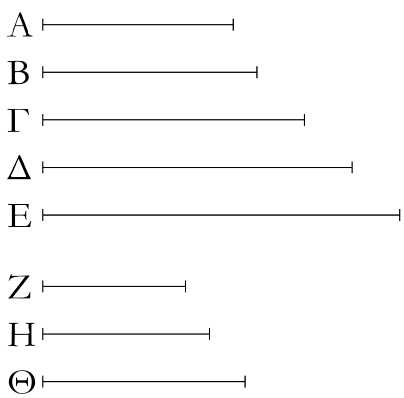
τὸν συγκείμενον ἐκ τῶν πλευρῶν· καὶ ὁ Α ἄρα πρὸς τὸν Β λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν· ὅπερ ἔδει δεῖξαι.

(Which is) the very thing it was required to show.

† *i.e.*, multiplied.

ϛ΄.

Ἐὰν ὦσιν ὁποσοιοῦν ἀριθμοὶ ἑξῆς ἀνάλογον, ὁ δὲ πρῶτος τὸν δεύτερον μὴ μετρῇ, οὐδὲ ἄλλος οὐδεὶς οὐδένα μετρήσει.

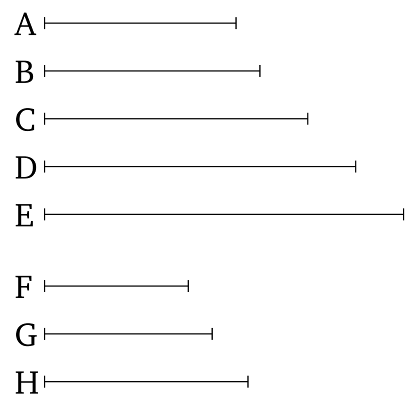


Ἔστωσαν ὁποσοιοῦν ἀριθμοὶ ἑξῆς ἀνάλογον οἱ Α, Β, Γ, Δ, Ε, ὁ δὲ Α τὸν Β μὴ μετρείτω· λέγω, ὅτι οὐδὲ ἄλλος οὐδεὶς οὐδένα μετρήσει.

Ὅτι μὲν οὖν οἱ Α, Β, Γ, Δ, Ε ἑξῆς ἀλλήλους οὐ μετροῦσιν, φανερόν· οὐδὲ γὰρ ὁ Α τὸν Β μετρεῖ. λέγω δή, ὅτι οὐδὲ ἄλλος οὐδεὶς οὐδένα μετρήσει. εἰ γὰρ δυνατόν, μετρείτω ὁ Α τὸν Γ. καὶ ὅσοι εἰσὶν οἱ Α, Β, Γ, τοσοῦτοι εἰλήφθωσαν ἐλάχιστοι ἀριθμοὶ τῶν τὸν αὐτὸν λόγον ἐχόντων τοῖς Α, Β, Γ οἱ Ζ, Η, Θ. καὶ ἐπεὶ οἱ Ζ, Η, Θ ἐν τῷ αὐτῷ λόγῳ εἰσὶ τοῖς Α, Β, Γ, καί ἐστιν ἴσον τὸ πλῆθος τῶν Α, Β, Γ τῷ πλήθει τῶν Ζ, Η, Θ, δι᾽ ἴσου ἄρα ἐστὶν ὡς ὁ Α πρὸς τὸν Γ, οὕτως ὁ Ζ πρὸς τὸν Θ. καὶ ἐπεί ἐστιν ὡς ὁ Α πρὸς τὸν Β, οὕτως ὁ Ζ πρὸς τὸν Η, οὐ μετρεῖ δὲ ὁ Α τὸν Β, οὐ μετρεῖ ἄρα οὐδὲ ὁ Ζ τὸν Η· οὐκ ἄρα μονάς ἐστιν ὁ Ζ· ἡ γὰρ μονὰς πάντα ἀριθμὸν μετρεῖ. καί εἰσιν οἱ Ζ, Θ πρῶτοι πρὸς ἀλλήλους [οὐδὲ ὁ Ζ ἄρα τὸν Θ μετρεῖ]. καί ἐστιν ὡς ὁ Ζ πρὸς τὸν Θ, οὕτως ὁ Α πρὸς τὸν Γ· οὐδὲ ὁ Α ἄρα τὸν Γ μετρεῖ. ὁμοίως δὴ δείξομεν, ὅτι οὐδὲ ἄλλος οὐδεὶς οὐδένα μετρήσει· ὅπερ ἔδει δεῖξαι.

Proposition 6

If there are any multitude whatsoever of continuously proportional numbers, and the first does not measure the second, then no other (number) will measure any other (number) either.



Let A, B, C, D, E be any multitude whatsoever of continuously proportional numbers, and let A not measure B. I say that no other (number) will measure any other (number) either.

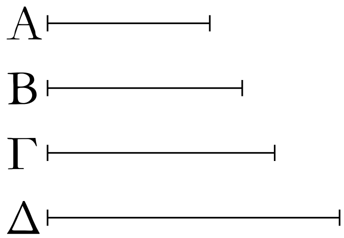
Now, (it is) clear that A, B, C, D, E do not successively measure one another. For A does not even measure B. So I say that no other (number) will measure any other (number) either. For, if possible, let A measure C. And as many (numbers) as are A, B, C, let so many of the least numbers, F, G, H, have been taken of those (numbers) having the same ratio as A, B, C [Prop. 7.33]. And since F, G, H are in the same ratio as A, B, C, and the multitude of A, B, C is equal to the multitude of F, G, H, thus, via equality, as A is to C, so F (is) to H [Prop. 7.14]. And since as A is to B, so F (is) to G, and A does not measure B, F does not measure G either [Def. 7.20]. Thus, F is not a unit. For a unit measures all numbers. And F and H are prime to one another [Prop. 8.3] [and thus F does not measure H]. And as F is to H, so A (is) to C. And thus A does not measure C either [Def. 7.20]. So, similarly, we can show that no other (number) can measure any other (number) either. (Which is) the very thing it was required to show.

ζ΄.

Ἐὰν ὦσιν ὁποσοιοῦν ἀριθμοὶ [ἑξῆς] ἀνάλογον, ὁ δὲ πρῶτος τὸν ἔσχατον μετρῇ, καὶ τὸν δεύτερον μετρήσει.

Proposition 7

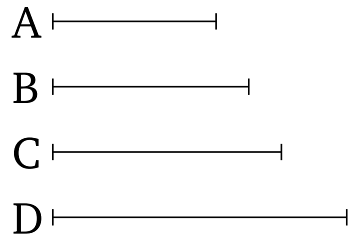
If there are any multitude whatsoever of [continuously] proportional numbers, and the first measures the



Ἔστωσαν ὁποσοιοῦν ἀριθμοὶ ἑξῆς ἀνάλογον οἱ Α, Β, Γ, Δ, ὁ δὲ Α τὸν Δ μετρείτω· λέγω, ὅτι καὶ ὁ Α τὸν Β μετρεῖ.

Εἰ γὰρ οὐ μετρεῖ ὁ Α τὸν Β, οὐδὲ ἄλλος οὐδεὶς οὐδένα μετρήσει· μετρεῖ δὲ ὁ Α τὸν Δ. μετρεῖ ἄρα καὶ ὁ Α τὸν Β· ὅπερ ἔδει δεῖξαι.

last, then (the first) will also measure the second.

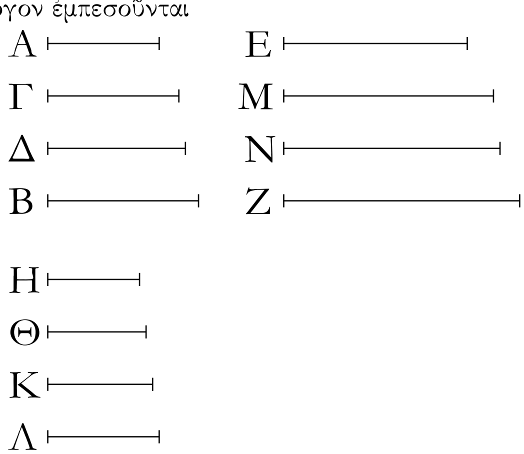


Let A, B, C, D be any number whatsoever of continuously proportional numbers. And let A measure D. I say that A also measures B.

For if A does not measure B then no other (number) will measure any other (number) either [Prop. 8.6]. But A measures D. Thus, A also measures B. (Which is) the very thing it was required to show.

η΄.

Ἐὰν δύο ἀριθμῶν μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπίπτωσιν ἀριθμοί, ὅσοι εἰς αὐτοὺς μεταξὺ κατὰ τὸ συνεχὲς ἀνόλογον ἐμπίπτουσιν ἀριθμοί, τοσοῦτοι καὶ εἰς τοὺς τὸν αὐτὸν λόγον ἔχοντας [αὐτοῖς] μεταξὺ κατὰ τὸ συνὲχες ἀνάλογον ἐμπεσοῦνται

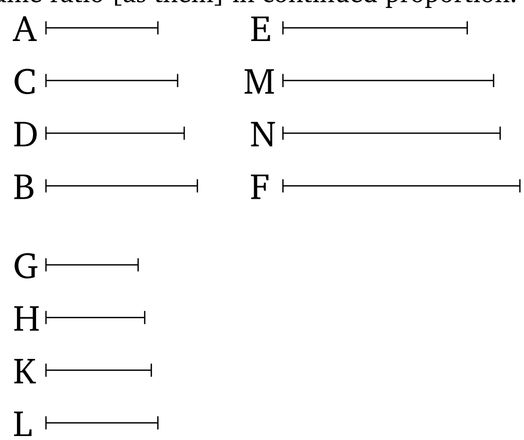


Δύο γὰρ ἀριθμῶν τῶν Α, Β μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπιπτέτωσαν ἀριθμοὶ οἱ Γ, Δ, καὶ πεποιήσθω ὡς ὁ Α πρὸς τὸν Β, οὕτως ὁ Ε πρὸς τὸν Ζ· λέγω, ὅτι ὅσοι εἰς τοὺς Α, Β μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεπτώκασιν ἀριθμοί, τοσοῦτοι καὶ εἰς τοὺς Ε, Ζ μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεσοῦνται.

Ὅσοι γάρ εἰσι τῷ πλήθει οἱ Α, Β, Γ, Δ, τοσοῦτοι εἰλήφθωσαν ἐλάχιστοι ἀριθμοὶ τῶν τὸν αὐτὸν λόγον ἐχόντων τοῖς Α, Γ, Δ, Β οἱ Η, Θ, Κ, Λ· οἱ ἄρα ἄκροι αὐτῶν οἱ Η, Λ πρῶτοι πρὸς ἀλλήλους εἰσίν. καὶ ἐπεὶ οἱ Α, Γ, Δ, Β τοῖς Η, Θ, Κ, Λ ἐν τῷ αὐτῷ λόγῳ εἰσίν, καί ἐστιν ἴσον τὸ πλῆθος τῶν Α, Γ, Δ, Β τῷ πλήθει τῶν Η, Θ, Κ, Λ, δι᾽ ἴσου ἄρα ἐστὶν ὡς ὁ Α πρὸς τὸν Β, οὕτως ὁ Η πρὸς τὸν Λ. ὡς δὲ ὁ Α πρὸς τὸν Β, οὕτως ὁ Ε πρὸς τὸν Ζ· καὶ

Proposition 8

If between two numbers there fall (some) numbers in continued proportion then, as many numbers as fall in between them in continued proportion, so many (numbers) will also fall in between (any two numbers) having the same ratio [as them] in continued proportion.



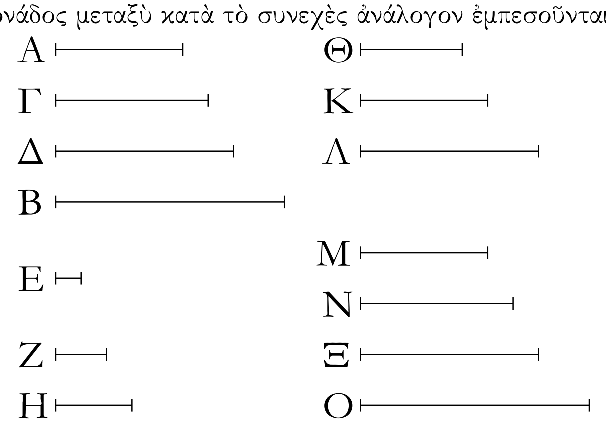
For let the numbers, C and D, fall between two numbers, A and B, in continued proportion, and let it have been contrived (that) as A (is) to B, so E (is) to F. I say that, as many numbers as have fallen in between A and B in continued proportion, so many (numbers) will also fall in between E and F in continued proportion.

