Cosmo. Collider as an Interaction Probe
Scale-dependence and Diagrams

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Based on:

2312.09642 with Shuntaro Aoki, Toshifumi Noumi, Masahide Yamaguchi

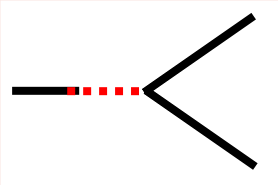
2404.09547 with Shuntaro Aoki, Lucas Pinol, Masahide Yamaguchi, Yuhang Zhu

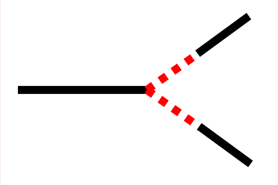
Outline

- ❑ **Introduction**

- ❑ **Non-shift-symmetric interactions of ϕ** (based on 2312.09642)
  - ➢ Time-dependent coupling and scale dependence

- ❑ **Exact calculation of double-exchange diagrams** (based on 2404.09547)
  - ➢ Comparison to single-exchange diagrams





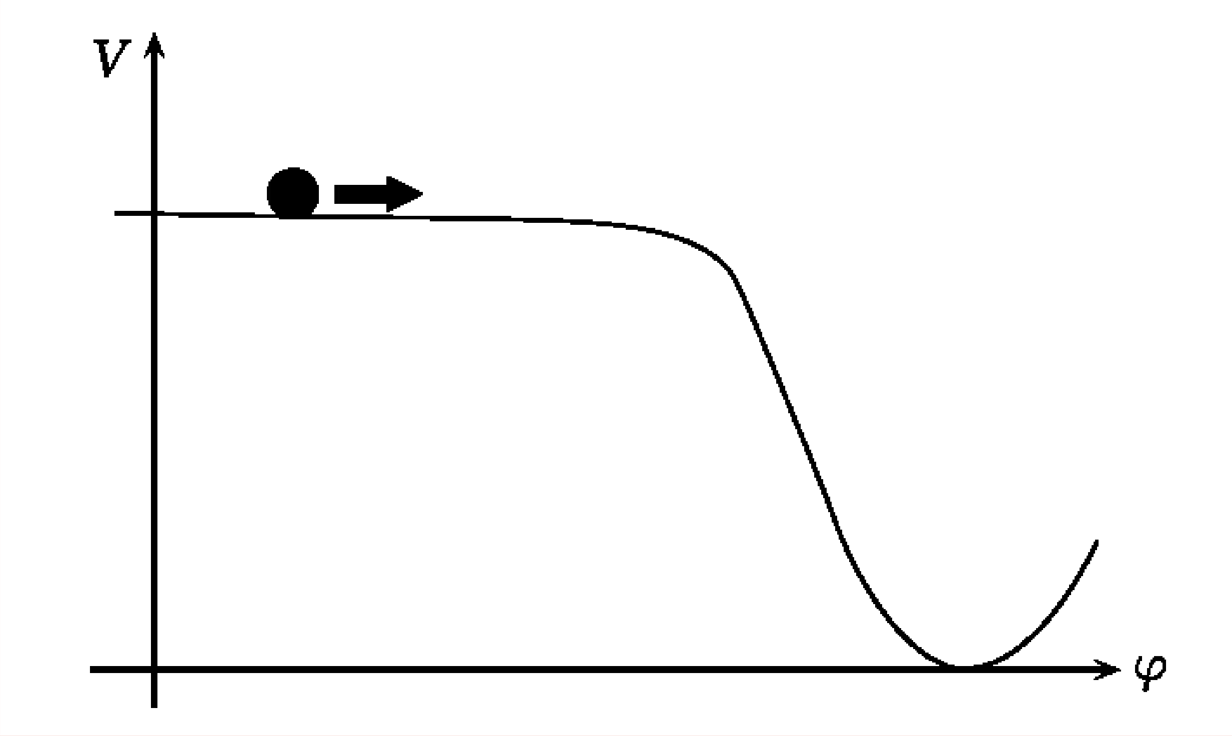
Inflation as a Source for Cosmological Perturbations

- **Slow-roll inflation**

$$\mathcal{L}_{\rm m} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$: Inflaton

$$\epsilon = M_{\rm pl}^2\left(\frac{V'}{V}\right)^2 \ll 1, \quad |\eta| = M_{\rm pl}^2\left|\frac{V''}{V}\right| \ll 1$$

$\phi = \phi_0(t) + \delta\phi(x)$ $\Longleftrightarrow$ Curvature perturb. $h_{ij} = (e^{\zeta(x)}a(t))^2\delta_{ij}$



Inflation as a Source for Cosmological Perturbations

- Slow-roll inflation can be an answer for ......

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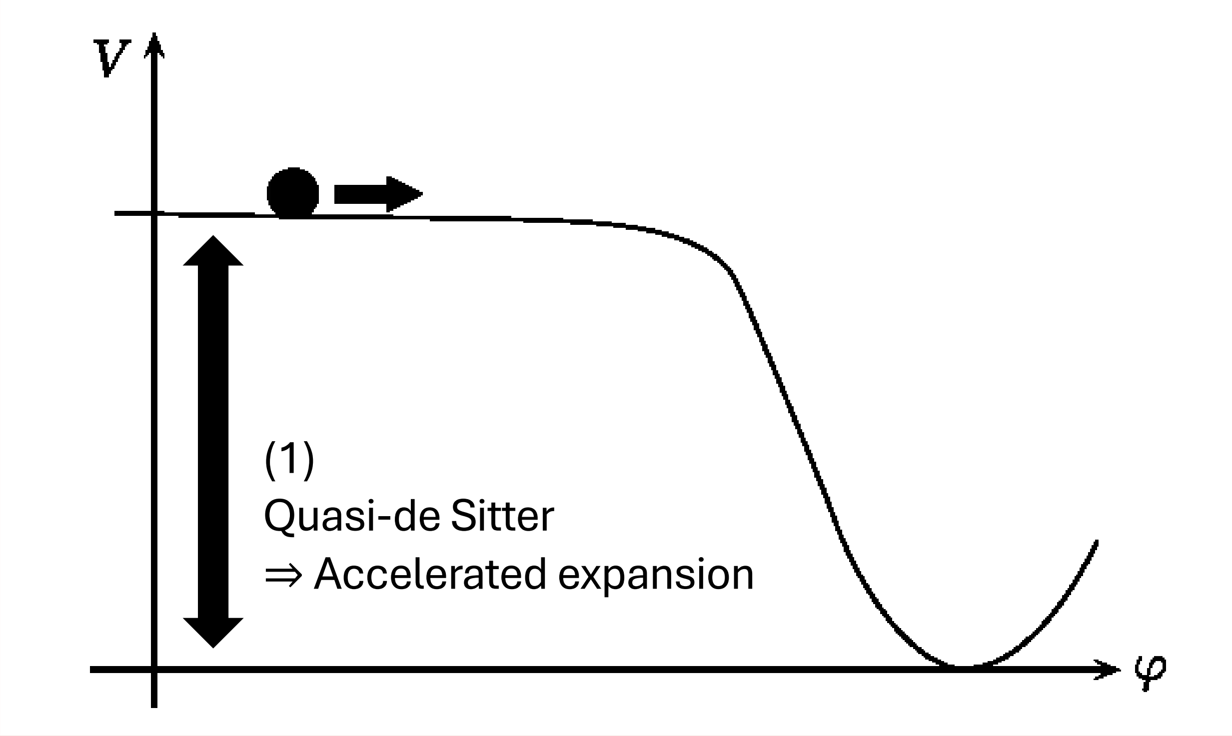
1) Flatness, horizon problem etc. in big bang model

2)

3)



Inflation as a Source for Cosmological Perturbations

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2) Transition from inflation to big bang
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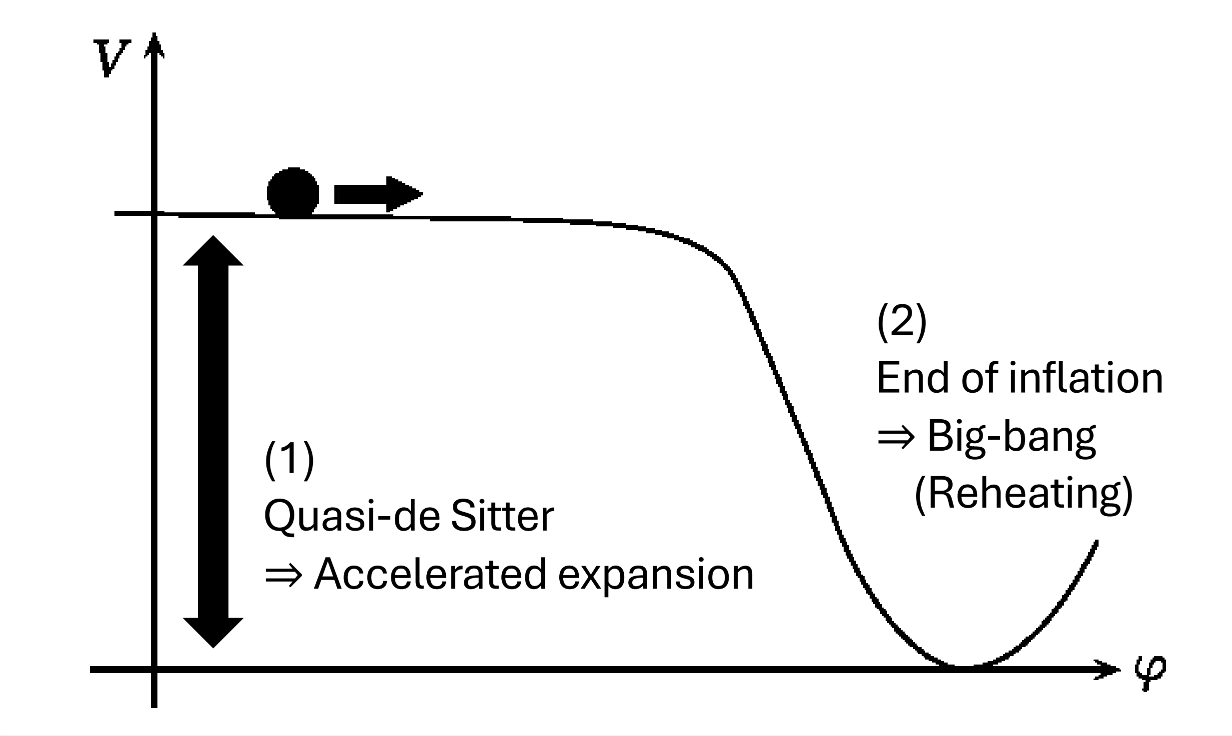

Inflation as a Source for Cosmological Perturbations

- **Slow-roll inflation can be an answer for ……**
  1) Flatness, horizon problem etc. in big bang model
  2) Transition from inflation to big bang
  3) **Origin of cosmological structures**

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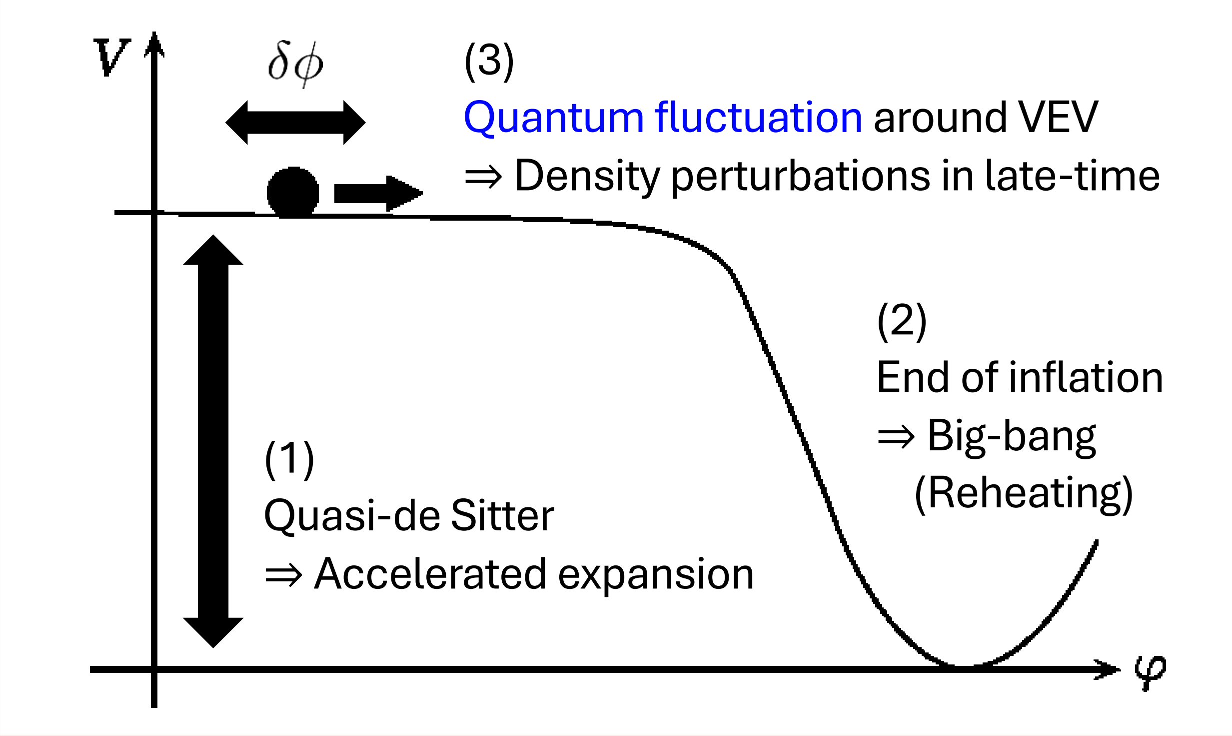
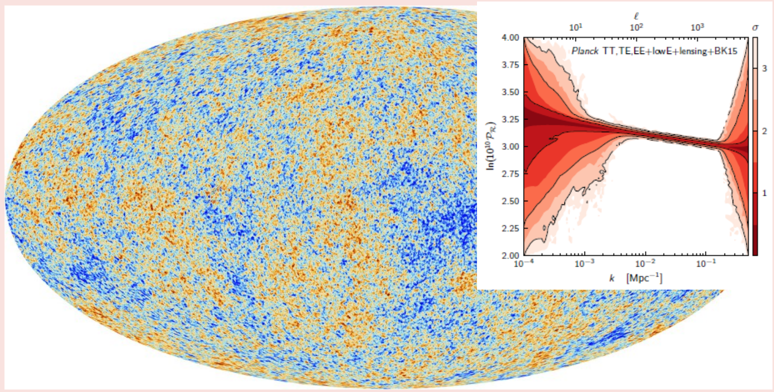
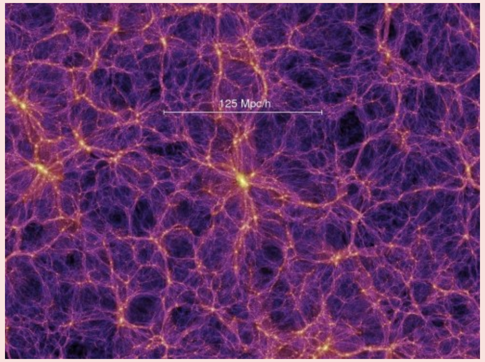




