Cosmological Collider as an Interaction Probe
—— Scale-dependence and Diagrams

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Seminar talk
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Based on
JHEP03(2024)073 with S. Aoki, T. Noumi, M. Yamaguchi
JHEP09(2024)176 with S. Aoki, L. Pinol, M. Yamaguchi, Y. Zhu

SCIENCE TOKYO

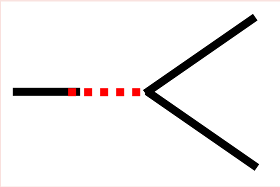
Outline

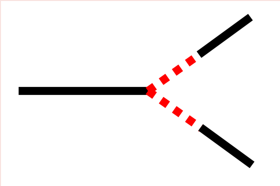
- **Motivation: Primordial universe as a particle collider**
  - **Preparing observational templates for various interactions**

- **Computational method: Cosmological bootstrap equation**

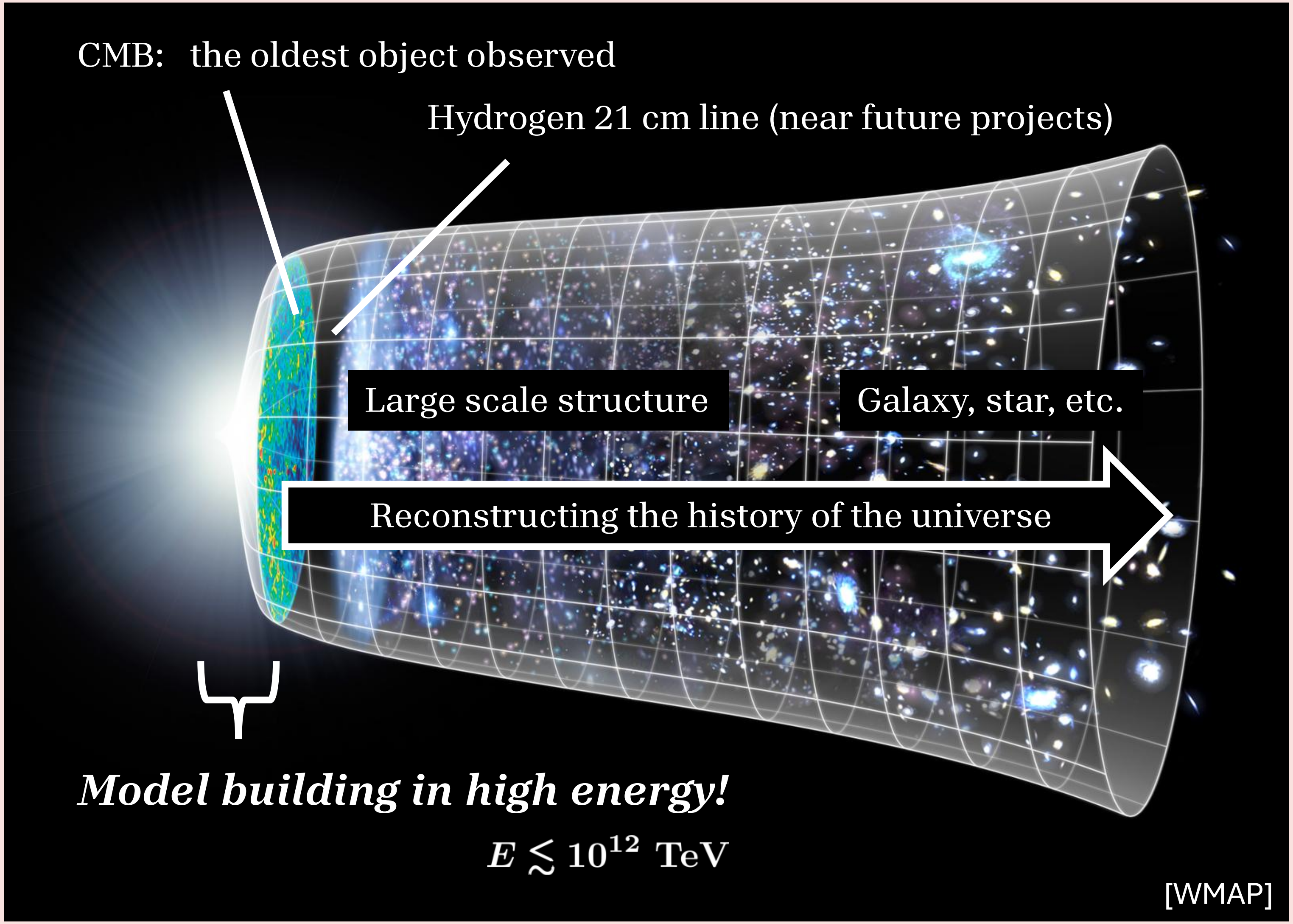
- **Derivative vs. non-derivative interactions** (based on JHEP03(2024)073)
  - **Time-dependent mass and back-reaction**

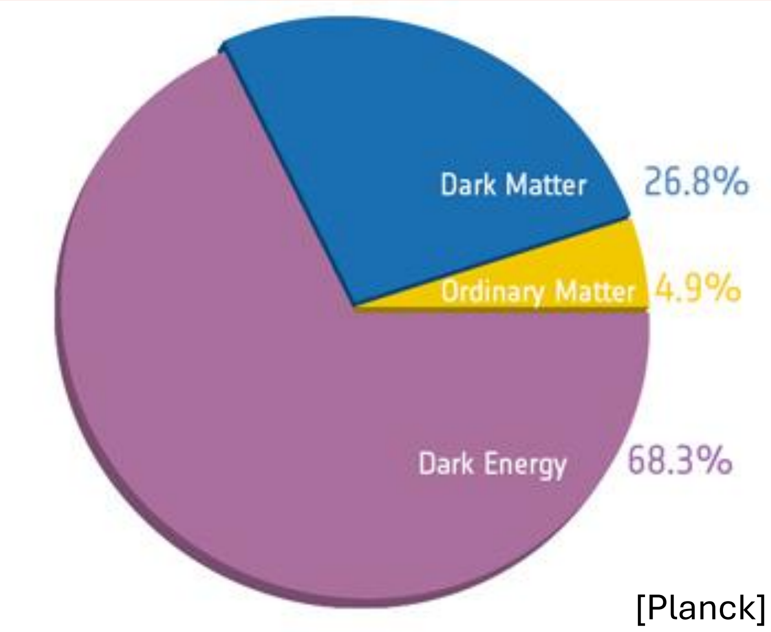
- **Simple scatterings** (based on JHEP09(2024)176)
  - **Which is the leading contribution?**





Cosmology for high energy physics





Plenty of unresolved problems

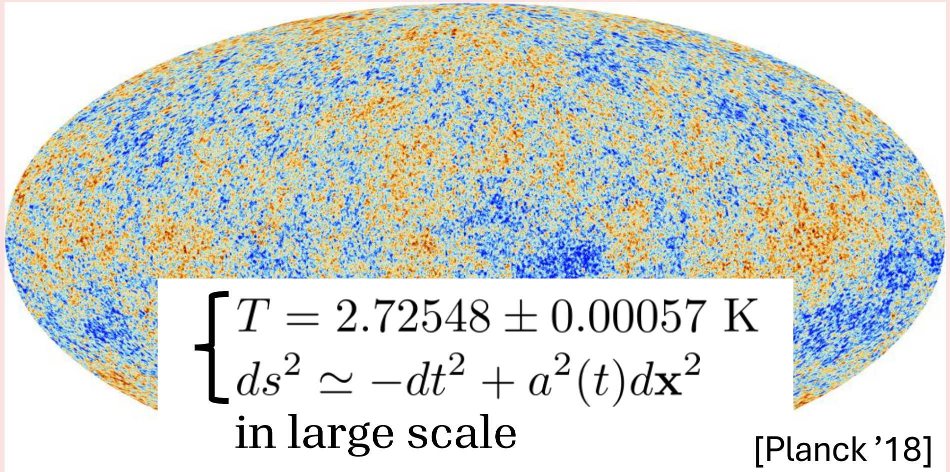
- ✓ Dark matter
- ✓ Dark energy
- ✓ The dawn

⋮

Interplay with particle physics

Inflation: An approach to the "dawn"

- **Requirement for initial conditions of the universe:**
  1) Flat and causally connected universe
  2) Transition mechanism to big bang
  3) Origin of cosmological perturbations



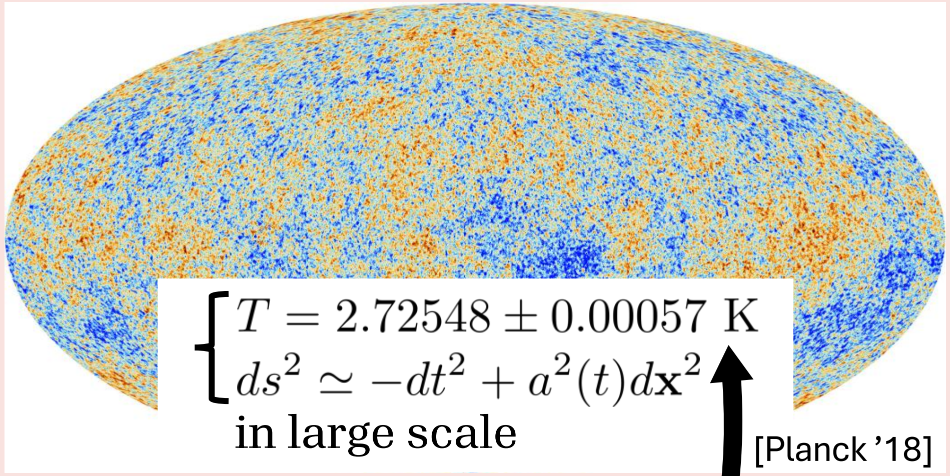
Inflation: An approach to the "dawn"

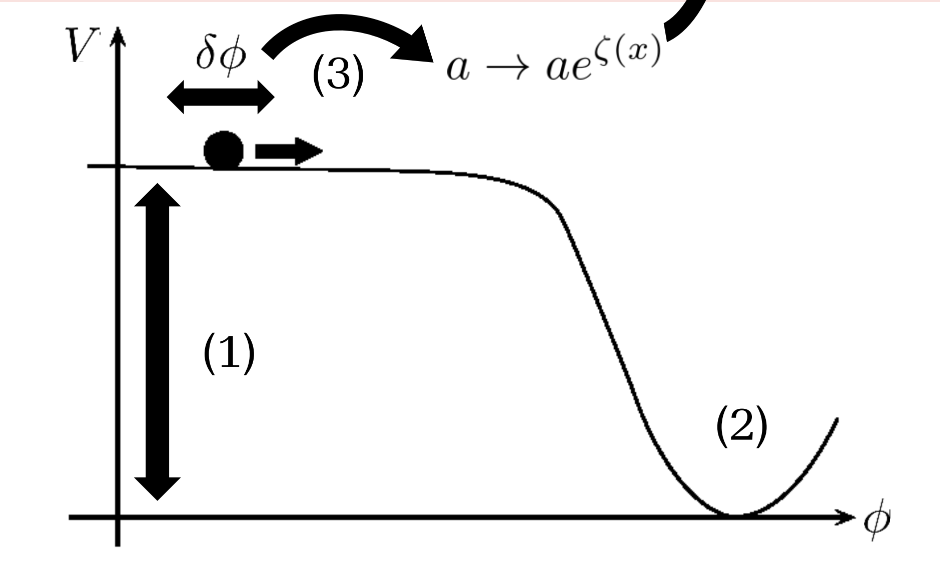
- **Requirement for initial conditions of the universe:**
  1) Flat and causally connected universe
  2) Transition mechanism to big bang
  3) Origin of cosmological perturbations

