

ΠΡ ὁποτέρῳ τῶν ΜΖ, ΝΘ ὁμοίον τε καὶ ὁμοίως κείμενον εὐθύγραμμον τὸ ΣΡ.

Ἐπεὶ οὖν ἐστὶν ὡς ἡ ΑΒ πρὸς τὴν ΓΔ, οὕτως ἡ ΕΖ πρὸς τὴν ΠΡ, καὶ ἀναγέγραπται ἀπὸ μὲν τῶν ΑΒ, ΓΔ ὁμοιά τε καὶ ὁμοίως κείμενα τὰ ΚΑΒ, ΛΓΔ, ἀπὸ δὲ τῶν ΕΖ, ΠΡ ὁμοιά τε καὶ ὁμοίως κείμενα τὰ ΜΖ, ΣΡ, ἔστιν ἄρα ὡς τὸ ΚΑΒ πρὸς τὸ ΛΓΔ, οὕτως τὸ ΜΖ πρὸς τὸ ΣΡ. ὑπόκειται δὲ καὶ ὡς τὸ ΚΑΒ πρὸς τὸ ΛΓΔ, οὕτως τὸ ΜΖ πρὸς τὸ ΝΘ· καὶ ὡς ἄρα τὸ ΜΖ πρὸς τὸ ΣΡ, οὕτως τὸ ΜΖ πρὸς τὸ ΝΘ. τὸ ΜΖ ἄρα πρὸς ἐκάτερον τῶν ΝΘ, ΣΡ τὸν αὐτὸν ἔχει λόγον· ἴσον ἄρα ἐστὶ τὸ ΝΘ τῷ ΣΡ. ἔστι δὲ αὐτῷ καὶ ὁμοιον καὶ ὁμοίως κείμενον· ἴση ἄρα ἡ ΗΘ τῇ ΠΡ. καὶ ἐπεὶ ἐστὶν ὡς ἡ ΑΒ πρὸς τὴν ΓΔ, οὕτως ἡ ΕΖ πρὸς τὴν ΠΡ, ἴση δὲ ἡ ΠΡ τῇ ΗΘ, ἔστιν ἄρα ὡς ἡ ΑΒ πρὸς τὴν ΓΔ, οὕτως ἡ ΕΖ πρὸς τὴν ΗΘ.

Ἐάν ἄρα τέσσαρες εὐθεῖαι ἀνάλογον ᾧσιν, καὶ τὰ ἀπ' αὐτῶν εὐθύγραμμα ὁμοιά τε καὶ ὁμοίως ἀναγεγραμμένα ἀνάλογον ἔσται· καθ' ὅτι τὰ ἀπ' αὐτῶν εὐθύγραμμα ὁμοιά τε καὶ ὁμοίως ἀναγεγραμμένα ἀνάλογον ᾧ, καὶ αὐταὶ αἱ εὐθεῖαι ἀνάλογον ἔσονται· ὅπερ ἔδει δεῖξαι.

(is) to QR [Prop. 6.12]. And let the rectilinear figure SR , similar, and similarly laid down, to either of MF or NH , have been described on QR [Props. 6.18, 6.21].

Therefore, since as AB is to CD , so EF (is) to QR , and the similar, and similarly laid out, (rectilinear figures) KAB and LCD have been described on AB and CD (respectively), and the similar, and similarly laid out, (rectilinear figures) MF and SR on EF and QR (respectively), thus as KAB is to LCD , so MF (is) to SR (see above). And it was also assumed that as KAB (is) to LCD , so MF (is) to NH . Thus, also, as MF (is) to SR , so MF (is) to NH [Prop. 5.11]. Thus, MF has the same ratio to each of NH and SR . Thus, NH is equal to SR [Prop. 5.9]. And it is also similar, and similarly laid out, to it. Thus, GH (is) equal to QR .[†] And since AB is to CD , as EF (is) to QR , and QR (is) equal to GH , thus as AB is to CD , so EF (is) to GH .

Thus, if four straight-lines are proportional, then similar, and similarly described, rectilinear figures (drawn) on them will also be proportional. And if similar, and similarly described, rectilinear figures (drawn) on them are proportional then the straight-lines themselves will also be proportional. (Which is) the very thing it was required to show.

[†] Here, Euclid assumes, without proof, that if two similar figures are equal then any pair of corresponding sides is also equal.

κγ'.

Τὰ ἰσογώνια παραλληλόγραμμα πρὸς ἄλληλα λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν.

Ἐστω ἰσογώνια παραλληλόγραμμα τὰ ΑΓ, ΓΖ ἴσην ἔχοντα τὴν ὑπὸ ΒΓΔ γωνίαν τῇ ὑπὸ ΕΓΗ· λέγω, ὅτι τὸ ΑΓ παραλληλόγραμμον πρὸς τὸ ΓΖ παραλληλόγραμμον λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν.

Κείσθω γὰρ ὥστε ἐπ' εὐθείας εἶναι τὴν ΒΓ τῇ ΓΗ· ἐπ' εὐθείας ἄρα ἐστὶ καὶ ἡ ΔΓ τῇ ΓΕ. καὶ συμπληρώσθω τὸ ΔΗ παραλληλόγραμμον, καὶ ἐκκείσθω τις εὐθεῖα ἡ Κ, καὶ γεγονέτω ὡς μὲν ἡ ΒΓ πρὸς τὴν ΓΗ, οὕτως ἡ Κ πρὸς τὴν Λ, ὡς δὲ ἡ ΔΓ πρὸς τὴν ΓΕ, οὕτως ἡ Λ πρὸς τὴν Μ.

Οἱ ἄρα λόγοι τῆς τε Κ πρὸς τὴν Λ καὶ τῆς Λ πρὸς τὴν Μ οἱ αὐτοὶ εἰσι τοῖς λόγοις τῶν πλευρῶν, τῆς τε ΒΓ πρὸς τὴν ΓΗ καὶ τῆς ΔΓ πρὸς τὴν ΓΕ. ἀλλ' ὁ τῆς Κ πρὸς Μ λόγος σύγκειται ἐκ τε τοῦ τῆς Κ πρὸς Λ λόγου καὶ τοῦ τῆς Λ πρὸς Μ· ὥστε καὶ ἡ Κ πρὸς τὴν Μ λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν. καὶ ἐπεὶ ἐστὶν ὡς ἡ ΒΓ πρὸς τὴν ΓΗ, οὕτως τὸ ΑΓ παραλληλόγραμμον πρὸς τὸ ΓΘ, ἀλλ' ὡς ἡ ΒΓ πρὸς τὴν ΓΗ, οὕτως ἡ Κ πρὸς τὴν Λ, καὶ ὡς ἄρα ἡ Κ πρὸς τὴν Λ, οὕτως τὸ ΑΓ πρὸς τὸ ΓΘ. πάλιν, ἐπεὶ ἐστὶν ὡς ἡ ΔΓ πρὸς τὴν ΓΕ, οὕτως τὸ ΓΘ παραλληλόγραμμον πρὸς τὸ ΓΖ, ἀλλ' ὡς ἡ ΔΓ πρὸς τὴν ΓΕ,

Proposition 23

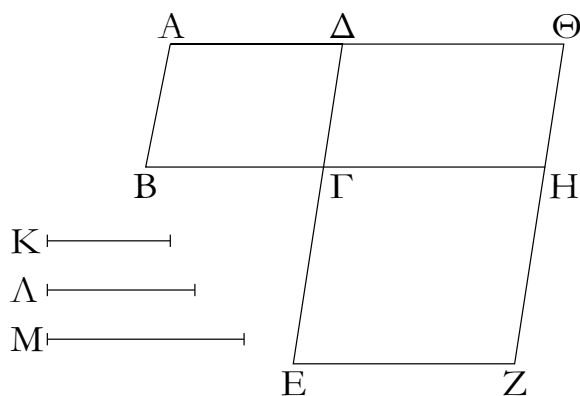
Equiangular parallelograms have to one another the ratio compounded[†] out of (the ratios of) their sides.

Let AC and CF be equiangular parallelograms having angle BCD equal to ECG . I say that parallelogram AC has to parallelogram CF the ratio compounded out of (the ratios of) their sides.

For let BC be laid down so as to be straight-on to CG . Thus, DC is also straight-on to CE [Prop. 1.14]. And let the parallelogram DG have been completed. And let some straight-line K have been laid down. And let it be contrived that as BC (is) to CG , so K (is) to L , and as DC (is) to CE , so L (is) to M [Prop. 6.12].

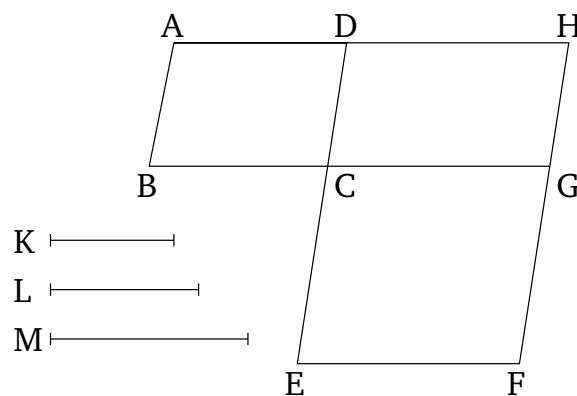
Thus, the ratios of K to L and of L to M are the same as the ratios of the sides, (namely), BC to CG and DC to CE (respectively). But, the ratio of K to M is compounded out of the ratio of K to L and (the ratio) of L to M . Hence, K also has to M the ratio compounded out of (the ratios of) the sides (of the parallelograms). And since as BC is to CG , so parallelogram AC (is) to CH [Prop. 6.1], but as BC (is) to CG , so K (is) to L , thus, also, as K (is) to L , so (parallelogram) AC (is) to CH . Again, since as DC (is) to CE , so parallelogram

οὕτως ἡ Λ πρὸς τὴν M , καὶ ὥς ἄρα ἡ Λ πρὸς τὴν M , οὕτως τὸ $\Gamma\Theta$ παραλληλόγραμμον πρὸς τὸ ΓZ παραλληλόγραμμον. ἐπεὶ οὖν ἐδείχθη, ὡς μὲν ἡ K πρὸς τὴν Λ , οὕτως τὸ $ΑΓ$ παραλληλόγραμμον πρὸς τὸ $\Gamma\Theta$ παραλληλόγραμμον, ὡς δὲ ἡ Λ πρὸς τὴν M , οὕτως τὸ $\Gamma\Theta$ παραλληλόγραμμον πρὸς τὸ ΓZ παραλληλόγραμμον, δι' ἴσου ἄρα ἐστὶν ὡς ἡ K πρὸς τὴν M , οὕτως τὸ $ΑΓ$ πρὸς τὸ ΓZ παραλληλόγραμμον. ἡ δὲ K πρὸς τὴν M λόγον ἔχει τὸν συγχείμενον ἐκ τῶν πλευρῶν· καὶ τὸ $ΑΓ$ ἄρα πρὸς τὸ ΓZ λόγον ἔχει τὸν συγχείμενον ἐκ τῶν πλευρῶν.



Τὰ ἄρα ἰσογώνια παραλληλόγραμμα πρὸς ἀλλήλα λόγον ἔχει τὸν συγχείμενον ἐκ τῶν πλευρῶν· ὅπερ ἔδει δεῖξαι.