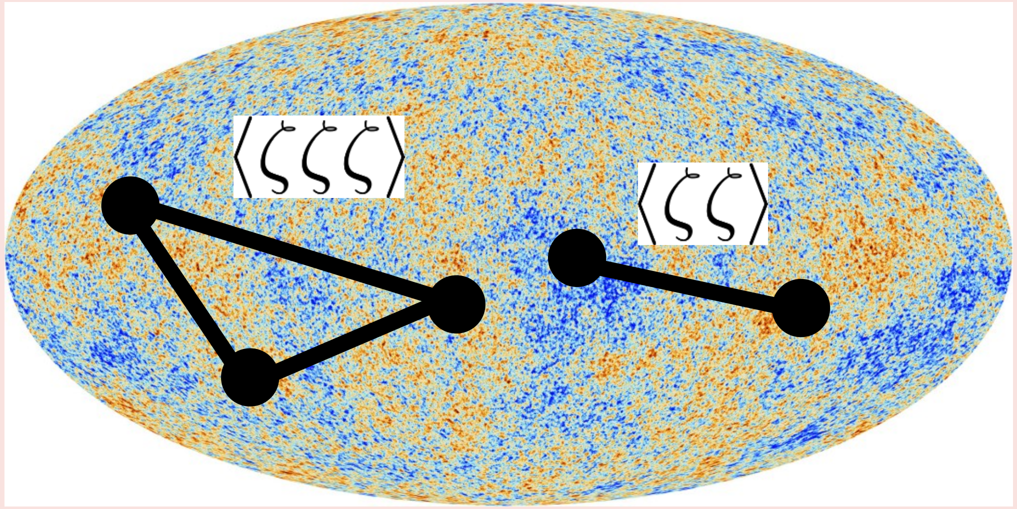
[Planck 1807.06211]



[Millennium Simulation 2005]

Observables:
Late time anisotropy and inhomogeneity

Observables for Inflationary Cosmology



❑ 2pt. correlation function (power spectrum)

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle_{\text{inf. end}} = (2\pi)^3 \delta^3(\boldsymbol{k} + \boldsymbol{k}') \frac{2\pi^2}{k^3} P_\zeta$$

$$P_\zeta \simeq \frac{H^2}{8\pi^2 \epsilon} \left(\frac{k}{k_*} \right)^{n_s - 1} \qquad n_s \simeq 0.965, \quad \frac{dn_s}{d \log k} \simeq 0.002$$

[Planck '18]

✓ CMB observation can be explained solely by curvature perturbations.

➡ Single field inflation is favored.

❑ 3pt. correlation function (bispectrum)

$$\langle \zeta_1 \zeta_2 \zeta_3 \rangle_{\text{inf. end}} = (2\pi)^7 \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \frac{P_\zeta^2}{(k_1 k_2 k_3)^2} S\left(\frac{k_1}{k_3}, \frac{k_2}{k_3} \right)$$

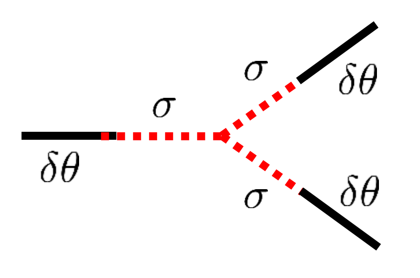
✓ 3pt.: effects of interactions ➡ **Probe for BSM physics** and inflation models

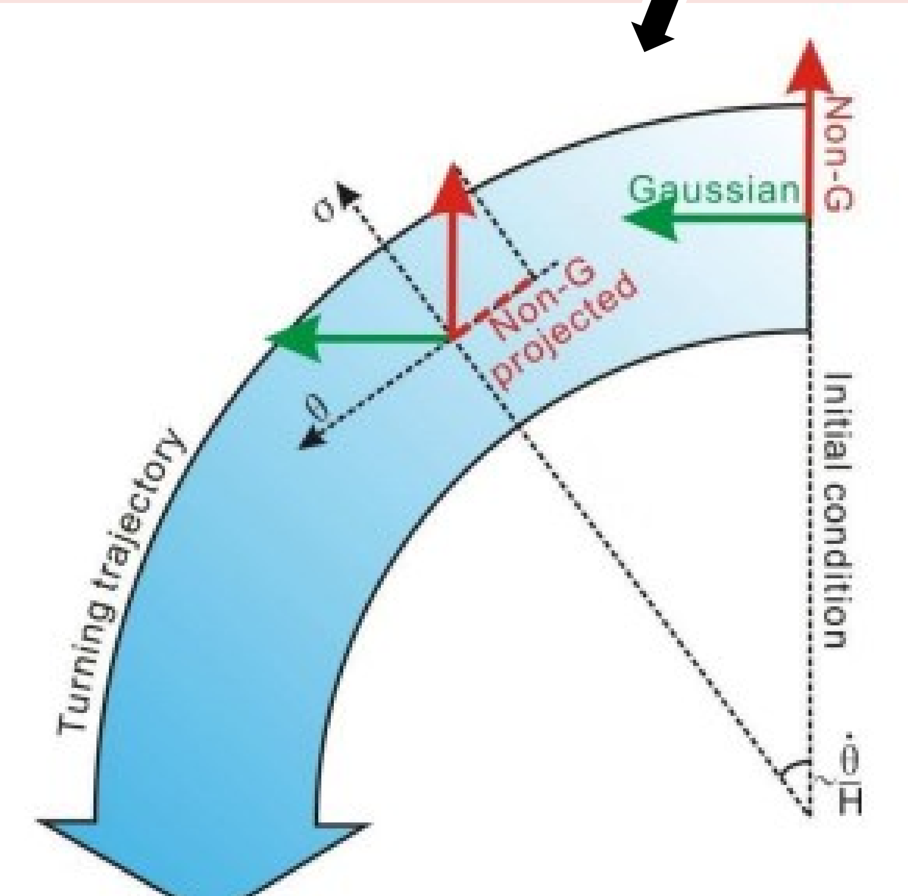
Quasi-single Field Inflation

[Chen, Wang '09, Noumi, Yamaguchi, Yokoyama '12 etc.]

- ❑ Inflaton + **heavy isocurvature mode**

$$\mathcal{L} = -\frac{1}{2}(r+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)$$





[Chen, Wang '09]

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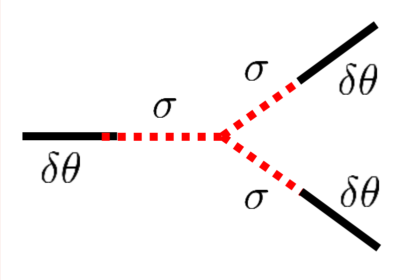
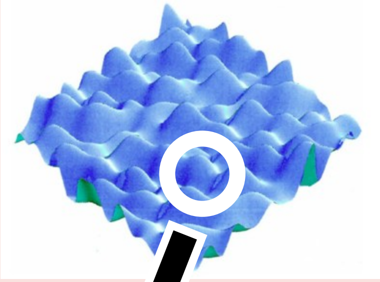
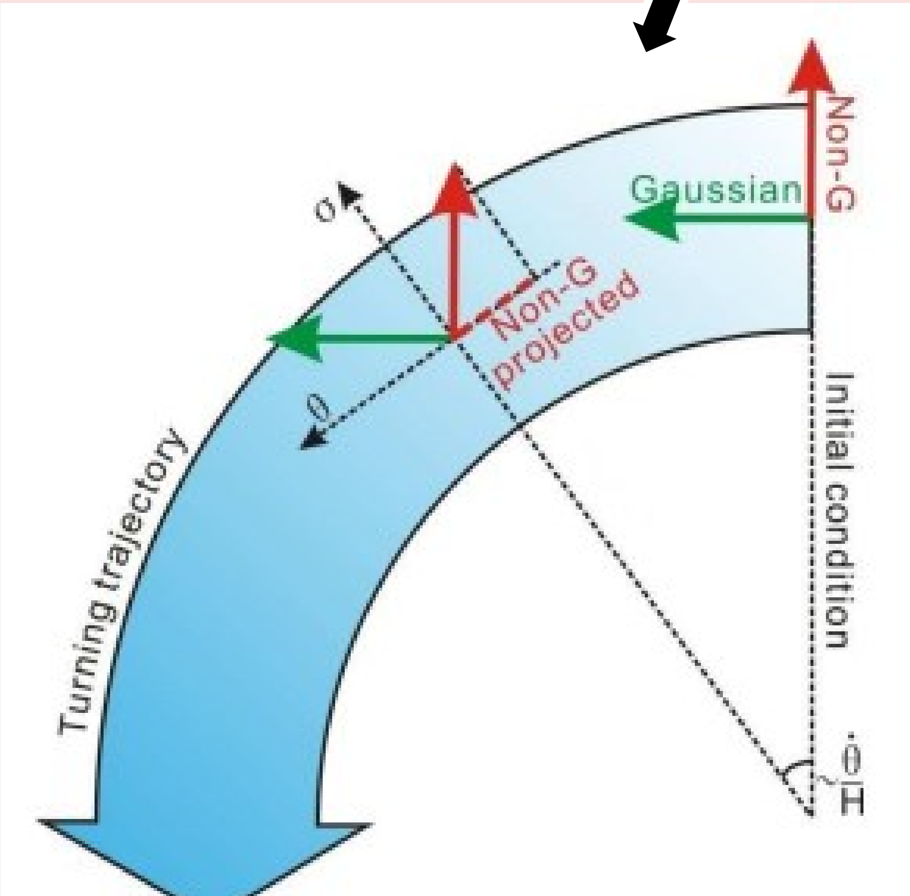
➢ UV contribution ~ single field EFT $\left(\frac{1}{\mu^2}\right)^3 \frac{k_{\rm L}}{k_{\rm S}}$

➢ IR contribution: leading in $k_{\rm L} \to 0$ when $m_\sigma \sim H$

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log\frac{k_{\rm L}}{k_{\rm S}} + \delta\right) \qquad \mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$







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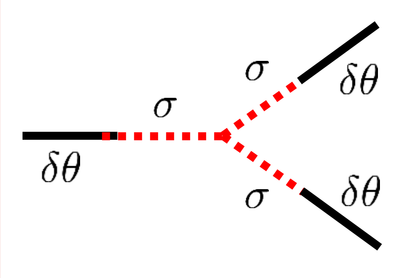
✓ Quantum interference $\left(\frac{k_{\rm S}}{k_{\rm L}}\right)^{i\mu} \sim \left(\frac{\tau_{\rm L}}{\tau_{\rm S}}\right)^{i\mu} \sim e^{im(t_{\rm S}-t_{\rm L})}$

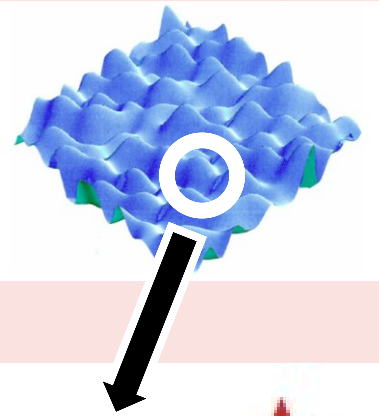
$\tau = -e^{-Ht}/H$
Conformal time

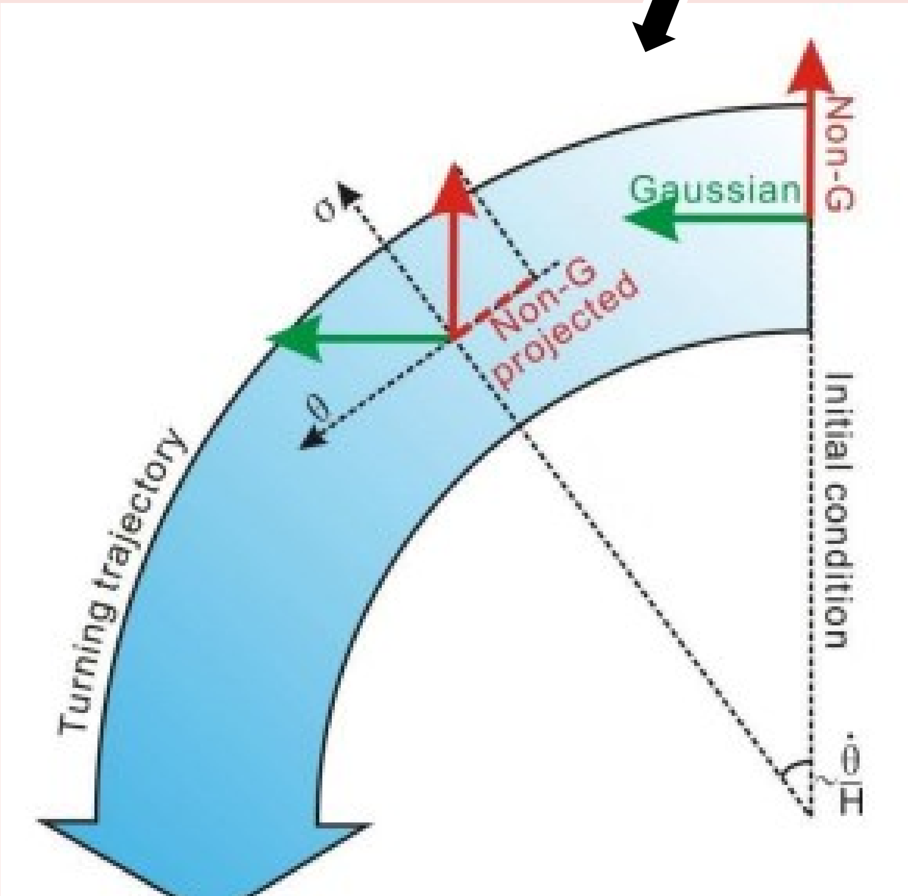
✓ Gravitational Boltzmann suppression

$$e^{-E/T} \sim e^{-m/T_{\rm H}} \sim e^{-2\pi\mu} \qquad T_{\rm H} = \frac{H}{2\pi}$$

➡ Dominant: interference term $e^{-\pi\mu}$



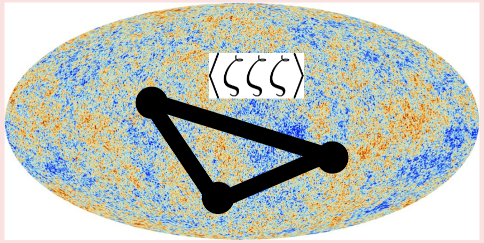




[Chen, Wang '09]

Cosmological Collider Physics

[Chen, Wang ’09, Noumi, Yamaguchi, Yokoyama ’12, Arkani-Hamed, Maldacena, ’15, Lee, Baumann, Pimentel ’16 etc.]



