Since $3834 \not\equiv \pm 36 \ mod\ 9073$, we obtain the nontrivial factor $g.c.d.(3834 + 36, 9073) = 43$. Thus, $9073 = 43 \cdot 211$.

Example 3. Factor 17873.

Solution. As in Example 2, we start out with a table

i	0	1	2	3	4	5
a_i	133	1	2	4	2	3
b_i	133	134	401	1738	3877	13369
$b_i^2 \ mod\ n$	−184	83	−56	107	−64	161

If we set $B = \{-1, 2, 7, 23\}$, we have B-numbers when $i = 0, 2, 4, 5$; the corresponding vectors $\vec{\alpha}_i$ are, respectively, $\{1,3,0,1\}$, $\{1,3,1,0\}$, $\{1,6,0,0\}$ and $\{0,0,1,1\}$. The sum of the first, second and fourth of these four vectors is zero modulo 2. However, if we compute $b = 133 \cdot 401 \cdot 13369 \equiv 1288 \ mod\ 17873$ and $c = 2^3 \cdot 7 \cdot 23 = 1288$, we find that $b \equiv c \ mod\ 17873$. Thus, we must continue to look for more B-numbers with vectors that sum to zero modulo 2. Continuing the table, we have

i	6	7	8
a_i	1	2	1
b_i	17246	12115	11488
$b_i^2 \ mod\ n$	−77	149	−88

If we now enlarge B to include the prime 11, i.e., $B = \{-1, 2, 7, 11, 23\}$, then for $i = 0, 2, 4, 5, 6, 8$ we obtain B-numbers with vectors $\vec{\alpha}_i$ as follows: $\{1,3,0,0,1\}$, $\{1,3,1,0,0\}$, $\{1,6,0,0,0\}$, $\{0,0,1,0,1\}$, $\{1,0,1,1,0\}$, $\{1,3,0,1,0\}$. We now note that the sum of the second, third, fifth and sixth of these six vectors is zero modulo 2. This leads to $b = 7272$, $c = 4928$, and we finally find a nontrivial factor $g.c.d.(7272 + 4928, 17873) = 61$. We obtain: $17873 = 61 \cdot 293$.

Exercises

1. Find the continued fraction representation of the following rational numbers: (a) 45/89; (b) 55/89; (c) 1.13.
2. (a) Suppose that x is a real number whose continued fraction expansion consists of the positive integer a repeated infinitely:
$$x = a + \frac{1}{a+}\,\frac{1}{a+}\,\frac{1}{a+}\,\frac{1}{a+}\cdots.$$
What real number is x (written in a simple closed form)?
(b) Prove that if $a = 1$ in part (a), then x is the golden ratio and the numerators and denominators of the convergents are Fibonacci numbers.
3. Expand e in a continued fraction, and try to guess a pattern in the integers a_i.