$$\begin{pmatrix} D^{-1}d & -D^{-1}b \\ -D^{-1}c & D^{-1}a \end{pmatrix}.$$

We have a similar situation when we work over an arbitrary ring R.

Namely, suppose that

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(R)$$

and $D = det(A) =_{def} ad - bc$ is in R^*. Let D^{-1} denote the multiplicative inverse of D in R. Then

$$\begin{pmatrix} D^{-1}d & -D^{-1}b \\ -D^{-1}c & D^{-1}a \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} D^{-1}(da - bc) & 0 \\ 0 & D^{-1}(-cb + ad) \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

and we obtain the same result

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

if we multiply in the opposite order. Thus, A has an inverse matrix given by the same formula as in the real number case:

$$A^{-1} = \begin{pmatrix} D^{-1}d & -D^{-1}b \\ -D^{-1}c & D^{-1}a \end{pmatrix}.$$

Example 1. Find the inverse of

$$A = \begin{pmatrix} 2 & 3 \\ 7 & 8 \end{pmatrix} \in M_2(\mathbf{Z}/26\mathbf{Z}).$$

Solution. Here $D = 2 \cdot 8 - 3 \cdot 7 = -5 = 21$ in $\mathbf{Z}/26\mathbf{Z}$. Since $g.c.d.(21, 26) = 1$, the determinant D has an inverse, namely $21^{-1} = 5$. Thus,

$$A^{-1} = \begin{pmatrix} 5 \cdot 8 & -5 \cdot 3 \\ -5 \cdot 7 & 5 \cdot 2 \end{pmatrix} = \begin{pmatrix} 40 & -15 \\ -35 & 10 \end{pmatrix} = \begin{pmatrix} 14 & 11 \\ 17 & 10 \end{pmatrix}.$$

We check that $\begin{pmatrix} 14 & 11 \\ 17 & 10 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 105 & 130 \\ 104 & 131 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Here, since we are working in $\mathbf{Z}/26\mathbf{Z}$, we are using "=" to mean that the entries are congruent modulo 26.

Just as in the real number case, a 2×2–matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$