- **Slow-roll inflation as a possible solution**
  - $\mathcal{L}_{\mathrm{m}} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\,, \quad \epsilon = \left(\frac{V'}{V}\right)^2 \ll 1,\ |\eta| = \left|\frac{V''}{V}\right| \ll 1$
    - ✓ Effectively cosmo. const. $ds^2 \simeq -dt^2 + e^{2Ht}d\mathbf{x}^2$

      ⟹ Huge region was initially prepared. ⇐ (1)
    - ✓ Dynamical transition to the big bang universe ⇐ (2)
  - Quantum fluctuation $\phi = \phi_0(t) + \delta\phi \longleftrightarrow a \to e^{Ht+\zeta(x)}$

    sources cosmological perturbations δT_{CMB} etc. ⇐ (3)





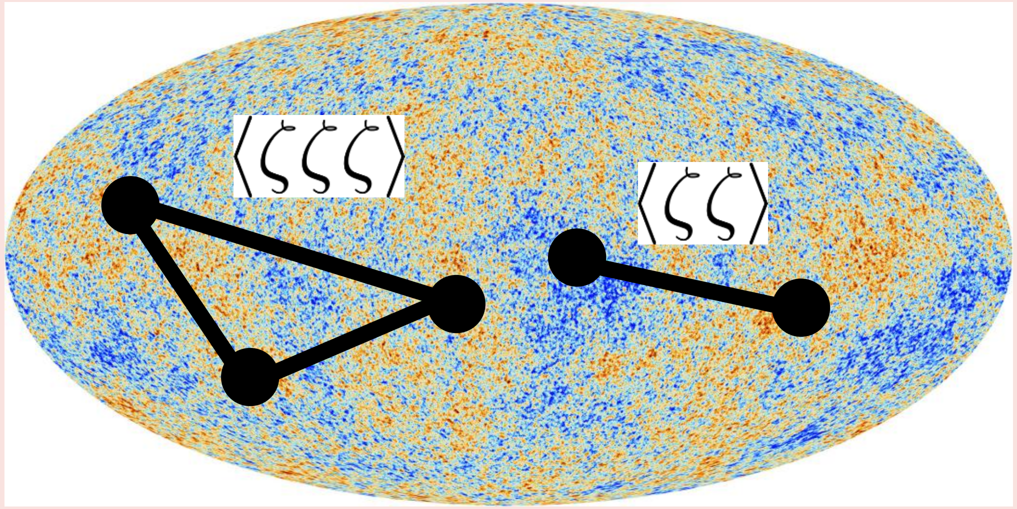
Observables for inflationary cosmology

◆ **2pt. correlation function (power spectrum)**

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle_{\text{inf. end}} = (2\pi)^3 \delta^3(\boldsymbol{k} + \boldsymbol{k}') \frac{2\pi^2}{k^3} P_\zeta$$

$$P_\zeta \simeq \frac{H^2}{8\pi^2 \epsilon} \left(\frac{k}{k_*} \right)^{n_s - 1} \qquad n_s \simeq 0.965, \quad \frac{dn_s}{d\log k} \simeq 0.002$$

[Planck '18]



$\left\{ \begin{array}{l} \text{Consistent with slow-roll inflation} \\ \text{Free propagation is dominant} \end{array} \right.$

◆ **3pt. correlation function (bispectrum)**

$$\langle \zeta_1 \zeta_2 \zeta_3 \rangle_{\text{inf. end}} = (2\pi)^7 \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \frac{P_\zeta^2}{(k_1 k_2 k_3)^2} S\left(\frac{k_1}{k_3}, \frac{k_2}{k_3} \right)$$

✓ 3pt.: effects of interactions ➡ ***Probe for BSM physics*** and inflation models

Particles during inflation

- **Mass spectra** [Copeland et al. ’94, Chen, Wang, Xianyu ’16 etc.]
  - Loop resummation $\Delta m^2 \propto H^2$
  - SUGRA $\mathcal{L} \supset e^K V(\phi) \simeq V + 3cH^2\sigma^2$
  - Non-minimal coupling $\mathcal{L} \supset \xi\sigma^2 R \simeq 12\xi H^2\sigma^2$

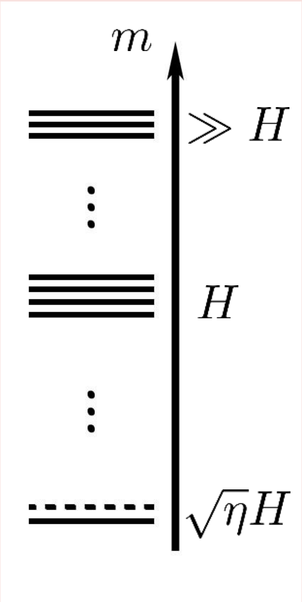
  What kind of (group) structure exists in UV theory?

  How is inflaton interpreted from particle physics?

UV theory (decouple) →

Hubble induced (quasi-single) →

Hubble induced? (inflaton(s)) →



- **Massive particles during inflation**
  - De Sitter symmetry in 3+1-dim $\sim$ conformal symmetry in 3-dim $(SO(1,4))$

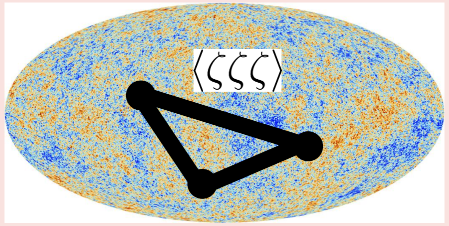
    ➡ Scaling behavior is universally determined

    e.g., scalar field $\lim_{k\tau\to 0} \sigma_{\mathbf{k}}(\tau) \sim (-k\tau)^{3/2+i\mu} + e^{-\pi\mu}(-k\tau)^{3/2-i\mu}$ $\mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}, \quad ad\tau = dt$

    The heavy fields oscillate in time with ***wavelength being their mass.***

Cosmological Collider physics

[Chen, Wang '09, Noumi, Yamaguchi, Yokoyama '12, Arkani-Hamed, Maldacena, '15 etc.]



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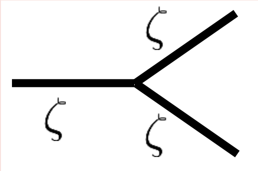

[Maldacena '02]

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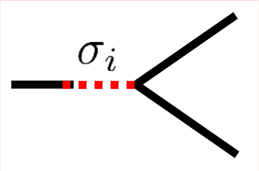

[Arkani-Hamed et al. '18, Qin and Xianyu '22 and '23]

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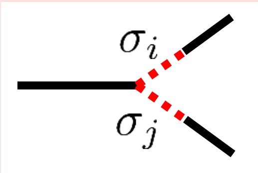

[Xianyu and Zang '24, **Aoki et al. '24**]

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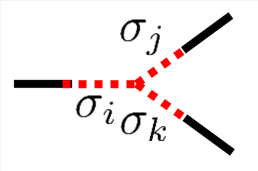


$+ \cdots$

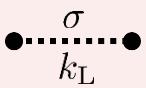
Higher spin, loops, ...

- **Imprints in correlation functions**

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log\frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$$

$k_{\rm L} \equiv k_3 \ll k_1 \simeq k_2 \equiv k_{\rm S}$

$\mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$



$\Rightarrow e^{-\pi\mu}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu}$

✓ Dictionary for particles of $m \sim H \lesssim 10^9$ TeV

Supersymmetry, [Baumann, Green '12] RH neutrino, [Chen et al. '18] CP violation, [Liu et al. '19] gauge theory, [Maru, Okawa '21] extra dimension, ... [Reece et al. '22]

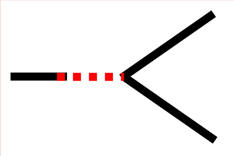
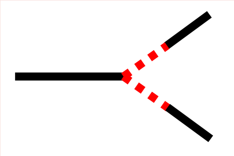
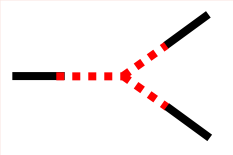
✓ Expected as a target of (near) future observations

Observational templates? ⟵ Even tree diagrams are not fully understood...

How do interactions appear?

$$S \sim \left(\prod_i \lambda_i\right)\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu}\cos\left(\mu\log\frac{k_{\rm L}}{k_{\rm S}}+\delta\right)$$

Interactions in CC-signal

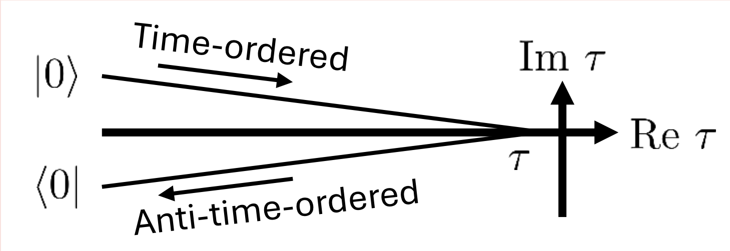
- **Diagrams** [Chen, Wang, Xianyu '17, Qin, Xianyu '22]
  -  ,  ,  $\sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu}\cos\left(\mu\log\frac{k_{\rm L}}{k_{\rm S}}+\delta\right)$ Qualitatively the same in squeezed limit ...
  - Phase information δ: $\mathcal{A}(\mu)\times\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{i\mu} = |\mathcal{A}(\mu)|e^{i\mu\ln(k_{\rm L}/k_{\rm S})+i{\rm Arg}[\mathcal{A}(\mu)]}$
- **Shift symmetric vs. non-shift symmetric**
  - Shift-sym. ints.: respecting dS symmetry
  - Non-shift-sym. ints.: breaking dS $\Rightarrow$ scale dependence $S\left(\frac{k_1}{k_3},\frac{k_2}{k_3}\right) \longrightarrow S\left(\frac{k_1}{k_3},\frac{k_2}{k_3},\frac{k_{1,2,3}}{k_*}\right)$
    - ✓ Numerical work exists, but analytical calculation does not.
      [Wang '18, Reece, Wang, Xianyu '22]

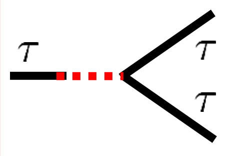
Precise analytical solutions lead to $\left\{\begin{array}{l}\text{Templates valid in any momentum configuration}\\ \text{Clarifying the distinguishability of these processes}\end{array}\right.$

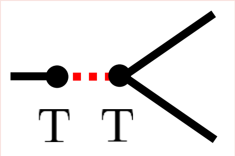
Difficulty in analytical computations

- **Perturbative expansion for correlators**

$$\langle\Omega|\zeta_1\zeta_2\zeta_3(\tau)|\Omega\rangle = \left\langle 0\left|\left(\overline{\mathrm{T}}e^{i\int_{-\infty}^{\tau}\mathrm{d}\tau' H_{\mathrm{I}}}\right)\zeta_1\zeta_2\zeta_3(\tau)\left(\mathrm{T}e^{-i\int_{-\infty}^{\tau}\mathrm{d}\tau' H_{\mathrm{I}}}\right)\right|0\right\rangle$$





$= \mathrm{Re}\Big\{$


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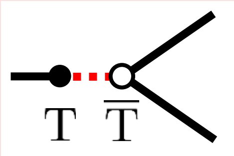
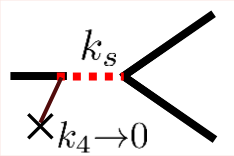


$$\Big\} \propto \frac{1}{8k_1k_2k_3^4}\lim_{k_4\to 0}\sum_{\mathrm{a,b}=\pm}\mathcal{I}_{\mathrm{ab}}^{0,-2} + (k_3 \to k_1, k_2)$$



✓ Seed integral

$$\mathcal{I}_{\mathrm{ab}}^{p_1p_2} = -\mathrm{ab}k_s^{5+p_{12}}\int_{-\infty}^{0}\mathrm{d}\tau_1\,\mathrm{d}\tau_2\,\underbrace{(-\tau_1)^{p_1}(-\tau_2)^{p_2}e^{\mathrm{i}\mathrm{a}k_{12}\tau_1+\mathrm{i}\mathrm{b}k_{34}\tau_2}}_{\text{Scale factor and propagators of }\zeta}\,\underbrace{D_{\mathrm{ab}}(k_s;\tau_1,\tau_2)}_{\text{Propagators of }\sigma}$$

$$\mathrm{a,b} = \pm \qquad \begin{array}{l} + : \mathrm{T} \\ - : \overline{\mathrm{T}} \end{array}$$

$$D_{++}(k_s;\tau_1,\tau_2) \sim \theta(\tau_1-\tau_2)H_{i\mu}^{(1)}(-k_s\tau_1)H_{i\mu}^{(1)*}(-k_s\tau_2)$$

No special fn. is developed for the integral ...

