

# **Cosmo. Collider as an Interaction Probe**

## **Scale-dependence and Diagrams**

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Based on:

**2312.09642** with Shuntaro Aoki, Toshifumi Noumi, Masahide Yamaguchi

**2404.09547** with Shuntaro Aoki, Lucas Pinol, Masahide Yamaguchi, Yuhang Zhu

# Outline

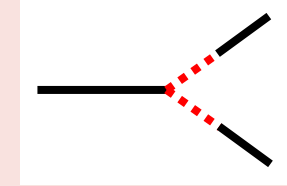
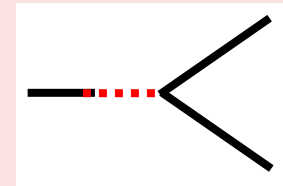
## □ Introduction

## □ Non-shift-symmetric interactions of $\phi$ (based on 2312.09642)

- Time-dependent coupling and scale dependence

## □ Exact calculation of double-exchange diagrams (based on 2404.09547)

- Comparison to single-exchange diagrams



# Inflation as a Source for Cosmological Perturbations

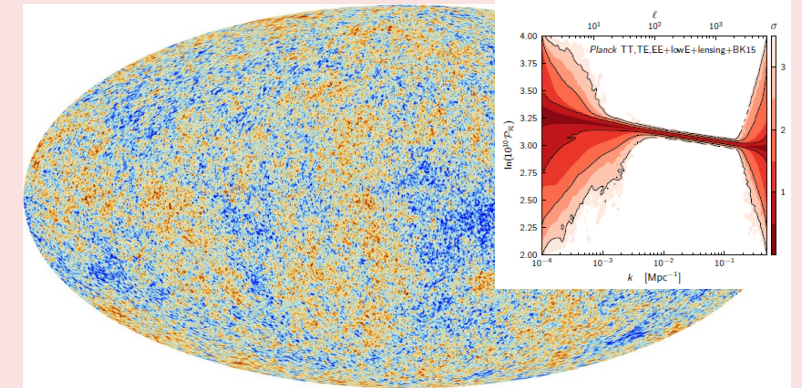
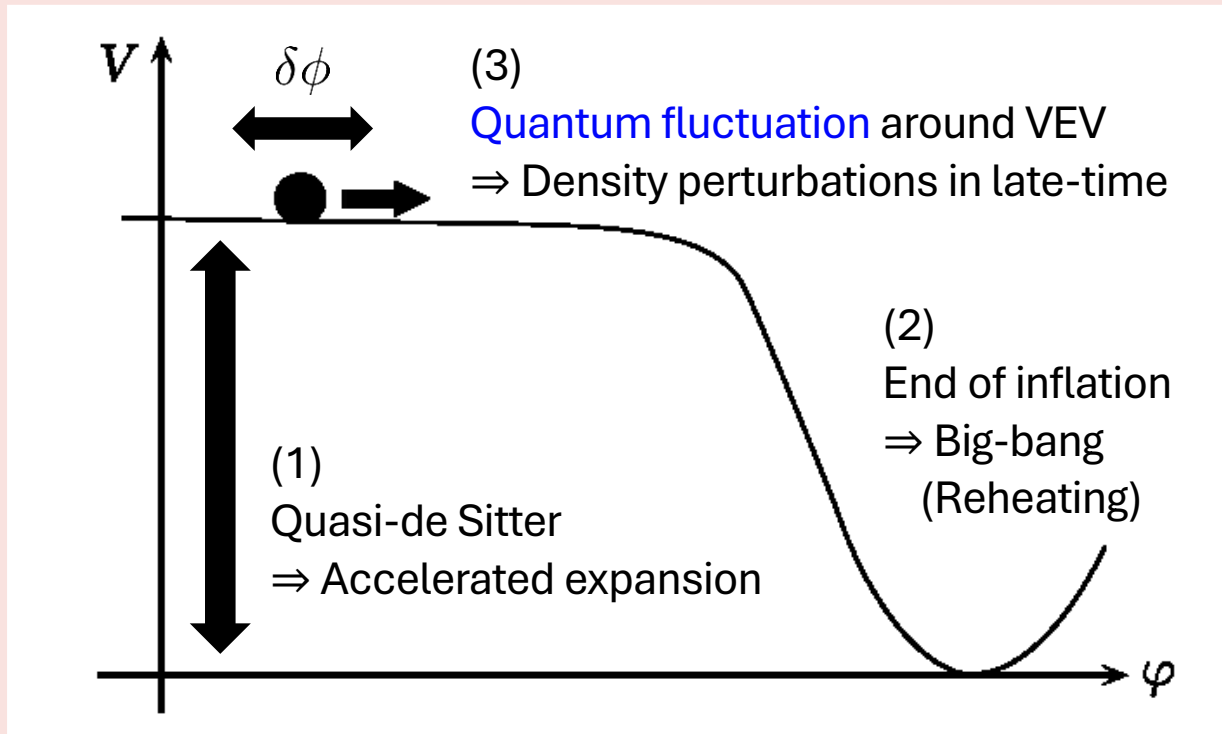
❑ Slow-roll inflation can be an answer for .....

$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) : \text{Inflaton}$$

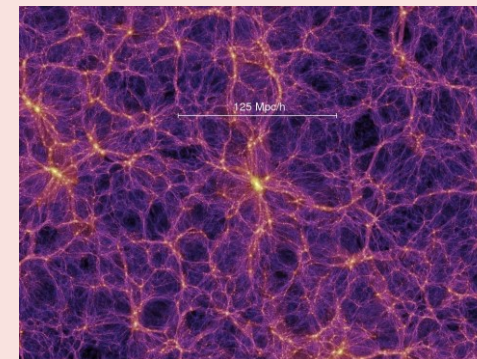
$$\epsilon = M_{\text{pl}}^2 \left(\frac{V'}{V}\right)^2 \ll 1, \quad |\eta| = M_{\text{pl}}^2 \left|\frac{V''}{V}\right| \ll 1$$

$$\phi = \phi_0(t) + \delta\phi(x) \longleftrightarrow \text{Curvature perturb. } h_{ij} = (e^{\zeta(x)}a(t))^2\delta_{ij}$$

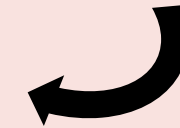
- 1) Flatness, horizon problem etc. in big bang model
- 2) Transition from inflation to big bang
- 3) **Origin of cosmological structures**



[Planck 1807.06211]



[Millennium Simulation 2005]



**Observables:**

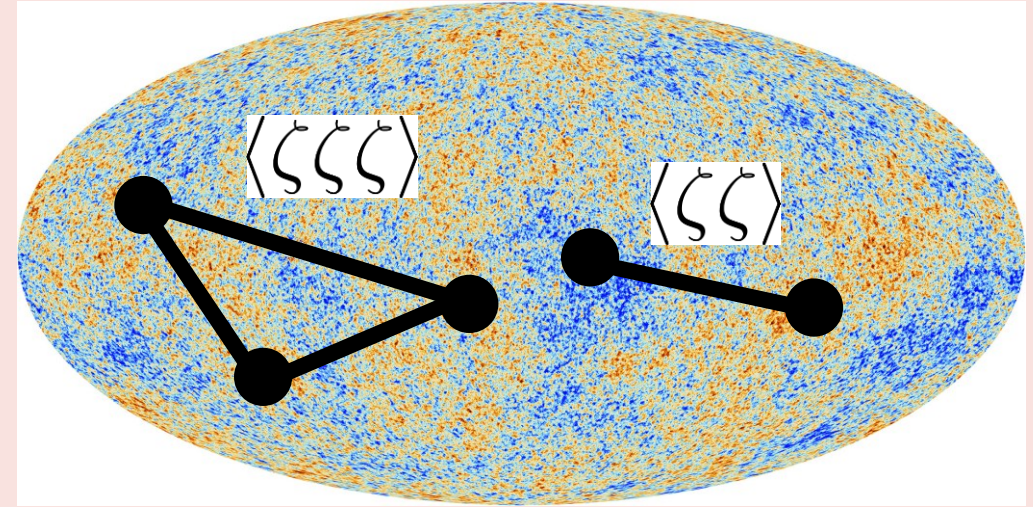
Late time anisotropy  
and inhomogeneity

# Observables for Inflationary Cosmology

## □ 2pt. correlation function (power spectrum)

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle_{\text{inf. end}} = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} P_\zeta$$
$$P_\zeta \simeq \frac{H^2}{8\pi^2 \epsilon} \left( \frac{k}{k_*} \right)^{n_s - 1} \quad n_s \simeq 0.965, \quad \frac{dn_s}{d \log k} \simeq 0.002$$

[Planck '18]



- ✓ CMB observation can be explained solely by curvature perturbations.  
➡ Single field inflation is favored.

## □ 3pt. correlation function (bispectrum)

$$\langle \zeta_1 \zeta_2 \zeta_3 \rangle_{\text{inf. end}} = (2\pi)^7 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{P_\zeta^2}{(k_1 k_2 k_3)^2} S\left(\frac{k_1}{k_3}, \frac{k_2}{k_3}\right)$$

- ✓ 3pt.: effects of interactions ➡ Probe for BSM physics and inflation models

# Quasi-single Field Inflation

[Chen, Wang '09, Noumi, Yamaguchi, Yokoyama '12 etc.]

## □ Inflaton + heavy isocurvature mode

$$\mathcal{L} = -\frac{1}{2}(r + \sigma)^2 g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V_{\text{sr}}(\theta) - V(\sigma)$$

- UV contribution  $\sim$  single field EFT  $\left(\frac{1}{\mu^2}\right)^3 \frac{k_L}{k_S}$
- IR contribution: leading in  $k_L \rightarrow 0$  when  $m_\sigma \sim H$

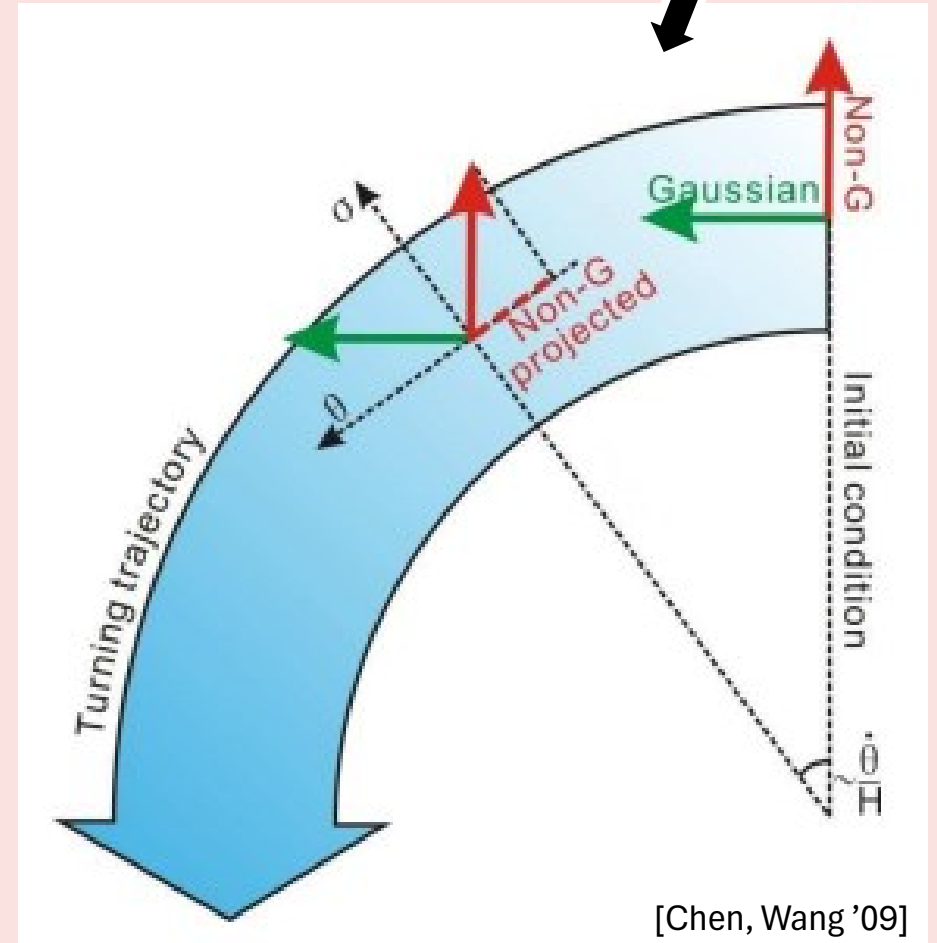
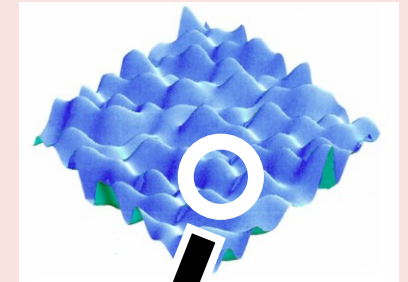
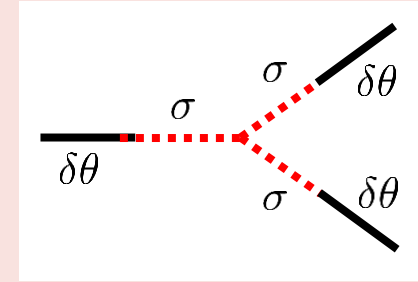
$$S \sim \left(\frac{k_L}{k_S}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_L}{k_S} + \delta\right) \quad \mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$

