L a laeua iaceat, necessario eam sitam esse aut inter (1) et (3), aut inter (3) et (5), aut inter (5) et (7) etc. (quoniam L est irrationalis, adeoque ipsi $\mathfrak{M}$ certo inaequalis, fractionesque (1), (3), (5) etc. quauis quantitate data, ipsi L inaequali, propius ad L accedere possunt). Si vero $\mathfrak{M}$ ipsi L a dextra iacet: necessario iacebit aut inter (2) et (4), aut inter (4) et (6) aut inter (6) et (8) etc. Ponamus itaque $\mathfrak{M}$ iacere inter (m) et $(m+2)$, patetque quantitates $\mathfrak{M}$, (m), $(m+1)$, $(m+2)$, L iacere sequenti ordine, (II)*): (m), $(\mathfrak{M})$, $(m+2)$, L, $(m+1)$. Tum erit necessario $\mathfrak{N} = (m+1)$. Iacebit enim $\mathfrak{N}$ ipsi L a dextra; si vero etiam ipsi $(m+1)$ a dextra iaceret, $(m+1)$ iaceret inter $\mathfrak{M}$ et $\mathfrak{N}$, vnde $\gamma^{m+1} > \mathfrak{C}$, $\mathfrak{M}$ vero inter (m) et $(m+1)$ vnde $\mathfrak{C} > \gamma^{m+1}$ (art. 190), *Q. E. A.*; si vero $\mathfrak{N}$ ipsi $(m+1)$ a laeua iaceret, siue inter $(m+2)$ et $(m+1)$, foret $\mathfrak{D} > \gamma^{m+2}$, et quia $(m+2)$ inter $\mathfrak{M}$ et $\mathfrak{N}$, foret $\gamma^{m+2} > \mathfrak{D}$, *Q. E. A.* Erit itaque $\mathfrak{N} = (m+1)$, siue $\frac{\mathfrak{B}}{\mathfrak{D}} = \frac{\alpha^{m+1}}{\gamma^{m+1}} = \frac{\mathfrak{c}^m}{\mathfrak{d}^m}$

Quia $\mathfrak{A}\mathfrak{D} - \mathfrak{B}\mathfrak{C} = 1$, $\mathfrak{B}$ erit primus ad $\mathfrak{D}$ et ex simili ratione $\mathfrak{c}^m$ primus ad $\mathfrak{d}^m$. Vnde facile perspicitur aequationem $\frac{\mathfrak{B}}{\mathfrak{D}} = \frac{\mathfrak{c}^m}{\mathfrak{d}^m}$ consistere non posse, nisi fuerit aut $\mathfrak{B} = \mathfrak{c}^m$, $\mathfrak{D} = \mathfrak{d}^m$, aut $\mathfrak{B} = -\mathfrak{c}^m$, $\mathfrak{D} = -\mathfrak{d}^m$. Iam

*) Nihil hic refert, siue ordo in (II) idem sit vt in (I), siue huic oppositus, *i. e.* siue (m) etiam in (I) ipsi L a laeua iaceat siue a dextra.

quum forma f per substitutionem propriam α^m, β^m, γ^m, δ^m in formam f^m transmutetur, quae est $(\pm a^m, b^m, \mp a^{m+1})$: habebuntur aequationes $a\alpha^m\alpha^m + 2b\alpha^m\gamma^m - a'\gamma^m\gamma^m = \mp a^m$ . ... [5]; $a\alpha^m\beta^m + b(\alpha^m\delta^m + \beta^m\gamma^m) - a'\gamma^m\delta^m = b^m$ ... [6]; $a\beta^m\beta^m + 2b\beta^m\delta^m - a'\delta^m\delta^m = \mp a^{m+1}$ .... [7]; $\alpha^m\delta^m - \beta^m\gamma^m = 1$ ... [8]. Hinc fit: (ex aequ. 7 et 3), $\mp a^{m+1} = -A'$. Porro multiplicando aequationem [2] per $\alpha^m\delta^m - \beta^m\gamma^m$, aequationem [6] per $\mathfrak{A}\mathfrak{D} - \mathfrak{B}\mathfrak{C}$ et subtrahendo facile per euolutionem confirmatur esse $B - b^m = (\mathfrak{C}\alpha^m - \mathfrak{A}\gamma^m)(a\mathfrak{B}\beta^m + b(\mathfrak{D}\beta^m + \mathfrak{B}\delta^m) - a'\mathfrak{D}\delta^m) + (\mathfrak{B}\delta^m - \mathfrak{D}\beta^m)(a\mathfrak{A}\alpha^m + b(\mathfrak{C}\alpha^m + \mathfrak{A}\gamma^m) - a'\mathfrak{C}\gamma^m)$ ... [9] siue quoniam vel $\beta^m = \mathfrak{B}$, $\delta^m = \mathfrak{D}$ vel $\beta^m = -\mathfrak{B}$, $\delta^m = -\mathfrak{D}$), $B - b_m = \pm(\mathfrak{C}\alpha^m - \mathfrak{A}\gamma^m)(a\mathfrak{B}\mathfrak{B} + 2b\mathfrak{B}\mathfrak{D} - a'\mathfrak{D}\mathfrak{D}) = \mp(\mathfrak{C}\alpha^m - \mathfrak{A}\gamma^m)A'$. Hinc $B \equiv b^m$ (mod. A'); quia vero tum B tum b^m, inter $\sqrt{D}$ et $\sqrt{D} \mp A'$ iacent, necessario erit $B = b^m$ adeoque $\mathfrak{C}\alpha^m - \mathfrak{A}\gamma^m = 0$, siue $\frac{\mathfrak{A}}{\mathfrak{C}} = \frac{\alpha^m}{\gamma^m}$, *i. e.* $\mathfrak{M} = (m)$.