➡ *Cosmological bootstrap with series expansion* realize a simple expression
[Series of papers by Baumann, Lee, Pimentel et al. '18, '20, '21]

Analytical method: De Sitter bootstrap equations

[Series of papers by Baumann, Lee, Pimentel et al. ’18, ’20, ’21, Qin, Xianyu ’22 and ’23]

◆ De Sitter symmetry $\sim$CFT

Translation $P_i = \partial_i$, Rotation $J_{ij} = x_i\partial_j - x_j\partial_i$, Dilatation $D = -\tau\partial_\tau - x_i\partial_i$,

dS boosts $K_i = \left(2x^j x_i + (\tau^2 - x^2)\delta_i^j\right)\partial_j + 2x_i\tau\partial_\tau$

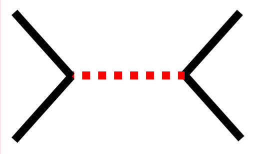
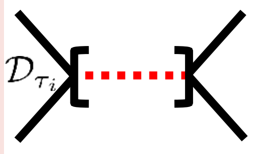
➢ Ward identity: Symmetry $\hat{S} \longrightarrow \langle 0|[\hat{S}, \hat{\mathcal{O}}]|0\rangle = 0$ (assuming $\hat{S}\,|0\rangle = 0$)

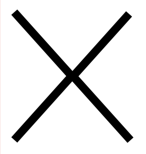
◆ Bootstrap equations for seed integrals

➢ Equations of motion: quadratic Casimir operator $\nabla_\mu\nabla^\mu$

$$(\nabla^2 + a^2m^2)\sigma = 0 \quad \Rightarrow \quad \begin{aligned} &\mathcal{D}_{\tau_i}\widetilde{D}^{\sigma}_{\rm ab}(k_s\tau_1, k_s\tau_2) = -\mathrm{i}aH^2(k_s\tau_1)^2(k_s\tau_2)^2\delta_{\rm ab}\delta(k_s\tau_1 - k_s\tau_2) \\ &\mathcal{D}_{\tau_i} = \tau_i\partial_{\tau_i}(\tau_i\partial_{\tau_i}) - 3\tau_i\partial_{\tau_i} + k_s^2\tau_i^2 + \mu^2 + \frac{9}{4}, \quad \mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}, \quad \widetilde{D} = k_s^3 D \end{aligned}$$

➢ Dilatation: $\tau\partial_\tau(\cdots) = k\partial_k(\cdots) \longrightarrow \mathcal{D}_\tau\widetilde{D} = \mathcal{D}_k\widetilde{D}$

$\longrightarrow \widetilde{\mathcal{D}}_{k_s}\Big[$  $\Big] \sim$ 

$\sim$  $\longrightarrow \mathcal{I}^{p_1p_2}_{\rm ab} \sim {}_2F_1,\ \sum_n^{\infty}\left(\frac{k_i}{\sum_j k_j}\right)^n {}_3F_2$

Boundary conditions: Mellin-Barnes representation

[Qin, Xianyu '22 and '23]

- **Bootstrap:** Boundary conditions are not fixed.

- **Direct integration using MB rep.**

$$H_{i\mu}^{(1)}(-k\tau) = \frac{1}{\pi}\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\left(\frac{-k\tau}{2}\right)^{-2s} e^{(2s-1-i\mu)\pi i/2}\Gamma(s-i\mu)\Gamma(s+i\mu)$$

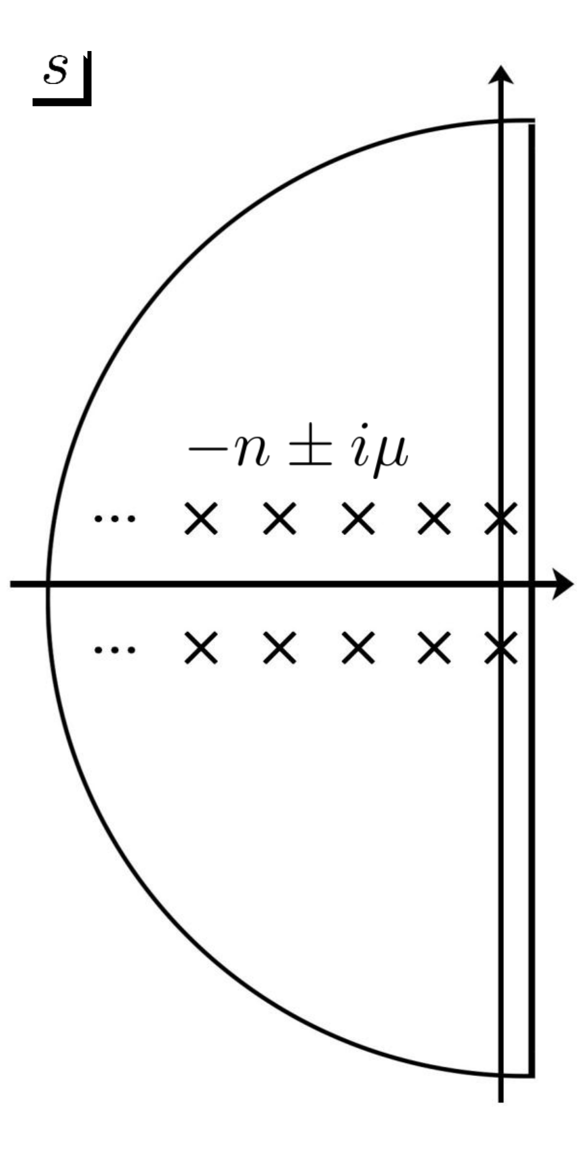
$$\Rightarrow \mathcal{I} \sim \int d\tau_1 d\tau_2 e^{ik_{12}\tau_1+ik_{34}\tau_2}(-\tau_1)^{p_1}(-\tau_2)^{p_2} H_{i\mu}^{(1)}(-k_s\tau_1) H_{i\mu}^{(1)*}(-k_s\tau_2)\theta(\tau_1-\tau_2)$$

$$\sim \sum_{\substack{n_1,n_2 \\ s_i=-n_i\pm i\mu}} \mathcal{A}_{n_1,n_2}(k,k')\mathrm{Res}[\Gamma(s_1\pm i\mu)]\mathrm{Res}[\Gamma(s_2\pm i\mu)]$$

✓ MB rep.: double sum. but boundary conditions are chosen in mode fn.

✓ Bootstrap: single sum. but boundary conditions are not fixed.

Matching them in some limits and obtaining simple expression (e.g., $k_s \to 0$)



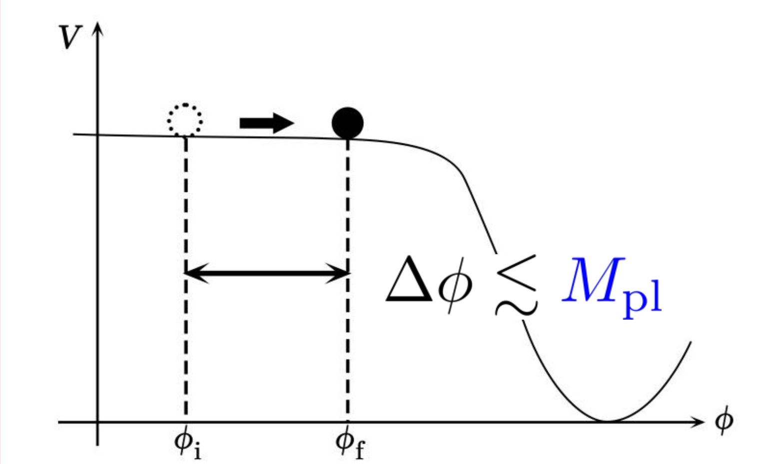
Detection of
non-shift-symmetric interactions

Classical backreaction from non-derivative ints.

[Wang '19, Reece, Wang, Xianyu '22]

◆ **Demonstration: scale-dependent mass of heavy fields**

$$\frac{\mathcal{L}_{\rm int}}{\sqrt{-g}} = yH\phi\sigma^2 \quad \Longrightarrow \quad m^2_{\sigma,\rm eff} = m^2_{\sigma,0} \underline{\underline{+ 2yH\phi_0}}$$

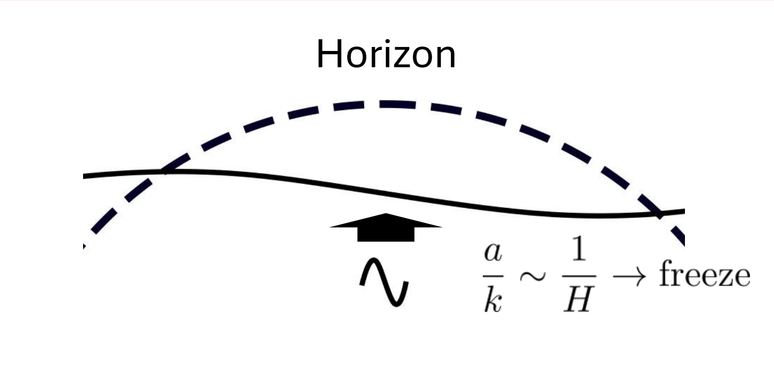


➢ Slow-roll approximation

$$|\phi_0| \simeq \sqrt{2\epsilon}M_{\rm pl}H(t-t_*) \simeq \sqrt{2\epsilon}M_{\rm pl}\log\left(\frac{\tau_*}{\tau}\right)$$

$$\sim \sqrt{2\epsilon}M_{\rm pl}\log\frac{k}{k_*} \qquad \text{(Horizon crossing } |k\tau| \simeq 1)$$

$$\Longrightarrow \Delta m^2_\sigma(k) \sim y\sqrt{\epsilon}HM_{\rm pl}\log\frac{k}{k_*}$$



* Shift symmetric couplings

$$\frac{\mathcal{L}_{\rm int}}{\sqrt{-g}} = \frac{1}{\Lambda}(\Box\phi)\sigma^2 \quad \Longrightarrow \quad \frac{|\partial_t^2\phi_0|}{\Lambda} \sim \epsilon^{3/2}HM_{\rm pl}\frac{H}{\Lambda}\log\frac{k}{k_*}$$