CH (is) to CF [Prop. 6.1], but as DC (is) to CE , so L (is) to M , thus, also, as L (is) to M , so parallelogram CH (is) to parallelogram CF . Therefore, since it was shown that as K (is) to L , so parallelogram AC (is) to parallelogram CH , and as L (is) to M , so parallelogram CH (is) to parallelogram CF , thus, via equality, as K is to M , so (parallelogram) AC (is) to parallelogram CF [Prop. 5.22]. And K has to M the ratio compounded out of (the ratios of) the sides (of the parallelograms). Thus, (parallelogram) AC also has to (parallelogram) CF the ratio compounded out of (the ratio of) their sides.



Thus, equiangular parallelograms have to one another the ratio compounded out of (the ratio of) their sides. (Which is) the very thing it was required to show.

† In modern terminology, if two ratios are “compounded” then they are multiplied together.

κδ'.

Proposition 24

Παντὸς παραλληλογράμμου τὰ περὶ τὴν διάμετρον παραλληλόγραμμα ὁμοία ἐστὶ τῷ τε ὅλῳ καὶ ἀλλήλοις.

Ἐστω παραλληλόγραμμον τὸ $ΑΒΓΔ$, διάμετρος δὲ αὐτοῦ ἡ $ΑΓ$, περὶ δὲ τὴν $ΑΓ$ παραλληλόγραμμα ἔστω τὰ $ΕΗ$, $ΘΚ$. λέγω, ὅτι ἐκάτερον τῶν $ΕΗ$, $ΘΚ$ παραλληλογράμμων ὁμοίον ἐστὶ ὅλῳ τῷ $ΑΒΓΔ$ καὶ ἀλλήλοις.

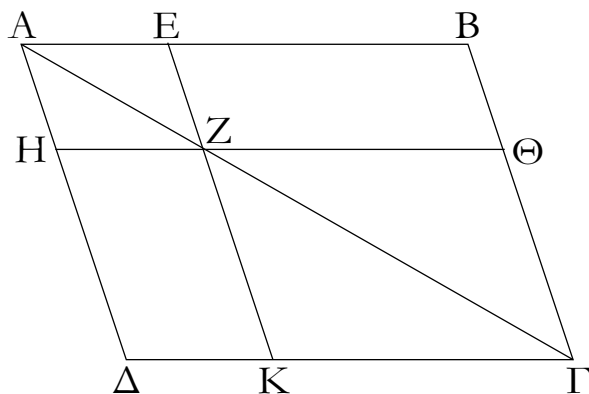
Ἐπεὶ γὰρ τριγώνου τοῦ $ΑΒΓ$ παρὰ μίαν τῶν πλευρῶν τὴν $ΒΓ$ ῥηταὶ ἡ $ΕΖ$, ἀνάλογόν ἐστιν ὡς ἡ $ΒΕ$ πρὸς τὴν $ΕΑ$, οὕτως ἡ $ΓΖ$ πρὸς τὴν $ΖΑ$. πάλιν, ἐπεὶ τριγώνου τοῦ $ΑΓΔ$ παρὰ μίαν τὴν $ΓΔ$ ῥηταὶ ἡ $ΖΗ$, ἀνάλογόν ἐστιν ὡς ἡ $ΓΖ$ πρὸς τὴν $ΖΑ$, οὕτως ἡ $ΔΗ$ πρὸς τὴν $ΗΑ$. ἀλλ' ὡς ἡ $ΓΖ$ πρὸς τὴν $ΖΑ$, οὕτως ἐδείχθη καὶ ἡ $ΒΕ$ πρὸς τὴν $ΕΑ$ · καὶ ὥς ἄρα ἡ $ΒΕ$ πρὸς τὴν $ΕΑ$, οὕτως ἡ $ΔΗ$ πρὸς τὴν $ΗΑ$, καὶ συνθέντι ἄρα ὡς ἡ $ΒΑ$ πρὸς $ΑΕ$, οὕτως ἡ $ΔΑ$ πρὸς $ΑΗ$, καὶ ἐναλλάξ ὡς ἡ $ΒΑ$ πρὸς τὴν $ΑΔ$, οὕτως ἡ $ΕΑ$ πρὸς τὴν $ΑΗ$. τῶν ἄρα $ΑΒΓΔ$, $ΕΗ$ παραλληλογράμμων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὴν κοινὴν γωνίαν τὴν ὑπὸ $ΒΑΔ$. καὶ ἐπεὶ παράλληλός ἐστιν ἡ $ΗΖ$ τῇ $ΔΓ$, ἴση ἐστὶν ἡ μὲν ὑπὸ $ΑΖΗ$ γωνία τῇ ὑπὸ $ΔΓΑ$ · καὶ κοινὴ τῶν δύο

In any parallelogram the parallelograms about the diagonal are similar to the whole, and to one another.

Let $ABCD$ be a parallelogram, and AC its diagonal. And let EG and HK be parallelograms about AC . I say that the parallelograms EG and HK are each similar to the whole (parallelogram) $ABCD$, and to one another.

For since EF has been drawn parallel to one of the sides BC of triangle ABC , proportionally, as BE is to EA , so CF (is) to FA [Prop. 6.2]. Again, since FG has been drawn parallel to one (of the sides) CD of triangle ACD , proportionally, as CF is to FA , so DG (is) to GA [Prop. 6.2]. But, as CF (is) to FA , so it was also shown (is) BE to EA . And thus as BE (is) to EA , so DG (is) to GA . And, thus, compounding, as BA (is) to AE , so DA (is) to AG [Prop. 5.18]. And, alternately, as BA (is) to AD , so EA (is) to AG [Prop. 5.16]. Thus, in parallelograms $ABCD$ and EG the sides about the common angle BAD are proportional. And since GF is parallel to DC , angle AFG is equal to DCA [Prop. 1.29].

τριγώνων τῶν $\triangle A\Delta\Gamma$, $\triangle AHZ$ ἡ ὑπὸ $\triangle A\Gamma$ γωνία· ἰσογώνιον ἄρα ἐστὶ τὸ $\triangle A\Delta\Gamma$ τριγώνον τῷ $\triangle AHZ$ τριγώνῳ. διὰ τὰ αὐτὰ δὴ καὶ τὸ $\triangle A\Gamma B$ τριγώνον ἰσογώνιον ἐστὶ τῷ $\triangle AZE$ τριγώνῳ, καὶ ὅλον τὸ $\triangle AB\Gamma$ παραλληλόγραμμον τῷ $\triangle EH$ παραλληλογράμμῳ ἰσογώνιον ἐστίν. ἀνάλογον ἄρα ἐστὶν ὡς ἡ $A\Delta$ πρὸς τὴν $\Delta\Gamma$, οὕτως ἡ AH πρὸς τὴν HZ , ὡς δὲ ἡ $\Delta\Gamma$ πρὸς τὴν ΓA , οὕτως ἡ HZ πρὸς τὴν ZA , ὡς δὲ ἡ $A\Gamma$ πρὸς τὴν ΓB , οὕτως ἡ AZ πρὸς τὴν ZE , καὶ ἔτι ὡς ἡ ΓB πρὸς τὴν BA , οὕτως ἡ ZE πρὸς τὴν EA . καὶ ἐπεὶ ἐδείχθη ὡς μὲν ἡ $\Delta\Gamma$ πρὸς τὴν ΓA , οὕτως ἡ HZ πρὸς τὴν ZA , ὡς δὲ ἡ $A\Gamma$ πρὸς τὴν ΓB , οὕτως ἡ AZ πρὸς τὴν ZE , δι' ἴσου ἄρα ἐστὶν ὡς ἡ $\Delta\Gamma$ πρὸς τὴν ΓB , οὕτως ἡ HZ πρὸς τὴν ZE . τῶν ἄρα $\triangle AB\Gamma$, $\triangle EH$ παραλληλογράμμων ἀνάλογον εἰσὶν αἱ πλευраὶ αἱ περὶ τὰς ἴσας γωνίας· ὁμοιον ἄρα ἐστὶ τὸ $\triangle AB\Gamma$ παραλληλόγραμμον τῷ $\triangle EH$ παραλληλογράμμῳ. διὰ τὰ αὐτὰ δὴ τὸ $\triangle AB\Gamma$ παραλληλόγραμμον καὶ τῷ $\triangle K\Theta$ παραλληλογράμμῳ ὁμοιον ἐστίν· ἐκάτερον ἄρα τῶν $\triangle EH$, $\triangle K\Theta$ παραλληλογράμμων τῷ $\triangle AB\Gamma$ [παραλληλογράμμῳ] ὁμοιον ἐστίν. τὰ δὲ τῷ αὐτῷ εὐθυγράμμῳ ὁμοια καὶ ἀλλήλοις ἐστὶν ὁμοια· καὶ τὸ $\triangle EH$ ἄρα παραλληλόγραμμον τῷ $\triangle K\Theta$ παραλληλογράμμῳ ὁμοιον ἐστίν.

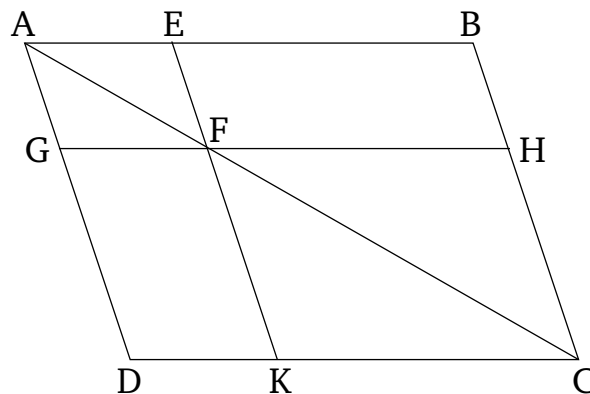


Παντὸς ἄρα παραλληλογράμμου τὰ περὶ τὴν διάμετρον παραλληλόγραμμα ὁμοιά ἐστὶ τῷ τε ὅλῳ καὶ ἀλλήλοις· ὅπερ ἔδει δεῖξαι.

κε'.

Τῷ δοθέντι εὐθυγράμμῳ ὁμοιον καὶ ἄλλῳ τῷ δοθέντι ἴσον τὸ αὐτὸ συστήσασθαι.

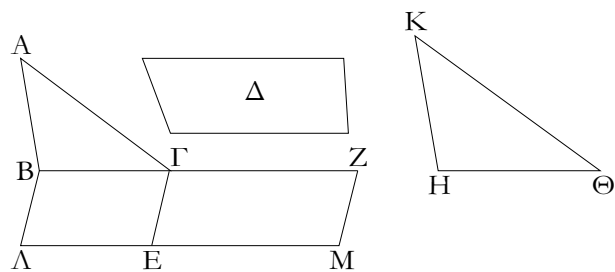
And angle $\angle DAC$ (is) common to the two triangles $\triangle ADC$ and $\triangle AGF$. Thus, triangle $\triangle ADC$ is equiangular to triangle $\triangle AGF$ [Prop. 1.32]. So, for the same (reasons), triangle $\triangle ACB$ is equiangular to triangle $\triangle AFE$, and the whole parallelogram $ABCD$ is equiangular to parallelogram EG . Thus, proportionally, as AD (is) to DC , so AG (is) to GF , and as DC (is) to CA , so GF (is) to FA , and as AC (is) to CB , so AF (is) to FE , and, further, as CB (is) to BA , so FE (is) to EA [Prop. 6.4]. And since it was shown that as DC is to CA , so GF (is) to FA , and as AC (is) to CB , so AF (is) to FE , thus, via equality, as DC is to CB , so GF (is) to FE [Prop. 5.22]. Thus, in parallelograms $ABCD$ and EG the sides about the equal angles are proportional. Thus, parallelogram $ABCD$ is similar to parallelogram EG [Def. 6.1]. So, for the same (reasons), parallelogram $ABCD$ is also similar to parallelogram KH . Thus, parallelograms EG and KH are each similar to [parallelogram] $ABCD$. And (rectilinear figures) similar to the same rectilinear figure are also similar to one another [Prop. 6.21]. Thus, parallelogram EG is also similar to parallelogram KH .



