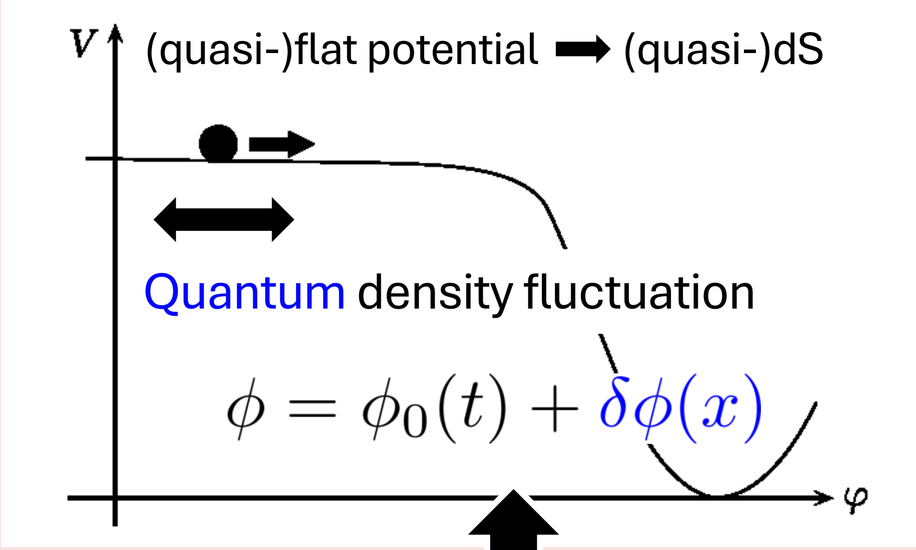
[Planck 1807.06211]



[Millennium Simulation 2005]

Classical anisotropy and inhomogeneity

Inflation as a source for cosmological perturbations



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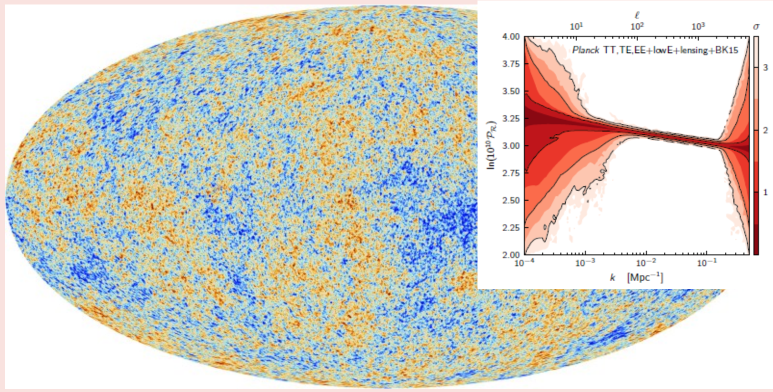
How (fast) classicalized?

Can we test quantum gravity from bottom-up?



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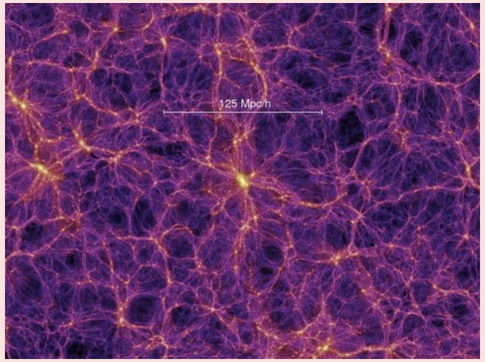

[Millennium Simulation 2005]

Classical anisotropy and inhomogeneity

Inflationary perturbations in a nutshell

[Maldacena astro-ph/0210603]

- ❑ **Expanding** $S_{\rm EH} = \frac{1}{2}\int dx^4 \sqrt{-g}R \; + S_{\rm GHY}$ **using perturbations around flat FLRW metric** $h_{ij} = (e^{\zeta(x)}a(t))^2\delta_{ij}$

✓ 2nd order

$$S_2 = \int dt d^3x \left\{ \epsilon a^3 H \dot{\zeta}^2 + \epsilon a (\partial\zeta)^2 - \partial_t\left(9a^3\zeta^2 + \frac{a}{H}(\partial\zeta)^2\right)\right\}$$

✓ 3rd order

$$S_3 = \int dt d^3x \left\{ a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + 2f(\zeta)\left.\frac{\delta\mathcal{L}}{\delta\zeta}\right|_1 + \mathcal{L}_b \right\}, \quad \partial^2\chi \equiv a^2\epsilon\dot{\zeta}$$

$$\begin{aligned}\mathcal{L}_b = \partial_t\Big[& -9a^3H\zeta^3 + \frac{a}{H}\zeta(\partial\zeta)^2 \\ & - \frac{1}{4aH^3}(\partial\zeta)^2\partial^2\zeta - \frac{a\epsilon}{H}\zeta(\partial\zeta)^2 \\ & - \frac{\epsilon a^3}{H}\zeta\dot{\zeta}^2 + \frac{1}{2aH^2}\zeta(\partial_i\partial_j\zeta\partial_i\partial_j\chi - \partial^2\zeta\partial^2\chi) \\ & - \frac{\eta a}{2}\zeta^2\partial^2\chi - \frac{1}{2aH}\zeta(\partial_i\partial_j\chi\partial_i\partial_j\chi - \partial^2\chi\partial^2\chi)\Big]\end{aligned}$$

⋮

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1,$$

$$\eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \ll 1$$

slow-roll parameter

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slow-roll parameter

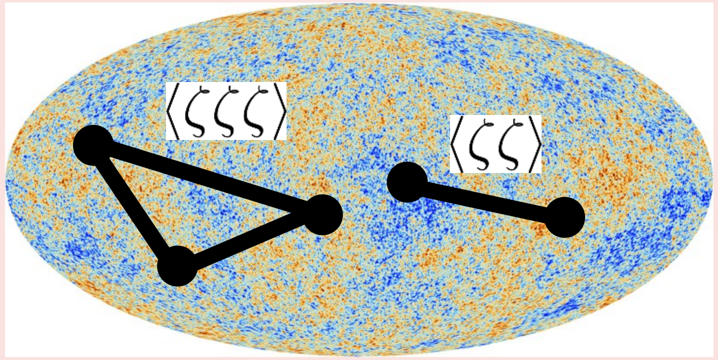
- **Initial condition for the universe after inflation:** $\langle 0_{\rm ini}|U^\dagger \hat{\mathcal{O}}(t_{\rm f})U|0_{\rm ini}\rangle$

  - ✓ 2 points $\langle \zeta_{\boldsymbol{k}}\zeta_{\boldsymbol{k}'}(t_{\rm f})\rangle = (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')P_\zeta$

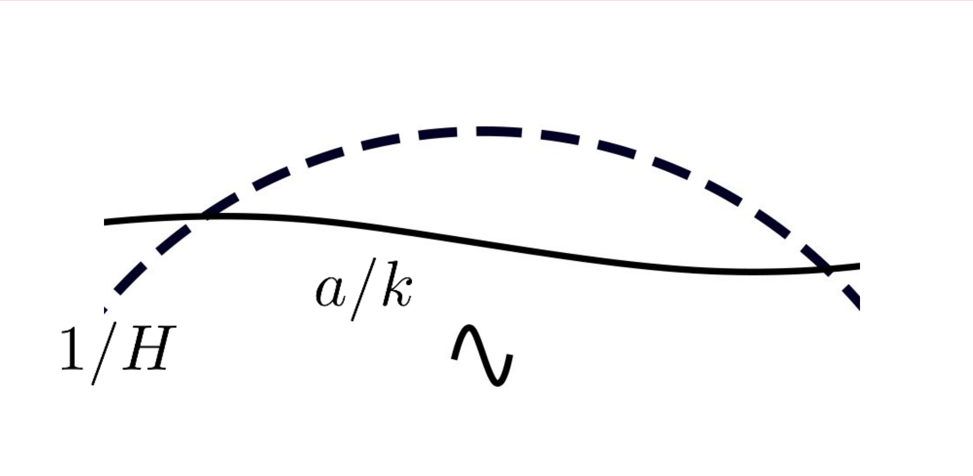
$$P_\zeta \simeq \frac{H^2}{4\epsilon k^3}\left(\frac{k}{k_*}\right)^{n_s-1} \qquad n_s = 1 - 2\epsilon - \eta \simeq 0.965, \quad \frac{dn_s}{d\log k} \simeq 0.002$$

[Planck 2018]

  - ✓ 3 or higher: perturbatively calculable. Expected in future observations.



