Solutions to Problems from Project Euler

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Problem #1: This problem can be solved by hand. To sum up all of the multiples of 3 or 5 which are less than 1000 (less than or equal to 999), we note that $\lfloor \frac{999}{3} \rfloor = 333$ and $\lfloor \frac{999}{5} \rfloor = 199$, so we wish to consider the sums

$$\sum_{i=1}^{333} 3i$$

and

$$\sum_{j=1}^{199} 5j.$$

Note that each of these sums count all of the multiples of 15 (i.e., those numbers which are multiples of 3 *and* 5), and so these are double-counted. As such, to find the desired sum we must subtract the sum of all multiples of 15. As, $\lfloor \frac{999}{15} \rfloor = 66$, this gives us the sum

$$S = \sum_{i=1}^{333} 3i + \sum_{j=1}^{199} 5j - \sum_{k=1}^{66} 15k$$

which, by using $\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$, gives us

$$S = 3\frac{333(333+1)}{2} + 5\frac{199(199+1)}{2} - 15\frac{66(66+1)}{2}$$
$$= 3 * 333 * 167 + 5 * 199 * 100 - 15 * 33 * 67$$
$$= 233,168.$$