Analytical setup for time-dependent mass

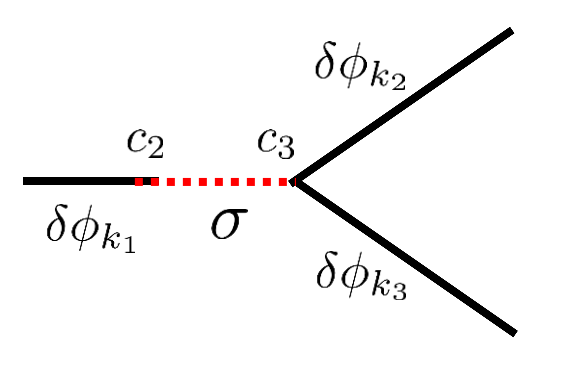
[Aoki et al. '24]

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) - \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}g(\phi)\sigma^2 + \mathcal{L}_{\rm diag}\right]$$

◆Diagram: Single-exchange with derivative coupling

$$\mathcal{L}_{\rm diag} \supset c_2(-\tau)^{-3}\sigma\delta\phi' + c_3(-\tau)^{-2}\sigma(\delta\phi')^2$$

Extracting the effect of the time-dependent mass



◆Time-dependent mass

➢ $\sigma_k'' - \dfrac{2}{\tau}\sigma_k' + \left(k^2 + \dfrac{m_{\rm eff}^2}{H^2\tau^2}\right)\sigma_k = 0$, $\quad m_{\rm eff}^2 = g_* - g_{\phi,*}\sqrt{2\epsilon}M_{\rm pl}(1+k\tau)$

➡ $v_k = \dfrac{e^{\pi\gamma/2}}{\sqrt{2k}}(-H\tau)W_{-i\gamma,i\mu}(2ik\tau)$ $\quad \mu^2 = \dfrac{g_*}{H^2}\left(1 - \dfrac{\sqrt{2\epsilon}g_{*,\phi}M_{\rm pl}}{g_*}\right) - \dfrac{9}{4},\quad \gamma = -\dfrac{\sqrt{2\epsilon}g_{*,\phi}M_{\rm pl}}{2H^2}$

(cf. $W_{0,i\mu} \sim H_{i\mu}^{(1)}$)

➡ Calculable through bootstrap eq. and MB rep. for Whittaker diff. eq. / function

Bispectrum: Mass at horizon-crossing

[Aoki et al. '24]

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_3}{k_1}\right)$$

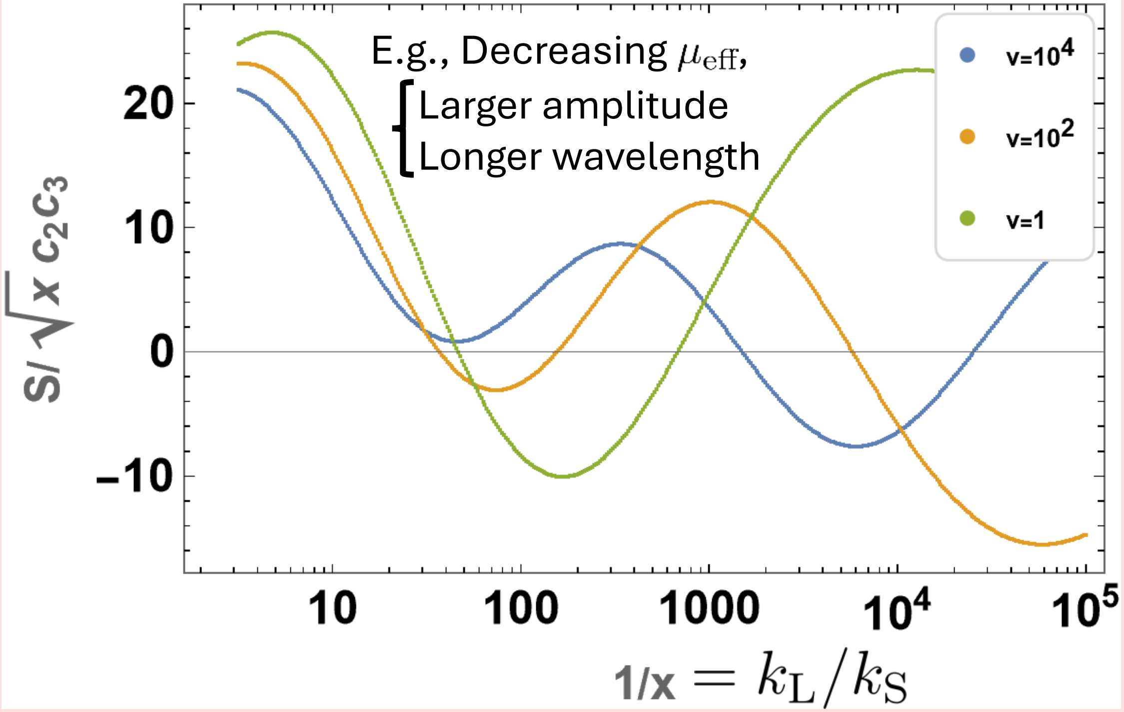
$$S \sim \left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)} \cos\left[\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)\log\frac{k_\mathrm{L}}{k_\mathrm{S}} + \delta\left(\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)\right)\right]$$

$$\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right) = \frac{m_0}{H}\sqrt{1-\alpha\sqrt{2\epsilon}\left(1+\log\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)\right)-\frac{9}{4}}$$

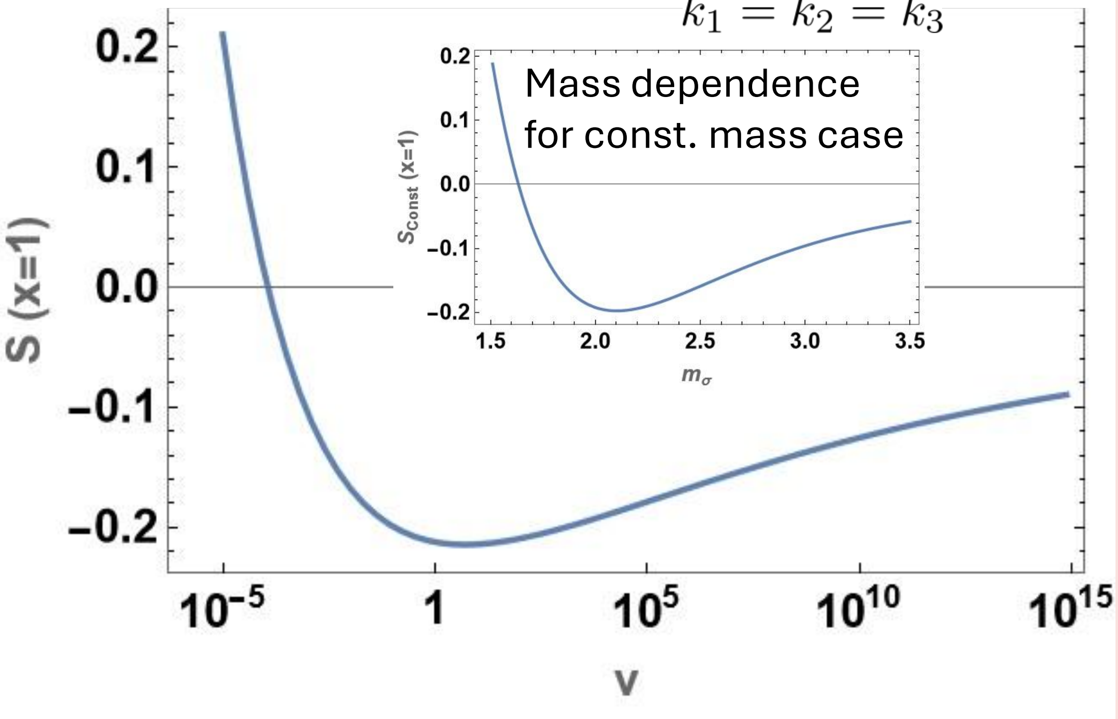
With the interaction $m_0^2\left(1+\alpha\frac{\phi}{M_\mathrm{pl}}\right)\sigma^2, \quad \Delta\phi \sim \sqrt{\epsilon}M_\mathrm{pl}\Delta N$

$v \equiv k_\mathrm{S}/k_*$: Scale dependence

CC signal



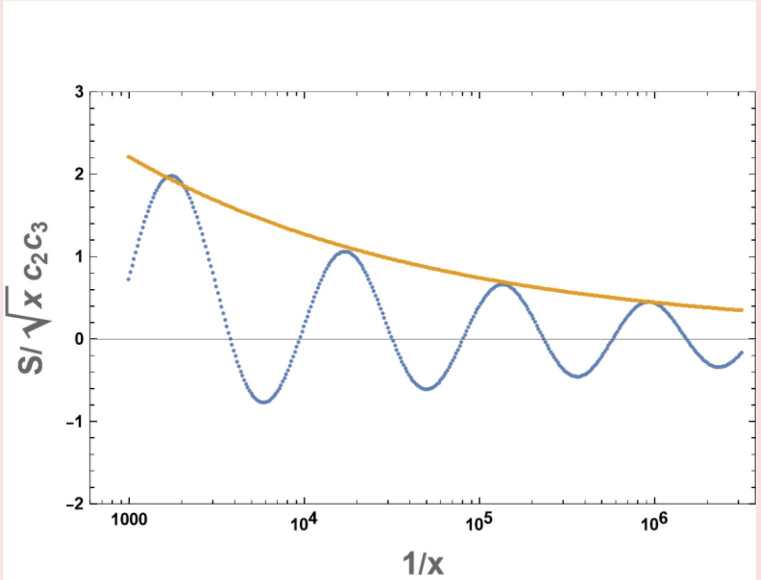
v dependence (e.g., equilateral) $k_1 = k_2 = k_3$



Interaction distinction using scale dependence

[Aoki et al. '24]

$$S \sim \left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)} \cos\left[\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)\log\frac{k_\mathrm{L}}{k_\mathrm{S}} + \delta\left(\mu\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)\right)\right]$$



- $\overset{\phi_0\sigma^2}{\Delta\mu^2_\mathrm{NSS}(k)} \lesssim \sqrt{\epsilon}\frac{M_\mathrm{pl}}{H}$ vs. $\overset{\ddot{\phi}_0\sigma^2}{\Delta\mu^2_\mathrm{SS}(k)} \lesssim \epsilon^{3/2}\frac{M_\mathrm{pl}}{\Lambda}$
- $e^{-\pi\mu} \sim \exp\left[-\frac{\pi}{H}\sqrt{m_0^2 - \frac{9H^2}{4} + g\left(M_\mathrm{pl}\sqrt{2\epsilon}\log\left(v\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)\right)}\right]$ for $\frac{\mathcal{L}_\mathrm{int}}{\sqrt{-g}} = g(\phi)\sigma^2$

✓ Scale dependence (suppression / enhancement etc.) is characterized by the interaction

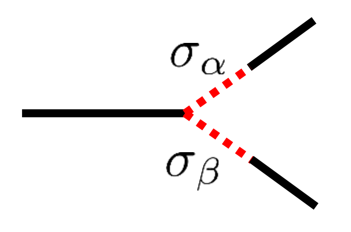
Non-shift-sym. ints: distinguishable through scale-dependence

Single-Exchange Diagrams
vs.
Double-Exchange Diagrams

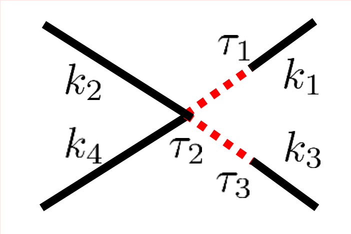
Method: Bootstrap Equations and MB Representations

