Holographic Entanglement Entropy in the FLRW Universe

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Contribute talk
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Based on
2504.10457 with Toshifumi Noumi (UTokyo), Yu-ki Suzuki (YITP)

Friedmann-Lemaitre-Robertson-Walker spacetime

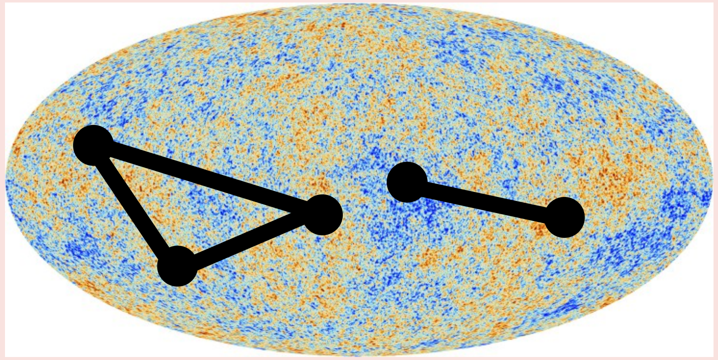
- ❑ **Cosmological principle: the universe is isotropic and homogeneous**

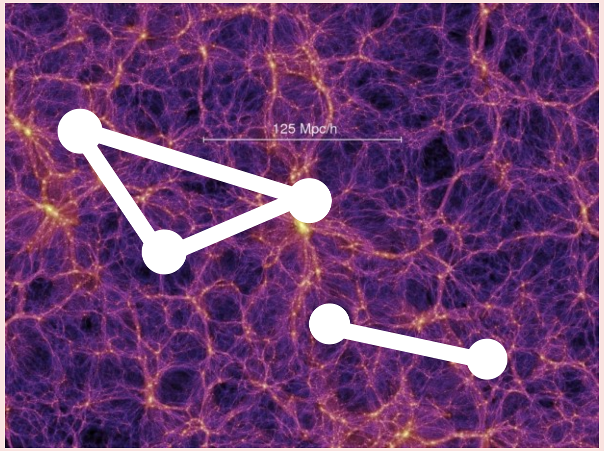
$$ds^2 = a^2(\eta)\left(-d\eta^2 + \frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \quad \begin{cases} k>0 & : \text{Sphere} \\ k=0 & : \text{Flat} \\ k<0 & : \text{Hyperbolic} \end{cases}$$

pressure energy density

Matter fields: perfect fluid $T^\mu_\nu = (\rho + p)u^\mu u_\nu + p\delta^\mu_\nu \underset{\text{rest}}{=} \mathrm{diag}(-\rho, p, p, p)$ with equation of state $p = w\rho$

- ❑ **Anisotropy and inhomogeneity**





There exist deviation from FLRW metric

➡ We observe correlations of the fluctuations

Cosmic inflation and current observations

- **A natural initial condition of the universe: cosmic inflation**

$$ds^2 \simeq \frac{1}{H^2\eta^2}(-d\eta^2 + e^{2\zeta}d\mathbf{x}^2)$$: quasi-de Sitter spacetime with curvature perturbations

- **Background evolution**: quasi-de Sitter prepares flat and causally connected initial condition for the universe.
- **Cosmo. perturbation**: sourced by vacuum fluctuation and gravitational particle production of ζ.
  - ✓ Correlation functions $\langle 0|\zeta^n(\eta_{\rm e})|0\rangle$ at inflation end are transported to observables.

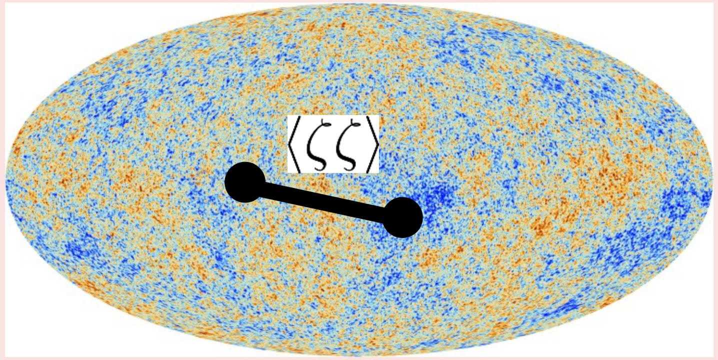
- **Current observations: power spectrum of curvature perturbations**

$$\langle \zeta_{\mathbf{k}}\zeta_{\mathbf{k}'}(\eta_{\rm e})\rangle = (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}P_\zeta \qquad P_\zeta \simeq \frac{H^2}{8\pi^2\epsilon}\left(\frac{k}{k_*}\right)^{n_s-1}$$

$$n_s \simeq 0.965, \qquad \frac{dn_s}{d\log k} \simeq 0.002, \ldots$$

[Planck '18]

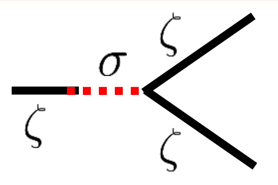
Inflation is consistent to observation.



Future observations and cosmological bootstrap

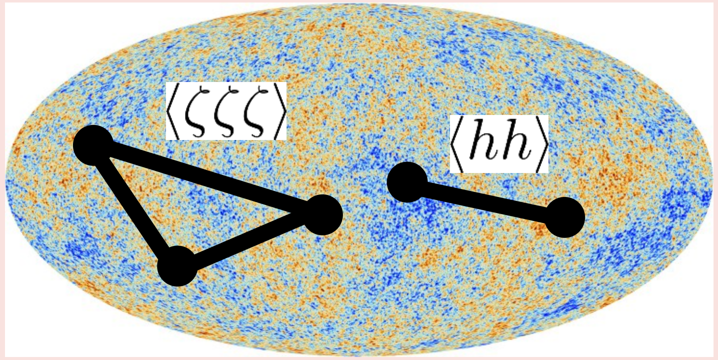
- **Next observations:** tensor power spectrum and precise observation of scalar bispectrum

- **Bispectrum:** "collider" signal of heavy particles
  [Chen and Wang 0911.3380, Noumi et al. 1211.1624, Arkani-Hamed and Maldacena 1503.08043]



$$\propto \cos\left(\frac{m_\sigma}{H}\log\frac{k_{\rm long}}{k_{\rm short}}\right)$$

Mass of heavy particles can be determined.



