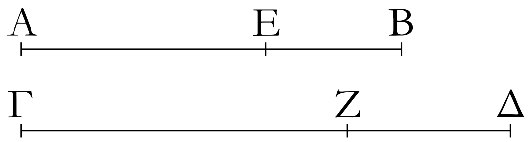
σύμμετρος ἔστω ἡ ΓΔ· λέγω, ὅτι ἡ ΓΔ ἐκ δύο ὀνομάτων ἐστὶ καὶ τῇ τάξει ἡ αὐτὴ τῇ ΑΒ.

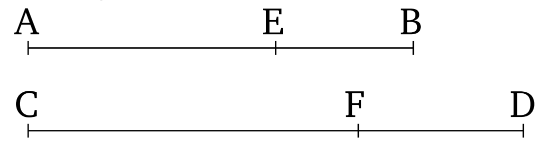


Ἐπεὶ γὰρ ἐκ δύο ὀνομάτων ἐστὶν ἡ ΑΒ, διῃρήσθω εἰς τὰ ὀνόματα κατὰ τὸ Ε, καὶ ἔστω μεῖζον ὄνομα τὸ ΑΕ· αἱ ΑΕ, ΕΒ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι. γεγονέτω ὡς ἡ ΑΒ πρὸς τὴν ΓΔ, οὕτως ἡ ΑΕ πρὸς τὴν ΓΖ· καὶ λοιπὴ ἄρα ἡ ΕΒ πρὸς λοιπὴν τὴν ΖΔ ἐστιν, ὡς ἡ ΑΒ πρὸς τὴν ΓΔ. σύμμετρος δὲ ἡ ΑΒ τῇ ΓΔ μήκει· σύμμετρος ἄρα ἐστὶ καὶ ἡ μὲν ΑΕ τῇ ΓΖ, ἡ δὲ ΕΒ τῇ ΖΔ. καί εἰσι ῥηταὶ αἱ ΑΕ, ΕΒ· ῥηταὶ ἄρα εἰσὶ καὶ αἱ ΓΖ, ΖΔ. καὶ ἐστιν ὡς ἡ ΑΕ πρὸς ΓΖ, ἡ ΕΒ πρὸς ΖΔ. ἐναλλὰξ ἄρα ἐστὶν ὡς ἡ ΑΕ πρὸς ΕΒ, ἡ ΓΖ πρὸς ΖΔ. αἱ δὲ ΑΕ, ΕΒ δυνάμει μόνον [εἰσὶ] σύμμετροι· καὶ αἱ ΓΖ, ΖΔ ἄρα δυνάμει μόνον εἰσὶ σύμμετροι. καί εἰσι ῥηταί· ἐκ δύο ἄρα ὀνομάτων ἐστὶν ἡ ΓΔ. λέγω δή, ὅτι τῇ τάξει ἐστὶν ἡ αὐτὴ τῇ ΑΒ.

Ἡ γὰρ ΑΕ τῆς ΕΒ μεῖζον δύναται ἤτοι τῷ ἀπὸ συμμέτρου ἑαυτῇ ἢ τῷ ἀπὸ ἀσυμμέτρου. εἰ μὲν οὖν ἡ ΑΕ τῆς ΕΒ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ, καὶ ἡ ΓΖ τῆς ΖΔ μεῖζον δυνήσεται τῷ ἀπὸ συμμέτρου ἑαυτῇ. καὶ εἰ μὲν σύμμετρός ἐστιν ἡ ΑΕ τῇ ἐκκειμένῃ ῥητῇ, καὶ ἡ ΓΖ σύμμετρος αὐτῇ ἔσται, καὶ διὰ τοῦτο ἑκατέρα τῶν ΑΒ, ΓΔ ἐκ δύο ὀνομάτων ἐστὶ πρώτη, τουτέστι τῇ τάξει ἡ αὐτή. εἰ δὲ ἡ ΕΒ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ, καὶ ἡ ΖΔ σύμμετρός ἐστιν αὐτῇ, καὶ διὰ τοῦτο πάλιν τῇ τάξει ἡ αὐτὴ ἔσται τῇ ΑΒ· ἑκατέρα γὰρ αὐτῶν ἔσται ἐκ δύο ὀνομάτων δευτέρα. εἰ δὲ οὐδετέρα τῶν ΑΕ, ΕΒ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ, οὐδετέρα τῶν ΓΖ, ΖΔ σύμμετρος αὐτῇ ἔσται, καί ἐστιν ἑκατέρα τρίτη. εἰ δὲ ἡ ΑΕ τῆς ΕΒ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ, καὶ ἡ ΓΖ τῆς ΖΔ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. καὶ εἰ μὲν ἡ ΑΕ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητη, καὶ ἡ ΓΖ σύμμετρός ἐστιν αὐτῇ, καὶ ἐστιν ἑκατέρα τετάρτη. εἰ δὲ ἡ ΕΒ, καὶ ἡ ΖΔ, καὶ ἔσται ἑκατέρα πέμπτη. εἰ δὲ οὐδετέρα τῶν ΑΕ, ΕΒ, καὶ τῶν ΓΖ, ΖΔ οὐδετέρα σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ, καὶ ἔσται ἑκατέρα ἕκτη.

Ὥστε ἡ τῇ ἐκ δύο ὀνομάτων μήκει σύμμετρος ἐκ δύο ὀνομάτων ἐστὶ καὶ τῇ τάξει ἡ αὐτή· ὅπερ ἔδει δεῖξαι.

Let AB be a binomial (straight-line), and let CD be commensurable in length with AB. I say that CD is a binomial (straight-line), and (is) the same in order as AB.



For since AB is a binomial (straight-line), let it have been divided into its (component) terms at E, and let AE be the greater term. AE and EB are thus rational (straight-lines which are) commensurable in square only [Prop. 10.36]. Let it have been contrived that as AB (is) to CD, so AE (is) to CF [Prop. 6.12]. Thus, the remainder EB is also to the remainder FD, as AB (is) to CD [Props. 6.16, 5.19 corr.]. And AB (is) commensurable in length with CD. Thus, AE is also commensurable (in length) with CF, and EB with FD [Prop. 10.11]. And AE and EB are rational. Thus, CF and FD are also rational. And as AE is to CF, (so) EB (is) to FD [Prop. 5.11]. Thus, alternately, as AE is to EB, (so) CF (is) to FD [Prop. 5.16]. And AE and EB [are] commensurable in square only. Thus, CF and FD are also commensurable in square only [Prop. 10.11]. And they are rational. CD is thus a binomial (straight-line) [Prop. 10.36]. So, I say that it is the same in order as AB.

