# tensors-v5

April 6, 2020

# 1 Tensors, version5

## 1.1 Setup packages.

[1]:
```
require(abind)
require(data.table)
require(magrittr)
require(rTensor)

require(ggplot2)
require(GGally)
```

```
Loading required package: abind
Loading required package: data.table
Loading required package: magrittr
Loading required package: rTensor
Loading required package: ggplot2
Loading required package: GGally
Registered S3 method overwritten by 'GGally':
  method from
  +.gg   ggplot2
```

## 1.2 Read data tables.

### 1.2.1 3D dataset.

[2]:
```
xs3d <- fread("xs-3d-20200322a.csv")[, case:=factor(case)]
xs3d %>% dim
```

1. 1331 2. 4

[3]:
```
ys3d <- fread("ys-3d-20200322a.csv")[, case:=factor(case)]
ys3d %>% dim
```

1. 27951 2. 5

[4]:
```
s3d <- merge(xs3d, ys3d)
s3d %>% dim
```

1. 27951 2. 8

### 1.3 Reorganize as a tensor.

[5]:
```
y <- array(
    as.matrix(s3d[order(t, x3, x2, x1)][, .(y1, y2, y3)]),
    dim = c(
        length(unique(s3d$x1)),
        length(unique(s3d$x2)),
        length(unique(s3d$x3)),
        length(unique(s3d$t )),
        3
    ),
    dimnames = list(
        x1=sort(unique(s3d$x1)),
        x2=sort(unique(s3d$x2)),
        x3=sort(unique(s3d$x3)),
        t =sort(unique(s3d$t )),
        i =1:3
    )
)
y %>% dim
```

1. 11 2. 11 3. 11 4. 21 5. 3

### 1.4 Compute differences.

#### 1.4.1 First differences (Jacobians).

[6]:
```
dy1 <- y[2:11,     ,     , , ] - y[1:10,     ,     , , ]
dy2 <- y[    , 2:11,     , , ] - y[    , 1:10,     , , ]
dy3 <- y[    ,     , 2:11, , ] - y[    ,     , 1:10, , ]

dy <- abind(
    (dy1[    , 2:11, 2:11, , ] + dy1[    , 1:10, 1:10, , ]) / 2,
    (dy2[2:11,     , 2:11, , ] + dy2[1:10,     , 1:10, , ]) / 2,
    (dy3[2:11, 2:11,     , , ] + dy3[1:10, 1:10,     , , ]) / 2,
    along=6
)
dy %>% dim
```

1. 10 2. 10 3. 10 4. 21 5. 3 6. 3

#### 1.4.2 Second differences (Hessians).

[7]:
```
ddy1 <- dy[2:10,     ,     , , , ] - dy[1:9,    ,    , , , ]
ddy2 <- dy[    , 2:10,     , , , ] - dy[   , 1:9,    , , , ]
ddy3 <- dy[    ,     , 2:10, , , ] - dy[   ,    , 1:9, , , ]

ddy <- abind(
    (ddy1[    , 2:10, 2:10, , , ] + ddy1[   , 1:9, 1:9, , , ]) / 2,
```

```
    (ddy2[2:10,     , 2:10, , , ] + ddy2[1:9,    , 1:9, , , ]) / 2,
    (ddy3[2:10, 2:10,     , , , ] + ddy3[1:9, 1:9,    , , , ]) / 2,
    along=7
)
ddy %>% dim
```

1. 9 2. 9 3. 9 4. 21 5. 3 6. 3 7. 3

### 1.5 Eigenvalues of the Hessian.

This is just an experiment, and I didn't pursue this far because it requires too much regularity in sampling.

#### 1.5.1 Compute the tensor.

[8]:
```
cy <- array(
    0,
    dim = c(
        dim(ddy)[1],
        dim(ddy)[2],
        dim(ddy)[3],
        dim(ddy)[4],
        dim(ddy)[5],
        3
    ),
    dimnames = list(
        x1=dimnames(y)$x1[2:10],
        x2=dimnames(y)$x2[2:10],
        x3=dimnames(y)$x3[2:10],
        t =dimnames(y)$t,
        i =dimnames(y)$i,
        e =c("e1", "e2", "e3")
    )
)
for (x1 in 1:dim(cy)[1])
    for (x2 in 1:dim(cy)[2])
        for (x3 in 1:dim(cy)[3])
            for(t in 1:dim(cy)[4])
                for (i in 1:dim(cy)[5])
                    cy[x1, x2, x3, t, i, ] <- sort(eigen(ddy[x1, x2, x3, t, i,␣
 ↪, ])$values)
cy %>% dim
```