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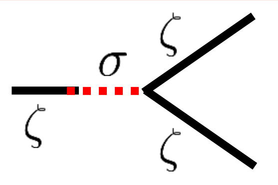

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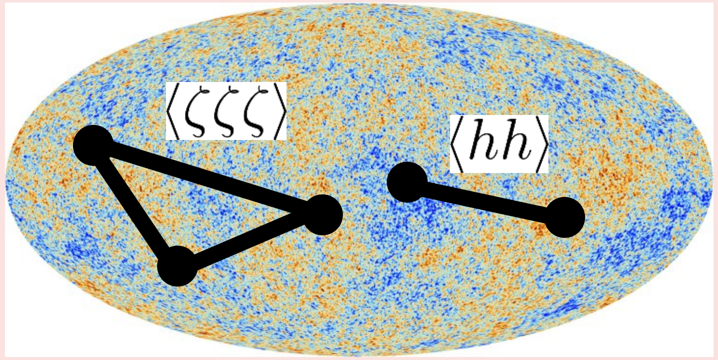

- **Calculation is hard.** ➡ **bootstrap approach**

  - de Sitter symmetry $SO(1,4)$ for "conformal" bootstrap [Baumann, Lee, Pimentel et al. 1811.00024, 1910.14051, 2005.04234]

  - Unitarity and (non-)analyticity: "collider" signal is on branch cut.

    - ✓ Cosmological cutting rules, pole information, and bootstrapping full signal from them
      [Goodhew et al. 2009.02898, Melville and Pajer 2103.09832, Baumann et al. 2106.05294, Liu et al. 2407.12299, etc.]
    - ✓ Perturbative unitarity bounds [Kim et al. 2102.04101, Pueyo et al. 2410.23709 etc.]

Cosmology meets holography

- **dS/CFT and cosmological bootstrap**
  - $\Psi_{\mathrm{dS}} \sim Z_{\mathrm{ECFT}}$ on the future boundary [Maldacena astro-ph/0210603]
    - ✓ How is unitarity of Euclidean CFT on the boundary? How to generalize it to quasi-de Sitter?
  - Constraints on dS amplitudes from holography? [AdS/CFT: Hartman et al. 1509.00014, Caron-Huot et al. 2106.10274, etc.]

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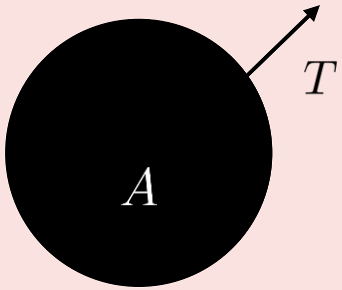
- **Topics in the late time universe**
  - GW and amplitude methods in Minkowski background ⬅ Gravitational EFT, amplitudes for BH merger, …
    [Tokuda et al. 2007.15009, Bern et al. 1901.04424, etc.]
  - Dark matter, dark energy, structure formation, … ⬅ EFT, entropy growth of the universe, …
    [Fan et al. 1008.1591, Gubitoni et al. 1210.0201, Hawking '85, etc.]

  ⟹ (How) is holography useful for constraining/UV completing/calculating these phenomena in cosmology?

Holographic principle and entropy

['t Hooft gr-qc/9310026, Susskind hep-th/9409089]

❑ Black hole entropy (Schwarzschild)



$$T = \frac{1}{8\pi M}$$

- Bekenstein-Hawking entropy $S_{\mathrm{BH}} = \dfrac{A}{4G}$
- First law: $dM = TdS$, Second law: $dS_{\mathrm{BH}} + dS_{\mathrm{rad.}} \geq 0$

➡ Event horizon have all information of internal region of BH.

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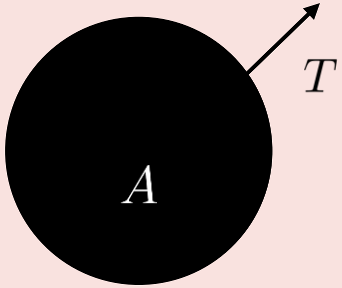

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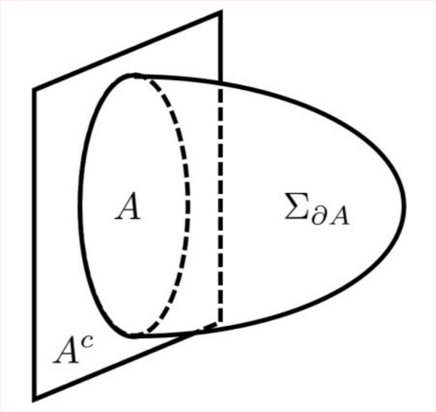
- **AdS/CFT** [Maldacena hep-th/9711200]



$$ds^2 = \frac{1}{z^2}(-dt^2 + dz^2 + d\mathbf{x}^2), \quad -\infty < z < 0$$

- CFT on the boundary $z = 0$ reproduce bulk phenomena ($Z_{\mathrm{CFT}} = Z_{\mathrm{AdS}}$).
- Ryu-Takayanagi entropy $S = \mathrm{Tr}[\rho_A \ln \rho_A] = \dfrac{\mathrm{Area}(\Sigma_{\partial A})}{4G}$

[Ryu and Takayanagi hep-th/0603001, hep-th/0605073]