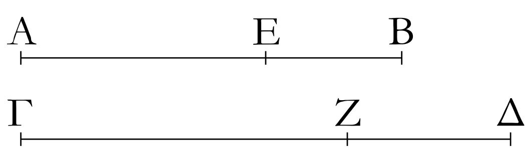
For the square on AE is greater than (the square on) EB by the (square) on (some straight-line) either commensurable or incommensurable (in length) with (AE). Therefore, if the square on AE is greater than (the square on) EB by the (square) on (some straight-line) commensurable (in length) with (AE) then the square on CF will also be greater than (the square on) FD by the (square) on (some straight-line) commensurable (in length) with (CF) [Prop. 10.14]. And if AE is commensurable (in length) with (some previously) laid down rational (straight-line) then CF will also be commensurable (in length) with it [Prop. 10.12]. And, on account of this, AB and CD are each first binomial (straight-lines) [Def. 10.5]—that is to say, the same in order. And if EB is commensurable (in length) with the (previously) laid down rational (straight-line) then FD is also commensurable (in length) with it [Prop. 10.12], and, again, on account of this, (CD) will be the same in order as AB. For each of them will be second binomial (straight-lines) [Def. 10.6]. And if neither of AE and EB is commensurable (in length) with the (previously) laid down rational (straight-line) then neither of CF and FD will be commensurable (in length) with it [Prop. 10.13], and each (of AB and CD) is a third (binomial straight-line)

[Def. 10.7]. And if the square on AE is greater than (the square on) EB by the (square) on (some straight-line) incommensurable (in length) with (AE) then the square on CF is also greater than (the square on) FD by the (square) on (some straight-line) incommensurable (in length) with (CF) [Prop. 10.14]. And if AE is commensurable (in length) with the (previously) laid down rational (straight-line) then CF is also commensurable (in length) with it [Prop. 10.12], and each (of AB and CD) is a fourth (binomial straight-line) [Def. 10.8]. And if EB (is commensurable in length with the previously laid down rational straight-line) then FD (is) also (commensurable in length with it), and each (of AB and CD) will be a fifth (binomial straight-line) [Def. 10.9]. And if neither of AE and EB (is commensurable in length with the previously laid down rational straight-line) then also neither of CF and FD is commensurable (in length) with the laid down rational (straight-line), and each (of AB and CD) will be a sixth (binomial straight-line) [Def. 10.10].

Hence, a (straight-line) commensurable in length with a binomial (straight-line) is a binomial (straight-line), and the same in order. (Which is) the very thing it was required to show.

ξζ'.

Ἡ τῇ ἐκ δύο μέσων μήκει σύμμετρος καὶ αὐτὴ ἐκ δύο μέσων ἐστὶ καὶ τῇ τάξει ἡ αὐτή.



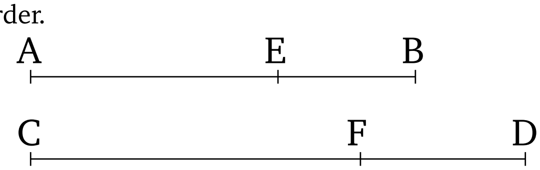
Ἔστω ἐκ δύο μέσων ἡ ΑΒ, καὶ τῇ ΑΒ σύμμετρος ἔστω μήκει ἡ ΓΔ· λέγω, ὅτι ἡ ΓΔ ἐκ δύο μέσων ἐστὶ καὶ τῇ τάξει ἡ αὐτὴ τῇ ΑΒ.

Ἐπεὶ γὰρ ἐκ δύο μέσων ἐστὶν ἡ ΑΒ, διῃρήσθω εἰς τὰς μέσας κατὰ τὸ Ε· αἱ ΑΕ, ΕΒ ἄρα μέσαι εἰσὶ δυνάμει μόνον σύμμετροι. καὶ γεγονέτω ὡς ἡ ΑΒ πρὸς ΓΔ, ἡ ΑΕ πρὸς ΓΖ· καὶ λοιπὴ ἄρα ἡ ΕΒ πρὸς λοιπὴν τὴν ΖΔ ἐστιν, ὡς ἡ ΑΒ πρὸς ΓΔ. σύμμετρος δὲ ἡ ΑΒ τῇ ΓΔ μήκει· σύμμετρος ἄρα καὶ ἑκατέρα τῶν ΑΕ, ΕΒ ἑκατέρᾳ τῶν ΓΖ, ΖΔ. μέσαι δὲ αἱ ΑΕ, ΕΒ· μέσαι ἄρα καὶ αἱ ΓΖ, ΖΔ. καὶ ἐπεί ἐστιν ὡς ἡ ΑΕ πρὸς ΕΒ, ἡ ΓΖ πρὸς ΖΔ, αἱ δὲ ΑΕ, ΕΒ δυνάμει μόνον σύμμετροί εἰσιν, καὶ αἱ ΓΖ, ΖΔ [ἄρα] δυνάμει μόνον σύμμετροί εἰσιν, ἐδείχθησαν δὲ καὶ μέσαι· ἡ ΓΔ ἄρα ἐκ δύο μέσων ἐστίν. λέγω δή, ὅτι καὶ τῇ τάξει ἡ αὐτή ἐστι τῇ ΑΒ.

Ἐπεὶ γάρ ἐστιν ὡς ἡ ΑΕ πρὸς ΕΒ, ἡ ΓΖ πρὸς ΖΔ, καὶ ὡς ἄρα τὸ ἀπὸ τῆς ΑΕ πρὸς τὸ ὑπὸ τῶν ΑΕΒ, οὕτως τὸ ἀπὸ τῆς ΓΖ πρὸς τὸ ὑπὸ τῶν ΓΖΔ· ἐναλλὰξ ὡς τὸ ἀπὸ τῆς

Proposition 67

A (straight-line) commensurable in length with a bimedial (straight-line) is itself also bimedial, and the same in order.



Let AB be a bimedial (straight-line), and let CD be commensurable in length with AB. I say that CD is bimedial, and the same in order as AB.

For since AB is a bimedial (straight-line), let it have been divided into its (component) medial (straight-lines) at E. Thus, AE and EB are medial (straight-lines which are) commensurable in square only [Props. 10.37, 10.38]. And let it have been contrived that as AB (is) to CD, (so) AE (is) to CF [Prop. 6.12]. And thus as the remainder EB is to the remainder FD, so AB (is) to CD [Props. 5.19 corr., 6.16]. And AB (is) commensurable in length with CD. Thus, AE and EB are also commensurable (in length) with CF and FD, respectively [Prop. 10.11]. And AE and EB (are) medial. Thus, CF and FD (are) also medial [Prop. 10.23]. And since as AE is to EB, (so) CF (is) to FD, and AE and EB are commensurable in square only, CF and FD are [thus]

ΑΕ πρὸς τὸ ἀπὸ τῆς ΓΖ, οὕτως τὸ ὑπὸ τῶν ΑΕΒ πρὸς τὸ ὑπὸ τῶν ΓΖΔ. σύμμετρον δὲ τὸ ἀπὸ τῆς ΑΕ τῷ ἀπὸ τῆς ΓΖ· σύμμετρον ἄρα καὶ τὸ ὑπὸ τῶν ΑΕΒ τῷ ὑπὸ τῶν ΓΖΔ. εἴτε οὖν ῥητόν ἐστι τὸ ὑπὸ τῶν ΑΕΒ, καὶ τὸ ὑπὸ τῶν ΓΖΔ ῥητόν ἐστιν [καὶ διὰ τοῦτό ἐστιν ἐκ δύο μέσων πρώτη]. εἴτε μέσον, μέσον, καί ἐστιν ἑκατέρα δευτέρα.

