

Dubrovnik Skein Theory and Power Sum Elements

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INTRODUCTION

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BACKGROUND

POWER SUM ELEMENTS

THE ALGEBRA $\mathcal{D}(T^2)$

TYPE B/C/D SCHUR FUNCTIONS

(FRAMED) LINK INVARIANTS

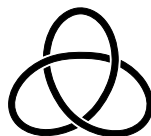


?

=



(FRAMED) LINK INVARIANTS

 $\stackrel{?}{=}$ 

Dubrovnik Polynomial

 $p_1(s, v)$ \neq $p_2(s, v)$

DUBROVNIK SKEIN RELATIONS

$$\begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \bigcirc = \begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \bigcirc + (s - s^{-1}) \left(\begin{array}{c} \diagup \diagup \\ \diagdown \diagdown \end{array} \bigcirc - \begin{array}{c} \diagdown \diagup \\ \diagup \diagdown \end{array} \bigcirc \right) \quad (1)$$

$$\begin{array}{c} \diagup \\ \diagdown \end{array} \bigcirc = v \begin{array}{c} \diagup \\ \diagdown \end{array} \bigcirc \quad (2)$$

$$\bigcirc \bigcirc = \left(1 - \frac{v - v^{-1}}{s - s^{-1}} \right) \bigcirc \quad (3)$$

DUBROVNIK SKEIN RELATIONS

$$\text{Diagram 1} = \text{Diagram 2} + (s - s^{-1}) \left(\text{Diagram 3} - \text{Diagram 4} \right) \quad (1)$$

$$\text{Diagram 5} = v \text{Diagram 6} \quad (2)$$

$$\text{Diagram 7} = \left(1 - \frac{v - v^{-1}}{s - s^{-1}} \right) \text{Diagram 8} \quad (3)$$

$$\text{Diagram 9} = \text{Diagram 10} + (s - s^{-1}) \left(\text{Diagram 11} - \text{Diagram 12} \right)$$

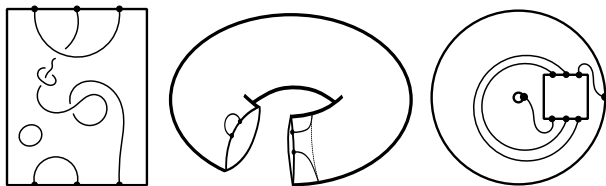
SKEIN MODULES

Observation: Skein relations are defined locally.

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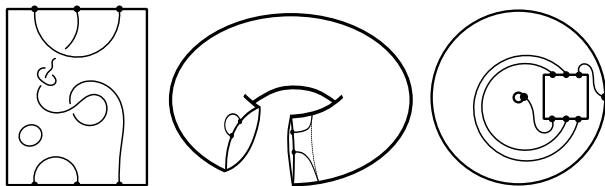
Consequence: May impose skein relations on tangles in arbitrary 3-dimensional manifolds.



SKEIN MODULES

Observation: Skein relations are defined locally.

Consequence: May impose skein relations on tangles in arbitrary 3-dimensional manifolds.



Definition

Let M be an oriented 3-manifold and $R := \mathbb{Q}(s, v)$.

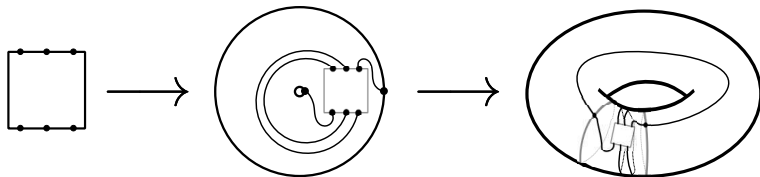
$$\mathcal{D}(M, N) := R\{\text{Tangles in } M \text{ relative to } N\} / \sim$$

FUNCTORIALITY

A “nice” embedding $f : M \rightarrow M'$

+ A wiring diagram in image complement

= A linear transformation $D(f) : \mathcal{D}(M, N) \rightarrow \mathcal{D}(M', N')$

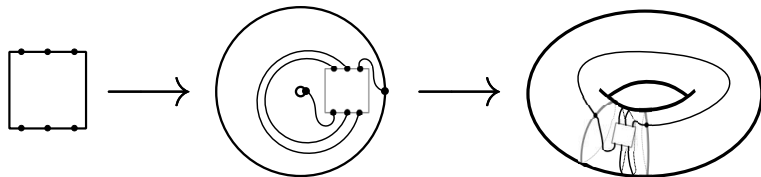


FUNCTORIALITY

A “nice” embedding $f : M \rightarrow M'$

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= A linear transformation $D(f) : \mathcal{D}(M, N) \rightarrow \mathcal{D}(M', N')$



Consequence: Dubrovnik skein theory is a type of algebraic topology for smooth, oriented, 3-manifolds.