Guessing holographic dual for a given bulk theory

(Complementary to bulk reconstruction [e.g., Hamilton et al. hep-th/0506118])

❑ RT prescription

➢ Derivation using Euclidean gravitational path integral [Lewkowycz and Maldacena 1304.4926]

$$Z_{\mathrm{bd}} = Z_{\mathrm{grav}} \quad \longrightarrow \quad S = \frac{\partial}{\partial n} \ln Z(n) \Big|_{n=1} ,$$

$Z(n)$: generating functional for n-copied theory (replica trick)

$$\Rightarrow \frac{\partial}{\partial n} \ln Z_{\mathrm{grav}}(n) \Big|_{n=1} = \frac{1}{4G} \int_{\Sigma} \sqrt{g}$$

Putting Ryu-Takayanagi formula as a conjecture of holography

➡ How to guess dual boundary theory?

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➡ How to guess dual boundary theory?

❑ Subadditivity $S_{AB} \leq S_A + S_B$

➢ Concavity of entropy regarding subsystem. E.g., $\frac{\partial^2 S}{\partial x^2} \leq 0$ for 1-dim subsystem $(\overset{AB}{x} \to \overset{A}{\lambda x} + \overset{B}{(1-\lambda)x})$

➢ Quantum mechanical derivation: unitarity is assumed.

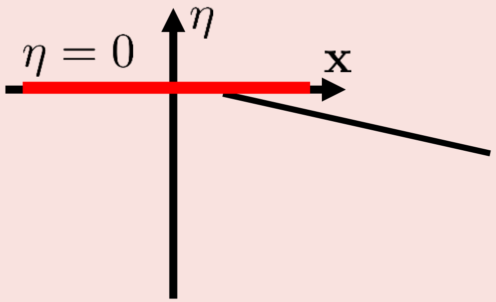
➢ Super-area law of RT entropy (SA violation) imply non-local dual QFT.
[Li and Takayanagi 1010.3700]

} Constraining boundary theory from RT prescription

de Sitter holographies (in 2+1-dim.)

❑ Original dS/CFT at future horizon

[Strominger hep-th/0106113]



Euclidean CFT
on spacelike boundary

- Bulk and boundary do not share time direction.

- Ryu-Takayanagi prescription gives imaginary entropy,

$$S = \frac{iL}{2G} \ln\left(\frac{l}{\epsilon}\right)$$ for dS_3/CFT_2

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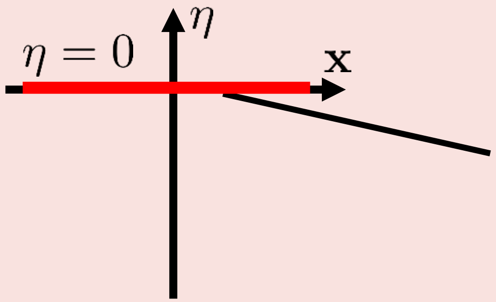

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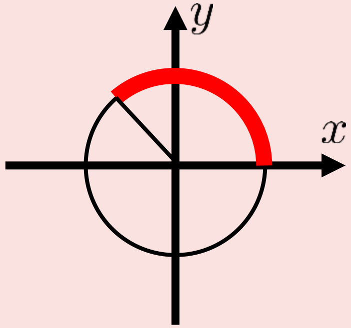
❑ Timelike boundary: time directions are shared.

- Static patch holography [Susskind 2106.03964]

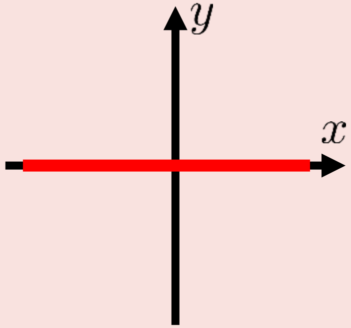


Boundary: event horizon $ar = \frac{1}{H}$

Subsystem: $0 < \phi < \phi_*, \ \eta = \eta_*$

RT entropy: area law $S = \frac{\phi_*}{4G}$

- Half holography [Kawamoto et al. 2306.07575]



Boundary: $y = \mathrm{Const.}$

Subsystem: $-x_* < x < x_*, \ \eta = \eta_*$

RT entropy: $S = \frac{\arccos(1 - 2x_*^2)}{4G}$ (convex)

Extension to FLRW holography

[Nomura et al. 1607.02508, 1611.02702, Franken et al. 2310.20652]

❑ Introducing matter fields

$$a(\eta) = L(w\eta)^{1/w} \quad \text{with } \; p = w\rho.$$

$\equiv 1$

($w = -1$ is de Sitter)

$$\begin{cases} \text{Accelerating} \;\; \ddot{a} > 0: & w < 0, \;\; -\infty < \eta < 0 \\ \text{Decelerating} \;\; \ddot{a} < 0: & w > 0, \;\; 0 < \eta < \infty \end{cases}$$

❑ Weak energy condition

$$T_{\mu\nu} p^\mu p^\nu \geq 0, \quad p^\mu \; : \; \text{timelike} \quad \Rightarrow \quad (1+w)\rho > 0 \; : \; \text{positivity of energy}$$

(dS saturates the ineq.: vacuum solution)

❑ Event horizon and apparent horizon

- Apparent horizon: a surface not feeling expansion $\partial_\mu(ar)\partial^\mu(ar) = 0 \Rightarrow \dot{a}r = 1 \Rightarrow r_{\mathrm{AH}} = w\eta$
- Event horizon: limitation in propagation of light $r_{\mathrm{EH}} = \displaystyle\int_\eta^0 d\eta = -\eta$

  (no EH for decelerating universe)

Identical in dS

$r = \mathrm{Const.}$

We consider half holography and horizon-type holography in 2+1 dim. FLRW spacetime.