Καὶ διὰ τοῦτο ἔσται ἡ ΓΔ τῇ ΑΒ τῇ τάξει ἡ αὐτή· ὅπερ ἔδει δεῖξαι.

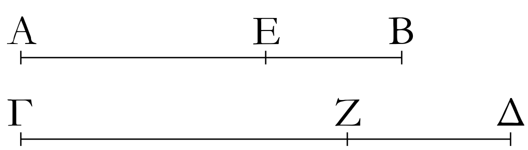
also commensurable in square only [Prop. 10.11]. And they were also shown (to be) medial. Thus, CD is a bimedial (straight-line). So, I say that it is also the same in order as AB.

For since as AE is to EB, (so) CF (is) to FD, thus also as the (square) on AE (is) to the (rectangle contained) by AEB, so the (square) on CF (is) to the (rectangle contained) by CFD [Prop. 10.21 lem.]. Alternately, as the (square) on AE (is) to the (square) on CF, so the (rectangle contained) by AEB (is) to the (rectangle contained) by CFD [Prop. 5.16]. And the (square) on AE (is) commensurable with the (square) on CF. Thus, the (rectangle contained) by AEB (is) also commensurable with the (rectangle contained) by CFD [Prop. 10.11]. Therefore, either the (rectangle contained) by AEB is rational, and the (rectangle contained) by CFD is rational [and, on account of this, (AE and CD) are first bimedial (straight-lines)], or (the rectangle contained by AEB is) medial, and (the rectangle contained by CFD is) medial, and (AB and CD) are each second (bimedial straight-lines) [Props. 10.23, 10.37, 10.38].

And, on account of this, CD will be the same in order as AB. (Which is) the very thing it was required to show.

ξη΄.

Ἡ τῇ μείζονι σύμμετρος καὶ αὐτὴ μείζων ἐστίν.

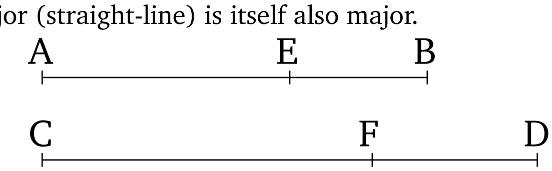


Ἔστω μείζων ἡ ΑΒ, καὶ τῇ ΑΒ σύμμετρος ἔστω ἡ ΓΔ· λέγω, ὅτι ἡ ΓΔ μείζων ἐστίν.

Διῃρήσθω ἡ ΑΒ κατὰ τὸ Ε· αἱ ΑΕ, ΕΒ ἄρα δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὸ μὲν συγκείμενον ἐκ τῶν ἀπ᾿ αὐτῶν τετραγώνων ῥητόν, τὸ δ᾿ ὑπ᾿ αὐτῶν μέσον· καὶ γεγονέτω τὰ αὐτὰ τοῖς πρότερον. καὶ ἐπεί ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΓΔ, οὕτως ἥ τε ΑΕ πρὸς τὴν ΓΖ καὶ ἡ ΕΒ πρὸς τὴν ΖΔ, καὶ ὡς ἄρα ἡ ΑΕ πρὸς τὴν ΓΖ, οὕτως ἡ ΕΒ πρὸς τὴν ΖΔ. σύμμετρος δὲ ἡ ΑΒ τῇ ΓΔ· σύμμετρος ἄρα καὶ ἑκατέρα τῶν ΑΕ, ΕΒ ἑκατέρᾳ τῶν ΓΖ, ΖΔ. καὶ ἐπεί ἐστιν ὡς ἡ ΑΕ πρὸς τὴν ΓΖ, οὕτως ἡ ΕΒ πρὸς τὴν ΖΔ, καὶ ἐναλλὰξ ὡς ἡ ΑΕ πρὸς ΕΒ, οὕτως ἡ ΓΖ πρὸς ΖΔ, καὶ συνθέντι ἄρα ἐστὶν ὡς ἡ ΑΒ πρὸς τὴν ΒΕ, οὕτως ἡ ΓΔ πρὸς τὴν ΔΖ· καὶ ὡς ἄρα τὸ ἀπὸ τῆς ΑΒ πρὸς τὸ ἀπὸ τῆς ΒΕ, οὕτως τὸ ἀπὸ τῆς ΓΔ πρὸς τὸ ἀπὸ τῆς ΔΖ. ὁμοίως δὴ δείξομεν, ὅτι καὶ ὡς τὸ ἀπὸ τῆς ΑΒ πρὸς τὸ ἀπὸ τῆς ΑΕ, οὕτως τὸ ἀπὸ τῆς ΓΔ πρὸς τὸ ἀπὸ τῆς ΓΖ. καὶ ὡς ἄρα τὸ ἀπὸ τῆς ΑΒ πρὸς τὰ ἀπὸ τῶν ΑΕ, ΕΒ, οὕτως τὸ ἀπὸ τῆς ΓΔ πρὸς τὰ ἀπὸ τῶν ΓΖ, ΖΔ·

Proposition 68

A (straight-line) commensurable (in length) with a major (straight-line) is itself also major.



Let AB be a major (straight-line), and let CD be commensurable (in length) with AB. I say that CD is a major (straight-line).

Let AB have been divided (into its component terms) at E. AE and EB are thus incommensurable in square, making the sum of the squares on them rational, and the (rectangle contained) by them medial [Prop. 10.39]. And let (the) same (things) have been contrived as in the previous (propositions). And since as AB is to CD, so AE (is) to CF and EB to FD, thus also as AE (is) to CF, so EB (is) to FD [Prop. 5.11]. And AB (is) commensurable (in length) with CD. Thus, AE and EB (are) also commensurable (in length) with CF and FD, respectively [Prop. 10.11]. And since as AE is to CF, so EB (is) to FD, also, alternately, as AE (is) to EB, so CF (is) to FD [Prop. 5.16], and thus, via composition, as AB is to BE, so CD (is) to DF [Prop. 5.18]. And thus as the (square) on AB (is) to the (square) on BE, so the καὶ ἐναλλὰξ ἄρα ἐστὶν ὡς τὸ ἀπὸ τῆς ΑΒ πρὸς τὸ ἀπὸ τῆς ΓΔ, οὕτως τὰ ἀπὸ τῶν ΑΕ, ΕΒ πρὸς τὰ ἀπὸ τῶν ΓΖ, ΖΔ. σύμμετρον δὲ τὸ ἀπὸ τῆς ΑΒ τῷ ἀπὸ τῆς ΓΔ· σύμμετρα ἄρα καὶ τὰ ἀπὸ τῶν ΑΕ, ΕΒ τοῖς ἀπὸ τῶν ΓΖ, ΖΔ. καί ἐστι τὰ ἀπὸ τῶν ΑΕ, ΕΒ ἅμα ῥητόν, καὶ τὰ ἀπὸ τῶν ΓΖ, ΖΔ ἅμα ῥητόν ἐστιν. ὁμοίως δὲ καὶ τὸ δὶς ὑπὸ τῶν ΑΕ, ΕΒ σύμμετρόν ἐστι τῷ δὶς ὑπὸ τῶν ΓΖ, ΖΔ. καί ἐστι μέσον τὸ δὶς ὑπὸ τῶν ΑΕ, ΕΒ· μέσον ἄρα καὶ τὸ δὶς ὑπὸ τῶν ΓΖ, ΖΔ. αἱ ΓΖ, ΖΔ ἄρα δυνάμει ἀσύμμετροί εἰσι ποιοῦσαι τὸ μὲν συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων ἅμα ῥητόν, τὸ δὲ δὶς ὑπ᾽ αὐτῶν μέσον· ὅλη ἄρα ἡ ΓΔ ἄλογός ἐστιν ἡ καλουμένη μείζων.