✓ Quantum interference  $\left(\frac{k_S}{k_L}\right)^{i\mu} \sim \left(\frac{\tau_L}{\tau_S}\right)^{i\mu} \sim e^{im(t_S - t_L)}$   
 $\tau = -e^{-Ht}/H$   
 Conformal time

## ✓ Gravitational Boltzmann suppression

$$e^{-E/T} \sim e^{-m/T_H} \sim e^{-2\pi\mu} \quad T_H = \frac{H}{2\pi}$$

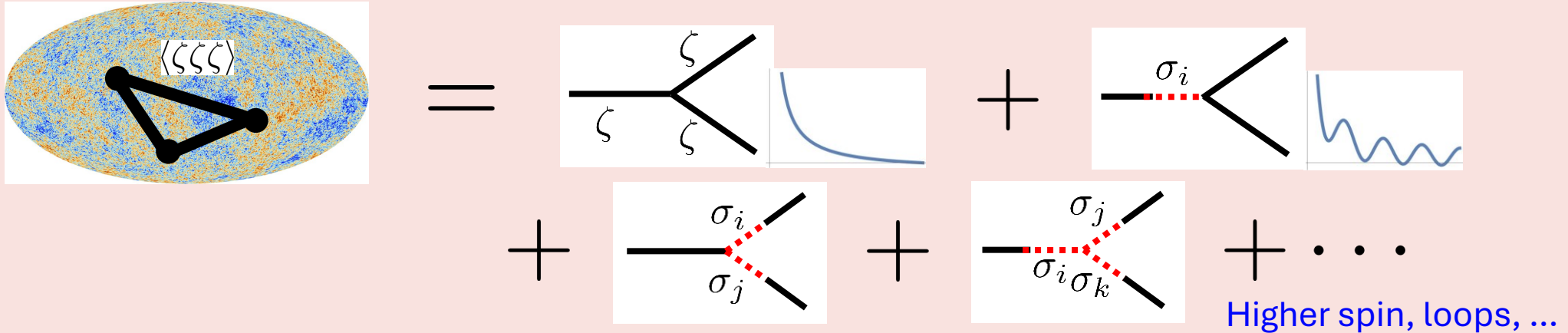
➡ Dominant: interference term  $e^{-\pi\mu}$



[Chen, Wang '09]

# Cosmological Collider Physics

[Chen, Wang '09, Noumi, Yamaguchi, Yokoyama '12, Arkani-Hamed, Maldacena, '15, Lee, Baumann, Pimentel '16 etc.]



## □ Cosmological Collider Signal

$$S \sim \left( \frac{k_L}{k_S} \right)^{1/2} e^{-\pi\mu} \cos \left( \mu \log \frac{k_L}{k_S} + \delta \right) \quad \begin{matrix} k_L \equiv k_3 \ll k_1 \simeq k_2 \equiv k_S \\ \mu = \sqrt{\left( \frac{m_\sigma}{H} \right)^2 - \frac{9}{4}} \end{matrix} \quad \boxed{\bullet \cdots \frac{\sigma}{k_L} \cdots \bullet \Rightarrow e^{-\pi\mu} \left( \frac{k_L}{k_S} \right)^{1/2+i\mu}}$$

✓ Dictionary of particles at the energy scale  $\lesssim 10^{15}$  GeV

Supersymmetry,	RH neutrino,	CP violation,	gauge symmetry,	swampland, ...
[Baumann, Green '12]	[Chen et al. '18]	[Liu et al. '19]	[Maru, Okawa '21]	[Reece et al. '22]



# Observational Expectation

## □ Observable range of the amplitude

➤ CMB:  $f_{\text{NL}}^{\text{sq}} \sim \mathcal{O}(1)$ , galaxy survey:  $f_{\text{NL}}^{\text{sq}} \sim \mathcal{O}(0.1)$ , 21cm line from dark age:  $f_{\text{NL}}^{\text{sq}} \sim \mathcal{O}(0.01)$ ?  
( $f_{\text{NL}} \sim (k_{\text{S}}/k_{\text{L}})S$ )

➤ Theoretical predictions:  $f_{\text{NL}}^{\text{CC}} \sim (\text{coupling consts.}) \times e^{-\pi\mu} \times (k_{\text{L}}/k_{\text{S}})^{3/2} \times \mathcal{O}(1)$

➡ Fields with  $m \sim H$  can have observably large signals.

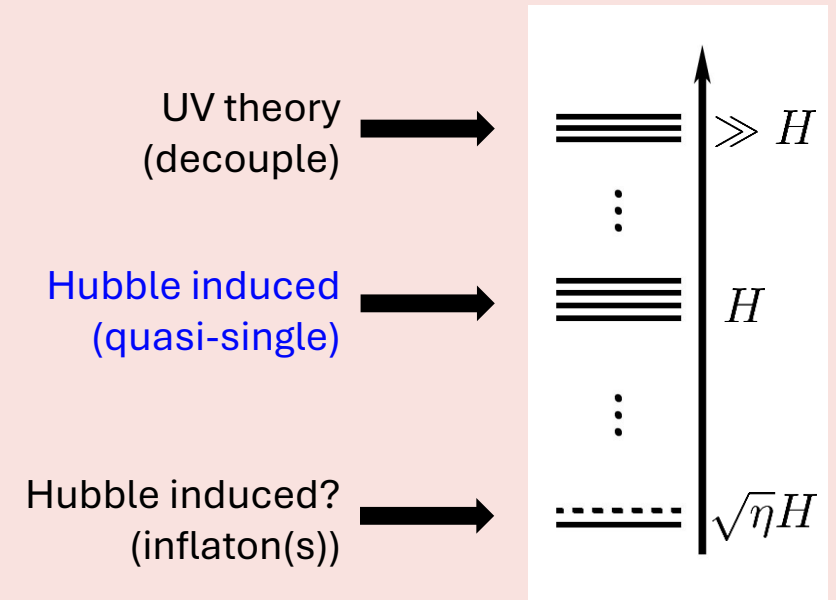
## □ Mass spectra [Copeland et al. '94, Chen, Wang, Xianyu '16 etc.]

### ➤ Hubble scale mass

✓ “Thermal” correction  $T_{\text{H}} = H/2\pi \longrightarrow \Delta m^2 \propto T_{\text{H}}^2$

✓ SUGRA  $\mathcal{L} \supset e^K V(\phi) \simeq V + \frac{c\sigma^2}{M_{\text{pl}}^2} V \simeq V + 3cH^2\sigma^2$

✓ Non-minimal coupling  $\mathcal{L} \supset \xi\sigma^2 R \simeq 12\xi H^2\sigma^2$



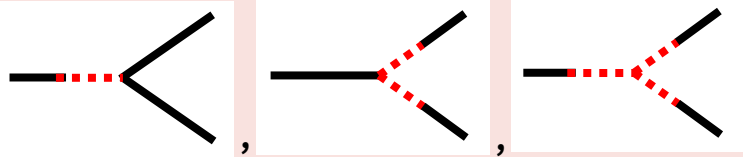
How interactions appear?

$$S \sim \left( \prod_i \lambda_i \right) \left( \frac{k_L}{k_S} \right)^{1/2} e^{-\pi\mu} \cos \left( \mu \log \frac{k_L}{k_S} + \delta \right)$$



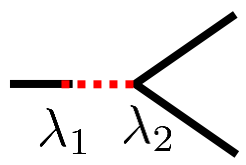
# Interactions in CC-signal

## □ Diagrams [Chen, Wang, Xianyu '17, Qin, Xianyu '22]

➤   $\sim \left(\frac{k_L}{k_S}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_L}{k_S} + \delta\right)$

➤ Phase information  $\delta$ :  $\mathcal{A}(\mu) \times \left(\frac{k_L}{k_S}\right)^{i\mu} = |\mathcal{A}(\mu)| e^{i\mu \ln(k_L/k_S) + i\text{Arg}[\mathcal{A}(\mu)]}$

## □ Shift symmetric vs. non-shift symmetric

➤ Signal size   $S \propto \lambda_1 \lambda_2$

Exact dS: shift-symmetric in terms of  $\phi$

➡ Non-shift sym. ints.:  $\lambda_1, \lambda_2$  are bounded by  $\eta, \epsilon$ .

✓ How can we distinguish diagrams?

✓ Can we detect non-shift sym. ints.?

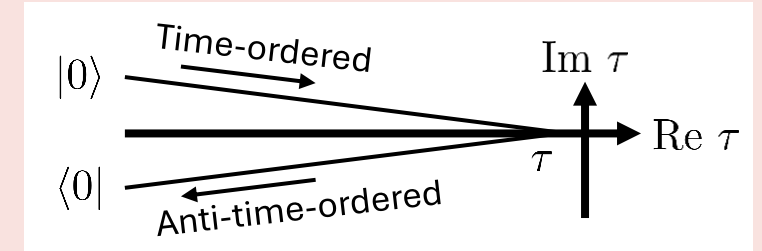


Analytic templates of signals?