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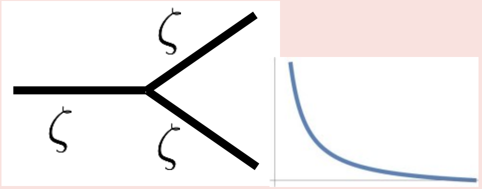

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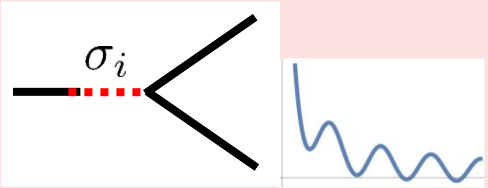

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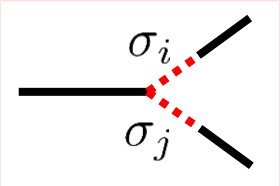

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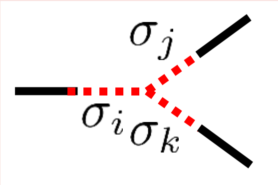

$+ \cdots$

Higher spin, loops, ...

❑ Cosmological Collider Signal

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$$

$$k_{\rm L} \equiv k_3 \ll k_1 \simeq k_2 \equiv k_{\rm S}$$

$$\mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$

$$\bullet\overset{\sigma}{\underset{k_{\rm L}}{\cdots\cdots}}\bullet \Rightarrow e^{-\pi\mu}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu}$$

✓ Dictionary of particles at the energy scale $\lesssim 10^{15}\ \mathrm{GeV}$

Supersymmetry, [Baumann, Green ’12] RH neutrino, [Chen et al. ‘18] CP violation, [Liu et al. ’19] gauge symmetry, [Maru, Okawa ’21] swampland, ... [Reece et al. ’22]

Observational Expectation

❑ Observable range of the amplitude

- ➢ CMB: $f_{\rm NL}^{\rm sq} \sim \mathcal{O}(1)$, galaxy survey: $f_{\rm NL}^{\rm sq} \sim \mathcal{O}(0.1)$, 21cm line from dark age: $f_{\rm NL}^{\rm sq} \sim \mathcal{O}(0.01)$?
  $(f_{\rm NL} \sim (k_{\rm S}/k_{\rm L})S)$
- ➢ Theoretical predictions: $f_{\rm NL}^{\rm CC} \sim (\text{coupling consts.}) \times e^{-\pi\mu} \times (k_{\rm L}/k_{\rm S})^{3/2} \times \mathcal{O}(1)$

➡ Fields with $m \sim H$ can have observably large signals.

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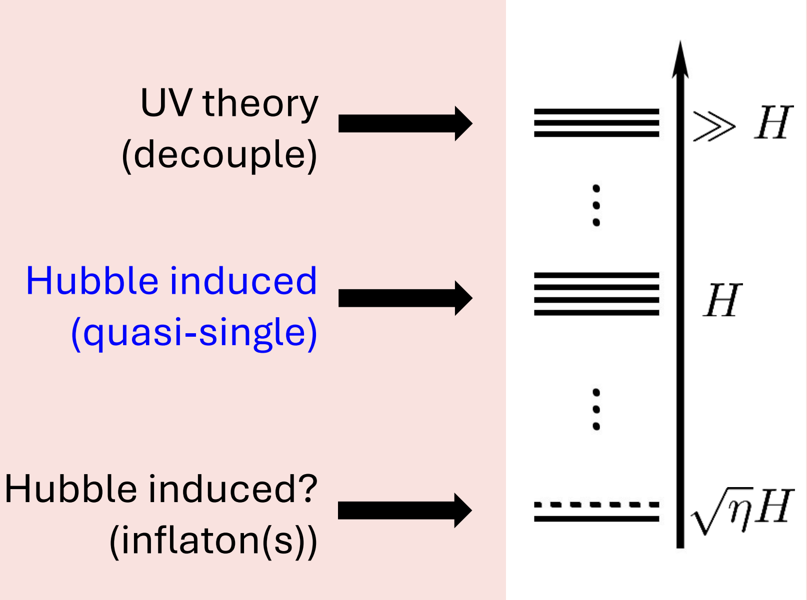
❑ Mass spectra [Copeland et al. '94, Chen, Wang, Xianyu '16 etc.]

- Hubble scale mass
  - ✓ "Thermal" correction $T_{\rm H} = H/2\pi \longrightarrow \Delta m^2 \propto T_{\rm H}^2$
  - ✓ SUGRA $\mathcal{L} \supset e^K V(\phi) \simeq V + \dfrac{c\sigma^2}{M_{\rm pl}^2} V \simeq V + 3cH^2\sigma^2$
  - ✓ Non-minimal coupling $\mathcal{L} \supset \xi\sigma^2 R \simeq 12\xi H^2\sigma^2$



How interactions appear?

$$S \sim \left(\prod_i \lambda_i\right)\left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_\mathrm{L}}{k_\mathrm{S}} + \delta\right)$$

Interactions in CC-signal

- ❑ **Diagrams** [Chen, Wang, Xianyu ’17, Qin, Xianyu ’22]

- ❑ **Shift symmetric vs. non-shift symmetric**

Interactions in CC-signal

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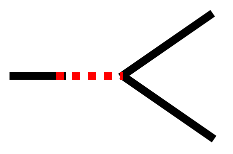
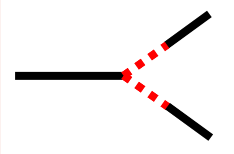
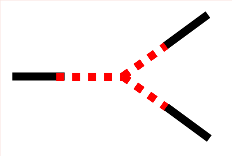
  -  ,  ,  $\sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$
  - Phase information δ : $\mathcal{A}(\mu) \times \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{i\mu} = |\mathcal{A}(\mu)| e^{i\mu \ln(k_{\rm L}/k_{\rm S}) + i\mathrm{Arg}[\mathcal{A}(\mu)]}$
- **Shift symmetric vs. non-shift symmetric**

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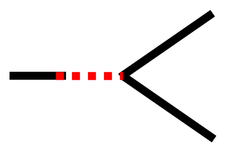
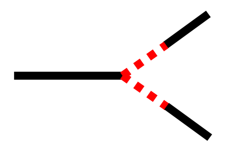
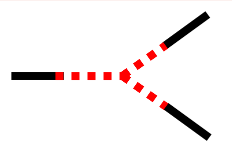
  - , ,  $\sim \left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_\mathrm{L}}{k_\mathrm{S}} + \delta\right)$
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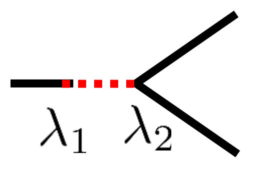
- **Shift symmetric vs. non-shift symmetric**
  - Signal size 

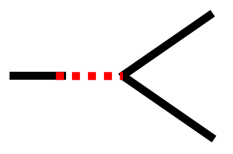
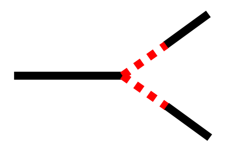
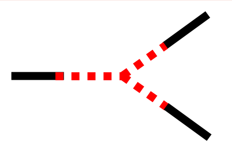
$S \propto \lambda_1 \lambda_2$

Exact dS: shift-symmetric in terms of ϕ

$\Rightarrow$ Non-shift sym. ints.: λ_1, λ_2 are bounded by η, ϵ .

Interactions in CC-signal

❑ Diagrams [Chen, Wang, Xianyu '17, Qin, Xianyu '22]

➢  ,  ,  $\sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$

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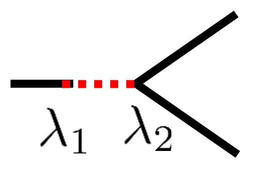
❑ Shift symmetric vs. non-shift symmetric

➢ Signal size 

$S \propto \lambda_1 \lambda_2$

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➡ Non-shift sym. ints.: λ_1, λ_2 are bounded by η, ϵ .

✓ How can we distinguish diagrams?

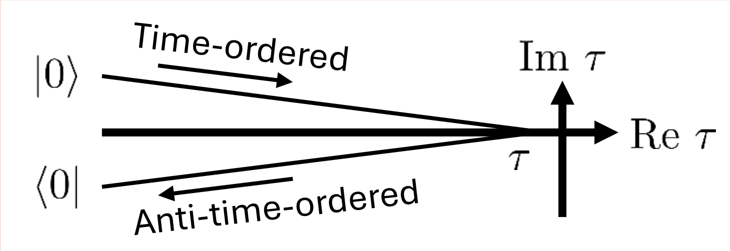
✓ Can we detect non-shift sym. ints.?

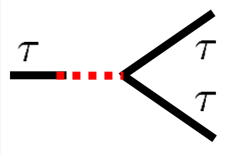
➡ Analytic templates of signals?

Difficulty in Analytical Computations

- Perturbative expansion for correlators

$$\langle \Omega | \zeta_1 \zeta_2 \zeta_3(\tau) | \Omega \rangle = \left\langle 0 \middle| \left(\overline{\mathrm{T}} e^{i \int_{-\infty}^{\tau} \mathrm{d}\tau' H_{\mathrm{I}}} \right) \zeta_1 \zeta_2 \zeta_3(\tau) \left(\mathrm{T} e^{-i \int_{-\infty}^{\tau} \mathrm{d}\tau' H_{\mathrm{I}}} \right) \middle| 0 \right\rangle$$





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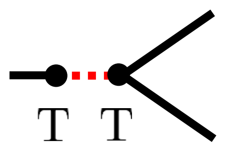

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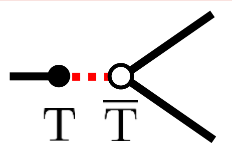
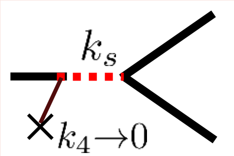


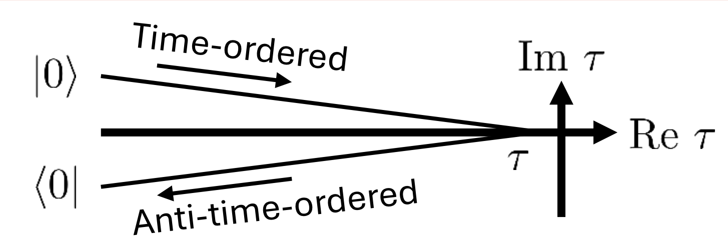
$$\Big\} \propto \frac{1}{8 k_1 k_2 k_3^4} \lim_{k_4 \to 0} \sum_{\mathrm{a,b}=\pm} \mathcal{I}_{\mathrm{ab}}^{0,-2} + (k_3 \to k_1, k_2)$$

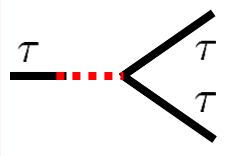


Difficulty in Analytical Computations

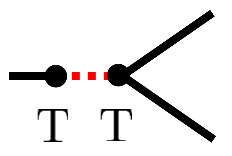
- **Perturbative expansion for correlators**

$$\langle\Omega|\zeta_1\zeta_2\zeta_3(\tau)|\Omega\rangle = \left\langle 0\left|\left(\overline{\mathrm{T}}e^{i\int_{-\infty}^{\tau}\mathrm{d}\tau' H_{\mathrm{I}}}\right)\zeta_1\zeta_2\zeta_3(\tau)\left(\mathrm{T}e^{-i\int_{-\infty}^{\tau}\mathrm{d}\tau' H_{\mathrm{I}}}\right)\right|0\right\rangle$$





$= \mathrm{Re}\Bigg\{$



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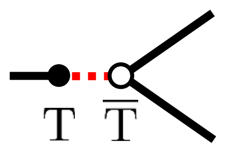

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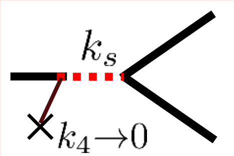

✓ Seed integral

$$\mathcal{I}_{\mathrm{ab}}^{p_1p_2} = -\mathrm{ab}k_s^{5+p_{12}}\int_{-\infty}^{0}\mathrm{d}\tau_1\ \mathrm{d}\tau_2\ \underbrace{(-\tau_1)^{p_1}(-\tau_2)^{p_2}e^{\mathrm{i}\mathrm{a}k_{12}\tau_1+\mathrm{i}\mathrm{b}k_{34}\tau_2}}_{\text{Scale factor and propagators of }\phi}\ \underbrace{D_{\mathrm{ab}}(k_s;\tau_1,\tau_2)}_{\text{Propagators of }\sigma} \qquad \mathrm{a,b}=\pm \quad \begin{matrix} +:\mathrm{T} \\ -:\overline{\mathrm{T}} \end{matrix}$$

$$D_{++}(k_s;\tau_1,\tau_2) \sim \theta(\tau_1-\tau_2)H_{i\mu}^{(1)}(-k_s\tau_1)H_{i\mu}^{(1)*}(-k_s\tau_1) \Longrightarrow$$ No formula for integration...

