erit $CF = CG = \sqrt{(aa+bb)}$. Hinc erit $FP = x - \sqrt{(aa+bb)}$ & $GP = x + \sqrt{(aa+bb)}$; unde, ob $yy = -bb + \frac{bbxx}{aa}$, fiet $FM = \sqrt{(aa + xx + \frac{bbxx}{aa} - 2x\sqrt{(aa+bb)})} = \frac{x\sqrt{(aa+bb)}}{a} - a$ & $GM = \sqrt{(aa + xx + \frac{bbxx}{aa} + 2x\sqrt{(aa+bb)})} = \frac{x\sqrt{(aa+bb)}}{a} + a$. Ductis ergo ex utroque Foco ad Curvæ punctum M rectis FM, GM erit $FM + AC = \frac{CP.CF}{CA}$ & $GM - AC = \frac{CP.CF}{CA}$, harum ergo rectarum differentia $GM - FM$ æqualis est $2AC$. Quemadmodum ergo in Ellipsi summa harum duarum Linearum æquatur Axi principali AB, ita pro Hyperbola differentia æqualis est Axi principali AB. Cap. VI.

155. Hinc etiam positio tangentis MT definiri potest, est enim perpetuo pro Lineis secundi ordinis $CP : CA = CA . CT$: unde fit $CT = \frac{aa}{x}$, & $PT = \frac{xx - aa}{x} = \frac{aayy}{bbx}$; hincque $MT = \frac{y}{bbx}\sqrt{(b^4x^2 + a^4y^2)} = \frac{y}{bx}\sqrt{(aaxx + bbxx - a^4)}$. At est $FM.GM = \frac{aaxx + bbxx - a^4}{aa}$, ergo $MT = \frac{ay}{bx}\sqrt{FM.GM}$. Deinde est $FT = \sqrt{(aa+bb)} - \frac{aa}{x}$, & $GT = \sqrt{(aa+bb)} + \frac{aa}{x}$ ergo $FT : FM = a : x$, & $GT : GM = a : x$, unde sequitur $FT : GT = FM : GM$, quæ proportio indicat angulum FMG per tangentem MT bisecari, esseque $FMT = GMT$. Recta autem CM producta erit Diameter obliquangula omnes Ordinatas tangenti MT parallelas bisecans.

156. Demittatur ex Centro C in tangentem perpendicularis CQ, erit $TM : PT : PM = CT : TQ : CQ$ seu $\frac{ay}{bx}\sqrt{FM.GM} : \frac{aayy}{bbx} : y = \frac{aa}{x} : TQ : CQ$; unde ori-

tur $TQ = \frac{a^3y}{bx\sqrt{FM.GM}}$ & $CQ = \frac{ab}{\sqrt{FM.GM}}$. Demittatur ſimili modo ex Foco F in tangentem perpendiculum FS, erit $TM : PT : PM = FT : TS : FS$, ſeu $\frac{ay}{bx}\sqrt{FM.GM}$: $\frac{aayy}{bbx} : y = \frac{a.FM}{x} : TS : FS$: unde oritur $TS = \frac{aayFM}{bx\sqrt{FM.GM}}$ & $FS = \frac{b.FM}{\sqrt{FM.GM}}$; pariterque, ſi ex altero Foco G in tangentem ducatur perpendicularis Gs, erit $Ts = \frac{aay.GM}{bx\sqrt{FM.GM}}$, & $Gs = \frac{b.GM}{\sqrt{FM.GM}}$. Hinc ergo habetur $TS.Ts = \frac{a^4yy}{bbxx} = \frac{aa(xx - aa)}{xx} = CT.PT$, & $TS : CT = PT : Ts$. Deinde fit $FS.Gs = bb$. Quia porro eſt $QS = Qs$ erit $QS = \frac{TS + Ts}{2} = \frac{aay(FM + GM)}{2bx\sqrt{FM.GM}} = \frac{ay\sqrt{(aa+bb)}}{b\sqrt{FM.GM}} = Qs$, unde ſequitur $CS^2 = CQ^2 + QS^2 = \frac{aab^4 + a^4yy + aabbyy}{bb.FM.GM} = \frac{aab^4 + (aa+bb)(bbxx - aabb)}{bb.FM.GM} = \frac{(aa+bb)xx - a^4}{FM.GM} = aa$. Erit ergo, uti in Ellipſi, recta $CS = a = CA$. Deinde eſt $CQ + FS = \frac{bx\sqrt{(aa+bb)}}{a\sqrt{FM.GM}}$, ideoque $(CQ + FS)^2 - CQ^2 = \frac{bbxx(aa+bb) - a^4bb}{aa.FM.GM} = bb$. Quare; ſi ducatur ex Foco F tangenti parallela FX, ſecans perpendiculum CQ productum in X, erit $CX = \sqrt{(bb + CQ^2)}$, cui ſimilis proprietas pro Ellipſi eſt inventa.