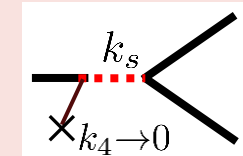
# Difficulty in Analytical Computations

## □ Perturbative expansion for correlators

$$\langle \Omega | \zeta_1 \zeta_2 \zeta_3(\tau) | \Omega \rangle = \left\langle 0 \left| \left( \bar{T} e^{i \int_{-\infty}^{\tau} d\tau' H_I} \right) \zeta_1 \zeta_2 \zeta_3(\tau) \left( T e^{-i \int_{-\infty}^{\tau} d\tau' H_I} \right) \right| 0 \right\rangle$$



$$\text{Diagram} = \text{Re} \left\{ \text{Diagram}_1 + \text{Diagram}_2 \right\} \propto \frac{1}{8k_1 k_2 k_3^4} \lim_{k_4 \rightarrow 0} \sum_{a,b=\pm} \mathcal{I}_{ab}^{0,-2} + (k_3 \rightarrow k_1, k_2)$$



## ✓ Seed integral

$$\mathcal{I}_{ab}^{p_1 p_2} = -ab k_s^{5+p_{12}} \int_{-\infty}^0 d\tau_1 d\tau_2 \underbrace{(-\tau_1)^{p_1} (-\tau_2)^{p_2} e^{iak_{12}\tau_1 + ibk_{34}\tau_2}}_{\text{Scale factor and propagators of } \phi} \underbrace{D_{ab}(k_s; \tau_1, \tau_2)}_{\text{Propagators of } \sigma} \quad a,b = \pm \begin{matrix} + : T \\ - : \bar{T} \end{matrix}$$

$$D_{++}(k_s; \tau_1, \tau_2) \sim \theta(\tau_1 - \tau_2) H_{i\mu}^{(1)}(-k_s \tau_1) H_{i\mu}^{(1)*}(-k_s \tau_1) \longrightarrow \text{No formula for integration...}$$

➡ **Cosmological bootstrap is proposed as an analytical method.**

[Series of papers by Baumann, Lee, Pimentel et al. '18, '20, '21]

# Analytical Method: De Sitter “Bootstrap” Equations

[Series of papers by Baumann, Lee, Pimentel et al. '18, '20, '21, Qin, Xianyu '22 and '23]

## □ De Sitter symmetries $\sim$ CFT

Translation  $P_i = \partial_i$ , Rotation  $J_{ij} = x_i \partial_j - x_j \partial_i$ , Dilatation  $D = -\tau \partial_\tau - x_i \partial_i$ ,

dS boosts  $K_i = \left(2x^j x_i + (\tau^2 - x^2) \delta_i^j\right) \partial_j + 2x_i \tau \partial_\tau$

✓ Ward identity: Symmetry  $\hat{S} \longrightarrow \langle 0 | [\hat{S}, \hat{\mathcal{O}}] | 0 \rangle = 0$  (assuming  $\hat{S} | 0 \rangle = 0$ )

## □ “Bootstrap” equations for seed integrals

➤ Equations of motion: quadratic Casimir operator  $\nabla_\mu \nabla^\mu$

$$(\nabla^2 + a^2 m^2) \sigma = 0 \quad \longrightarrow \quad \begin{aligned} \mathcal{D}_{\tau_i} \tilde{D}_{\text{ab}}^\sigma(k_s \tau_1, k_s \tau_2) &= -i a H^2 (k_s \tau_1)^2 (k_s \tau_2)^2 \delta_{\text{ab}} \delta(k_s \tau_1 - k_s \tau_2) \\ \mathcal{D}_{\tau_i} &= \tau_i \partial_{\tau_i} (\tau_i \partial_{\tau_i}) - 3 \tau_i \partial_{\tau_i} + k_s^2 \tau_i^2 + \mu^2 + \frac{9}{4}, \quad \mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}, \quad \tilde{D} = k_s^3 D \end{aligned}$$

➤ Dilatation:  $\tau \partial_\tau (\dots) = k \partial_k (\dots) \longrightarrow \mathcal{D}_\tau \tilde{D} = \mathcal{D}_k \tilde{D}$

$$\longrightarrow \tilde{\mathcal{D}}_{k_s} \left[ \begin{array}{c} \text{Diagram 1: Two vertices connected by a red dashed line} \end{array} \right] \sim \begin{array}{c} \text{Diagram 2: Two vertices connected by a red dashed line, with a crossed-out box around the first vertex} \end{array} \sim \begin{array}{c} \text{Diagram 3: A crossed box} \end{array} \longrightarrow \mathcal{I}_{\text{ab}}^{p_1 p_2} \sim {}_2F_1, \quad \sum_n \left( \frac{k_i}{\sum_j k_j} \right)^n {}_3F_2 \quad \in D_{\pm\pm}$$

# Boundary Conditions: Mellin-Barnes Representation

[Qin, Xianyu '22 and '23]

❑ **Bootstrap:** overall factors (integration constants) are not fixed.

➡ Reference points are necessary

❑ **Direct integration using MB rep.**

$$H_{i\mu}^{(1)}(-k\tau) = \frac{1}{\pi} \int_{-i\infty}^{i\infty} \frac{ds}{2\pi i} \left( \frac{-k\tau}{2} \right)^{-2s} e^{(2s-1-i\mu)\pi i/2} \Gamma(s-i\mu) \Gamma(s+i\mu)$$

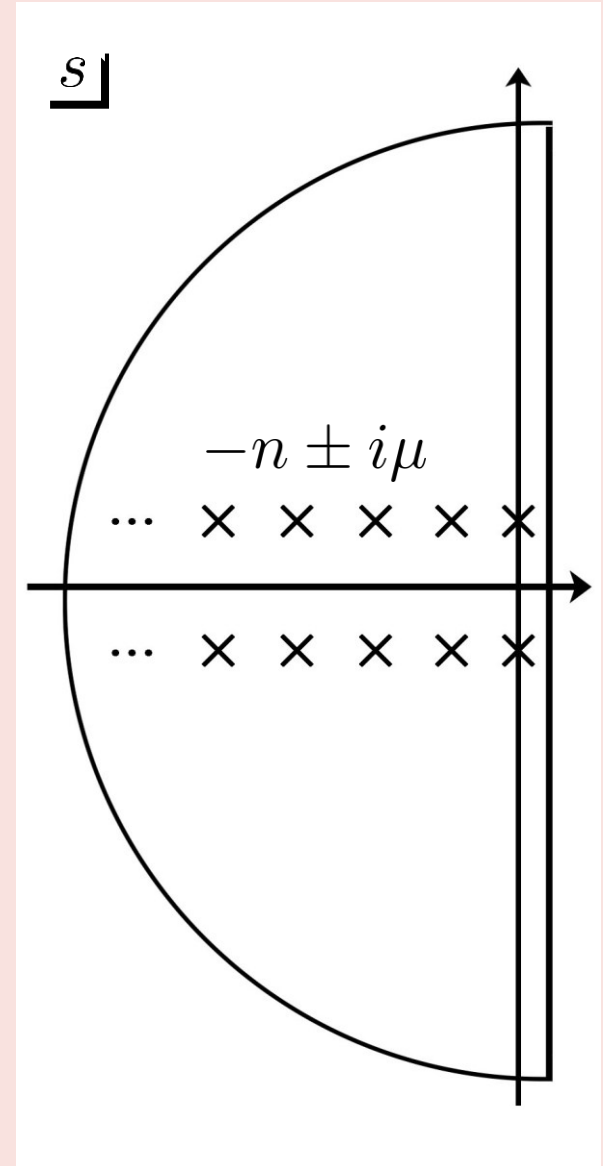
$$\begin{aligned} \Rightarrow \mathcal{I} &\sim \int d\tau_1 d\tau_2 e^{ik_{12}\tau_1 + ik_{34}\tau_2} (-\tau_1)^{p_1} (-\tau_2)^{p_2} H_{i\mu}^{(1)}(-k_s\tau_1) H_{i\mu}^{(1)*}(-k_s\tau_2) \theta(\tau_1 - \tau_2) \\ &\sim \sum_{\substack{n_1, n_2 \\ s_i = -n_i \pm i\mu}} \mathcal{A}_{n_1, n_2}(k, k') \text{Res}[\Gamma(s_1 \pm i\mu)] \text{Res}[\Gamma(s_2 \pm i\mu)] \end{aligned}$$

- ✓ MB rep.: double summation but boundary conditions are chosen in mode fn.
- ✓ Bootstrap: single summation but boundary conditions are non-trivial.

Equating them in some limit determines integration constants in bootstrap.

(e.g.,  $k_s \rightarrow 0$ )

[Qin, Xianyu '22 and '23]



# **Detectability of Non-shift-symmetric Interactions**

# Scale Dependence of Couplings from Slow-roll

[Wang '19, Reece, Wang, Xianyu '22]

## □ Example: mass of isocurvature modes

$$\frac{\mathcal{L}_{\text{int}}}{\sqrt{-g}} = yH\phi\sigma^2 \quad \longrightarrow \quad m_{\sigma,\text{eff}}^2 = m_{\sigma,0}^2 + \underline{\underline{2yH\phi_0}}$$

### ➤ Slow-roll approximation

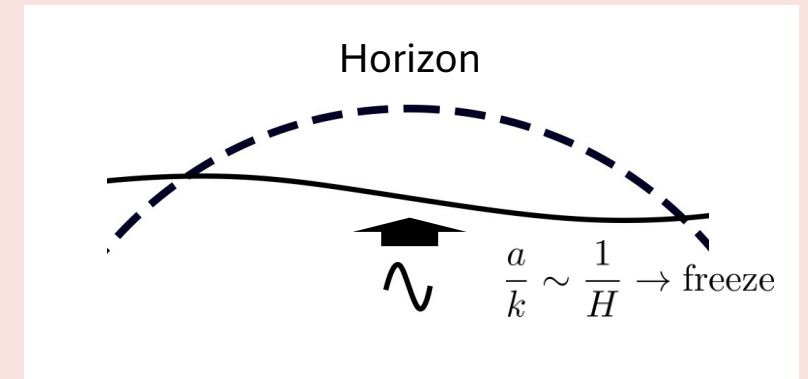
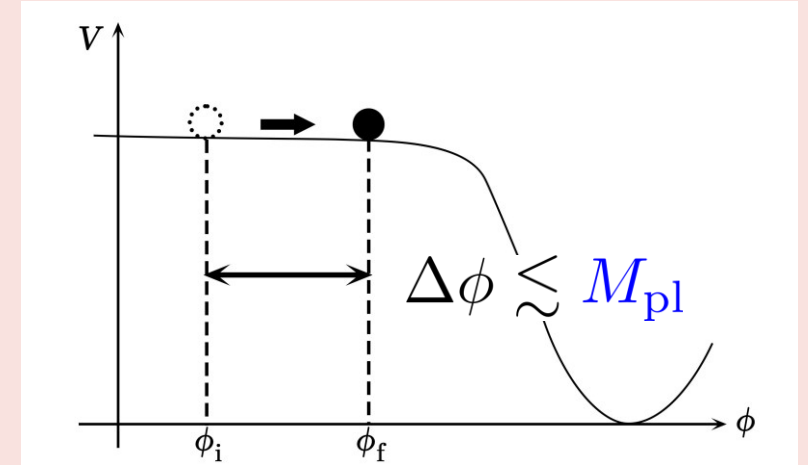
$$|\phi_0| \simeq \sqrt{2\epsilon} M_{\text{pl}} H(t - t_*) \simeq \sqrt{2\epsilon} M_{\text{pl}} \log\left(\frac{\tau_*}{\tau}\right)$$

$$\sim \sqrt{2\epsilon} M_{\text{pl}} \log \frac{k}{k_*} \quad (\text{Horizon crossing } |k\tau| \simeq 1)$$

$$\longrightarrow \Delta m_\sigma^2(k) \sim y\sqrt{\epsilon} H M_{\text{pl}} \log \frac{k}{k_*}$$

### \* Shift symmetric couplings

$$\frac{\mathcal{L}_{\text{int}}}{\sqrt{-g}} = \frac{1}{\Lambda} (\square\phi) \sigma^2 \quad \longrightarrow \quad \frac{|\partial_t^2 \phi_0|}{\Lambda} \sim \epsilon^{3/2} H M_{\text{pl}} \frac{H}{\Lambda} \log \frac{k}{k_*}$$