SPECIAL CASE: SKEIN ALGEBRAS

If

- ▶ $M = \Sigma \times I$
- ▶ $N = (X \times \{0\}) \sqcup (X \times \{1\})$

Then $\mathcal{D}(M, N)$ is naturally an algebra.

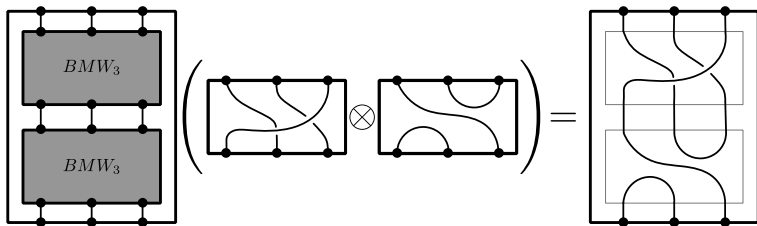
SPECIAL CASE: SKEIN ALGEBRAS

If

- ▶ $M = \Sigma \times I$
- ▶ $N = (X \times \{0\}) \sqcup (X \times \{1\})$

Then $\mathcal{D}(M, N)$ is naturally an algebra.

e.g.: $\Sigma = \text{Square}$, $N = 2n$ points $\rightsquigarrow \mathcal{D}(\Sigma, N) \cong \text{BMW}_n$



IDEMPOTENTS IN BIRMAN-MURAKAMI-WENZL ALGEBRAS

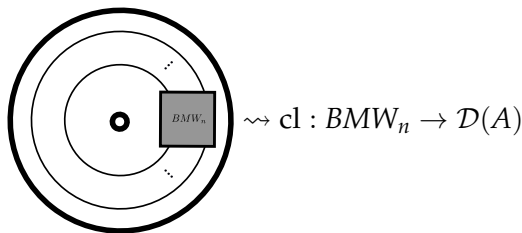
Theorem (Ram-Wenzl 1992, Beliakova-Blanchet 2001)

For each partition $\lambda \vdash n$, there is a minimal idempotent $\tilde{y}_\lambda \in BMW_n$.

IDEMPOTENTS IN BIRMAN-MURAKAMI-WENZL ALGEBRAS

Theorem (Ram-Wenzl 1992, Beliakova-Blanchet 2001)

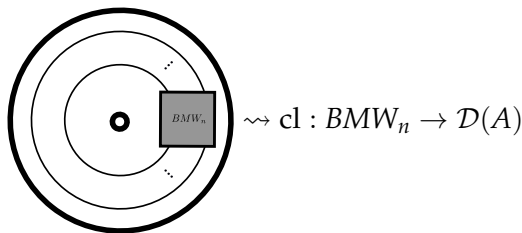
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For each partition $\lambda \vdash n$, there is a minimal idempotent $\tilde{y}_\lambda \in BMW_n$.



Theorem (Lu-Zhong 2002)

The elements $\tilde{Q}_\lambda := \text{cl}(\tilde{y}_\lambda)$ form a basis of $\mathcal{D}(A)$.

DUBROVNIK POWER SUM ELEMENTS

Define a family of elements $\tilde{P}_k \in \mathcal{D}(A)$ for $k \in \mathbb{Z}_{\geq 1}$ via

$$\sum_{k \geq 1} \frac{\tilde{P}_k}{k} t^k = \ln \left(1 + \sum_{n \geq 1} \tilde{Q}_{(n)} t^n \right)$$

Idea:

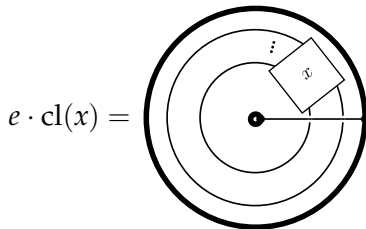
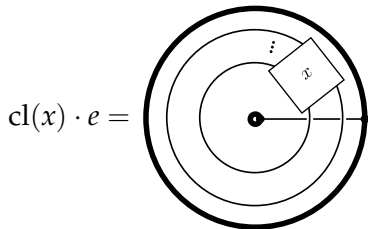
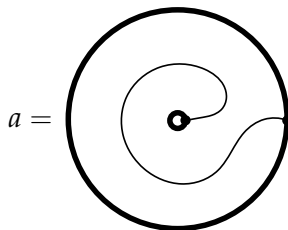
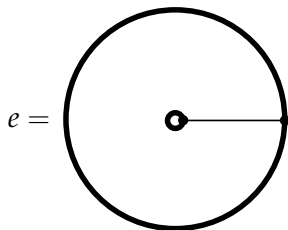
$\tilde{Q}_{(n)}$ are “like” complete homogeneous symmetric functions.

$\rightsquigarrow \tilde{P}_k$ are “like” power sum symmetric functions.

(Will make more precise later)

A RELATIVE SKEIN ALGEBRA

Let $\mathcal{A} := \mathcal{D}(A, 1)$.



COMMUTATION RELATIONS

Theorem (Morton-P.-Samuelson)

$$e \cdot \tilde{P}_k - \tilde{P}_k \cdot e = (s^k - s^{-k})(a^k - a^{-k})$$

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Theorem (P.)

$$e \cdot \tilde{Q}_{(n)} - \tilde{Q}_{(n)} \cdot e = \sum_{i=1}^n d_i (e \cdot \tilde{Q}_{(n-i)})$$

where

$$d_i = \sum_{l=0}^{i-1} (s^2 - 1) s^{2l-i} a^{i-2l} + (s^{-2} - 1) s^{i-2l} a^{2l-i}$$

IDEA OF PROOF

1) By power series manipulations, the statement is equivalent to

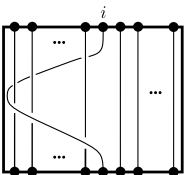
$$\begin{aligned}
 e \cdot (\tilde{Q}_{(n+2)} + \tilde{Q}_{(n)}) - (\tilde{Q}_{(n+2)} + \tilde{Q}_{(n)}) \cdot e \\
 = \\
 (sa + s^{-1}a^{-1})(e \cdot \tilde{Q}_{(n+1)}) - (s^{-1}a + sa^{-1})(\tilde{Q}_{(n+1)} \cdot e)
 \end{aligned}$$

2) [Shelly, 2016] The $\tilde{y}_{(n)}$ satisfy a skein-theoretic recurrence relation.

$$[n+1] \begin{array}{|c|} \hline \tilde{y}_{n+1} \\ \hline \end{array} = [n]s^{-1} \begin{array}{|c|} \hline \tilde{y}_n \\ \hline \end{array} + \begin{array}{|c|} \hline \tilde{y}_n \\ \hline \end{array} + [n]s^{-1}\beta_n \begin{array}{|c|} \hline \tilde{y}_n \\ \hline \end{array}$$

APPLICATION: CENTRAL ELEMENTS OF BMW_n

The Jucys-Murphy elements $M_{n,i}$ generate a commutative subalgebra of BMW_n .

$$M_{n,i} =$$


$$2 \leq i \leq n$$

APPLICATION: CENTRAL ELEMENTS OF BMW_n

The Jucys-Murphy elements $M_{n,i}$ generate a commutative subalgebra of BMW_n .

$$M_{n,i} = \text{Diagram} \quad 2 \leq i \leq n$$

Proposition (P.)

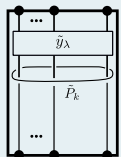
$$\text{Diagram} = \langle \tilde{P}_k \rangle 1 + (s^k - s^{-k}) \sum_{i=2}^n v^{-k} M_{n,i}^k - v^k M_{n,i}^{-k}$$

APPLICATION: CENTRAL ELEMENTS OF BMW_n Sketch: Apply commutation relation for \tilde{P}_k .