1. 9 2. 9 3. 9 4. 21 5. 3 6. 3

### 1.5.2 Look at some slices.

[9]:
```
cy[, 5, 5, 21, 3, ]
```

A matrix: 9 x 3 of type dbl

| | e1 | e2 | e3 |
|---|---|---|---|
| 0.1 | 0.000000 | 0.00000000 | 0.05303377 |
| 0.2 | 0.000000 | 0.00000000 | 0.05303377 |
| 0.3 | 0.000000 | 0.00000000 | 0.05303377 |
| 0.4 | 0.000000 | 0.00000000 | 0.05303377 |
| 0.5 | -1.088146 | 0.00000000 | 0.22180704 |
| 0.6 | 0.000000 | 0.09492435 | 1.12622275 |
| 0.7 | 0.000000 | 0.00000000 | 0.30177402 |
| 0.8 | 0.000000 | 0.00000000 | 0.30177402 |
| 0.9 | 0.000000 | 0.00000000 | 0.30177402 |

[10]:
```
cy[5, , 5, 21, 3, ]
```

A matrix: 9 x 3 of type dbl

| | e1 | e2 | e3 |
|---|---|---|---|
| 0.1 | -1.088146 | 0.00000000 | 0.2218070 |
| 0.2 | -1.088146 | 0.00000000 | 0.2218070 |
| 0.3 | -1.088146 | 0.00000000 | 0.2218070 |
| 0.4 | -1.088146 | 0.00000000 | 0.2218070 |
| 0.5 | -1.088146 | 0.00000000 | 0.2218070 |
| 0.6 | -1.088146 | 0.00000000 | 0.2218070 |
| 0.7 | -1.088146 | 0.00000000 | 0.2218070 |
| 0.8 | -1.088146 | 0.00000000 | 0.2218070 |
| 0.9 | -1.047872 | 0.05756947 | 0.2189628 |

[11]:
```
cy[5, 5, , 21, 3, ]
```

A matrix: 9 x 3 of type dbl

| | e1 | e2 | e3 |
|---|---|---|---|
| 0.1 | -2.20301709 | 0.0000000 | 0.001645644 |
| 0.2 | -2.13405883 | 0.0000000 | 0.012367898 |
| 0.3 | -1.95182311 | 0.0000000 | 0.043062267 |
| 0.4 | -1.60836159 | 0.0000000 | 0.105934198 |
| 0.5 | -1.08814634 | 0.0000000 | 0.221807040 |
| 0.6 | -0.48770813 | 0.0000000 | 0.404491013 |
| 0.7 | -0.01462897 | 0.0000000 | 0.591342061 |
| 0.8 | 0.00000000 | 0.1138212 | 0.639566089 |
| 0.9 | -0.31335437 | 0.0000000 | 0.400191869 |

## 1.6 Tucker decomposition.

This is just an experiment, and I didn't pursue this far because it requires too much regularity in sampling.

### 1.6.1 Try a decomposition.

[12]:
```
yt <- tucker(as.tensor(y), c(3, 3, 3, 3, 3))
```

```
|======================================================================| 100%
```