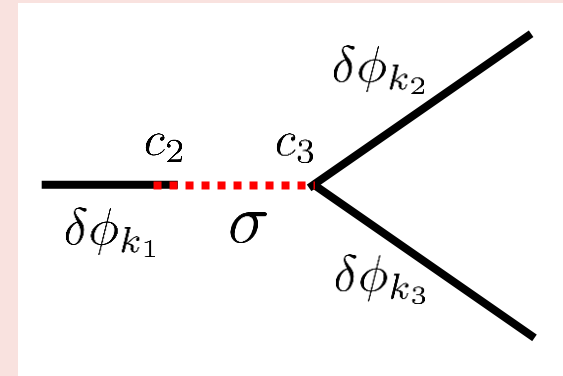
# Analytical Setup for Time-dependent Mass

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) - \frac{1}{2} (\partial_\mu \sigma)^2 - \frac{1}{2} g(\phi) \sigma^2 + \mathcal{L}_{\text{diag}} \right]$$

## □ Single-exchange with derivative coupling

$$\mathcal{L}_{\text{diag}} \supset \underline{c_2} (-\tau)^{-3} \sigma \delta\phi' + \underline{c_3} (-\tau)^{-2} \sigma (\delta\phi')^2$$

Unbounded by slow-roll (shift symmetric)



## □ Time-dependent mass

$$\triangleright \sigma_k'' - \frac{2}{\tau} \sigma_k' + \left( k^2 + \frac{m_{\text{eff}}^2}{H^2 \tau^2} \right) \sigma_k = 0 \quad , \quad \sigma_k = v_k a_k + v_k^* a_{-k}^\dagger$$

$$\longrightarrow v_k = \frac{e^{\pi\gamma/2}}{\sqrt{2k}} (-H\tau) W_{-i\gamma, i\mu}(2ik\tau) \quad \mu^2 = \frac{g_*}{H^2} \left( 1 - \frac{\sqrt{2\epsilon} g_{*,\phi} M_{\text{pl}}}{g_*} \right) - \frac{9}{4}, \quad \gamma = -\frac{\sqrt{2\epsilon} g_{*,\phi} M_{\text{pl}}}{2H^2}$$

(cf.  $W_{0,i\mu} \sim H_{i\mu}^{(1)}$ )

➡ Analytically calculable using MB rep. for Whittaker functions! (full result: see our paper)



# Bispectrum: Mass at Horizon-crossing

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_3}{k_1}\right)$$

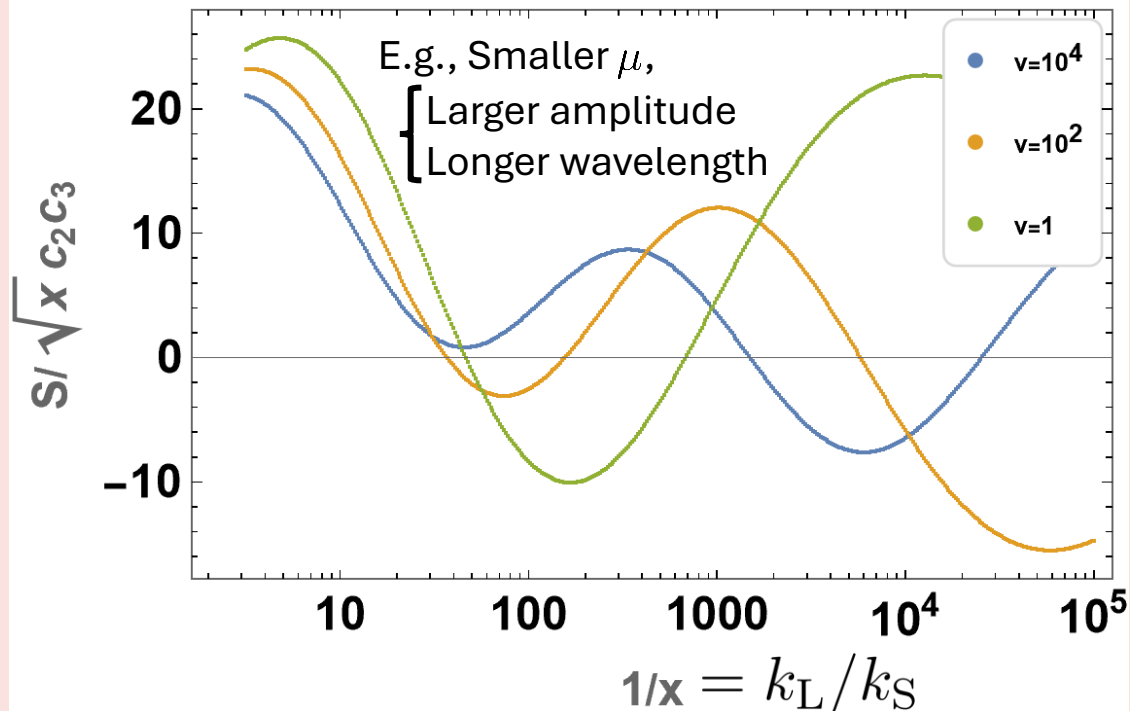
$$S \sim \left(\frac{k_L}{k_S}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_L}{k_S}\right)} \cos\left[\mu\left(v\frac{k_L}{k_S}\right) \log \frac{k_L}{k_S} + \delta\left(\mu\left(v\frac{k_L}{k_S}\right)\right)\right]$$

$$\mu\left(v\frac{k_L}{k_S}\right) = \frac{m_0}{H} \sqrt{1 - \alpha\sqrt{2\epsilon}\left(1 + \log\left(v\frac{k_L}{k_S}\right)\right) - \frac{9}{4}}$$

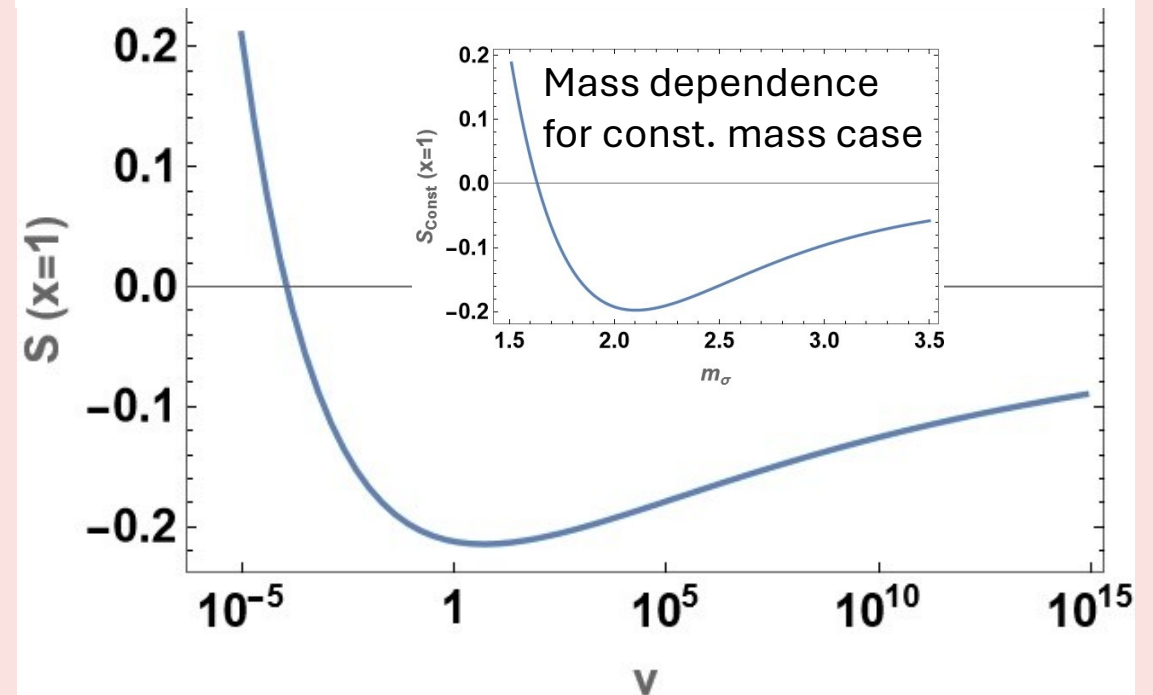
With the interaction  $m_0^2\left(1 + \alpha\frac{\phi}{M_{\text{pl}}}\right)\sigma^2$ ,  $\Delta\phi \sim \sqrt{\epsilon}M_{\text{pl}}\Delta N$

$v \equiv k_S/k_*$ : Scale dependence

CC signal



$v$  dependence (e.g., equilateral)



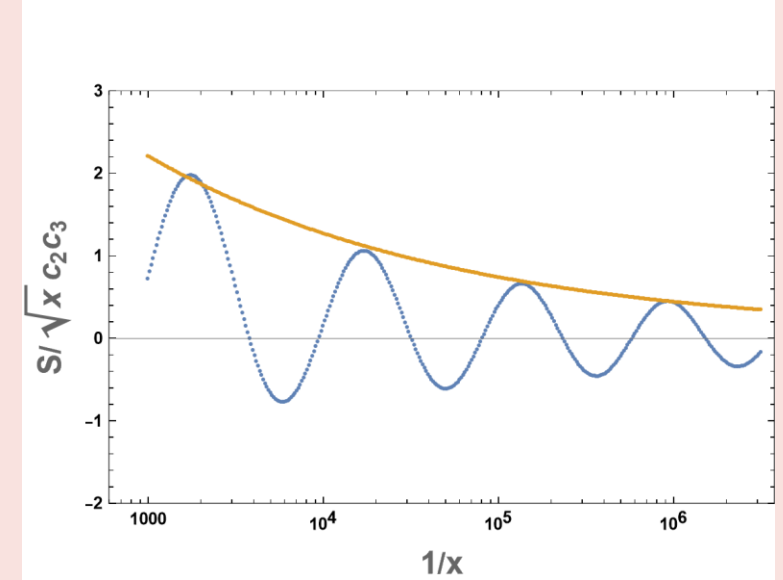
# Interaction distinction using scale dependence

$$S \sim \left( \frac{k_L}{k_S} \right)^{1/2} e^{-\pi \mu \left( v \frac{k_L}{k_S} \right)} \cos \left[ \mu \left( v \frac{k_L}{k_S} \right) \log \frac{k_L}{k_S} + \delta \left( \mu \left( v \frac{k_L}{k_S} \right) \right) \right]$$

$$\square \quad \overset{\phi_0 \sigma^2}{\Delta \mu_{\text{NSS}}^2(k)} \lesssim \sqrt{\epsilon} \frac{M_{\text{pl}}}{H} \quad \text{vs.} \quad \overset{\ddot{\phi}_0 \sigma^2}{\Delta \mu_{\text{SS}}^2(k)} \lesssim \epsilon^{3/2} \frac{M_{\text{pl}}}{\Lambda}$$

$$\square \quad e^{-\pi \mu} \sim \exp \left[ -\frac{\pi}{H} \sqrt{m_0^2 - \frac{9H^2}{4} + g \left( M_{\text{pl}} \sqrt{2\epsilon} \log \left( v \frac{k_L}{k_S} \right) \right)} \right] \quad \text{for} \quad \frac{\mathcal{L}_{\text{int}}}{\sqrt{-g}} = g(\phi) \sigma^2$$

✓ Scale dependence (suppression / enhancement etc.) is characterized by the interaction



*Non-shift-sym. ints: detectable through scale-dependence*

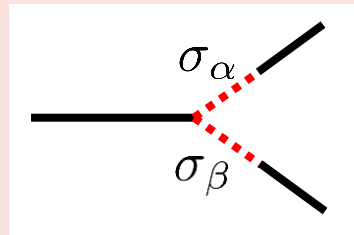
**Single-Exchange Diagrams  
vs.  
Double-Exchange Diagrams**

# Method: Bootstrap Equations and MB Representations

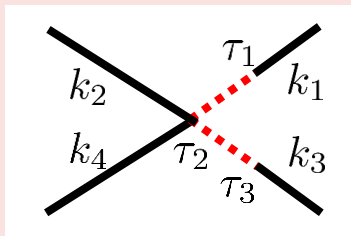
[Xianyu, Zang '24: MB rep.]