Simplification in matter entropy

- When are matters negligible?

$$S_{\mathrm{RT}} + S_m \sim \frac{\mathcal{A}}{4G} + s_m \mathcal{V} + \mathcal{O}(G^0)$$

Entropy of background matters is, in general, leading effect.

- Single component
  - ✓ Perfect fluid
    $$\begin{cases} s_m = \dfrac{\rho + p}{T} \sim \dfrac{\rho}{T} \\ \text{Einstein eq.} \quad \rho \sim \dfrac{H^2}{G} \end{cases}$$
  - ✓ d+1-dim. $\mathcal{A} \sim L^{d-1}, \mathcal{V} \sim L^d$

$S_m \ll S_{\mathrm{RT}}$ ➡ $L \ll \dfrac{1}{G s_m} \sim \dfrac{T}{H^2}$

Horizon scale subsystem $L \sim \dfrac{1}{H}$

$T \gg H$

We assume
the (natural) condition

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  - ✓ d+1-dim. $\mathcal{A} \sim L^{d-1}, \mathcal{V} \sim L^d$

$S_m \ll S_{\rm RT} \Rightarrow L \ll \dfrac{1}{G s_m} \sim \dfrac{T}{H^2}$

Horizon scale subsystem $L \sim \dfrac{1}{H}$

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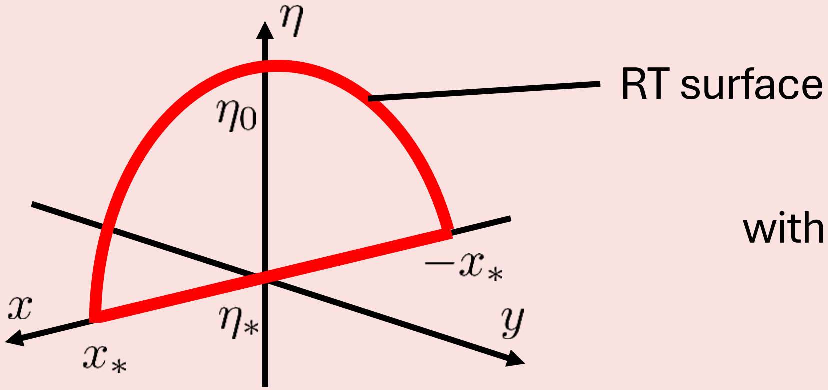
We assume the (natural) condition

- **Outlook: multiple components, perturbations, and realistic entropic growth of the universe.**
  - Dominant components in entropy and energy can be different. E.g., radiation vs. non-relativistic matter.
  - Contributions from structure formations (e.g., dark matter halo) and BH entropy?
    [Egan and Lineweaver 0909.3983, Profumo et al. 2412.11282]

Half FLRW holography

❑ RT prescription for linear subsystem



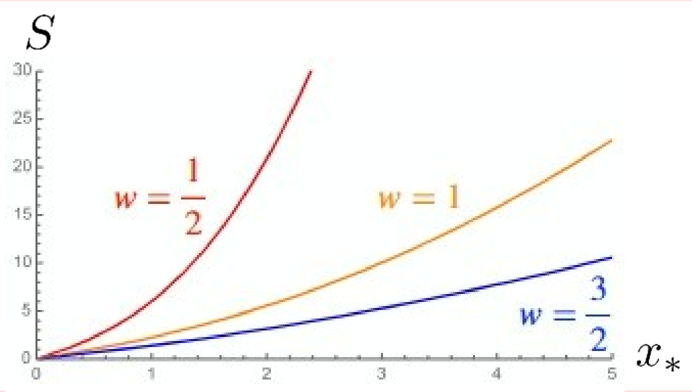
$$4GS = 2\int_0^{x_*} dx\ a(\eta(x))\sqrt{1-\eta'(x)} = 2\int_{\eta_*}^{\eta_0(x_*)} \frac{a^2(\tilde{\eta})d\tilde{\eta}}{\sqrt{a^2(\eta_0)-a^2(\tilde{\eta})}}$$

with $$x(\eta,\eta_0) = \pm\int_{\eta}^{\eta_0} \frac{a(\eta_0)d\tilde{\eta}}{\sqrt{a^2(\eta_0)-a^2(\tilde{\eta})}}$$

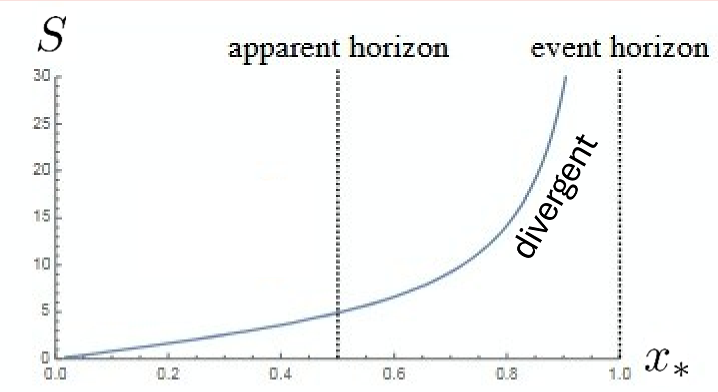
Parametric plot $(x_*(\eta_0), S(\eta_0))$

➢ Subadditivity is violated for any w and any subsystem size (non-standard dual).