$\Longrightarrow$ Cosmological bootstrap is proposed as an analytical method.

[Series of papers by Baumann, Lee, Pimentel et al. ’18, ’20, ’21]

Analytical Method: De Sitter "Bootstrap" Equations

[Series of papers by Baumann, Lee, Pimentel et al. '18, '20, '21, Qin, Xianyu '22 and '23]

- **De Sitter symmetries** $\sim$ CFT

  Translation $P_i = \partial_i$, Rotation $J_{ij} = x_i\partial_j - x_j\partial_i$, Dilatation $D = -\tau\partial_\tau - x_i\partial_i$,

  dS boosts $K_i = \left(2x^j x_i + (\tau^2 - x^2)\delta_i^j\right)\partial_j + 2x_i\tau\partial_\tau$

  ✓ Ward identity: Symmetry $\hat{S}$ $\Longrightarrow$ $\langle 0|[\hat{S}, \hat{\mathcal{O}}]|0\rangle = 0$ (assuming $\hat{S}\,|0\rangle = 0$)

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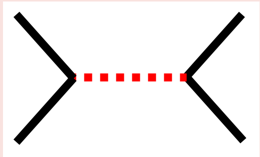
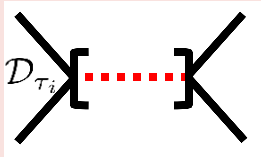
❑ "Bootstrap" equations for seed integrals

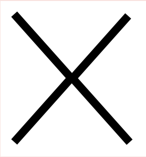
➢ Equations of motion: quadratic Casimir operator $\nabla_\mu\nabla^\mu$

$$(\nabla^2 + a^2m^2)\sigma = 0 \quad \Longrightarrow \quad \mathcal{D}_{\tau_i}\widetilde{D}^{\sigma}_{\mathrm{ab}}(k_s\tau_1, k_s\tau_2) = -\mathrm{i}\mathrm{a}H^2(k_s\tau_1)^2(k_s\tau_2)^2\delta_{\mathrm{ab}}\delta(k_s\tau_1 - k_s\tau_2)$$

$$\mathcal{D}_{\tau_i} = \tau_i\partial_{\tau_i}(\tau_i\partial_{\tau_i}) - 3\tau_i\partial_{\tau_i} + k_s^2\tau_i^2 + \mu^2 + \frac{9}{4}, \quad \mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}, \quad \widetilde{D} = k_s^3 D$$

➢ Dilatation: $\tau\partial_\tau(\cdots) = k\partial_k(\cdots)$ $\longrightarrow$ $\mathcal{D}_\tau\widetilde{D} = \mathcal{D}_k\widetilde{D}$

$\longrightarrow$ $\widetilde{\mathcal{D}}_{k_s}\Big[$  $\Big]$ $\sim$ 

$\sim$  $\longrightarrow$ $\mathcal{I}^{p_1p_2}_{\mathrm{ab}} \sim {}_2F_1, \; \sum_n^\infty \left(\frac{k_i}{\sum_j k_j}\right)^{n\,\in D^{\pm\pm}} {}_3F_2$

Boundary Conditions: Mellin-Barnes Representation

[Qin, Xianyu '22 and '23]

- **Bootstrap:** overall factors (integration constants) are not fixed.

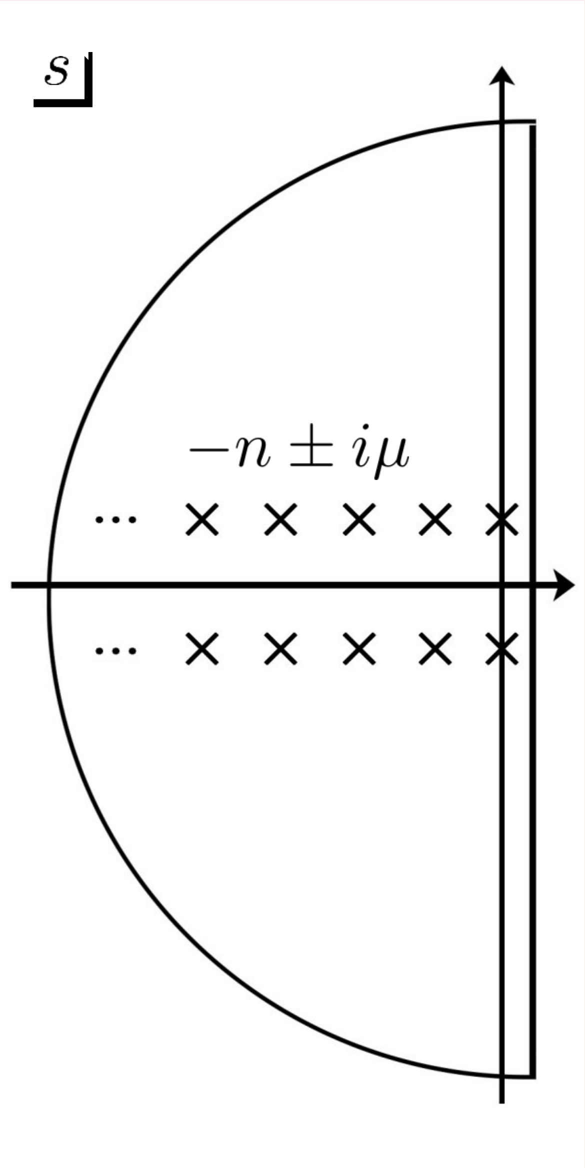
  ⟶ Reference points are necessary

- **Direct integration using MB rep.**

$$H_{i\mu}^{(1)}(-k\tau) = \frac{1}{\pi}\int_{-i\infty}^{i\infty} \frac{ds}{2\pi i}\left(\frac{-k\tau}{2}\right)^{-2s} e^{(2s-1-i\mu)\pi i/2}\Gamma(s-i\mu)\Gamma(s+i\mu)$$

$$\Rightarrow \mathcal{I} \sim \int d\tau_1 d\tau_2 e^{ik_{12}\tau_1 + ik_{34}\tau_2}(-\tau_1)^{p_1}(-\tau_2)^{p_2} H_{i\mu}^{(1)}(-k_s\tau_1)H_{i\mu}^{(1)*}(-k_s\tau_2)\theta(\tau_1-\tau_2)$$

$$\sim \sum_{\substack{n_1,n_2 \\ s_i = -n_i \pm i\mu}} \mathcal{A}_{n_1,n_2}(k,k')\mathrm{Res}[\Gamma(s_1 \pm i\mu)]\mathrm{Res}[\Gamma(s_2 \pm i\mu)]$$



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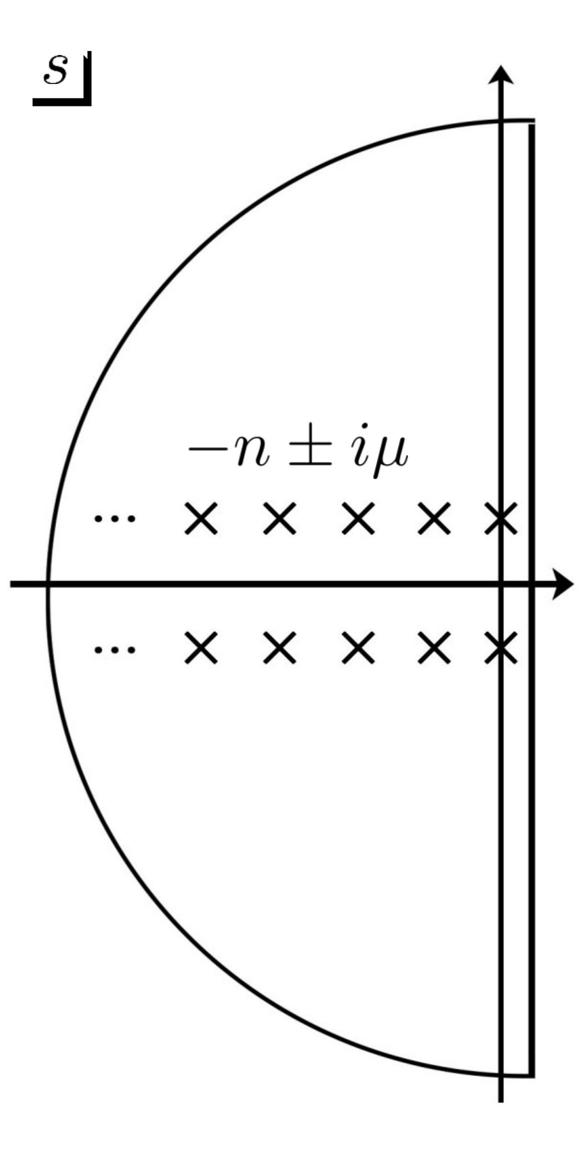


- ✓ MB rep.: double summation but boundary conditions are chosen in mode fn.
- ✓ Bootstrap: single summation but boundary conditions are non-trivial.

Equating them in some limit determines integration constants in bootstrap.
(e.g., $k_s \to 0$)
[Qin, Xianyu '22 and '23]

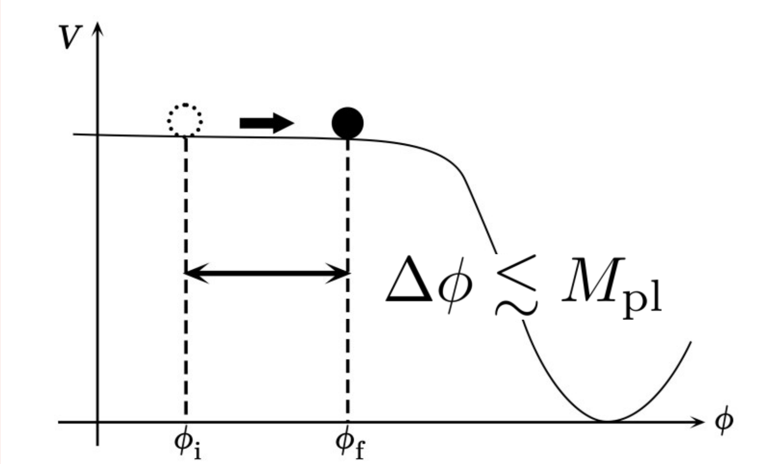
Detectability of
Non-shift-symmetric Interactions

Scale Dependence of Couplings from Slow-roll

[Wang '19, Reece, Wang, Xianyu '22]

- **Example: mass of isocurvature modes**

$$\frac{\mathcal{L}_{\rm int}}{\sqrt{-g}} = yH\phi\sigma^2 \Longrightarrow m^2_{\sigma,\rm eff} = m^2_{\sigma,0} \underline{\underline{+\, 2yH\phi_0}}$$



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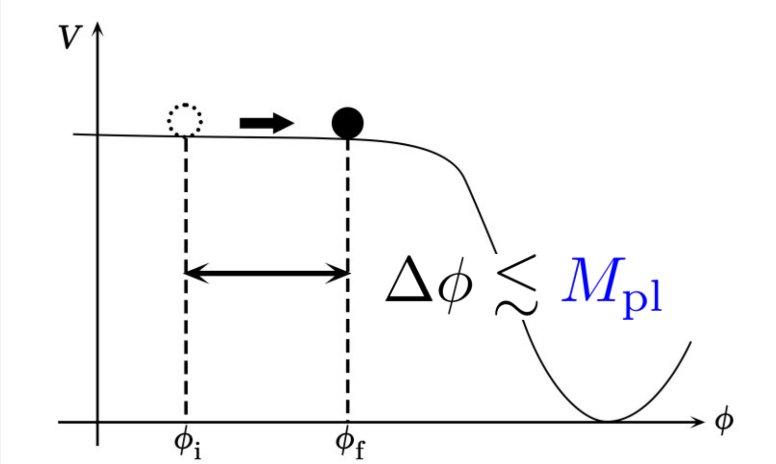
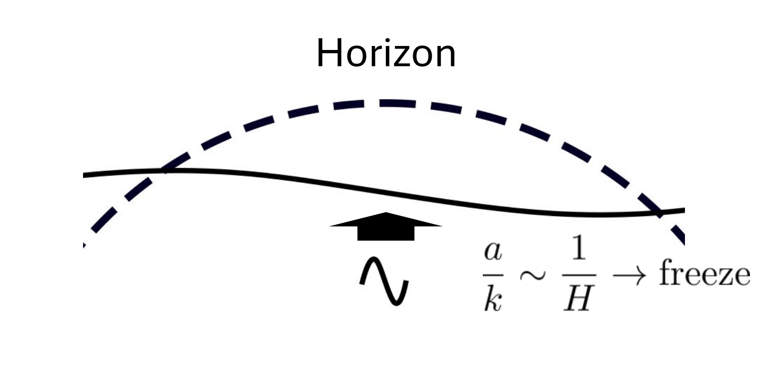


- Slow-roll approximation

$$|\phi_0| \simeq \sqrt{2\epsilon} M_{\rm pl} H(t - t_*) \simeq \sqrt{2\epsilon} M_{\rm pl} \log\left(\frac{\tau_*}{\tau}\right)$$

$$\sim \sqrt{2\epsilon} M_{\rm pl} \log\frac{k}{k_*} \qquad \text{(Horizon crossing } |k\tau| \simeq 1 \text{)}$$

$$\Rightarrow \quad \Delta m^2_\sigma(k) \sim y\sqrt{\epsilon} H M_{\rm pl} \log\frac{k}{k_*}$$