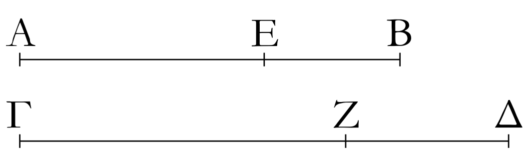
Ἡ ἄρα τῇ μείζονι σύμμετρος μείζων ἐστίν· ὅπερ ἔδει δεῖξαι.

(square) on CD (is) to the (square) on DF [Prop. 6.20]. So, similarly, we can also show that as the (square) on AB (is) to the (square) on AE, so the (square) on CD (is) to the (square) on CF. And thus as the (square) on AB (is) to (the sum of) the (squares) on AE and EB, so the (square) on CD (is) to (the sum of) the (squares) on CF and FD. And thus, alternately, as the (square) on AB is to the (square) on CD, so (the sum of) the (squares) on AE and EB (is) to (the sum of) the (squares) on CF and FD [Prop. 5.16]. And the (square) on AB (is) commensurable with the (square) on CD. Thus, (the sum of) the (squares) on AE and EB (is) also commensurable with (the sum of) the (squares) on CF and FD [Prop. 10.11]. And the (squares) on AE and EB (added) together are rational. The (squares) on CF and FD (added) together (are) thus also rational. So, similarly, twice the (rectangle contained) by AE and EB is also commensurable with twice the (rectangle contained) by CF and FD. And twice the (rectangle contained) by AE and EB is medial. Therefore, twice the (rectangle contained) by CF and FD (is) also medial [Prop. 10.23 corr.]. CF and FD are thus (straight-lines which are) incommensurable in square [Prop 10.13], simultaneously making the sum of the squares on them rational, and twice the (rectangle contained) by them medial. The whole, CD, is thus that irrational (straight-line) called major [Prop. 10.39].

Thus, a (straight-line) commensurable (in length) with a major (straight-line) is major. (Which is) the very thing it was required to show.

ξθ´.

Ἡ τῇ ῥητὸν καὶ μέσον δυναμένῃ σύμμετρος [καὶ αὐτὴ] ῥητὸν καὶ μέσον δυναμένη ἐστίν.

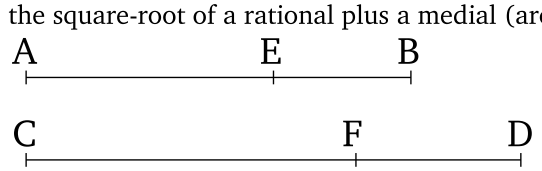


Ἔστω ῥητὸν καὶ μέσον δυναμένη ἡ ΑΒ, καὶ τῇ ΑΒ σύμμετρος ἔστω ἡ ΓΔ· δεικτέον, ὅτι καὶ ἡ ΓΔ ῥητὸν καὶ μέσον δυναμένη ἐστίν.

Διῃρήσθω ἡ ΑΒ εἰς τὰς εὐθείας κατὰ τὸ Ε· αἱ ΑΕ, ΕΒ ἄρα δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὸ μὲν συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων μέσον, τὸ δ᾽ ὑπ᾽ αὐτῶν ῥητόν· καὶ τὰ αὐτὰ κατεσκευάσθω τοῖς πρότερον. ὁμοίως δὴ δείξομεν, ὅτι καὶ αἱ ΓΖ, ΖΔ δυνάμει εἰσὶν ἀσύμμετροι, καὶ σύμμετρον τὸ μὲν συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΕ, ΕΒ τῷ συγκειμένῳ ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ, τὸ δὲ ὑπὸ ΑΕ, ΕΒ τῷ ὑπὸ ΓΖ, ΖΔ· ὥστε καὶ τὸ [μὲν] συγκείμενον ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ τετραγώνων ἐστὶ μέσον, τὸ δ᾽ ὑπὸ τῶν ΓΖ,

Proposition 69

A (straight-line) commensurable (in length) with the square-root of a rational plus a medial (area) is [itself also] the square-root of a rational plus a medial (area).



Let AB be the square-root of a rational plus a medial (area), and let CD be commensurable (in length) with AB. We must show that CD is also the square-root of a rational plus a medial (area).

Let AB have been divided into its (component) straight-lines at E. AE and EB are thus incommensurable in square, making the sum of the squares on them medial, and the (rectangle contained) by them rational [Prop. 10.40]. And let the same construction have been made as in the previous (propositions). So, similarly, we can show that CF and FD are also incommensurable in square, and that the sum of the (squares) on AE and

ΖΔ ῥητόν.

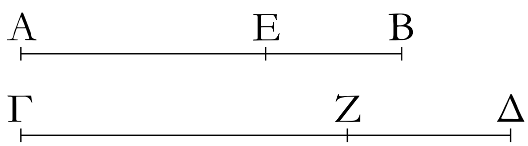
Ῥητὸν ἄρα καὶ μέσον δυναμένη ἐστὶν ἡ ΓΔ· ὅπερ ἔδει δεῖξαι.

EB (is) commensurable with the sum of the (squares) on CF and FD, and the (rectangle contained) by AE and EB with the (rectangle contained) by CF and FD. And hence the sum of the squares on CF and FD is medial, and the (rectangle contained) by CF and FD (is) rational.

Thus, CD is the square-root of a rational plus a medial (area) [Prop. 10.40]. (Which is) the very thing it was required to show.

ο´.

Ἡ τῇ δύο μέσα δυναμένῃ σύμμετρος δύο μέσα δυναμένη ἐστίν.



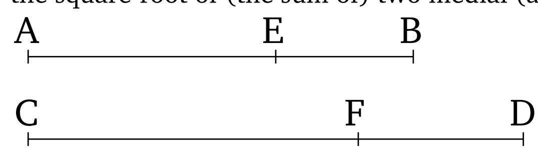
Ἔστω δύο μέσα δυναμένη ἡ ΑΒ, καὶ τῇ ΑΒ σύμμετρος ἡ ΓΔ· δεικτέον, ὅτι καὶ ἡ ΓΔ δύο μέσα δυναμένη ἐστίν.