$$\begin{aligned}
 & \text{Diagram 1: A rectangle with } k \text{ vertical lines. A horizontal oval labeled } \tilde{P}_k \text{ encircles the middle of the lines.} \\
 &= \text{Diagram 2: Same as Diagram 1.} + (s^k - s^{-k}) \text{Diagram 3: A rectangle with } k \text{ vertical lines. A horizontal oval encircles the middle of the lines, with a crossing on the right side.} - (s^k - s^{-k}) \text{Diagram 4: A rectangle with } k \text{ vertical lines. A horizontal oval encircles the middle of the lines, with a crossing on the left side.} \\
 &= \text{Diagram 5: Same as Diagram 2.} + (s^k - s^{-k})v^{-k} \text{Diagram 6: Same as Diagram 3, but with a crossing on the left side.} - (s^k - s^{-k})v^k \text{Diagram 7: Same as Diagram 4, but with a crossing on the right side.}
 \end{aligned}$$

APPLICATION: MERIDIANS OF \tilde{y}_λ

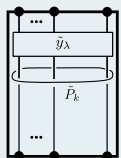
Proposition (P.)



$$= \left(\langle \tilde{P}_k \rangle + (s^k - s^{-k}) \sum_{\square \in \lambda} \left(v^{-k} s^{2\text{cn}(\square)} - v^k s^{-2\text{cn}(\square)} \right) \right) \tilde{y}_\lambda$$

APPLICATION: MERIDIANS OF \tilde{y}_λ

Proposition (P.)

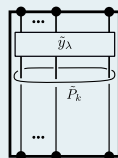


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Observation: For any fixed k , the eigenvalues of \tilde{y}_λ are distinct.

APPLICATION: MERIDIANS OF \tilde{y}_λ

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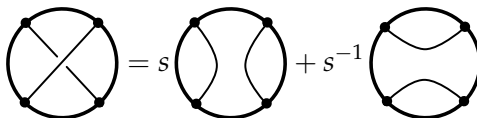
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Observation: For any fixed k , the eigenvalues of \tilde{y}_λ are distinct.

Consequence: The basis $\{\tilde{Q}_\lambda\}$ of $\mathcal{D}(A)$ is an eigenbasis with 1-dimensional eigenspaces. Setting $k = 1$ recovers the result from [Lu-Zhong, 2002].

COMPATIBILITY WITH KAUFFMAN BRACKET SKEIN THEORY \mathcal{K}

The Kauffman bracket skein relation \rightsquigarrow Jones polynomial

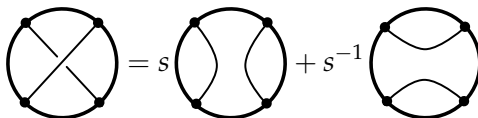


The diagram illustrates the Kauffman bracket skein relation. It shows a circle with four marked points (top, bottom, left, right) and two internal diagonal lines forming an 'X' (crossing). This is equal to s times a circle with two internal arcs connecting top-left to top-right and bottom-left to bottom-right, plus s^{-1} times a circle with two internal arcs connecting top-right to top-left and bottom-right to bottom-left.

$$\text{Crossing} = s \cdot \text{Arcs} + s^{-1} \cdot \text{Arcs}$$

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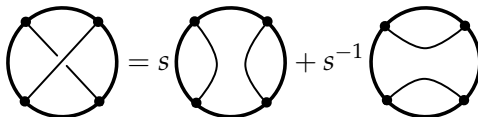
The diagram illustrates the Kauffman bracket skein relation. It shows a circle with four marked points (top, bottom, left, right). On the left, the circle is divided into four quadrants by two straight lines crossing at the center. This is followed by an equals sign, then the variable s , then a circle with two vertical arcs (left and right) connecting the top and bottom points. This is followed by a plus sign, then s^{-1} , then a circle with two horizontal arcs (top and bottom) connecting the left and right points.

$$\text{Crossing} = s \text{ (Vertical Arcs)} + s^{-1} \text{ (Horizontal Arcs)}$$

Fact: The Dubrovnik skein relation satisfies the Kauffman bracket skein relation.

COMPATIBILITY WITH KAUFFMAN BRACKET SKEIN THEORY \mathcal{K}

The Kauffman bracket skein relation \rightsquigarrow Jones polynomial



The diagram illustrates the Kauffman bracket skein relation. It shows a circle with four marked points on its boundary. On the left, the circle is divided into four quadrants by two straight lines crossing at the center. This is followed by an equals sign, then the term s multiplied by a circle where the two lines are curved away from each other, creating two separate regions. This is followed by a plus sign, then the term s^{-1} multiplied by a circle where the two lines are curved towards each other, also creating two separate regions.

