Quum ${}'f$ transeat in f per substitutionem propriam $0,\ 1,\ 1,\ h$ (art. 161): f transibit in ${}'f$ per subst. propr. $h,\ 1,\ -1,\ 0$. Ex simili ratione ${}'f$ transibit in ${}''f$ per subst. propr. ${}'h,\ 1,\ -1,\ 0$; ${}''f$ in ${}'''f$ per subst. pr. ${}''h,\ 1,\ -1,\ 0$ etc. Hinc per art. 159 eodem modo vt art. 177 colligitur, si numeri ${}'\alpha$, ${}''\alpha$, ${}'''\alpha$ etc. ${}'\beta$, ${}''\beta$, ${}'''\beta$ etc. etc. formentur secundum algorithmum sequentem

$$\begin{array}{l|l|l|l}
{}'\alpha = h & {}'\beta = 1 & {}'\gamma = -1 & {}'\delta = 0 \\
{}''\alpha = {}'h\,{}'\alpha - 1 & {}''\beta = {}'\alpha & {}''\gamma = {}'h\,{}'\gamma & {}''\delta = {}'\gamma \\
{}'''\alpha = {}''h\,{}''\alpha - {}''\alpha & {}'''\beta = {}''\alpha & {}'''\gamma = {}''h\,{}''\gamma - {}'\gamma & {}'''\delta = {}''\gamma \\
{}^{\mathrm{iv}}\alpha = {}'''h\,{}'''\alpha - {}'''\alpha & {}^{\mathrm{iv}}\beta = {}'''\alpha & {}^{\mathrm{iv}}\gamma = {}'''h\,{}'''\gamma - {}''\gamma & {}^{\mathrm{iv}}\delta = {}'''\gamma
\end{array}$$

etc.

f transformatum iri

in	per substitutionem
${}'f$	${}'\alpha,\ {}'\beta,\ {}'\gamma,\ {}'\delta$
${}''f$	${}''\alpha,\ {}''\beta,\ {}''\gamma,\ {}''\delta$
${}'''f$	${}'''\alpha,\ {}'''\beta,\ {}'''\gamma,\ {}'''\delta$ etc.

omnesque has transformationes fore proprias.

Si ponitur $\alpha = 1,\ \beta = 0,\ \gamma = 0,\ \delta = 1$: hi numeri eandem relationem habebunt ad formam f, quam habent $\alpha',\ \beta',\ \gamma',\ \delta'$ ad f'; $\alpha'',\ \beta'',\ \gamma'',\ \delta''$ ad f'' etc.; ${}'\alpha,\ {}'\beta,\ {}'\gamma,\ {}'\delta$ ad ${}'f$ etc. Scilicet per substitutionem $\alpha,\ \beta,\ \gamma,\ \delta$ forma f transibit in f. Tum vero progressiones infinitae α', α'', α''' etc., ${}'\alpha$, ${}''\alpha$, ${}'''\alpha$ etc., per intercalationem termini α, concinne iungentur ita vt vnam continuam vtrimque infinitam constituere concipi possint secundum eandem legem vbique progredientem ... $'''\alpha$, $''\alpha$, $'\alpha$, α, α', α'', α''' ... Lex progressionis haec est: $'''\alpha + '\alpha = ''h''\alpha$, $''\alpha + \alpha = 'h'\alpha$, $'\alpha + \alpha' = h\alpha$, $\alpha + \alpha'' = h'\alpha'$, $\alpha' + \alpha''' = h''\alpha''$ etc., siue generaliter (si indicem negatiuum a dextra scriptum idem designare supponimus, vt positiuum a laeua) $\alpha^{m-1} + \alpha^{m+1} = h^m\alpha^m$. Simili modo progressio ... $''\beta$, $'\beta$, β, β' β'' ... continua erit, cuius lex $\beta^{m-1} + \beta^{m+1} = h^{m+1}\beta^m$; et proprie cum praecedente identica, omnibus terminis vno loco promotis, $\beta'' = '\alpha$, $'\beta = \alpha$, $\beta = \alpha'$ etc, Lex progressionis continuae .... $''\gamma$, $'\gamma$, γ, γ', γ'' ..... erit haec $\gamma^{m-1} + \gamma^{m+1} = h^m\gamma^m$, et lex huius ... $''\delta$, $'\delta$, δ, δ', δ'' ... erit $\delta^{m-1} + \delta^{m+1} = h^{m+1}\delta^m$ insuperque generaliter $\delta^m = \gamma^{m+1}$.