Thus, in any parallelogram the parallelograms about the diagonal are similar to the whole, and to one another. (Which is) the very thing it was required to show.

Proposition 25

To construct a single (rectilinear figure) similar to a given rectilinear figure, and equal to a different given rectilinear figure.



Ἐστω τὸ μὲν δοθὲν εὐθύγραμμον, ᾧ δεῖ ὁμοιον συστήσασθαι, τὸ $AB\Gamma$, ᾧ δὲ δεῖ ἴσον, τὸ Δ . δεῖ δὴ τῶ μὲν $AB\Gamma$ ὁμοιον, τῶ δὲ Δ ἴσον τὸ αὐτὸ συστήσασθαι.

Παραβεβλήσθω γὰρ παρὰ μὲν τὴν $B\Gamma$ τῶ $AB\Gamma$ τριγώνω ἴσον παραλληλόγραμμον τὸ BE , παρὰ δὲ τὴν GE τῶ Δ ἴσον παραλληλόγραμμον τὸ GM ἐν γωνίᾳ τῇ ὑπὸ ZGE , ἥ ἐστίν ἴση τῇ ὑπὸ GBA . ἐπ' εὐθείας ἄρα ἐστὶν ἡ μὲν $B\Gamma$ τῇ ΓZ , ἡ δὲ AE τῇ EM . καὶ εἰλήφθω τῶν $B\Gamma$, ΓZ μέση ἀνάλογον ἡ $H\Theta$, καὶ ἀναγεγράφθω ἀπὸ τῆς $H\Theta$ τῶ $AB\Gamma$ ὁμοίον τε καὶ ὁμοίως κείμενον τὸ $KH\Theta$.

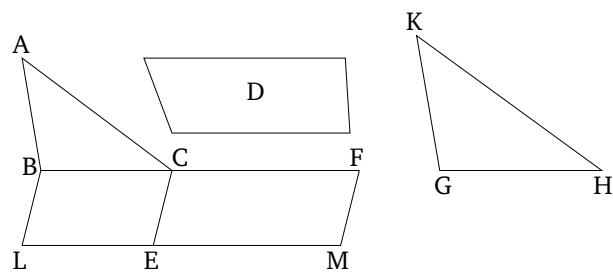
Καὶ ἐπεὶ ἐστὶν ὡς ἡ $B\Gamma$ πρὸς τὴν $H\Theta$, οὕτως ἡ $H\Theta$ πρὸς τὴν ΓZ , ἐὰν δὲ τρεῖς εὐθεῖαι ἀνάλογον ὦσιν, ἔστιν ὡς ἡ πρώτη πρὸς τὴν τρίτην, οὕτως τὸ ἀπὸ τῆς πρώτης εἶδος πρὸς τὸ ἀπὸ τῆς δευτέρας τὸ ὁμοιον καὶ ὁμοίως ἀναγεγόμενον, ἔστιν ἄρα ὡς ἡ $B\Gamma$ πρὸς τὴν ΓZ , οὕτως τὸ $AB\Gamma$ τρίγωνον πρὸς τὸ $KH\Theta$ τρίγωνον. ἀλλὰ καὶ ὡς ἡ $B\Gamma$ πρὸς τὴν ΓZ , οὕτως τὸ BE παραλληλόγραμμον πρὸς τὸ EZ παραλληλόγραμμον. καὶ ὡς ἄρα τὸ $AB\Gamma$ τρίγωνον πρὸς τὸ $KH\Theta$ τρίγωνον, οὕτως τὸ BE παραλληλόγραμμον πρὸς τὸ EZ παραλληλόγραμμον. ἐναλλάξ ἄρα ὡς τὸ $AB\Gamma$ τρίγωνον πρὸς τὸ BE παραλληλόγραμμον, οὕτως τὸ $KH\Theta$ τρίγωνον πρὸς τὸ EZ παραλληλόγραμμον. ἴσον δὲ τὸ $AB\Gamma$ τρίγωνον τῶ BE παραλληλογράμμῳ· ἴσον ἄρα καὶ τὸ $KH\Theta$ τρίγωνον τῶ EZ παραλληλογράμμῳ. ἀλλὰ τὸ EZ παραλληλόγραμμον τῶ Δ ἐστὶν ἴσον· καὶ τὸ $KH\Theta$ ἄρα τῶ Δ ἐστὶν ἴσον. ἔστι δὲ τὸ $KH\Theta$ καὶ τῶ $AB\Gamma$ ὁμοιον.

Τῶ ἄρα δοθέντι εὐθυγράμμῳ τῶ $AB\Gamma$ ὁμοιον καὶ ἄλλω τῶ δοθέντι τῶ Δ ἴσον τὸ αὐτὸ συνέσταται τὸ $KH\Theta$. ὅπερ ἔδει ποιῆσαι.

κς'.

Ἐὰν ἀπὸ παραλληλογράμμου παραλληλόγραμμον ἀφαιρεθῇ ὁμοίον τε τῶ ὅλῳ καὶ ὁμοίως κείμενον κοινὴν γωνίαν ἔχον αὐτῶ, περὶ τὴν αὐτὴν διάμετρον ἔστι τῶ ὅλῳ.

Ἀπὸ γὰρ παραλληλογράμμου τοῦ $AB\Gamma\Delta$ παραλληλόγραμμον ἀφηρήσθω τὸ AZ ὁμοιον τῶ $AB\Gamma\Delta$ καὶ ὁμοίως κείμενον κοινὴν γωνίαν ἔχον αὐτῶ τὴν ὑπὸ ΔAB . λέγω,



Let ABC be the given rectilinear figure to which it is required to construct a similar (rectilinear figure), and D the (rectilinear figure) to which (the constructed figure) is required (to be) equal. So it is required to construct a single (rectilinear figure) similar to ABC , and equal to D .

For let the parallelogram BE , equal to triangle ABC , have been applied to (the straight-line) BC [Prop. 1.44], and the parallelogram CM , equal to D , (have been applied) to (the straight-line) CE , in the angle FCE , which is equal to CBL [Prop. 1.45]. Thus, BC is straight-on to CF , and LE to EM [Prop. 1.14]. And let the mean proportion GH have been taken of BC and CF [Prop. 6.13]. And let KGH , similar, and similarly laid out, to ABC have been described on GH [Prop. 6.18].

And since as BC is to GH , so GH (is) to CF , and if three straight-lines are proportional then as the first is to the third, so the figure (described) on the first (is) to the similar, and similarly described, (figure) on the second [Prop. 6.19 corr.], thus as BC is to CF , so triangle ABC (is) to triangle KGH . But, also, as BC (is) to CF , so parallelogram BE (is) to parallelogram EF [Prop. 6.1]. And, thus, as triangle ABC (is) to triangle KGH , so parallelogram BE (is) to parallelogram EF . Thus, alternately, as triangle ABC (is) to parallelogram BE , so triangle KGH (is) to parallelogram EF [Prop. 5.16]. And triangle ABC (is) equal to parallelogram BE . Thus, triangle KGH (is) also equal to parallelogram EF . But, parallelogram EF is equal to D . Thus, KGH is also equal to D . And KGH is also similar to ABC .

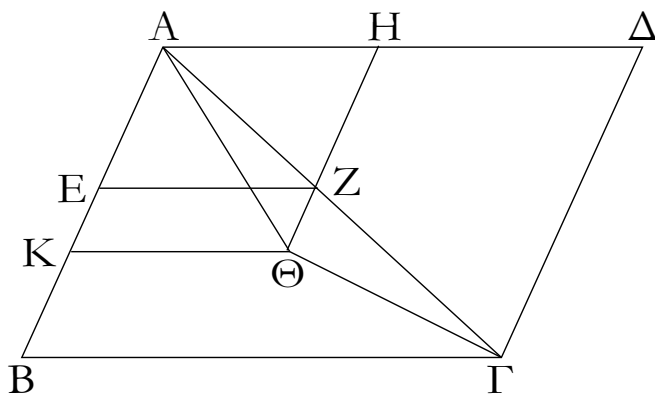
Thus, a single (rectilinear figure) KGH has been constructed (which is) similar to the given rectilinear figure ABC , and equal to a different given (rectilinear figure) D . (Which is) the very thing it was required to do.

Proposition 26

If from a parallelogram a(nother) parallelogram is subtracted (which is) similar, and similarly laid out, to the whole, having a common angle with it, then (the subtracted parallelogram) is about the same diagonal as the whole.

For, from parallelogram $ABCD$, let (parallelogram)

ὅτι περὶ τὴν αὐτὴν διάμετρον ἐστὶ τὸ $AB\Gamma\Delta$ τῷ AZ .



Μὴ γάρ, ἀλλ' εἰ δυνατόν, ἔστω [αὐτῶν] διάμετρος ἡ $A\Theta\Gamma$, καὶ ἐκβληθεῖσα ἡ HZ διήχθω ἐπὶ τὸ Θ , καὶ ἤχθω διὰ τοῦ Θ ὁπορέρα τῶν $A\Delta$, $B\Gamma$ παράλληλος ἡ ΘK .

Ἐπεὶ οὖν περὶ τὴν αὐτὴν διάμετρον ἐστὶ τὸ $AB\Gamma\Delta$ τῷ KH , ἔστιν ἄρα ὡς ἡ ΔA πρὸς τὴν AB , οὕτως ἡ HA πρὸς τὴν AK . ἔστι δὲ καὶ διὰ τὴν ὁμοιότητα τῶν $AB\Gamma\Delta$, EH καὶ ὡς ἡ ΔA πρὸς τὴν AB , οὕτως ἡ HA πρὸς τὴν AE · καὶ ὡς ἄρα ἡ HA πρὸς τὴν AK , οὕτως ἡ HA πρὸς τὴν AE . ἡ HA ἄρα πρὸς ἑκατέραν τῶν AK , AE τὸν αὐτὸν ἔχει λόγον. ἴση ἄρα ἐστὶν ἡ AE τῇ AK ἢ ἐλάττω τῇ μείζονι· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα οὐκ ἐστὶ περὶ τὴν αὐτὴν διάμετρον τὸ $AB\Gamma\Delta$ τῷ AZ · περὶ τὴν αὐτὴν ἄρα ἐστὶ διάμετρον τὸ $AB\Gamma\Delta$ παραλληλόγραμμον τῷ AZ παραλληλογράμμῳ.

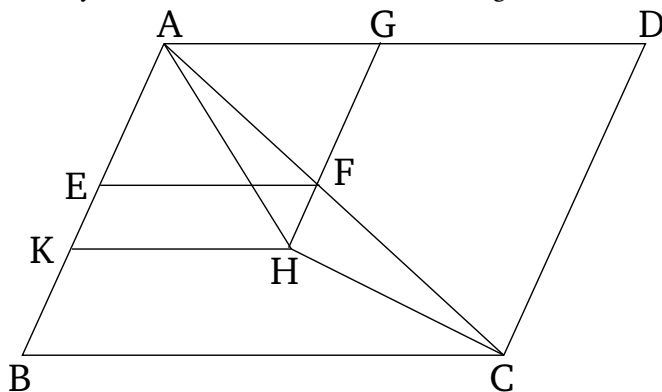
Ἐὰν ἄρα ἀπὸ παραλληλογράμμου παραλληλόγραμμον ἀφαιρεθῇ ὁμοίον τε τῷ ὅλῳ καὶ ὁμοίως κείμενον κοινὴν γωνίαν ἔχον αὐτῷ, περὶ τὴν αὐτὴν διάμετρον ἐστὶ τῷ ὅλῳ· ὅπερ ἔδει δεῖξαι.