### 1.6.2 Examine the results.

[13]:
```
yt
```

```
$Z
Numeric Tensor of 5 Modes
Modes:  3 3 3 3 3
Data:
[1] -4.155819e+02  3.229759e+01  1.159814e-13 -7.646852e-01  1.591227e+00
[6]  3.377517e-16

$U
$U[[1]]
            [,1]       [,2]        [,3]
 [1,] -0.1207642 -0.3899778  0.21973641
 [2,] -0.1207642 -0.3899778 -0.04394728
 [3,] -0.1207642 -0.3899778 -0.04394728
 [4,] -0.1207642 -0.3899778 -0.04394728
 [5,] -0.1207642 -0.3899778 -0.04394728
 [6,] -0.1207642 -0.3899778 -0.04394728
 [7,] -0.4271993  0.1322905  0.44962672
 [8,] -0.4271993  0.1322905  0.30273000
 [9,] -0.4271993  0.1322905  0.30273000
[10,] -0.4271993  0.1322905 -0.52754336
[11,] -0.4271993  0.1322905 -0.52754336

$U[[2]]
            [,1]        [,2]          [,3]
 [1,] -0.3016936 -0.09476814  2.891288e-01
 [2,] -0.3016936 -0.09476814 -1.443700e-01
 [3,] -0.3016936 -0.09476814 -1.443700e-01
 [4,] -0.3016936 -0.09476814 -1.443700e-01
 [5,] -0.3016936 -0.09476814 -1.443700e-01
 [6,] -0.3016936 -0.09476814 -1.443700e-01
 [7,] -0.3016936 -0.09476814 -1.443700e-01
 [8,] -0.3016936 -0.09476814 -1.443700e-01
 [9,] -0.3016936 -0.09476814 -1.443700e-01
[10,] -0.3016936 -0.09476814  8.658308e-01
[11,] -0.2996832  0.95403878  1.124101e-15

$U[[3]]
            [,1]         [,2]        [,3]
 [1,] -0.3457802 -0.210799669 -0.20073935
 [2,] -0.3457802 -0.210799669 -0.20073935
 [3,] -0.3453728 -0.206080123 -0.18501506
 [4,] -0.3432972 -0.189671151 -0.13203296
 [5,] -0.3370863 -0.156103486 -0.03051859
 [6,] -0.3229730 -0.095539119  0.13265221
```

```
 [7,] -0.2978970  0.004849792  0.34688044
 [8,] -0.2647734  0.152583504  0.51216571
 [9,] -0.2356062  0.336672723  0.41482711
[10,] -0.2194217  0.519637115 -0.03901869
[11,] -0.2080728  0.626506437 -0.54126927

$U[[4]]
             [,1]         [,2]         [,3]
 [1,] -0.02610281 -0.169724812 -0.457859119
 [2,] -0.03250529 -0.179026646 -0.407357288
 [3,] -0.04028317 -0.191386861 -0.351927601
 [4,] -0.04968550 -0.206722046 -0.291038353
 [5,] -0.06094416 -0.224410278 -0.224597685
 [6,] -0.07423946 -0.243080649 -0.153316297
 [7,] -0.08966383 -0.260523556 -0.079043243
 [8,] -0.10719346 -0.273843004 -0.004912115
 [9,] -0.12667626 -0.279884969  0.064837036
[10,] -0.14783954 -0.275833746  0.125392598
[11,] -0.17031404 -0.259760263  0.172135543
[12,] -0.19366878 -0.230925907  0.201399990
[13,] -0.21744857 -0.189777236  0.211003069
[14,] -0.24120952 -0.137708167  0.200423074
[15,] -0.26454755 -0.076725461  0.170636642
[16,] -0.28711770 -0.009132149  0.123733417
[17,] -0.30864404  0.062722215  0.062455623
[18,] -0.32892161  0.136619674 -0.010216154
[19,] -0.34781234  0.210591962 -0.091367600
[20,] -0.36523707  0.282990270 -0.178335666
[21,] -0.38116540  0.352514815 -0.268796842

$U[[5]]
            [,1]       [,2]       [,3]
[1,] -0.18590666  0.8869320 -0.4228360
[2,] -0.98198948 -0.1529557  0.1109108
[3,]  0.03369518  0.4358396  0.8993934


$conv
[1] TRUE

$est
Numeric Tensor of 5 Modes
Modes:  11 11 11 21 3
Data:
[1] -0.2018135 -0.2018135 -0.2018135 -0.2018135 -0.2018135 -0.2018135

$norm_percent
[1] 96.51951
```

```
$fnorm_resid
[1] 16.83547

$all_resids
[1] 16.83548 16.83547
```

### 1.7 Experiment with discrepancies across diagonals.

Let's fall back to something simpler. Each cell has three diagonals that cross in the center. Averaging the endpoints of each diagonal gives an estimate for the value at the center point. If the variance of the estimates based on each diagonal is a measure of the curvature over the cell. Score each cell by the maximum variance over time and over the y1, y2, and y3. Before doing this, it's best to scale the scale y dimensions separately.

[14]:
```
y %>% dim
```

1. 11 2. 11 3. 11 4. 21 5. 3

[15]:
```
midpoints <- function(x) (x[-1] + x[-length(x)]) / 2
```

[16]:
```
y.scaled <- apply(y, 1:4, scale)
y.scaled %>% dim
```

1. 3 2. 11 3. 11 4. 11 5. 21

[17]:
```
y.deltas <- array(
    0,
    dim = c(
        dim(y.scaled)[1],
        dim(y.scaled)[2] - 1,
        dim(y.scaled)[3] - 1,
        dim(y.scaled)[4] - 1,
        dim(y.scaled)[5],
        3
    ),
    dimnames = list(
        i = dimnames(y)$i,
        x1 = midpoints(as.numeric(dimnames(y.scaled)$x1)),
        x2 = midpoints(as.numeric(dimnames(y.scaled)$x2)),
        x3 = midpoints(as.numeric(dimnames(y.scaled)$x3)),
        t = dimnames(y)$t,
        d = c("+++", "++-", "+--")
    )
)
im <- 1 : dim(y.deltas)[2]
ip <- 1 + im
y.deltas[, , , , , 1] <- (y.scaled[, ip, ip, ip, ] + y.scaled[, im, im, im, ]) /
 ↪ 2
```

```
y.deltas[, , , , , 2] <- (y.scaled[, ip, ip, im, ] + y.scaled[, im, im, ip, ]) /
 ↪ 2
y.deltas[, , , , , 3] <- (y.scaled[, ip, im, im, ] + y.scaled[, im, ip, ip, ]) /
 ↪ 2
y.deltas %>% dim
```