For as many as A, B, C, D are in multitude, let so many of the least numbers, G, H, K, L, having the same ratio as A, B, C, D, have been taken [Prop. 7.33]. Thus, the outermost of them, G and L, are prime to one another [Prop. 8.3]. And since A, B, C, D are in the same ratio as G, H, K, L, and the multitude of A, B, C, D is equal to the multitude of G, H, K, L, thus, via equality, as A is to B, so G (is) to L [Prop. 7.14]. And as A (is) to B, so ὡς ἄρα ὁ Η πρὸς τὸν Λ, οὕτως ὁ Ε πρὸς τὸν Ζ. οἱ δὲ Η, Λ πρῶτοι, οἱ δὲ πρῶτοι καὶ ἐλάχιστοι, οἱ δὲ ἐλάχιστοι ἀριθμοὶ μετροῦσι τοὺς τὸν αὐτὸν λόγον ἔχοντας ἰσάκις ὅ τε μείζων τὸν μείζονα καὶ ὁ ἐλάσσων τὸν ἐλάσσονα, τουτέστιν ὅ τε ἡγούμενος τὸν ἡγούμενον καὶ ὁ ἑπόμενος τὸν ἑπόμενον. ἰσάκις ἄρα ὁ Η τὸν Ε μετρεῖ καὶ ὁ Λ τὸν Ζ. ὁσάκις δὴ ὁ Η τὸν Ε μετρεῖ, τοσαυτάκις καὶ ἑκάτερος τῶν Θ, Κ ἑκάτερον τῶν Μ, Ν μετρείτω· οἱ Η, Θ, Κ, Λ ἄρα τοὺς Ε, Μ, Ν, Ζ ἰσάκις μετροῦσιν. οἱ Η, Θ, Κ, Λ ἄρα τοῖς Ε, Μ, Ν, Ζ ἐν τῷ αὐτῷ λόγῳ εἰσίν. ἀλλὰ οἱ Η, Θ, Κ, Λ τοῖς Α, Γ, Δ, Β ἐν τῷ αὐτῷ λόγῳ εἰσίν· καὶ οἱ Α, Γ, Δ, Β ἄρα τοῖς Ε, Μ, Ν, Ζ ἐν τῷ αὐτῷ λόγῳ εἰσίν. οἱ δὲ Α, Γ, Δ, Β ἑξῆς ἀνάλογόν εἰσιν· καὶ οἱ Ε, Μ, Ν, Ζ ἄρα ἑξῆς ἀνάλογόν εἰσιν. ὅσοι ἄρα εἰς τοὺς Α, Β μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεπτώκασιν ἀριθμοί, τοσοῦτοι καὶ εἰς τοὺς Ε, Ζ μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεπτώκασιν ἀριθμοί· ὅπερ ἔδει δεῖξαι.

E (is) to F. And thus as G (is) to L, so E (is) to F. And G and L (are) prime (to one another). And (numbers) prime (to one another are) also the least (numbers having the same ratio as them) [Prop. 7.21]. And the least numbers measure those (numbers) having the same ratio (as them) an equal number of times, the greater (measuring) the greater, and the lesser the lesser—that is to say, the leading (measuring) the leading, and the following the following [Prop. 7.20]. Thus, G measures E the same number of times as L (measures) F. So as many times as G measures E, so many times let H, K also measure M, N, respectively. Thus, G, H, K, L measure E, M, N, F (respectively) an equal number of times. Thus, G, H, K, L are in the same ratio as E, M, N, F [Def. 7.20]. But, G, H, K, L are in the same ratio as A, C, D, B. Thus, A, C, D, B are also in the same ratio as E, M, N, F. And A, C, D, B are continuously proportional. Thus, E, M, N, F are also continuously proportional. Thus, as many numbers as have fallen in between A and B in continued proportion, so many numbers have also fallen in between E and F in continued proportion. (Which is) the very thing it was required to show.

ϑ΄.

Ἐὰν δύο ἀριθμοὶ πρῶτοι πρὸς ἀλλήλους ὦσιν, καὶ εἰς αὐτοὺς μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπίπτωσιν ἀριθμοί, ὅσοι εἰς αὐτοὺς μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπίπτουσιν ἀριθμοί, τοσοῦτοι καὶ ἑκατέρου αὐτῶν καὶ μονάδος μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεσοῦνται.

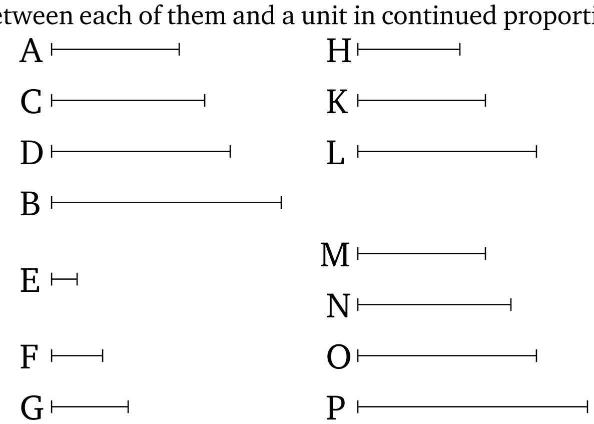


Ἔστωσαν δύο ἀριθμοὶ πρῶτοι πρὸς ἀλλήλους οἱ Α, Β, καὶ εἰς αὐτοὺς μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπιπτέτωσαν οἱ Γ, Δ, καὶ ἐκκείσθω ἡ Ε μονάς· λέγω, ὅτι ὅσοι εἰς τοὺς Α, Β μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεπτώκασιν ἀριθμοί, τοσοῦτοι καὶ ἑκατέρου τῶν Α, Β καὶ τῆς μονάδος μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεσοῦνται.

Εἰλήφθωσαν γὰρ δύο μὲν ἀριθμοὶ ἐλάχιστοι ἐν τῷ τῶν Α, Γ, Δ, Β λόγῳ ὄντες οἱ Ζ, Η, τρεῖς δὲ οἱ Θ, Κ, Λ, καὶ ἀεὶ

Proposition 9

If two numbers are prime to one another and there fall in between them (some) numbers in continued proportion then, as many numbers as fall in between them in continued proportion, so many (numbers) will also fall between each of them and a unit in continued proportion.



Let A and B be two numbers (which are) prime to one another, and let the (numbers) C and D fall in between them in continued proportion. And let the unit E be set out. I say that, as many numbers as have fallen in between A and B in continued proportion, so many (numbers) will also fall between each of A and B and the unit in continued proportion.

For let the least two numbers, F and G, which are in the ratio of A, C, D, B, have been taken [Prop. 7.33].

ἑξῆς ἑνὶ πλείους, ἕως ἂν ἴσον γένηται τὸ πλῆθος αὐτῶν τῷ πλήθει τῶν Α, Γ, Δ, Β. εἰλήφθωσαν, καὶ ἔστωσαν οἱ Μ, Ν, Ξ, Ο. φανερὸν δή, ὅτι ὁ μὲν Ζ ἑαυτὸν πολλαπλασιάσας τὸν Θ πεποίηκεν, τὸν δὲ Θ πολλαπλασιάσας τὸν Μ πεποίηκεν, καὶ ὁ Η ἑαυτὸν μὲν πολλαπλασιάσας τὸν Λ πεποίηκεν, τὸν δὲ Λ πολλαπλασιάσας τὸν Ο πεποίηκεν. καὶ ἐπεὶ οἱ Μ, Ν, Ξ, Ο ἐλάχιστοί εἰσι τῶν τὸν αὐτὸν λόγον ἐχόντων τοῖς Ζ, Η, εἰσὶ δὲ καὶ οἱ Α, Γ, Δ, Β ἐλάχιστοι τῶν τὸν αὐτὸν λόγον ἐχόντων τοῖς Ζ, Η, καί ἐστιν ἴσον τὸ πλῆθος τῶν Μ, Ν, Ξ, Ο τῷ πλήθει τῶν Α, Γ, Δ, Β, ἕκαστος ἄρα τῶν Μ, Ν, Ξ, Ο ἑκάστῳ τῶν Α, Γ, Δ, Β ἴσος ἐστίν· ἴσος ἄρα ἐστὶν ὁ μὲν Μ τῷ Α, ὁ δὲ Ο τῷ Β. καὶ ἐπεὶ ὁ Ζ ἑαυτὸν πολλαπλασιάσας τὸν Θ πεποίηκεν, ὁ Ζ ἄρα τὸν Θ μετρεῖ κατὰ τὰς ἐν τῷ Ζ μονάδας. μετρεῖ δὲ καὶ ἡ Ε μονὰς τὸν Ζ κατὰ τὰς ἐν αὐτῷ μονάδας· ἰσάκις ἄρα ἡ Ε μονὰς τὸν Ζ ἀριθμὸν μετρεῖ καὶ ὁ Ζ τὸν Θ. ἔστιν ἄρα ὡς ἡ Ε μονὰς πρὸς τὸν Ζ ἀριθμόν, οὕτως ὁ Ζ πρὸς τὸν Θ. πάλιν, ἐπεὶ ὁ Ζ τὸν Θ πολλαπλασιάσας τὸν Μ πεποίηκεν, ὁ Θ ἄρα τὸν Μ μετρεῖ κατὰ τὰς ἐν τῷ Ζ μονάδας. μετρεῖ δὲ καὶ ἡ Ε μονὰς τὸν Ζ ἀριθμὸν κατὰ τὰς ἐν αὐτῷ μονάδας· ἰσάκις ἄρα ἡ Ε μονὰς τὸν Ζ ἀριθμὸν μετρεῖ καὶ ὁ Θ τὸν Μ. ἔστιν ἄρα ὡς ἡ Ε μονὰς πρὸς τὸν Ζ ἀριθμόν, οὕτως ὁ Θ πρὸς τὸν Μ. ἐδείχθη δὲ καὶ ὡς ἡ Ε μονὰς πρὸς τὸν Ζ ἀριθμόν, οὕτως ὁ Ζ πρὸς τὸν Θ· καὶ ὡς ἄρα ἡ Ε μονὰς πρὸς τὸν Ζ ἀριθμόν, οὕτως ὁ Ζ πρὸς τὸν Θ καὶ ὁ Θ πρὸς τὸν Μ. ἴσος δὲ ὁ Μ τῷ Α· ἔστιν ἄρα ὡς ἡ Ε μονὰς πρὸς τὸν Ζ ἀριθμόν, οὕτως ὁ Ζ πρὸς τὸν Θ καὶ ὁ Θ πρὸς τὸν Α. διὰ τὰ αὐτὰ δὴ καὶ ὡς ἡ Ε μονὰς πρὸς τὸν Η ἀριθμόν, οὕτως ὁ Η πρὸς τὸν Λ καὶ ὁ Λ πρὸς τὸν Β. ὅσοι ἄρα εἰς τοὺς Α, Β μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεπτώκασιν ἀριθμοί, τοσοῦτοι καὶ ἑκατέρου τῶν Α, Β καὶ μονάδος τῆς Ε μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεπτώκασιν ἀριθμοί· ὅπερ ἔδει δεῖξαι.

And the (least) three (numbers), H, K, L. And so on, successively increasing by one, until the multitude of the (least numbers taken) is made equal to the multitude of A, C, D, B [Prop. 8.2]. Let them have been taken, and let them be M, N, O, P. So (it is) clear that F has made H (by) multiplying itself, and has made M (by) multiplying H. And G has made L (by) multiplying itself, and has made P (by) multiplying L [Prop. 8.2 corr.]. And since M, N, O, P are the least of those (numbers) having the same ratio as F, G, and A, C, D, B are also the least of those (numbers) having the same ratio as F, G [Prop. 8.2], and the multitude of M, N, O, P is equal to the multitude of A, C, D, B, thus M, N, O, P are equal to A, C, D, B, respectively. Thus, M is equal to A, and P to B. And since F has made H (by) multiplying itself, F thus measures H according to the units in F [Def. 7.15]. And the unit E also measures F according to the units in it. Thus, the unit E measures the number F as many times as F (measures) H. Thus, as the unit E is to the number F, so F (is) to H [Def. 7.20]. Again, since F has made M (by) multiplying H, H thus measures M according to the units in F [Def. 7.15]. And the unit E also measures the number F according to the units in it. Thus, the unit E measures the number F as many times as H (measures) M. Thus, as the unit E is to the number F, so H (is) to M [Prop. 7.20]. And it was shown that as the unit E (is) to the number F, so F (is) to H. And thus as the unit E (is) to the number F, so F (is) to H, and H (is) to M. And M (is) equal to A. Thus, as the unit E is to the number F, so F (is) to H, and H to A. And so, for the same (reasons), as the unit E (is) to the number G, so G (is) to L, and L to B. Thus, as many (numbers) as have fallen in between A and B in continued proportion, so many numbers have also fallen between each of A and B and the unit E in continued proportion. (Which is) the very thing it was required to show.

ι'.

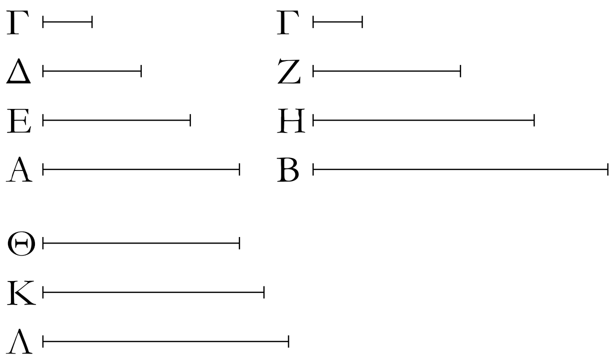
Ἐάν δύο ἀριθμῶν ἑκατέρου καὶ μονάδος μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπίπτωσιν ἀριθμοί, ὅσοι ἑκατέρου αὐτῶν καὶ μονάδος μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπίπτουσιν ἀριθμοί, τοσοῦτοι καὶ εἰς αὐτοὺς μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεσοῦνται.