[Aoki et al. '24]

$$\mathcal{L}_{\rm int} = a^3 \sum_\alpha \rho_\alpha \sigma_\alpha \delta\phi' + a^3 \sum_{\alpha,\beta} \lambda_{\alpha\beta} \sigma_\alpha \sigma_\beta \delta\phi'$$



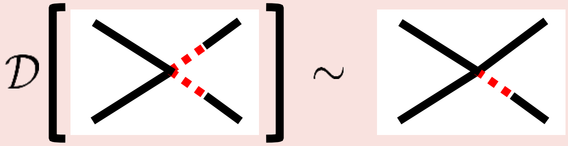
Seed integral



($k_4 \to 0$: bispectrum)

$$\mathcal{I}^{p_1 p_2 p_3}_{\rm abc,\alpha\beta} = H^{-4} k_{24}^{9+p_{123}} (-{\rm iabc}) \int_{-\infty}^{0} d\tau_1\, d\tau_2\, d\tau_3\, (-\tau_1)^{p_1} (-\tau_2)^{p_2} (-\tau_3)^{p_3} \times e^{{\rm i}ak_1\tau_1 + {\rm i}bk_{24}\tau_2 + {\rm i}ck_3\tau_3} D^\alpha_{\rm ab}(k_1;\tau_1,\tau_2)\, D^\beta_{\rm bc}(k_3;\tau_2,\tau_3)$$

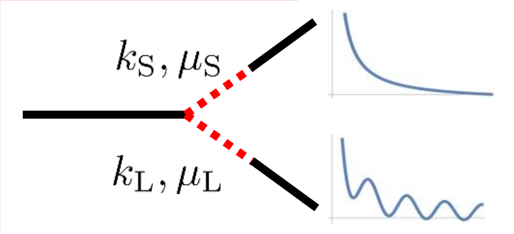
- **Bootstrap equations**



$$\mathcal{I} \sim F_4, \sum_n^\infty \left(\frac{k_i}{\sum_j k_j}\right)^n ({}_3F_2 + {}_2F_1)$$

Analytical expression for arbitrary momentum configuration

- **Bispectrum in squeezed region**



$$S \sim \left(\frac{k_L}{k_S}\right)^{1/2} e^{-\pi\mu_L} \cos\left(\mu_L \log\frac{k_L}{k_S} + \delta\right)$$

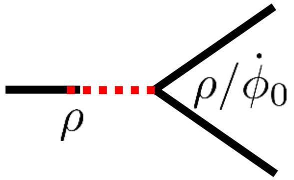
Qualitatively same as single-exchange?

Difference between SE and DE 1: Size of Signals

[Pinol, Renaux-Petel, Werth '23, **Aoki et al. '24**]

◆ Single-exchange (SE)

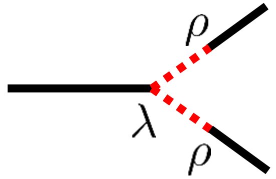
$$\frac{\rho}{\dot{\phi}_0}(\partial_\mu \phi)^2 \sigma \longrightarrow \rho\, \delta\phi' \sigma + \frac{\rho}{\dot{\phi}_0}(\delta\phi')^2 \sigma$$



$$S_{\rm SE} \sim \frac{\rho^2}{\dot{\phi}_0} \times P_\zeta^{-1/2}$$

◆ Double-exchange (DE)

$$\rho\, \delta\phi' \sigma + \lambda\, \delta\phi' \sigma^2$$

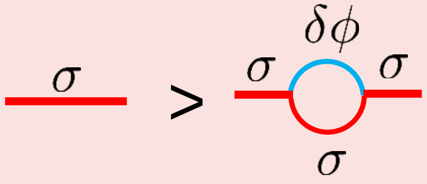


$$S_{\rm DE} \sim \lambda \frac{\rho^2}{H^2} \times P_\zeta^{-1/2}$$

◆ Constraints

✓ Perturbativity $\lambda \lesssim 1$

✓ Naturalness $\lambda \lesssim P_\zeta^{1/4}$



$$\frac{S_{\rm DE}}{S_{\rm SE}} \sim \lambda \frac{\dot{\phi}_0}{H^2} \sim \lambda P_\zeta^{-1/2} \lesssim P_\zeta^{-1/4} \sim 10^2$$ **Naturally larger than single-exchange**

Difference between SE and DE 2: Phase Information

[Aoki et al. '24]

◆ Phase information

➢ SE and single isocurvature DE

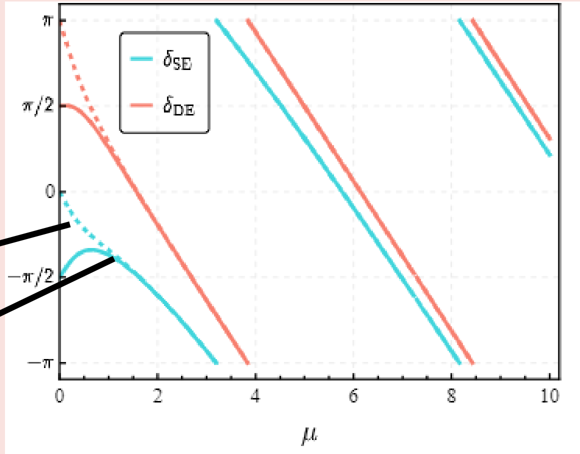
$$S^{\rm single}_{\rm DE,CC} = \frac{\rho^2}{H^2}\frac{\lambda}{2\pi P_\zeta^{1/2}}{\rm Re}\left[\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu}\mathcal{A}_{\rm DE}(\mu)e^{i\delta(\mu)}\right]$$

$$S_{\rm SE,CC} = \frac{\rho^2}{\dot\phi}\frac{1}{2\pi P_\zeta^{1/2}}{\rm Re}\left[\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu}\mathcal{A}_{\rm SE}(\mu)e^{i\delta(\mu)}\right]$$

Consistency between phase and wavelength

Leading of μ

Full order



➢ DE with multiple isocurvature modes

$$S^{\rm multi}_{\rm DE,CC} = \sum_{\alpha,\beta}^{N}\frac{\rho_\alpha\rho_\beta}{H^2}\frac{\lambda_{\alpha\beta}}{2\pi P_\zeta^{1/2}}{\rm Re}\left[\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu_\alpha}\mathcal{A}_{\mu_\alpha,\mu_\beta}e^{i\delta_{\mu_\alpha,\mu_\beta}}\right]$$

$$= \sum_{\alpha}^{N}\frac{\rho_\alpha}{H}{\rm Re}\left[\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu_\alpha}\mathcal{B}_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta}e^{i\vartheta_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta}}\right]$$

$$\left(a\sin\theta + b\sin(\theta+\Delta\theta) = \sqrt{a^2+b^2+2ab\cos\Delta\theta}\,\sin(\theta+\alpha)\right)$$

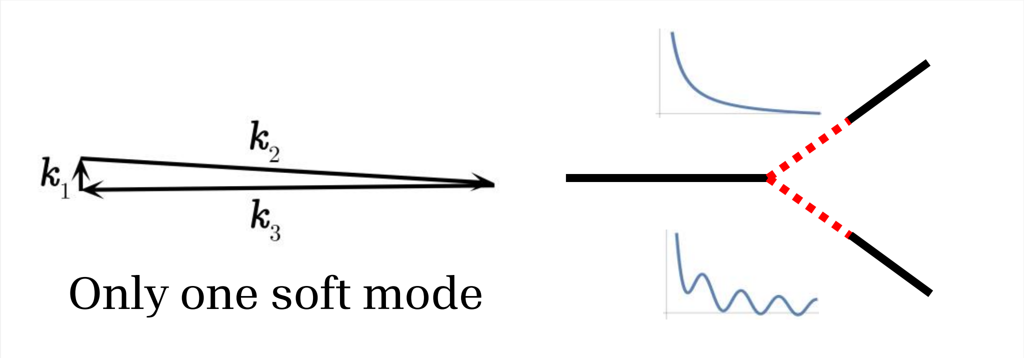
✓ Information in squeezed limit

# of observables	$\leq$	# of parameters	
✓ Amplitude N		✓ ρ_α	N
✓ Wavelength N		✓ μ_α	N
✓ Phase N		✓ $\lambda_{\alpha\beta}$	$N(N+1)/2$

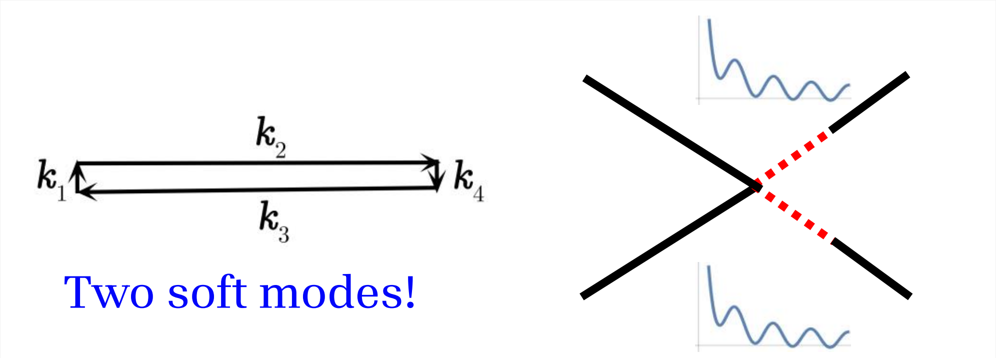
Difference between SE and DE 3: Trispectrum

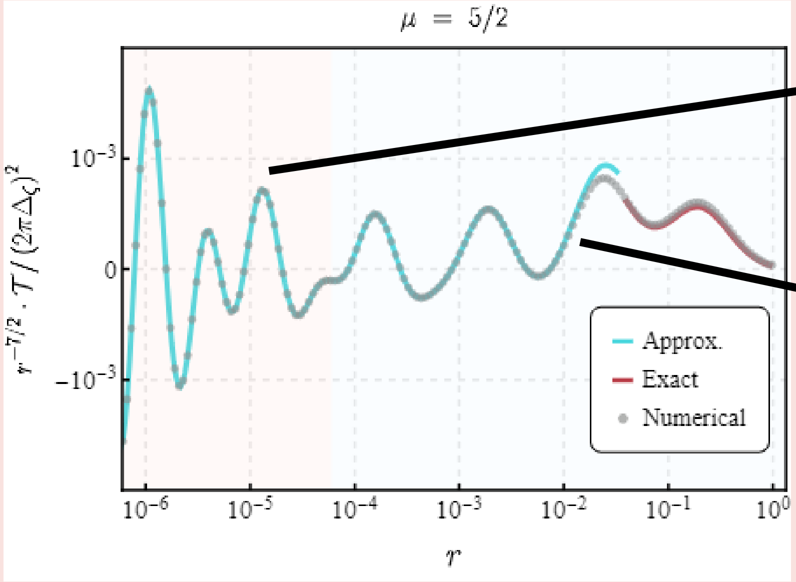
[Aoki et al. ’24]

◆ Bispectrum



◆ Trispectrum

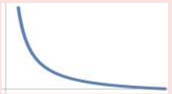




$$\mu_\alpha^{3/2}\mu_\beta^{3/2}e^{-\pi(\mu_\alpha+\mu_\beta)}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{3+i(\mu_\alpha+\mu_\beta)}$$

No such signals in SE

$$\frac{\mu_\alpha^{3/2}}{\mu_\beta^{2}}e^{-\pi\mu_\alpha}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{7/2+i\mu_\alpha}$$

*  $\sim \mu^{3/2}e^{-\pi\mu}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{3/2+i\mu}$,  $\sim \frac{1}{\mu^2}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{2}$