Ἐπεὶ γὰρ δύο μέσα δυναμένη ἐστὶν ἡ ΑΒ, διῃρήσθω εἰς τὰς εὐθείας κατὰ τὸ Ε· αἱ ΑΕ, ΕΒ ἄρα δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τό τε συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν [τετραγώνων] μέσον καὶ τὸ ὑπ᾽ αὐτῶν μέσον καὶ ἔτι ἀσύμμετρον τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΕ, ΕΒ τετραγώνων τῷ ὑπὸ τῶν ΑΕ, ΕΒ· καὶ κατεσκευάσθω τὰ αὐτὰ τοῖς πρότερον. ὁμοίως δὴ δείξομεν, ὅτι καὶ αἱ ΓΖ, ΖΔ δυνάμει εἰσὶν ἀσύμμετροι καὶ σύμμετρον τὸ μὲν συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΕ, ΕΒ τῷ συγκειμένῳ ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ, τὸ δὲ ὑπὸ τῶν ΑΕ, ΕΒ τῷ ὑπὸ τῶν ΓΖ, ΖΔ· ὥστε καὶ τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ τετραγώνων μέσον ἐστὶ καὶ τὸ ὑπὸ τῶν ΓΖ, ΖΔ μέσον καὶ ἔτι ἀσύμμετρον τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ τετραγώνων τῷ ὑπὸ τῶν ΓΖ, ΖΔ.

Ἡ ἄρα ΓΔ δύο μέσα δυναμένη ἐστίν· ὅπερ ἔδει δεῖξαι.

Proposition 70

A (straight-line) commensurable (in length) with the square-root of (the sum of) two medial (areas) is (itself also) the square-root of (the sum of) two medial (areas).



Let AB be the square-root of (the sum of) two medial (areas), and (let) CD (be) commensurable (in length) with AB. We must show that CD is also the square-root of (the sum of) two medial (areas).

For since AB is the square-root of (the sum of) two medial (areas), let it have been divided into its (component) straight-lines at E. Thus, AE and EB are incommensurable in square, making the sum of the [squares] on them medial, and the (rectangle contained) by them medial, and, moreover, the sum of the (squares) on AE and EB incommensurable with the (rectangle) contained by AE and EB [Prop. 10.41]. And let the same construction have been made as in the previous (propositions). So, similarly, we can show that CF and FD are also incommensurable in square, and (that) the sum of the (squares) on AE and EB (is) commensurable with the sum of the (squares) on CF and FD, and the (rectangle contained) by AE and EB with the (rectangle contained) by CF and FD. Hence, the sum of the squares on CF and FD is also medial, and the (rectangle contained) by CF and FD (is) medial, and, moreover, the sum of the squares on CF and FD (is) incommensurable with the (rectangle contained) by CF and FD.

Thus, CD is the square-root of (the sum of) two medial (areas) [Prop. 10.41]. (Which is) the very thing it was required to show.

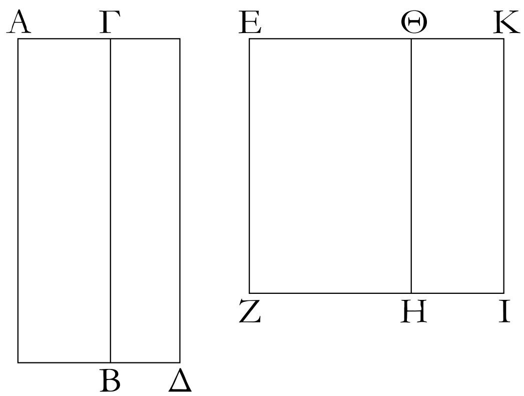
οα´.

Ῥητοῦ καὶ μέσου συντιθεμένου τέσσαρες ἄλογοι γίγνονται ἤτοι ἐκ δύο ὀνομάτων ἢ ἐκ δύο μέσων πρώτη ἢ μείζων ἢ ῥητὸν καὶ μέσον δυναμένη.

Proposition 71

When a rational and a medial (area) are added together, four irrational (straight-lines) arise (as the square-roots of the total area)—either a binomial, or a first bi-

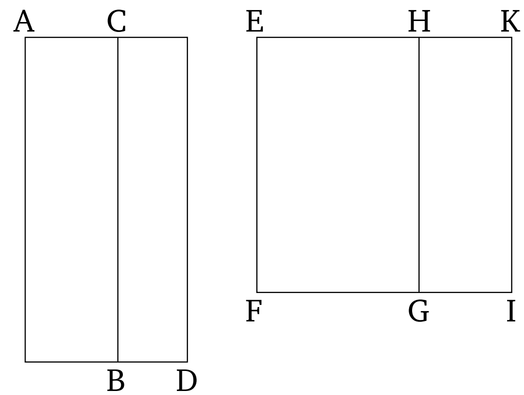
Ἔστω ῥητὸν μὲν τὸ ΑΒ, μέσον δὲ τὸ ΓΔ· λέγω, ὅτι ἡ τὸ ΑΔ χωρίον δυναμένη ἤτοι ἐκ δύο ὀνομάτων ἐστὶν ἢ ἐκ δύο μέσων πρώτη ἢ μείζων ἢ ῥητὸν καὶ μέσον δυναμένη.