Ex. Sit forma proposita f haec (3, 8, — 5) quae transformabitur

	in formam	per substitutionem
^{vii}f	(— 10, 7, 3)	— 805, — 152, + 143, + 27
^{vi}f	(3, 8, — 5)	— 152, + 45, + 27, — 8
^{v}f	(— 5, 7, 6)	+ 45, + 17, — 8, — 3
^{iv}f	(6, 5, — 9)	+ 17, — 11, — 3, + 2
$'''f$	(— 9, 4, 7)	— 11, — 6, + 2, + 1
$''f$	(7, 3, — 10)	— 6, + 5, + 1, — 1
$'f$	(— 10, 7, 3)	+ 5, + 1, — 1, 0
f	(3, 8, — 5)	+ 1, 0, 0, + 1
f'	(— 5, 7, 6)	0, — 1, + 1, — 3
f''	(6, 5, — 9)	— 1, — 2, — 3, — 7
f'''	(— 9, 4, 7)	— 2, + 3, — 7, + 10
f^{iv}	(7, 3, — 10)	+ 3, + 5, + 10, + 17
f^{v}	(— 10, 7, 3)	+ 5, — 8, + 17, — 27
f^{vi}	(3 8 — 5)	— 8, — 45, — 27, — 152
f^{vii}	(— 5, 7, 6)	— 45, + 143, — 152, + 483

etc.

189. Circa hunc algorithmum sequentia sunt annotanda.

1) Omnes a, a' a'' etc., $'a$, $''a$ etc. eadem signa habebunt; omnes b, b', b'' etc. $'b$, $''b$ etc. erunt positiui; in progressione .... $''h$, $'h$, h, h', h'' ... signa alternabunt, scilicet si omnes a, a' etc. sunt positiui, h^m vel ${}^mh$ erit positiuus quando m est par, negatiuus quando m impar; si vero a, a' etc. sunt negatiui, h^m vel ${}^mh$ pro m pari erit negatiuus, pro impari positiuus.

2) Si a est positiuus adeoque h' negatiuus, h'' positiuus etc., erit $\alpha'' = -1$ neg., $\alpha''' = h''\alpha''$ neg. et $> \alpha''$ (vel $= \alpha''$ si $h'' = 1$); $\alpha^{IV} = h'''\alpha''' - ''$ pos. et $> \alpha'''$ (quia $h'''\alpha'''$ pos., α'' neg); $\alpha^V = h^{IV}\alpha^{IV} - \alpha'''$ pos. et $> \alpha^{IV}$ (quia $h^{IV}\alpha^{IV}$ pos.) etc. Hinc facile concluditur, progressionem α', α'', α''' etc. in infinitum crescere duoque signa positiua semper duo negatiua excipere ita vt α^m habeat signum $+$, $+$, $-$, $-$ prout $m \equiv 0, 1, 2, 3 \pmod{4}$. — Si a est negatiuus, per simile ratiocinium inuenitur α'' neg., α''' pos. et vel $>$ vel $=$ α''; α^{IV} pos. $> \alpha'''$; α^V neg. $> \alpha^{IV}$ etc., ita vt progressio α', α'', α''' etc. continuo crescat, signumque termini α^m sit $+$, $-$, $-$, $+$ prout $m \equiv 0, 1, 2, 3 \pmod{4}$

3) Hoc modo inuenitur, omnes quatuor progressiones infinitas α', α'', α''' etc. γ, γ', γ'' etc.; α', α, $'\alpha$, $''\alpha$ etc.; γ, $'\gamma$, $''\gamma$ etc. continuo crescere, adeoque etiam sequentes cum illis identicas: β, β', β'' etc.; $'\delta$, δ, δ', δ'' etc.; β, $'\beta$, $''\beta$ etc.; $'\delta$, $''\delta$ etc.; et, prout $m \equiv 0, 1, 2, 3 \pmod{4}$,