“Quantumness” and “classicalization”



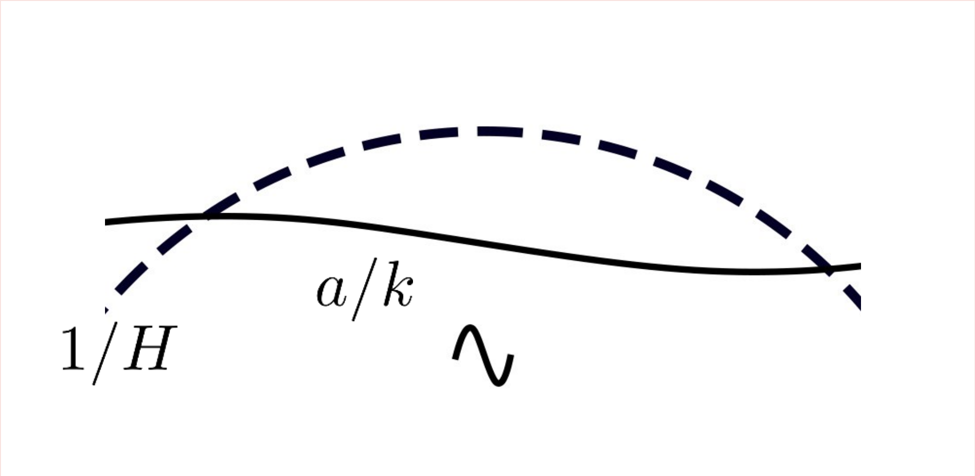
- Intuitively...

Large scale → Classical

$a/k \gg 1/H$

Formally?

"Quantumness" and "classicalization"



- Intuitively...

Large scale $a/k \gg 1/H$ ⟶ Classical

Formally?

- Coherence, Entanglement, Uncertainty, ...

Coherence

$$\hat{\rho}[\zeta, \tilde{\zeta}] \text{ vs. } P(\zeta)$$

✓ Quantum vs. classical dist.
[Martin and Vennin 1801.09949, 1805.05609, Green and Porto 2001.09149, etc.]

✓ Stochastic formalism, PBH
[Weenink and Prokopec 1108.3994]

Entanglement

$$|\Psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\varphi\rangle_B$$

✓ Bell test
[Martin and Vennin 1706.04516, 2203.03505 etc.
Sou et al. 2405.07141]

$$\mathcal{H}_{\rm tot} = \bigotimes_i \mathcal{H}_i$$

i ⟵ k? x? fields? e^{ikx} vs Y_{lm}?

Quantumness can be sensitive to the system.

Uncertainty

$$\Delta\zeta\Delta\pi \gtrsim \hbar$$

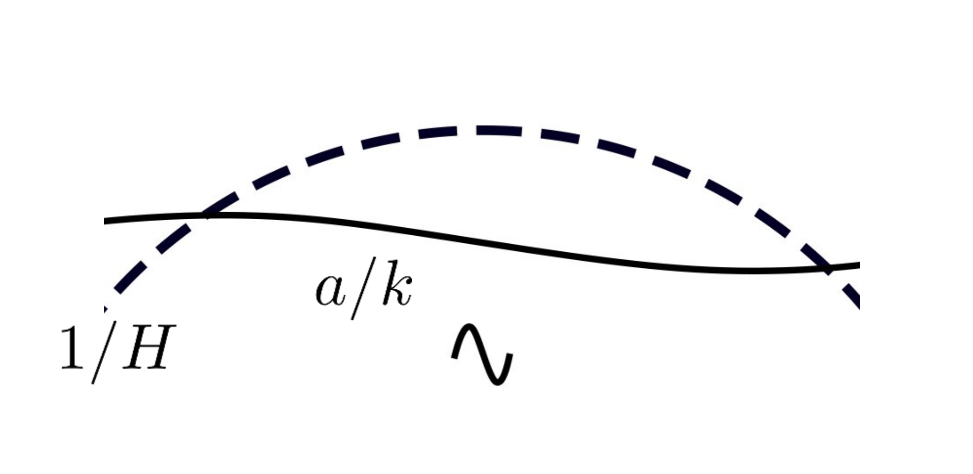
$$\Leftrightarrow [\zeta_{\mathbf{k}}, \pi_{\mathbf{k}'}] = i\hbar(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')$$

✓ Gaussian ⟶ minimal uncertainty

Two mode squeezed state
[Polarski and Starobinsky gr-qc/9504030]

$$|\Psi\rangle = \prod_{\mathbf{k}} \left(\sum_n \alpha_{n,\mathbf{k}} |n_{\mathbf{k}}\rangle \otimes |n_{-\mathbf{k}}\rangle \right)$$

"Quantumness" and "classicalization"



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- Coherence, This talk

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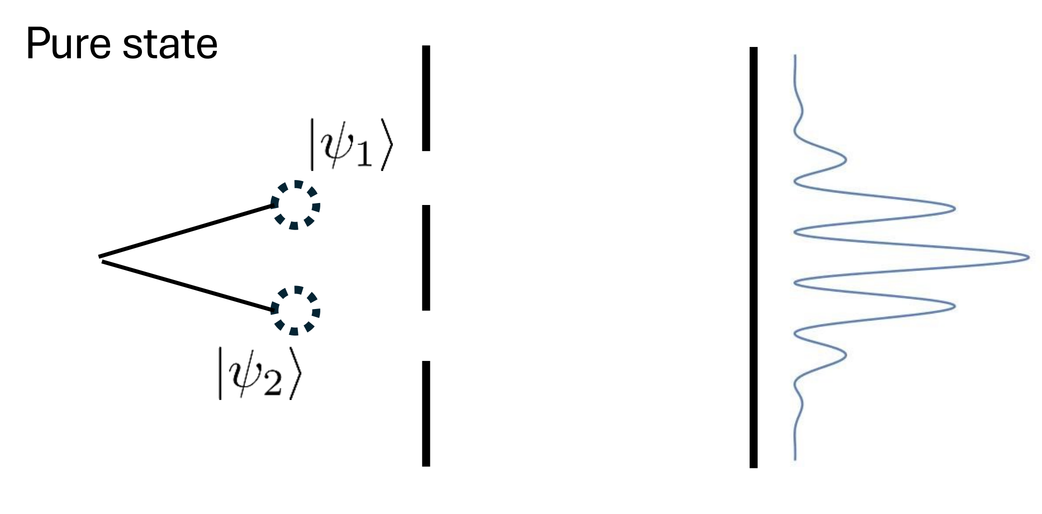
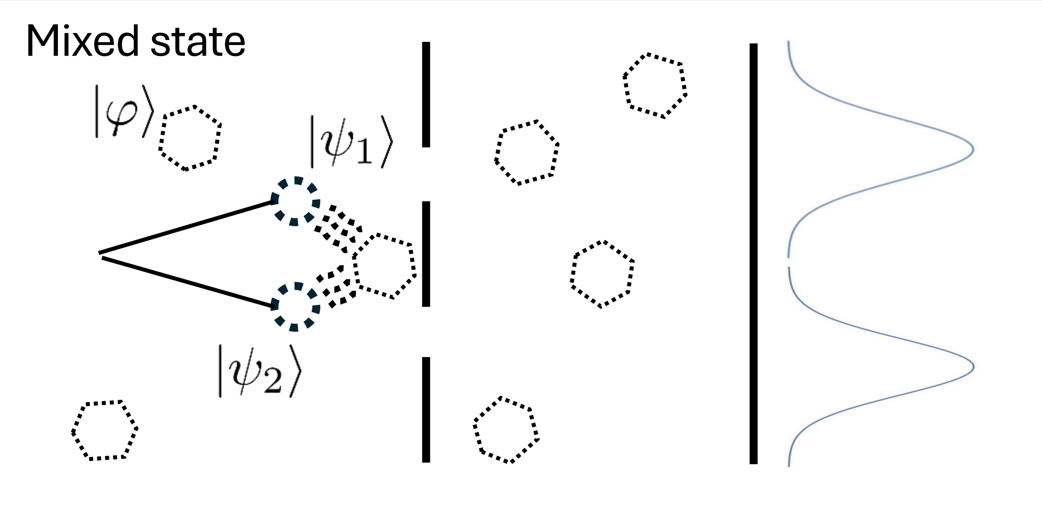
Quantum interference and decoherence



$$|\Psi\rangle = \alpha\,|\psi_1\rangle + \beta\,|\psi_2\rangle$$

$$\langle\Psi|\widehat{A}|\Psi\rangle = |\alpha|^2\,\langle\psi_1|\widehat{A}|\psi_1\rangle + |\beta|^2\,\langle\psi_2|\widehat{A}|\psi_2\rangle$$
$$+\,(\alpha\beta^*\,\langle\psi_2|\widehat{A}|\psi_1\rangle + \text{c.c.})$$



$$|\Psi\rangle = \alpha\,|\psi_1\rangle\,|\varphi_1\rangle + \beta\,|\psi_2\rangle\,|\varphi_2\rangle$$

$$\rho_\psi = \mathrm{Tr}_\varphi[|\Psi\rangle\,\langle\Psi|] = \begin{pmatrix} |\alpha|^2 & \alpha\beta^*\langle\varphi_2|\varphi_1\rangle \\ \alpha^*\beta\langle\varphi_1|\varphi_2\rangle & |\beta|^2 \end{pmatrix}$$

$\langle\varphi_2|\varphi_1\rangle \sim 0$ if scattered to independent states.