* Shift symmetric couplings

$$\frac{\mathcal{L}_{\rm int}}{\sqrt{-g}} = \frac{1}{\Lambda}(\Box\phi)\sigma^2 \quad \Rightarrow \quad \frac{|\partial_t^2\phi_0|}{\Lambda} \sim \epsilon^{3/2} H M_{\rm pl} \frac{H}{\Lambda} \log\frac{k}{k_*}$$

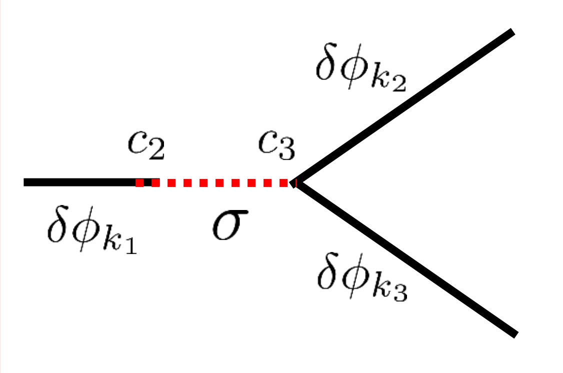
Analytical Setup for Time-dependent Mass

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) - \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}g(\phi)\sigma^2 + \mathcal{L}_{\rm diag}\right]$$

- **Single-exchange with derivative coupling**

$$\mathcal{L}_{\rm diag} \supset \underline{c_2}(-\tau)^{-3}\sigma\delta\phi' + \underline{c_3}(-\tau)^{-2}\sigma(\delta\phi')^2$$

Unbounded by slow-roll (shift symmetric)



- **Time-dependent mass**

  - $\sigma_k'' - \frac{2}{\tau}\sigma_k' + \left(k^2 + \frac{m_{\rm eff}^2}{H^2\tau^2}\right)\sigma_k = 0$, $\quad \sigma_k = v_k a_k + v_k^* a_{-k}^\dagger$

$\Rightarrow$ $v_k = \frac{e^{\pi\gamma/2}}{\sqrt{2k}}(-H\tau)W_{-i\gamma,i\mu}(2ik\tau)$ $\quad \mu^2 = \frac{g_*}{H^2}\left(1 - \frac{\sqrt{2\epsilon}g_{*,\phi}M_{\rm pl}}{g_*}\right) - \frac{9}{4}, \quad \gamma = -\frac{\sqrt{2\epsilon}g_{*,\phi}M_{\rm pl}}{2H^2}$

(cf. $W_{0,i\mu} \sim H_{i\mu}^{(1)}$)

$\Rightarrow$ Analytically calculable using MB rep. for Whittaker functions! (full result: see our paper)

Bispectrum: Mass at Horizon-crossing

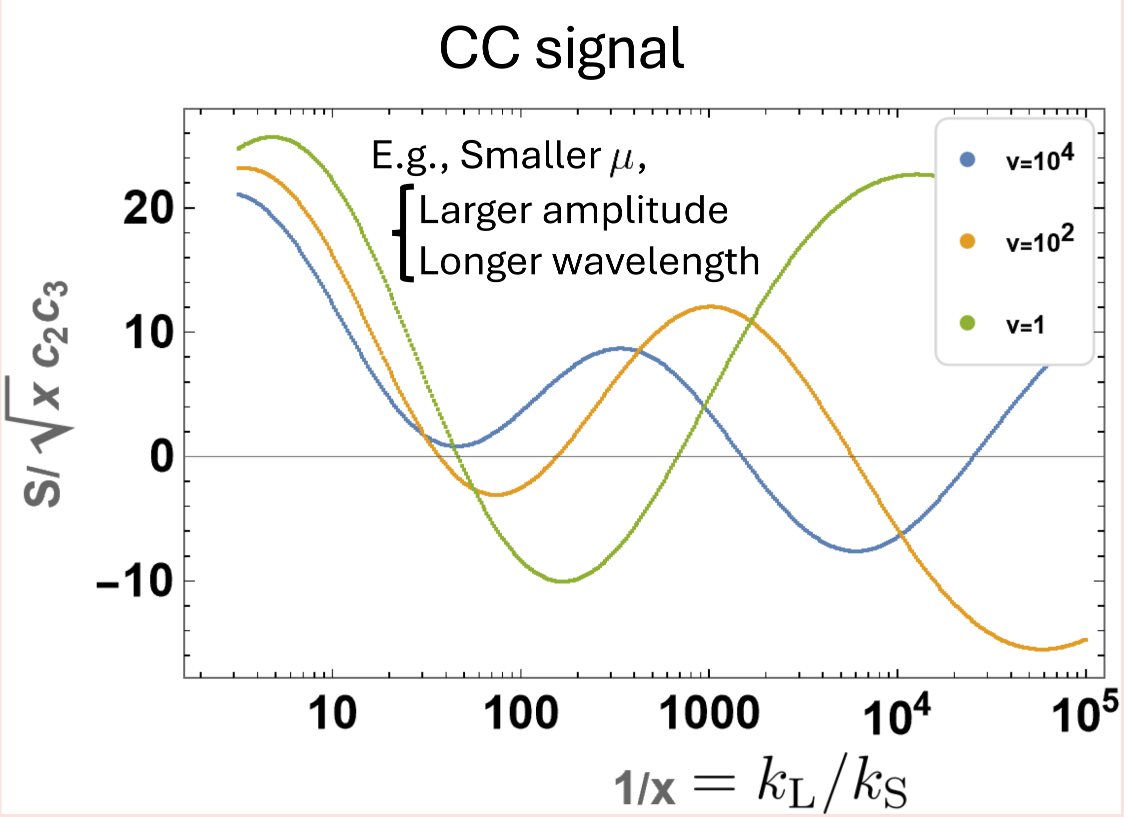
$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_3}{k_1}\right)$$

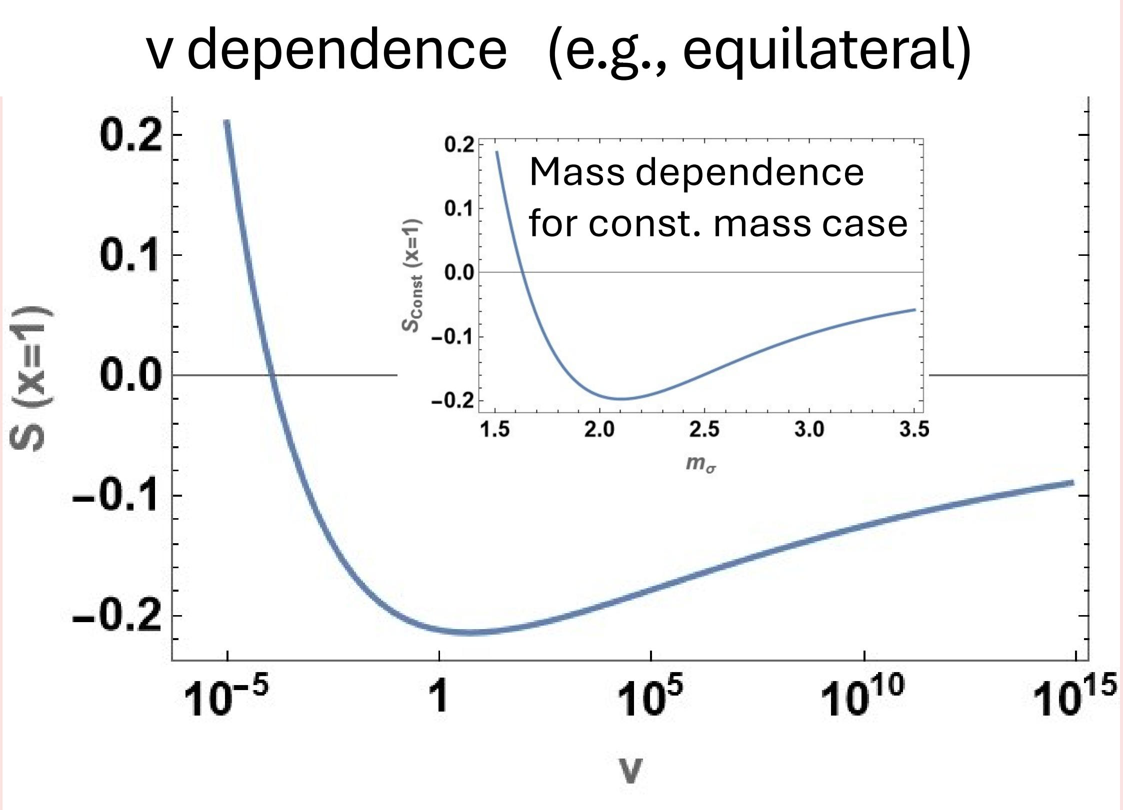
$$S \sim \left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)} \cos\left[\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)\log\frac{k_\mathrm{L}}{k_\mathrm{S}} + \delta\left(\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)\right)\right]$$

$$\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right) = \frac{m_0}{H}\sqrt{1-\alpha\sqrt{2\epsilon}\left(1+\log\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)\right) - \frac{9}{4}}$$

With the interaction $m_0^2\left(1+\alpha\frac{\phi}{M_\mathrm{pl}}\right)\sigma^2, \quad \Delta\phi \sim \sqrt{\epsilon}M_\mathrm{pl}\Delta N$

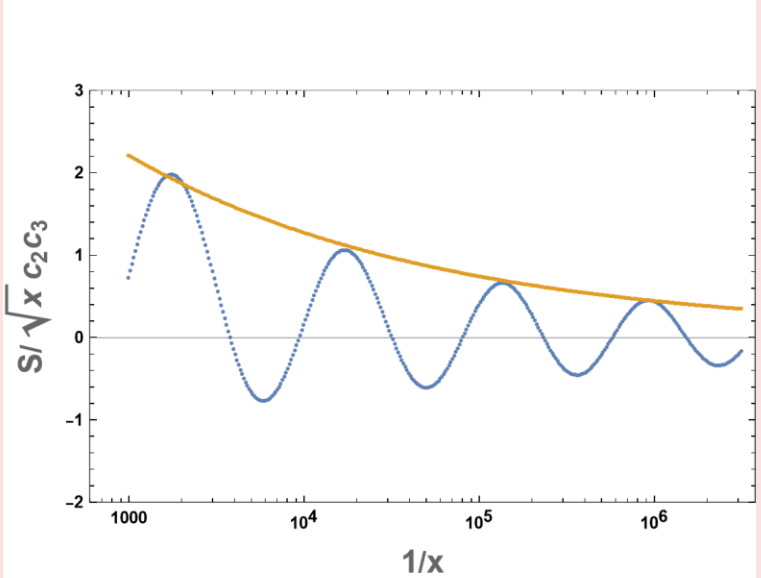
$v \equiv k_\mathrm{S}/k_*$: Scale dependence





Interaction distinction using scale dependence

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)} \cos\left[\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\log\frac{k_{\rm L}}{k_{\rm S}} + \delta\left(\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\right)\right]$$



- ❑ $\overset{\phi_0\sigma^2}{\Delta\mu^2_{\rm NSS}(k)} \lesssim \sqrt{\epsilon}\frac{M_{\rm pl}}{H}$ vs. $\overset{\ddot{\phi}_0\sigma^2}{\Delta\mu^2_{\rm SS}(k)} \lesssim \epsilon^{3/2}\frac{M_{\rm pl}}{\Lambda}$

- ❑ $e^{-\pi\mu} \sim \exp\left[-\frac{\pi}{H}\sqrt{m_0^2 - \frac{9H^2}{4} + g\left(M_{\rm pl}\sqrt{2\epsilon}\log\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\right)}\right]$ for $\frac{\mathcal{L}_{\rm int}}{\sqrt{-g}} = g(\phi)\sigma^2$

✓ Scale dependence (suppression / enhancement etc.) is characterized by the interaction

Non-shift-sym. ints: detectable through scale-dependence

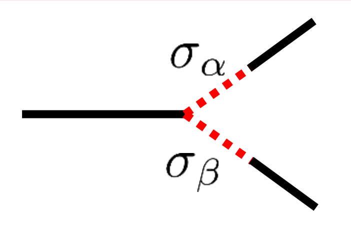
Single-Exchange Diagrams
vs.
Double-Exchange Diagrams

Method: Bootstrap Equations and MB Representations

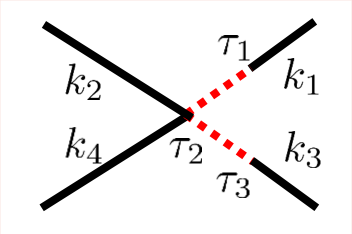
[Xianyu, Zang ’24: MB rep.]