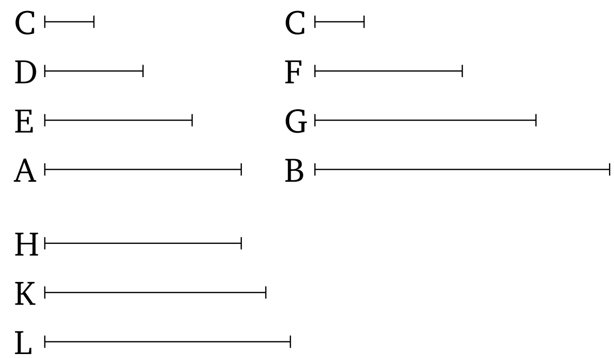
Δύο γὰρ ἀριθμῶν τῶν Α, Β καὶ μονάδος τῆς Γ μεταξύ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπιπτέτωσαν ἀριθμοὶ οἵ τε Δ, Ε καὶ οἱ Ζ, Η· λέγω, ὅτι ὅσοι ἑκατέρου τῶν Α, Β καὶ μονάδος τῆς Γ μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεπτώκασιν ἀριθμοί, τοσοῦτοι καὶ εἰς τοὺς Α, Β μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπεσοῦνται.

Proposition 10

If (some) numbers fall between each of two numbers and a unit in continued proportion then, as many (numbers) as fall between each of the (two numbers) and the unit in continued proportion, so many (numbers) will also fall in between the (two numbers) themselves in continued proportion.

For let the numbers D, E and F, G fall between the numbers A and B (respectively) and the unit C in continued proportion. I say that, as many numbers as have fallen between each of A and B and the unit C in continued proportion, so many will also fall in between A and B in continued proportion.





Ὁ Δ γὰρ τὸν Ζ πολλαπλασιάσας τὸν Θ ποιείτω, ἑκάτερος δὲ τῶν Δ, Ζ τὸν Θ πολλαπλασιάσας ἑκάτερον τῶν Κ, Λ ποιείτω.

Καὶ ἐπεί ἐστιν ὡς ἡ Γ μονὰς πρὸς τὸν Δ ἀριθμόν, οὕτως ὁ Δ πρὸς τὸν Ε, ἰσάκις ἄρα ἡ Γ μονὰς τὸν Δ ἀριθμὸν μετρεῖ καὶ ὁ Δ τὸν Ε. ἡ δὲ Γ μονὰς τὸν Δ ἀριθμὸν μετρεῖ κατὰ τὰς ἐν τῷ Δ μονάδας· καὶ ὁ Δ ἄρα ἀριθμὸς τὸν Ε μετρεῖ κατὰ τὰς ἐν τῷ Δ μονάδας· ὁ Δ ἄρα ἑαυτὸν πολλαπλασιάσας τὸν Ε πεποίηκεν. πάλιν, ἐπεί ἐστιν ὡς ἡ Γ [μονὰς] πρὸς τὸν Δ ἀριθμόν, οὕτως ὁ Ε πρὸς τὸν Α, ἰσάκις ἄρα ἡ Γ μονὰς τὸν Δ ἀριθμὸν μετρεῖ καὶ ὁ Ε τὸν Α. ἡ δὲ Γ μονὰς τὸν Δ ἀριθμὸν μετρεῖ κατὰ τὰς ἐν τῷ Δ μονάδας· καὶ ὁ Ε ἄρα τὸν Α μετρεῖ κατὰ τὰς ἐν τῷ Δ μονάδας· ὁ Δ ἄρα τὸν Ε πολλαπλασιάσας τὸν Α πεποίηκεν. διὰ τὰ αὐτὰ δὴ καὶ ὁ μὲν Ζ ἑαυτὸν πολλαπλασιάσας τὸν Η πεποίηκεν, τὸν δὲ Η πολλαπλασιάσας τὸν Β πεποίηκεν. καὶ ἐπεὶ ὁ Δ ἑαυτὸν μὲν πολλαπλασιάσας τὸν Ε πεποίηκεν, τὸν δὲ Ζ πολλαπλασιάσας τὸν Θ πεποίηκεν, ἔστιν ἄρα ὡς ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Ε πρὸς τὸν Θ. διὰ τὰ αὐτὰ δὴ καὶ ὡς ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Θ πρὸς τὸν Η. καὶ ὡς ἄρα ὁ Ε πρὸς τὸν Θ, οὕτως ὁ Θ πρὸς τὸν Η. πάλιν, ἐπεὶ ὁ Δ ἑκάτερον τῶν Ε, Θ πολλαπλασιάσας ἑκάτερον τῶν Α, Κ πεποίηκεν, ἔστιν ἄρα ὡς ὁ Ε πρὸς τὸν Θ, οὕτως ὁ Α πρὸς τὸν Κ. ἀλλ᾽ ὡς ὁ Ε πρὸς τὸν Θ, οὕτως ὁ Δ πρὸς τὸν Ζ· καὶ ὡς ἄρα ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Α πρὸς τὸν Κ. πάλιν, ἐπεὶ ἑκάτερος τῶν Δ, Ζ τὸν Θ πολλαπλασιάσας ἑκάτερον τῶν Κ, Λ πεποίηκεν, ἔστιν ἄρα ὡς ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Κ πρὸς τὸν Λ. ἀλλ᾽ ὡς ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Α πρὸς τὸν Κ· καὶ ὡς ἄρα ὁ Α πρὸς τὸν Κ, οὕτως ὁ Κ πρὸς τὸν Λ. ἔτι ἐπεὶ ὁ Ζ ἑκάτερον τῶν Θ, Η πολλαπλασιάσας ἑκάτερον τῶν Λ, Β πεποίηκεν, ἔστιν ἄρα ὡς ὁ Θ πρὸς τὸν Η, οὕτως ὁ Λ πρὸς τὸν Β. ὡς δὲ ὁ Θ πρὸς τὸν Η, οὕτως ὁ Δ πρὸς τὸν Ζ· καὶ ὡς ἄρα ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Λ πρὸς τὸν Β. ἐδείχθη δὲ καὶ ὡς ὁ Δ πρὸς τὸν Ζ, οὕτως ὅ τε Α πρὸς τὸν Κ καὶ ὁ Κ πρὸς τὸν Λ· καὶ ὡς ἄρα ὁ Α πρὸς τὸν Κ, οὕτως ὁ Κ πρὸς τὸν Λ καὶ ὁ Λ πρὸς τὸν Β. οἱ Α, Κ, Λ, Β ἄρα κατὰ τὸ συνεχὲς ἑξῆς εἰσιν ἀνάλογον. ὅσοι ἄρα ἑκατέρου τῶν Α, Β καὶ τῆς Γ μονάδος μεταξὺ κατὰ τὸ συνεχὲς ἀνάλογον ἐμπίπτουσιν ἀριθμοί, τοσοῦτοι καὶ εἰς τοὺς Α, Β μεταξὺ κατὰ τὸ συνεχὲς ἐμπεσοῦνται· ὅπερ ἔδει

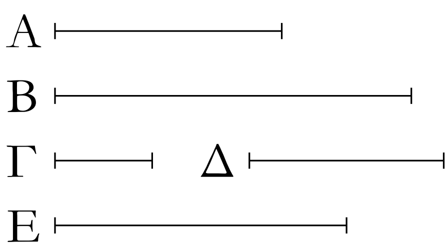
For let D make H (by) multiplying F. And let D, F make K, L, respectively, by multiplying H.

As since as the unit C is to the number D, so D (is) to E, the unit C thus measures the number D as many times as D (measures) E [Def. 7.20]. And the unit C measures the number D according to the units in D. Thus, the number D also measures E according to the units in D. Thus, D has made E (by) multiplying itself. Again, since as the [unit] C is to the number D, so E (is) to A, the unit C thus measures the number D as many times as E (measures) A [Def. 7.20]. And the unit C measures the number D according to the units in D. Thus, E also measures A according to the units in D. Thus, D has made A (by) multiplying E. And so, for the same (reasons), F has made G (by) multiplying itself, and has made B (by) multiplying G. And since D has made E (by) multiplying itself, and has made H (by) multiplying F, thus as D is to F, so E (is) to H [Prop 7.17]. And so, for the same reasons, as D (is) to F, so H (is) to G [Prop. 7.18]. And thus as E (is) to H, so H (is) to G. Again, since D has made A, K (by) multiplying E, H, respectively, thus as E is to H, so A (is) to K [Prop 7.17]. But, as E (is) to H, so D (is) to F. And thus as D (is) to F, so A (is) to K. Again, since D, F have made K, L, respectively, (by) multiplying H, thus as D is to F, so K (is) to L [Prop. 7.18]. But, as D (is) to F, so A (is) to K. And thus as A (is) to K, so K (is) to L. Further, since F has made L, B (by) multiplying H, G, respectively, thus as H is to G, so L (is) to B [Prop 7.17]. And as H (is) to G, so D (is) to F. And thus as D (is) to F, so L (is) to B. And it was also shown that as D (is) to F, so A (is) to K, and K to L. And thus as A (is) to K, so K (is) to L, and L to B. Thus, A, K, L, B are successively in continued proportion. Thus, as many numbers as fall between each of A and B and the unit C in continued proportion, so many will also fall in between A and B in continued proportion. (Which is) the very thing it was required to show.

δεῖξαι.

ια΄.

Δύο τετραγώνων ἀριθμῶν εἷς μέσος ἀνάλογόν ἐστιν ἀριθμός, καὶ ὁ τετράγωνος πρὸς τὸν τετράγωνον διπλασίονα λόγον ἔχει ἤπερ ἡ πλευρὰ πρὸς τὴν πλευράν.



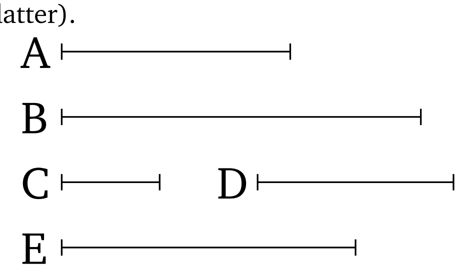
Ἔστωσαν τετράγωνοι ἀριθμοὶ οἱ Α, Β, καὶ τοῦ μὲν Α πλευρὰ ἔστω ὁ Γ, τοῦ δὲ Β ὁ Δ· λέγω, ὅτι τῶν Α, Β εἷς μέσος ἀνάλογόν ἐστιν ἀριθμός, καὶ ὁ Α πρὸς τὸν Β διπλασίονα λόγον ἔχει ἤπερ ὁ Γ πρὸς τὸν Δ.

Ὁ Γ γὰρ τὸν Δ πολλαπλασιάσας τὸν Ε ποιείτω. καὶ ἐπεὶ τετράγωνός ἐστιν ὁ Α, πλευρὰ δὲ αὐτοῦ ἐστιν ὁ Γ, ὁ Γ ἄρα ἑαυτὸν πολλαπλασιάσας τὸν Α πεποίηκεν. διὰ τὰ αὐτὰ δὴ καὶ ὁ Δ ἑαυτὸν πολλαπλασιάσας τὸν Β πεποίηκεν. ἐπεὶ οὖν ὁ Γ ἑκάτερον τῶν Γ, Δ πολλαπλασιάσας ἑκάτερον τῶν Α, Ε πεποίηκεν, ἔστιν ἄρα ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Α πρὸς τὸν Ε. διὰ τὰ αὐτὰ δὴ καὶ ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Ε πρὸς τὸν Β. καὶ ὡς ἄρα ὁ Α πρὸς τὸν Ε, οὕτως ὁ Ε πρὸς τὸν Β. τῶν Α, Β ἄρα εἷς μέσος ἀνάλογόν ἐστιν ἀριθμός.

Λέγω δή, ὅτι καὶ ὁ Α πρὸς τὸν Β διπλασίονα λόγον ἔχει ἤπερ ὁ Γ πρὸς τὸν Δ. ἐπεὶ γὰρ τρεῖς ἀριθμοὶ ἀνάλογόν εἰσιν οἱ Α, Ε, Β, ὁ Α ἄρα πρὸς τὸν Β διπλασίονα λόγον ἔχει ἤπερ ὁ Α πρὸς τὸν Ε. ὡς δὲ ὁ Α πρὸς τὸν Ε, οὕτως ὁ Γ πρὸς τὸν Δ. ὁ Α ἄρα πρὸς τὸν Β διπλασίονα λόγον ἔχει ἤπερ ἡ Γ πλευρὰ πρὸς τὴν Δ· ὅπερ ἔδει δεῖξαι.

Proposition 11

There exists one number in mean proportion to two (given) square numbers.[†] And (one) square (number) has to the (other) square (number) a squared[‡] ratio with respect to (that) the side (of the former has) to the side (of the latter).



Let A and B be square numbers, and let C be the side of A, and D (the side) of B. I say that there exists one number in mean proportion to A and B, and that A has to B a squared ratio with respect to (that) C (has) to D.

For let C make E (by) multiplying D. And since A is square, and C is its side, C has thus made A (by) multiplying itself. And so, for the same (reasons), D has made B (by) multiplying itself. Therefore, since C has made A, E (by) multiplying C, D, respectively, thus as C is to D, so A (is) to E [Prop. 7.17]. And so, for the same (reasons), as C (is) to D, so E (is) to B [Prop. 7.18]. And thus as A (is) to E, so E (is) to B. Thus, one number (namely, E) is in mean proportion to A and B.

So I say that A also has to B a squared ratio with respect to (that) C (has) to D. For since A, E, B are three (continuously) proportional numbers, A thus has to B a squared ratio with respect to (that) A (has) to E [Def. 5.9]. And as A (is) to E, so C (is) to D. Thus, A has to B a squared ratio with respect to (that) side C (has) to (side) D. (Which is) the very thing it was required to show.

† In other words, between two given square numbers there exists a number in continued proportion.

‡ Literally, “double”.

ιβ΄.