More scattering, more independent, less interference.

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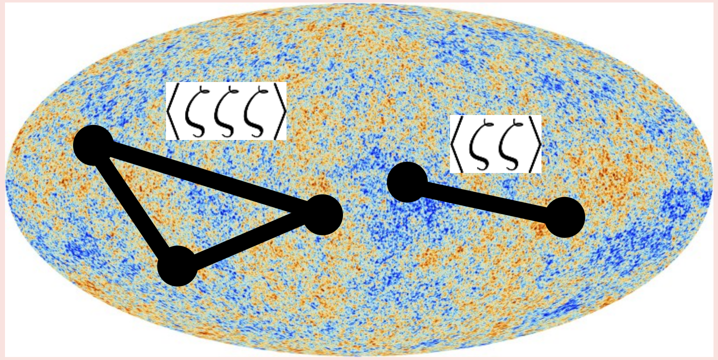
- ❑ UV divergence: time-smeared observables

Wavefunction formalism

- ❑ **Observables: correlation functions**

$$\langle\Omega|\widehat{\zeta}^n(t)|\Omega\rangle = \int \mathcal{D}\zeta(t)\langle\Omega|\zeta;t\rangle\,\langle\zeta;t|\Omega\rangle\zeta^n \equiv \int \mathcal{D}\zeta(t)|\Psi[\zeta(t)]|^2\zeta^n$$

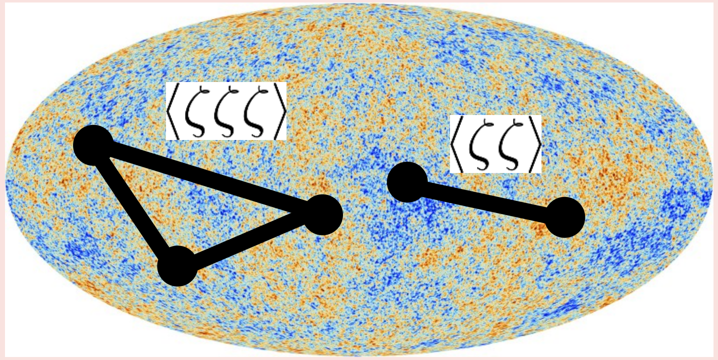
$$\widehat{\zeta}(t)\,|\zeta;t\rangle = \zeta(t)\,|\zeta;t\rangle$$



- ✓ System: single mode $\pm\mathbf{k}_{\rm S} \in \{\mathbf{k}_{\rm CMB}\}$, $\quad \mathcal{H} = \mathcal{H}_{\mathbf{k}_{\rm S}} \otimes \mathcal{H}_{\mathbf{k}_{\rm E}}$

Wavefunction formalism

- **Observables: correlation functions**



$$\langle\Omega|\hat{\zeta}^n(t)|\Omega\rangle = \int \mathcal{D}\zeta(t)\langle\Omega|\zeta;t\rangle\,\langle\zeta;t|\Omega\rangle\zeta^n \equiv \int \mathcal{D}\zeta(t)|\Psi[\zeta(t)]|^2\zeta^n$$

$$\hat{\zeta}(t)\,|\zeta;t\rangle = \zeta(t)\,|\zeta;t\rangle$$

✓ System: single mode $\pm\mathbf{k}_\mathrm{S} \in \{\mathbf{k}_\mathrm{CMB}\}\,, \quad \mathcal{H} = \mathcal{H}_{\mathbf{k}_\mathrm{S}} \otimes \mathcal{H}_{\mathbf{k}_\mathrm{E}}$

- **Wavefunction at a certain time slice**

Gaussian

Gravitational non-linearity

$$\Psi[\zeta(t)] \equiv \langle\zeta;t|\Omega\rangle = \exp\left[-\frac{1}{2}\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\psi_2\zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\frac{d^3\mathbf{k}_3}{(2\pi)^3}\psi_3\zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots\right]$$

$$\left(= \int_\Omega^\zeta \mathscr{D}\zeta' e^{iS[\zeta']}\right)$$

ψ_n : coefficient of the expansion

✓ Free propagation: $e^{-\int_\mathbf{k}\psi_2\zeta_\mathbf{k}\zeta_{-\mathbf{k}}}$ ➡ no entanglement between $\mathbf{k}_\mathrm{S}$ and $\mathbf{k}_\mathrm{E}$ (no scattering)

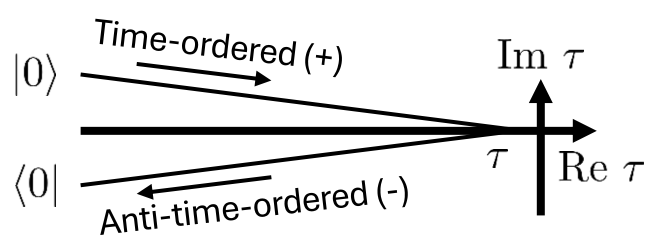
➡ Non-linearities cause decoherence.

Unitarity and Schwinger-Keldysh formalism

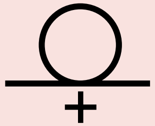
❑ Expectation values at a time slice

✓ Perturbation theory in interaction picture (2 pt.)

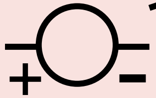
$$\langle\Omega|U^\dagger\zeta^2(\tau_0)U|\Omega\rangle = \left\langle 0\left|\left(\overline{\mathrm{T}}e^{i\int_{\tau_0}^{\tau}\mathrm{d}\tau' H_{\mathrm{I}}}\right)\zeta_{\mathrm{I}}^2(\tau)\left(\mathrm{T}e^{-i\int_{\tau_0}^{\tau}\mathrm{d}\tau' H_{\mathrm{I}}}\right)\right|0\right\rangle$$

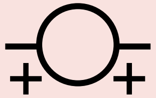


$$= \langle 0|\zeta_{\mathrm{I}}^2|0\rangle - i\int_{\tau_0}^{\tau} d\tau'\,\langle 0|\zeta_{\mathrm{I}}^2(\tau)H_{\mathrm{I}}(\tau')|0\rangle + \text{c.c.}$$

 etc.

$$+\int_{\tau_0}^{\tau} d\tau' d\tau''\,\langle 0|H_{\mathrm{I}}(\tau')\zeta_{\mathrm{I}}^2(\tau)H_{\mathrm{I}}(\tau'')|0\rangle - \int_{\tau_0}^{\tau} d\tau' d\tau''\,\langle 0|\zeta_{\mathrm{I}}^2(\tau)\mathrm{T}[H_{\mathrm{I}}(\tau')H_{\mathrm{I}}(\tau'')]|0\rangle + \text{c.c.}$$

 etc.

 etc.

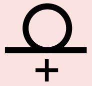
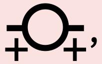
✓ Comparison with density matrix

$$\langle\Omega|U^\dagger\mathcal{O}_{0,\mathrm{S}}U|\Omega\rangle = \mathrm{Tr}[\rho(\tau)\mathcal{O}_{0,\mathrm{S}}] = \mathrm{Tr}_{\mathrm{S}}[\rho_{\mathrm{S}}(\tau)\mathcal{O}_{0,\mathrm{S}}]$$

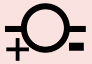
Defined in a subsystem

$$\begin{cases}\rho(\tau) = U\,|\Omega\rangle\langle\Omega|\,U^\dagger \\ \rho_{\mathrm{S}}(\tau) = \mathrm{Tr}_{\mathrm{E}}[U\,|\Omega\rangle\langle\Omega|\,U^\dagger]\end{cases}$$

Path integral

$\sim\Psi$. Unitary evolution within “+” contour (corrections in $\langle\zeta^2\rangle$: , , ...)

$$\rho_{\mathrm{S}}[\zeta,\tilde{\zeta};\tau] = \int_{\Omega}^{\zeta}\mathscr{D}\zeta_+\int_{\Omega}^{\tilde{\zeta}}\mathscr{D}\zeta_-\,e^{iS[\zeta_+]-iS[\zeta_-]+iS_{\mathrm{IF}}[\zeta_+,\zeta_-]}$$

Non-unitarity
(contributions in $\langle\zeta^2\rangle$: , ...)