Numerical check: CosmoFlow  
[Pinol, Renaux-Petel, Werth '23, '24]

$$\mathcal{L}_{\text{int}} = a^3 \sum_{\alpha} \rho_{\alpha} \sigma_{\alpha} \delta\phi' + a^3 \sum_{\alpha, \beta} \lambda_{\alpha\beta} \sigma_{\alpha} \sigma_{\beta} \delta\phi'$$



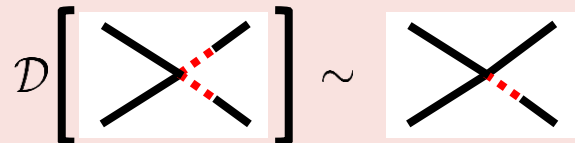
Seed integral



( $k_4 \rightarrow 0$ : bispectrum)

$$\mathcal{I}_{\text{abc}, \alpha\beta}^{p_1 p_2 p_3} = H^{-4} k_{24}^{9+p_{123}} (-i\text{abc}) \int_{-\infty}^0 d\tau_1 d\tau_2 d\tau_3 (-\tau_1)^{p_1} (-\tau_2)^{p_2} (-\tau_3)^{p_3} \\ \times e^{i a k_1 \tau_1 + i b k_{24} \tau_2 + i c k_3 \tau_3} D_{\text{ab}}^{\alpha}(k_1; \tau_1, \tau_2) D_{\text{bc}}^{\beta}(k_3; \tau_2, \tau_3)$$

## ❑ “Bootstrap” equations



$$\mathcal{I} \sim F_4, \sum_n \left( \frac{k_i}{\sum_j k_j} \right)^n ({}_3F_2 + {}_2F_1)$$

Analytical expression for  
arbitrary momentum configuration

## ❑ Bispectrum in squeezed region

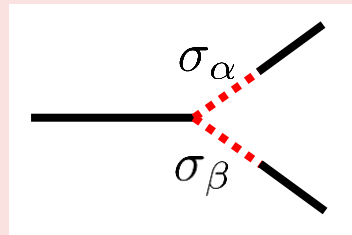
$$\langle \delta\phi_{k_1} \delta\phi_{k_2} \delta\phi_{k_3} \rangle' \xrightarrow{k_3 \rightarrow 0} \sum_{\alpha, \beta} \frac{\rho_{\alpha} \rho_{\beta} \lambda_{\alpha\beta} H}{(k_1 k_2 k_3)^2} \cdot \text{Re} \left\{ \left[ i \frac{\pi^{3/2}}{2^{4+2i\mu_{\alpha}}} \text{sech}(\pi\mu_{\beta}) [1 + \tanh(\pi\mu_{\alpha})] \times \Gamma \left[ \begin{matrix} -i\mu_{\alpha} \\ -1 - i\mu_{\alpha} + i\mu_{\beta}, -1 - i\mu_{\alpha} - i\mu_{\beta} \end{matrix} \right] \right. \right. \\ \left. \left. \times {}_3F_2 \left[ \begin{matrix} -\frac{3}{2} - i\mu_{\alpha}, -1 - i\mu_{\alpha} - i\mu_{\beta}, -1 - i\mu_{\alpha} + i\mu_{\beta} \\ -\frac{1}{2} - i\mu_{\alpha}, -\frac{1}{2} - i\mu_{\alpha} \end{matrix} \middle| 1 \right] + \mathcal{O}(e^{-2\pi\mu_{\alpha}}, e^{-2\pi\mu_{\beta}}) \right] \left( \frac{k_1}{k_3} \right)^{\frac{1}{2} + i\mu_{\alpha}} + \mathcal{O}\left( \frac{k_1}{k_3} \right) \right\}$$

# Method: Bootstrap Equations and MB Representations

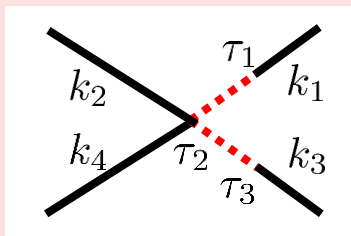
[Xianyu, Zang '24: MB rep.]

Numerical check: CosmoFlow  
[Pinol, Renaux-Petel, Werth '23, '24]

$$\mathcal{L}_{\text{int}} = a^3 \sum_{\alpha} \rho_{\alpha} \sigma_{\alpha} \delta\phi' + a^3 \sum_{\alpha, \beta} \lambda_{\alpha\beta} \sigma_{\alpha} \sigma_{\beta} \delta\phi'$$



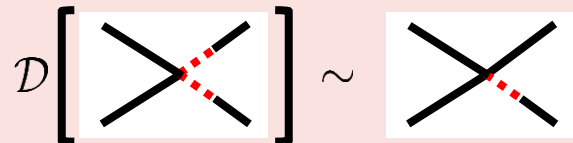
Seed integral



( $k_4 \rightarrow 0$ : bispectrum)

$$\mathcal{I}_{\text{abc}, \alpha\beta}^{p_1 p_2 p_3} = H^{-4} k_{24}^{9+p_{123}} (-i\text{abc}) \int_{-\infty}^0 d\tau_1 d\tau_2 d\tau_3 (-\tau_1)^{p_1} (-\tau_2)^{p_2} (-\tau_3)^{p_3} \\ \times e^{iak_1\tau_1 + ibk_{24}\tau_2 + ick_3\tau_3} D_{\text{ab}}^{\alpha}(k_1; \tau_1, \tau_2) D_{\text{bc}}^{\beta}(k_3; \tau_2, \tau_3)$$

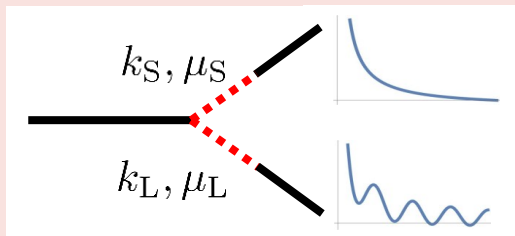
## ❑ “Bootstrap” equations



$$\mathcal{I} \sim F_4, \sum_n \left( \frac{k_i}{\sum_j k_j} \right)^n ({}_3F_2 + {}_2F_1)$$

Analytical expression for  
arbitrary momentum configuration

## ❑ Bispectrum in squeezed region



$$S \sim \left( \frac{k_L}{k_S} \right)^{1/2} e^{-\pi\mu_L} \cos \left( \mu_L \log \frac{k_L}{k_S} + \delta \right)$$

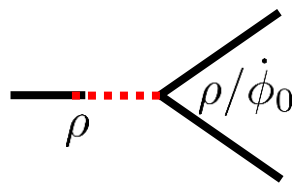
Qualitatively same as  
single-exchange?

# Difference between SE and DE 1: Size of Signals

[Pinol, Renaux-Petel, Werth '23]

## □ Single-exchange (SE)

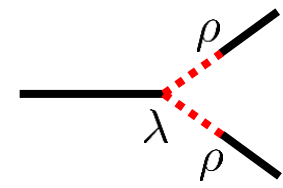
$$\frac{\rho}{\dot{\phi}_0} (\partial_\mu \phi)^2 \sigma \longrightarrow \rho \delta\phi' \sigma + \frac{\rho}{\dot{\phi}_0} (\delta\phi')^2 \sigma$$



$$S_{\text{SE}} \sim \frac{\rho^2}{\dot{\phi}_0} \times P_\zeta^{-1/2}$$

## □ Double-exchange (DE)

$$\rho \delta\phi' \sigma + \lambda \delta\phi' \sigma^2$$

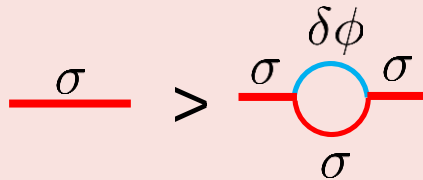


$$S_{\text{DE}} \sim \lambda \frac{\rho^2}{H^2} \times P_\zeta^{-1/2}$$

## □ Constraints

✓ Perturbativity  $\lambda \lesssim 1$

✓ Naturalness  $\lambda \lesssim P_\zeta^{1/4}$



$$\sigma > \sigma \delta\phi \sigma$$

$$\frac{S_{\text{DE}}}{S_{\text{SE}}} \sim \lambda \frac{\dot{\phi}_0}{H^2} \sim \lambda P_\zeta^{-1/2} \lesssim P_\zeta^{-1/4} \sim 10^2 \text{ Naturally larger than single-exchange}$$

# Difference between SE and DE 2: Phase Information

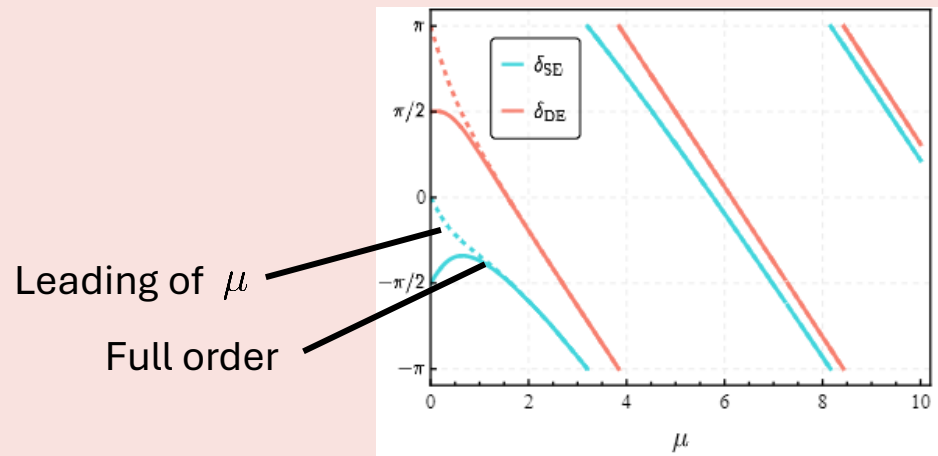
## □ Phase information

### ➤ SE and single isocurvature DE

$$S_{\text{DE,CC}}^{\text{single}} = \frac{\rho^2}{H^2} \frac{\lambda}{2\pi P_\zeta^{1/2}} \text{Re} \left[ \left( \frac{k_L}{k_S} \right)^{1/2+i\mu} \mathcal{A}_{\text{DE}}(\mu) e^{i\delta(\mu)} \right]$$

$$S_{\text{SE,CC}} = \frac{\rho^2}{\phi} \frac{1}{2\pi P_\zeta^{1/2}} \text{Re} \left[ \left( \frac{k_L}{k_S} \right)^{1/2+i\mu} \mathcal{A}_{\text{SE}}(\mu) e^{i\delta(\mu)} \right]$$

Consistency between phase and wavelength



### ➤ DE with multiple isocurvature modes

$$S_{\text{DE,CC}}^{\text{multi}} = \sum_{\alpha,\beta}^N \frac{\rho_\alpha \rho_\beta}{H^2} \frac{\lambda_{\alpha\beta}}{2\pi P_\zeta^{1/2}} \text{Re} \left[ \left( \frac{k_L}{k_S} \right)^{1/2+i\mu_\alpha} \mathcal{A}_{\mu_\alpha,\mu_\beta} e^{i\delta_{\mu_\alpha,\mu_\beta}} \right]$$

$$= \sum_{\alpha}^N \frac{\rho_\alpha}{H} \text{Re} \left[ \left( \frac{k_L}{k_S} \right)^{1/2+i\mu_\alpha} \mathcal{B}_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta} e^{i\vartheta_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta}} \right]$$

$$\left( a \sin \theta + b \sin(\theta + \Delta\theta) = \sqrt{a^2 + b^2 + 2ab \cos \Delta\theta} \sin(\theta + \alpha) \right)$$