Τὸ γὰρ ΑΒ τοῦ ΓΔ ἤτοι μεῖζόν ἐστιν ἢ ἔλασσον. ἔστω πρότερον μεῖζον· καὶ ἐκκείσθω ῥητὴ ἡ ΕΖ, καὶ παραβεβλήσθω παρὰ τὴν ΕΖ τῷ ΑΒ ἴσον τὸ ΕΗ πλάτος ποιοῦν τὴν ΕΘ· τῷ δὲ ΔΓ ἴσον παρὰ τὴν ΕΖ παραβεβλήσθω τὸ ΘΙ πλάτος ποιοῦν τὴν ΘΚ. καὶ ἐπεὶ ῥητόν ἐστι τὸ ΑΒ καί ἐστιν ἴσον τῷ ΕΗ, ῥητὸν ἄρα καὶ τὸ ΕΗ. καὶ παρὰ [ῥητὴν] τὴν ΕΖ παραβέβληται πλάτος ποιοῦν τὴν ΕΘ· ἡ ΕΘ ἄρα ῥητή ἐστι καὶ σύμμετρος τῇ ΕΖ μήκει. πάλιν, ἐπεὶ μέσον ἐστὶ τὸ ΓΔ καί ἐστιν ἴσον τῷ ΘΙ, μέσον ἄρα ἐστὶ καὶ τὸ ΘΙ. καὶ παρὰ ῥητὴν τὴν ΕΖ παράκειται πλάτος ποιοῦν τὴν ΘΚ· ῥητὴ ἄρα ἐστὶν ἡ ΘΚ καὶ ἀσύμμετρος τῇ ΕΖ μήκει. καὶ ἐπεὶ μέσον ἐστὶ τὸ ΓΔ, ῥητὸν δὲ τὸ ΑΒ, ἀσύμμετρον ἄρα ἐστὶ τὸ ΑΒ τῷ ΓΔ· ὥστε καὶ τὸ ΕΗ ἀσύμμετρόν ἐστι τῷ ΘΙ. ὡς δὲ τὸ ΕΗ πρὸς τὸ ΘΙ, οὕτως ἐστὶν ἡ ΕΘ πρὸς τὴν ΘΚ· ἀσύμμετρος ἄρα ἐστὶ καὶ ἡ ΕΘ τῇ ΘΚ μήκει. καί εἰσιν ἀμφότεραι ῥηταί· αἱ ΕΘ, ΘΚ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἐκ δύο ἄρα ὀνομάτων ἐστὶν ἡ ΕΚ διῃρημένη κατὰ τὸ Θ. καὶ ἐπεὶ μεῖζόν ἐστι τὸ ΑΒ τοῦ ΓΔ, ἴσον δὲ τὸ μὲν ΑΒ τῷ ΕΗ, τὸ δὲ ΓΔ τῷ ΘΙ, μεῖζον ἄρα καὶ τὸ ΕΗ τοῦ ΘΙ· καὶ ἡ ΕΘ ἄρα μείζων ἐστὶ τῆς ΘΚ. ἤτοι οὖν ἡ ΕΘ τῆς ΘΚ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ μήκει ἢ τῷ ἀπὸ ἀσυμμέτρου. δυνάσθω πρότερον τῷ ἀπὸ συμμέτρου ἑαυτῇ· καί ἐστιν ἡ μείζων ἡ ΘΕ σύμμετρος τῇ ἐκκειμένῃ ῥητῃ τῇ ΕΖ· ἡ ἄρα ΕΚ ἐκ δύο ὀνομάτων ἐστὶ πρώτη. ῥητὴ δὲ ἡ ΕΖ· ἐὰν δὲ χωρίον περιέχηται ὑπὸ ῥητῆς καὶ τῆς ἐκ δύο ὀνομάτων πρώτης, ἡ τὸ χωρίον δυναμένη ἐκ δύο ὀνομάτων ἐστίν. ἡ ἄρα τὸ ΕΙ δυναμένη ἐκ δύο ὀνομάτων ἐστίν· ὥστε καὶ ἡ τὸ ΑΔ δυναμένη ἐκ δύο ὀνομάτων ἐστίν. ἀλλὰ δὴ δυνάσθω ἡ ΕΘ τῆς ΘΚ μεῖζον τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ· καί ἐστιν ἡ μείζων ἡ ΕΘ σύμμετρος τῇ ἐκκειμένῃ ῥητῇ τῇ ΕΖ μήκει· ἡ ἄρα ΕΚ ἐκ δύο ὀνομάτων ἐστὶ τετάρτη. ῥητὴ δὲ ἡ ΕΖ· ἐὰν δὲ χωρίον περιέχηται ὑπὸ ῥητῆς καὶ τῆς ἐκ δύο

medial, or a major, or the square-root of a rational plus a medial (area).

Let AB be a rational (area), and CD a medial (area). I say that the square-root of area AD is either binomial, or first bimedial, or major, or the square-root of a rational plus a medial (area).



For AB is either greater or less than CD. Let it, first of all, be greater. And let the rational (straight-line) EF be laid down. And let (the rectangle) EG, equal to AB, have been applied to EF, producing EH as breadth. And let (the recatangle) HI, equal to DC, have been applied to EF, producing HK as breadth. And since AB is rational, and is equal to EG, EG is thus also rational. And it has been applied to the [rational] (straight-line) EF, producing EH as breadth. EH is thus rational, and commensurable in length with EF [Prop. 10.20]. Again, since CD is medial, and is equal to HI, HI is thus also medial. And it is applied to the rational (straight-line) EF, producing HK as breadth. HK is thus rational, and incommensurable in length with EF [Prop. 10.22]. And since CD is medial, and AB rational, AB is thus incommensurable with CD. Hence, EG is also incommensurable with HI. And as EG (is) to HI, so EH is to HK [Prop. 6.1]. Thus, EH is also incommensurable in length with HK [Prop. 10.11]. And they are both rational. Thus, EH and HK are rational (straight-lines which are) commensurable in square only. EK is thus a binomial (straight-line), having been divided (into its component terms) at H [Prop. 10.36]. And since AB is greater than CD, and AB (is) equal to EG, and CD to HI, EG (is) thus also greater than HI. Thus, EH is also greater than HK [Prop. 5.14]. Therefore, the square on EH is greater than (the square on) HK either by the (square) on (some straight-line) commensurable in length with (EH), or by the (square) on (some straight-line) incommensurable (in length with EH). Let it, first of all, be greater by the (square) on (some straight-line) commensurable (in length with EH). And the greater ὀνομάτων τετάρτης, ἡ τὸ χωρίον δυναμένη ἄλογός ἐστιν ἡ καλουμένη μείζων. ἡ ἄρα τὸ ΕΙ χωρίον δυναμένη μείζων ἐστίν· ὥστε καὶ ἡ τὸ ΑΔ δυναμένη μείζων ἐστίν.

Ἀλλὰ δὴ ἔστω ἔλασσον τὸ ΑΒ τοῦ ΓΔ· καὶ τὸ ΕΗ ἄρα ἔλασσόν ἐστι τοῦ ΘΙ· ὥστε καὶ ἡ ΕΘ ἐλάσσων ἐστὶ τῆς ΘΚ. ἤτοι δὲ ἡ ΘΚ τῆς ΕΘ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ ἢ τῷ ἀπὸ ἀσυμμέτρου. δυνάσθω πρότερον τῷ ἀπὸ συμμέτρου ἑαυτῇ μήκει· καί ἐστιν ἡ ἐλάσσων ἡ ΕΘ σύμμετρος τῇ ἐκκειμένῃ ῥητῇ τῇ ΕΖ μήκει· ἡ ἄρα ΕΚ ἐκ δύο ὀνομάτων ἐστὶ δευτέρα. ῥητὴ δὲ ἡ ΕΖ· ἐὰν δὲ χωρίον περιέχηται ὑπὸ ῥητῆς καὶ τῆς ἐκ δύο ὀνομάτων δευτέρας, ἡ τὸ χωρίον δυναμένη ἐκ δύο μέσων ἐστὶ πρώτη. ἡ ἄρα τὸ ΕΙ χωρίον δυναμένη ἐκ δύο μέσων ἐστὶ πρώτη· ὥστε καὶ ἡ τὸ ΑΔ δυναμένη ἐκ δύο μέσων ἐστὶ πρώτη. ἀλλὰ δὴ ἡ ΘΚ τῆς ΘΕ μεῖζον δυνάσθω τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. καί ἐστιν ἡ ἐλάσσων ἡ ΕΘ σύμμετρος τῇ ἐκκειμένῃ ῥητῇ τῇ ΕΖ· ἡ ἄρα ΕΚ ἐκ δύο ὀνομάτων ἐστὶ πέμπτη. ῥητὴ δὲ ἡ ΕΖ· ἐὰν δὲ χωρίον περιέχηται ὑπὸ ῥητῆς καὶ τῆς ἐκ δύο ὀνομάτων πέμπτης, ἡ τὸ χωρίον δυναμένη ῥητὸν καὶ μέσον δυναμένη ἐστίν. ἡ ἄρα τὸ ΕΙ χωρίον δυναμένη ῥητὸν καὶ μέσον δυναμένη ἐστίν· ὥστε καὶ ἡ τὸ ΑΔ χωρίον δυναμένη ῥητὸν καὶ μέσον δυναμένη ἐστίν.