Tracing out environmental modes

[Nelson 1601.03734]

- **Gaussian approximation**

$$
\begin{aligned}
\rho_{\rm S}[\zeta_{\rm S},\tilde{\zeta}_{\rm S}] &= \int \mathcal{D}\zeta_{\rm E}(t)\Psi[\zeta_{\rm S},\zeta_{\rm E}]\Psi^*[\tilde{\zeta}_{\rm S},\zeta_{\rm E}] \\
&= \Psi_{\rm G}[\zeta_{\rm S}]\Psi^*_{\rm G}[\tilde{\zeta}_{\rm S}] \int \mathcal{D}\zeta_{\rm E}|\Psi_{\rm G}[\zeta_{\rm E}]|^2 e^{-\frac{1}{6}\int(\psi_3\zeta^3+\psi_3^*\tilde{\zeta}^3)-\frac{1}{24}\int(\psi_4\zeta^4+\psi_4^*\tilde{\zeta}^4)+\cdots} \\
&= \Psi_{\rm G}[\zeta_{\rm S}]\Psi^*_{\rm G}[\tilde{\zeta}_{\rm S}] \exp\left[\sum_{n=1}^{\infty}\left\langle\left(-\frac{1}{6}\int(\psi_3\zeta^3+\psi_3^*\tilde{\zeta}^3)-\frac{1}{24}\int(\psi_4\zeta^4+\psi_4^*\tilde{\zeta}^4)+\cdots\right)^n\right\rangle_{\rm G,E}\right] \\
&\equiv N\exp\Big[\underbrace{-A_{k_{\rm S}}|\zeta_{\mathbf{k}_{\rm S}}|^2-A^*_{k_{\rm S}}|\tilde{\zeta}_{\mathbf{k}_{\rm S}}|^2}_{\text{“Pure state” part }\Psi\Psi^*}+\underbrace{\frac{C_{k_{\rm S}}}{2}(\zeta_{\mathbf{k}_{\rm S}}\tilde{\zeta}^*_{\mathbf{k}_{\rm S}}+\zeta^*_{\mathbf{k}_{\rm S}}\tilde{\zeta}_{\mathbf{k}_{\rm S}})}_{\text{Mixed part due to non-unitarity}}+\cdots\Big]
\end{aligned}
$$

(~influence functional)

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[Nelson 1601.03734]

❑ Gaussian approximation

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\rho_{\mathrm{S}}[\zeta_{\mathrm{S}},\widetilde{\zeta}_{\mathrm{S}}] &= \int \mathcal{D}\zeta_{\mathrm{E}}(t)\Psi[\zeta_{\mathrm{S}},\zeta_{\mathrm{E}}]\Psi^*[\widetilde{\zeta}_{\mathrm{S}},\zeta_{\mathrm{E}}] \\
&= \Psi_{\mathrm{G}}[\zeta_{\mathrm{S}}]\Psi^*_{\mathrm{G}}[\widetilde{\zeta}_{\mathrm{S}}]\int \mathcal{D}\zeta_{\mathrm{E}}|\Psi_{\mathrm{G}}[\zeta_{\mathrm{E}}]|^2 e^{-\frac{1}{6}\int(\psi_3\zeta^3+\psi_3^*\widetilde{\zeta}^3)-\frac{1}{24}\int(\psi_4\zeta^4+\psi_4^*\widetilde{\zeta}^4)+\cdots} \\
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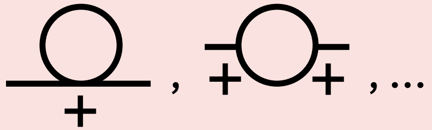
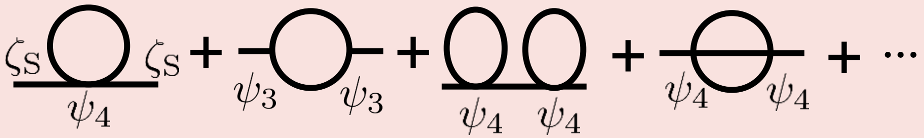
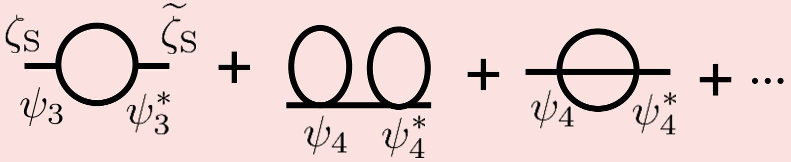
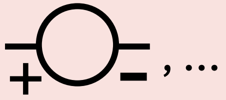


✓ $A_{k_{\mathrm{S}}}$: $\psi_2^{\text{tree}}+\psi_2^{\text{loop}}$ +  ⬅ Time ordered in Schwinger-Keldysh

✓ $C_{k_{\mathrm{S}}}$:  ⬅ Wightman functions in Schwinger-Keldysh 

* Purity: $P=\mathrm{Tr}\left[\rho^2\right]\simeq\dfrac{1}{1+\Gamma}$ where $\Gamma=4P_{k_{\mathrm{S}}}C_{k_{\mathrm{S}}}$

Tracing out environmental modes

[Nelson 1601.03734]

❑ Gaussian approximation

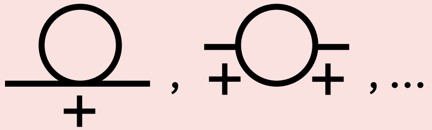
$$\rho_{\rm S}[\zeta_{\rm S},\tilde\zeta_{\rm S}]=\int\mathcal{D}\zeta_{\rm E}(t)\Psi[\zeta_{\rm S},\zeta_{\rm E}]\Psi^*[\tilde\zeta_{\rm S},\zeta_{\rm E}]$$

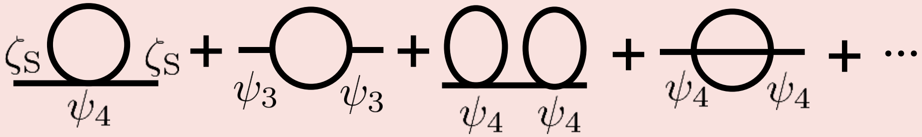
$$=\Psi_{\rm G}[\zeta_{\rm S}]\Psi^*_{\rm G}[\tilde\zeta_{\rm S}]\int\mathcal{D}\zeta_{\rm E}|\Psi_{\rm G}[\zeta_{\rm E}]|^2e^{-\frac{1}{6}\int(\psi_3\zeta^3+\psi_3^*\tilde\zeta^3)-\frac{1}{24}\int(\psi_4\zeta^4+\psi_4^*\tilde\zeta^4)+\cdots}$$

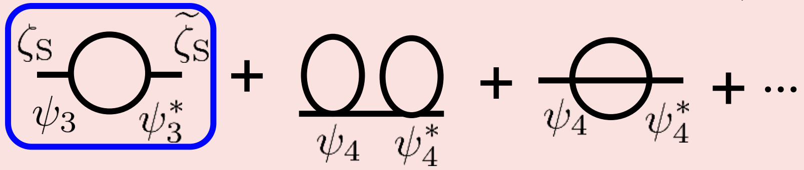
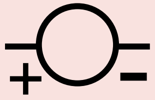
$$=\Psi_{\rm G}[\zeta_{\rm S}]\Psi^*_{\rm G}[\tilde\zeta_{\rm S}]\exp\left[\sum_{n=1}^{\infty}\left\langle\left(-\frac{1}{6}\int(\psi_3\zeta^3+\psi_3^*\tilde\zeta^3)-\frac{1}{24}\int(\psi_4\zeta^4+\psi_4^*\tilde\zeta^4)+\cdots\right)^n\right\rangle_{\rm G,E}\right]$$

$$\equiv N\exp\Big[\underbrace{-A_{k_{\rm S}}|\zeta_{\mathbf{k}_{\rm S}}|^2-A^*_{k_{\rm S}}|\tilde\zeta_{\mathbf{k}_{\rm S}}|^2}_{\text{“Pure state” part }\ \Psi\Psi^*}+\underbrace{\frac{C_{k_{\rm S}}}{2}(\zeta_{\mathbf{k}_{\rm S}}\tilde\zeta^*_{\mathbf{k}_{\rm S}}+\zeta^*_{\mathbf{k}_{\rm S}}\tilde\zeta_{\mathbf{k}_{\rm S}})}_{\text{Mixed part due to non-unitarity}}+\cdots\Big]$$