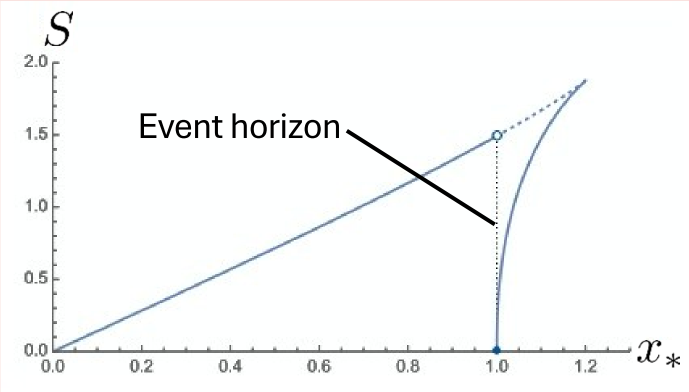
$w > 0$



$-1 < w < 0$



$w < -1$

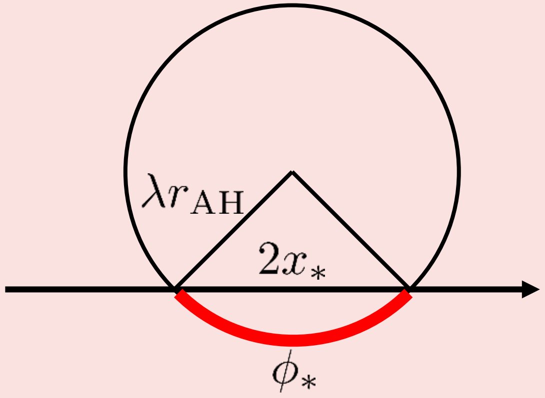


* Detailed discussions for behaviors of the entropy is postponed to horizon-type holography.

Horizon-type holography: intro. and pre-summary

- **Parametrization of constant-radius subsystem**

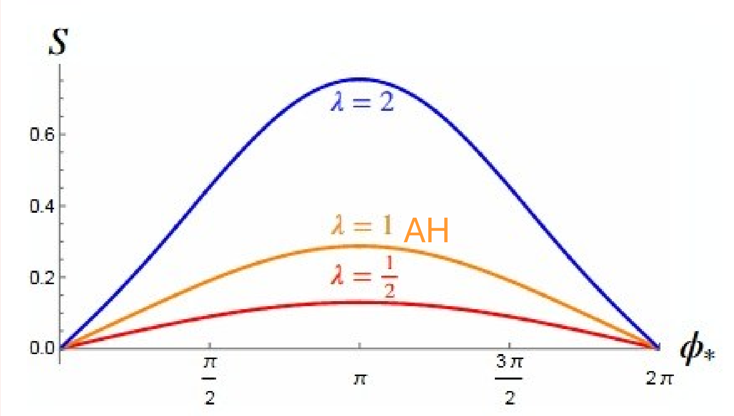
  - $r_* = \lambda r_{\mathrm{AH}}$ where $r_{\mathrm{AH}} = w\eta_*$ is apparent horizon. $\begin{cases} \lambda = 1 & : \text{AH} \\ \lambda = 1/|w| & : \text{EH if exist} \end{cases}$



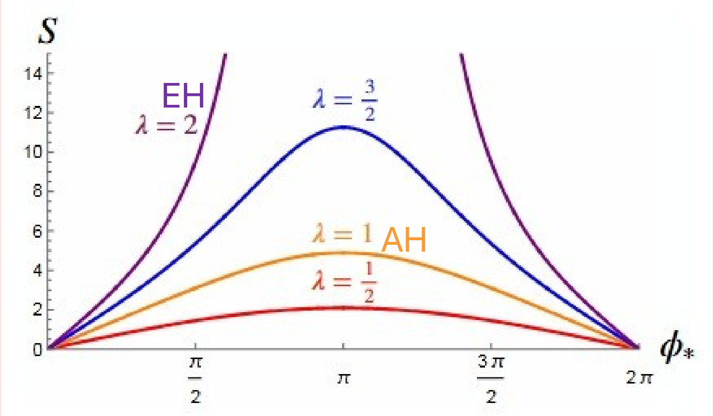
- **RT prescription for horizon-type holography:** $x_* \to \lambda r_{\mathrm{H}} \sin\dfrac{\phi_*}{2}$

  - $\begin{cases} \text{Small scale subadditivity} \\ \text{Large scale subadditivity} \end{cases}$ ⬅ inside AH (defined from local quantity) / ⬅ inside EH (defined from global structure)

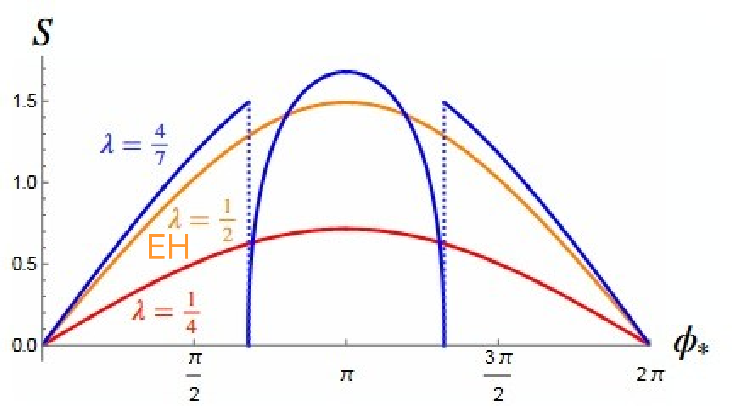
$w > 0$ (no EH)



$w = -1/2$ (EH>AH)



$w = -2$ (AH>EH)



Horizon-type 1: $w > 0$

- **No event horizon. Apparent horizon is at $r_* = w\eta_*$.**
  (parametrization: $r_* = \lambda r_{\mathrm{AH}}$)