# of observables:  $3N$  ≤ # of parameters:  $N(N+5)/2$

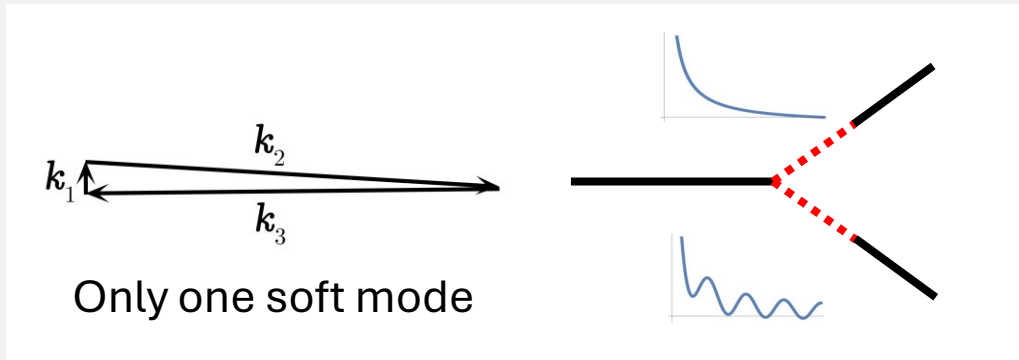
✓ Amplitude	$N$	✓ $\rho_\alpha$	$N$
✓ Wavelength	$N$	✓ $\mu_\alpha$	$N$
✓ Phase	$N$	✓ $\lambda_{\alpha\beta}$	$N(N+1)/2$

Parameters are not determined only from CC signal.  
Analytic template for arbitrary  $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$  is important!

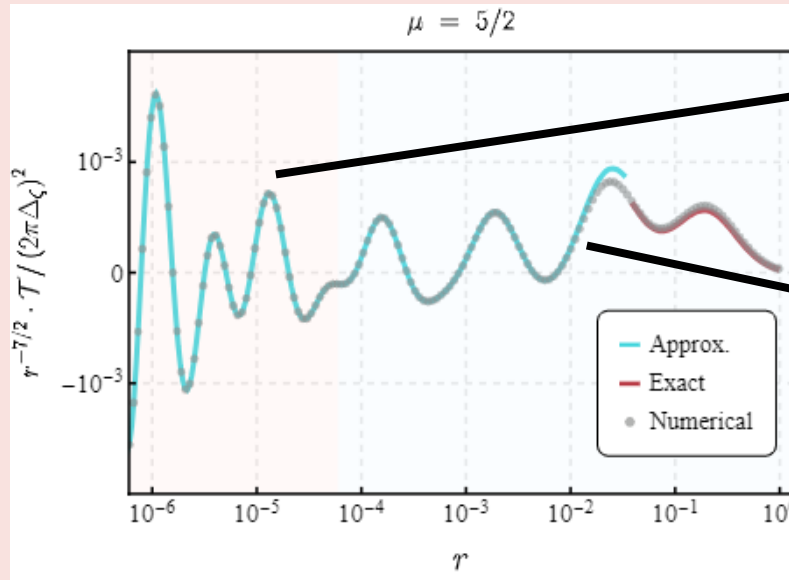
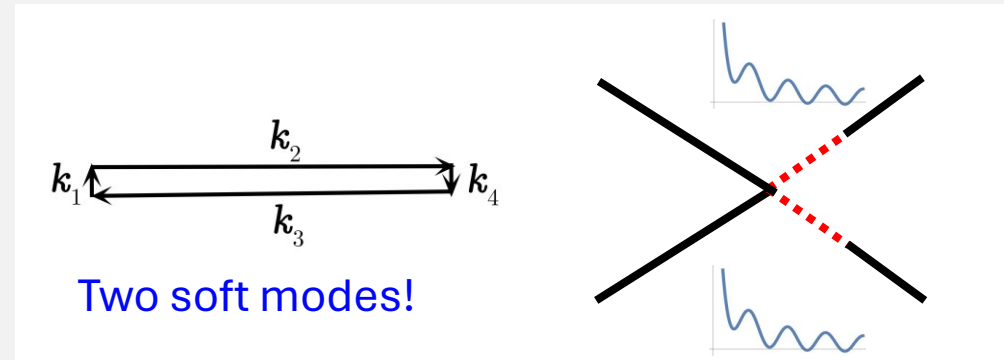


# Difference between SE and DE 3: Trispectrum

## □ Bispectrum



## □ Trispectrum



$$\mu_\alpha^{3/2} \mu_\beta^{3/2} e^{-\pi(\mu_\alpha + \mu_\beta)} \left( \frac{k_L}{k_S} \right)^{3+i(\mu_\alpha + \mu_\beta)}$$

No such signals in SE

$$\frac{\mu_\alpha^{3/2}}{\mu_\beta^2} e^{-\pi\mu_\alpha} \left( \frac{k_L}{k_S} \right)^{7/2+i\mu_\alpha}$$

$$\star \quad \text{[wavy plot]} \sim \mu^{3/2} e^{-\pi\mu} \left( \frac{k_L}{k_S} \right)^{3/2+i\mu}, \quad \text{[smooth plot]} \sim \frac{1}{\mu^2} \left( \frac{k_L}{k_S} \right)^2$$

# Summary

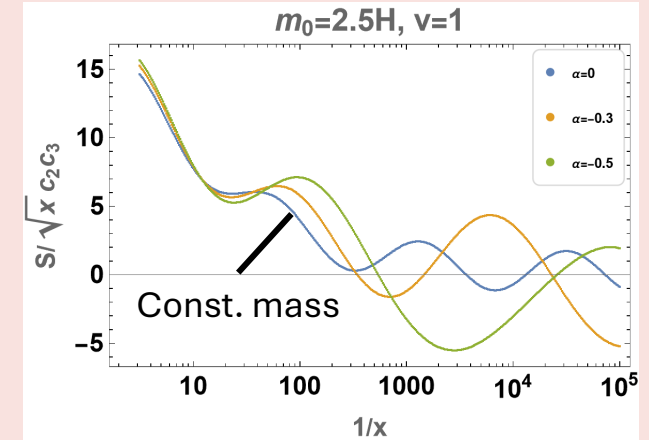
## □ Non-shift-symmetric interactions

- Large excursion of inflaton  $\Delta\phi \sim \sqrt{\epsilon} M_{\text{pl}} \Delta N$  introduces scale-dependence for non-shift sym. couplings.

$$S \sim \left(\frac{k_L}{k_S}\right)^{1/2} e^{-\pi \mu_k} \cos \left[ \mu_k \log \frac{k_L}{k_S} + \delta(\mu_k) \right] \quad \mathcal{L}_{\text{int}} = y f(\phi) \sigma^2$$

$$\Delta\mu^2 \sim \frac{y}{H} \Delta f(\phi)$$

$$\phi_0 \sigma^2 \quad \Delta\mu_{\text{NSS}}^2(k) \lesssim \sqrt{\epsilon} \frac{M_{\text{pl}}}{H} \quad \text{vs.} \quad \ddot{\phi}_0 \sigma^2 \quad \Delta\mu_{\text{SS}}^2(k) \lesssim \epsilon^{3/2} \frac{M_{\text{pl}}}{\Lambda}$$



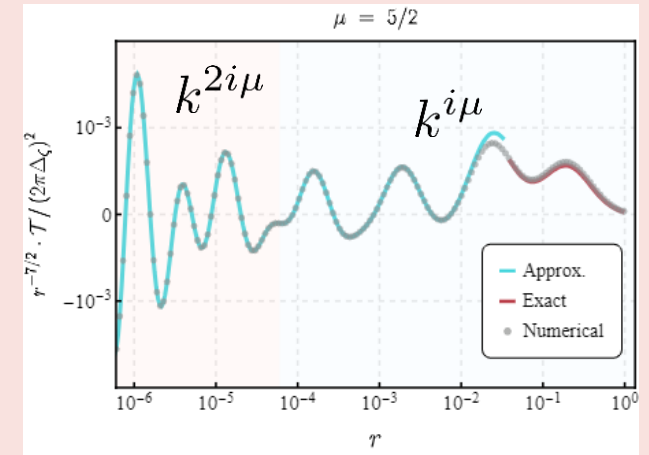
## □ Double-exchange vs. single-exchange

- Large size  $\frac{S_{\text{DE}}}{S_{\text{SE}}} \sim \lambda P_\zeta^{-1/2} \frac{1}{\mu^2} \lesssim P_\zeta^{-1/4} \frac{1}{\mu^2}$
- Trispectrum

- Coupling constants are not determined only from CC signals.

$$S_{\text{DE,CC}}^{\text{multi}} = \sum_{\alpha} \frac{\rho_{\alpha}}{H} \text{Re} \left[ \left( \frac{k_L}{k_S} \right)^{1/2+i\mu_{\alpha}} \mathcal{B}_{\mu_{\alpha}, \mu_{\beta}, \lambda_{\alpha\beta}, \rho_{\beta}} e^{i\vartheta_{\mu_{\alpha}, \mu_{\beta}, \lambda_{\alpha\beta}, \rho_{\beta}}} \right]$$

Analytic template we obtained is important!



**Back-up**

# Bispectrum in Single Field Inflation

## □ Perturbative expansion of the action

$$S_{\text{EH}} = \frac{1}{2} \int dx^4 \sqrt{-g} R \quad \text{with} \quad ds^2 = -dt^2 + e^{2\zeta} a^2(t) d\mathbf{x}^2, \quad \phi = \phi_0(t)$$

$$\mathcal{L} = \mathcal{L}_{\text{BG}} + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \dots$$

Homogeneous  
and isotropic

$\propto$  EoM of BG  
 $\longrightarrow 0$

$\mathcal{L}_2$  : EoM for the perturbations

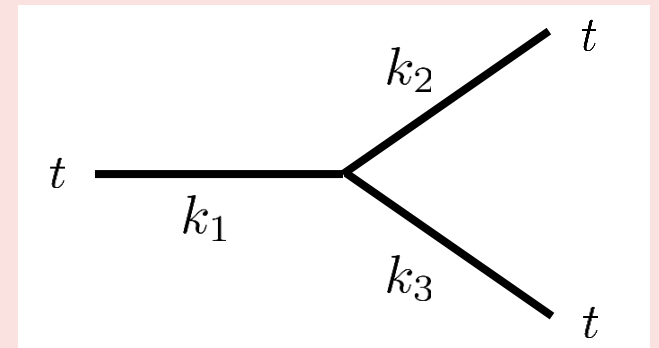
$\mathcal{L}_3$  : Interaction terms

## □ Maldacena's consistency relation in bispectrum [Maldacena '02]

$$\mathcal{L}_3^{\text{EH}} = a^3 \epsilon^2 \zeta \dot{\zeta}^2 + a \epsilon^2 \zeta (\partial \zeta)^2 - 2a \epsilon \dot{\zeta} \partial \zeta \partial \chi + \partial_t \left( -\frac{\epsilon \eta}{2} a^3 \zeta^2 \dot{\zeta} \right) + \dots \quad \text{where } \partial^2 \chi \equiv a^2 \epsilon \dot{\zeta}$$

$$\langle \zeta_1 \zeta_2 \zeta_3 \rangle_{\text{inf. end}} = (2\pi)^7 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{P_\zeta^2}{(k_1 k_2 k_3)^2} S \left( \frac{k_1}{k_3}, \frac{k_2}{k_3} \right)$$