(~influence functional)



✓ $A_{k_{\rm S}}$: $\psi_2^{\rm tree}+\psi_2^{\rm loop}$ +  + ⋯ ⬅ Time ordered in Schwinger-Keldysh

✓ $C_{k_{\rm S}}$:  + ⋯ ⬅ Wightman functions in Schwinger-Keldysh  , …

* Purity: $P={\rm Tr}[\rho^2]\simeq\dfrac{1}{1+\Gamma}$ where $\Gamma=4P_{k_{\rm S}}C_{k_{\rm S}}\simeq 2P_{k_{\rm S}}\displaystyle\int_{\mathbf{q}}P_qP_{|\mathbf{k}_{\rm S}-\mathbf{q}|}|\psi_{3,(\mathbf{k}_{\rm S},-\mathbf{q},\mathbf{q}-\mathbf{k}_{\rm S})}|^2$

Estimations in previous work

[Nelson 1601.03734, Sou et al. 2207.04435]

Another method: quantum master equation [Burgess et al. astro-ph/061646, Burgess et al. 2211.11046, etc.]

❑ **Dependence on scale factor** $(\rho_{\text{off-diag}} \sim e^{-\Gamma})$

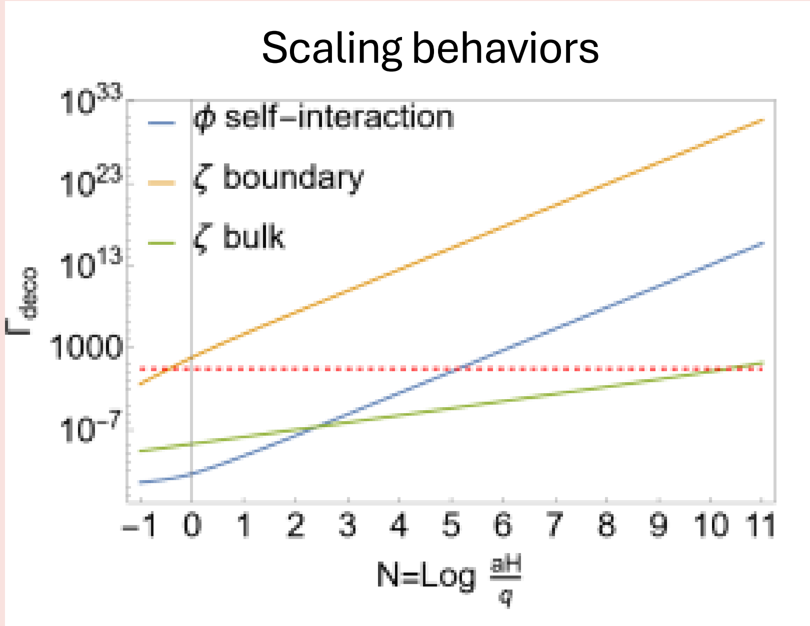
$$\Gamma \simeq 2P_{k_S} \int_{\mathbf{q}} P_q P_{|\mathbf{k}_S - \mathbf{q}|} |\psi_{3,(\mathbf{k}_S,-\mathbf{q},\mathbf{q}-\mathbf{k}_S)}|^2$$

$$\sim \frac{H^2}{M_{\text{pl}}^2}\left[\left(\underbrace{\frac{1}{\epsilon^2}\left(\frac{aH}{k_S}\right)^6}_{\partial_t(9aH\zeta^3)} + \underbrace{\epsilon^2\left(\frac{aH}{k_S}\right)^3}_{a^2\epsilon^2\zeta(\partial\zeta)^2}\right)\underbrace{(1+\log(k_{\text{IR}}/k_S))}_{\text{IR cutoff}} + \underbrace{\left(\frac{\Lambda_{\text{phys}}}{H}\right)^{\#}}_{\text{UV cutoff}}\right]$$

Is the quantum state sensitive to the duration of inflation?

Do sub- and super-horizon modes strongly correlate?

(Are subhorizon modes decohered?)



[Sou et al. 2207.04435]

✓ Proper observables should be insensitive to deep IR and deep UV contributions.

(e.g., adiabaticity: rapid modes decouple to slow modes. [Unruh 1110.2199 in the context of coherence])

IR: local observer's coordinate

UV: time averaged observables as well as renormalization

Consistency condition for loop calculations

ψ_3 ⟵

[Nelson 1601.03734]

$$S_3 = \int dt d^3x \Big\{ a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + 2f(\zeta)\left.\frac{\delta\mathcal{L}}{\delta\zeta}\right|_1 + \mathcal{L}_b \Big\}\,, \quad \partial^2\chi \equiv a^2\epsilon\dot{\zeta}$$

[Sou et al. 2207.04435]

$$\begin{aligned}\mathcal{L}_b = \partial_t\Big[& -9a^3H\zeta^3 + \frac{a}{H}\zeta(\partial\zeta)^2 \\ & - \frac{1}{4aH^3}(\partial\zeta)^2\partial^2\zeta - \frac{a\epsilon}{H}\zeta(\partial\zeta)^2 \\ & - \frac{\epsilon a^3}{H}\zeta\dot{\zeta}^2 + \frac{1}{2aH^2}\zeta(\partial_i\partial_j\zeta\partial_i\partial_j\chi - \partial^2\zeta\partial^2\chi) \\ & - \frac{\eta a}{2}\zeta^2\partial^2\chi - \frac{1}{2aH}\zeta(\partial_i\partial_j\chi\partial_i\partial_j\chi - \partial^2\chi\partial^2\chi)\Big]\end{aligned}$$

Necessary for correlation function

😒

Consistency condition for loop calculations

$$\psi_3 \longleftarrow$$

[Nelson 1601.03734]

$$S_3 = \int dt d^3x \Big\{ a^3\epsilon^2\zeta\dot\zeta^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot\zeta\partial\zeta\partial\chi + 2f(\zeta)\left.\frac{\delta\mathcal{L}}{\delta\zeta}\right|_1 + \mathcal{L}_b \Big\}\,, \quad \partial^2\chi \equiv a^2\epsilon\dot\zeta$$

[Sou et al. 2207.04435]

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Necessary for correlation function

- **Maldacena's consistency condition for wavefunction** [Maldacena astro-ph/0210603, Pimentel 1309.1793]

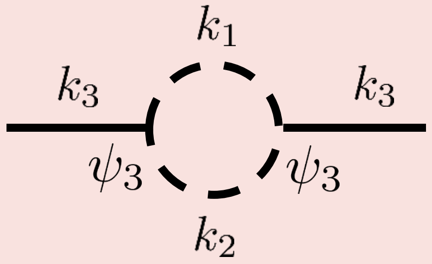
$$\lim_{k_1\to 0}\psi_3(k_1,k_3) = \left(3 - k_3\frac{d}{dk_3}\right)\psi_2(k_3)$$

Cf.

$$\lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle = -\langle\zeta_1\zeta_1\rangle\left(3 + k_3\frac{d}{dk_3}\right)\langle\zeta_3\zeta_3\rangle$$

$$\langle\zeta_1\zeta_2\rangle = \frac{1}{2\,\mathrm{Re}[\psi_2(k_1)]}, \qquad \langle\zeta_1\zeta_2\zeta_3\rangle = -\frac{2\,\mathrm{Re}[\psi_3]}{\prod_{i=1}^3 2\,\mathrm{Re}[\psi_2(k_i)]}$$

- **Loop diagram at a time slice**



IR: $k_1 \ll k_2 \simeq k_3 \ll aH \longrightarrow \log k_1$ from $\int \langle\zeta_1\zeta_1\rangle\, k_1^2 dk_1$

UV: $k_1 \simeq k_2 \gg aH \gg k_3 \longrightarrow k_1^5$ from $\partial_t(a\zeta(\partial_i\zeta)^2/H)$

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- ❑ Introduction
- ❑ Decoherence in cosmology
  - ➢ Wavefunction formalism
  - ➢ Decoherence rate and divergences
- ❑ **IR divergence: local observer effect**
- ❑ UV divergence: time-averaged observables

Approaches to IR divergence

❑ Cut-off

[Sou et al. 2207.04435]

✓ $k_{\rm IR}$ as the largest scale

➡ Finite duration of inflation

✓ The easiest way

✓ Works for every observables

❑ Resummation

[Real part of ψ_n: Céspedes et al. 2311.17990 etc.]

