

In 1704, in an appendix to a book on optics, he gave a study of ‘fluents’, his name for indefinite integrals or antiderivatives. He showed the connection between fluents and ‘quadratures’, that is, definite integrals. He also treated maxima and minima, tangents to curves and lengths of curves.

In 1696, Newton abandoned his Cambridge professorship for a government position in London and, three years later, he was Master of the Mint.

We defer the discussion of Leibniz to the next chapter, ending this chapter with a quotation from Newton:

I do not know what I may appear to the world; but to myself I seem to have been only like a boy, playing on the sea-shore, and diverting myself, in now and then finding a smoother pebble, or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

## Exercises

1. Show that if the three Laws of Indices are to hold for negative and fractional exponents, Wallis’s way of using such exponents is the only way. (The three Laws of Indices are  $x^{m+n} = x^m x^n$ ,  $(x^m)^n = x^{mn}$ ,  $(xy)^m = x^m y^m$ .)
2. Show that, when  $m$  is odd,

$$\int_0^{\pi/2} \sin^m x \, dx = \frac{(m-1)(m-3)\cdots 2}{m(m-2)\cdots 3 \cdot 1}.$$

Derive a similar formula for the case in which  $m$  is even. Finally, use the above two formulas to give a derivation of Wallis’s product formula for  $\pi$ .

3. Suppose the wheel in the definition of the cycloid is rolling along the positive  $x$  axis, with the fixed point starting at the origin. What is the equation of the resulting cycloid if the radius of the wheel is 1?
4. How many terms of Gregory’s series do you have to add up to get  $\pi$  accurate to two decimal places?
5. According to Newton’s calculations, your weight on a planet of uniform density is determined only by the matter which is closer to the center than you are. Suppose that you dig down towards the center of such a planet (assumed spherical). Draw a graph showing how your weight varies as you descend towards the center. (Hint: at the center, your weight is 0.)

6. You are on a planet consisting of a spherical inner core of density 5 and radius  $R$ . This inner core is in the middle of a spherical outer core of radius  $5R$  and density 1. Show that, as you dig down towards the center of this planet, your weight at first decreases, then increases, and, finally, decreases. Where does your weight start to increase?

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## Leibniz

Gottfried Wilhelm Leibniz (1646–1716) was born in Leipzig. His father died when he was six. Leibniz educated himself, using his late father's library, and entered the university in Leipzig when he was only fifteen. In 1666 he was refused the degree of Doctor of Law on the grounds that he was too young. In the same year, Leibniz conceived the idea of symbolic logic, a universal language in which all rational thinking could be expressed.

Leibniz worked as a diplomat for the Elector of Mainz. In this capacity, he went to Paris, where Louis XIV rejected his idea of attacking Egypt instead of another European country. Here Leibniz met Huygens, who introduced him to geometry and physics.

Huygens challenged Leibniz to sum the series

$$\sum_{n=1}^{\infty} \frac{2}{n(n+1)} = 1 + 1/3 + 1/6 + \dots$$

Leibniz solved the problem thus:  $2/n(n+1) = 2(1/n - 1/(n+1))$ , so the series equals  $2(1 - 1/2 + 1/2 - 1/3 + 1/3 - 1/4 + 1/4 - \dots) = 2(1 + 0) = 2$ . In the 20th century, we would object to this on the grounds that Leibniz might equally well have written

$$2(2 - 3/2 + 3/2 - 4/3 + 4/3 - 5/4 + 5/4 - \dots) = 2(2 + 0) = 4.$$

We now prefer to solve this problem by first showing that the  $m$ th partial sum of the series is  $2(1 - 1/(m+1))$  and then taking the limit as  $m \rightarrow \infty$ .

In 1673, Leibniz visited England and became a fellow of the Royal Society. If he got his ideas about the Calculus from Newton, he never acknowl-