Mathematics was one of his many interests outside his profession. Cardano also secured a niche in the history of cryptography for an encoding device known as the Cardano grille [see Kahn (1967), pp. 143–145] and in the history of probability, where he was the first to make calculations, though not always correctly [see David (1962), pp. 40–60, and Ore (1953), which contains a translation of Cardano's book on games of chance].

The violence and intrigue of Renaissance Italy soured Cardano's life just as much as Tartaglia's, though in a different way. An uncle died of poisoning, attempts were made to poison both Cardano and his father (so Cardano claimed), and in 1560 Cardano's oldest son was beheaded for the crime of poisoning his wife. Cardano, who believed his son's only fault was to marry the girl in the first place, never got over this calamity. He could no longer bear to live in Milan and moved to Bologna. There he suffered another blow when his protégé Ferrari died in 1565—poisoned by his sister, it was said. In 1570 Cardano was imprisoned by the Inquisition for heresy. After a few months he recanted, was released, and moved to Rome.

In the year before he died, Cardano wrote *The Book of My Life* [Cardano (1575)], which is not so much autobiography as self-advertisement. It contains a few scenes from his childhood and returns again and again to the tragedy of his oldest son, but most of the book is devoted to boasting. There is a chapter of testimonials from patients, a chapter on important people who sought his services, a list of authors who cited his works, a list of his sayings he considered quotable, and a collection of tall stories that would have done Baron von Münchhausen proud. Admittedly, there is also a (very short) chapter called "Things in Which I Have Failed" and frequent warnings about the vanity of earthly things, but Cardano invariably tramples all such outbreaks of humility in his rush to admire other facets of his excellent self.

On the quarrel with Tartaglia, *The Book of My Life* is almost silent. Among the authors who have cited him, Cardano lumps Tartaglia with those of whom he "cannot understand by what impertinence they have managed to get themselves into the ranks of the learned." Only at the end of the book does Cardano concede that "in mathematics I received a few suggestions, but very few, from brother Niccolò." Thus we are forced back to the *Cartelli* and Tartaglia's writings. The most accessible analysis of these works, with translations of relevant passages, is in Ore (1953), Chapter 4.

François Viète (Figure 6.5) was born in 1540 in Fontenay-le-Comte, a town in what is now the Vendèe department of France. His father, Etienne, was a lawyer and his mother, Marguerite Dupont, was well connected to ruling circles in France. Viète was educated by the Franciscans in Fontenay and at the University of Poitiers. He received his bachelor's degree in law in 1560 and then returned to Fontenay to commence practice.

Figure 6.5: Viète

For the rest of his life he was engaged mainly in law or related judicial and court services, doing mathematics only in periods of leisure. His clients are said to have included Queen Mary of England and Eleanor of Austria, and from 1574 to 1584 he acted as an advisor and negotiator for

King Henry III of France. At that stage he was banished through the efforts of political rivals, but he returned to court in 1589 when Henry III moved his seat of government from Paris to Tours. Following the assassination of Henry III in 1589, he served Henry IV until 1602. Viète died in 1603.

The most famous exploit of Viète's professional career was his deciphering of Spanish dispatches for Henry IV during the war against Spain. King Philip II of Spain, unable to believe that this was humanly possible, protested to the pope that the French were using black magic. The pope may well have been impressed, but not enough to believe that magic was involved, as the Vatican's own experts had broken one of Philip's codes 30 years earlier [see Kahn (1967), pp. 116–118].

An equally famous mathematical feat of Viète's, and equally magical to his contemporaries, was his solution of a 45th-degree equation posed by Adriaen van Roomen in 1593:

$$45x - 3795x^3 + 95634x^5 - \cdots + 945x^{41} - 45x^{43} + x^{45} = N.$$

Viète saw immediately that this equation resulted from the expansion of $\sin 45\theta$ in powers of $\sin\theta$, and he was able to give 23 solutions (he did not recognize negative solutions). This was one contest, incidentally, that did not generate any bitterness—it led to a firm friendship between the two mathematicians.

7

Analytic Geometry

7.1 Steps toward Analytic Geometry

The basic idea of analytic geometry is the representation of curves by equations, but this is not the whole idea. If it were, then the Greeks would be considered the first analytic geometers. Menaechmus was perhaps the first to discover equations of curves, along with his discovery of the conic sections, and we have seen how he used equations to obtain $\sqrt[3]{2}$ as the intersection of a parabola and a hyperbola (Section 2.4). Apollonius' study of conics used equations obtained as by-products of geometric arguments.

What was lacking in Greek mathematics was both the inclination and the technique to manipulate equations to obtain information about curves. The Greeks used curves to study algebra rather than the other way around. Menaechmus' construction of $\sqrt[3]{2}$ is an excellent example of this: extraction of roots was not a given operation but one that had to be secured by geometric construction. Similarly, an equation was not an entity in its own right but a property of a curve that could be discovered after the curve had been constructed geometrically. This was a natural state of affairs as long as equations were written out in words. When, as in Apollonius, an equation takes half a page to write out, it is difficult to form a general concept of equation, function, or curve. Hence the lack of a general concept of curve in Greek mathematics—it was just too complicated to handle in their language.

In the Middle Ages the idea of coordinates emerged in a different way in the work of Oresme (around 1323–1382). Coordinates had been used in astronomy and geography since Hipparchus (around 150 BCE); in fact,