1. 3 2. 10 3. 10 4. 10 5. 21 6. 3

[18]:
```
y.maxdelta <- apply(
    apply(y.deltas, 1:5, sd),
    2:4,
    max
)
y.maxdelta %>% dim
```

1. 10 2. 10 3. 10

### 1.7.1 Organize as a data table.

[19]:
```
z <- data.table(melt(y.maxdelta))[, rank := rank(value, ties="random")]
z %>% head
```

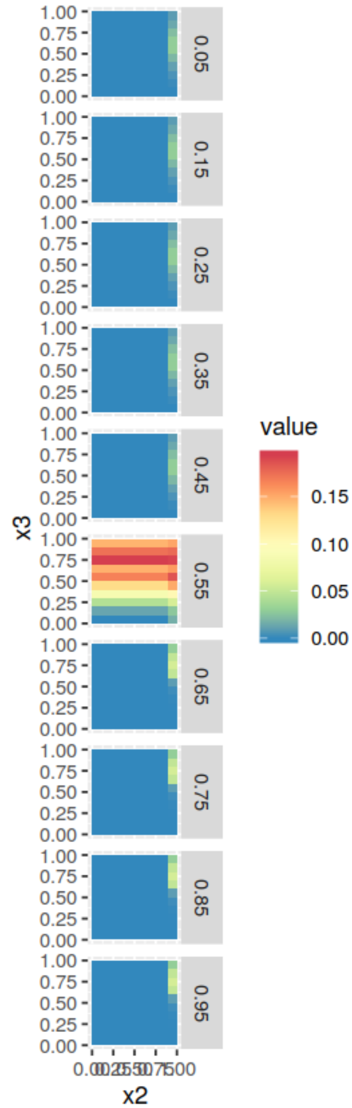
A data.table: 6 x 5

| x1 <dbl> | x2 <dbl> | x3 <dbl> | value <dbl> | rank <int> |
|---|---|---|---|---|
| 0.05 | 0.05 | 0.05 | 0 | 473 |
| 0.15 | 0.05 | 0.05 | 0 | 482 |
| 0.25 | 0.05 | 0.05 | 0 | 140 |
| 0.35 | 0.05 | 0.05 | 0 | 324 |
| 0.45 | 0.05 | 0.05 | 0 | 321 |
| 0.55 | 0.05 | 0.05 | 0 | 41 |

### 1.7.2 Plot slices.

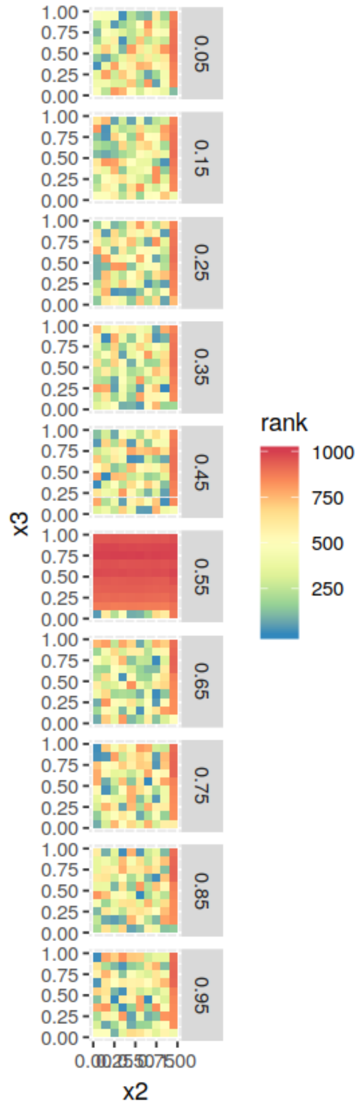
**By value.**

[20]:
```
ggplot(z, aes(x=x2, y=x3, fill=value)) +
    geom_tile() +
    coord_fixed() +
    scale_fill_distiller(palette = "Spectral") +
    facet_grid(x1 ~ .)
```




**By rank.**

[21]:
```
ggplot(z, aes(x=x2, y=x3, fill=rank)) +
    geom_tile() +
    coord_fixed() +
    scale_fill_distiller(palette = "Spectral") +
    facet_grid(x1 ~ .)
```




**Add a reference plot.**

[22]:
```
ggpairs(s3d, columns=c(2:4, 6:8), mapping=aes(color=factor(t)))
```

```
Warning message in cor(x, y, method = method, use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
```

```
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
the standard deviation is zeroWarning message in cor(x, y, method = method,
use = use):
```