Summary

◆Cosmological Collider physics

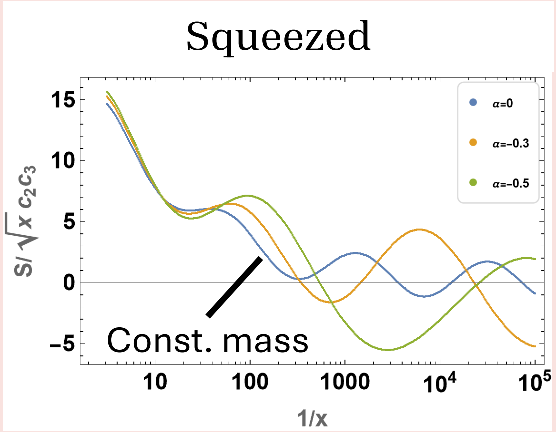
$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$$

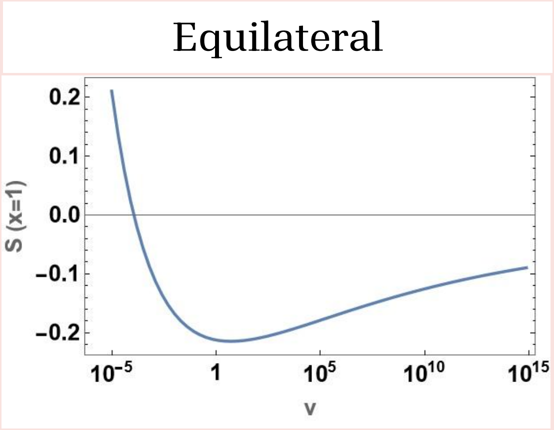
Goal: mass spectrum of particles during inflation
Task: preparing precise observational templates
➡ Cosmological bootstrap for analytical method

◆Non-shift-symmetric interactions

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu_k} \cos\left(\mu_k \log \frac{k_{\rm L}}{k_{\rm S}} + \delta(\mu_k)\right)$$

Breaking dS sym. ⇒ time-dependent mass
⇒ Scale dep.: mass at horizon-crossing

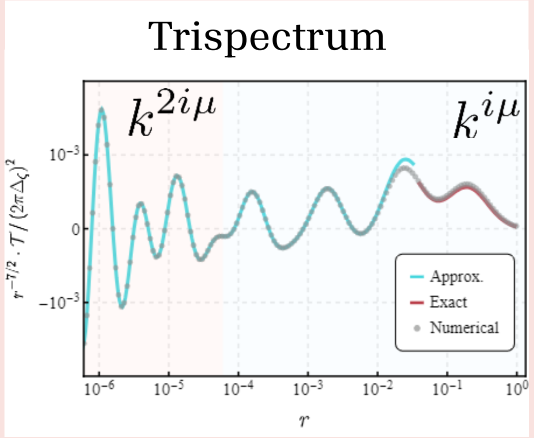


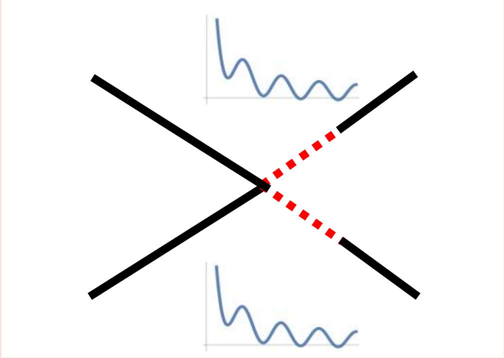


◆Double-exchange vs. single-exchange

➢ Larger signal $\frac{S_{\rm DE}}{S_{\rm SE}} \sim \lambda P_\zeta^{-1/2}\frac{1}{\mu^2} \lesssim P_\zeta^{-1/4}\frac{1}{\mu^2}$

➢ Distinctive feature in trispectrum





Back-up

Bispectrum in Single Field Inflation

❑ Perturbative expansion of the action

$$S_{\rm EH} = \frac{1}{2}\int dx^4 \sqrt{-g}R \quad \text{with} \quad ds^2 = -dt^2 + e^{2\zeta}a^2(t)d\boldsymbol{x}^2, \quad \phi = \phi_0(t)$$

$$\mathcal{L} = \underbrace{\mathcal{L}_{\rm BG}}_{\text{Homogeneous and isotropic}} + \underbrace{\mathcal{L}_1}_{\propto \text{ EoM of BG } \longrightarrow 0} + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$$

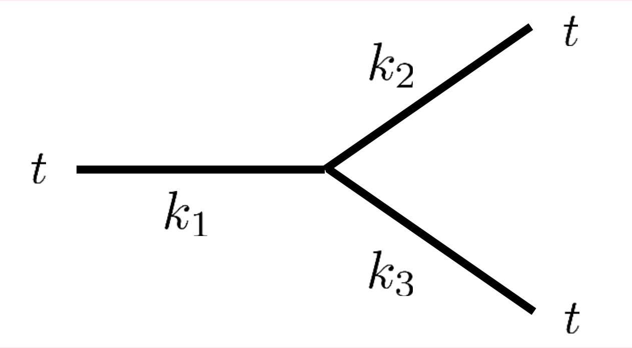
$\mathcal{L}_2$: EoM for the perturbations

$\mathcal{L}_3$: Interaction terms

❑ Maldacena's consistency relation in bispectrum [Maldacena '02]

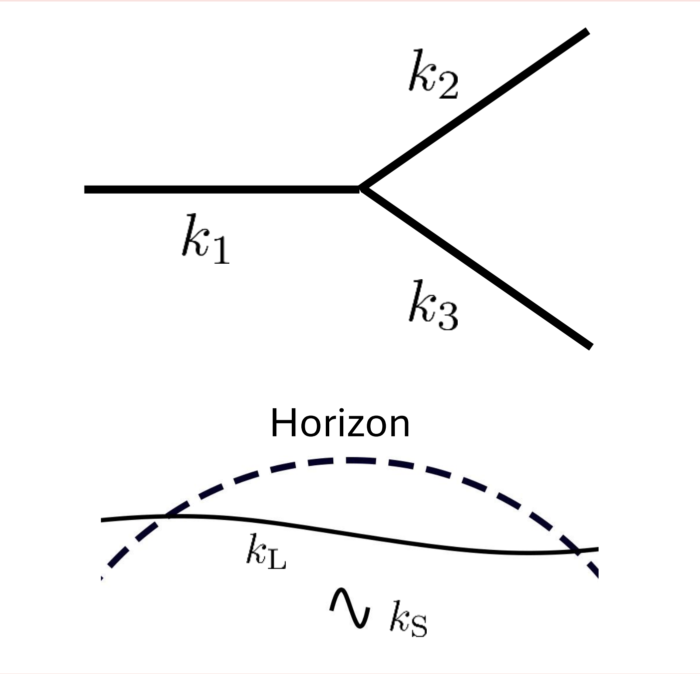
$$\mathcal{L}_3^{\rm EH} = a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + \partial_t\left(-\frac{\epsilon\eta}{2}a^3\zeta^2\dot{\zeta}\right) + \cdots \qquad \text{where } \partial^2\chi \equiv a^2\epsilon\dot{\zeta}$$

$$\langle\zeta_1\zeta_2\zeta_3\rangle_{\rm inf.\ end} = (2\pi)^7\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{P_\zeta^2}{(k_1k_2k_3)^2}S\left(\frac{k_1}{k_3},\frac{k_2}{k_3}\right)$$



- ➢ Squeezed limit $k_{\rm L} \equiv k_3 \ll k_1 \simeq k_2 \equiv k_{\rm S}$ ➡ $S \xrightarrow[\rm sq.]{} \frac{k_{\rm S}}{4k_{\rm L}}(1-n_s)$

Bispectrum in Single Field Inflation



❑ Maldacena’s consistency relation [Maldacena ’02]

$$\mathcal{L}_3^{\zeta} = a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + \partial_t\left(-\frac{\epsilon\eta}{2}a^3\zeta^2\dot{\zeta}\right) + \cdots$$

➢ Squeezed limit $\overset{\equiv k_{\mathrm{L}}}{k_3} \ll \overset{\equiv k_{\mathrm{S}}}{k_1} \simeq k_2$ with $\delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$

$$S \longrightarrow \frac{k_{\mathrm{S}}}{4k_{\mathrm{L}}}(1 - n_s) + \mathcal{O}\left(\left(\frac{k_{\mathrm{L}}}{k_{\mathrm{S}}}\right)^0\right)$$

But… $\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle \sim \int \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9}\frac{S}{k_1^2k_2^2k_3^2}\delta^3\left(\sum k_i\right) \to \int_{k_{\mathrm{L}}\ll k_{\mathrm{S}}} \frac{dk_{\mathrm{S}}dk_{\mathrm{L}}}{k_{\mathrm{S}}k_{\mathrm{L}}} \to \infty$?

➢ Geodesic coordinate (local observer’s effect) [Tanaka, Urakawa ’11, Pajer et al. ’13]

$$ds^2 = -dt^2 + e^{2\zeta}a^2(t)d\boldsymbol{x}^2$$

$$\boldsymbol{x}_{\mathrm{F}} \simeq (1+\zeta)\boldsymbol{x}, \;\; \zeta_{\mathrm{F}}(\boldsymbol{x}_{\mathrm{F}}) = \zeta(\boldsymbol{x}) \simeq \zeta(\boldsymbol{x}_{\mathrm{F}}) - \zeta(1 + \boldsymbol{x}\cdot\partial_{\boldsymbol{x}}\zeta)$$

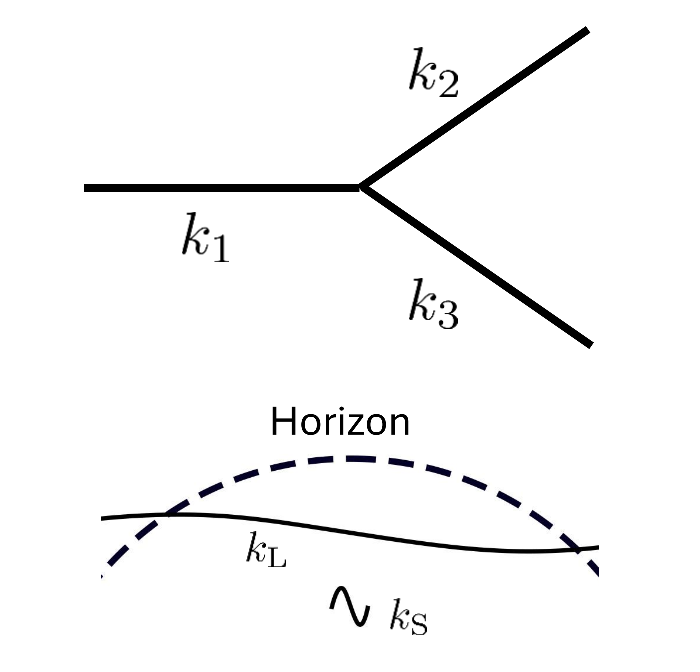
$$= -dt^2 + a^2(t)d\boldsymbol{x}_{\mathrm{F}}^2 + \cdots$$

(conformal Fermi normal coordinate)

Bispectrum in Single Field Inflation

❑ Maldacena’s consistency relation [Maldacena ’02]

$$\mathcal{L}_3^\zeta = a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + \partial_t\left(-\frac{\epsilon\eta}{2}a^3\zeta^2\dot{\zeta}\right) + \cdots$$