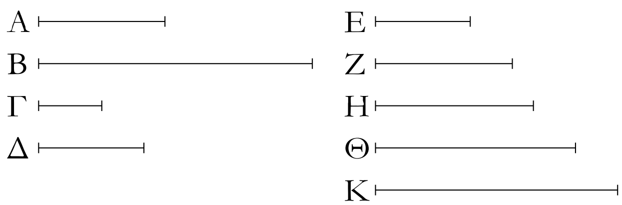
Δύο κύβων ἀριθμῶν δύο μέσοι ἀνάλογόν εἰσιν ἀριθμοί, καὶ ὁ κύβος πρὸς τὸν κύβον τριπλασίονα λόγον ἔχει ἤπερ ἡ πλευρὰ πρὸς τὴν πλευράν.

Ἔστωσαν κύβοι ἀριθμοὶ οἱ Α, Β καὶ τοῦ μὲν Α πλευρὰ ἔστω ὁ Γ, τοῦ δὲ Β ὁ Δ· λέγω, ὅτι τῶν Α, Β δύο μέσοι ἀνάλογόν εἰσιν ἀριθμοί, καὶ ὁ Α πρὸς τὸν Β τριπλασίονα λόγον ἔχει ἤπερ ὁ Γ πρὸς τὸν Δ.

Proposition 12

There exist two numbers in mean proportion to two (given) cube numbers.[†] And (one) cube (number) has to the (other) cube (number) a cubed[‡] ratio with respect to (that) the side (of the former has) to the side (of the latter).

Let A and B be cube numbers, and let C be the side of A, and D (the side) of B. I say that there exist two numbers in mean proportion to A and B, and that A has

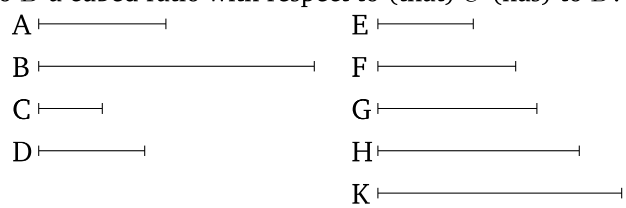


Ὁ γὰρ Γ ἑαυτὸν μὲν πολλαπλασιάσας τὸν Ε ποιείτω, τὸν δὲ Δ πολλαπλασιάσας τὸν Ζ ποιείτω, ὁ δὲ Δ ἑαυτὸν πολλαπλασιάσας τὸν Η ποιείτω, ἑκάτερος δὲ τῶν Γ, Δ τὸν Ζ πολλαπλασιάσας ἑκάτερον τῶν Θ, Κ ποιείτω.

Καὶ ἐπεὶ κύβος ἐστὶν ὁ Α, πλευρὰ δὲ αὐτοῦ ὁ Γ, καὶ ὁ Γ ἑαυτὸν μὲν πολλαπλασιάσας τὸν Ε πεποίηκεν, ὁ Γ ἄρα ἑαυτὸν μὲν πολλαπλασιάσας τὸν Ε πεποίηκεν, τὸν δὲ Ε πολλαπλασιάσας τὸν Α πεποίηκεν. διὰ τὰ αὐτὰ δὴ καὶ ὁ Δ ἑαυτὸν μὲν πολλαπλασιάσας τὸν Η πεποίηκεν, τὸν δὲ Η πολλαπλασιάσας τὸν Β πεποίηκεν. καὶ ἐπεὶ ὁ Γ ἑκάτερον τῶν Γ, Δ πολλαπλασιάσας ἑκάτερον τῶν Ε, Ζ πεποίηκεν, ἔστιν ἄρα ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Ε πρὸς τὸν Ζ. διὰ τὰ αὐτὰ δὴ καὶ ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Ζ πρὸς τὸν Η. πάλιν, ἐπεὶ ὁ Γ ἑκάτερον τῶν Ε, Ζ πολλαπλασιάσας ἑκάτερον τῶν Α, Θ πεποίηκεν, ἔστιν ἄρα ὡς ὁ Ε πρὸς τὸν Ζ, οὕτως ὁ Α πρὸς τὸν Θ. ὡς δὲ ὁ Ε πρὸς τὸν Ζ, οὕτως ὁ Γ πρὸς τὸν Δ· καὶ ὡς ἄρα ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Α πρὸς τὸν Θ. πάλιν, ἐπεὶ ἑκάτερος τῶν Γ, Δ τὸν Ζ πολλαπλασιάσας ἑκάτερον τῶν Θ, Κ πεποίηκεν, ἔστιν ἄρα ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Θ πρὸς τὸν Κ. πάλιν, ἐπεὶ ὁ Δ ἑκάτερον τῶν Ζ, Η πολλαπλασιάσας ἑκάτερον τῶν Κ, Β πεποίηκεν, ἔστιν ἄρα ὡς ὁ Ζ πρὸς τὸν Η, οὕτως ὁ Κ πρὸς τὸν Β. ὡς δὲ ὁ Ζ πρὸς τὸν Η, οὕτως ὁ Γ πρὸς τὸν Δ· καὶ ὡς ἄρα ὁ Γ πρὸς τὸν Δ, οὕτως ὅ τε Α πρὸς τὸν Θ καὶ ὁ Θ πρὸς τὸν Κ καὶ ὁ Κ πρὸς τὸν Β. τῶν Α, Β ἄρα δύο μέσοι ἀνάλογόν εἰσιν οἱ Θ, Κ.

Λέγω δή, ὅτι καὶ ὁ Α πρὸς τὸν Β τριπλασίονα λόγον ἔχει ἤπερ ὁ Γ πρὸς τὸν Δ. ἐπεὶ γὰρ τέσσαρες ἀριθμοὶ ἀνάλογόν εἰσιν οἱ Α, Θ, Κ, Β, ὁ Α ἄρα πρὸς τὸν Β τριπλασίονα λόγον ἔχει ἤπερ ὁ Α πρὸς τὸν Θ. ὡς δὲ ὁ Α πρὸς τὸν Θ, οὕτως ὁ Γ πρὸς τὸν Δ· καὶ ὁ Α [ἄρα] πρὸς τὸν Β τριπλασίονα λόγον ἔχει ἤπερ ὁ Γ πρὸς τὸν Δ· ὅπερ ἔδει δεῖξαι.

to B a cubed ratio with respect to (that) C (has) to D.



For let C make E (by) multiplying itself, and let it make F (by) multiplying D. And let D make G (by) multiplying itself, and let C, D make H, K, respectively, (by) multiplying F.

And since A is cube, and C (is) its side, and C has made E (by) multiplying itself, C has thus made E (by) multiplying itself, and has made A (by) multiplying E. And so, for the same (reasons), D has made G (by) multiplying itself, and has made B (by) multiplying G. And since C has made E, F (by) multiplying C, D, respectively, thus as C is to D, so E (is) to F [Prop. 7.17]. And so, for the same (reasons), as C (is) to D, so F (is) to G [Prop. 7.18]. Again, since C has made A, H (by) multiplying E, F, respectively, thus as E is to F, so A (is) to H [Prop. 7.17]. And as E (is) to F, so C (is) to D. And thus as C (is) to D, so A (is) to H. Again, since C, D have made H, K, respectively, (by) multiplying F, thus as C is to D, so H (is) to K [Prop. 7.18]. Again, since D has made K, B (by) multiplying F, G, respectively, thus as F is to G, so K (is) to B [Prop. 7.17]. And as F (is) to G, so C (is) to D. And thus as C (is) to D, so A (is) to H, and H to K, and K to B. Thus, H and K are two (numbers) in mean proportion to A and B.

So I say that A also has to B a cubed ratio with respect to (that) C (has) to D. For since A, H, K, B are four (continuously) proportional numbers, A thus has to B a cubed ratio with respect to (that) A (has) to H [Def. 5.10]. And as A (is) to H, so C (is) to D. And [thus] A has to B a cubed ratio with respect to (that) C (has) to D. (Which is) the very thing it was required to show.

† In other words, between two given cube numbers there exist two numbers in continued proportion.

‡ Literally, "triple".

ιγ΄.

Ἐὰν ὦσιν ὁσοιδηποτοῦν ἀριθμοὶ ἑξῆς ἀνάλογον, καὶ πολλαπλασιάσας ἕκαστος ἑαυτὸν ποιῇ τινα, οἱ γενόμενοι ἐξ αὐτῶν ἀνάλογον ἔσονται· καὶ ἐὰν οἱ ἐξ ἀρχῆς τοὺς γενομένους πολλαπλασιάσαντες ποιῶσί τινας, καὶ αὐτοὶ ἀνάλογον ἔσονται [καὶ ἀεὶ περὶ τοὺς ἄκρους τοῦτο συμβαίνει].

Ἔστωσαν ὁποσοιοῦν ἀριθμοὶ ἑξῆς ἀνάλογον, οἱ Α, Β,

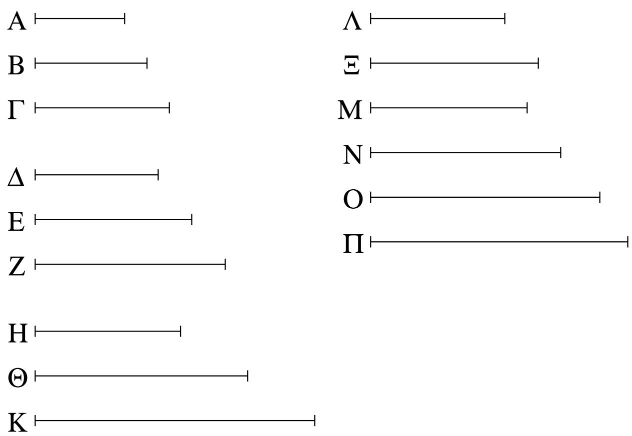
Proposition 13

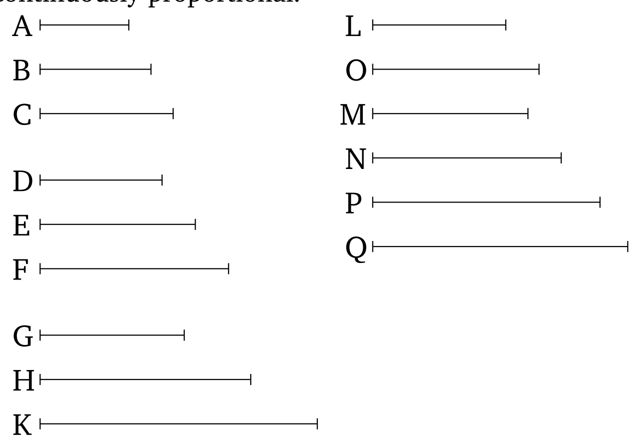
If there are any multitude whatsoever of continuously proportional numbers, and each makes some (number by) multiplying itself, then the (numbers) created from them will (also) be (continuously) proportional. And if the original (numbers) make some (more numbers by) multiplying the created (numbers) then these will also

Γ, ὡς ὁ Α πρὸς τὸν Β, οὕτως ὁ Β πρὸς τὸν Γ, καὶ οἱ Α, Β, Γ ἑαυτοὺς μὲν πολλαπλασιάσαντες τοὺς Δ, Ε, Ζ ποιείτωσαν, τοὺς δὲ Δ, Ε, Ζ πολλαπλασιάσαντες τοὺς Η, Θ, Κ ποιείτωσαν· λέγω, ὅτι οἵ τε Δ, Ε, Ζ καὶ οἱ Η, Θ, Κ ἑξῆς ἀνάλογον εἰσιν.

be (continuously) proportional [and this always happens with the extremes].

Let A, B, C be any multitude whatsoever of continuously proportional numbers, (such that) as A (is) to B, so B (is) to C. And let A, B, C make D, E, F (by) multiplying themselves, and let them make G, H, K (by) multiplying D, E, F. I say that D, E, F and G, H, K are continuously proportional.





Ὁ μὲν γὰρ Α τὸν Β πολλαπλασιάσας τὸν Λ ποιείτω, ἑκάτερος δὲ τῶν Α, Β τὸν Λ πολλαπλασιάσας ἑκάτερον τῶν Μ, Ν ποιείτω. καὶ πάλιν ὁ μὲν Β τὸν Γ πολλαπλασιάσας τὸν Ξ ποιείτω, ἑκάτερος δὲ τῶν Β, Γ τὸν Ξ πολλαπλασιάσας ἑκάτερον τῶν Ο, Π ποιείτω.

Ὁμοίως δὴ τοῖς ἐπάνω δείξομεν, ὅτι οἱ Δ, Λ, Ε καὶ οἱ Η, Μ, Ν, Θ ἑξῆς εἰσιν ἀνάλογον ἐν τῷ τοῦ Α πρὸς τὸν Β λόγῳ, καὶ ἔτι οἱ Ε, Ξ, Ζ καὶ οἱ Θ, Ο, Π, Κ ἑξῆς εἰσιν ἀνάλογον ἐν τῷ τοῦ Β πρὸς τὸν Γ λόγῳ. καί ἐστιν ὡς ὁ Α πρὸς τὸν Β, οὕτως ὁ Β πρὸς τὸν Γ· καὶ οἱ Δ, Λ, Ε ἄρα τοῖς Ε, Ξ, Ζ ἐν τῷ αὐτῷ λόγῳ εἰσὶ καὶ ἔτι οἱ Η, Μ, Ν, Θ τοῖς Θ, Ο, Π, Κ. καί ἐστιν ἴσον τὸ μὲν τῶν Δ, Λ, Ε πλῆθος τῷ τῶν Ε, Ξ, Ζ πλήθει, τὸ δὲ τῶν Η, Μ, Ν, Θ τῷ τῶν Θ, Ο, Π, Κ· δι᾽ ἴσου ἄρα ἐστὶν ὡς μὲν ὁ Δ πρὸς τὸν Ε, οὕτως ὁ Ε πρὸς τὸν Ζ, ὡς δὲ ὁ Η πρὸς τὸν Θ, οὕτως ὁ Θ πρὸς τὸν Κ· ὅπερ ἔδει δεῖξαι.