Numerical check: CosmoFlow
[Pinol, Renaux-Petel, Werth ’23, ’24]

$$\mathcal{L}_{\text{int}} = a^3 \sum_{\alpha} \rho_\alpha \sigma_\alpha \delta\phi' + a^3 \sum_{\alpha,\beta} \lambda_{\alpha\beta} \sigma_\alpha \sigma_\beta \delta\phi'$$



Seed integral



($k_4 \to 0$: bispectrum)

$$\mathcal{I}^{p_1p_2p_3}_{\text{abc},\alpha\beta} = H^{-4} k_{24}^{9+p_{123}} (-\text{iabc}) \int_{-\infty}^{0} \mathrm{d}\tau_1\, \mathrm{d}\tau_2\, \mathrm{d}\tau_3\, (-\tau_1)^{p_1} (-\tau_2)^{p_2} (-\tau_3)^{p_3}$$
$$\times\, e^{\text{i}ak_1\tau_1 + \text{i}bk_{24}\tau_2 + \text{i}ck_3\tau_3} D^{\alpha}_{\text{ab}}(k_1;\tau_1,\tau_2)\, D^{\beta}_{\text{bc}}(k_3;\tau_2,\tau_3)$$

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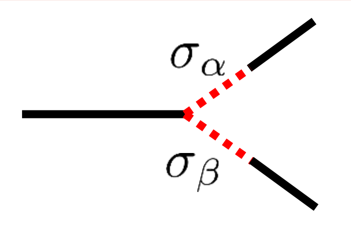

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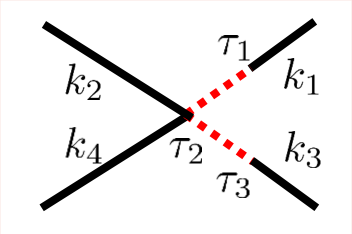

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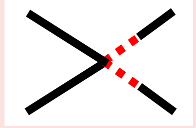
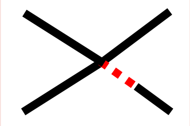
- **"Bootstrap" equations**

$\mathcal{D}\Big[$  $\Big] \sim$ 

$$\mathcal{I} \sim F_4, \sum_{n}^{\infty}\left(\frac{k_i}{\sum_j k_j}\right)^n ({}_3F_2 + {}_2F_1)$$

Analytical expression for arbitrary momentum configuration

- **Bispectrum in squeezed region**

$$\langle \delta\phi_{k1}\delta\phi_{k2}\delta\phi_{k3}\rangle' \xrightarrow[k_3\to 0]{} \sum_{\alpha,\beta} \frac{\rho_\alpha\rho_\beta\lambda_{\alpha\beta}H}{(k_1k_2k_3)^2}\cdot \mathrm{Re}\left\{\left[\mathrm{i}\frac{\pi^{3/2}}{2^{4+2\mathrm{i}\mu_\alpha}}\,\mathrm{sech}\,(\pi\mu_\beta)\,[1+\tanh(\pi\mu_\alpha)]\times\Gamma\left[\begin{matrix}-\mathrm{i}\mu_\alpha\\ -1-\mathrm{i}\mu_\alpha+\mathrm{i}\mu_\beta, -1-\mathrm{i}\mu_\alpha-\mathrm{i}\mu_\beta\end{matrix}\right]\right.\right.$$
$$\left.\left.\times\,{}_3\mathcal{F}_2\left[\left.\begin{matrix}-\frac{3}{2}-\mathrm{i}\mu_\alpha, -1-\mathrm{i}\mu_\alpha-\mathrm{i}\mu_\beta, -1-\mathrm{i}\mu_\alpha+\mathrm{i}\mu_\beta\\ -\frac{1}{2}-\mathrm{i}\mu_\alpha, -\frac{1}{2}-\mathrm{i}\mu_\alpha\end{matrix}\,\right|\,1\right] + \mathcal{O}\left(e^{-2\pi\mu_\alpha}, e^{-2\pi\mu_\beta}\right)\right]\left(\frac{k_1}{k_3}\right)^{\frac{1}{2}+\mathrm{i}\mu_\alpha} + \mathcal{O}\left(\frac{k_1}{k_3}\right)\right\}$$

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Numerical check: CosmoFlow
[Pinol, Renaux-Petel, Werth '23, '24]

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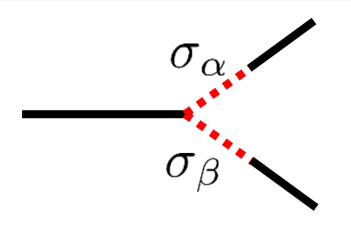
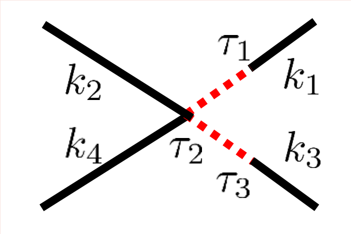


Seed integral →



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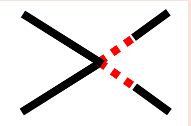
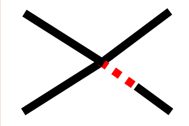
$$\mathcal{I}^{p_1p_2p_3}_{\text{abc},\alpha\beta} = H^{-4}k_{24}^{9+p_{123}}(-\text{iabc})\int_{-\infty}^{0} \mathrm{d}\tau_1\,\mathrm{d}\tau_2\,\mathrm{d}\tau_3\,(-\tau_1)^{p_1}(-\tau_2)^{p_2}(-\tau_3)^{p_3}$$
$$\times\, e^{\text{i}ak_1\tau_1+\text{i}bk_{24}\tau_2+\text{i}ck_3\tau_3} D^{\alpha}_{\text{ab}}(k_1;\tau_1,\tau_2)\, D^{\beta}_{\text{bc}}(k_3;\tau_2,\tau_3)$$

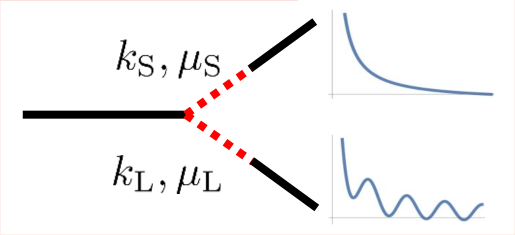
- **"Bootstrap" equations**

$\mathcal{D}\Big[$  $\Big] \sim$  →

$$\mathcal{I} \sim F_4,\ \sum_n^\infty \left(\frac{k_i}{\sum_j k_j}\right)^n ({}_3F_2 + {}_2F_1)$$

Analytical expression for arbitrary momentum configuration

- **Bispectrum in squeezed region**



$$S \sim \left(\frac{k_L}{k_S}\right)^{1/2} e^{-\pi\mu_L}\cos\left(\mu_L \log\frac{k_L}{k_S} + \delta\right)$$

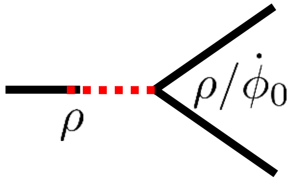
Qualitatively same as single-exchange?

Difference between SE and DE 1: Size of Signals

[Pinol, Renaux-Petel, Werth ’23]

❑ Single-exchange (SE)

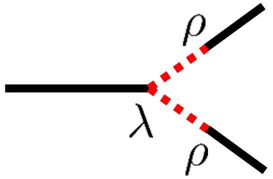
$$\frac{\rho}{\dot\phi_0}(\partial_\mu\phi)^2\sigma \longrightarrow \rho\ \delta\phi'\sigma + \frac{\rho}{\dot\phi_0}\ (\delta\phi')^2\sigma$$



$$S_{\rm SE} \sim \frac{\rho^2}{\dot\phi_0} \times P_\zeta^{-1/2}$$

❑ Double-exchange (DE)

$$\rho\ \delta\phi'\sigma + \lambda\ \delta\phi'\sigma^2$$

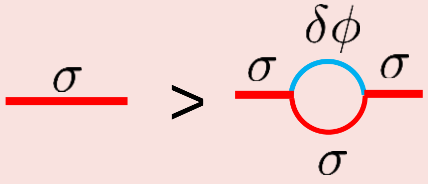


$$S_{\rm DE} \sim \lambda\frac{\rho^2}{H^2} \times P_\zeta^{-1/2}$$

❑ Constraints

✓ Perturbativity $\quad \lambda \lesssim 1$

✓ Naturalness $\quad \lambda \lesssim P_\zeta^{1/4}$



$$\boxed{\frac{S_{\rm DE}}{S_{\rm SE}} \sim \lambda\frac{\dot\phi_0}{H^2} \sim \lambda P_\zeta^{-1/2} \lesssim P_\zeta^{-1/4} \sim 10^2 \quad \textbf{Naturally larger than single-exchange}}$$

Difference between SE and DE 2: Phase Information

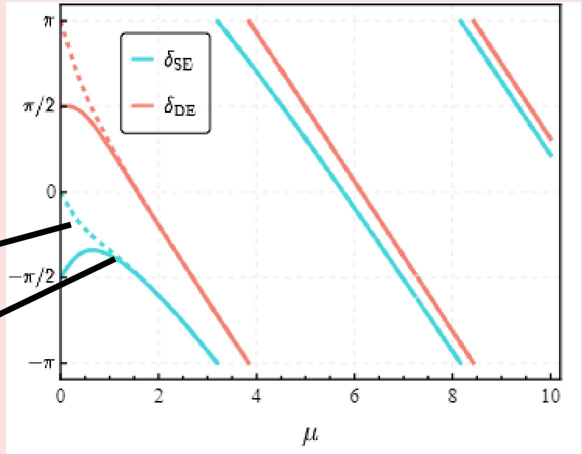
❑ Phase information

➢ SE and single isocurvature DE

$$S^{\rm single}_{\rm DE,CC} = \frac{\rho^2}{H^2}\frac{\lambda}{2\pi P_\zeta^{1/2}}{\rm Re}\left[\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu}\mathcal{A}_{\rm DE}(\mu)e^{i\delta(\mu)}\right]$$

$$S_{\rm SE,CC} = \frac{\rho^2}{\dot\phi}\frac{1}{2\pi P_\zeta^{1/2}}{\rm Re}\left[\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu}\mathcal{A}_{\rm SE}(\mu)e^{i\delta(\mu)}\right]$$

Consistency between phase and wavelength



Leading of μ

Full order

➢ DE with multiple isocurvature modes

$$S^{\rm multi}_{\rm DE,CC} = \sum_{\alpha,\beta}^{N}\frac{\rho_\alpha\rho_\beta}{H^2}\frac{\lambda_{\alpha\beta}}{2\pi P_\zeta^{1/2}}{\rm Re}\left[\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu_\alpha}\mathcal{A}_{\mu_\alpha,\mu_\beta}e^{i\delta_{\mu_\alpha,\mu_\beta}}\right]$$

$$= \sum_{\alpha}^{N}\frac{\rho_\alpha}{H}{\rm Re}\left[\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu_\alpha}\mathcal{B}_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta}e^{i\vartheta_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta}}\right]$$

$$\left(a\sin\theta + b\sin(\theta+\Delta\theta) = \sqrt{a^2+b^2+2ab\cos\Delta\theta}\,\sin(\theta+\alpha)\right)$$

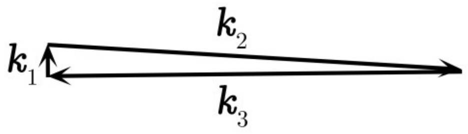
# of observables: $3N$ $\leq$ # of parameters: $N(N+5)/2$

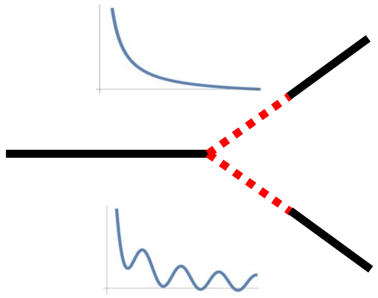
✓ Amplitude	N	✓ ρ_α	N
✓ Wavelength	N	✓ μ_α	N
✓ Phase	N	✓ $\lambda_{\alpha\beta}$	$N(N+1)/2$

Parameters are not determined only from CC signal.
Analytic template for arbitrary $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ is important!

Difference between SE and DE 3: Trispectrum

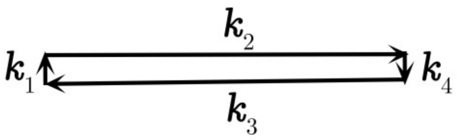
❑ Bispectrum

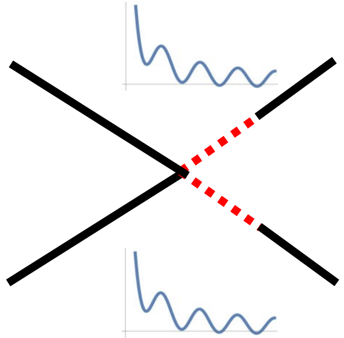




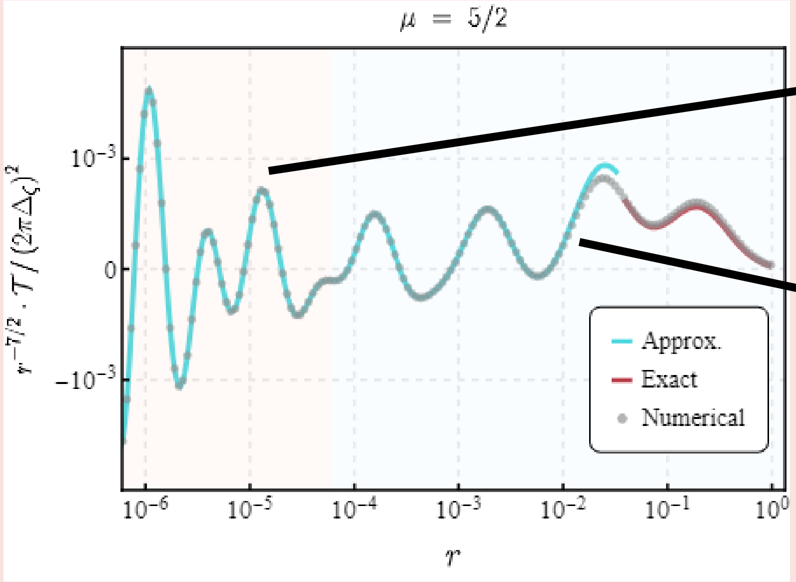
Only one soft mode

❑ Trispectrum





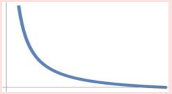
Two soft modes!



$$\mu_\alpha^{3/2}\mu_\beta^{3/2}e^{-\pi(\mu_\alpha+\mu_\beta)}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{3+i(\mu_\alpha+\mu_\beta)}$$

No such signals in SE

$$\frac{\mu_\alpha^{3/2}}{\mu_\beta^2}e^{-\pi\mu_\alpha}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{7/2+i\mu_\alpha}$$

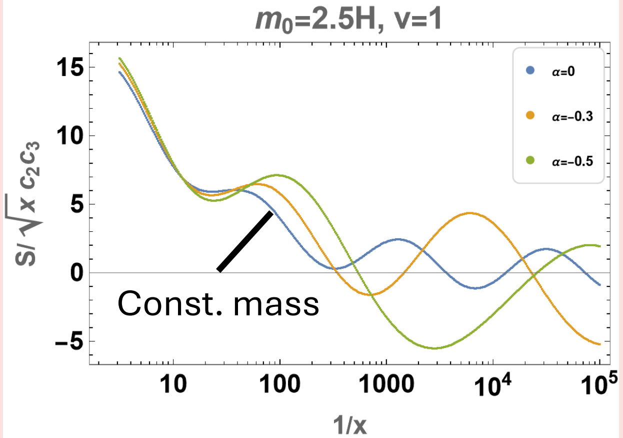
*  $\sim \mu^{3/2}e^{-\pi\mu}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{3/2+i\mu}$,  $\sim \frac{1}{\mu^2}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^2$

Summary

❑ Non-shift-symmetric interactions

➢ Large excursion of inflaton $\Delta\phi \sim \sqrt{\epsilon}M_{\rm pl}\Delta N$ introduces scale-dependence for non-shift sym. couplings.