157. Si in Verticibus A & B ad Axem perpendiculares erigantur donec tangenti occurrant in V & v, ob $AT = \frac{a(x-a)}{x}$ & $BT = \frac{a(x+a)}{x}$, $PT : PM = AT : AV = BT : Bv$, hinc fit $AV = \frac{bb(x-a)}{ay}$ & $Bv = \frac{bb(x+a)}{ay}$; ergo

AV.

$AV.Bv = \frac{b^4(xx - aa)}{aayy} = bb$, seu $AV.Bv = FS.Gs$. Cap. VI.
Deinde $PT:TM = AT:TV = BT:Tv$; ergo $TV = \frac{b(x-a)}{xy}\sqrt{FM.GM}$ & $Tv = \frac{b(x+a)}{xy}\sqrt{FM.GM}$: unde fit $TV.Tv = \frac{aa}{xx}FM.GM = FT.GT$. Simili autem modo hinc plura alia consectaria deduci possunt.

158. Quia est $CT = \frac{aa}{x}$, patet quo major capiatur Abscissa $CP = x$, eo minus futurum esse intervallum CT: atque adeo tangens, quæ Curvam in infinitum productam tangit, per ipsum Centrum C transibit, fietque $CT = 0$. Cum autem sit $tang.PTM = \frac{PM}{PT} = \frac{bbx}{aay}$, puncto M in infinitum abeunte, seu posito $x = \infty$, fit $y = \frac{b}{a}\sqrt{(xx - aa)} = \frac{bx}{a}$. Tangens ergo Curvæ in infinitum productæ, & per Centrum C transibit, & cum Axe angulum constituet ACD cujus tangens $= \frac{b}{a}$. Posita ergo in Vertice A ad Axem normali $AD = b$, tum recta CD in infinitum utrinque producta, Curvam nusquam quidem tanget, at Curva continuo magis ad eam appropinquabit, donec in infinitum tota cum recta CI confundatur. Hoc idem valebit de parte Ck, quæ tandem cum ramo Bk confundetur. Atque si ad alteram partem sub eodem angulo ducatur recta KCi, ea cum ramis BK & Bi in infinitum productis conveniet. Hujusmodi autem Lineæ rectæ, ad quas Linea quæpiam Curva continuo propius accedit, in infinitum autem excurrens demum attingit, Asymptotæ vocantur, unde Lineæ rectæ ICk, KCi sunt binæ Asymtotæ Hyperbolæ.

159. Asymtotæ ergo se mutuo in Centro C Hyperbolæ decussant, atque ad Axem inclinantur angulo $ACD = ACd$, cujus tangens $= \frac{b}{a}$, angulique dupli DCd tangens $= \frac{2ab}{aa - bb}$, unde

LIB. II. unde patet si fuerit $b = a$, fore angulum, sub quo Asymtotæ se intersecant, $DCd =$ recto; quo casu Hyperbola *æquilatera* dicitur. Cum autem sit $AC = a$, $AD = b$, erit $CD = Cd = \sqrt{(aa + bb)}$; quare, si ex Foco G in utramvis Asymtotam perpendiculum GH demittatur, ob $CG = \sqrt{(aa + bb)} = CD$, erit $CH = AC = BC = a$, & $GH = b$.