For let A make L (by) multiplying B. And let A, B make M, N, respectively, (by) multiplying L. And, again, let B make O (by) multiplying C. And let B, C make P, Q, respectively, (by) multplying O.

So, similarly to the above, we can show that D, L, E and G, M, N, H are continuously proportional in the ratio of A to B, and, further, (that) E, O, F and H, P, Q, K are continuously proportional in the ratio of B to C. And as A is to B, so B (is) to C. And thus D, L, E are in the same ratio as E, O, F, and, further, G, M, N, H (are in the same ratio) as H, P, Q, K. And the multitude of D, L, E is equal to the multitude of E, O, F, and that of G, M, N, H to that of H, P, Q, K. Thus, via equality, as D is to E, so E (is) to F, and as G (is) to H, so H (is) to K [Prop. 7.14]. (Which is) the very thing it was required to show.

ιδ΄.

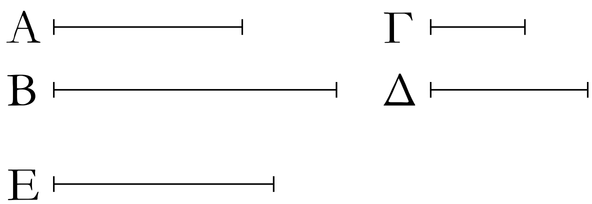
Proposition 14

Ἐὰν τετράγωνος τετράγωνον μετρῇ, καὶ ἡ πλευρὰ τὴν πλευρὰν μετρήσει· καὶ ἐὰν ἡ πλευρὰ τὴν πλευρὰν μετρῇ, καὶ ὁ τετράγωνος τὸν τετράγωνον μετρήσει.

Ἔστωσαν τετράγωνοι ἀριθμοὶ οἱ Α, Β, πλευραὶ δὲ αὐτῶν ἔστωσαν οἱ Γ, Δ, ὁ δὲ Α τὸν Β μετρείτω· λέγω, ὅτι καὶ ὁ Γ τὸν Δ μετρεῖ.

If a square (number) measures a(nother) square (number) then the side (of the former) will also measure the side (of the latter). And if the side (of a square number) measures the side (of another square number) then the (former) square (number) will also measure the (latter) square (number).

Let A and B be square numbers, and let C and D be their sides (respectively). And let A measure B. I say that C also measures D.

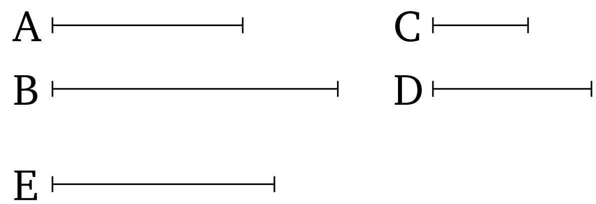


Ὁ Γ γὰρ τὸν Δ πολλαπλασιάσας τὸν Ε ποιείτω· οἱ Α, Ε, Β ἄρα ἑξῆς ἀνάλογόν εἰσιν ἐν τῷ τοῦ Γ πρὸς τὸν Δ λόγῳ. καὶ ἐπεὶ οἱ Α, Ε, Β ἑξῆς ἀνάλογόν εἰσιν, καὶ μετρεῖ ὁ Α τὸν Β, μετρεῖ ἄρα καὶ ὁ Α τὸν Ε. καί ἐστιν ὡς ὁ Α πρὸς τὸν Ε, οὕτως ὁ Γ πρὸς τὸν Δ· μετρεῖ ἄρα καὶ ὁ Γ τὸν Δ.

Πάλιν δὴ ὁ Γ τὸν Δ μετρείτω· λέγω, ὅτι καὶ ὁ Α τὸν Β μετρεῖ.

Τῶν γὰρ αὐτῶν κατασκευασθέντων ὁμοίως δείξομεν, ὅτι οἱ Α, Ε, Β ἑξῆς ἀνάλογόν εἰσιν ἐν τῷ τοῦ Γ πρὸς τὸν Δ λόγῳ. καὶ ἐπεί ἐστιν ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Α πρὸς τὸν Ε, μετρεῖ δὲ ὁ Γ τὸν Δ, μετρεῖ ἄρα καὶ ὁ Α τὸν Ε. καί εἰσιν οἱ Α, Ε, Β ἑξῆς ἀνάλογον· μετρεῖ ἄρα καὶ ὁ Α τὸν Β.

Ἐὰν ἄρα τετράγωνος τετράγωνον μετρῇ, καὶ ἡ πλευρὰ τὴν πλευρὰν μετρήσει· καὶ ἐὰν ἡ πλευρὰ τὴν πλευρὰν μετρῇ, καὶ ὁ τετράγωνος τὸν τετράγωνον μετρήσει· ὅπερ ἔδει δεῖξαι.



For let C make E (by) multiplying D. Thus, A, E, B are continuously proportional in the ratio of C to D [Prop. 8.11]. And since A, E, B are continuously proportional, and A measures B, A thus also measures E [Prop. 8.7]. And as A is to E, so C (is) to D. Thus, C also measures D [Def. 7.20].

So, again, let C measure D. I say that A also measures B.

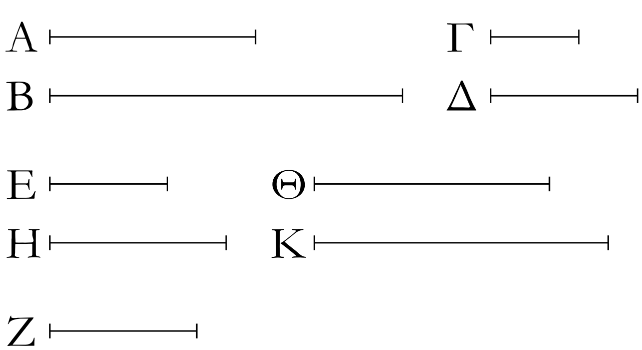
For similarly, with the same construction, we can show that A, E, B are continuously proportional in the ratio of C to D. And since as C is to D, so A (is) to E, and C measures D, A thus also measures E [Def. 7.20]. And A, E, B are continuously proportional. Thus, A also measures B.

Thus, if a square (number) measures a(nother) square (number) then the side (of the former) will also measure the side (of the latter). And if the side (of a square number) measures the side (of another square number) then the (former) square (number) will also measure the (latter) square (number). (Which is) the very thing it was required to show.

ιε′.

Ἐὰν κύβος ἀριθμὸς κύβον ἀριθμὸν μετρῇ, καὶ ἡ πλευρὰ τὴν πλευρὰν μετρήσει· καὶ ἐὰν ἡ πλευρὰ τὴν πλευρὰν μετρῇ, καὶ ὁ κύβος τὸν κύβον μετρήσει.

Κύβος γὰρ ἀριθμὸς ὁ Α κύβον τὸν Β μετρείτω, καὶ τοῦ μὲν Α πλευρὰ ἔστω ὁ Γ, τοῦ δὲ Β ὁ Δ· λέγω, ὅτι ὁ Γ τὸν Δ μετρεῖ.

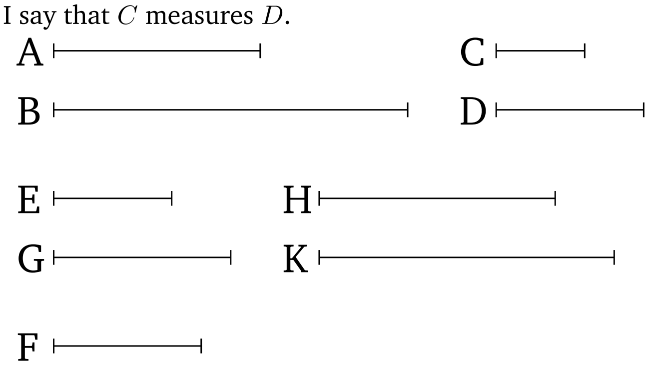


Ὁ Γ γὰρ ἑαυτὸν πολλαπλασιάσας τὸν Ε ποιείτω, ὁ δὲ Δ

Proposition 15

If a cube number measures a(nother) cube number then the side (of the former) will also measure the side (of the latter). And if the side (of a cube number) measures the side (of another cube number) then the (former) cube (number) will also measure the (latter) cube (number).

For let the cube number A measure the cube (number) B, and let C be the side of A, and D (the side) of B. I say that C measures D.



For let C make E (by) multiplying itself. And let ἑαυτὸν πολλαπλασιάσας τὸν Η ποιείτω, καὶ ἔτι ὁ Γ τὸν Δ πολλαπλασιάσας τὸν Ζ [ποιείτω], ἑκάτερος δὲ τῶν Γ, Δ τὸν Ζ πολλαπλασιάσας ἑκάτερον τῶν Θ, Κ ποιείτω. φανερὸν δή, ὅτι οἱ Ε, Ζ, Η καὶ οἱ Α, Θ, Κ, Β ἑξῆς ἀνάλογόν εἰσιν ἐν τῷ τοῦ Γ πρὸς τὸν Δ λόγῳ. καὶ ἐπεὶ οἱ Α, Θ, Κ, Β ἑξῆς ἀνάλογόν εἰσιν, καὶ μετρεῖ ὁ Α τὸν Β, μετρεῖ ἄρα καὶ τὸν Θ. καί ἐστιν ὡς ὁ Α πρὸς τὸν Θ, οὕτως ὁ Γ πρὸς τὸν Δ· μετρεῖ ἄρα καὶ ὁ Γ τὸν Δ.

Ἀλλὰ δὴ μετρείτω ὁ Γ τὸν Δ· λέγω, ὅτι καὶ ὁ Α τὸν Β μετρήσει.

Τῶν γὰρ αὐτῶν κατασκευασθέντων ὁμοίως δὴ δείξομεν, ὅτι οἱ Α, Θ, Κ, Β ἑξῆς ἀνάλογόν εἰσιν ἐν τῷ τοῦ Γ πρὸς τὸν Δ λόγῳ. καὶ ἐπεὶ ὁ Γ τὸν Δ μετρεῖ, καί ἐστιν ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Α πρὸς τὸν Θ, καὶ ὁ Α ἄρα τὸν Θ μετρεῖ· ὥστε καὶ τὸν Β μετρεῖ ὁ Α· ὅπερ ἔδει δεῖξαι.

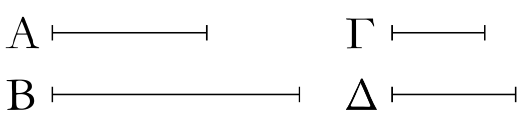
D make G (by) multiplying itself. And, further, [let] C [make] F (by) multiplying D, and let C, D make H, K, respectively, (by) multiplying F. So it is clear that E, F, G and A, H, K, B are continuously proportional in the ratio of C to D [Prop. 8.12]. And since A, H, K, B are continuously proportional, and A measures B, (A) thus also measures H [Prop. 8.7]. And as A is to H, so C (is) to D. Thus, C also measures D [Def. 7.20].

And so let C measure D. I say that A will also measure B.

For similarly, with the same construction, we can show that A, H, K, B are continuously proportional in the ratio of C to D. And since C measures D, and as C is to D, so A (is) to H, A thus also measures H [Def. 7.20]. Hence, A also measures B. (Which is) the very thing it was required to show.

ιϛ'.

Ἐὰν τετράγωνος ἀριθμὸς τετράγωνον ἀριθμὸν μὴ μετρῇ, οὐδὲ ἡ πλευρὰ τὴν πλευρὰν μετρήσει· κἂν ἡ πλευρὰ τὴν πλευρὰν μὴ μετρῇ, οὐδὲ ὁ τετράγωνος τὸν τετράγωνον μετρήσει.



Ἔστωσαν τετράγωνοι ἀριθμοὶ οἱ Α, Β, πλευραὶ δὲ αὐτῶν ἔστωσαν οἱ Γ, Δ, καὶ μὴ μετρείτω ὁ Α τὸν Β· λὲγω, ὅτι οὐδὲ ὁ Γ τὸν Δ μετρεῖ.

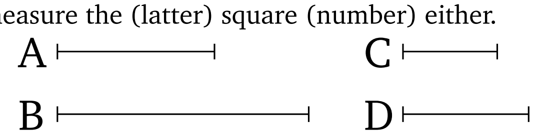
Εἰ γὰρ μετρεῖ ὁ Γ τὸν Δ, μετρήσει καὶ ὁ Α τὸν Β. οὐ μετρεῖ δὲ ὁ Α τὸν Β· οὐδὲ ἄρα ὁ Γ τὸν Δ μετρήσει.

Μὴ μετρείτω [δὴ] πάλιν ὁ Γ τὸν Δ· λέγω, ὅτι οὐδὲ ὁ Α τὸν Β μετρήσει.

Εἰ γὰρ μετρεῖ ὁ Α τὸν Β, μετρήσει καὶ ὁ Γ τὸν Δ. οὐ μετρεῖ δὲ ὁ Γ τὸν Δ· οὐδ᾽ ἄρα ὁ Α τὸν Β μετρήσει· ὅπερ ἔδει δεῖξαι.

Proposition 16

If a square number does not measure a(nother) square number then the side (of the former) will not measure the side (of the latter) either. And if the side (of a square number) does not measure the side (of another square number) then the (former) square (number) will not measure the (latter) square (number) either.



Let A and B be square numbers, and let C and D be their sides (respectively). And let A not measure B. I say that C does not measure D either.