✓ $\sum_n (n\text{-loop}) \xrightarrow[\rm IR]{} \sum_n \alpha_n (\log k)^n$

✓ Requires higher order loops

✓ Less physical subtlety

❑ Local observer effect

[Correlators: Tanaka and Urakawa 1103.1251, Pajer et al. 1305.0824 etc.]

✓ Renormalizes constant IR modes to metric

➡ Turning off interactions with IR modes

✓ Interpreted as free-falling observer's coordinate

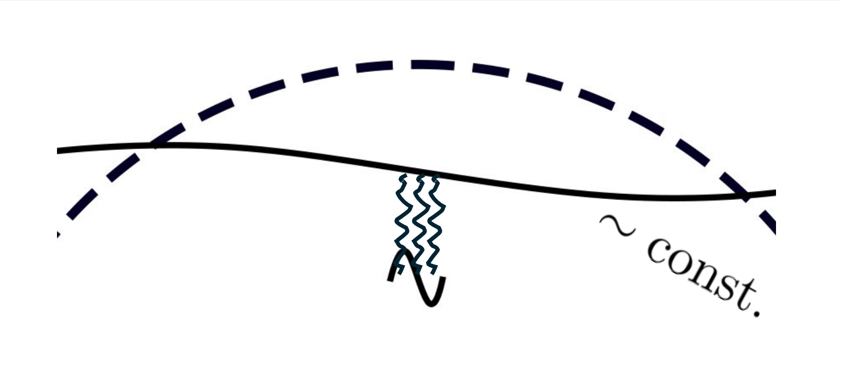
✓ Enables us order-by-order calculation

Correlation functions for a local observer

[Tanaka and Urakawa 1103.1251, Pajer et al. 1305.0824]

$$\langle \zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle \supset \int_{k_1 \ll k_3} \frac{k_1^2 dk_1 \; k_3^2 dk_3}{k_1^3 k_3^3} \sim \log k_1 \Big|_{k_1 \to 0}$$

Short modes strongly correlates with constant long modes (?)



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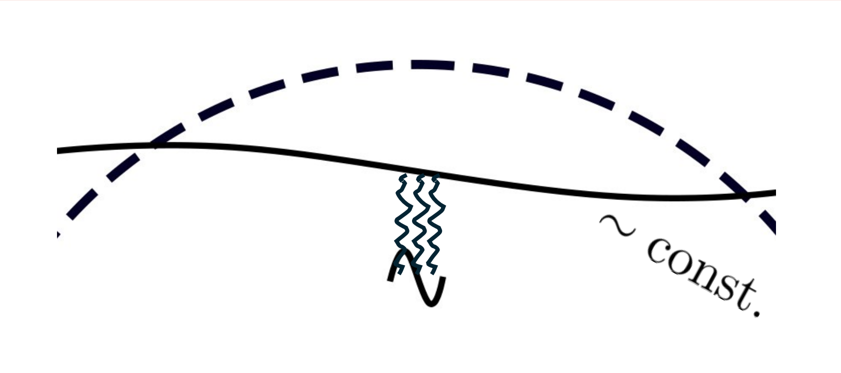


Short modes strongly correlates with constant long modes (?)

- **Conformal free-falling observer** $\mathbf{x}_{\mathrm{F}} \simeq (1+\zeta_{\mathrm{L}})\mathbf{x}$ **,** $ds^2 = a^2(-d\tau^2 + d\mathbf{x}_{\mathrm{F}}^2) + \cdots$

  (Conformal Fermi normal coordinate)

➡ $$\zeta_{\mathrm{F},\mathbf{k}} \simeq \zeta_{\mathbf{k}} + \zeta_{\mathrm{L}}(3 + k\partial_k)\zeta_{\mathbf{k}}$$

➡ $$\lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle_{\mathrm{F}} = \lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle + \langle\zeta_1\zeta_1\rangle\left(3 + k_3\frac{d}{dk_3}\right)\langle\zeta_3\zeta_3\rangle = \underline{0}$$ IR correlations are turned off

➡ $$\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle_{\mathrm{F}} \xrightarrow[\mathrm{IR}]{} \int_{k_1\ll k_3} \frac{k_1}{k_3^3} dk_1 \; dk_3$$ Finite result

Wavefunction for a local observer

[Sano and Tokuda 2504.10472]

❑ Wavefunction in free-falling coordinate

$$\Psi[\zeta] = \exp\left[-\frac{1}{2}\int_{\mathbf{k}_1,\mathbf{k}_2}\psi_2\zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3}\psi_3\zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots\right]$$

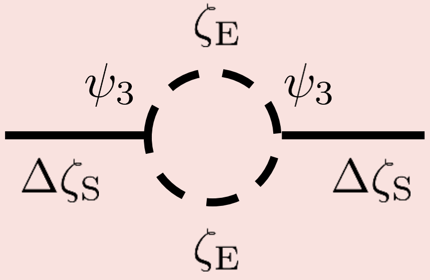
$$\zeta_{\mathbf{k}} \simeq \zeta_{\mathrm{F},\mathbf{k}} - \zeta_{\mathrm{L}}(3 + k\partial_k)\zeta_{\mathrm{F},\mathbf{k}}$$

Changing the expansion basis

$$= \Psi_{\mathrm{F}}[\zeta_{\mathrm{F}}] = \exp\left[-\frac{1}{2}\int_{\mathbf{k}_1,\mathbf{k}_2}\psi_{\mathrm{F},2}\zeta_{\mathrm{F},k_1}\zeta_{\mathrm{F},k_2} - \frac{1}{3!}\int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3}\psi_{\mathrm{F},3}\zeta_{\mathrm{F},k_1}\zeta_{\mathrm{F},k_2}\zeta_{\mathrm{F},k_3} - \cdots\right]$$

➡ $$\lim_{k_1\to 0}\psi_{\mathrm{F},3} = \lim_{k_1\to 0}\psi_3 - \left(3 - k_3\frac{d}{dk_3}\right)\psi_2 = \underline{0}$$

➡ $\Gamma_{\mathrm{IR}} \sim$



$\sim \cancel{\log(k_{\mathrm{IR}}/k_{\mathrm{S}})}$

$(k_{\mathrm{E}}/k_{\mathrm{S}})^2$ moderation for each ψ_3

$$\Gamma_{\mathrm{IR,F}} \sim (k_{\mathrm{IR}}/k_{\mathrm{S}})^4 \sim 0$$

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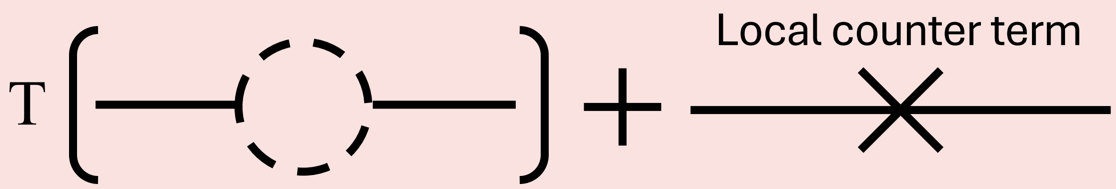
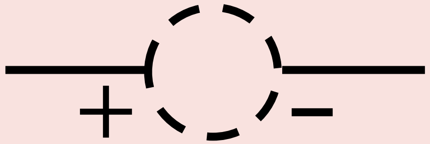
UV divergence in equal time

➢ Unitary evolution



➢ Non-unitary evolution



Counter term?

❑ Equal time correlators in 3d momentum space

✓ Composite operators in 4d position or 4d momentum space [e.g., Ch.6, "Renormalization", Collins 2023]

Loops are renormalized through counter terms: $\phi_R^2(x) = Z_a \phi^2(x) + \mu^{-1} Z_b m^2 \phi(x) + \mu^{-1} Z_c \Box \phi(x)\,.$

✓ Inconsistent treatment in time and space? [e.g., Balasubramanian et al. 1108.3568, Bucciotti 2410.01903]

$\langle \mathcal{O}_1^{\mathbf{k}} \mathcal{O}_2^{-\mathbf{k}}(t) \rangle \sim \int d(\mathbf{x}_1 - \mathbf{x}_2) \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|^{2\Delta}}$ diverges even at tree level when $\Delta \geq \frac{3}{2}$.

