

changing any preliminary information. All of the information necessary to send an enciphered message is publicly available.

**Classical versus public key.** By a *classical* cryptosystem (also called a *private key* cryptosystem or a *symmetrical* cryptosystem), we mean a cryptosystem in which, once the enciphering information is known, the deciphering transformation can be implemented in approximately the same order of magnitude of time as the enciphering transformation. All of the cryptosystems in Chapter III are classical. Occasionally, it takes a little longer for the deciphering — because one needs to apply the Euclidean algorithm to find an inverse modulo  $N$  or one must invert a matrix (and this can take a fairly long time if we work with  $k \times k$  -matrices for  $k$  larger than 2) — nevertheless, the additional time required is not prohibitive. (Moreover, usually the additional time is required only once — to find  $K_D$  — after which it takes no longer to decipher than to encipher.) For example, we might need only  $O(\log^2 B)$  to encipher a message unit, and  $O(\log^3 B)$  bit operations to decipher one by finding  $K_D$  from  $K_E$ , where  $B$  is a bound on the size of the key parameters. Notice the role of big-O estimates here.

If, on the other hand, the enciphering time were polynomial in  $\log B$  and the deciphering time (based on knowledge of  $K_E$  but not  $K_D$ ) were, say, polynomial in  $B$  but not in  $\log B$ , then we would have a *public key* rather than a classical cryptosystem.

**Authentication.** Often, one of the most important parts of a message is the *signature*. A person's signature — hopefully, written with an idiosyncratic flourish of the pen which is hard to duplicate — lets the recipient know that the message really is from the person whose name is typed below. If the message is particularly important, it might be necessary to use additional methods to *authenticate* the communication. And in electronic communication, where one does not have a physical signature, one has to rely entirely on other methods. For example, when an officer of a corporation wants to withdraw money from the corporate account by telephone, he/she is often asked to give some personal information (e.g., mother's maiden name) which the corporate officer knows and the bank knows (from data submitted when the account was opened) but which an imposter would not be likely to know.

In public key cryptography there is an especially easy way to identify oneself in such a way that no one could be simply pretending to be you. Let  $A$  (Alice) and  $B$  (Bob) be two users of the system. Let  $f_A$  be the enciphering transformation with which any user of the system sends a message to Alice, and let  $f_B$  be the same for Bob. For simplicity, we shall assume that the set  $\mathcal{P}$  of all possible plaintext message units and the set  $\mathcal{C}$  of all possible ciphertext message units are equal, and are the same for all users. Let  $P$  be Alice's "signature" (perhaps including an identification number, a statement of the time the message was sent, etc.). It would not be enough for Alice to send Bob the encoded message  $f_B(P)$ , since *everyone* knows how to do that, so there would be no way of knowing that the signature was not