Ῥητοῦ ἄρα καὶ μέσου συντιθεμένου τέσσαρες ἄλογοι γίγνονται ἤτοι ἐκ δύο ὀνομάτων ἢ ἐκ δύο μέσων πρώτη ἢ μείζων ἢ ῥητὸν καὶ μέσον δυναμένη· ὅπερ ἔδει δεῖξαι.

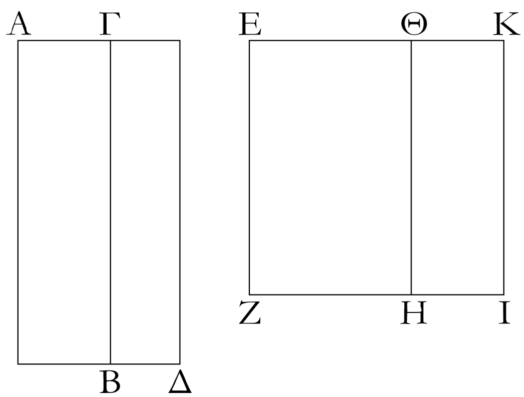
(of the two components of EK) HE is commensurable (in length) with the (previously) laid down (straight-line) EF. EK is thus a first binomial (straight-line) [Def. 10.5]. And EF (is) rational. And if an area is contained by a rational (straight-line) and a first binomial (straight-line) then the square-root of the area is a binomial (straight-line) [Prop. 10.54]. Thus, the square-root of EI is a binomial (straight-line). Hence the square-root of AD is also a binomial (straight-line). And, so, let the square on EH be greater than (the square on) HK by the (square) on (some straight-line) incommensurable (in length) with (EH). And the greater (of the two components of EK) EH is commensurable in length with the (previously) laid down rational (straight-line) EF. Thus, EK is a fourth binomial (straight-line) [Def. 10.8]. And EF (is) rational. And if an area is contained by a rational (straight-line) and a fourth binomial (straight-line) then the square-root of the area is the irrational (straight-line) called major [Prop. 10.57]. Thus, the square-root of area EI is a major (straight-line). Hence, the square-root of AD is also major.

And so, let AB be less than CD. Thus, EG is also less than HI. Hence, EH is also less than HK [Props. 6.1, 5.14]. And the square on HK is greater than (the square on) EH either by the (square) on (some straight-line) commensurable (in length) with (HK), or by the (square) on (some straight-line) incommensurable (in length) with (HK). Let it, first of all, be greater by the square on (some straight-line) commensurable in length with (HK). And the lesser (of the two components of EK) EH is commensurable in length with the (previously) laid down rational (straight-line) EF. Thus, EK is a second binomial (straight-line) [Def. 10.6]. And EF (is) rational. And if an area is contained by a rational (straight-line) and a second binomial (straight-line) then the square-root of the area is a first bimedial (straight-line) [Prop. 10.55]. Thus, the square-root of area EI is a first bimedial (straight-line). Hence, the square-root of AD is also a first bimedial (straight-line). And so, let the square on HK be greater than (the square on) HE by the (square) on (some straight-line) incommensurable (in length) with (HK). And the lesser (of the two components of EK) EH is commensurable (in length) with the (previously) laid down rational (straight-line) EF. Thus, EK is a fifth binomial (straight-line) [Def. 10.9]. And EF (is) rational. And if an area is contained by a rational (straight-line) and a fifth binomial (straight-line) then the square-root of the area is the square-root of a rational plus a medial (area) [Prop. 10.58]. Thus, the square-root of area EI is the square-root of a rational plus a medial (area). Hence, the square-root of area AD is also the square-root of a rational plus a medial (area).

Thus, when a rational and a medial area are added together, four irrational (straight-lines) arise (as the square-roots of the total area)—either a binomial, or a first bimedial, or a major, or the square-root of a rational plus a medial (area). (Which is) the very thing it was required to show.

ξβ'.

Δύο μέσων ἀσυμμέτρων ἀλλήλοις συντιθεμένων αἱ λοιπαὶ δύο ἄλογοι γίγνονται ἤτοι ἐκ δύο μέσων δευτέρα ἢ [ἡ] δύο μέσα δυναμένη.

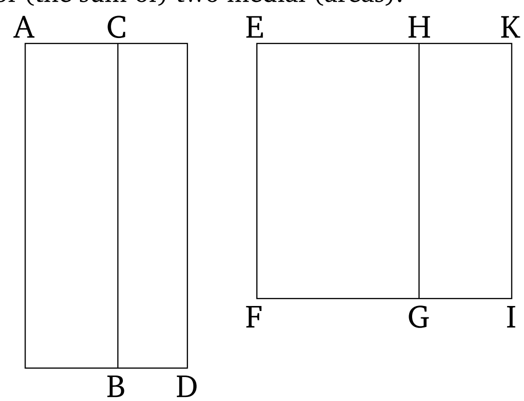


Συγκείσθω γὰρ δύο μέσα ἀσύμμετρα ἀλλήλοις τὰ ΑΒ, ΓΔ· λέγω, ὅτι ἡ τὸ ΑΔ χωρίον δυναμένη ἤτοι ἐκ δύο μέσων ἐστὶ δευτέρα ἢ δύο μέσα δυναμένη.