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu_k} \cos\left[\mu_k \log\frac{k_{\rm L}}{k_{\rm S}} + \delta(\mu_k)\right] \qquad \mathcal{L}_{\rm int} = y f(\phi)\sigma^2 \qquad \Delta\mu^2 \sim \frac{y}{H}\Delta f(\phi)$$

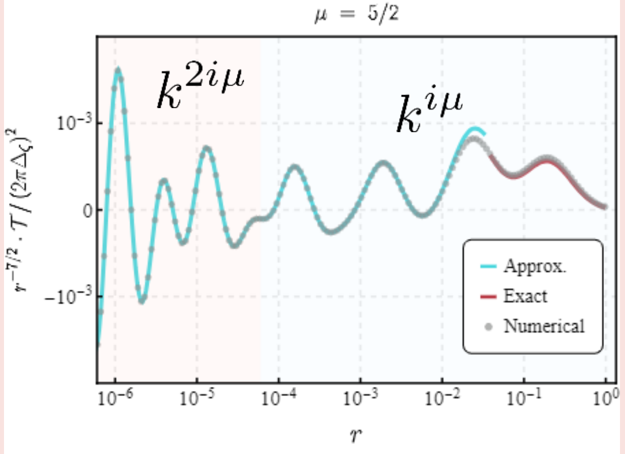


$$\overset{\phi_0\sigma^2}{\Delta\mu^2_{\rm NSS}(k) \lesssim \sqrt{\epsilon}\frac{M_{\rm pl}}{H}} \quad \text{vs.} \quad \overset{\ddot{\phi}_0\sigma^2}{\Delta\mu^2_{\rm SS}(k) \lesssim \epsilon^{3/2}\frac{M_{\rm pl}}{\Lambda}}$$

❑ Double-exchange vs. single-exchange

➢ Large size $\dfrac{S_{\rm DE}}{S_{\rm SE}} \sim \lambda P_\zeta^{-1/2}\dfrac{1}{\mu^2} \lesssim P_\zeta^{-1/4}\dfrac{1}{\mu^2}$

➢ Trispectrum



➢ Coupling constants are not determined only from CC signals.

$$S^{\rm multi}_{\rm DE,CC} = \sum_{\alpha}^{N}\frac{\rho_\alpha}{H}{\rm Re}\left[\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu_\alpha}\mathcal{B}_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta}e^{i\vartheta_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta}}\right]$$

Analytic template we obtained is important!

Back-up

Bispectrum in Single Field Inflation

❑ Perturbative expansion of the action

$$S_{\rm EH} = \frac{1}{2}\int dx^4 \sqrt{-g}R \quad \text{with} \quad ds^2 = -dt^2 + e^{2\zeta}a^2(t)d\boldsymbol{x}^2, \quad \phi = \phi_0(t)$$

$$\mathcal{L} = \mathcal{L}_{\rm BG} + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$$

$\mathcal{L}_{\rm BG}$: Homogeneous and isotropic

$\mathcal{L}_1$: $\propto$ EoM of BG $\longrightarrow 0$

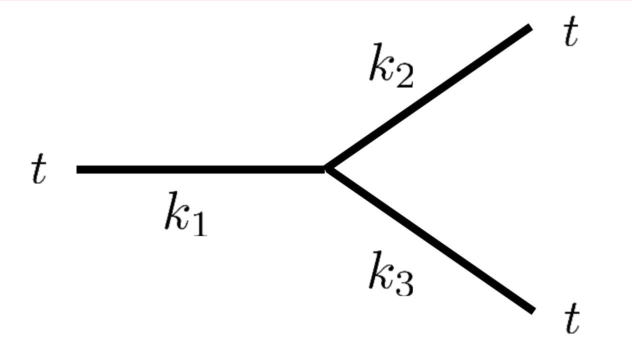
$\mathcal{L}_2$: EoM for the perturbations

$\mathcal{L}_3$: Interaction terms

❑ Maldacena’s consistency relation in bispectrum [Maldacena ’02]

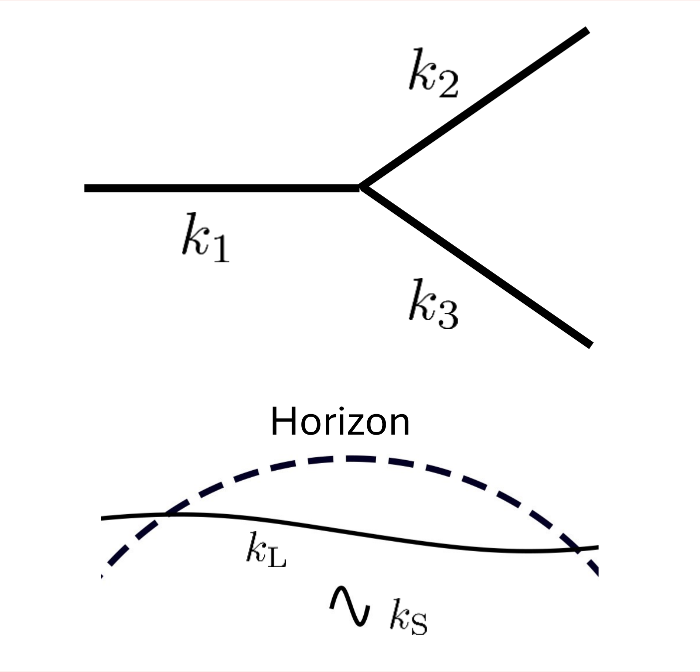
$$\mathcal{L}_3^{\rm EH} = a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + \partial_t\left(-\frac{\epsilon\eta}{2}a^3\zeta^2\dot{\zeta}\right) + \cdots \qquad \text{where } \partial^2\chi \equiv a^2\epsilon\dot{\zeta}$$

$$\langle\zeta_1\zeta_2\zeta_3\rangle_{\rm inf.\ end} = (2\pi)^7\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{P_\zeta^2}{(k_1k_2k_3)^2}S\left(\frac{k_1}{k_3}, \frac{k_2}{k_3}\right)$$



➢ Squeezed limit $k_{\rm L} \equiv k_3 \ll k_1 \simeq k_2 \equiv k_{\rm S}$ ➡ $S \xrightarrow[\rm sq.]{} \frac{k_{\rm S}}{4k_{\rm L}}(1-n_s)$

Bispectrum in Single Field Inflation



❑ Maldacena's consistency relation [Maldacena '02]

$$\mathcal{L}_3^{\zeta} = a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + \partial_t\left(-\frac{\epsilon\eta}{2}a^3\zeta^2\dot{\zeta}\right) + \cdots$$

➢ Squeezed limit $\underset{\equiv k_L}{k_3} \ll \underset{\equiv k_S}{k_1} \simeq k_2$ with $\delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$

$$S \longrightarrow \frac{k_S}{4k_L}(1 - n_s) + \mathcal{O}\left(\left(\frac{k_L}{k_S}\right)^0\right)$$

But... $\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle \sim \int \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9}\frac{S}{k_1^2k_2^2k_3^2}\delta^3\left(\sum k_i\right) \to \int_{k_L \ll k_S}\frac{dk_S dk_L}{k_S k_L} \to \infty$?

➢ Geodesic coordinate (local observer's effect) [Tanaka, Urakawa '11, Pajer et al. '13]

$$ds^2 = -dt^2 + e^{2\zeta}a^2(t)d\boldsymbol{x}^2$$

$$\boldsymbol{x}_F \simeq (1+\zeta)\boldsymbol{x},\ \ \zeta_F(\boldsymbol{x}_F) = \zeta(\boldsymbol{x}) \simeq \zeta(\boldsymbol{x}_F) - \zeta(1 + \boldsymbol{x}\cdot\partial_{\boldsymbol{x}}\zeta)$$

$$= -dt^2 + a^2(t)d\boldsymbol{x}_F^2 + \cdots$$

(conformal Fermi normal coordinate)

Bispectrum in Single Field Inflation

❑ Maldacena's consistency relation [Maldacena '02]

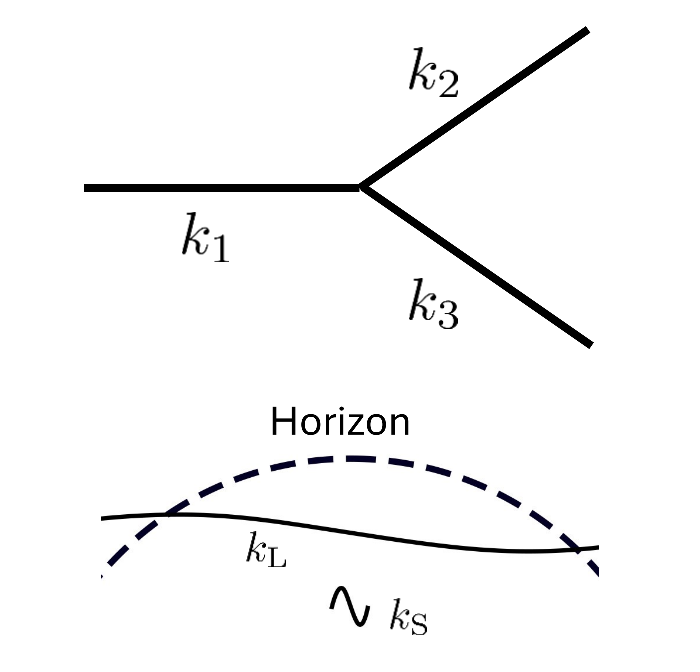
$$\mathcal{L}_3^\zeta = a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + \partial_t\left(-\frac{\epsilon\eta}{2}a^3\zeta^2\dot{\zeta}\right) + \cdots$$

- Squeezed limit $\overset{\equiv k_\mathrm{L}}{k_3} \ll \overset{\equiv k_\mathrm{S}}{k_1} \simeq k_2$ with $\delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$

$$S_\mathrm{F} \longrightarrow \cancel{\frac{k_\mathrm{S}}{4k_\mathrm{L}}(1-n_s) + \mathcal{O}\left(\left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^0\right)} + \mathcal{O}\left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)$$

$$\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle_\mathrm{F} \sim \int \frac{d^3k_1d^3k_2d^3k_3}{(2\pi)^9}\frac{S_\mathrm{F}}{k_1^2k_2^2k_3^2}\delta^3\left(\sum k_i\right) \to \cancel{\int_{k_\mathrm{L}\ll k_\mathrm{S}} \frac{dk_\mathrm{S}dk_\mathrm{L}}{k_\mathrm{S}k_\mathrm{L}} \to \infty}$$

$$\to \int_{k_\mathrm{L}\ll k_\mathrm{S}} dk_\mathrm{S}dk_\mathrm{L}\frac{k_\mathrm{L}}{k_\mathrm{S}^3} \quad : \text{ finite}$$



- Geodesic coordinate (local observer's effect) [Tanaka, Urakawa '11, Pajer et al. '13]

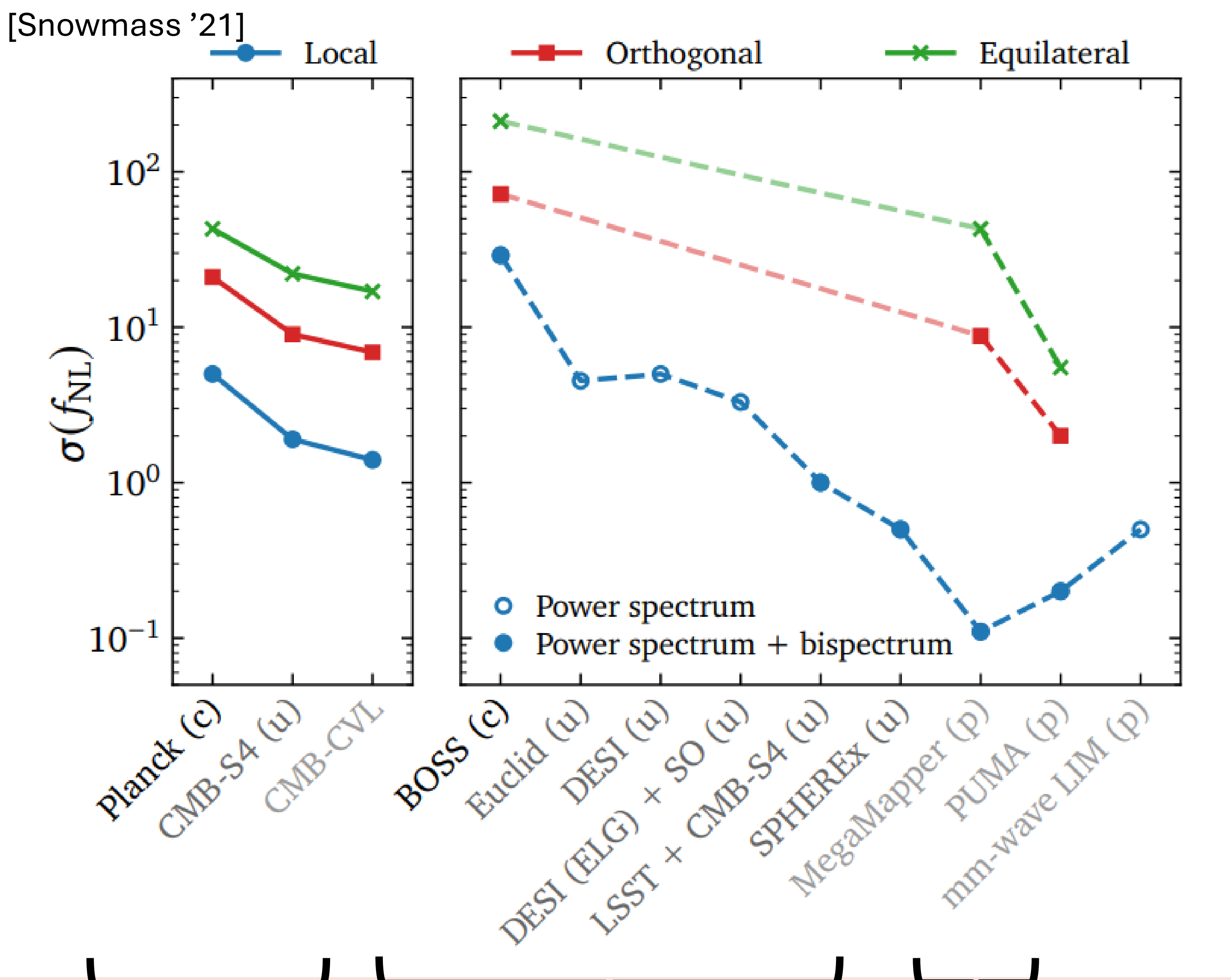
$$ds^2 = -dt^2 + e^{2\zeta}a^2(t)d\boldsymbol{x}^2$$
$$= -dt^2 + a^2(t)d\boldsymbol{x}_\mathrm{F}^2 + \cdots$$

