

Undergraduate Texts in Mathematics

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Undergraduate Texts in Mathematics

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Fleming: Functions of Several Variables. Second edition.

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Foulds: Optimization Techniques: An Introduction.

(continued after index)

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Mathematics and Its History

Second Edition

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To Elaine, Michael, and Robert

Preface to the Second Edition

This edition has been completely retyped in \LaTeX , and many of the figures redone using the `PSTricks` package, to improve accuracy and make revision easier in the future. In the process, several substantial additions have been made.

- There are three new chapters, on Chinese and Indian number theory, on hypercomplex numbers, and on algebraic number theory. These fill some gaps in the first edition and give more insight into later developments.
- There are many more exercises. This, I hope, corrects a weakness of the first edition, which had too few exercises, and some that were too hard. Some of the monster exercises in the first edition, such as the one in Section 2.2 comparing volume and surface area of the icosahedron and dodecahedron, have now been broken into manageable parts. Nevertheless, there are still a few challenging questions for those who want them.
- Commentary has been added to the exercises to explain how they relate to the preceding section, and also (when relevant) how they foreshadow later topics.
- The index has been given extra structure to make searching easier. To find Euler's work on Fermat's last theorem, for example, one no longer has to look at 41 different pages under "Euler." Instead, one can find the entry "Euler, and Fermat's last theorem" in the index.
- The bibliography has been redone, giving more complete publication data for many works previously listed with little or none. I have found the online catalogue of the Burndy Library of the Dibner Institute at MIT helpful in finding this information, particularly for

early printed works. For recent works I have made extensive use of MathSciNet, the online version of *Mathematical Reviews*.

There are also many small changes, some prompted by recent mathematical events, such as the proof of Fermat's last theorem. (Fortunately, this one did not force a major rewrite, because the background theory of elliptic curves was covered in the first edition.)

I thank the many friends, colleagues, and reviewers who drew my attention to faults in the first edition, and helped me in the process of revision. Special thanks go to the following people.

- My sons Michael and Robert, who did most of the typing, and my wife, Elaine, who did a great deal of the proofreading.
- My students in Math 310 at the University of San Francisco, who tried out many of the exercises, and to Tristan Needham, who invited me to USF in the first place.
- Mark Aarons, David Cox, Duane DeTemple, Wes Hughes, Christine Muldoon, Martin Muldoon, and Abe Shenitzer, for corrections and suggestions.

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2001

Preface to the First Edition

One of the disappointments experienced by most mathematics students is that they never get a course on mathematics. They get courses in calculus, algebra, topology, and so on, but the division of labor in teaching seems to prevent these different topics from being combined into a whole. In fact, some of the most important and natural questions are stifled because they fall on the wrong side of topic boundary lines. Algebraists do not discuss the fundamental theorem of algebra because “that’s analysis” and analysts do not discuss Riemann surfaces because “that’s topology,” for example. Thus if students are to feel they really know mathematics by the time they graduate, there is a need to unify the subject.

This book aims to give a unified view of undergraduate mathematics by approaching the subject through its history. Since readers should have had some mathematical experience, certain basics are assumed and the mathematics is not developed formally as in a standard text. On the other hand, the mathematics is pursued more thoroughly than in most general histories of mathematics, because mathematics is our main goal and history only the means of approaching it. Readers are assumed to know basic calculus, algebra, and geometry, to understand the language of set theory, and to have met some more advanced topics such as group theory, topology, and differential equations. I have tried to pick out the dominant themes of this body of mathematics, and to weave them together as strongly as possible by tracing their historical development.

In doing so, I have also tried to tie up some traditional loose ends. For example, undergraduates can solve quadratic equations. Why not cubics? They can integrate $1/\sqrt{1-x^2}$ but are told not to worry about $1/\sqrt{1-x^4}$. Why? Pursuing the history of these questions turns out to be very fruitful, leading to a deeper understanding of complex analysis and algebraic geometry, among other things. Thus I hope that the book will be not only a

bird's-eye view of undergraduate mathematics but also a glimpse of wider horizons.

Some historians of mathematics may object to my anachronistic use of modern notation and (fairly) modern interpretations of classical mathematics. This has certain risks, such as making the mathematics look simpler than it really was in its time, but the risk of obscuring ideas by cumbersome, unfamiliar notation is greater, in my opinion. Indeed, it is practically a truism that mathematical ideas generally arise before there is notation or language to express them clearly, and that ideas are implicit before they become explicit. Thus the historian, who is presumably trying to be both clear and explicit, often has no choice but to be anachronistic when tracing the origins of ideas.

