



A First Course in  
**Abstract  
Algebra**

8<sup>th</sup> EDITION

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# A First Course in Abstract Algebra

Eighth Edition

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<sup>†</sup> Not required for the remainder of the text.

<sup>‡</sup> This section is a prerequisite for Sections 17 and 36 only.

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