➤ Squeezed limit  $k_{\text{L}} \equiv k_3 \ll k_1 \simeq k_2 \equiv k_{\text{S}} \quad \longrightarrow \quad S \xrightarrow{\text{sq.}} \frac{k_{\text{S}}}{4k_{\text{L}}} (1 - n_s)$



# Bispectrum in Single Field Inflation

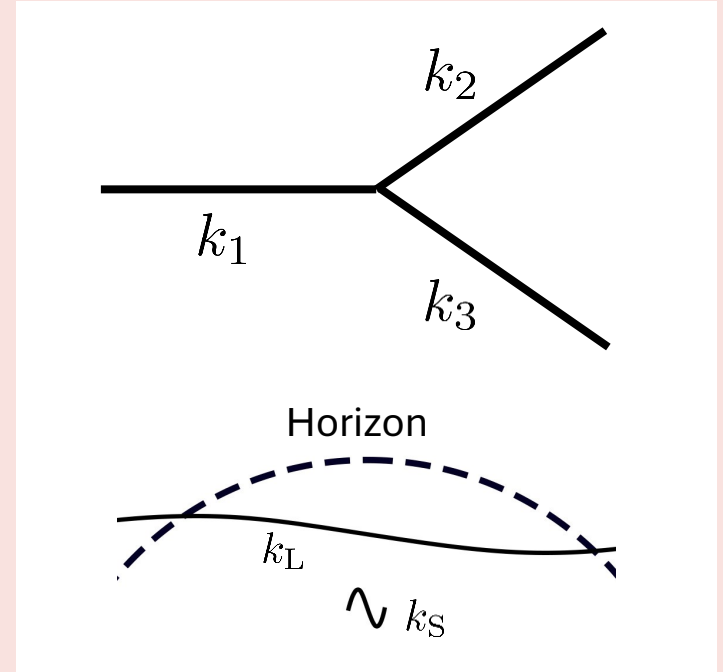
## □ Maldacena's consistency relation [Maldacena '02]

$$\mathcal{L}_3^\zeta = a^3 \epsilon^2 \zeta \dot{\zeta}^2 + a \epsilon^2 \zeta (\partial \zeta)^2 - 2a \epsilon \dot{\zeta} \partial \zeta \partial \chi + \partial_t \left( -\frac{\epsilon \eta}{2} a^3 \zeta^2 \dot{\zeta} \right) + \dots$$

➤ Squeezed limit  $k_3 \stackrel{\equiv k_L}{\ll} k_1 \simeq k_2 \stackrel{\equiv k_S}{}$  with  $\delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$

$$S \longrightarrow \frac{k_S}{4k_L} (1 - n_s) + \mathcal{O} \left( \left( \frac{k_L}{k_S} \right)^0 \right)$$

But...  $\langle \zeta(x_1) \zeta(x_2) \zeta(x_3) \rangle \sim \int \frac{d^3 k_1 d^3 k_2 d^3 k_3}{(2\pi)^9} \frac{S}{k_1^2 k_2^2 k_3^2} \delta^3(\sum k_i) \rightarrow \int_{k_L \ll k_S} \frac{dk_S dk_L}{k_S k_L} \rightarrow \infty \quad ?$



➤ Geodesic coordinate (local observer's effect) [Tanaka, Urakawa '11, Pajer et al. '13]

$$\begin{aligned} ds^2 &= -dt^2 + e^{2\zeta} a^2(t) d\mathbf{x}^2 \quad \xrightarrow{\quad} \quad \mathbf{x}_F \simeq (1 + \zeta) \mathbf{x}, \quad \zeta_F(\mathbf{x}_F) = \zeta(\mathbf{x}) \simeq \zeta(\mathbf{x}_F) - \zeta(1 + \mathbf{x} \cdot \partial_{\mathbf{x}} \zeta) \\ &= -dt^2 + a^2(t) d\mathbf{x}_F^2 + \dots \quad \text{(conformal Fermi normal coordinate)} \end{aligned}$$

# Bispectrum in Single Field Inflation

## □ Maldacena's consistency relation [Maldacena '02]

$$\mathcal{L}_3^\zeta = a^3 \epsilon^2 \zeta \dot{\zeta}^2 + a \epsilon^2 \zeta (\partial \zeta)^2 - 2a \epsilon \dot{\zeta} \partial \zeta \partial \chi + \partial_t \left( -\frac{\epsilon \eta}{2} a^3 \zeta^2 \dot{\zeta} \right) + \dots$$

➤ Squeezed limit  $k_3 \stackrel{\equiv k_L}{\ll} k_1 \simeq k_2 \stackrel{\equiv k_S}{\sim} k_S$  with  $\delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$

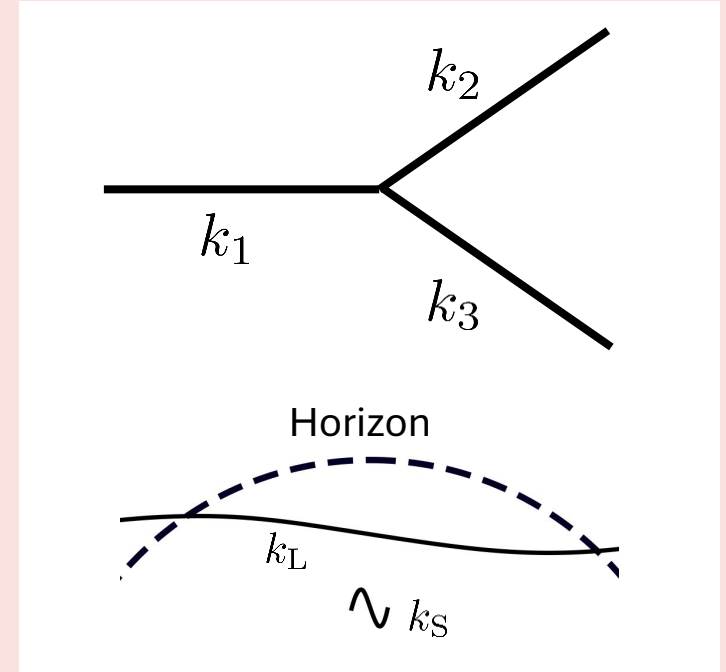
$$S_F \longrightarrow \frac{k_S}{4k_L} (1 - n_s) + \mathcal{O}\left(\left(\frac{k_L}{k_S}\right)^0\right) + \mathcal{O}\left(\frac{k_L}{k_S}\right)$$

$$\langle \zeta(x_1) \zeta(x_2) \zeta(x_3) \rangle_F \sim \int \frac{d^3 k_1 d^3 k_2 d^3 k_3}{(2\pi)^9} \frac{S_F}{k_1^2 k_2^2 k_3^2} \delta^3(\sum k_i) \rightarrow \int_{k_L \ll k_S} \frac{dk_S dk_L}{k_S k_L} \rightarrow \infty$$

$$\rightarrow \int_{k_L \ll k_S} dk_S dk_L \frac{k_L}{k_S^3} : \text{finite}$$

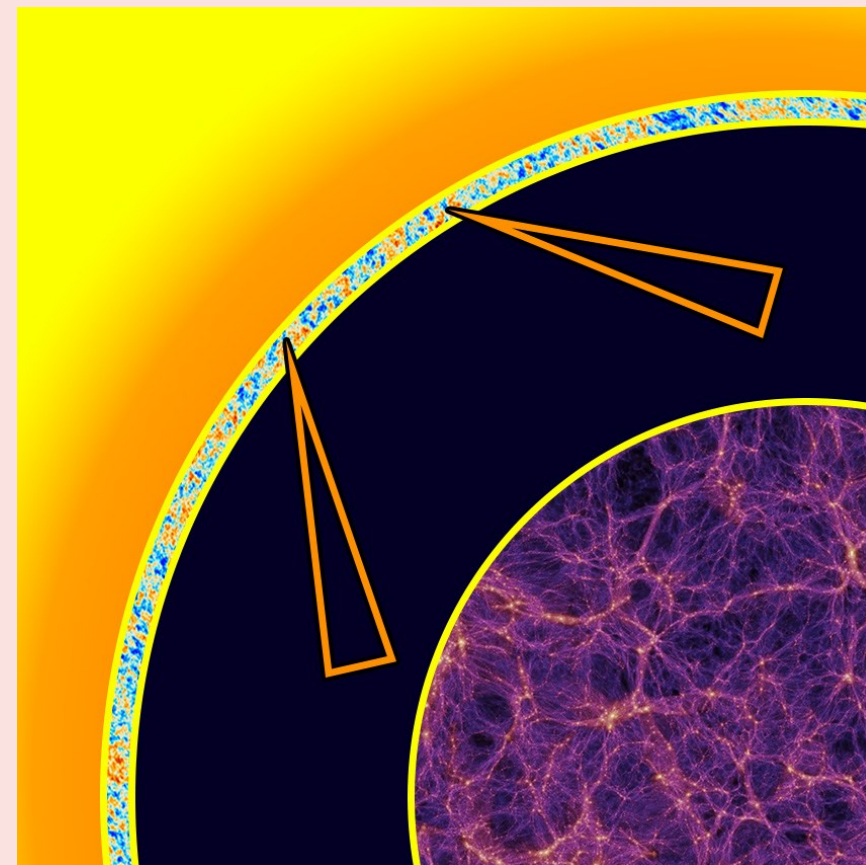
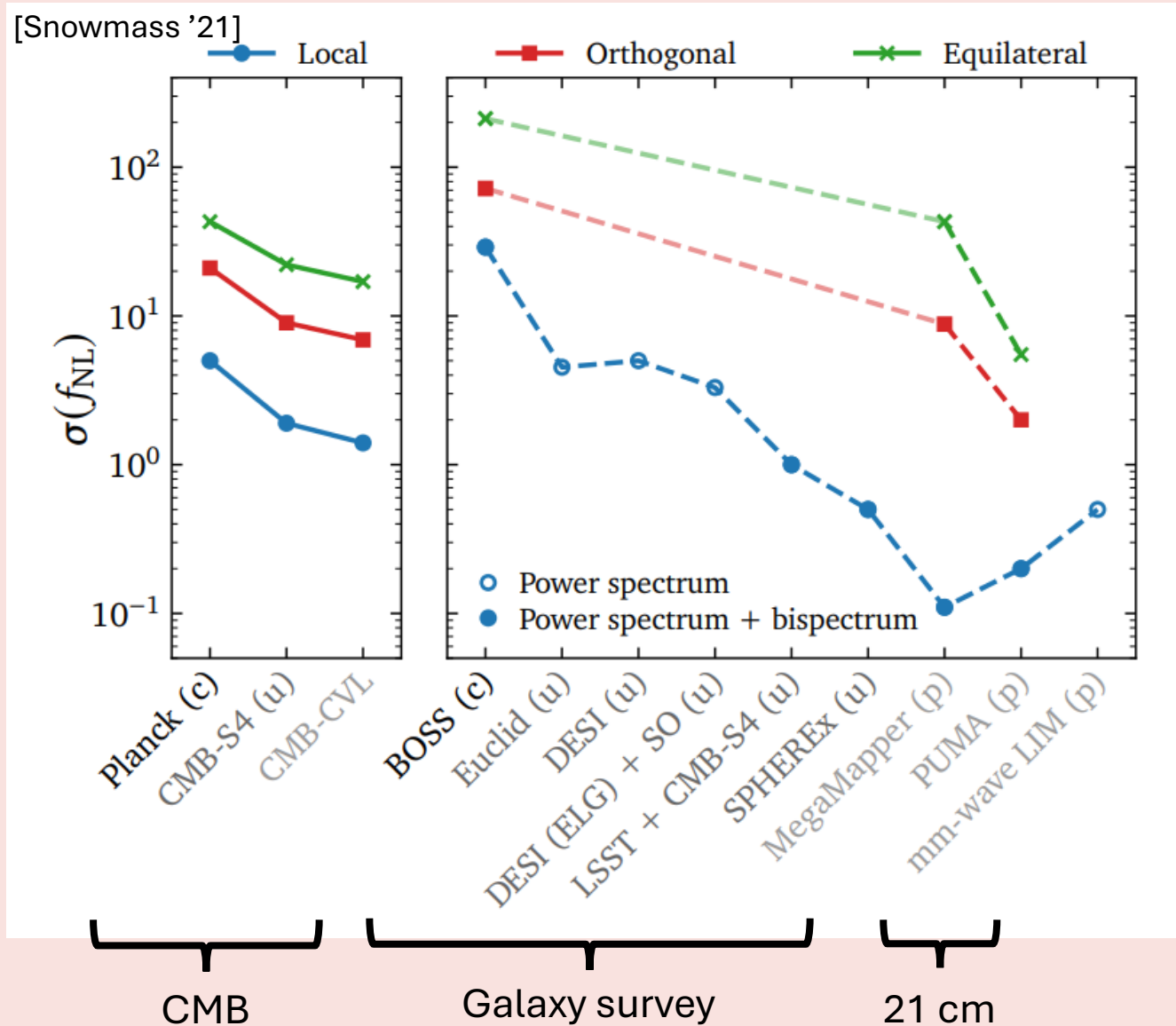
➤ Geodesic coordinate (local observer's effect) [Tanaka, Urakawa '11, Pajer et al. '13]