$$\boldsymbol{x}_\mathrm{F} \simeq (1+\zeta)\boldsymbol{x},\ \ \zeta_\mathrm{F}(\boldsymbol{x}_\mathrm{F}) = \zeta(\boldsymbol{x}) \simeq \zeta(\boldsymbol{x}_\mathrm{F}) - \zeta(1 + \boldsymbol{x}\cdot\partial_{\boldsymbol{x}}\zeta)$$

(conformal Fermi normal coordinate)

Future Observations

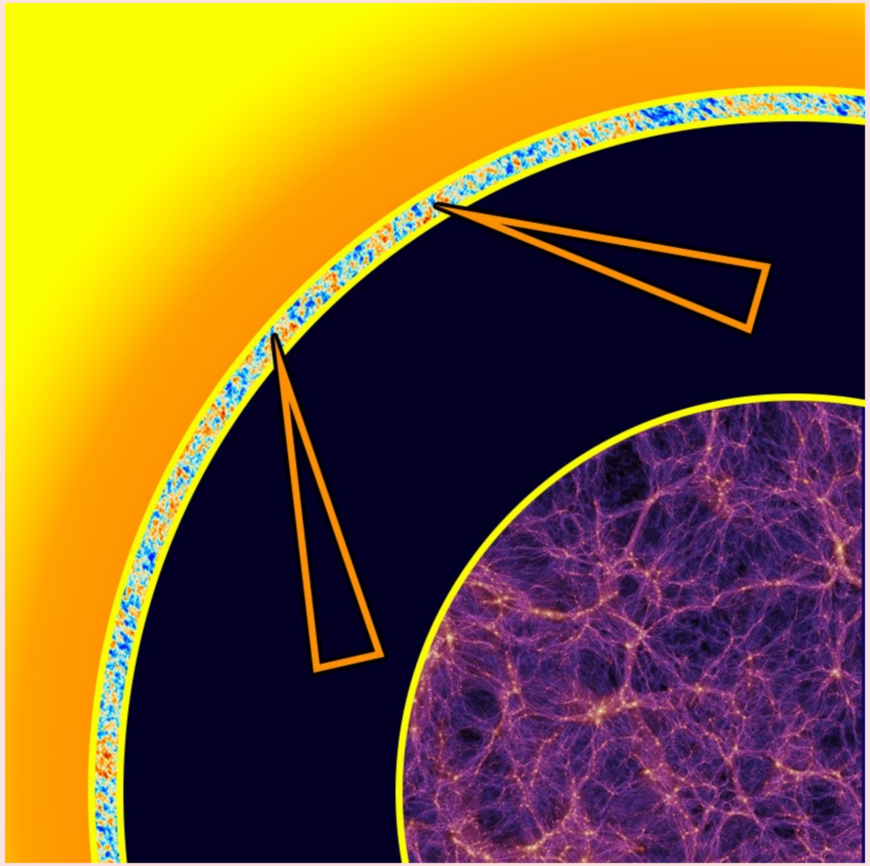
(c): completed
(u): upcoming
(p): projected



CMB

Galaxy survey

21 cm



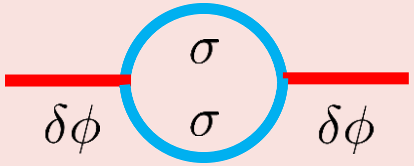
CMB Dark age Galaxies

21cm-21cm-CMB cross-correlation

$f_{\rm NL}^{\rm local} \sim 6 \times 10^{-3}$ [Orlando et al. '23]

Naturalness Conditions from Inflaton Mass for NSS Ints.

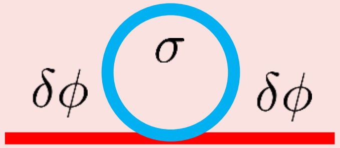
- $yH\phi\sigma^2$



$$\sim y^2H^2 \lesssim \mathcal{O}(\eta,\epsilon)H^2 \longrightarrow y \lesssim \sqrt{\epsilon} \longrightarrow \Delta m_\sigma^2(k) \lesssim \epsilon H M_{\rm pl} \log\frac{k}{k_{\rm i}}$$

- $\lambda\phi^2\sigma^2$

$$\Delta m_{\phi_0}^2 \sim \lambda\langle\sigma^2\rangle \sim \lambda H^2 \lesssim \mathcal{O}(\eta,\epsilon)H^2 \longrightarrow \lambda \lesssim \epsilon$$



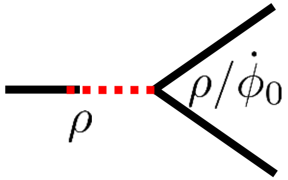
$$\sim \lambda\Lambda^2 \lesssim \mathcal{O}(\eta,\epsilon)H^2 \longrightarrow \lambda \lesssim \epsilon\frac{H^2}{\Lambda^2}$$

$$\Rightarrow \Delta m_\sigma^2(k) \lesssim \epsilon^2 H M_{\rm pl}\frac{HM_{\rm pl}}{\Lambda^2}\log^2\frac{k}{k_{\rm i}}$$

* Shift sym. Couplings: $\dfrac{|\partial_t^2\phi|}{\Lambda} \sim \epsilon^{3/2}HM_{\rm pl}\dfrac{H}{\Lambda}\log\dfrac{k}{k_{\rm i}}, \quad \dfrac{|\partial_t\phi_0|^2}{\Lambda^2} \sim \epsilon HM_{\rm pl}\dfrac{HM_{\rm pl}}{\Lambda^2}\log^2\dfrac{k}{k_{\rm i}}$

Size Estimation of Single-exchange Diagrams

$$\frac{\rho}{\dot\phi_0}(\partial_\mu\phi)^2\sigma \longrightarrow \rho\,\delta\phi'\sigma + \frac{\rho}{\dot\phi_0}\,(\delta\phi')^2\sigma$$

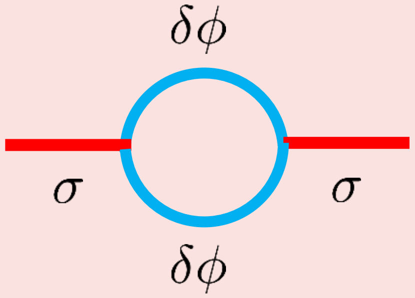


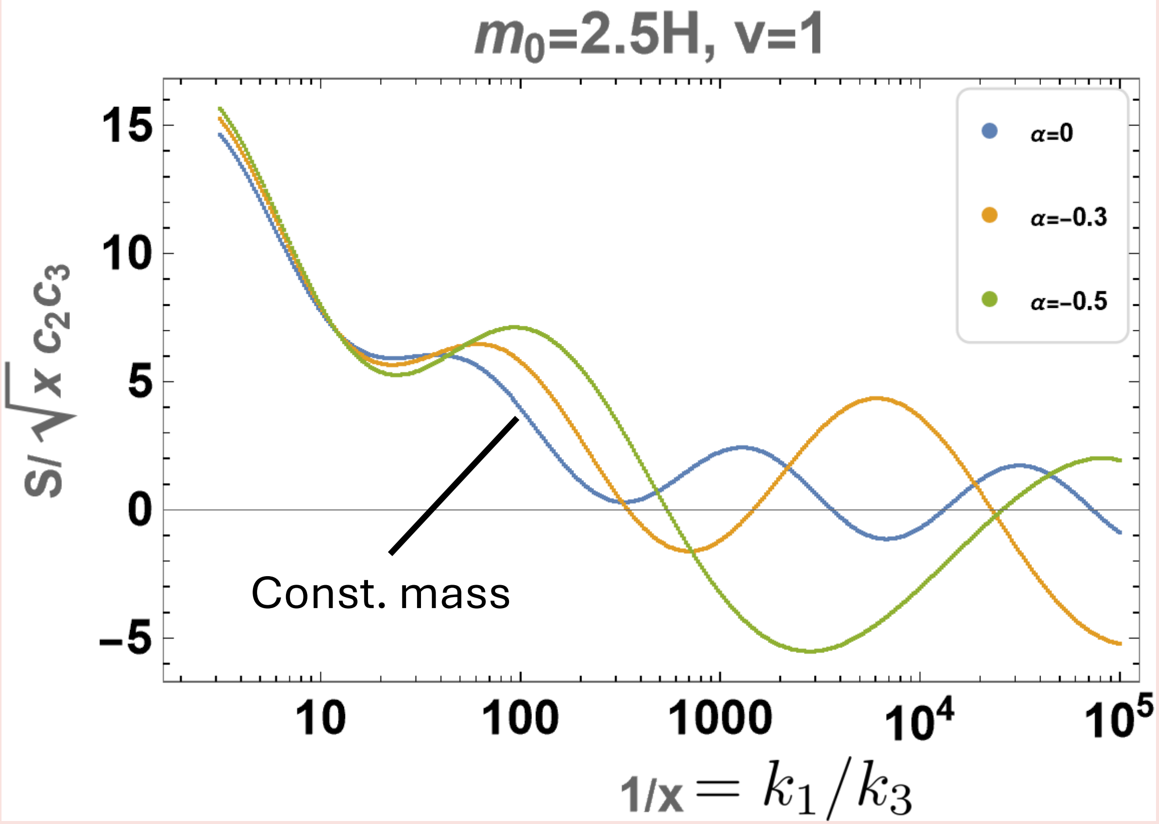
$$S_{\rm SE} \sim \frac{\rho^2}{\dot\phi_0} P_\zeta^{-1/2} e^{-\pi\mu}\mathcal{O}(1)$$

$$\left.\begin{array}{l} \dot\phi_0 \sim H^2 P_\zeta^{-1/2} \\ \rho \equiv \alpha H \end{array}\right\} \quad \frac{\rho^2}{\dot\phi_0} \sim \alpha^2 P_\zeta^{1/2}$$

Naturalness $\alpha \lesssim 1$

[Pinol, Renaux-Petel, Werth '23]





$$\Longrightarrow \quad S_{\rm SE} \lesssim e^{-\pi\mu} \times \mathcal{O}(1)$$

$$m_\sigma = 2.5H \to e^{-\pi\mu} \sim 10^{-3}$$

Observational Signals in Bispectrum

Consistency check: CosmoFlow
[Pinol, Renaux-Petel, Werth '23, '24]

- **Squeezed limit** $k_3 \ll k_1 \simeq k_2$

$$\langle \delta\phi_{k1}\delta\phi_{k2}\delta\phi_{k3}\rangle' \xrightarrow[k_3\to 0]{} \sum_{\alpha,\beta} \frac{\rho_\alpha\rho_\beta\lambda_{\alpha\beta}H}{(k_1k_2k_3)^2}\cdot \mathrm{Re}\left\{\left[\mathrm{i}\frac{\pi^{3/2}}{2^{4+2\mathrm{i}\mu_\alpha}}\,\mathrm{sech}\left(\pi\mu_\beta\right)\left[1+\tanh\left(\pi\mu_\alpha\right)\right]\times\Gamma\left[\begin{matrix}-\mathrm{i}\mu_\alpha\\ -1-\mathrm{i}\mu_\alpha+\mathrm{i}\mu_\beta,-1-\mathrm{i}\mu_\alpha-\mathrm{i}\mu_\beta\end{matrix}\right]\right.\right.$$

$$\left.\left.\times\,{}_3\mathcal{F}_2\left[\begin{matrix}-\frac{3}{2}-\mathrm{i}\mu_\alpha,-1-\mathrm{i}\mu_\alpha-\mathrm{i}\mu_\beta,-1-\mathrm{i}\mu_\alpha+\mathrm{i}\mu_\beta\\ -\frac{1}{2}-\mathrm{i}\mu_\alpha,-\frac{1}{2}-\mathrm{i}\mu_\alpha\end{matrix}\middle|\,1\right]+\mathcal{O}\left(e^{-2\pi\mu_\alpha},e^{-2\pi\mu_\beta}\right)\right]\left(\frac{k_1}{k_3}\right)^{\frac{1}{2}+\mathrm{i}\mu_\alpha}+\mathcal{O}\left(\frac{k_1}{k_3}\right)\right\}$$

- **Size in equilateral limit** $k_1 = k_2 = k_3 = k$