➡ Possible solution: Averaging/cut-off in time (time resolution of detectors)
[Agón et al. 1412.3148, Bucciotti 2410.01903, Burgess et al. 2411.09000 for Minkowski spacetime]

➡ We calculated time-averaged observables in single-field inflation.

Tomographic approach to quantum state

[Sano and Tokuda 2504.10472]

- ❑ Wavefunction $\Psi[\zeta(t)] = \langle \zeta(t)|\psi\rangle$: defined in equal time. How to consider time averaging?

- ❑ **Quantum state tomography**

$$\left.\begin{aligned} \langle \zeta_1\zeta_2\rangle &= \frac{1}{2\,\mathrm{Re}[\psi_2(k_1)]}, \quad \langle \zeta_1\zeta_2\zeta_3\rangle = -\frac{2\,\mathrm{Re}[\psi_3]}{\prod_{i=1}^3 2\,\mathrm{Re}[\psi_2(k_i)]} \\ \langle \pi_1\zeta_2\rangle &= -\frac{\mathrm{Im}[\psi_2(k_1)]}{2\,\mathrm{Re}[\psi_2(k_1)]}, \quad \langle \pi_1\zeta_2\zeta_3\rangle = \frac{2\,\mathrm{Im}[\psi_2(k_1)\psi_3^*]}{\prod_{i=1}^3 2\,\mathrm{Re}[\psi_2(k_i)]} \end{aligned}\right\} \Psi[\zeta] = \exp\left[-\frac{1}{2}\int_{k_1,k_2}\psi_2\zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int_{k_1,k_2,k_3}\psi_3\zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots\right]$$

➡ Quantum state is identified as a probability distribution of canonical variables.

✓ E.g., tree-level of averaged quantum fields

$$\langle \bar\zeta_1\bar\zeta_2\rangle \equiv \frac{1}{2\mathrm{Re}[\bar\psi_2(k_1)]}, \quad \langle \bar\pi_1\bar\zeta_2\rangle \equiv \frac{\mathrm{Im}[\bar\psi_2(k_1)]}{2\,\mathrm{Re}[\bar\psi_2(k_1)]}, \cdots \quad \Longleftrightarrow \quad \Psi[\bar\zeta] \equiv \exp\left[-\frac{1}{2}\int_{k_1,k_2}\bar\psi_2\bar\zeta_{k_1}\bar\zeta_{k_2} - \cdots\right]$$

Mathematical identity

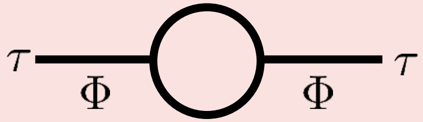
with $\lim_{k\tau\to 0}[\bar\zeta_{\mathbf{k}}, \bar\pi_{\mathbf{k}'}] = i\hbar(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')$

➡ The correlation functions $\langle \Phi(\tau)\Phi(\tau')\rangle$ in perturbative QFT is the task.

$\Phi = \zeta$ or π

Time averaged observables

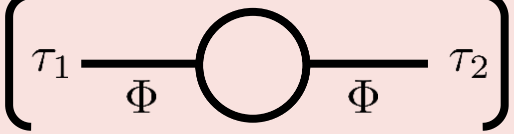
[Sano and Tokuda 2504.10472]

$\langle \Phi_1 \Phi_2(\tau) \rangle \supset$ 

$\Phi = \zeta$ or π

$$\langle \overline{\Phi}_1 \overline{\Phi}_2(\tau) \rangle = \int d\tau_1 d\tau_2 W_\tau(\tau_1) W_\tau(\tau_2) \langle \Phi_1(\tau_1) \Phi_2(\tau_2) \rangle$$

$$\supset \int d\tau_1 d\tau_2 W_\tau(\tau_1) W_\tau(\tau_2) \left[\quad \right]$$



Time averaging

$$\int^\Lambda k^\# dk \longrightarrow \Lambda^\#$$

$$\frac{1}{|\tau_1 - \tau_2|^\#}$$

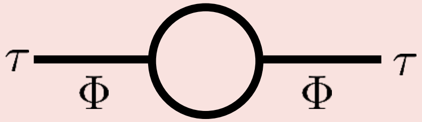
This is (expected to be) renormalized.

$$\frac{e^{-ik(\tau_1 - \tau_2)}}{|\tau_1 - \tau_2|^\#}$$

Included in Wightman function.
Not renormalized in standard procedure.

Time averaged observables

[Sano and Tokuda 2504.10472]

$\langle \Phi_1 \Phi_2(\tau) \rangle \supset$ 

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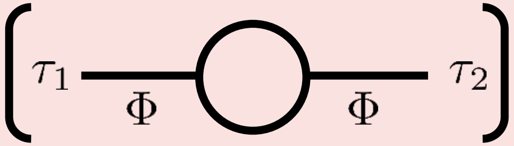
$$\supset \int d\tau_1 d\tau_2 W_\tau(\tau_1) W_\tau(\tau_2) \Big[\quad \Big]$$



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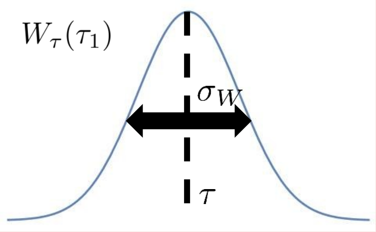
Included in Wightman function.
Not renormalized in standard procedure.

❑ Time averaging

$$W_\tau(\tau_1) = \frac{e^{-(\tau_1 - \tau)^2 / 2\sigma_W^2}}{\sqrt{2\pi\sigma_W^2}},$$

$$G(k; \tau_1, \tau_2) \propto e^{-\underline{ik}(\tau_1 - \tau_2)}$$



$$\Gamma_{\rm UV} \sim \int_{k > aH} dk \; k^\# e^{-\underline{k^2} \sigma_W^2}$$

Exponential decay in sub-horizon

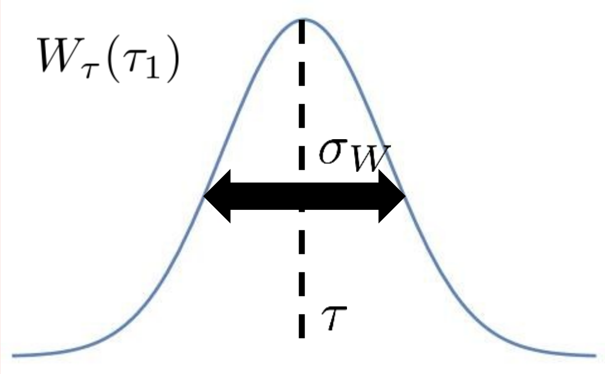
Averaging scale?

[Sano and Tokuda 2504.10472 and ongoing]

$$\Gamma_{\rm UV} \sim \int_{k>aH} dk\ k^{\#} e^{-k^2\sigma_W^2}$$

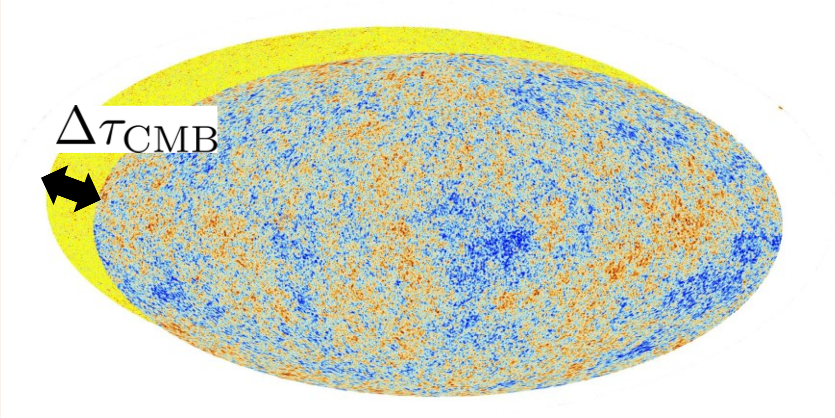
$$(a\Lambda_{\rm UV})^{-1} \lesssim \sigma_W \ll k_{\rm S}^{-1}$$

to ensure the time smearing only affect UV contributions



- **What is σ_W ?**
  - ✓ Theoretical resolution $\sigma_W \sim \dfrac{1}{a\Lambda_{\rm UV}}$
  - ✓ Phenomenological scale? E.g., $\Delta\tau_{\rm CMB}$
  - ✓ Observational device's resolution?

