haps because of the repeated impact of epidemic diseases. About 800 AD, mathematics in the Alexandrian tradition resurfaced in India, which had a long mathematical tradition of its own. The Arabs, aided by translations of Greek texts, developed and transmitted mathematical knowledge from India back to the mediterranean area and ultimately to Europe. During the so-called 'renaissance', mathematics flourished in Italy and, aided by the Chinese invention of printing, spread to Western and Central Europe. Of course, today mathematics is being pursued in all the industrial countries of the world.

## Introduction to the number system

The historical and pedagogical development of the number system goes somewhat like this:

$$\mathbf{N}^+ \to \mathbf{Q}^+ \to \mathbf{R}^+ \to \mathbf{R} \to \mathbf{C} \to \mathbf{H}\,.$$

Here $\mathbf{N}^+$ is the set of positive integers, the *numbers* used for counting, known to all societies. $\mathbf{Q}^+$ is the set of positive rationals, namely, *quotients* of positive integers, surely known to all agricultural civilizations. At one time, they were believed to exhaust all the numbers, until the Pythagoreans discovered that the diagonal of a square was not a rational multiple of its side. We use $\mathbf{R}^+$ to denote the positive *reals*; these were certainly used effectively by Thales, though the Greeks originally tended to regard them as ratios of geometric quantities. A formal treatment, anticipating the nineteenth century definition by Dedekind, was first given by Eudoxus in Athens. The transition from $\mathbf{R}^+$ to $\mathbf{R}$, the set of all reals, positive, zero and negative, took place in India and may be ascribed to Brahmagupta. The set $\mathbf{C}$ of *complex* numbers was first considered by Cardano to describe the intermediate steps in solving a cubic equation with real coefficients and three real solutions. The set $\mathbf{H}$ of *quaternions* is named after their inventor William Hamilton, who may have been preceded by Olinde Rodrigues and perhaps even by Carl Friedrich Gauss.

Most of the advances in the development of the number system may have been motivated by the desire to solve equations. Thus, the equations $2x = 3$, $x^2 = 2$, $x + 1 = 0$ and $x^2 + 1 = 0$ led to the successive introduction of $\mathbf{Q}^+, \mathbf{R}^+, \mathbf{R}$ and $\mathbf{C}$, respectively. However, all polynomial equations with complex coefficients do have complex solutions, so the introduction of quaternions requires a different justification. They were motivated by the desire to pass from the plane, describable by complex numbers, to three or four dimensions.

# Part I
# Topics in the History and Philosophy of Mathematics

# 1
# Egyptian Mathematics

The Greeks believed that mathematics originated in Egypt. As to the reason for this, opinion was divided. Aristotle thought that mathematics was developed by priests, 'because the priestly class was allowed leisure' (*Metaphysics* 981b 23-24). Herodotus believed that the annual flooding of the Nile necessitated surveying to redetermine field boundaries, and thus led to the invention of geometry. In fact, Democritus referred to Egyptian mathematicians as 'rope stretchers'. It may be of interest to note that the Egyptians themselves believed that mathematics had been given to them by the god Thoth. Our only original sources of information on the mathematics of ancient Egypt are the Moscow Mathematical Papyrus and the Rhind Mathematical Papyrus.

The Moscow Papyrus dates from 1850 BC, about the time the Bible dates the life of the patriarch Abraham. In 1893 it was acquired by V. S. Golenishchev and brought to Moscow (Gillings [1972], p. 246). Problem 14 of this papyrus is by far the most interesting. It is the computation of a *truncated pyramid*, a square pyramid with a similar pyramid cut off its top. If a side of the base has length $a$ and a side of the top has length $b$, then the volume of the truncated pyramid of vertical height $h$ is

$$V = \frac{h}{3}(a^2 + ab + b^2).$$

This is exactly the formula used by the Egyptians. Note that, if $b = 0$, we get the formula for the volume of the complete pyramid.

The Rhind Mathematical Papyrus seems to be based on an earlier work. It was written by one Ahmose in 1650 BC, about the time when, accord-