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$l(D)$	maximum length of path	$\alpha(G)$	independence number
$l(F)$	length of a face	$\alpha'(G)$	maximum size of matching
$\lg x$	logarithm base 2	$\beta(G)$	vertex cover number
$\ln x$	natural logarithm	$\beta'(G)$	edge cover number
M	matching	$\gamma(G)$	genus, domination number
$M(G)$	incidence matrix	$\Delta(G)$	maximum degree
$M(G)$	cycle matroid of G	$\Delta^+(G), \Delta^-(G)$	maximum out-, in-degree
M^*	dual hereditary system	$\delta(G)$	minimum degree
$M.F$	contraction of M to F	$\delta^+(G), \delta^-(G)$	minimum out-, in-degree
$M F$	restriction of M to F	$\partial(v)$	demand at a vertex
\mathbb{N}	set of natural numbers	$\epsilon_G(u)$	eccentricity of u in G
N	network	$\Theta(f)$	growth rate
$N(v)N_G(v)$	(open) neighborhood	$\theta(G)$	clique cover number
$N[v]$	closed neighborhood	$\theta'(G)$	intersection number
$N^+(v), N^-(v)$	out-, in-neighborhood	$\kappa(G)$	(vertex) connectivity
$n(G)$	order (number of vertices)	$\kappa'(G)$	edge-connectivity
$O(f), o(f)$	growth rate	$\kappa(x, y)$	local connectivity
$o(H)$	number of odd components	$\kappa'(x, y)$	local edge-connectivity
$P(A)$	probability of an event	$\kappa(r; G)$	local-global connectivity
P_n	path with n vertices	$\lambda(x, y)$	max # disjoint paths
$\text{pdim } G$	product dimension	$\lambda'(x, y)$	max # edge-disjoint paths
$\text{qdim } G$	squashed-cube dimension	$\lambda_1, \dots, \lambda_n$	eigenvalues
Q_k	k -dimensional hypercube	μ_1, \dots, μ_n	eigenvalues
$\text{rad } G$	radius	$\mu(e), \mu(G)$	edge multiplicity
$R(k, l)$	Ramsey number	$v(G)$	crossing number
$R(G, H)$	graph Ramsey number	\prod	product
\mathbb{R}	set of real numbers	$\rho(G)$	maximum density
\mathbb{R}^2	$\mathbb{R} \times \mathbb{R}$	\sum	summation
r_M	rank function of matroid	σ, π, τ	permutation
S_γ	surface with γ handles	$\sigma(v)$	supply at a vertex
$\text{Spec}(G)$	spectrum (eigenvalues)	σ_M	span function
A^T	transpose of matrix	$\tau(G)$	number of spanning trees
T	tree, tournament	$\Upsilon(G)$	arboricity
$T_{n,i}$	Turán graph	$\phi(G; \lambda)$	characteristic polynomial
$t_r(n)$	size of Turán graph	$\chi(G)$	chromatic number
$U_{k,n}$	uniform matroid	$\chi'(G)$	edge-chromatic number
$u(e)$	upper bound on flow	$\chi(G; k)$	chromatic polynomial
$\text{val}(f)$	value of a flow f	$\chi_l(G)$	list chromatic number
$V(G)$	vertex set	$\psi(G; \lambda)$	minimum polynomial
W_n	wheel with n vertices	$\Omega(f), \omega(f)$	growth rate
$w(e)$	weight of edge	$\omega(G)$	clique number
\mathbb{Z}	set of integers		
\mathbb{Z}_p	integers modulo p		