Hoc modo itaque ex suppositione, $\mathfrak{M}$ nulli quantitatum (2), (3), (4) etc. aequalem esse, deduximus, eam reuera alicui aequalem esse. Quodsi vero ab initio supponimus, esse $\mathfrak{M} = (m)$, manifesto erit vel $\mathfrak{A} = \alpha^m$, $\mathfrak{C} = \gamma^m$, vel $-\mathfrak{A} = \alpha^m$, $-\mathfrak{C} = \gamma^m$. In vtroque casu fit ex [1] et [5] $A = \pm a^m$, et ex [9] $B - b^m = \pm(\mathfrak{B}\delta^m - \mathfrak{D}\beta^m)A$, siue $B \equiv b^m$ (mod. A). Hinc simili modo vt supra concluditur $B = b^m$, et hinc $\mathfrak{B}\delta^m = \mathfrak{D}\beta^m$; quare quum $\mathfrak{B}$ ad $\mathfrak{D}$ primus sit et ${}^{m}\beta$ ad ${}^{m}\delta$: erit aut $\mathfrak{B} = {}^{m}\beta$, $\mathfrak{D} = {}^{m}\delta$ aut $-\mathfrak{B} = {}^{m}\beta$, $-\mathfrak{D} = {}^{m}\delta$, et proin ex [7] $-A' = \mp\, {}^{m+1}a$. Quamobrem formae F, ${}^{m}f$ identicae erunt. Adiumento aequationis $\mathfrak{A}\mathfrak{D} - \mathfrak{B}\mathfrak{C} = {}^{m}\alpha\, {}^{m}\delta - {}^{m}\beta\, {}^{m}\gamma$ autem nullo negotio probatur, poni debere $+\mathfrak{B} = {}^{m}\beta$, $+\mathfrak{D} = {}^{m}\delta$, quando $+\mathfrak{A} = {}^{m}\alpha$, $+\mathfrak{C} = {}^{m}\gamma$; contra $-\mathfrak{B} = {}^{m}\beta$, $-\mathfrak{D} = -{}^{m}\delta$, quando $-\mathfrak{A} = {}^{m}\alpha$, $-\mathfrak{C} = {}^{m}\gamma$. *Q. E. D.*

III. Si signum quantitatum $\frac{\mathfrak{A}}{\mathfrak{C}}$ signo ipsius a oppositum: demonstratio praecedenti tam similis est, vt praecipua tantum momenta addigitauisse sufficiat. Iacebit $\frac{-\sqrt{D}+b}{a'}$ inter $\frac{\mathfrak{C}}{\mathfrak{A}}$ et $\frac{\mathfrak{D}}{\mathfrak{B}}$. Fractio $\frac{\mathfrak{D}}{\mathfrak{B}}$ alicui fractionum $\frac{'\delta}{'\beta}$, $\frac{''\delta}{''\beta}$, $\frac{'''\delta}{'''\beta}$ etc. aequalis erit.... (I), qua posita $= \frac{{}^{m}\delta}{{}^{m}\beta}$, $\frac{\mathfrak{C}}{\mathfrak{A}}$ erit $= \frac{{}^{m}\gamma}{{}^{m}\alpha}$... (II). Demonstratur autem (I) ita: Si $\frac{\mathfrak{D}}{\mathfrak{B}}$ nulli illarum fractionum aequalis esse supponitur: inter duas tales $\frac{{}^{m}\delta}{{}^{m}\beta}$ et $\frac{{}^{m+2}\delta}{{}^{m+2}\beta}$ iacere debebit. Hinc vero eodem modo vt supra deducitur, necessario esse $\frac{\mathfrak{C}}{\mathfrak{A}} = \frac{{}^{m+1}\delta}{{}^{m+1}\beta} = \frac{{}^{m}\gamma}{{}^{m}\alpha}$, atque vel $\mathfrak{A} = {}^{m}\alpha$, $\mathfrak{C} = {}^{m}\gamma$, vel $-\mathfrak{A} = {}^{m}\alpha$, $-\mathfrak{C} = {}^{m}\gamma$. Quoniam vero f per substitutionem propriam ${}^{m}\alpha$, ${}^{m}\beta$, ${}^{m}\gamma$, ${}^{m}\delta$ in formam ${}^{m}f = (\pm\, {}^{m}a, {}^{m}b, \pm\, {}^{m-1}a)$ transit: hinc emergunt tres aequa-