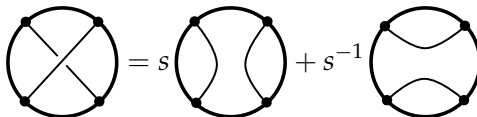
$$\text{Crossing} = s \text{ (Skein 1)} + s^{-1} \text{ (Skein 2)}$$

Fact: The Dubrovnik skein relation satisfies the Kauffman bracket skein relation.

Consequence: There is a natural transformation of skein theories $\eta : \mathcal{D} \Rightarrow \mathcal{K}$.

COMPATIBILITY WITH KAUFFMAN BRACKET SKEIN THEORY \mathcal{K}

The Kauffman bracket skein relation \rightsquigarrow Jones polynomial



Fact: The Dubrovinik skein relation satisfies the Kauffman bracket skein relation.

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Theorem (Morton-P.-Samuelson)

The image of $\tilde{P}_k \in \mathcal{D}(A)$ under η_A is the Chebyshev polynomial $T_k \in \mathcal{K}(A)$.

SKEIN ALGEBRAS OF T^2

Theorem (Frohman-Gelca 2000)

The algebra $\mathcal{K}(T^2)$ is presented by generators $T_{\mathbf{x}}$ for $\mathbf{x} \in \mathbb{Z}^2 / \langle \mathbf{x} = -\mathbf{x} \rangle$ subject to the relations

$$T_{\mathbf{x}}T_{\mathbf{y}} = s^{\det(\mathbf{x},\mathbf{y})}T_{\mathbf{x}+\mathbf{y}} + s^{-\det(\mathbf{x},\mathbf{y})}T_{\mathbf{x}-\mathbf{y}}$$

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Theorem (Morton-Samuelson 2017)

The algebra $\mathcal{H}(T^2)$ is presented by generators $P_{\mathbf{x}}$ for $\mathbf{x} \in \mathbb{Z}^2$ subject to the relations

$$[P_{\mathbf{x}}, P_{\mathbf{y}}] = (s^{\det(\mathbf{x},\mathbf{y})} - s^{-\det(\mathbf{x},\mathbf{y})})P_{\mathbf{x}+\mathbf{y}}$$

A PRESENTATION OF $\mathcal{D}(T^2)$

Let $\mathbf{x} = (a, b)$, $k = \gcd(\mathbf{x})$.

Define $\tilde{P}_{\mathbf{x}} \in \mathcal{D}(T^2)$ be the embedding of \tilde{P}_k along the closed curve of slope a/b .

Theorem (Morton-P.-Samuelson)

The algebra $\mathcal{D}(T^2)$ is presented by generators $\tilde{P}_{\mathbf{x}}$ for $\mathbf{x} \in \mathbb{Z}^2 / \langle \mathbf{x} = -\mathbf{x} \rangle$ subject to the relations

$$[\tilde{P}_{\mathbf{x}}, \tilde{P}_{\mathbf{y}}] = (s^{\det(\mathbf{x}, \mathbf{y})} - s^{-\det(\mathbf{x}, \mathbf{y})})(\tilde{P}_{\mathbf{x}+\mathbf{y}} - \tilde{P}_{\mathbf{x}-\mathbf{y}})$$

A PRESENTATION OF $\mathcal{D}(T^2)$

Proof:

1. $[\tilde{P}_{\mathbf{x}}, \tilde{P}_{\mathbf{y}}] = (s^{\det(\mathbf{x}, \mathbf{y})} - s^{-\det(\mathbf{x}, \mathbf{y})})(\tilde{P}_{\mathbf{x}+\mathbf{y}} - \tilde{P}_{\mathbf{x}-\mathbf{y}}).$
 - 1.1 $[\tilde{P}_{1,0}, \tilde{P}_{0,n}] = (s^n - s^{-n})(\tilde{P}_{1,n} - \tilde{P}_{1,-n})$
 - 1.2 $[\tilde{P}_{1,0}, \tilde{P}_{1,n}] = (s^n - s^{-n})(\tilde{P}_{2,n} - \tilde{P}_{0,n})$
 - 1.3 Use $SL_2(\mathbb{Z})$ -action on $\mathcal{D}(T^2)$. Induct on $|\det(\mathbf{x}, \mathbf{y})|$.
2. The $\tilde{P}_{\mathbf{x}}$ generate $\mathcal{D}(T^2)$.
 - 2.1 Each link in $\mathcal{D}(T^2)$ is a sum of products of annular knots.
 - 2.2 The \tilde{P}_k generate $\mathcal{D}(A)$.
3. The $\tilde{P}_{\mathbf{x}}$ are linearly independent.
 - 3.1 Diamond lemma type argument.