Mathematicians may object to my choice of topics, since a book of this size is necessarily incomplete. My preference has been for topics with elementary roots and strong interconnections. The major themes are the concepts of number and space: their initial separation in Greek mathematics, their union in the geometry of Fermat and Descartes, and the fruits of this union in calculus and analytic geometry. Certain important topics of today, such as Lie groups and functional analysis, are omitted on the grounds of their comparative remoteness from elementary roots. Others, such as probability theory, are mentioned only briefly, as most of their development seems to have occurred outside the mainstream. For any other omissions or slights I can only plead personal taste and a desire to keep the book within the bounds of a one- or two-semester course.

The book has grown from notes for a course given to senior undergraduates at Monash University over the past few years. The course was of half-semester length and a little over half the book was covered (Chapters 1–11 one year and Chapters 5–15 another year). Naturally I will be delighted if other universities decide to base a course on the book. There is plenty of scope for custom course design by varying the periods or topics discussed. However, the book should serve equally well as general reading for the student or professional mathematician.

Biographical notes have been inserted at the end of each chapter, partly to add human interest but also to help trace the transmission of ideas from one mathematician to another. These notes have been distilled mainly from secondary sources, the *Dictionary of Scientific Biography* (DSB) normally being used in addition to the sources cited explicitly. I have followed the DSB's practice of describing the subject's mother by her maiden name.

References are cited in the name (year) form, for example, Newton (1687) refers to the *Principia*, and the references are collected at the end of the book.

The manuscript has been read carefully and critically by John Crossley, Jeremy Gray, George Odifreddi, and Abe Shenitzer. Their comments have resulted in innumerable improvements, and any flaws remaining may be due to my failure to follow all their advice. To them, and to Anne-Marie Vandenberg for her usual excellent typing, I offer my sincere thanks.

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1989

Contents

Preface to the Second Edition	vii
Preface to the First Edition	ix
1 The Theorem of Pythagoras	1
1.1 Arithmetic and Geometry	1
1.2 Pythagorean Triples	3
1.3 Rational Points on the Circle	5
1.4 Right-angled Triangles	8
1.5 Irrational Numbers	10
1.6 The Definition of Distance	12
1.7 Biographical Notes: Pythagoras	15
2 Greek Geometry	17
2.1 The Deductive Method	17
2.2 The Regular Polyhedra	20
2.3 Ruler and Compass Constructions	25
2.4 Conic Sections	28
2.5 Higher-Degree Curves	31
2.6 Biographical Notes: Euclid	35
3 Greek Number Theory	37
3.1 The Role of Number Theory	37
3.2 Polygonal, Prime, and Perfect Numbers	38
3.3 The Euclidean Algorithm	41
3.4 Pell's Equation	43
3.5 The Chord and Tangent Methods	48
3.6 Biographical Notes: Diophantus	49

4	Infinity in Greek Mathematics	51
4.1	Fear of Infinity	51
4.2	Eudoxus' Theory of Proportions	53
4.3	The Method of Exhaustion	55
4.4	The Area of a Parabolic Segment	61
4.5	Biographical Notes: Archimedes	64
5	Number Theory in Asia	66
5.1	The Euclidean Algorithm	66
5.2	The Chinese Remainder Theorem	68
5.3	Linear Diophantine Equations	70
5.4	Pell's Equation in Brahmagupta	72
5.5	Pell's Equation in Bhâskara II	74
5.6	Rational Triangles	77
5.7	Biographical Notes: Brahmagupta and Bhâskara	80
6	Polynomial Equations	82
6.1	Algebra	82
6.2	Linear Equations and Elimination	84
6.3	Quadratic Equations	86
6.4	Quadratic Irrationals	90
6.5	The Solution of the Cubic	91
6.6	Angle Division	93
6.7	Higher-Degree Equations	96
6.8	Biographical Notes: Tartaglia, Cardano, and Viète	97
7	Analytic Geometry	104
7.1	Steps toward Analytic Geometry	104
7.2	Fermat and Descartes	105
7.3	Algebraic Curves	107
7.4	Newton's Classification of Cubics	110
7.5	Construction of Equations and Bézout's Theorem	111
7.5	The Arithmetization of Geometry	115
7.6	Biographical Notes: Descartes	116
8	Projective Geometry	120
8.1	Perspective	120
8.2	Anamorphosis	123
8.3	Desargues' Projective Geometry	125