- Squeezed limit $\overset{\equiv k_\mathrm{L}}{k_3} \ll \overset{\equiv k_\mathrm{S}}{k_1} \simeq k_2$ with $\delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$

$$S_\mathrm{F} \longrightarrow \cancel{\frac{k_\mathrm{S}}{4k_\mathrm{L}}(1-n_s) + \mathcal{O}\left(\left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^0\right)} + \mathcal{O}\left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)$$

$$\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle_\mathrm{F} \sim \int \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9}\frac{S_\mathrm{F}}{k_1^2k_2^2k_3^2}\delta^3\left(\sum k_i\right) \to \cancel{\int_{k_\mathrm{L}\ll k_\mathrm{S}} \frac{dk_\mathrm{S}dk_\mathrm{L}}{k_\mathrm{S}k_\mathrm{L}} \to \infty}$$

$$\to \int_{k_\mathrm{L}\ll k_\mathrm{S}} dk_\mathrm{S}dk_\mathrm{L}\frac{k_\mathrm{L}}{k_\mathrm{S}^3} \quad : \ \text{finite}$$

- Geodesic coordinate (local observer’s effect) [Tanaka, Urakawa ’11, Pajer et al. ’13]

$$ds^2 = -dt^2 + e^{2\zeta}a^2(t)d\boldsymbol{x}^2 \qquad \boldsymbol{x}_\mathrm{F} \simeq (1+\zeta)\boldsymbol{x},\ \ \zeta_\mathrm{F}(\boldsymbol{x}_\mathrm{F}) = \zeta(\boldsymbol{x}) \simeq \zeta(\boldsymbol{x}_\mathrm{F}) - \zeta(1 + \boldsymbol{x}\cdot\partial_{\boldsymbol{x}}\zeta)$$

$$= -dt^2 + a^2(t)d\boldsymbol{x}_\mathrm{F}^2 + \cdots$$

(conformal Fermi normal coordinate)

Observational Expectation

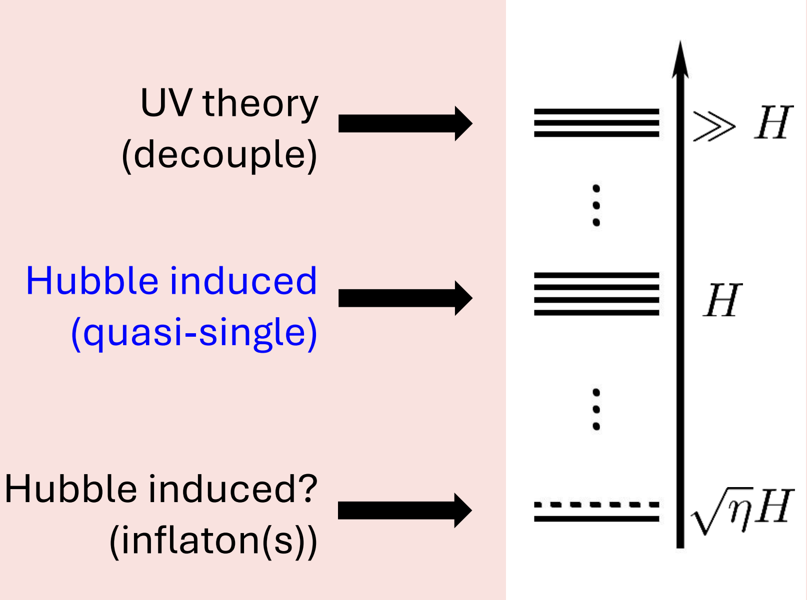
◆ Observable range of the amplitude

- CMB: $f_{\rm NL}^{\rm sq} \sim \mathcal{O}(1)$, galaxy survey: $f_{\rm NL}^{\rm sq} \sim \mathcal{O}(0.1)$, 21cm line from dark age: $f_{\rm NL}^{\rm sq} \sim \mathcal{O}(0.01)$?
  $(f_{\rm NL} \sim (k_{\rm S}/k_{\rm L})S)$
- Theoretical predictions: $f_{\rm NL}^{\rm CC} \sim (\text{coupling consts.}) \times e^{-\pi\mu} \times (k_{\rm L}/k_{\rm S})^{3/2} \times \mathcal{O}(1)$

  ➡ Fields with $m \sim H$ can have observably large signals.

◆ Mass spectra [Copeland et al. '94, Chen, Wang, Xianyu '16 etc.]

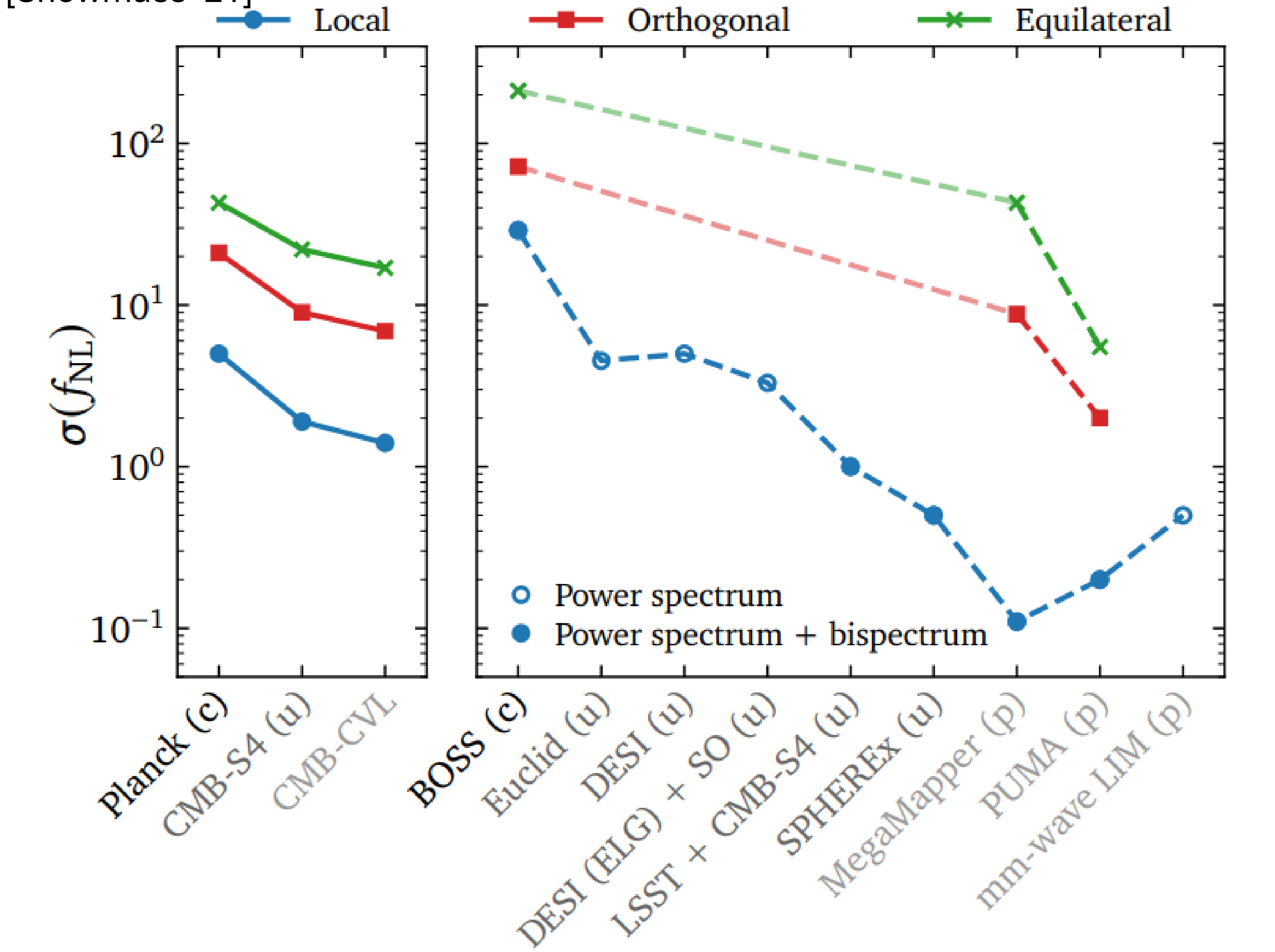
- Hubble scale mass
  - ✓ "Thermal" correction $T_{\rm H} = H/2\pi \longrightarrow \Delta m^2 \propto T_{\rm H}^2$
  - ✓ SUGRA $\mathcal{L} \supset e^K V(\phi) \simeq V + \dfrac{c\sigma^2}{M_{\rm pl}^2} V \simeq V + 3cH^2\sigma^2$
  - ✓ Non-minimal coupling $\mathcal{L} \supset \xi\sigma^2 R \simeq 12\xi H^2\sigma^2$



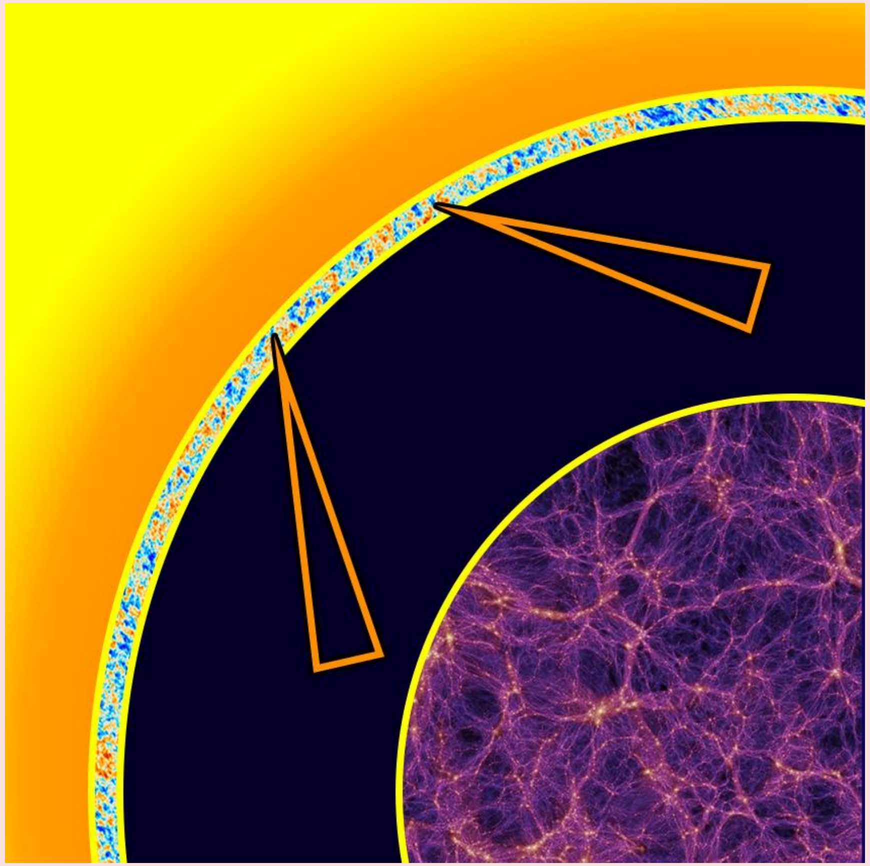
Future Observations

(c): completed
(u): upcoming
(p): projected

[Snowmass '21]



CMB — Galaxy survey — 21 cm



CMB Dark age Galaxies

21cm-21cm-CMB cross-correlation

$\sigma(f_{NL}^{local}) \sim 6 \times 10^{-3}$ [Orlando et al. '23]