RELATIONSHIP WITH $\mathcal{K}(T^2)$

Corollary (Morton-P.-Samuelson)

There is a surjective algebra homomorphism $\mathcal{D}(T^2) \rightarrow \mathcal{K}(T^2)$ defined by

$$\tilde{P}_x \mapsto T_x.$$

Proof: Use the natural transformation $\eta : \mathcal{D} \Rightarrow \mathcal{K}$. Recall

$$\eta_A(\tilde{P}_k) = T_k.$$

Note: $\mathcal{D}(T^2)$ is much bigger than $\mathcal{K}(T^2)$.

RELATIONSHIP WITH $\mathcal{H}(T^2)$

Corollary (P.)

The algebras $\mathcal{D}(T^2)$ and $\mathcal{H}(T^2)$ are universal enveloping algebras of some Lie algebras $\mathfrak{g}_{\mathcal{D}}$ and $\mathfrak{g}_{\mathcal{H}}$ generated by the $\tilde{P}_{\mathbf{x}}$ and $T_{\mathbf{x}}$, respectively. There is an injective Lie algebra homomorphism $\mathfrak{g}_{\mathcal{D}} \rightarrow \mathfrak{g}_{\mathcal{H}}$ defined by

$$\tilde{P}_{\mathbf{x}} \mapsto P_{\mathbf{x}} + P_{-\mathbf{x}}.$$

Proof: $[\tilde{P}_{\mathbf{x}}, \tilde{P}_{\mathbf{y}}] - (s^{\det(\mathbf{x}, \mathbf{y})} - s^{-\det(\mathbf{x}, \mathbf{y})})(\tilde{P}_{\mathbf{x}+\mathbf{y}} - \tilde{P}_{\mathbf{x}-\mathbf{y}})$ is sent to 0.

Note: This restricts to an algebra homomorphism

$$\mathcal{D}(A) \rightarrow \mathcal{H}(A).$$

Note: $\mathcal{D}(A)$ and $\mathcal{H}(A)$ are related to characters of classical Lie groups.

A $\mathcal{D}(T^2)$ -ACTION ON $\mathcal{D}(D^2 \times S^1)$

Lemma (P.)

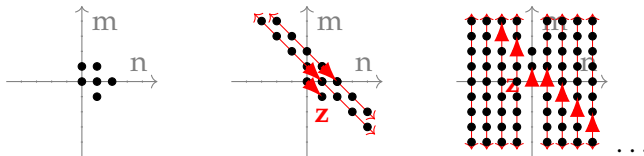
$\mathcal{D}(T^2)$ is generated by the identity and the elements

$$\tilde{P}_{1,0}, \tilde{P}_{0,1}, \tilde{P}_{1,1}, \tilde{P}_{2,0}$$

Proof: Can generate a “line” of \tilde{P}_x by

$$\tilde{P}_{\mathbf{y}+n\mathbf{z}} = (s^d - s^{-d})^{-1} \left([\tilde{P}_{\mathbf{y}+(n-1)\mathbf{z}}, \tilde{P}_{\mathbf{z}}] + \{d\} \tilde{P}_{\mathbf{y}+(n-2)\mathbf{z}} \right)$$

as long as $d := \det(\mathbf{y} + n\mathbf{z}, \mathbf{z}) = \det(\mathbf{y}, \mathbf{z}) \neq 0$.



A $\mathcal{D}(T^2)$ -ACTION ON $\mathcal{D}(D^2 \times S^1)$

Proposition (P.)