κζ'.

Πάντων τῶν παρὰ τὴν αὐτὴν εὐθεῖαν παραβαλλομένων παραλληλογράμμων καὶ ἐλλειπόντων εἶδεσι παραλληλογράμμοις ὁμοίοις τε καὶ ὁμοίως κειμένους τῷ ἀπὸ τῆς ἡμισείας ἀναγραφόμενῳ μέγιστόν ἐστὶ τὸ ἀπὸ τῆς ἡμισείας παραβαλλόμενον [παραλληλόγραμμον] ὁμοίον δὲ τῷ ἐλλείμμενῳ.

Ἐστω εὐθεῖα ἡ AB καὶ τετμήσθω δίχα κατὰ τὸ Γ , καὶ παραβεβλήσθω παρὰ τὴν AB εὐθεῖαν τὸ $A\Delta$ παραλληλόγραμμον ἐλλείπον εἶδει παραλληλογράμμῳ τῷ ΔB ἀναγραφέντι ἀπὸ τῆς ἡμισείας τῆς AB , τουτέστι τῆς ΓB · λέγω, ὅτι πάντων τῶν παρὰ τὴν AB παραβαλλομένων παραλληλογράμμων καὶ ἐλλειπόντων εἶδεσι [παραλληλογράμμοις] ὁμοίοις τε καὶ ὁμοίως κειμένους τῷ ΔB μέγιστόν ἐστὶ τὸ

AF have been subtracted (which is) similar, and similarly laid out, to $ABCD$, having the common angle DAB with it. I say that $ABCD$ is about the same diagonal as AF .



For (if) not, then, if possible, let AHC be [$ABCD$'s] diagonal. And producing GF , let it have been drawn through (point) H . And let HK have been drawn through (point) H , parallel to either of AD or BC [Prop. 1.31].

Therefore, since $ABCD$ is about the same diagonal as KG , thus as DA is to AB , so GA (is) to AK [Prop. 6.24]. And, on account of the similarity of $ABCD$ and EG , also, as DA (is) to AB , so GA (is) to AE . Thus, also, as GA (is) to AK , so GA (is) to AE . Thus, GA has the same ratio to each of AK and AE . Thus, AE is equal to AK [Prop. 5.9], the lesser to the greater. The very thing is impossible. Thus, $ABCD$ is not not about the same diagonal as AF . Thus, parallelogram $ABCD$ is about the same diagonal as parallelogram AF .

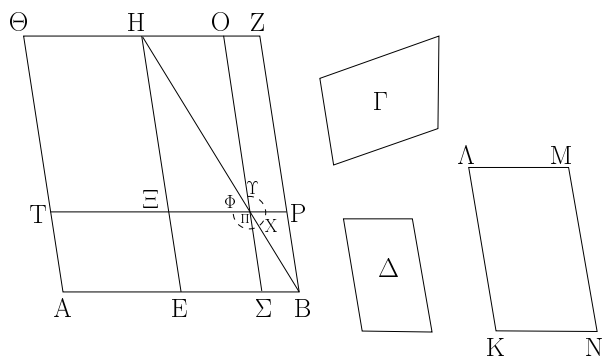
Thus, if from a parallelogram a (nother) parallelogram is subtracted (which is) similar, and similarly laid out, to the whole, having a common angle with it, then (the subtracted parallelogram) is about the same diagonal as the whole. (Which is) the very thing it was required to show.

Proposition 27

Of all the parallelograms applied to the same straight-line, and falling short by parallelogrammic figures similar, and similarly laid out, to the (parallelogram) described on half (the straight-line), the greatest is the [parallelogram] applied to half (the straight-line) which (is) similar to (that parallelogram) by which it falls short.

Let AB be a straight-line, and let it have been cut in half at (point) C [Prop. 1.10]. And let the parallelogram AD have been applied to the straight-line AB , falling short by the parallelogrammic figure DB (which is) applied to half of AB —that is to say, CB . I say that of all the parallelograms applied to AB , and falling short by

παρὰ τὴν δοθεῖσαν εὐθεῖαν τὴν AB τῷ δοθέντι εὐθυγράμμῳ τῷ Γ ἴσον παραλληλόγραμμον παραβαλεῖν ἐλλείπον εἶδει παραλληλογράμμῳ ὁμοίῳ ὄντι τῷ Δ .



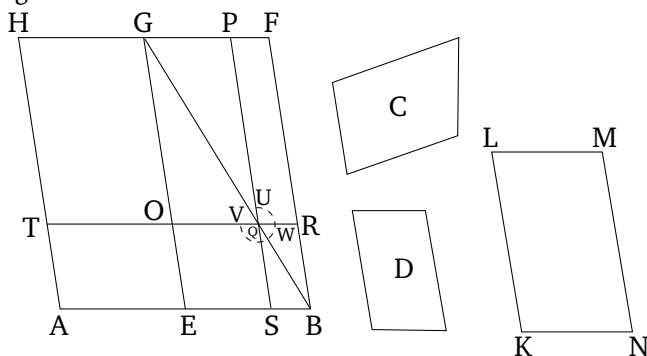
Τετμήσθω ἡ AB δίχα κατὰ τὸ E σημεῖον, καὶ ἀναγεγράφθω ἀπὸ τῆς EB τῷ Δ ὁμοιον καὶ ὁμοίως κείμενον τὸ $EBZH$, καὶ συμπεπληρώσθω τὸ AH παραλληλόγραμμον.

Εἰ μὲν οὖν ἴσον ἐστὶ τὸ AH τῷ Γ , γεγονός ἂν εἴη τὸ ἐπιταχθέν· παραβέβληται γὰρ παρὰ τὴν δοθεῖσαν εὐθεῖαν τὴν AB τῷ δοθέντι εὐθυγράμμῳ τῷ Γ ἴσον παραλληλόγραμμον τὸ AH ἐλλείπον εἶδει παραλληλογράμμῳ τῷ HB ὁμοίῳ ὄντι τῷ Δ . εἰ δὲ οὐ, μείζον ἔστω τὸ ΘE τοῦ Γ . ἴσον δὲ τὸ ΘE τῷ HB · μείζον ἄρα καὶ τὸ HB τοῦ Γ . ὅ δὲ μείζον ἐστὶ τὸ HB τοῦ Γ , ταύτῃ τῇ ὑπεροχῇ ἴσον, τῷ δὲ Δ ὁμοιον καὶ ὁμοίως κείμενον τὸ αὐτὸ συνεστάτω τὸ $KLMN$. ἀλλὰ τὸ Δ τῷ HB [ἐστίν] ὁμοιον· καὶ τὸ KM ἄρα τῷ HB ἐστὶν ὁμοιον. ἔστω οὖν ὁμόλογος ἡ μὲν KA τῇ HE , ἡ δὲ AM τῇ HZ . καὶ ἐπεὶ ἴσον ἐστὶ τὸ HB τοῖς Γ , KM , μείζον ἄρα ἐστὶ τὸ HB τοῦ KM · μείζων ἄρα ἐστὶ καὶ ἡ μὲν HE τῆς KA , ἡ δὲ HZ τῆς AM . κείσθω τῇ μὲν KA ἴση ἡ HE , τῇ δὲ AM ἴση ἡ HO , καὶ συμπεπληρώσθω τὸ $\Xi H O \Pi$ παραλληλόγραμμον· ἴσον ἄρα καὶ ὁμοιον ἐστὶ [τὸ $H \Pi$] τῷ KM [ἀλλὰ τὸ KM τῷ HB ὁμοιον ἐστίν]. καὶ τὸ $H \Pi$ ἄρα τῷ HB ὁμοιον ἐστίν· περὶ τὴν αὐτὴν ἄρα διάμετρον ἐστὶ τὸ $H \Pi$ τῷ HB . ἔστω αὐτῶν διάμετρος ἡ $H \Pi B$, καὶ καταγεγράφθω τὸ σχῆμα.

Ἐπεὶ οὖν ἴσον ἐστὶ τὸ BH τοῖς Γ , KM , ὣν τὸ $H \Pi$ τῷ KM ἐστὶν ἴσον, λοιπὸς ἄρα ὁ $\Gamma X \Phi$ γνόμενος λοιπῷ τῷ Γ ἴσος ἐστίν. καὶ ἐπεὶ ἴσον ἐστὶ τὸ OP τῷ $\Xi \Sigma$, κοινὸν προσκείσθω τὸ ΠB · ὅλον ἄρα τὸ OB ὅλῳ τῷ ΞB ἴσον ἐστίν. ἀλλὰ τὸ ΞB τῷ TE ἐστὶν ἴσον, ἐπεὶ καὶ πλευρὰ ἡ AE πλευρᾷ τῇ EB ἐστὶν ἴση· καὶ τὸ TE ἄρα τῷ OB ἐστὶν ἴσον. κοινὸν προσκείσθω τὸ $\Xi \Sigma$ · ὅλον ἄρα τὸ $T \Sigma$ ὅλῳ τῷ $\Phi X \Upsilon$ γνόμενόν ἐστιν ἴσον. ἀλλ' ὁ $\Phi X \Upsilon$ γνόμενος τῷ Γ ἐδείχθη ἴσος· καὶ τὸ $T \Sigma$ ἄρα τῷ Γ ἐστὶν ἴσον.

Παρὰ τὴν δοθεῖσαν ἄρα εὐθεῖαν τὴν AB τῷ δοθέντι εὐθυγράμμῳ τῷ Γ ἴσον παραλληλόγραμμον παραβέβληται τὸ ΣT ἐλλείπον εἶδει παραλληλογράμμῳ τῷ ΠB ὁμοίῳ ὄντι

AB is required (to be) equal, [being] not greater than the (parallelogram) described on half of AB and similar to the deficit, and D the (parallelogram) to which the deficit is required (to be) similar. So it is required to apply a parallelogram, equal to the given rectilinear figure C , to the straight-line AB , falling short by a parallelogrammic figure which is similar to D .



Let AB have been cut in half at point E [Prop. 1.10], and let (parallelogram) $EBFG$, (which is) similar, and similarly laid out, to (parallelogram) D , have been described on EB [Prop. 6.18]. And let parallelogram AG have been completed.

Therefore, if AG is equal to C then the thing prescribed has happened. For a parallelogram AG , equal to the given rectilinear figure C , has been applied to the given straight-line AB , falling short by a parallelogrammic figure GB which is similar to D . And if not, let HE be greater than C . And HE (is) equal to GB [Prop. 6.1]. Thus, GB (is) also greater than C . So, let (parallelogram) $KLMN$ have been constructed (so as to be) both similar, and similarly laid out, to D , and equal to the excess by which GB is greater than C [Prop. 6.25]. But, GB [is] similar to D . Thus, KM is also similar to GB [Prop. 6.21]. Therefore, let KL correspond to GE , and LM to GF . And since (parallelogram) GB is equal to (figure) C and (parallelogram) KM , GB is thus greater than KM . Thus, GE is also greater than KL , and GF than LM . Let GO be made equal to KL , and GP to LM [Prop. 1.3]. And let the parallelogram $OGPQ$ have been completed. Thus, [GQ] is equal and similar to KM [but, KM is similar to GB]. Thus, GQ is also similar to GB [Prop. 6.21]. Thus, GQ and GB are about the same diagonal [Prop. 6.26]. Let GQB be their (common) diagonal, and let the (remainder of the) figure have been described.