For if C measures D then A will also measure B [Prop. 8.14]. And A does not measure B. Thus, C will not measure D either.

[So], again, let C not measure D. I say that A will not measure B either.

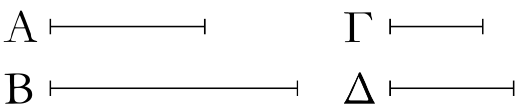
For if A measures B then C will also measure D [Prop. 8.14]. And C does not measure D. Thus, A will not measure B either. (Which is) the very thing it was required to show.

ιζ'.

Ἐὰν κύβος ἀριθμὸς κύβον ἀριθμὸν μὴ μετρῇ, οὐδὲ ἡ πλευρὰ τὴν πλευρὰν μετρήσει· κἂν ἡ πλευρὰ τὴν πλευρὰν μὴ μετρῇ, οὐδὲ ὁ κύβος τὸν κύβον μετρήσει.

Proposition 17

If a cube number does not measure a(nother) cube number then the side (of the former) will not measure the side (of the latter) either. And if the side (of a cube number) does not measure the side (of another cube number) then the (former) cube (number) will not measure the (latter) cube (number) either.

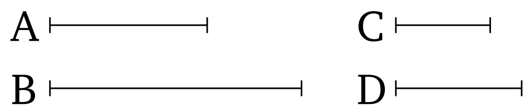


Κύβος γὰρ ἀριθμὸς ὁ Α κύβον ἀριθμὸν τὸν Β μὴ μετρείτω, καὶ τοῦ μὲν Α πλευρὰ ἔστω ὁ Γ, τοῦ δὲ Β ὁ Δ· λέγω, ὅτι ὁ Γ τὸν Δ οὐ μετρήσει.

Εἰ γὰρ μετρεῖ ὁ Γ τὸν Δ, καὶ ὁ Α τὸν Β μετρήσει. οὐ μετρεῖ δὲ ὁ Α τὸν Β· οὐδ᾿ ἄρα ὁ Γ τὸν Δ μετρεῖ.

Ἀλλὰ δὴ μὴ μετρείτω ὁ Γ τὸν Δ· λέγω, ὅτι οὐδὲ ὁ Α τὸν Β μετρήσει.

Εἰ γὰρ ὁ Α τὸν Β μετρεῖ, καὶ ὁ Γ τὸν Δ μετρήσει. οὐ μετρεῖ δὲ ὁ Γ τὸν Δ· οὐδ᾿ ἄρα ὁ Α τὸν Β μετρήσει· ὅπερ ἔδει δεῖξαι.



For let the cube number A not measure the cube number B. And let C be the side of A, and D (the side) of B. I say that C will not measure D.

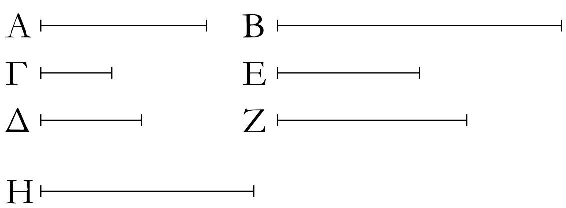
For if C measures D then A will also measure B [Prop. 8.15]. And A does not measure B. Thus, C does not measure D either.

And so let C not measure D. I say that A will not measure B either.

For if A measures B then C will also measure D [Prop. 8.15]. And C does not measure D. Thus, A will not measure B either. (Which is) the very thing it was required to show.

ιη´.

Δύο ὁμοίων ἐπιπέδων ἀριθμῶν εἷς μέσος ἀνάλογόν ἐστιν ἀριθμός· καὶ ὁ ἐπίπεδος πρὸς τὸν ἐπίπεδον διπλασίονα λόγον ἔχει ἤπερ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν.

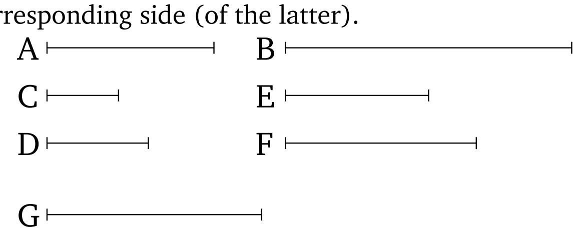


Ἔστωσαν δύο ὅμοιοι ἐπίπεδοι ἀριθμοὶ οἱ Α, Β, καὶ τοῦ μὲν Α πλευραὶ ἔστωσαν οἱ Γ, Δ ἀριθμοί, τοῦ δὲ Β οἱ Ε, Ζ. καὶ ἐπεὶ ὅμοιοι ἐπίπεδοί εἰσιν οἱ ἀνάλογον ἔχοντες τὰς πλευράς, ἔστιν ἄρα ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Ε πρὸς τὸν Ζ. λέγω οὖν, ὅτι τῶν Α, Β εἷς μέσος ἀνάλογόν ἐστιν ἀριθμός, καὶ ὁ Α πρὸς τὸν Β διπλασίονα λόγον ἔχει ἤπερ ὁ Γ πρὸς τὸν Ε ἢ ὁ Δ πρὸς τὸν Ζ, τουτέστιν ἤπερ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον [πλευράν].

Καὶ ἐπεί ἐστιν ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Ε πρὸς τὸν Ζ, ἐναλλὰξ ἄρα ἐστὶν ὡς ὁ Γ πρὸς τὸν Ε, ὁ Δ πρὸς τὸν Ζ. καὶ ἐπεὶ ἐπίπεδός ἐστιν ὁ Α, πλευραὶ δὲ αὐτοῦ οἱ Γ, Δ, ὁ Δ ἄρα τὸν Γ πολλαπλασιάσας τὸν Α πεποίηκεν. διὰ τὰ αὐτὰ δὴ καὶ ὁ Ε τὸν Ζ πολλαπλασιάσας τὸν Β πεποίηκεν. ὁ Δ δὴ τὸν Ε πολλαπλασιάσας τὸν Η ποιείτω. καὶ ἐπεὶ ὁ Δ τὸν μὲν Γ πολλαπλασιάσας τὸν Α πεποίηκεν, τὸν δὲ Ε πολλαπλασιάσας τὸν Η πεποίηκεν, ἔστιν ἄρα ὡς ὁ Γ πρὸς τὸν Ε, οὕτως ὁ Α πρὸς τὸν Η. ἀλλ᾿ ὡς ὁ Γ πρὸς τὸν Ε, [οὕτως] ὁ Δ πρὸς τὸν Ζ· καὶ ὡς ἄρα ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Α πρὸς τὸν Η. πάλιν, ἐπεὶ ὁ Ε τὸν μὲν Δ πολλαπλασιάσας τὸν Η πεποίηκεν, τὸν δὲ Ζ πολλαπλασιάσας τὸν Β πεποίηκεν, ἔστιν ἄρα ὡς ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Η πρὸς τὸν Β. ἐδείχθη δὲ καὶ ὡς ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Α πρὸς τὸν

Proposition 18

There exists one number in mean proportion to two similar plane numbers. And (one) plane (number) has to the (other) plane (number) a squared[†] ratio with respect to (that) a corresponding side (of the former has) to a corresponding side (of the latter).



Let A and B be two similar plane numbers. And let the numbers C, D be the sides of A, and E, F (the sides) of B. And since similar numbers are those having proportional sides [Def. 7.21], thus as C is to D, so E (is) to F. Therefore, I say that there exists one number in mean proportion to A and B, and that A has to B a squared ratio with respect to that C (has) to E, or D to F—that is to say, with respect to (that) a corresponding side (has) to a corresponding [side].

For since as C is to D, so E (is) to F, thus, alternately, as C is to E, so D (is) to F [Prop. 7.13]. And since A is plane, and C, D its sides, D has thus made A (by) multiplying C. And so, for the same (reasons), E has made B (by) multiplying F. So let D make G (by) multiplying E. And since D has made A (by) multiplying C, and has made G (by) multiplying E, thus as C is to E, so A (is) to G [Prop. 7.17]. But as C (is) to E, [so] D (is) to F. And thus as D (is) to F, so A (is) to G. Again, since E has made G (by) multiplying D, and has made B (by) multiplying F, thus as D is to F, so G (is) to B [Prop. 7.17]. And it was also shown that as D (is) to F, so A (is) to G. And thus as A (is) to G, so G (is) to B. Thus, A, G, B are

Η· καὶ ὡς ἄρα ὁ Α πρὸς τὸν Η, οὕτως ὁ Η πρὸς τὸν Β. οἱ Α, Η, Β ἄρα ἑξῆς ἀνάλογόν εἰσιν. τῶν Α, Β ἄρα εἷς μέσος ἀνάλογόν ἐστιν ἀριθμός.

Λέγω δή, ὅτι καὶ ὁ Α πρὸς τὸν Β διπλασίονα λόγον ἔχει ἤπερ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν, τουτέστιν ἤπερ ὁ Γ πρὸς τὸν Ε ἢ ὁ Δ πρὸς τὸν Ζ. ἐπεὶ γὰρ οἱ Α, Η, Β ἑξῆς ἀνάλογόν εἰσιν, ὁ Α πρὸς τὸν Β διπλασίονα λόγον ἔχει ἤπερ πρὸς τὸν Η. καί ἐστιν ὡς ὁ Α πρὸς τὸν Η, οὕτως ὅ τε Γ πρὸς τὸν Ε καὶ ὁ Δ πρὸς τὸν Ζ. καὶ ὁ Α ἄρα πρὸς τὸν Β διπλασίονα λόγον ἔχει ἤπερ ὁ Γ πρὸς τὸν Ε ἢ ὁ Δ πρὸς τὸν Ζ· ὅπερ ἔδει δεῖξαι.

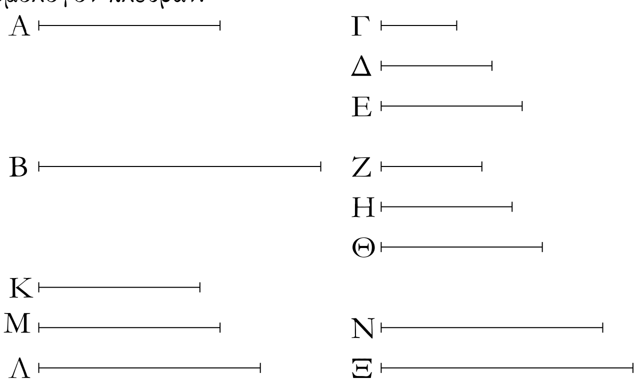
continously proportional. Thus, there exists one number (namely, G) in mean proportion to A and B.

So I say that A also has to B a squared ratio with respect to (that) a corresponding side (has) to a corresponding side—that is to say, with respect to (that) C (has) to E, or D to F. For since A, G, B are continuously proportional, A has to B a squared ratio with respect to (that A has) to G [Prop. 5.9]. And as A is to G, so C (is) to E, and D to F. And thus A has to B a squared ratio with respect to (that) C (has) to E, or D to F. (Which is) the very thing it was required to show.

† Literally, "double".

ιθ´.

Δύο ὁμοίων στερεῶν ἀριθμῶν δύο μέσοι ἀνάλογον ἐμπίπτουσιν ἀριθμοί· καὶ ὁ στερεὸς πρὸς τὸν ὅμοιον στερεὸν τριπλασίονα λόγον ἔχει ἤπερ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν.

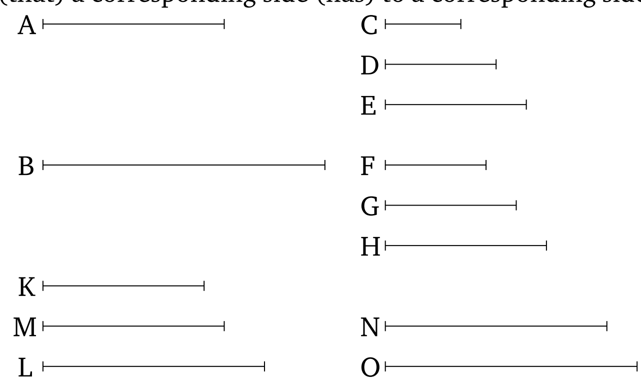


Ἔστωσαν δύο ὅμοιοι στερεοὶ οἱ Α, Β, καὶ τοῦ μὲν Α πλευραὶ ἔστωσαν οἱ Γ, Δ, Ε, τοῦ δὲ Β οἱ Ζ, Η, Θ. καὶ ἐπεὶ ὅμοιοι στερεοί εἰσιν οἱ ἀνάλογον ἔχοντες τὰς πλευράς, ἔστιν ἄρα ὡς μὲν ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Ζ πρὸς τὸν Η, ὡς δὲ ὁ Δ πρὸς τὸν Ε, οὕτως ὁ Η πρὸς τὸν Θ. λέγω, ὅτι τῶν Α, Β δύο μέσοι ἀνάλογόν ἐμπίπτουσιν ἀριθμοί, καὶ ὁ Α πρὸς τὸν Β τριπλασίονα λόγον ἔχει ἤπερ ὁ Γ πρὸς τὸν Ζ καὶ ὁ Δ πρὸς τὸν Η καὶ ἔτι ὁ Ε πρὸς τὸν Θ.