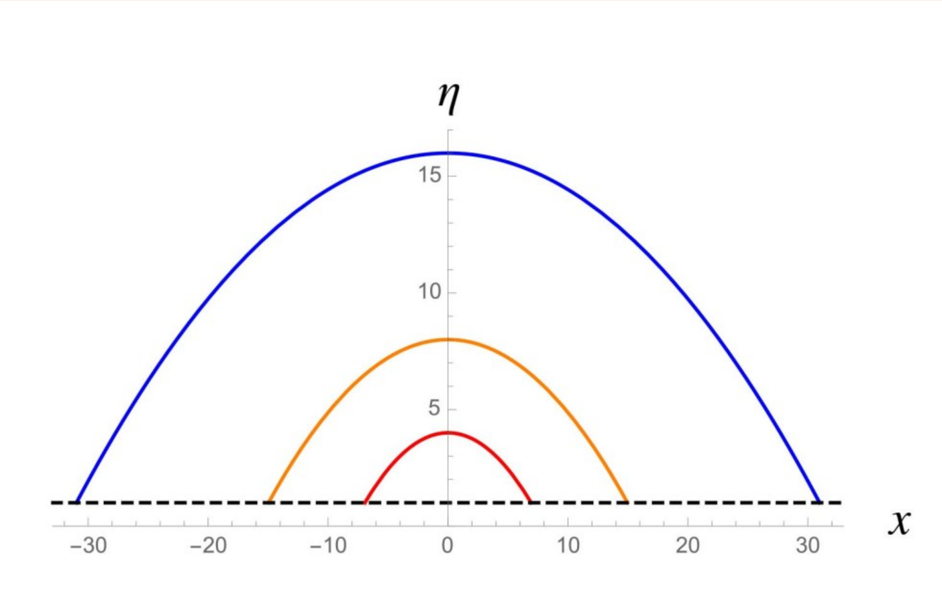
- **Subadditivity**
  - $\lambda < 1$: SA is satisfied.
  - $\lambda > 1$: SA is violated in small subsystems.

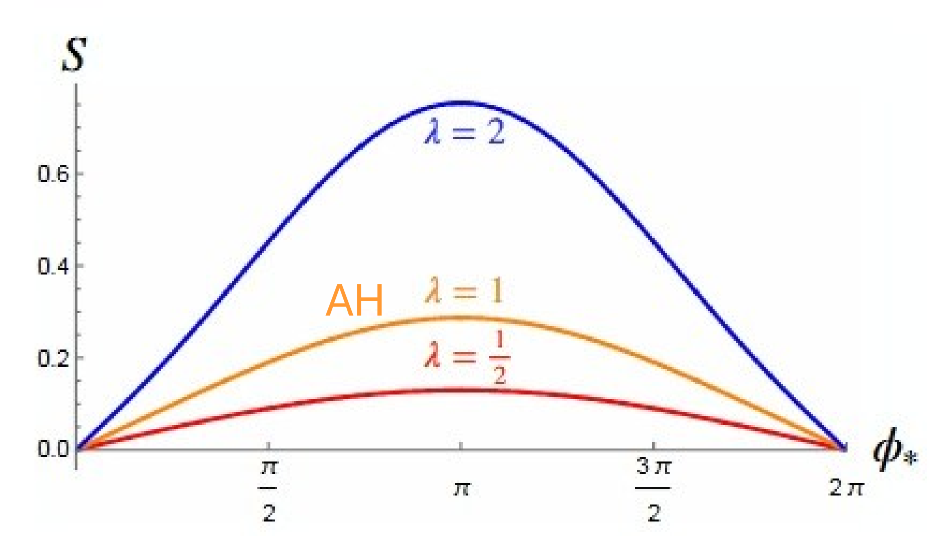
  ➡ AH (locally defined) characterizes small subsystems.

- **Small-subsystem expansion**

$$S \simeq \frac{a(\eta_*)\lambda r_{\mathrm{AH}}}{4G}\left(\phi_* - \frac{1}{24}(1-\lambda^2)\phi_*^3\right)$$

Sign of $\dfrac{\partial^2 S}{\partial \phi_*^2}$ changes at $\lambda = 1$.





Horizon-type 2: $-1 < w < 0$

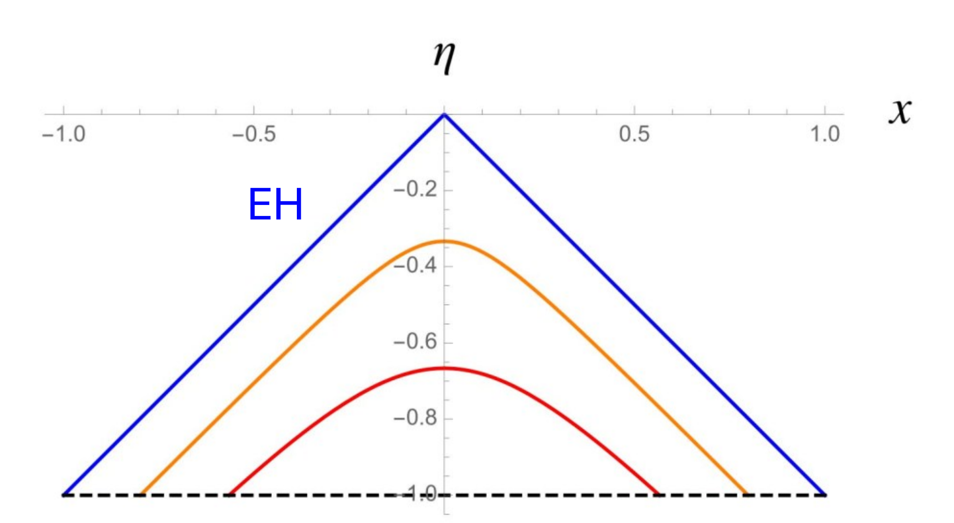
- Apparent horizon $r_* = w\eta_*$ is inside event horizon $r_* = -\eta_*$ $(r_* = \lambda r_{\mathrm{AH}})$.