Therefore, since BG is equal to C and KM , of which GQ is equal to KM , the remaining gnomon UWV is thus equal to the remainder C . And since (the complement) PR is equal to (the complement) OS [Prop. 1.43], let (parallelogram) QB have been added to both. Thus, the whole (parallelogram) PB is equal to the whole (par-

τῷ Δ [ἐπειδήπερ τὸ ΠB τῷ $ΗΠ$ ὁμοίον ἐστίν]· ὅπερ ἔδει ποιῆσαι.

allegorism) OB . But, OB is equal to TE , since side AE is equal to side EB [Prop. 6.1]. Thus, TE is also equal to PB . Let (parallelogram) OS have been added to both. Thus, the whole (parallelogram) TS is equal to the gnomon VWU . But, gnomon VWU was shown (to be) equal to C . Therefore, (parallelogram) TS is also equal to (figure) C .

Thus, the parallelogram ST , equal to the given rectilinear figure C , has been applied to the given straight-line AB , falling short by the parallelogrammic figure QB , which is similar to D [inasmuch as QB is similar to GQ [Prop. 6.24]]. (Which is) the very thing it was required to do.

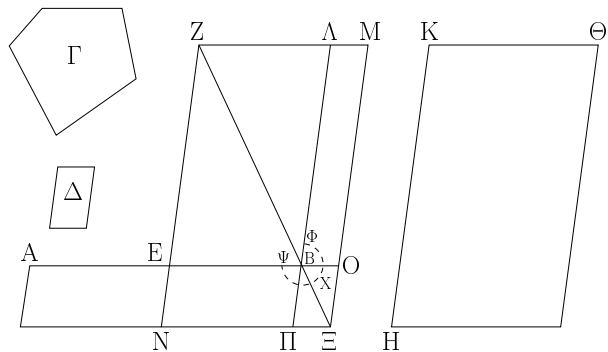
† This proposition is a geometric solution of the quadratic equation $x^2 - \alpha x + \beta = 0$. Here, x is the ratio of a side of the deficit to the corresponding side of figure D , α is the ratio of the length of AB to the length of that side of figure D which corresponds to the side of the deficit running along AB , and β is the ratio of the areas of figures C and D . The constraint corresponds to the condition $\beta < \alpha^2/4$ for the equation to have real roots. Only the smaller root of the equation is found. The larger root can be found by a similar method.

κθ'.

Proposition 29†

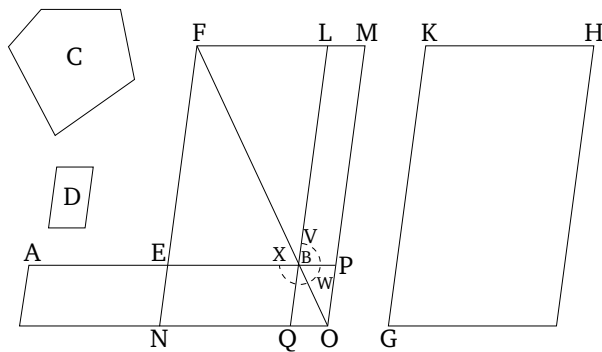
Παρά τὴν δοθεῖσαν εὐθεῖαν τῷ δοθέντι εὐθύγραμμῳ ἴσον παραλληλόγραμμον παραβαλεῖν ὑπερβάλλον εἶδει παραλληλογράμμῳ ὁμοίῳ τῷ δοθέντι.

To apply a parallelogram, equal to a given rectilinear figure, to a given straight-line, (the applied parallelogram) overshooting by a parallelogrammic figure similar to a given (parallelogram).



Ἐστω ἡ μὲν δοθεῖσα εὐθεῖα ἡ AB , τὸ δὲ δοθέν εὐθύγραμμον, ὃ δεῖ ἴσον παρὰ τὴν AB παραβαλεῖν, τὸ Γ , ὃ δὲ δεῖ ὁμοίον ὑπερβάλλειν, τὸ Δ . δὲ δὴ παρὰ τὴν AB εὐθεῖαν τῷ Γ εὐθύγραμμῳ ἴσον παραλληλόγραμμον παραβαλεῖν ὑπερβάλλον εἶδει παραλληλογράμμῳ ὁμοίῳ τῷ Δ .

Τετμήσθω ἡ AB δίχα κατὰ τὸ E , καὶ ἀναγεγράφθω ἀπὸ τῆς EB τῷ Δ ὁμοίον καὶ ὁμοίως κείμενον παραλληλόγραμμον τὸ BZ , καὶ συναμφοτέροις μὲν τοῖς BZ , Γ ἴσον, τῷ δὲ Δ ὁμοίον καὶ ὁμοίως κείμενον τὸ αὐτὸ συνεστάτω τὸ $H\Theta$. ὁμόλογος δὲ ἔστω ἡ μὲν $K\Theta$ τῇ ZA , ἡ δὲ KH τῇ ZE . καὶ ἐπεὶ μείζον ἐστὶ τὸ $H\Theta$ τοῦ ZB , μείζων ἄρα ἐστὶ καὶ ἡ μὲν $K\Theta$ τῆς ZA , ἡ δὲ KH τῇ ZE . ἐκβεβλήσθωσαν αἱ ZA , ZE , καὶ τῇ μὲν $K\Theta$ ἴση ἔστω ἡ ZAM , τῇ δὲ KH ἴση ἡ ZEN , καὶ συμπεπληρώσθω τὸ MN . τὸ MN ἄρα τῷ $H\Theta$ ἴσον τέ ἐστὶ καὶ ὁμοίον. ἀλλὰ τὸ $H\Theta$ τῷ EL ἐστὶν ὁμοίον.



Let AB be the given straight-line, and C the given rectilinear figure to which the (parallelogram) applied to AB is required (to be) equal, and D the (parallelogram) to which the excess is required (to be) similar. So it is required to apply a parallelogram, equal to the given rectilinear figure C , to the given straight-line AB , overshooting by a parallelogrammic figure similar to D .

Let AB have been cut in half at (point) E [Prop. 1.10], and let the parallelogram BF , (which is) similar, and similarly laid out, to D , have been described on EB [Prop. 6.18]. And let (parallelogram) GH have been constructed (so as to be) both similar, and similarly laid out, to D , and equal to the sum of BF and C [Prop. 6.25]. And let KH correspond to FL , and KG to FE . And since (parallelogram) GH is greater than (parallelogram) FB ,

καὶ τὸ MN ἄρα τῷ EL ὁμοίον ἐστίν· περὶ τὴν αὐτὴν ἄρα διάμετρον ἐστὶ τὸ EL τῷ MN . ἤχθω αὐτῶν διάμετρος ἡ $ZΞ$, καὶ καταγεγράφθω τὸ σχῆμα.

Ἐπεὶ ἴσον ἐστὶ τὸ $HΘ$ τοῖς $EL, Γ$, ἀλλὰ τὸ $HΘ$ τῷ MN ἴσον ἐστίν, καὶ τὸ MN ἄρα τοῖς $EL, Γ$ ἴσον ἐστίν. κοινὸν ἀφηρήσθω τὸ EL · λοιπὸς ἄρα ὁ $ΨΧΦ$ γνῶμων τῷ $Γ$ ἐστὶν ἴσος. καὶ ἐπεὶ ἴση ἐστὶν ἡ AE τῇ EB , ἴσον ἐστὶ καὶ τὸ AN τῷ NB , τουτέστι τῷ $ΛΟ$. κοινὸν προσκείσθω τὸ $EΞ$ · ὅλον ἄρα τὸ $AΞ$ ἴσον ἐστὶ τῷ $ΦΧΨ$ γνῶμονι. ἀλλὰ ὁ $ΦΧΨ$ γνῶμων τῷ $Γ$ ἴσος ἐστίν· καὶ τὸ $AΞ$ ἄρα τῷ $Γ$ ἴσον ἐστίν.

Παρὰ τὴν δοθεῖσαν ἄρα εὐθεῖαν τὴν AB τῷ δοθέντι εὐθυγράμμῳ τῷ $Γ$ ἴσον παραλληλόγραμμον παραβέβληται τὸ $AΞ$ ὑπερβάλλον εἶδει παραλληλογράμμῳ τῷ $ΠΟ$ ὁμοίῳ ὄντι τῷ $Δ$, ἐπεὶ καὶ τῷ EL ἐστὶν ὁμοίον τὸ $ΟΠ$ · ὅπερ ἔδει ποιῆσαι.

KH is thus also greater than FL , and KG than FE . Let FL and FE have been produced, and let FLM be (made) equal to KH , and FEN to KG [Prop. 1.3]. And let (parallelogram) MN have been completed. Thus, MN is equal and similar to GH . But, GH is similar to EL . Thus, MN is also similar to EL [Prop. 6.21]. EL is thus about the same diagonal as MN [Prop. 6.26]. Let their (common) diagonal FO have been drawn, and let the (remainder of the) figure have been described.

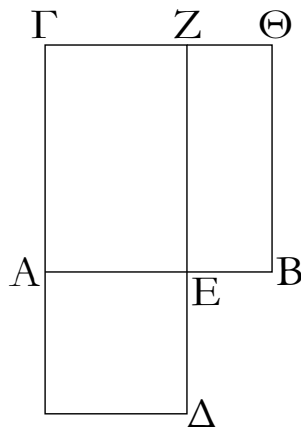
And since (parallelogram) GH is equal to (parallelogram) EL and (figure) C , but GH is equal to (parallelogram) MN , MN is thus also equal to EL and C . Let EL have been subtracted from both. Thus, the remaining gnomon XWV is equal to (figure) C . And since AE is equal to EB , (parallelogram) AN is also equal to (parallelogram) NB [Prop. 6.1], that is to say, (parallelogram) LP [Prop. 1.43]. Let (parallelogram) EO have been added to both. Thus, the whole (parallelogram) AO is equal to the gnomon VWX . But, the gnomon VWX is equal to (figure) C . Thus, (parallelogram) AO is also equal to (figure) C .

Thus, the parallelogram AO , equal to the given rectilinear figure C , has been applied to the given straight-line AB , overshooting by the parallelogrammic figure QP which is similar to D , since PQ is also similar to EL [Prop. 6.24]. (Which is) the very thing it was required to do.

† This proposition is a geometric solution of the quadratic equation $x^2 + \alpha x - \beta = 0$. Here, x is the ratio of a side of the excess to the corresponding side of figure D , α is the ratio of the length of AB to the length of that side of figure D which corresponds to the side of the excess running along AB , and β is the ratio of the areas of figures C and D . Only the positive root of the equation is found.

λ'.

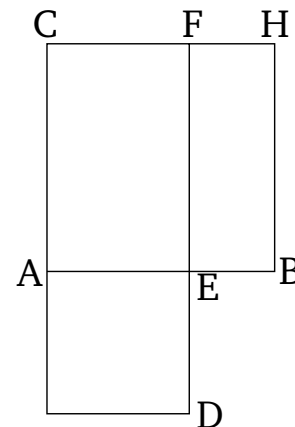
Τὴν δοθεῖσαν εὐθεῖαν πεπερασμένην ἄκρον καὶ μέσον λόγον τεμεῖν.