Cf. Seed integral of single-exchange diagram

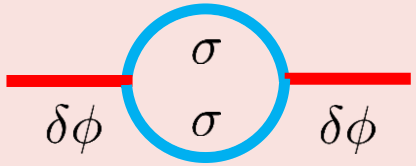
[Qin, Xianyu '22 and '23]

$$\begin{aligned}
&\mathcal{I}^{p_1p_2}_{\pm\mp}\\
&=\frac{-e^{\mp i\frac{\pi}{2}\bar{p}_{12}}\left[1+\cosh(2\pi\mu)\right]}{2\sinh^2(2\pi\mu)}\\
&\times\left\{2^{\pm i\mu}\left(\frac{u_1}{2}\right)^{\frac{5}{2}+p_1\pm i\mu}{}_2\mathcal{F}_1\left[\left.\begin{matrix}\frac{5}{2}+p_1\pm i\mu,\frac{1}{2}\pm i\mu\\1\pm 2i\mu\end{matrix}\right|u_1\right]-(\mu\to-\mu)\right\}\\
&\times\left\{2^{\pm i\mu}\left(\frac{u_2}{2}\right)^{\frac{5}{2}+p_2\pm i\mu}{}_2\mathcal{F}_1\left[\left.\begin{matrix}\frac{5}{2}+p_2\pm i\mu,\frac{1}{2}\pm i\mu\\1\pm 2i\mu\end{matrix}\right|u_2\right]-(\mu\to-\mu)\right\},
\end{aligned}$$

$$\begin{aligned}
&\mathcal{I}^{p_1p_2}_{\pm\pm}\\
&=\frac{\mp ie^{\mp i\frac{\pi}{2}p_{12}}\pi}{\Gamma\left[\frac{1}{2}-i\mu,\frac{1}{2}+i\mu\right]\sinh^2(2\pi\mu)}\\
&\times\left\{\frac{e^{\pi\mu}\cosh\left[\pi(-\mu)\right]}{2^{\mp i\mu}}\left(\frac{u_1}{2}\right)^{\frac{5}{2}+p_1\pm i\mu}{}_2\mathcal{F}_1\left[\left.\begin{matrix}\frac{5}{2}+p_1\pm i\mu,\frac{1}{2}\pm i\mu\\1\pm 2i\mu\end{matrix}\right|u_1\right]-(\mu\to-\mu)\right\}\\
&\times\left\{2^{\pm i\mu}\left(\frac{u_2}{2}\right)^{\frac{5}{2}+p_2\pm i\mu}{}_2\mathcal{F}_1\left[\left.\begin{matrix}\frac{5}{2}+p_2\pm i\mu,\frac{1}{2}\pm i\mu\\1\pm 2i\mu\end{matrix}\right|u_2\right]-(\mu\to-\mu)\right\}\\
&+\frac{e^{\mp i\frac{\pi}{2}p_{12}}\Gamma\left(p_{12}+5\right)}{2^{p_{12}+5}}\sum_{n=0}^{\infty}u_1^{n+p_{12}+5}\left(1-\frac{1}{u_2}\right)^n\begin{pmatrix}n+p_{12}+4\\n\end{pmatrix}\\
&\times\frac{1}{\mu^2+\left(\frac{5}{2}+n+p_2\right)^2}{}_3\mathrm{F}_2\left[\left.\begin{matrix}1,3+n+p_2,5+n+p_{12}\\\frac{7}{2}+n+p_2-i\mu,\frac{7}{2}+n+p_2+i\mu\end{matrix}\right|u_1\right].
\end{aligned}$$

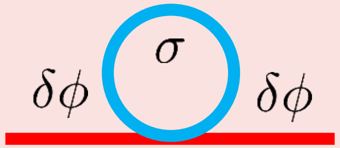
Quantum correction to inflaton mass

- $yH\phi\sigma^2$



$$\sim y^2H^2 \lesssim \mathcal{O}(\eta,\epsilon)H^2 \Longrightarrow y \lesssim \sqrt{\epsilon} \Longrightarrow \Delta m_\sigma^2(k) \lesssim \epsilon H M_{\rm pl} \log\frac{k}{k_{\rm i}}$$

- $\lambda\phi^2\sigma^2$

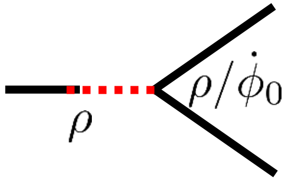


$$\sim \lambda\Lambda^2 \lesssim \mathcal{O}(\eta,\epsilon)H^2 \Longrightarrow \lambda \lesssim \epsilon\frac{H^2}{\Lambda^2} \Longrightarrow \Delta m_\sigma^2(k) \lesssim \epsilon^2 H M_{\rm pl}\frac{HM_{\rm pl}}{\Lambda^2}\log^2\frac{k}{k_{\rm i}}$$

* Shift sym. Couplings: $\frac{|\partial_t^2\phi|}{\Lambda} \sim \epsilon^{3/2}HM_{\rm pl}\frac{H}{\Lambda}\log\frac{k}{k_{\rm i}}, \quad \frac{|\partial_t\phi_0|^2}{\Lambda^2} \sim \epsilon HM_{\rm pl}\frac{HM_{\rm pl}}{\Lambda^2}\log^2\frac{k}{k_{\rm i}}$

Size Estimation of Single-exchange Diagrams

$$\frac{\rho}{\dot\phi_0}(\partial_\mu\phi)^2\sigma \longrightarrow \rho\,\delta\phi'\sigma + \frac{\rho}{\dot\phi_0}\,(\delta\phi')^2\sigma$$

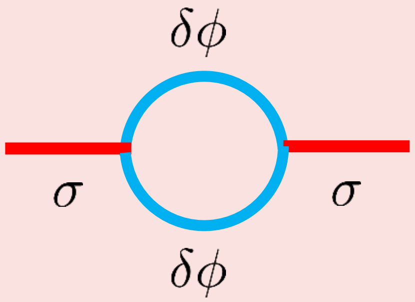


$$S_{\rm SE} \sim \frac{\rho^2}{\dot\phi_0} P_\zeta^{-1/2} e^{-\pi\mu}\mathcal{O}(1)$$

$$\left.\begin{array}{l}\dot\phi_0 \sim H^2 P_\zeta^{-1/2} \\ \rho \equiv \alpha H\end{array}\right\} \quad \frac{\rho^2}{\dot\phi_0} \sim \alpha^2 P_\zeta^{1/2}$$

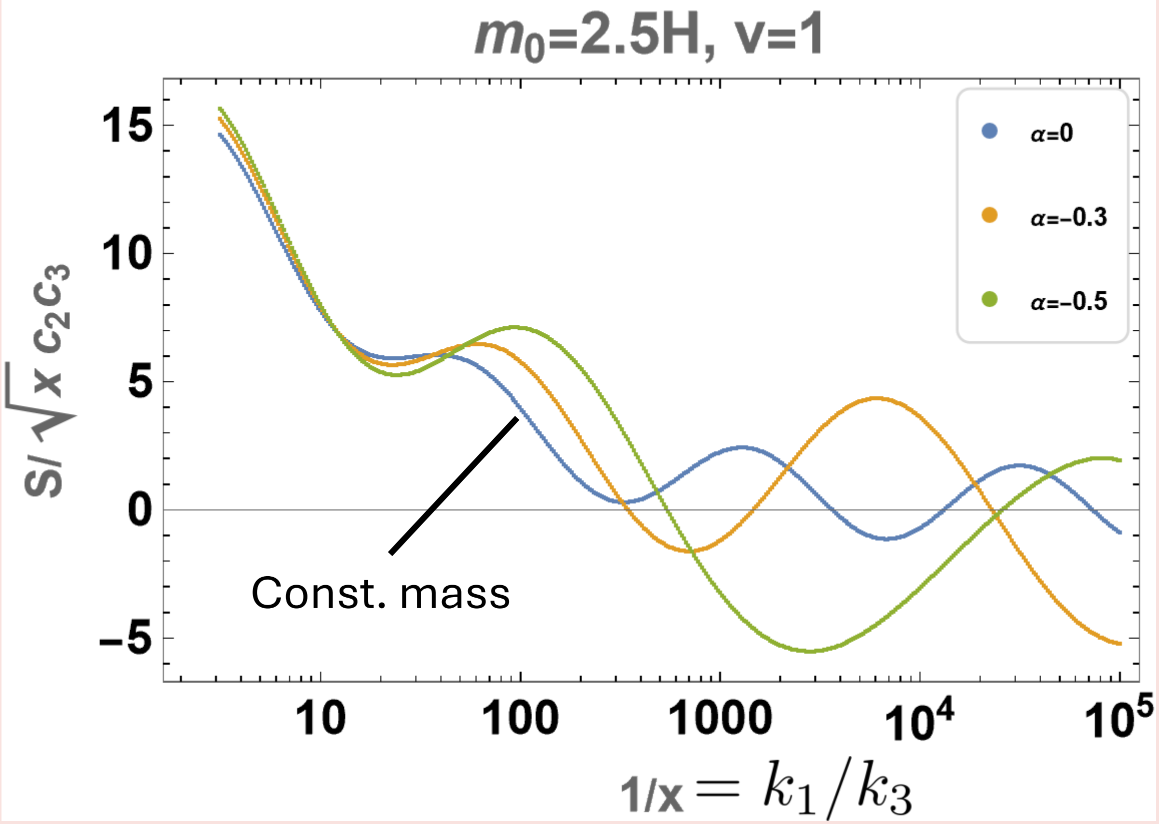
Naturalness $\alpha \lesssim 1$

[Pinol, Renaux-Petel, Werth '23]



$$\Longrightarrow \quad S_{\rm SE} \lesssim e^{-\pi\mu} \times \mathcal{O}(1)$$

$$m_\sigma = 2.5H \to e^{-\pi\mu} \sim 10^{-3}$$



Observational Signals in Bispectrum

Consistency check: CosmoFlow
[Pinol, Renaux-Petel, Werth '23, '24]

- **Squeezed limit** $k_3 \ll k_1 \simeq k_2$

$$\langle \delta\phi_{k1}\delta\phi_{k2}\delta\phi_{k3}\rangle' \xrightarrow[k_3\to 0]{} \sum_{\alpha,\beta} \frac{\rho_\alpha\rho_\beta\lambda_{\alpha\beta}H}{(k_1k_2k_3)^2}\cdot \mathrm{Re}\left\{\left[\mathrm{i}\frac{\pi^{3/2}}{2^{4+2\mathrm{i}\mu_\alpha}}\,\mathrm{sech}\,(\pi\mu_\beta)\,[1+\tanh(\pi\mu_\alpha)]\times\Gamma\left[\begin{matrix}-\mathrm{i}\mu_\alpha\\ -1-\mathrm{i}\mu_\alpha+\mathrm{i}\mu_\beta, -1-\mathrm{i}\mu_\alpha-\mathrm{i}\mu_\beta\end{matrix}\right]\right.\right.$$

$$\left.\left.\times\, {}_3\mathcal{F}_2\left[\begin{matrix}-\frac32-\mathrm{i}\mu_\alpha, -1-\mathrm{i}\mu_\alpha-\mathrm{i}\mu_\beta, -1-\mathrm{i}\mu_\alpha+\mathrm{i}\mu_\beta\\ -\frac12-\mathrm{i}\mu_\alpha, -\frac12-\mathrm{i}\mu_\alpha\end{matrix}\middle|\, 1\right] + \mathcal{O}\left(e^{-2\pi\mu_\alpha}, e^{-2\pi\mu_\beta}\right)\right]\left(\frac{k_1}{k_3}\right)^{\frac12+\mathrm{i}\mu_\alpha} + \mathcal{O}\left(\frac{k_1}{k_3}\right)\right\}$$

- **Size in equilateral limit** $k_1 = k_2 = k_3 = k$