Ὁ Γ γὰρ τὸν Δ πολλαπλασιάσας τὸν Κ ποιείτω, ὁ δὲ Ζ τὸν Η πολλαπλασιάσας τὸν Λ ποιείτω. καὶ ἐπεὶ οἱ Γ, Δ τοῖς Ζ, Η ἐν τῷ αὐτῷ λόγῳ εἰσίν, καὶ ἐκ μὲν τῶν Γ, Δ ἐστιν ὁ Κ, ἐκ δὲ τῶν Ζ, Η ὁ Λ, οἱ Κ, Λ [ἄρα] ὅμοιοι ἐπίπεδοί εἰσιν ἀριθμοί· τῶν Κ, Λ ἄρα εἷς μέσος ἀνάλογόν ἐστιν ἀριθμός. ἔστω ὁ Μ. ὁ Μ ἄρα ἐστὶν ὁ ἐκ τῶν Δ, Ζ, ὡς ἐν τῷ πρὸ τούτου θεωρήματι ἐδείχθη. καὶ ἐπεὶ ὁ Δ τὸν μὲν Γ πολλαπλασιάσας τὸν Κ πεποίηκεν, τὸν δὲ Ζ πολλαπλασιάσας τὸν Μ πεποίηκεν, ἔστιν ἄρα ὡς ὁ Γ πρὸς τὸν Ζ, οὕτως ὁ Κ πρὸς τὸν Μ. ἀλλ᾽ ὡς ὁ Κ πρὸς τὸν Μ, ὁ Μ πρὸς τὸν Λ. οἱ Κ, Μ, Λ ἄρα ἑξῆς εἰσιν ἀνάλογον ἐν

Proposition 19

Two numbers fall (between) two similar solid numbers in mean proportion. And a solid (number) has to a similar solid (number) a cubed† ratio with respect to (that) a corresponding side (has) to a corresponding side.



Let A and B be two similar solid numbers, and let C, D, E be the sides of A, and F, G, H (the sides) of B. And since similar solid (numbers) are those having proportional sides [Def. 7.21], thus as C is to D, so F (is) to G, and as D (is) to E, so G (is) to H. I say that two numbers fall (between) A and B in mean proportion, and (that) A has to B a cubed ratio with respect to (that) C (has) to F, and D to G, and, further, E to H.

For let C make K (by) multiplying D, and let F make L (by) multiplying G. And since C, D are in the same ratio as F, G, and K is the (number created) from (multiplying) C, D, and L the (number created) from (multiplying) F, G, [thus] K and L are similar plane numbers [Def. 7.21]. Thus, there exits one number in mean proportion to K and L [Prop. 8.18]. Let it be M. Thus, M is the (number created) from (multiplying) D, F, as shown in the theorem before this (one). And since D has made K (by) multiplying C, and has made M (by) multiplying F, thus as C is to F, so K (is) to M [Prop. 7.17]. But, as τῷ τοῦ Γ πρὸς τὸν Ζ λόγῳ. καὶ ἐπεί ἐστιν ὡς ὁ Γ πρὸς τὸν Δ, οὕτως ὁ Ζ πρὸς τὸν Η, ἐναλλὰξ ἄρα ἐστὶν ὡς ὁ Γ πρὸς τὸν Ζ, οὕτως ὁ Δ πρὸς τὸν Η. διὰ τὰ αὐτὰ δὴ καὶ ὡς ὁ Δ πρὸς τὸν Η, οὕτως ὁ Ε πρὸς τὸν Θ. οἱ Κ, Μ, Λ ἄρα ἑξῆς εἰσιν ἀνάλογον ἔν τε τῷ τοῦ Γ πρὸς τὸν Ζ λόγῳ καὶ τῷ τοῦ Δ πρὸς τὸν Η καὶ ἔτι τῷ τοῦ Ε πρὸς τὸν Θ. ἑκατερος δὴ τῶν Ε, Θ τὸν Μ πολλαπλασιάσας ἑκάτερον τῶν Ν, Ξ ποιείτω. καὶ ἐπεὶ στερεός ἐστιν ὁ Α, πλευραὶ δὲ αὐτοῦ εἰσιν οἱ Γ, Δ, Ε, ὁ Ε ἄρα τὸν ἐκ τῶν Γ, Δ πολλαπλασιάσας τὸν Α πεποίηκεν. ὁ δὲ ἐκ τῶν Γ, Δ ἐστιν ὁ Κ· ὁ Ε ἄρα τὸν Κ πολλαπλασιάσας τὸν Α πεποίηκεν. διὰ τὰ αὐτὰ δὴ καὶ ὁ Θ τὸν Λ πολλαπλασιάσας τὸν Β πεποίηκεν. καὶ ἐπεὶ ὁ Ε τὸν Κ πολλαπλασιάσας τὸν Α πεποίηκεν, ἀλλὰ μὴν καὶ τὸν Μ πολλαπλασιάσας τὸν Ν πεποίηκεν, ἔστιν ἄρα ὡς ὁ Κ πρὸς τὸν Μ, οὕτως ὁ Α πρὸς τὸν Ν. ὡς δὲ ὁ Κ πρὸς τὸν Μ, οὕτως ὅ τε Γ πρὸς τὸν Ζ καὶ ὁ Δ πρὸς τὸν Η καὶ ἔτι ὁ Ε πρὸς τὸν Θ· καὶ ὡς ἄρα ὁ Γ πρὸς τὸν Ζ καὶ ὁ Δ πρὸς τὸν Η καὶ ὁ Ε πρὸς τὸν Θ, οὕτως ὁ Α πρὸς τὸν Ν. πάλιν, ἐπεὶ ἑκάτερος τῶν Ε, Θ τὸν Μ πολλαπλασιάσας ἑκάτερον τῶν Ν, Ξ πεποίηκεν, ἔστιν ἄρα ὡς ὁ Ε πρὸς τὸν Θ, οὕτως ὁ Ν πρὸς τὸν Ξ. ἀλλ᾿ ὡς ὁ Ε πρὸς τὸν Θ, οὕτως ὅ τε Γ πρὸς τὸν Ζ καὶ ὁ Δ πρὸς τὸν Η· καὶ ὡς ἄρα ὁ Γ πρὸς τὸν Ζ καὶ ὁ Δ πρὸς τὸν Η καὶ ὁ Ε πρὸς τὸν Θ, οὕτως ὅ τε Α πρὸς τὸν Ν καὶ ὁ Ν πρὸς τὸν Ξ. πάλιν, ἐπεὶ ὁ Θ τὸν Μ πολλαπλασιάσας τὸν Ξ πεποίηκεν, ἀλλὰ μὴν καὶ τὸν Λ πολλαπλασιάσας τὸν Β πεποίηκεν, ἔστιν ἄρα ὡς ὁ Μ πρὸς τὸν Λ, οὕτως ὁ Ξ πρὸς τὸν Β. ἀλλ᾿ ὡς ὁ Μ πρὸς τὸν Λ, οὕτως ὅ τε Γ πρὸς τὸν Ζ καὶ ὁ Δ πρὸς τὸν Η καὶ ὁ Ε πρὸς τὸν Θ. καὶ ὡς ἄρα ὁ Γ πρὸς τὸν Ζ καὶ ὁ Δ πρὸς τὸν Η καὶ ὁ Ε πρὸς τὸν Θ, οὕτως οὐ μόνον ὁ Ξ πρὸς τὸν Β, ἀλλὰ καὶ ὁ Α πρὸς τὸν Ν καὶ ὁ Ν πρὸς τὸν Ξ. οἱ Α, Ν, Ξ, Β ἄρα ἑξῆς εἰσιν ἀνάλογον ἐν τοῖς εἰρημένοις τῶν πλευρῶν λόγοις.

Λέγω, ὅτι καὶ ὁ Α πρὸς τὸν Β τριπλασίονα λόγον ἔχει ἤπερ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν, τουτέστιν ἤπερ ὁ Γ ἀριθμὸς πρὸς τὸν Ζ ἢ ὁ Δ πρὸς τὸν Η καὶ ἔτι ὁ Ε πρὸς τὸν Θ. ἐπεὶ γὰρ τέσσαρες ἀριθμοὶ ἑξῆς ἀνάλογόν εἰσιν οἱ Α, Ν, Ξ, Β, ὁ Α ἄρα πρὸς τὸν Β τριπλασίονα λόγον ἔχει ἤπερ ὁ Α πρὸς τὸν Ν. ἀλλ᾿ ὡς ὁ Α πρὸς τὸν Ν, οὕτως ἐδείχθη ὅ τε Γ πρὸς τὸν Ζ καὶ ὁ Δ πρὸς τὸν Η καὶ ἔτι ὁ Ε πρὸς τὸν Θ. καὶ ὁ Α ἄρα πρὸς τὸν Β τριπλασίονα λόγον ἔχει ἤπερ ἡ ομόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν, τουτέστιν ἤπερ ὁ Γ ἀριθμὸς πρὸς τὸν Ζ καὶ ὁ Δ πρὸς τὸν Η καὶ ἔτι ὁ Ε πρὸς τὸν Θ· ὅπερ ἔδει δεῖξαι.

K (is) to M, (so) M (is) to L. Thus, K, M, L are continuously proportional in the ratio of C to F. And since as C is to D, so F (is) to G, thus, alternately, as C is to F, so D (is) to G [Prop. 7.13]. And so, for the same (reasons), as D (is) to G, so E (is) to H. Thus, K, M, L are continuously proportional in the ratio of C to F, and of D to G, and, further, of E to H. So let E, H make N, O, respectively, (by) multiplying M. And since A is solid, and C, D, E are its sides, E has thus made A (by) multiplying the (number created) from (multiplying) C, D. And K is the (number created) from (multiplying) C, D. Thus, E has made A (by) multiplying K. And so, for the same (reasons), H has made B (by) multiplying L. And since E has made A (by) multiplying K, but has, in fact, also made N (by) multiplying M, thus as K is to M, so A (is) to N [Prop. 7.17]. And as K (is) to M, so C (is) to F, and D to G, and, further, E to H. And thus as C (is) to F, and D to G, and E to H, so A (is) to N. Again, since E, H have made N, O, respectively, (by) multiplying M, thus as E is to H, so N (is) to O [Prop. 7.18]. But, as E (is) to H, so C (is) to F, and D to G. And thus as C (is) to F, and D to G, and E to H, so (is) A to N, and N to O. Again, since H has made O (by) multiplying M, but has, in fact, also made B (by) multiplying L, thus as M (is) to L, so O (is) to B [Prop. 7.17]. But, as M (is) to L, so C (is) to F, and D to G, and E to H. And thus as C (is) to F, and D to G, and E to H, so not only (is) O to B, but also A to N, and N to O. Thus, A, N, O, B are continuously proportional in the aforementioned ratios of the sides.

So I say that A also has to B a cubed ratio with respect to (that) a corresponding side (has) to a corresponding side—that is to say, with respect to (that) the number C (has) to F, or D to G, and, further, E to H. For since A, N, O, B are four continuously proportional numbers, A thus has to B a cubed ratio with respect to (that) A (has) to N [Def. 5.10]. But, as A (is) to N, so it was shown (is) C to F, and D to G, and, further, E to H. And thus A has to B a cubed ratio with respect to (that) a corresponding side (has) to a corresponding side—that is to say, with respect to (that) the number C (has) to F, and D to G, and, further, E to H. (Which is) the very thing it was required to show.

† Literally, "triple".

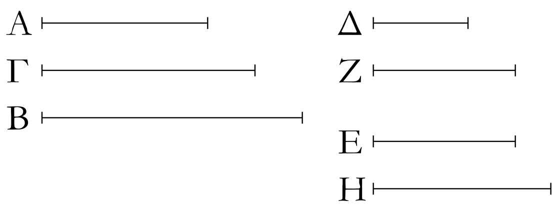
κ΄.

Ἐὰν δύο ἀριθμῶν εἷς μέσος ἀνάλογον ἐμπίπτῃ ἀριθμός, ὅμοιοι ἐπίπεδοι ἔσονται οἱ ἀριθμοί.

Proposition 20

If one number falls between two numbers in mean proportion then the numbers will be similar plane (num-

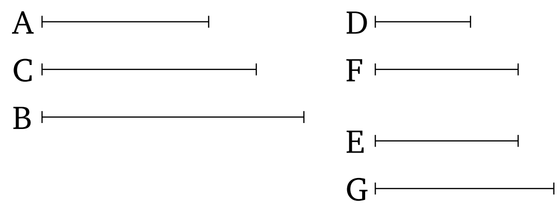
Δύο γὰρ ἀριθμῶν τῶν Α, Β εἷς μέσος ἀνάλογον ἐμπιπτέτω ἀριθμὸς ὁ Γ· λέγω, ὅτι οἱ Α, Β ὅμοιοι ἐπίπεδοί εἰσιν ἀριθμοί.