Ἐστω ἡ δοθεῖσα εὐθεῖα πεπερασμένη ἡ AB · δεῖ δὴ τὴν AB εὐθεῖαν ἄκρον καὶ μέσον λόγον τεμεῖν.

Proposition 30†

To cut a given finite straight-line in extreme and mean ratio.



Let AB be the given finite straight-line. So it is required to cut the straight-line AB in extreme and mean

Ἀναγεγράφθω ἀπὸ τῆς AB τετράγωνον τὸ $BΓ$, καὶ παραβεβλήσθω παρὰ τὴν $ΑΓ$ τῷ $BΓ$ ἴσον παραλληλόγραμμον τὸ $ΓΔ$ ὑπερβάλλον εἶδει τῷ $ΑΔ$ ὁμοίῳ τῷ $BΓ$.

Τετράγωνον δέ ἐστι τὸ $BΓ$ · τετράγωνον ἄρα ἐστὶ καὶ τὸ $ΑΔ$. καὶ ἐπεὶ ἴσον ἐστὶ τὸ $BΓ$ τῷ $ΓΔ$, κοινὸν ἀφηρήσθω τὸ $ΓΕ$ · λοιπὸν ἄρα τὸ $BΖ$ λοιπῷ τῷ $ΑΔ$ ἐστὶν ἴσον. ἐστὶ δὲ αὐτῷ καὶ ἰσογώνιον· τῶν $BΖ$, $ΑΔ$ ἄρα ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας· ἐστὶν ἄρα ὡς ἡ $ΖΕ$ πρὸς τὴν $ΕΔ$, οὕτως ἡ $ΑΕ$ πρὸς τὴν $ΕΒ$. ἴση δὲ ἡ μὲν $ΖΕ$ τῇ $ΑΒ$, ἡ δὲ $ΕΔ$ τῇ $ΑΕ$. ἐστὶν ἄρα ὡς ἡ $ΒΑ$ πρὸς τὴν $ΑΕ$, οὕτως ἡ $ΑΕ$ πρὸς τὴν $ΕΒ$. μείζων δὲ ἡ $ΑΒ$ τῆς $ΑΕ$ · μείζων ἄρα καὶ ἡ $ΑΕ$ τῆς $ΕΒ$.

Ἡ ἄρα $ΑΒ$ εὐθεῖα ἄκρον καὶ μέσον λόγον τέμνεται κατὰ τὸ $Ε$, καὶ τὸ μείζον αὐτῆς τμήμα ἐστὶ τὸ $ΑΕ$ · ὅπερ ἔδει ποιῆσαι.

ratio.

Let the square BC have been described on AB [Prop. 1.46], and let the parallelogram CD , equal to BC , have been applied to AC , overshooting by the figure AD (which is) similar to BC [Prop. 6.29].

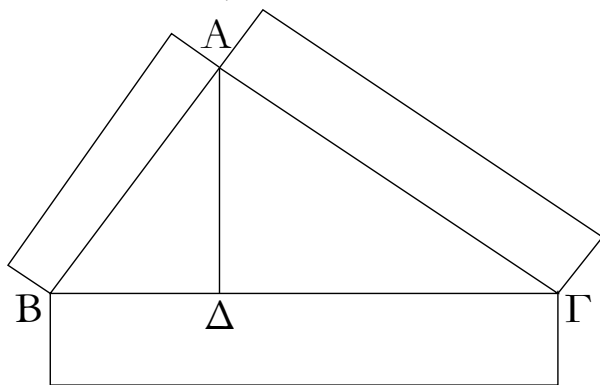
And BC is a square. Thus, AD is also a square. And since BC is equal to CD , let (rectangle) CE have been subtracted from both. Thus, the remaining (rectangle) BF is equal to the remaining (square) AD . And it is also equiangular to it. Thus, the sides of BF and AD about the equal angles are reciprocally proportional [Prop. 6.14]. Thus, as FE is to ED , so AE (is) to EB . And FE (is) equal to AB , and ED to AE . Thus, as BA is to AE , so AE (is) to EB . And AB (is) greater than AE . Thus, AE (is) also greater than EB [Prop. 5.14].

Thus, the straight-line AB has been cut in extreme and mean ratio at E , and AE is its greater piece. (Which is) the very thing it was required to do.

† This method of cutting a straight-line is sometimes called the “Golden Section”—see Prop. 2.11.

λα'.

Ἐν τοῖς ὀρθογωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτείνουσας πλευρᾶς εἶδος ἴσον ἐστὶ τοῖς ἀπὸ τῶν τὴν ὀρθὴν γωνίαν περιεχουσῶν πλευρῶν εἶδεσι τοῖς ὁμοίοις τε καὶ ὁμοίως ἀναγραφόμενοις.



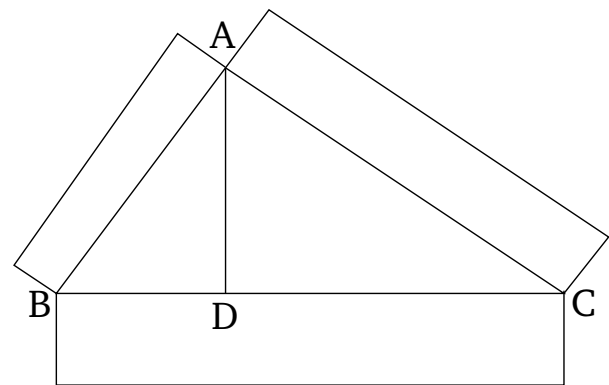
Ἐστω τρίγωνον ὀρθογώνιον τὸ $ΑΒΓ$ ὀρθὴν ἔχον τὴν ὑπὸ $ΒΑΓ$ γωνίαν· λέγω, ὅτι τὸ ἀπὸ τῆς $ΒΓ$ εἶδος ἴσον ἐστὶ τοῖς ἀπὸ τῶν $ΒΑ$, $ΑΓ$ εἶδεσι τοῖς ὁμοίοις τε καὶ ὁμοίως ἀναγραφόμενοις.

Ἦχθω κάθετος ἡ $ΑΔ$.

Ἐπεὶ οὖν ἐν ὀρθογωνίῳ τριγώνῳ τῷ $ΑΒΓ$ ἀπὸ τῆς πρὸς τῷ $Α$ ὀρθῆς γωνίας ἐπὶ τὴν $ΒΓ$ βάσιν κάθετος ἦκται ἡ $ΑΔ$, τὰ $ΑΒΔ$, $ΑΔΓ$ πρὸς τῇ καθετῷ τρίγωνα ὁμοιά ἐστὶ τῷ τε ὅλῳ τῷ $ΑΒΓ$ καὶ ἀλλήλοις. καὶ ἐπεὶ ὁμοίον ἐστὶ τὸ $ΑΒΓ$ τῷ $ΑΒΔ$, ἐστὶν ἄρα ὡς ἡ $ΓΒ$ πρὸς τὴν $ΒΑ$, οὕτως ἡ $ΑΒ$ πρὸς τὴν $ΒΔ$. καὶ ἐπεὶ τρεῖς εὐθεῖαι ἀνάλογόν εἰσιν, ἐστὶν ὡς ἡ πρώτη πρὸς τὴν τρίτην, οὕτως τὸ ἀπὸ τῆς πρώτης εἶδος πρὸς

Proposition 31

In right-angled triangles, the figure (drawn) on the side subtending the right-angle is equal to the (sum of the) similar, and similarly described, figures on the sides surrounding the right-angle.



Let ABC be a right-angled triangle having the angle BAC a right-angle. I say that the figure (drawn) on BC is equal to the (sum of the) similar, and similarly described, figures on BA and AC .

Let the perpendicular AD have been drawn [Prop. 1.12].

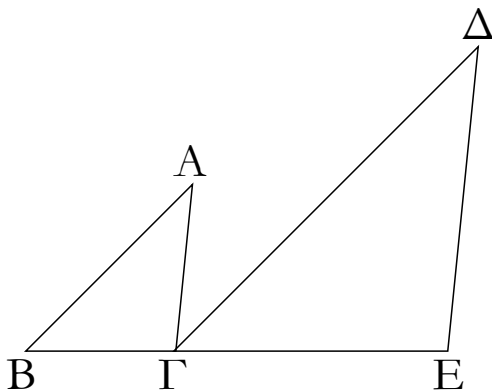
Therefore, since, in the right-angled triangle ABC , the (straight-line) AD has been drawn from the right-angle at A perpendicular to the base BC , the triangles ABD and ADC about the perpendicular are similar to the whole (triangle) ABC , and to one another [Prop. 6.8]. And since ABC is similar to ABD , thus

τὸ ἀπὸ τῆς δευτέρας τὸ ὁμοιον καὶ ὁμοίως ἀναγραφόμενον. ὥς ἄρα ἡ ΓΒ πρὸς τὴν ΒΔ, οὕτως τὸ ἀπὸ τῆς ΓΒ εἶδος πρὸς τὸ ἀπὸ τῆς ΒΑ τὸ ὁμοιον καὶ ὁμοίως ἀναγραφόμενον. διὰ τὰ αὐτὰ δὴ καὶ ὥς ἡ ΒΓ πρὸς τὴν ΓΔ, οὕτως τὸ ἀπὸ τῆς ΒΓ εἶδος πρὸς τὸ ἀπὸ τῆς ΓΑ. ὥστε καὶ ὥς ἡ ΒΓ πρὸς τὰς ΒΔ, ΔΓ, οὕτως τὸ ἀπὸ τῆς ΒΓ εἶδος πρὸς τὰ ἀπὸ τῶν ΒΑ, ΑΓ τὰ ὁμοια καὶ ὁμοίως ἀναγραφόμενα. ἴση δὲ ἡ ΒΓ ταῖς ΒΔ, ΔΓ· ἴσον ἄρα καὶ τὸ ἀπὸ τῆς ΒΓ εἶδος τοῖς ἀπὸ τῶν ΒΑ, ΑΓ εἶδεσι τοῖς ὁμοίοις τε καὶ ὁμοίως ἀναγραφόμενοις.

Ἐν ἄρα τοῖς ὀρθογωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτείνουσας πλευρᾶς εἶδος ἴσον ἐστὶ τοῖς ἀπὸ τῶν τὴν ὀρθὴν γωνίαν περιεχουσῶν πλευρῶν εἶδεσι τοῖς ὁμοίοις τε καὶ ὁμοίως ἀναγραφόμενοις· ὅπερ ἔδει δεῖξαι.

λβ'.

Ἐὰν δύο τρίγωνα συντεθῇ κατὰ μίαν γωνίαν τὰς δύο πλευρὰς ταῖς δυσὶ πλευραῖς ἀνάλογον ἔχοντα ὥστε τὰς ὁμολόγους αὐτῶν πλευρὰς καὶ παραλλήλους εἶναι, αἱ λοιπαὶ τῶν τριγώνων πλευραὶ ἐπ' εὐθείας ἔσσονται.



Ἐστω δύο τρίγωνα τὰ ΑΒΓ, ΔΓΕ τὰς δύο πλευρὰς τὰς ΒΑ, ΑΓ ταῖς δυσὶ πλευραῖς ταῖς ΔΓ, ΔΕ ἀνάλογον ἔχοντα, ὥς μὲν τὴν ΑΒ πρὸς τὴν ΑΓ, οὕτως τὴν ΔΓ πρὸς τὴν ΔΕ, παραλλήλων δὲ τὴν μὲν ΑΒ τῇ ΔΓ, τὴν δὲ ΑΓ τῇ ΔΕ· λέγω, ὅτι ἐπ' εὐθείας ἐστὶν ἡ ΒΓ τῇ ΓΕ.