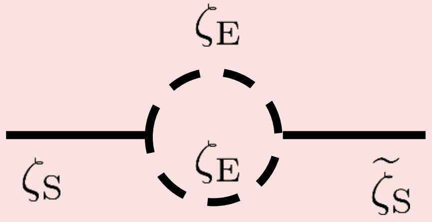
When is ζ "measured"?



Observational resolution on UV, rather than theoretical cut-off, can affect the signal

Summary

❑ False contributions $\quad (\rho_{\text{off-diag}} \sim e^{-\Gamma})$

$\Gamma \approx$ 

$$\sim \frac{H^2}{M_{\rm pl}^2}\left[\underbrace{(1+\log(k_{\rm IR}/k_{\rm S}))}_{\text{IR cutoff}}\left(\frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6+\epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3\right)+\underbrace{\left(\frac{\Lambda}{aH}\right)^{\#}}_{\text{UV cutoff}}\right]$$

Long mode is absorbed in geodesic coordinate.

$$ds^2 = a^2(-d\tau^2 + e^{2\zeta}d\mathbf{x}^2) = a^2(-d\tau^2 + d\mathbf{x}_{\rm F}^2) + \cdots$$

$$\lim_{k_1\to 0}\psi_{\rm F,3} = \lim_{k_1\to 0}\psi_3 - \left(3 - k_3\frac{d}{dk_3}\right)\psi_2 = 0$$

Classified to two contributions when averaging in time.

$$\frac{1}{|\tau_1-\tau_2|^{\#}}$$

Renormalized

$$\frac{e^{-ik(\tau_1-\tau_2)}}{|\tau_1-\tau_2|^{\#}}$$

Averaged out

$$\Gamma_{\rm UV} \sim \int_{k>aH} dk\ k^{\#} e^{-k^2\sigma_W^2}$$

✓ Leading scaling in the previous work is genuine

$$\Gamma_{\rm genuine} \sim \frac{H^2}{M_{\rm pl}^2}\left[\underbrace{\frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6}_{\partial_t(9aH\zeta^3)} + \underbrace{\epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3}_{a^2\epsilon^2\zeta(\partial\zeta)^2}\right]$$

Outlook: Importance of late time evolutions

❑ Boundary terms in late time [Sano and Tokuda 2504.10472]

✓ During inflation

$$\Gamma_{\rm inf} \sim \frac{H^2}{M_{\rm pl}^2}\left[\frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6 + \epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3\right]$$

➡

✓ Late time universe (but before re-entry)

$$\Gamma_{\rm rad.\ dom.} \sim \frac{H^2}{M_{\rm pl}^2}\left[\frac{1}{\epsilon^2}\left(\frac{a_{\rm f}H_{\rm f}}{k_{\rm S}}\right)^6\left(\frac{a}{a_{\rm f}}\right)^2 + \epsilon^2\left(\frac{a_{\rm f}H_{\rm f}}{k_{\rm S}}\right)^3\left(\frac{a}{a_{\rm f}}\right)^5\right]$$

❑ Time averaging scale?

❑ High-frequency gravitational wave [Takeda and Tanaka 2502.18560]

✓ GW with frequency $f_{\rm GW} \gtrsim 100\ {\rm Hz}$ (?) may be quantum even today!

* Estimation of thermal decoherence by a scalar field, keeping reheating in mind.

❑ Outlook

- ✓ Systematic approaches to sub-horizon evolution for more realistic models?
- ✓ Entanglement harvesting through detectors? Graviton-photon conversion?
- ✓ What is more than proving quantumness of gravity? QG from bottom up.

Back up slides

Jacobian and momentum correlators

- In general, correlation functions are expressed as

$$\langle \hat{\mathcal{O}}[\zeta,\pi]\rangle = \int \mathcal{D}\zeta_c \left(\mathcal{O}\left[\zeta_c, -i\frac{\delta}{\delta\zeta_\Delta}\right] \Psi\left[\zeta_c + \frac{\zeta_\Delta}{2}\right] \Psi^*\left[\zeta_c - \frac{\zeta_\Delta}{2}\right]\right)_{\zeta_\Delta=0} \qquad \zeta_c = \frac{\zeta+\widetilde{\zeta}}{2}, \quad \zeta_\Delta = \zeta - \widetilde{\zeta}$$

Coord. Transf.

$$\Rightarrow \langle \hat{\mathcal{O}}[\zeta_{\rm F},\pi_{\rm F}]\rangle = \int \mathcal{D}\zeta_{c,{\rm F}} \underbrace{\left|\frac{\delta\zeta_c}{\delta\zeta_{c,{\rm F}}}\right|}_{\text{Jacobian}} \left(\mathcal{O}\left[\zeta_{c,{\rm F}}, -i\frac{\delta}{\delta\zeta_{\Delta,{\rm F}}}\right] \Psi_{\rm F}\left[\zeta_{c,{\rm F}} + \frac{\zeta_{\Delta,{\rm F}}}{2}\right] \Psi^*_{\rm F}\left[\zeta_{c,{\rm F}} - \frac{\zeta_{\Delta,{\rm F}}}{2}\right]\right)_{\zeta_{\Delta,{\rm F}}=0}$$

- **Momentum correlators in the geodesic coordinate**

$$\left.\begin{aligned}
\lim_{k_1\to 0}\langle \pi_{1,{\rm F}}\zeta_{2,{\rm F}}\zeta_{3,{\rm F}}\rangle &= -\frac{(3-k_3\partial_{k_3})\,{\rm Im}\,\psi_2(k_3)}{4({\rm Re}\,\psi_2(k_3))^2}\\
\lim_{k_1\to 0}\langle \pi_{1,{\rm F}}\pi_{2,{\rm F}}\zeta_{3,{\rm F}}\rangle &= \frac{{\rm Re}[\psi_2(k_3)(3-k_3\partial_{k_3})\psi_2(k_3)]}{4({\rm Re}\,\psi_2(k_3))^2}\\
\lim_{k_1\to 0}\langle \pi_{1,{\rm F}}\pi_{2,{\rm F}}\pi_{3,{\rm F}}\rangle &= -\frac{{\rm Im}[\psi_2^2(k_3)(3-k_3\partial_{k_3})\psi_2(k_3)]}{4({\rm Re}\,\psi_2(k_3))^2}
\end{aligned}\right\}$$

Convergent but non-vanishing contributions in IR when the conjugate momentum is soft.

$\Rightarrow$ corresponding to Jacobian?

Purity as a quantumness monotone

[Streltsov et al. 1612.07570]

❑ Coherence is basis-dependent

- ✓ But the maximally mixed state cannot be coherent even when changing basis, $\frac{\hat{1}}{d} = U\frac{\hat{1}}{d}U^\dagger$.

➡ "Maximal coherence" exist for each quantum state.

❑ Coherence monotone

- ✓ Set of incoherent state $\mathcal{I}$: $\sigma = \sum_i p_i |i\rangle\langle i|$ in the basis $|i\rangle$ ➡ $\sigma \in \mathcal{I}$
- ✓ Monotone $\mathbb{C}(\rho) = \inf_{\sigma\in\mathcal{I}} D(\rho,\sigma)$ (example: L1 norm, Renyi relative entropy, ...)

  (quasi-)distance

  $\sum_{i,j,i\neq j} |\rho_{ij}|$
- ✓ Maximal coherence $\mathbb{C}_\mathrm{m}(\rho) = \sup_U \mathbb{C}(U\rho U^\dagger)$ (example: $S_\alpha(\rho||\hat{1}/d)$, which is written by **purity** when $\alpha = 2$)
- ✓ By definition, $\mathbb{C}_\mathrm{m} \geq \mathbb{C} \geq 0$ when we use the same distance.

❑ Comments on other quantumness

- ✓ Free states $\begin{cases} \text{Entanglement: separable states } \mathcal{S} & \sum_i p_i\rho_{A,i}\otimes\sigma_{B,i} \\ \text{Discord: pointer states } \mathcal{P} & \sum_i p_i\rho_{A,i}\otimes|i\rangle\langle i|_B \end{cases}$ ➡ $\mathrm{Tr}_\mathrm{E}[\mathcal{S}] \supset \mathrm{Tr}_\mathrm{E}[\mathcal{P}] \supset \mathcal{I}$ ➡ $\mathbb{C}_\mathrm{m} \geq \mathbb{C} \geq \mathbb{D} \geq \mathbb{E} \geq 0$

  Entanglement monotone

  Discord monotone

  Distance based