$$\begin{aligned} ds^2 &= -dt^2 + e^{2\zeta} a^2(t) d\mathbf{x}^2 \\ &= -dt^2 + a^2(t) d\mathbf{x}_F^2 + \dots \end{aligned} \quad \begin{array}{l} \curvearrowright \mathbf{x}_F \simeq (1 + \zeta) \mathbf{x}, \quad \zeta_F(\mathbf{x}_F) = \zeta(\mathbf{x}) \simeq \zeta(\mathbf{x}_F) - \zeta(1 + \mathbf{x} \cdot \partial_{\mathbf{x}} \zeta) \\ \text{(conformal Fermi normal coordinate)} \end{array}$$



# Future Observations

(c): completed  
(u): upcoming  
(p): projected



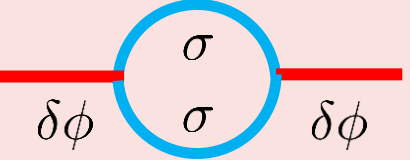
CMB Dark age Galaxies  
21cm-21cm-CMB cross-correlation

$$f_{\text{NL}}^{\text{local}} \sim 6 \times 10^{-3} \quad [\text{Orlando et al. '23}]$$



# Naturalness Conditions from Inflaton Mass for NSS Ints.

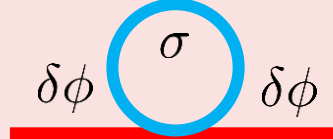
□  $yH\phi\sigma^2$



$$\sim y^2 H^2 \lesssim \mathcal{O}(\eta, \epsilon) H^2 \longrightarrow y \lesssim \sqrt{\epsilon} \longrightarrow \Delta m_\sigma^2(k) \lesssim \epsilon H M_{\text{pl}} \log \frac{k}{k_i}$$

□  $\lambda\phi^2\sigma^2$

$$\Delta m_{\phi_0}^2 \sim \lambda \langle \sigma^2 \rangle \sim \lambda H^2 \lesssim \mathcal{O}(\eta, \epsilon) H^2 \longrightarrow \lambda \lesssim \epsilon$$



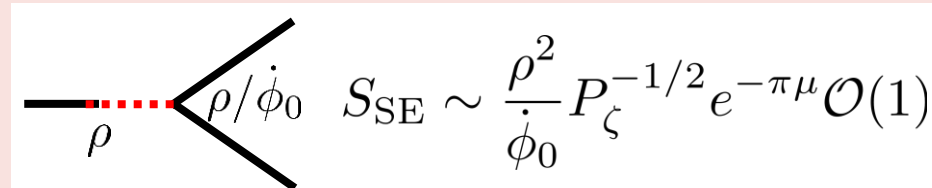
$$\sim \lambda \Lambda^2 \lesssim \mathcal{O}(\eta, \epsilon) H^2 \longrightarrow \lambda \lesssim \epsilon \frac{H^2}{\Lambda^2}$$

$$\Delta m_\sigma^2(k) \lesssim \epsilon^2 H M_{\text{pl}} \frac{H M_{\text{pl}}}{\Lambda^2} \log^2 \frac{k}{k_i}$$

\* Shift sym. Couplings:  $\frac{|\partial_t^2 \phi|}{\Lambda} \sim \epsilon^{3/2} H M_{\text{pl}} \frac{H}{\Lambda} \log \frac{k}{k_i}, \quad \frac{|\partial_t \phi_0|^2}{\Lambda^2} \sim \epsilon H M_{\text{pl}} \frac{H M_{\text{pl}}}{\Lambda^2} \log^2 \frac{k}{k_i}$

# Size Estimation of Single-exchange Diagrams

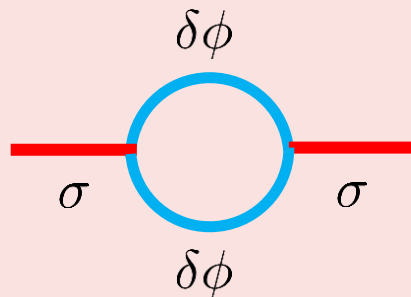
$$\frac{\rho}{\dot{\phi}_0} (\partial_\mu \phi)^2 \sigma \longrightarrow \rho \delta\phi' \sigma + \frac{\rho}{\dot{\phi}_0} (\delta\phi')^2 \sigma$$



$$S_{\text{SE}} \sim \frac{\rho^2}{\dot{\phi}_0} P_\zeta^{-1/2} e^{-\pi\mu} \mathcal{O}(1)$$

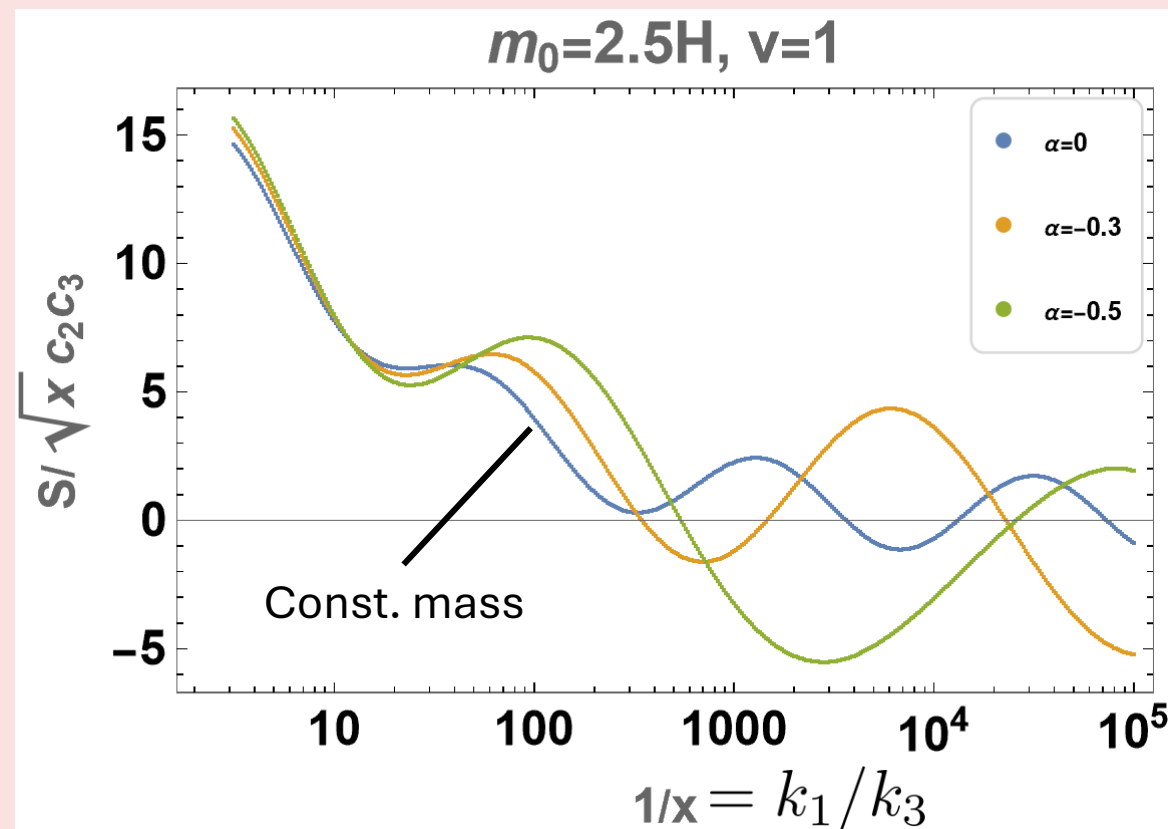
$$\left. \begin{array}{l} \dot{\phi}_0 \sim H^2 P_\zeta^{-1/2} \\ \rho \equiv \alpha H \end{array} \right\} \frac{\rho^2}{\dot{\phi}_0} \sim \alpha^2 P_\zeta^{1/2}$$

Naturalness  $\alpha \lesssim 1$   
[Pinol, Renaux-Petel, Werth '23]



➔  $S_{\text{SE}} \lesssim e^{-\pi\mu} \times \mathcal{O}(1)$

$$m_\sigma = 2.5H \rightarrow e^{-\pi\mu} \sim 10^{-3}$$



# Observational Signals in Bispectrum

Consistency check: CosmoFlow  
[Pinol, Renaux-Petel, Werth '23, '24]

❑ Squeezed limit  $k_3 \ll k_1 \simeq k_2$

$$\langle \delta\phi_{k_1} \delta\phi_{k_2} \delta\phi_{k_3} \rangle' \xrightarrow{k_3 \rightarrow 0} \sum_{\alpha, \beta} \frac{\rho_\alpha \rho_\beta \lambda_{\alpha\beta} H}{(k_1 k_2 k_3)^2} \cdot \text{Re} \left\{ \left[ i \frac{\pi^{3/2}}{2^{4+2i\mu_\alpha}} \text{sech}(\pi\mu_\beta) [1 + \tanh(\pi\mu_\alpha)] \times \Gamma \left[ \begin{matrix} -i\mu_\alpha \\ -1 - i\mu_\alpha + i\mu_\beta, -1 - i\mu_\alpha - i\mu_\beta \end{matrix} \right] \right. \right. \\ \left. \left. \times {}_3F_2 \left[ \begin{matrix} -\frac{3}{2} - i\mu_\alpha, -1 - i\mu_\alpha - i\mu_\beta, -1 - i\mu_\alpha + i\mu_\beta \\ -\frac{1}{2} - i\mu_\alpha, -\frac{1}{2} - i\mu_\alpha \end{matrix} \middle| 1 \right] + \mathcal{O}(e^{-2\pi\mu_\alpha}, e^{-2\pi\mu_\beta}) \right] \left( \frac{k_1}{k_3} \right)^{\frac{1}{2} + i\mu_\alpha} + \mathcal{O}\left( \frac{k_1}{k_3} \right) \right\}$$

❑ Size in equilateral limit  $k_1 = k_2 = k_3 = k$