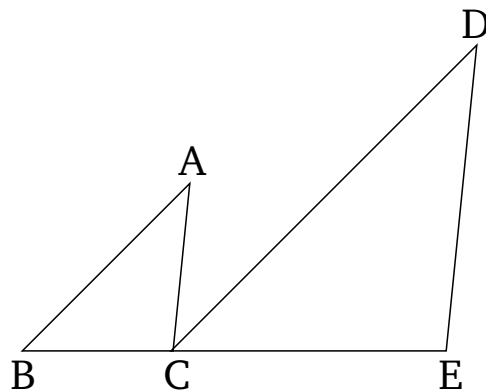
Ἐπεὶ γὰρ παράλληλός ἐστιν ἡ ΑΒ τῇ ΔΓ, καὶ εἰς αὐτὰς ἐμπίπτωκεν εὐθεῖα ἡ ΑΓ, αἱ ἐναλλάξ γωνίαι αἱ ὑπὸ ΒΑΓ, ΑΓΔ ἴσαι ἀλλήλαις εἰσίν. διὰ τὰ αὐτὰ δὴ καὶ ἡ ὑπὸ ΓΔΕ τῇ ὑπὸ ΑΓΔ ἴση ἐστίν. ὥστε καὶ ἡ ὑπὸ ΒΑΓ τῇ ὑπὸ ΓΔΕ ἐστὶν ἴση. καὶ ἐπεὶ δύο τρίγωνα ἐστί τὰ ΑΒΓ, ΔΓΕ μίαν γωνίαν τὴν πρὸς τῷ Α μιᾶ γωνίᾳ τῇ πρὸς τῷ Δ ἴσην ἔχοντα, περὶ

as CB is to BA , so AB (is) to BD [Def. 6.1]. And since three straight-lines are proportional, as the first is to the third, so the figure (drawn) on the first is to the similar, and similarly described, (figure) on the second [Prop. 6.19 corr.]. Thus, as CB (is) to BD , so the figure (drawn) on CB (is) to the similar, and similarly described, (figure) on BA . And so, for the same (reasons), as BC (is) to CD , so the figure (drawn) on BC (is) to the (figure) on CA . Hence, also, as BC (is) to BD and DC , so the figure (drawn) on BC (is) to the (sum of the) similar, and similarly described, (figures) on BA and AC [Prop. 5.24]. And BC is equal to BD and DC . Thus, the figure (drawn) on BC (is) also equal to the (sum of the) similar, and similarly described, figures on BA and AC [Prop. 5.9].

Thus, in right-angled triangles, the figure (drawn) on the side subtending the right-angle is equal to the (sum of the) similar, and similarly described, figures on the sides surrounding the right-angle. (Which is) the very thing it was required to show.

Proposition 32

If two triangles, having two sides proportional to two sides, are placed together at a single angle such that the corresponding sides are also parallel, then the remaining sides of the triangles will be straight-on (with respect to one another).



Let ABC and DCE be two triangles having the two sides BA and AC proportional to the two sides DC and DE —so that as AB (is) to AC , so DC (is) to DE —and (having side) AB parallel to DC , and AC to DE . I say that (side) BC is straight-on to CE .

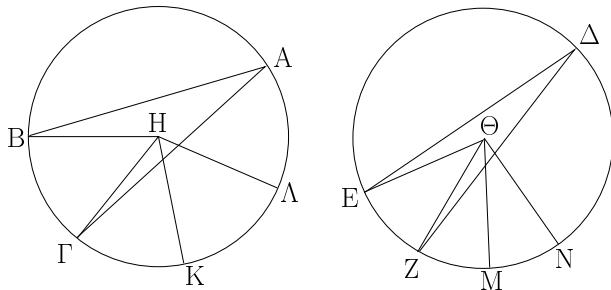
For since AB is parallel to DC , and the straight-line AC has fallen across them, the alternate angles BAC and ACD are equal to one another [Prop. 1.29]. So, for the same (reasons), CDE is also equal to ACD . And, hence, BAC is equal to CDE . And since ABC and DCE are two triangles having the one angle at A equal to the one

δὲ τὰς ἴσας γωνίας τὰς πλευράς ἀνάλογον, ὥς τὴν BA πρὸς τὴν AG , οὕτως τὴν $ΓΔ$ πρὸς τὴν $ΔΕ$, ἰσογώνιον ἄρα ἐστὶ τὸ $ABΓ$ τρίγωνον τῷ $ΔΓΕ$ τριγώνῳ· ἴση ἄρα ἡ ὑπὸ $ABΓ$ γωνία τῇ ὑπὸ $ΔΓΕ$. ἐδείχθη δὲ καὶ ἡ ὑπὸ $ΑΓΔ$ τῇ ὑπὸ $BAΓ$ ἴση· ὅλη ἄρα ἡ ὑπὸ $ΑΓΕ$ δυσὶ ταῖς ὑπὸ $ABΓ$, $BAΓ$ ἴση ἐστίν. κοινὴ προσκείσθω ἡ ὑπὸ $ΑΓΒ$ · αἱ ἄρα ὑπὸ $ΑΓΕ$, $ΑΓΒ$ ταῖς ὑπὸ $BAΓ$, $ΑΓΒ$, $ΓΒΑ$ ἴσαι εἰσίν. ἀλλ' αἱ ὑπὸ $BAΓ$, $ABΓ$, $ΑΓΒ$ δυσὶν ὀρθαῖς ἴσαι εἰσίν· καὶ αἱ ὑπὸ $ΑΓΕ$, $ΑΓΒ$ ἄρα δυσὶν ὀρθαῖς ἴσαι εἰσίν. πρὸς δὲ τινὶ εὐθείᾳ τῇ $ΑΓ$ καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ $Γ$ δύο εὐθεῖαι αἱ $ΒΓ$, $ΓΕ$ μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι τὰς ἐφεξῆς γωνίας τὰς ὑπὸ $ΑΓΕ$, $ΑΓΒ$ δυσὶν ὀρθαῖς ἴσας ποιοῦσιν· ἐπ' εὐθείας ἄρα ἐστὶν ἡ $ΒΓ$ τῇ $ΓΕ$.

Ἐὰν ἄρα δύο τρίγωνα συντεθῇ κατὰ μίαν γωνίαν τὰς δύο πλευράς ταῖς δυσὶ πλευραῖς ἀνάλογον ἔχοντα ὥστε τὰς ὁμολόγους αὐτῶν πλευράς καὶ παραλλήλους εἶναι, αἱ λοιπαὶ τῶν τριγώνων πλευραὶ ἐπ' εὐθείας ἔσσονται· ὅπερ ἔδει δεῖξαι.

λγ'.

Ἐν τοῖς ἴσοις κύκλοις αἱ γωνίαι τὸν αὐτὸν ἔχουσι λόγον ταῖς περιφερείαις, ἐφ' ὧν βεβήκασιν, ἐάν τε πρὸς τοῖς κέντροις ἐάν τε πρὸς ταῖς περιφερείαις ὥσι βεβηκυῖαι.



Ἐστωσαν ἴσοι κύκλοι οἱ $ABΓ$, $ΔΕΖ$, καὶ πρὸς μὲν τοῖς κέντροις αὐτῶν τοῖς H , $Θ$ γωνίαι ἔστωσαν αἱ ὑπὸ $BHΓ$, $EΘΖ$, πρὸς δὲ ταῖς περιφερείαις αἱ ὑπὸ $BAΓ$, $ΕΔΖ$ · λέγω, ὅτι ἐστὶν ὥς ἡ $ΒΓ$ περιφέρεια πρὸς τὴν $ΕΖ$ περιφέρειαν, οὕτως ἡ τε ὑπὸ $BHΓ$ γωνία πρὸς τὴν ὑπὸ $EΘΖ$ καὶ ἡ ὑπὸ $BAΓ$ πρὸς τὴν ὑπὸ $ΕΔΖ$.

Κείσθωσαν γὰρ τῇ μὲν $ΒΓ$ περιφέρειᾳ ἴσαι κατὰ τὸ ἐξῆς ὁσαυδηποτοῦν αἱ $ΓΚ$, $ΚΛ$, τῇ δὲ $ΕΖ$ περιφέρειᾳ ἴσαι ὁσαυδηποτοῦν αἱ $ΖΜ$, $ΜΝ$, καὶ ἐπεζεύχθωσαν αἱ $ΗΚ$, $ΗΛ$, $ΘΜ$, $ΘΝ$.

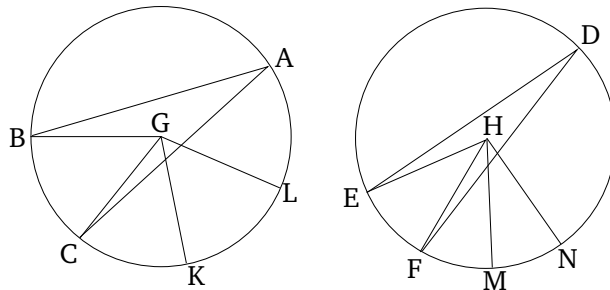
Ἐπεὶ οὖν ἴσαι εἰσὶν αἱ $ΒΓ$, $ΓΚ$, $ΚΛ$ περιφέρειαι ἀλλήλαις, ἴσαι εἰσὶ καὶ αἱ ὑπὸ $BHΓ$, $ΓΗΚ$, $ΚΗΛ$ γωνίαι ἀλλήλαις· ὁσαπλασίων ἄρα ἐστὶν ἡ $ΒΛ$ περιφέρεια τῆς $ΒΓ$, τοσαυταπλασίων ἐστὶ καὶ ἡ ὑπὸ $BHΛ$ γωνία τῆς ὑπὸ $BHΓ$. διὰ τὰ

angle at D , and the sides about the equal angles proportional, (so that) as BA (is) to AC , so CD (is) to DE , triangle ABC is thus equiangular to triangle DCE [Prop. 6.6]. Thus, angle ABC is equal to DCE . And (angle) ACD was also shown (to be) equal to BAC . Thus, the whole (angle) ACE is equal to the two (angles) ABC and BAC . Let ACB have been added to both. Thus, ACE and ACB are equal to BAC , ACB , and CBA . But, BAC , ABC , and ACB are equal to two right-angles [Prop. 1.32]. Thus, ACE and ACB are also equal to two right-angles. Thus, the two straight-lines BC and CE , not lying on the same side, make adjacent angles ACE and ACB (whose sum is) equal to two right-angles with some straight-line AC , at the point C on it. Thus, BC is straight-on to CE [Prop. 1.14].

Thus, if two triangles, having two sides proportional to two sides, are placed together at a single angle such that the corresponding sides are also parallel, then the remaining sides of the triangles will be straight-on (with respect to one another). (Which is) the very thing it was required to show.

Proposition 33

In equal circles, angles have the same ratio as the (ratio of the) circumferences on which they stand, whether they are standing at the centers (of the circles) or at the circumferences.



Let ABC and DEF be equal circles, and let BGC and EHF be angles at their centers, G and H (respectively), and BAC and EDF (angles) at their circumferences. I say that as circumference BC is to circumference EF , so angle BGC (is) to EHF , and (angle) BAC to EDF .

For let any number whatsoever of consecutive (circumferences), CK and KL , be made equal to circumference BC , and any number whatsoever, FM and MN , to circumference EF . And let GK , GL , HM , and HN have been joined.

