



A First Course in

Abstract Algebra

8th EDITION

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A First Course in Abstract Algebra

Eighth Edition

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[†] Not required for the remainder of the text.

[‡] This section is a prerequisite for Sections 17 and 36 only.

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