Εἰλήφθωσαν [γὰρ] ἐλάχιστοι ἀριθμοὶ τῶν τὸν αὐτὸν λόγον ἐχόντων τοῖς Α, Γ οἱ Δ, Ε· ἰσάκις ἄρα ὁ Δ τὸν Α μετρεῖ καὶ ὁ Ε τὸν Γ. ὁσάκις δὴ ὁ Δ τὸν Α μετρεῖ, τοσαῦται μονάδες ἔστωσαν ἐν τῷ Ζ· ὁ Ζ ἄρα τὸν Δ πολλαπλασιάσας τὸν Α πεποίηκεν. ὥστε ὁ Α ἐπίπεδός ἐστιν, πλευραὶ δὲ αὐτοῦ οἱ Δ, Ζ. πάλιν, ἐπεὶ οἱ Δ, Ε ἐλάχιστοί εἰσι τῶν τὸν αὐτὸν λόγον ἐχόντων τοῖς Γ, Β, ἰσάκις ἄρα ὁ Δ τὸν Γ μετρεῖ καὶ ὁ Ε τὸν Β. ὁσάκις δὴ ὁ Ε τὸν Β μετρεῖ, τοσαῦται μονάδες ἔστωσαν ἐν τῷ Η. ὁ Ε ἄρα τὸν Β μετρεῖ κατὰ τὰς ἐν τῷ Η μονάδας· ὁ Η ἄρα τὸν Ε πολλαπλασιάσας τὸν Β πεποίηκεν. ὁ Β ἄρα ἐπίπεδος ἐστι, πλευραὶ δὲ αὐτοῦ εἰσιν οἱ Ε, Η. οἱ Α, Β ἄρα ἐπίπεδοί εἰσιν ἀριθμοί. λέγω δή, ὅτι καὶ ὅμοιοι. ἐπεὶ γὰρ ὁ Ζ τὸν μὲν Δ πολλαπλασιάσας τὸν Α πεποίηκεν, τὸν δὲ Ε πολλαπλασιάσας τὸν Γ πεποίηκεν, ἔστιν ἄρα ὡς ὁ Δ πρὸς τὸν Ε, οὕτως ὁ Α πρὸς τὸν Γ, τουτέστιν ὁ Γ πρὸς τὸν Β. πάλιν, ἐπεὶ ὁ Ε ἑκάτερον τῶν Ζ, Η πολλαπλασιάσας τοὺς Γ, Β πεποίηκεν, ἔστιν ἄρα ὡς ὁ Ζ πρὸς τὸν Η, οὕτως ὁ Γ πρὸς τὸν Β. ὡς δὲ ὁ Γ πρὸς τὸν Β, οὕτως ὁ Δ πρὸς τὸν Ε· καὶ ὡς ἄρα ὁ Δ πρὸς τὸν Ε, οὕτως ὁ Ζ πρὸς τὸν Η· καὶ ἐναλλὰξ ὡς ὁ Δ πρὸς τὸν Ζ, οὕτως ὁ Ε πρὸς τὸν Η. οἱ Α, Β ἄρα ὅμοιοι ἐπίπεδοι ἀριθμοί εἰσιν· αἱ γὰρ πλευραὶ αὐτῶν ἀνάλογόν εἰσιν· ὅπερ ἔδει δεῖξαι.

bers).

For let one number C fall between the two numbers A and B in mean proportion. I say that A and B are similar plane numbers.



[For] let the least numbers, D and E, having the same ratio as A and C have been taken [Prop. 7.33]. Thus, D measures A as many times as E (measures) C [Prop. 7.20]. So as many times as D measures A, so many units let there be in F. Thus, F has made A (by) multiplying D [Def. 7.15]. Hence, A is plane, and D, F (are) its sides. Again, since D and E are the least of those (numbers) having the same ratio as C and B, D thus measures C as many times as E (measures) B [Prop. 7.20]. So as many times as E measures B, so many units let there be in G. Thus, E measures B according to the units in G. Thus, G has made B (by) multiplying E [Def. 7.15]. Thus, B is plane, and E, G are its sides. Thus, A and B are (both) plane numbers. So I say that (they are) also similar. For since F has made A (by) multiplying D, and has made C (by) multiplying E, thus as D is to E, so A (is) to C—that is to say, C to B [Prop. 7.17].† Again, since E has made C, B (by) multiplying F, G, respectively, thus as F is to G, so C (is) to B [Prop. 7.17]. And as C (is) to B, so D (is) to E. And thus as D (is) to E, so F (is) to G. And, alternately, as D (is) to F, so E (is) to G [Prop. 7.13]. Thus, A and B are similar plane numbers. For their sides are proportional [Def. 7.21]. (Which is) the very thing it was required to show.

† This part of the proof is defective, since it is not demonstrated that $F \times E = C$. Furthermore, it is not necessary to show that $D : E :: A : C$, because this is true by hypothesis.

κα′.

Ἐὰν δύο ἀριθμῶν δύο μέσοι ἀνάλογον ἐμπίπτωσιν ἀριθμοί, ὅμοιοι στερεοί εἰσιν οἱ ἀριθμοί.

Δύο γὰρ ἀριθμῶν τῶν Α, Β δύο μέσοι ἀνάλογον ἐμπιπτέτωσαν ἀριθμοὶ οἱ Γ, Δ· λέγω, ὅτι οἱ Α, Β ὅμοιοι στερεοί εἰσιν.

Εἰλήφθωσαν γὰρ ἐλάχιστοι ἀριθμοὶ τῶν τὸν αὐτὸν λόγον ἐχόντων τοῖς Α, Γ, Δ τρεῖς οἱ Ε, Ζ, Η· οἱ ἄρα ἄκροι αὐτῶν οἱ Ε, Η πρῶτοι πρὸς ἀλλήλους εἰσίν. καὶ ἐπεὶ τῶν Ε, Η εἷς μέσος ἀνάλογον ἐμπέπτωκεν ἀριθμὸς ὁ Ζ, οἱ Ε, Η ἄρα ἀριθμοὶ ὅμοιοι ἐπίπεδοί εἰσιν. ἔστωσαν οὖν τοῦ μὲν

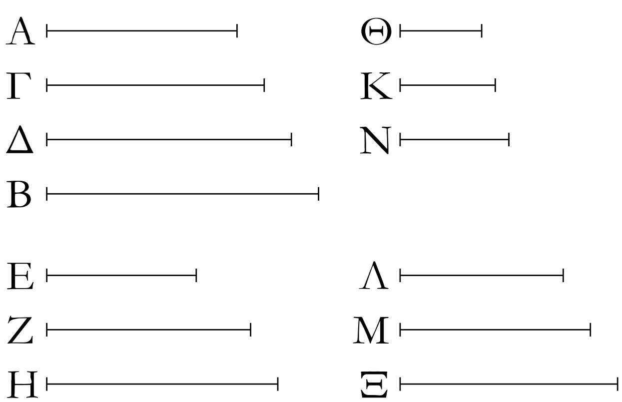
Proposition 21

If two numbers fall between two numbers in mean proportion then the (latter) are similar solid (numbers).

For let the two numbers C and D fall between the two numbers A and B in mean proportion. I say that A and B are similar solid (numbers).

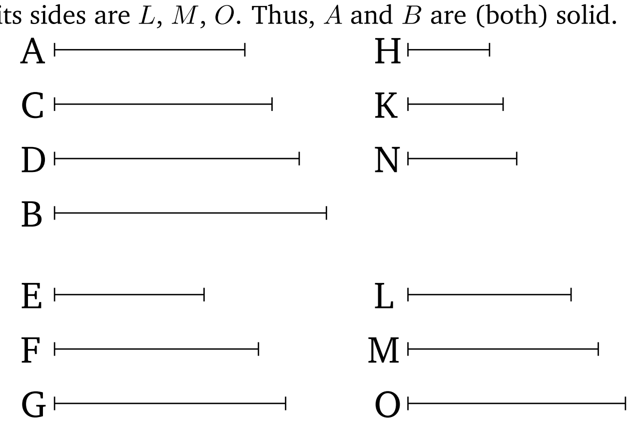
For let the three least numbers E, F, G having the same ratio as A, C, D have been taken [Prop. 8.2]. Thus, the outermost of them, E and G, are prime to one another [Prop. 8.3]. And since one number, F, has fallen (between) E and G in mean proportion, E and G are

Ε πλευραὶ οἱ Θ, Κ, τοῦ δὲ Η οἱ Λ, Μ. φανερὸν ἄρα ἐστὶν ἐκ τοῦ πρὸ τούτου, ὅτι οἱ Ε, Ζ, Η ἑξῆς εἰσιν ἀνάλογον ἔν τε τῷ τοῦ Θ πρὸς τὸν Λ λόγῳ καὶ τῷ τοῦ Κ πρὸς τὸν Μ. καὶ ἐπεὶ οἱ Ε, Ζ, Η ἐλάχιστοί εἰσι τῶν τὸν αὐτὸν λόγον ἐχόντων τοῖς Α, Γ, Δ, καί ἐστιν ἴσον τὸ πλῆθος τῶν Ε, Ζ, Η τῷ πλήθει τῶν Α, Γ, Δ, δι᾿ ἴσου ἄρα ἐστὶν ὡς ὁ Ε πρὸς τὸν Η, οὕτως ὁ Α πρὸς τὸν Δ. οἱ δὲ Ε, Η πρῶτοι, οἱ δὲ πρῶτοι καὶ ἐλάχιστοι, οἱ δὲ ἐλάχιστοι μετροῦσι τοὺς τὸν αὐτὸν λόγον ἔχοντας αὐτοῖς ἰσάκις ὅ τε μείζων τὸν μείζονα καὶ ὁ ἐλάσσων τὸν ἐλάσσονα, τουτέστιν ὅ τε ἡγούμενος τὸν ἡγούμενον καὶ ὁ ἑπόμενος τὸν ἑπόμενον· ἰσάκις ἄρα ὁ Ε τὸν Α μετρεῖ καὶ ὁ Η τὸν Δ. ὁσάκις δὴ ὁ Ε τὸν Α μετρεῖ, τοσαῦται μονάδες ἔστωσαν ἐν τῷ Ν. ὁ Ν ἄρα τὸν Ε πολλαπλασιάσας τὸν Α πεποίηκεν. ὁ δὲ Ε ἐστιν ὁ ἐκ τῶν Θ, Κ· ὁ Ν ἄρα τὸν ἐκ τῶν Θ, Κ πολλαπλασιάσας τὸν Α πεποίηκεν. στερεὸς ἄρα ἐστὶν ὁ Α, πλευραὶ δὲ αὐτοῦ εἰσιν οἱ Θ, Κ, Ν. πάλιν, ἐπεὶ οἱ Ε, Ζ, Η ἐλάχιστοί εἰσι τῶν τὸν αὐτὸν λόγον ἐχόντων τοῖς Γ, Δ, Β, ἰσάκις ἄρα ὁ Ε τὸν Γ μετρεῖ καὶ ὁ Η τὸν Β. ὁσάκις δὴ ὁ Ε τὸν Γ μετρεῖ, τοσαῦται μονάδες ἔστωσαν ἐν τῷ Ξ. ὁ Η ἄρα τὸν Β μετρεῖ κατὰ τὰς ἐν τῷ Ξ μονάδας· ὁ Ξ ἄρα τὸν Η πολλαπλασιάσας τὸν Β πεποίηκεν. ὁ δὲ Η ἐστιν ὁ ἐκ τῶν Λ, Μ· ὁ Ξ ἄρα τὸν ἐκ τῶν Λ, Μ πολλαπλασιάσας τὸν Β πεποίηκεν. στερεὸς ἄρα ἐστὶν ὁ Β, πλευραὶ δὲ αὐτοῦ εἰσιν οἱ Λ, Μ, Ξ· οἱ Α, Β ἄρα στερεοί εἰσιν.



Λέγω [δή], ὅτι καὶ ὅμοιοι. ἐπεὶ γὰρ οἱ Ν, Ξ τὸν Ε πολλαπλασιάσαντες τοὺς Α, Γ πεποιήκασιν, ἔστιν ἄρα ὡς ὁ Ν πρὸς τὸν Ξ, ὁ Α πρὸς τὸν Γ, τουτέστιν ὁ Ε πρὸς τὸν Ζ. ἀλλ᾿ ὡς ὁ Ε πρὸς τὸν Ζ, ὁ Θ πρὸς τὸν Λ καὶ ὁ Κ πρὸς τὸν Μ· καὶ ὡς ἄρα ὁ Θ πρὸς τὸν Λ, οὕτως ὁ Κ πρὸς τὸν Μ καὶ ὁ Ν πρὸς τὸν Ξ. καί εἰσιν οἱ μὲν Θ, Κ, Ν πλευραὶ τοῦ Α,

thus similar plane numbers [Prop. 8.20]. Therefore, let H, K be the sides of E, and L, M (the sides) of G. Thus, it is clear from the (proposition) before this (one) that E, F, G are continuously proportional in the ratio of H to L, and of K to M. And since E, F, G are the least (numbers) having the same ratio as A, C, D, and the multitude of E, F, G is equal to the multitude of A, C, D, thus, via equality, as E is to G, so A (is) to D [Prop. 7.14]. And E and G (are) prime (to one another), and prime (numbers) are also the least (of those numbers having the same ratio as them) [Prop. 7.21], and the least (numbers) measure those (numbers) having the same ratio as them an equal number of times, the greater (measuring) the greater, and the lesser the lesser—that is to say, the leading (measuring) the leading, and the following the following [Prop. 7.20]. Thus, E measures A the same number of times as G (measures) D. So as many times as E measures A, so many units let there be in N. Thus, N has made A (by) multiplying E [Def. 7.15]. And E is the (number created) from (multiplying) H and K. Thus, N has made A (by) multiplying the (number created) from (multiplying) H and K. Thus, A is solid, and its sides are H, K, N. Again, since E, F, G are the least (numbers) having the same ratio as C, D, B, thus E measures C the same number of times as G (measures) B [Prop. 7.20]. So as many times as E measures C, so many units let there be in O. Thus, G measures B according to the units in O. Thus, O has made B (by) multiplying G. And G is the (number created) from (multiplying) L and M. Thus, O has made B (by) multiplying the (number created) from (multiplying) L and M. Thus, B is solid, and its sides are L, M, O. Thus, A and B are (both) solid.



[So] I say that (they are) also similar. For since N, O have made A, C (by) multiplying E, thus as N is to O, so A (is) to C—that is to say, E to F [Prop. 7.18]. But, as E (is) to F, so H (is) to L, and K to M. And thus as H (is) to L, so K (is) to M, and N to O. And H, K, N are the sides of A, and L, M, O the sides of B. Thus, A and