Therefore, since circumferences BC , CK , and KL are equal to one another, angles BGC , CGK , and KGL are also equal to one another [Prop. 3.27]. Thus, as many times as circumference BL is (divisible) by BC , so many

αὐτὰ δὴ καὶ ὁσαπλασίων ἐστὶν ἡ NE περιφέρεια τῆς EZ , τοσαυταπλασίων ἐστὶ καὶ ἡ ὑπὸ $N\Theta E$ γωνία τῆς ὑπὸ $E\Theta Z$. εἰ ἄρα ἴση ἐστὶν ἡ BA περιφέρεια τῇ EN περιφερείᾳ, ἴση ἐστὶ καὶ γωνία ἡ ὑπὸ BHA τῇ ὑπὸ $E\Theta N$, καὶ εἰ μείζων ἐστὶν ἡ BA περιφέρεια τῆς EN περιφερείας, μείζων ἐστὶ καὶ ἡ ὑπὸ BHA γωνία τῆς ὑπὸ $E\Theta N$, καὶ εἰ ἐλάσσων, ἐλάσσων. τεσσάρων δὴ ὄντων μεγεθῶν, δύο μὲν περιφερειῶν τῶν $B\Gamma$, EZ , δύο δὲ γωνιῶν τῶν ὑπὸ $BH\Gamma$, $E\Theta Z$, εἴληπται τῆς μὲν $B\Gamma$ περιφερείας καὶ τῆς ὑπὸ $BH\Gamma$ γωνίας ἰσάκεις πολλαπλασίων ἢ τε BA περιφέρεια καὶ ἡ ὑπὸ BHA γωνία, τῆς δὲ EZ περιφερείας καὶ τῆς ὑπὸ $E\Theta Z$ γωνίας ἢ τε EN περιφέρεια καὶ ἡ ὑπὸ $E\Theta N$ γωνία. καὶ δέδεικται, ὅτι εἰ ὑπερέχει ἡ BA περιφέρεια τῆς EN περιφερείας, ὑπερέχει καὶ ἡ ὑπὸ BHA γωνία τῆς ὑπὸ $E\Theta N$ γωνίας, καὶ εἰ ἴση, ἴση, καὶ εἰ ἐλάσσων, ἐλάσσων. ἔστιν ἄρα, ὡς ἡ $B\Gamma$ περιφέρεια πρὸς τὴν EZ , οὕτως ἡ ὑπὸ $BH\Gamma$ γωνία πρὸς τὴν ὑπὸ $E\Theta Z$. ἀλλ' ὡς ἡ ὑπὸ $BH\Gamma$ γωνία πρὸς τὴν ὑπὸ $E\Theta Z$, οὕτως ἡ ὑπὸ $BA\Gamma$ πρὸς τὴν ὑπὸ $EA\Delta Z$. διπλασία γὰρ ἑκατέρα ἑκατέρας. καὶ ὡς ἄρα ἡ $B\Gamma$ περιφέρεια πρὸς τὴν EZ περιφέρειαν, οὕτως ἢ τε ὑπὸ $BH\Gamma$ γωνία πρὸς τὴν ὑπὸ $E\Theta Z$ καὶ ἡ ὑπὸ $BA\Gamma$ πρὸς τὴν ὑπὸ $EA\Delta Z$.

Ἐν ἄρα τοῖς ἴσοις κύκλοις αἱ γωνίαι τὸν αὐτὸν ἔχουσι λόγον ταῖς περιφερείαις, ἐφ' ὧν βεβήκασιν, ἐάν τε πρὸς τοῖς κέντροις ἐάν τε πρὸς ταῖς περιφερείαις ὥσι βεβηκῶσι· ὅπερ ἔδει δεῖξαι.

times is angle BGL also (divisible) by BGC . And so, for the same (reasons), as many times as circumference NE is (divisible) by EF , so many times is angle NHE also (divisible) by EHF . Thus, if circumference BL is equal to circumference EN then angle BGL is also equal to EHN [Prop. 3.27], and if circumference BL is greater than circumference EN then angle BGL is also greater than EHN ,[†] and if (BL is) less (than EN then BGL is also) less (than EHN). So there are four magnitudes, two circumferences BC and EF , and two angles BGC and EHF . And equal multiples have been taken of circumference BC and angle BGC , (namely) circumference BL and angle BGL , and of circumference EF and angle EHF , (namely) circumference EN and angle EHN . And it has been shown that if circumference BL exceeds circumference EN then angle BGL also exceeds angle EHN , and if (BL is) equal (to EN then BGL is also) equal (to EHN), and if (BL is) less (than EN then BGL is also) less (than EHN). Thus, as circumference BC (is) to EF , so angle BGC (is) to EHF [Def. 5.5]. But as angle BGC (is) to EHF , so (angle) BAC (is) to EDF [Prop. 5.15]. For the former (are) double the latter (respectively) [Prop. 3.20]. Thus, also, as circumference BC (is) to circumference EF , so angle BGC (is) to EHF , and BAC to EDF .

Thus, in equal circles, angles have the same ratio as the (ratio of the) circumferences on which they stand, whether they are standing at the centers (of the circles) or at the circumferences. (Which is) the very thing it was required to show.

[†] This is a straight-forward generalization of Prop. 3.27

ELEMENTS BOOK 7

Elementary Number Theory[†]

[†]The propositions contained in Books 7–9 are generally attributed to the school of Pythagoras.

Ὅροι.

- α'. Μονάς ἐστίν, καθ' ἣν ἕκαστον τῶν ὄντων ἐν λέγεται.
 β'. Ἀριθμὸς δὲ τὸ ἐκ μονάδων συγχείμενον πλῆθος.
 γ'. Μέρος ἐστίν ἀριθμὸς ἀριθμοῦ ὁ ἐλάσσων τοῦ μείζονος, ὅταν καταμετρηῇ τὸν μείζονα.
 δ'. Μέρη δέ, ὅταν μὴ καταμετρηῇ.
 ε'. Πολλαπλάσιος δὲ ὁ μείζων τοῦ ἐλάσσονος, ὅταν καταμετρηῇται ὑπὸ τοῦ ἐλάσσονος.
 ς'. Ἄρτιος ἀριθμὸς ἐστίν ὁ δίχα διαιρούμενος.
 ζ'. Περισσὸς δὲ ὁ μὴ διαιρούμενος δίχα ἢ [ὁ] μονάδι διαφέρων ἀρτίου ἀριθμοῦ.
 η'. Ἀρτιάκις ἄρτιος ἀριθμὸς ἐστίν ὁ ὑπὸ ἀρτίου ἀριθμοῦ μετρούμενος κατὰ ἄρτιον ἀριθμόν.
 θ'. Ἀρτιάκις δὲ περισσὸς ἐστίν ὁ ὑπὸ ἀρτίου ἀριθμοῦ μετρούμενος κατὰ περισσὸν ἀριθμόν.
 ι'. Περισσάκις δὲ περισσὸς ἀριθμὸς ἐστίν ὁ ὑπὸ περισσοῦ ἀριθμοῦ μετρούμενος κατὰ περισσὸν ἀριθμόν.
 ια'. Πρῶτος ἀριθμὸς ἐστίν ὁ μονάδι μόνῃ μετρούμενος.
 ιβ'. Πρῶτοι πρὸς ἀλλήλους ἀριθμοὶ εἰσιν οἱ μονάδι μόνῃ μετρούμενοι κοινῷ μέτρῳ.
 ιγ'. Σύνθετος ἀριθμὸς ἐστίν ὁ ἀριθμῷ τινι μετρούμενος.
 ιδ'. Σύνθετοι δὲ πρὸς ἀλλήλους ἀριθμοὶ εἰσιν οἱ ἀριθμῷ τινι μετρούμενοι κοινῷ μέτρῳ.
 ιε'. Ἀριθμὸς ἀριθμὸν πολλαπλασιάζειν λέγεται, ὅταν, ὅσαι εἰσὶν ἐν αὐτῷ μονάδες, τοσαυτάκις συντεθῇ ὁ πολλαπλασιαζόμενος, καὶ γέννηταί τις.
 ις'. Ὅταν δὲ δύο ἀριθμοὶ πολλαπλασιάσαντες ἀλλήλους ποιῶσιν τινα, ὁ γενόμενος ἐπίπεδος καλεῖται, πλευραὶ δὲ αὐτοῦ οἱ πολλαπλασιάσαντες ἀλλήλους ἀριθμοί.
 ιζ'. Ὅταν δὲ τρεῖς ἀριθμοὶ πολλαπλασιάσαντες ἀλλήλους ποιῶσιν τινα, ὁ γενόμενος στερεός ἐστίν, πλευραὶ δὲ αὐτοῦ οἱ πολλαπλασιάσαντες ἀλλήλους ἀριθμοί.
 ιη'. Τετράγωνος ἀριθμὸς ἐστίν ὁ ισάκις ἴσος ἢ [ὁ] ὑπὸ δύο ἴσων ἀριθμῶν περιεχόμενος.
 ιθ'. Κύβος δὲ ὁ ισάκις ἴσος ισάκις ἢ [ὁ] ὑπὸ τριῶν ἴσων ἀριθμῶν περιεχόμενος.
 κ'. Ἀριθμοὶ ἀνάλογόν εἰσιν, ὅταν ὁ πρῶτος τοῦ δευτέρου καὶ ὁ τρίτος τοῦ τετάρτου ισάκις ἢ πολλαπλάσιος ἢ τὸ αὐτὸ μέρος ἢ τὰ αὐτὰ μέρη ᾖσιν.
 κα'. Ὅμοιοι ἐπίπεδοι καὶ στερεοὶ ἀριθμοὶ εἰσιν οἱ ἀνάλογον ἔχοντες τὰς πλευράς.
 κβ'. Τέλεις ἀριθμὸς ἐστίν ὁ τοῖς ἑαυτοῦ μέρεσιν ἴσος ὢν.

Definitions

1. A unit is (that) according to which each existing (thing) is said (to be) one.
2. And a number (is) a multitude composed of units.[†]
3. A number is part of a(nother) number, the lesser of the greater, when it measures the greater.[‡]
4. But (the lesser is) parts (of the greater) when it does not measure it.[§]
5. And the greater (number is) a multiple of the lesser when it is measured by the lesser.
6. An even number is one (which can be) divided in half.
7. And an odd number is one (which can)not (be) divided in half, or which differs from an even number by a unit.
8. An even-times-even number is one (which is) measured by an even number according to an even number.[¶]
9. And an even-times-odd number is one (which is) measured by an even number according to an odd number.*
10. And an odd-times-odd number is one (which is) measured by an odd number according to an odd number.[§]
11. A prime^{||} number is one (which is) measured by a unit alone.
12. Numbers prime to one another are those (which are) measured by a unit alone as a common measure.
13. A composite number is one (which is) measured by some number.
14. And numbers composite to one another are those (which are) measured by some number as a common measure.
15. A number is said to multiply a(nother) number when the (number being) multiplied is added (to itself) as many times as there are units in the former (number), and (thereby) some (other number) is produced.
16. And when two numbers multiplying one another make some (other number) then the (number so) created is called plane, and its sides (are) the numbers which multiply one another.
17. And when three numbers multiplying one another make some (other number) then the (number so) created is (called) solid, and its sides (are) the numbers which multiply one another.
18. A square number is an equal times an equal, or (a plane number) contained by two equal numbers.
19. And a cube (number) is an equal times an equal times an equal, or (a solid number) contained by three equal numbers.