The $\mathcal{D}(T^2)$ -action on $\mathcal{D}(D^2 \times S^1)$ is determined by the equations

$$\tilde{P}_{1,0} \cdot \tilde{Q}_\lambda = \left(\langle \tilde{P}_1 \rangle + \{1\} \left(v^{-1} \sum_{\square \in \lambda} s^{2\text{cn}(\square)} - v \sum_{\square \in \lambda} s^{-2\text{cn}(\square)} \right) \right) \tilde{Q}_\lambda$$

$$\tilde{P}_{2,0} \cdot \tilde{Q}_\lambda = \left(\langle \tilde{P}_2 \rangle + \{2\} \left(v^{-2} \sum_{\square \in \lambda} s^{4\text{cn}(\square)} - v^2 \sum_{\square \in \lambda} s^{-4\text{cn}(\square)} \right) \right) \tilde{Q}_\lambda$$

$$\tilde{P}_{0,1} \cdot \tilde{Q}_\lambda = \sum_{\substack{\lambda \subset \mu \\ \mu = \lambda + \square}} \tilde{Q}_\mu + \sum_{\substack{\nu \subset \lambda \\ \lambda = \nu + \square}} \tilde{Q}_\nu$$

$$\tilde{P}_{1,1} \cdot \tilde{Q}_\lambda = v^{-1} \sum_{\substack{\lambda \subset \mu \\ \mu = \lambda + \square}} s^{2\text{cn}(\square)} \tilde{Q}_\mu + v \sum_{\substack{\nu \subset \lambda \\ \lambda = \nu + \square}} s^{-2\text{cn}(\square)} \tilde{Q}_\nu$$

SYMMETRIC FUNCTIONS AND $\mathcal{H}(A)^+$

Λ - The ring of symmetric functions

s_λ - Schur function

$\mathcal{H}(A)^+$ - Annular HOMFLYPT links oriented counter-clockwise

Theorem (Lukac 2005)

*There is a basis of elements Q_λ in $\mathcal{H}(A)^+$ indexed by partitions λ .
The assignment $\Lambda \rightarrow \mathcal{H}(A)^+$ defined by*

$$s_\lambda \mapsto Q_\lambda$$

is an algebra isomorphism.

Hope: Generalize this to Dubrovnik case.

ORTHOGONAL & SYMPLECTIC SCHUR FUNCTIONS

Partition λ \longrightarrow Symmetric functions sb_λ, sc_λ

ORTHOGONAL & SYMPLECTIC SCHUR FUNCTIONS

Partition λ \longrightarrow Symmetric functions sb_λ, sc_λ

Theorem (Koike-Terada 1987, Koike 1989)

The sets $\{sb_\lambda\}$ and $\{sc_\lambda\}$ are bases of Λ .

The structure constants of Λ with respect to each basis are identical natural numbers.

ORTHOGONAL & SYMPLECTIC SCHUR FUNCTIONS

Partition λ \longrightarrow Symmetric functions sb_λ, sc_λ

Theorem (Koike-Terada 1987, Koike 1989)

The sets $\{sb_\lambda\}$ and $\{sc_\lambda\}$ are bases of Λ .

The structure constants of Λ with respect to each basis are identical natural numbers.

Conjecture

Both of the assignments $\Lambda \rightarrow \mathcal{D}(A)$ defined by

$$sb_{(n)} \mapsto \tilde{Q}_{(n)}$$

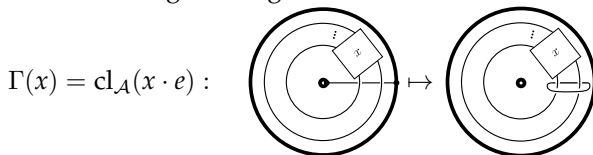
$$sc_{(n)} \mapsto \tilde{Q}_{(n)}$$

are algebra isomorphisms.

PARTIAL PROOF OF CONJECTURE

Idea: Let S_λ be the image of sc_λ . Show $S_\lambda = \tilde{Q}_\lambda$.

1. Show S_λ is in the same eigenspace of \tilde{Q}_λ with respect to the meridian map Γ .
 - 1.1 Use a Jacobi-Trudi type identity to write S_λ as a determinant whose entries are in terms of $S_{(n)} = \tilde{Q}_{(n)}$.
 - 1.2 Factor Γ through the algebra \mathcal{A} so that



- 1.3 ??? $\text{cl}_{\mathcal{A}}(e \cdot x) = \delta x$. Try to translate left-action to right-action. (Works if $|\lambda| \leq 2$)
 - 1.4 Compute eigenvalue.
2. Eigenspaces are 1-dimensional $\Rightarrow S_\lambda = dQ_\lambda$. Show $d = 1$ with branching rules: $S_{\square}^n = \sum c_\mu S_\mu$ and $\tilde{Q}_{\square}^n = \sum c_\mu \tilde{Q}_\mu$

FRAME 1