Hinc fit ex $d[1] - c[2]$, $(Aa + Bc)(ad - bc) = (Ad - Bc)e'e'$, adeoque $A(a + d) = o$. Porro ex $(a + d)[2] - b[1] - c[3]$ fit $(Ab + B(a + d) + Cc)(ad + bc) = (-Ab + B(a+d) - Cc)e'e'$, adeoque $B(a + d) = o$. Denique ex $a[3] - b[2]$ fit $(Bb + Cd)(ad - bc) = (-Bb + Ca)e'e'$ adeoque $C(a + d) = o$. Quare quum omnes A, B, C nequeant esse $= o$, necessario erit $a + d = o$ siue $a = -d$.

Ex $a[2] - b[1]$ fit $(Ba + Cc)(ad - bc) = (Ba - Ab)e'e'$, vnde $Ab - 2Ba - Cc = o$ [5]

Ex aequationibus $e + e' = o$, $a + d = o$ siue $\alpha\beta - \beta\gamma + \alpha'\delta' - \beta'\gamma' = o$, $\alpha\delta' - \beta\gamma' - \gamma\beta' + \delta\alpha' = o$ sequitur $(\alpha + \alpha')(\delta + \delta') = (\beta + \beta')(\gamma + \gamma')$ siue $(\alpha + \alpha') : (\gamma + \gamma') = (\beta + \beta') : (\delta + \delta')$. Sit rationi huic *) in numeris minimis aequalis ratio $m : n$, ita vt m, n inter se primi sint, accipianturque μ, ν ita vt fiat $\mu m + \nu n = 1$. Porro sit r diu. comm. max. numerorum a, b, c; cuius quadratum propterea metietur ipsum $aa + bc$ siue $bc - ad$ siue ee; quare r etiam ipsum e metietur. His ita factis, si forma F per substitutionem $= mt + \frac{\nu e}{r} u$, $y = nt - \frac{\mu e}{r} u$ in formam $Mtt + 2Ntu + Puu$ (G) transire supponitur, haec anceps erit formamque F' implicabit.

*) Si omnes $\alpha + \alpha'$, $\gamma + \gamma'$, $\beta + \beta'$, $\delta + \delta'$ essent $= o$, ratio indeterminata foret, adeoque methodus non applicabilis. Sed exigua attentio docet, hoc cum suppositionibus nostris consistere non posse. Foret enim $\alpha\delta - \beta\gamma = \alpha'\delta' - \beta'\gamma'$ i. e. $e = e'$ adeoque, quia $e = -e'$, $e = e' = o$. Hinc vero etiam $B'B' - A'C'$ i. e. determinans formae F' fieret $= o$, quales formas omnino exclusimus.

Dem. I. Quo pateat, formam G esse ancipitem, ostendemus esse $M(b\mu\mu - 2a\mu\nu - c\nu\nu) = 2Nr$ vnde quia r ipsos a, b, c metitur, $\frac{1}{r}(b\mu\mu - 2a\mu\nu - c\nu\nu)$ integer erit, adeoque $2N$ multiplum ipsius M. Erit autem $M = Amm + 2Bmn + Cnn$, $Nr = (Am\nu - B(m\mu - n\nu) - Cm\mu)e$. Porro per euolutionem facile confirmatur esse $2e + 2a = e - e' + a - d = (\alpha - \alpha')(\delta + \delta') - (\beta - \beta')(\gamma + \gamma')$, $2b = (\alpha + \alpha')(\beta - \beta') - (\alpha - \alpha')(\beta + \beta')$ Hinc quoniam $m(\gamma + \gamma') = n(\alpha + \alpha')$, $m(\delta + \delta') = n(\beta + \beta')$, erit $m(2e + 2a) = -2nb$ siue $me + ma + nb = 0 \ldots [7]$. Eodem modo erit $2e - 2a = e - e' - a + d = (\alpha + \alpha')(\delta - \delta') - (\beta + \beta')(\gamma - \gamma')$, $2c = (\gamma - \gamma')(\delta + \delta') - (\gamma + \gamma')(\delta - \delta')$, atque hinc $n(2e - 2a) = -2mc$, siue $ne - na + mc = 0 \ldots [8]$