160. Producatur Ordinata $MPN = 2y$ utrinque donec Asymtotas secet in m & n; erit $Pm = Pn = \frac{bx}{a}$, & $Cm = Cn = \frac{x\sqrt{(aa+bb)}}{a} = FM + AC = GM - AC$. Tum vero erit $Mm = Nn = \frac{bx - ay}{a}$ & $Nm = Mn = \frac{bx + ay}{a}$, unde fit $Mm . Nm = Mm . Mn = \frac{bbxx - aayy}{aa} = bb$, ob $aayy = bbxx - aabb$: erit ergo ubique $Mm . Nm = Mm \times Mn = Nn . Nm = Nn . Mn = bb = AD^2$. Ducatur ex M Asymtotæ Cd parallela Mr; erit $2b\sqrt{(aa + bb)} = Mm : mr\ (Mr)$, unde fit $mr = Mr = \frac{(bx - ay)\sqrt{(aa+bb)}}{2ab}$ & $Cm - mr = Cr = \frac{(bx + ay)\sqrt{(aa+bb)}}{2ab}$. Hinc ergo conficietur $Mr . Cr = \frac{(bbxx - aayy)(aa+bb)}{4aabb} = \frac{aa+bb}{4}$. Vel, ducta ex A Asymtotæ Cd parallela AE, erit $AE = CE = \frac{1}{2}\sqrt{(aa + bb)}$, ideoque erit $Mr . Cr = AE . CE$; quæ est proprietas primaria Hyperbolæ ad Asymtotas relatæ.

TAB. IX. Fig. 34. 161. Quod si ergo Abscissæ $CP = x$, in una Asymtota a Centro sumantur, & Applicatæ $PM = y$ alteri Asymtotæ parallelæ statuantur, erit $yx = \frac{aa + bb}{4}$, existente $AC = BC = a$, & $AD = Ad = b$: seu, si ponatur $AE = CE = h$, erit $yx = hh$, & $y = \frac{hh}{x}$. Posito ergo $x = 0$, fit $y = \infty$, ac vicissim facto $x = \infty$ fiet $y = 0$. Agatur jam

per

per punctum Curvæ M recta quæcunque $QMNR$, quæ parallela sit ductæ pro libitu rectæ GH, ac ponatur $CQ = t$, $QM = u$, erit $GH : CH : CG = u : PQ : PM$, ergo $PQ = \frac{CH}{GH}u$, $PM = \frac{CG}{GH}u$: unde $y = \frac{CG}{GH}u$ & $x = t - \frac{CH}{GH}u$; quibus valoribus substitutis, erit $\frac{CG}{GH}tu - \frac{CH \cdot CG}{GH^2} \times uu = hh$, seu $uu - \frac{GH}{CH}tu + \frac{GH^2}{CH \cdot CG}hh = 0$. Habebit ergo Applicata u duplicem valorem, nempe QM & QN, quarum summa erit $= \frac{GH}{CH}t = QR$, & rectangulum $QM \times QN = \frac{GH^2}{CH \cdot CG}hh$. CAP. VI.

162. Cum igitur sit $QM + QN = QR$, erit $QM = RN$ & $QN = RM$. Quare, si puncta M & N conveniant quo casu recta QR Curvam tanget, tum ea in ipso puncto contactus bisecabitur. Scilicet, si recta XY tangat Hyperbolam, punctum contactus Z in medio rectæ XY erit positum. Unde, si ex Z alteri Asymtotæ parallela ducatur ZV, erit $CV = VY$, hincque ad quodvis Hyperbolæ punctum Z expedite tangens ducetur. Sumatur scilicet $VY = CV$, ac recta per Y & Curvæ punctum Z ducta Hyperbolam in hoc puncto Z tanget.

Cum ergo sit $CV \cdot ZV = hh = \frac{aa + bb}{4}$, erit $CX \cdot CY = aa + bb = CD^2 = CD \cdot Cd$: quocirca, si rectæ DX & dY ducerentur, eæ inter se forent parallelæ; unde facillimus oritur modus quotcunque Curvæ tangentes ducendi.

163. Quoniam deinde est rectangulum $QM \cdot QN = \frac{GH^2}{CH \cdot CG} \cdot hh$, patet, ubicunque recta QR ipsi HG parallela ducatur, fore semper rectangulum $QM \cdot QN$ ejusdem magnitudinis. Erit ergo etiam $QM \cdot QN = QM \cdot MR = QN \times NR = \frac{CH^2}{CH \cdot CG}hh$. Quod, si ergo concipiatur ducta tan-