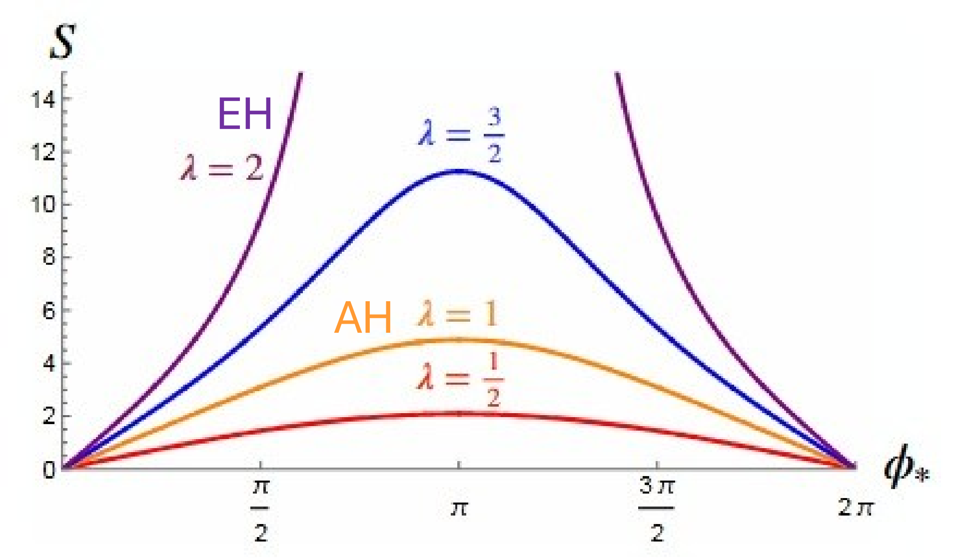
- Subadditivity

  - $\lambda < 1$: SA is satisfied.
  - $1 < \lambda < -1/w$: SA is violated in small subsystems.
  - $\lambda > -1/w$: Entropy diverges when it reaches $\eta = 0$ and becomes complex for larger systems.

  ➡ No geodesic exists beyond event horizon.

  ✓ EH (defined from global structure) characterizes large scale subsystems.