Iam si ad $mm(b\mu\mu - 2a\mu\nu - c\nu\nu)$ additur

$(1 - m\mu - n\nu)(m\nu(e - a) + (m\mu + 1)b)$

$+ (me + ma + nb)(m\mu\nu + \nu) + (ne - na + mc)m\nu\nu$

quod manifesto $= 0$, propter $1 - \mu m - \nu n = 0$, $me + ma + nb = 0$, $ne - na + mc = 0$: prodit productis rite euolutis partibusque se destruentibus deletis, $2m\nu e + b$. Quare erit

$$mm(b\mu\mu - 2a\mu\nu - c\nu\nu) = 2m\nu e + b \ldots\ldots [9]$$

Eodem modo addendo ad $mn(b\mu\mu - 2a\mu\nu - c\nu\nu)$ haec:

$(1 - m\mu - n\nu)((n\nu - m\mu)e - (1 + m\mu + n\nu)a)$

$- (me + ma + nb)m\mu\mu + (ne - na + mc)n\nu\nu$

inuenitur

$$mn(b\mu\mu - 2a\mu\nu - c\nu\nu) = (n\nu - m\mu)e - a \ldots [10]$$

Denique addendo ad $nn(b\mu\mu - 2a\mu\nu - c\nu\nu)$ haec:

$(m\mu + n\nu - 1)(n\mu(e + a) + (n\nu + 1)c)$

$-(me + ma + nb)n\mu\mu - (ne - na + mc)(n\mu\nu + \mu)$

fit

$nn(b\mu\mu - 2a\mu\nu - c\nu\nu) = -2n\mu e - c \ldots\ldots$ [11]

Iam ex 9, 10, 11, deducitur

$(Amm + 2Bmn + Cnn)(b\mu\mu - 2a\mu\nu - c\nu\nu) = 2e(Am\nu + B(n\nu - m\mu) - Cn\mu) + Ab - 2Ba - Cc$, siue propter [6],

$M(b\mu\mu - 2a\mu\nu - c\nu\nu) = 2Nr$. *Q. E. D.*

II. Vt probetur, formam G implicare formam F' demonstrabimus, *primo* G transire in F' ponendo $t = (\mu\alpha + \nu\gamma)x' + (\mu\beta + \nu\delta)y'$, $u = \frac{r}{e}(n\alpha - m\gamma)x' + \frac{r}{e}(n\beta - m\delta)y' \ldots (S)$; *secundo* $\frac{r}{e}(n\alpha - m\gamma)$, $\frac{r}{e}(n\beta - m\delta)$ esse integros.

1. Quoniam F transit in G ponendo $x = mt + \frac{\nu e}{r}u$, $y = nt - \frac{\mu e}{r}u$: forma G per substitutionem (S) transmutabitur in eandem formam in quam F transformatur ponendo $x = m((\mu\alpha + \nu\gamma)x' + (\mu\beta + \nu\delta)y') + \nu((n\alpha - m\gamma)x' + (n\beta - m\delta)y')$ i. e. $= \alpha(m\mu + n\nu)x' + \beta(m\mu + n\nu)y'$ siue $= \alpha x' + \beta y'$; et $y = n((\mu\alpha + \nu\gamma)x' + (\mu\beta + \nu\delta)y') - \mu((n\alpha - m\gamma)x' + (n\beta - m\delta)y')$ i. e. $= \gamma(n\nu + m\mu)x' + \delta(n\nu + m\mu)y'$ siue $= \gamma x' + \delta y'$. Per hanc vero substitutionem F transit in F': quare per substitutionem (S) etiam G transibit in F'.

2. Ex valoribus ipsorum e, b, d inuenitur $\alpha'e + \gamma b - \alpha d = 0$, siue propter $d = -a$, $n\alpha'e + n\alpha a + n\gamma b = 0$; hinc ex [8], $n\alpha'e + n\alpha a = m\gamma e + m\gamma a$ siue $(n\alpha - m\gamma)a = (m\gamma - n\alpha')e$ [12]