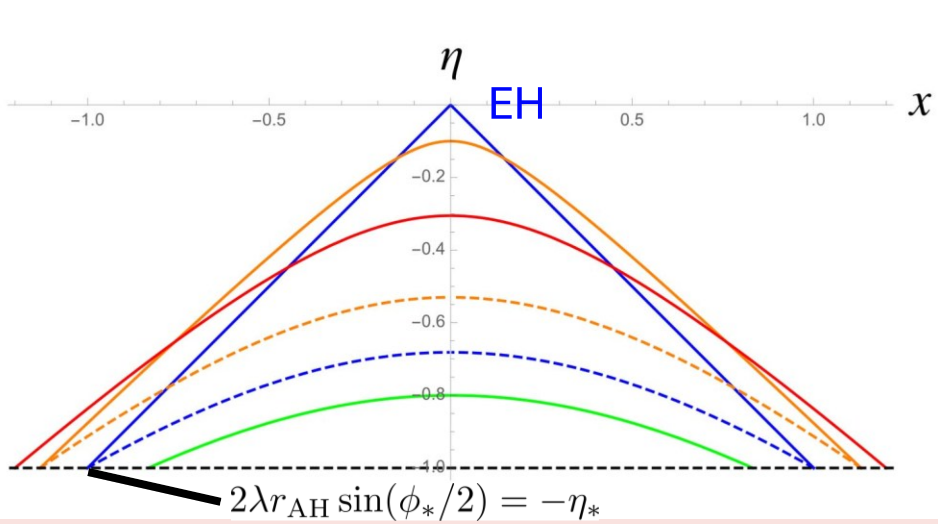


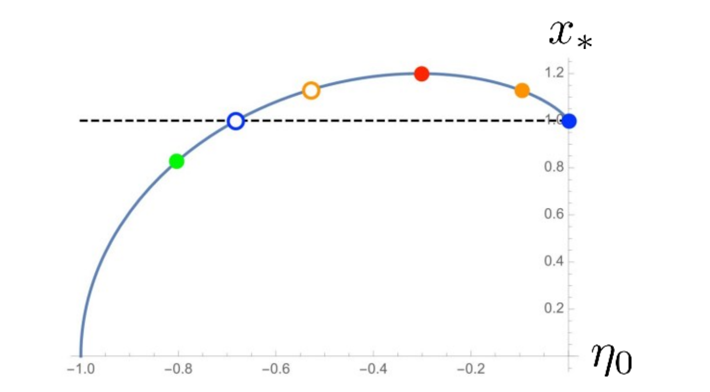


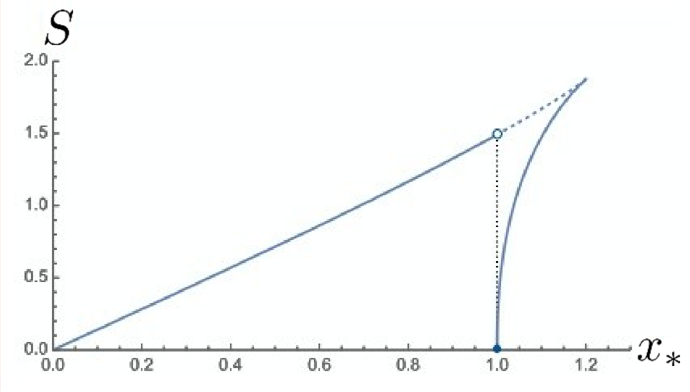
Horizon-type 3: $w < -1$

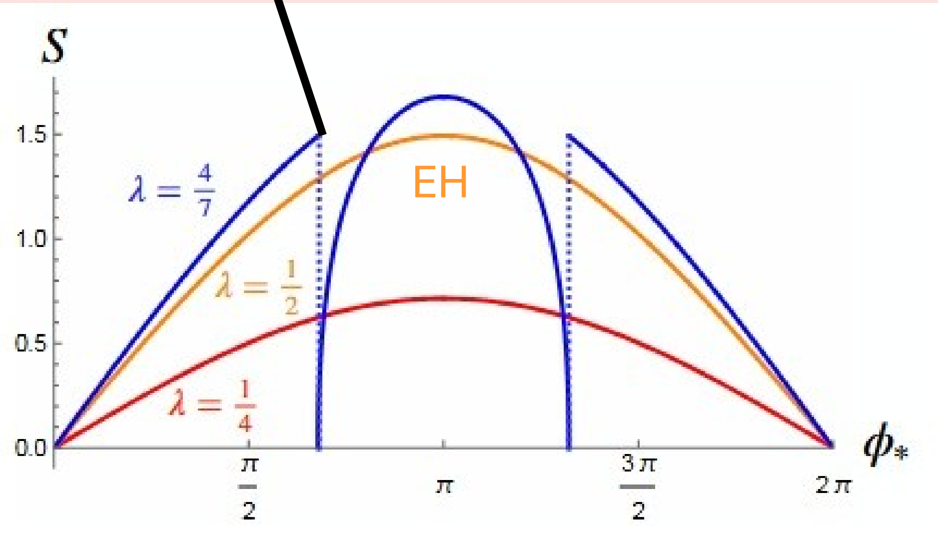
- **Apparent horizon $r_* = w\eta_*$ is outside event horizon $r_* = -\eta_*$.** $(r_* = \lambda r_{\mathrm{AH}})$

- **Subadditivity**
  - $\lambda < -1/w$: SA is satisfied.
  - $-1/w < \lambda < 1$: SA is violated due to the discrete transition.
    - ✓ There exist two solutions outside event horizon.
  - $\lambda > 1$: Entropy becomes complex at future horizon.









Implication (or outlook): weak energy condition and horizon

[see, e.g., Allais and Tonni 1110.1607 for other spacetime]

❑ Apparent horizon

- Small subsystem expansion $S \simeq \frac{a(\eta_*)\lambda r_{\mathrm{AH}}}{4G}\left(\phi_* - \frac{1}{24}(1-\lambda^2)\phi_*^3\right)$
- Apparent horizon outside event horizon ➡ some superluminal features in small scale?

❑ Weak energy condition

$$T_{\mu\nu}p^\mu p^\nu \geq 0, \quad p^\mu : \ \text{timelike}$$ ➡ $(1+w)\rho \geq 0$: positivity of energy

- WEC is also defined at each point of spacetime.
- Violation of WEC can be associated with superluminal propagation [e.g., Dubovsky et al. hep-th/0512260].

➡ Convex entropy (e.g., non-locality on bdy.)
WEC violation in bulk theory
AH outside EH in bulk theory

may be related to each other through superluminality.

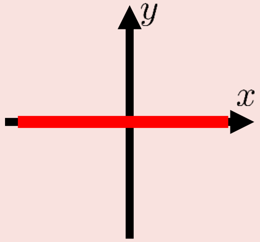
Summary

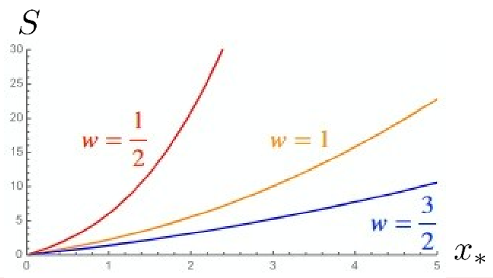
- **Q. What if there exist FLRW holography?** Motivation: growing interests of EFT and amplitude in cosmology.

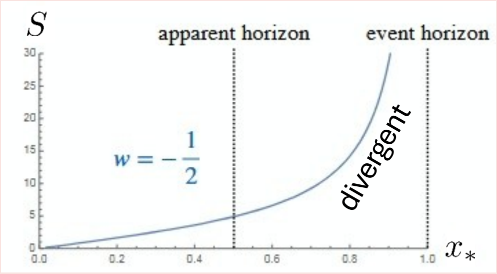
- **Constraining boundary theory from RT prescription and subadditivity** (~ unitarity and locality)

  - Half holography

    Subadditivity is violated.

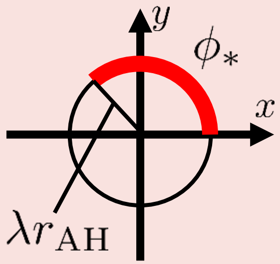


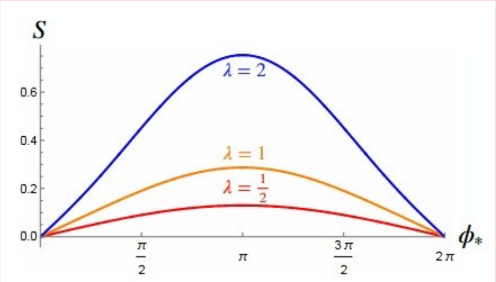


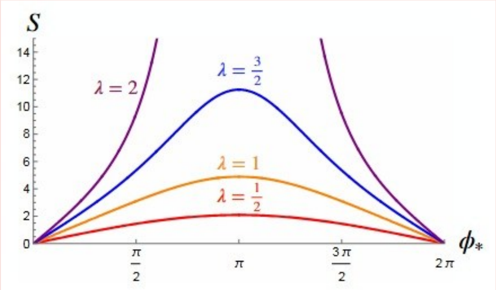


  - Horizon-type holography

    Apparent horizon is a suitable screen for standard matter $w > -1$.







  - Implication from horizon-type holography

$$\left\{\begin{array}{l}\text{Convex entropy (e.g., non-locality on bdy.)}\\ \text{WEC violation in bulk theory}\\ \text{AH outside EH in bulk theory}\end{array}\right.$$

may be related to each other through superluminality.