Τὸ γὰρ ΑΒ τοῦ ΓΔ ἤτοι μεῖζόν ἐστιν ἢ ἔλασσον. ἔστω, εἰ τύχον, πρότερον μεῖζον τὸ ΑΒ τοῦ ΓΔ· καὶ ἐκκείσθω ῥητὴ ἡ ΕΖ, καὶ τῷ μὲν ΑΒ ἴσον παρὰ τὴν ΕΖ παραβεβλήσθω τὸ ΕΗ πλάτος ποιοῦν τὴν ΕΘ, τῷ δὲ ΓΔ ἴσον τὸ ΘΙ πλάτος ποιοῦν τὴν ΘΚ. καὶ ἐπεὶ μέσον ἐστὶν ἑκάτερον τῶν ΑΒ, ΓΔ, μέσον ἄρα καὶ ἑκάτερον τῶν ΕΗ, ΘΙ. καὶ παρὰ ῥητὴν τὴν ΖΕ παράκειται πλάτος ποιοῦν τὰς ΕΘ, ΘΚ· ἑκατέρα ἄρα τῶν ΕΘ, ΘΚ ῥητή ἐστι καὶ ἀσύμμετρος τῇ ΕΖ μήκει. καὶ ἐπεὶ ἀσύμμετρόν ἐστι τὸ ΑΒ τῷ ΓΔ, καί ἐστιν ἴσον τὸ μὲν ΑΒ τῷ ΕΗ, τὸ δὲ ΓΔ τῷ ΘΙ, ἀσύμμετρον ἄρα ἐστὶ καὶ τὸ ΕΗ τῷ ΘΙ. ὡς δὲ τὸ ΕΗ πρὸς τὸ ΘΙ, οὕτως ἐστὶν ἡ ΕΘ πρὸς ΘΚ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΕΘ τῇ ΘΚ μήκει. αἱ ΕΘ, ΘΚ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἐκ δύο ἄρα ὀνομάτων ἐστὶν ἡ ΕΚ. ἤτοι δὲ ἡ ΕΘ τῆς ΘΚ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ ἢ τῷ ἀπὸ ἀσυμμέτρου. δυνάσθω πρότερον τῷ ἀπὸ συμμέτρου ἑαυτῇ μήκει· καὶ οὐδετέρα τῶν ΕΘ, ΘΚ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ τῇ ΕΖ μήκει· ἡ ΕΚ ἄρα ἐκ δύο ὀνομάτων ἐστὶ τρίτη. ῥητὴ δὲ ἡ ΕΖ· ἐὰν δὲ χωρίον περιέχηται ὑπὸ ῥητῆς καὶ τῆς ἐκ δύο ὀνομάτων τρίτης, ἡ τὸ χωρίον δυναμένη ἐκ δύο μέσων ἐστὶ δευτέρα· ἡ ἄρα τὸ ΕΙ, τουτέστι τὸ ΑΔ, δυναμένη ἐκ δύο μέσων ἐστὶ δευτέρα.

Proposition 72

When two medial (areas which are) incommensurable with one another are added together, the remaining two irrational (straight-lines) arise (as the square-roots of the total area)—either a second bimedial, or the square-root of (the sum of) two medial (areas).



For let the two medial (areas) AB and CD, (which are) incommensurable with one another, have been added together. I say that the square-root of area AD is either a second bimedial, or the square-root of (the sum of) two medial (areas).

For AB is either greater than or less than CD. By chance, let AB, first of all, be greater than CD. And let the rational (straight-line) EF be laid down. And let EG, equal to AB, have been applied to EF, producing EH as breadth, and HI, equal to CD, producing HK as breadth. And since AB and CD are each medial, EG and HI (are) thus also each medial. And they are applied to the rational straight-line FE, producing EH and HK (respectively) as breadth. Thus, EH and HK are each rational (straight-lines which are) incommensurable in length with EF [Prop. 10.22]. And since AB is incommensurable with CD, and AB is equal to EG, and CD to HI, EG is thus also incommensurable with HI. And as EG (is) to HI, so EH is to HK [Prop. 6.1]. EH is thus incommensurable in length with HK [Prop. 10.11]. Thus, EH and HK are rational (straight-lines which are) commensurable in square only. EK is thus a binomial (straight-line) [Prop. 10.36]. And the square on EH is greater than (the square on) HK either by the (square) ἀλλα δὴ ἡ ΕΘ τῆς ΘΚ μεῖζον δυνάσθω τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ μήκει· καὶ ἀσύμμετρός ἐστιν ἑκατέρα τῶν ΕΘ, ΘΚ τῇ ΕΖ μήκει· ἡ ἄρα ΕΚ ἐκ δύο ὀνομάτων ἐστὶν ἕκτη. ἐὰν δὲ χωρίον περιέχηται ὑπὸ ῥητῆς καὶ τῆς ἐκ δύο ὀνομάτων ἕκτης, ἡ τὸ χωρίον δυναμένη ἡ δύο μέσα δυναμένη ἐστίν· ὥστε καὶ ἡ τὸ ΑΔ χωρίον δυναμένη ἡ δύο μέσα δυναμένη ἐστίν.

[Ὁμοίως δὴ δείξομεν, ὅτι κἂν ἔλαττον ᾖ τὸ ΑΒ τοῦ ΓΔ, ἡ τὸ ΑΔ χωρίον δυναμένη ἢ ἐκ δύο μέσων δευτέρα ἐστὶν ἤτοι δύο μέσα δυναμένη].

Δύο ἄρα μέσων ἀσυμμέτρων ἀλλήλοις συντιθεμένων αἱ λοιπαὶ δύο ἄλογοι γίγνονται ἤτοι ἐκ δύο μέσων δευτέρα ἢ δύο μέσα δυναμένη.

Ἡ ἐκ δύο ὀνομάτων καὶ αἱ μετ᾽ αὐτὴν ἄλογοι οὔτε τῇ μέσῃ οὔτε ἀλλήλαις εἰσὶν αἱ αὐταί. τὸ μὲν γὰρ ἀπὸ μέσης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ῥητὴν καὶ ἀσύμμετρον τῇ παρ᾽ ἣν παράκειται μήκει. τὸ δὲ ἀπὸ τῆς ἐκ δύο ὀνομάτων παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ τὴν ἐκ δύο ὀνομάτων πρώτην. τὸ δὲ ἀπὸ τῆς ἐκ δύο μέσων πρώτης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ τὴν ἐκ δύο ὀνομάτων δευτέραν. τὸ δὲ ἀπὸ τῆς ἐκ δύο μέσων δευτέρας παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ τὴν ἐκ δύο ὀνομάτων τρίτην. τὸ δὲ ἀπὸ τῆς μείζονος παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ τὴν ἐκ δύο ὀνομάτων τετάρτην. τὸ δὲ ἀπὸ τῆς ῥητὸν καὶ μέσον δυναμένης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ τὴν ἐκ δύο ὀνομάτων πέμπτην. τὸ δὲ ἀπὸ τῆς δύο μέσα δυναμένης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ τὴν ἐκ δύο ὀνομάτων ἕκτην. τὰ δ᾽ εἰρημένα πλάτη διαφέρει τοῦ τε πρώτου καὶ ἀλλήλων, τοῦ μὲν πρώτου, ὅτι ῥητή ἐστιν, ἀλλήλων δέ, ὅτι τῇ τάξει οὐκ εἰσὶν αἱ αὐταί· ὥστε καὶ αὐταὶ αἱ ἄλογοι διαφέρουσιν ἀλλήλων.

on (some straight-line) commensurable (in length) with (EH), or by the (square) on (some straight-line) incommensurable (in length with EH). Let it, first of all, be greater by the square on (some straight-line) commensurable in length with (EH). And neither of EH or HK is commensurable in length with the (previously) laid down rational (straight-line) EF. Thus, EK is a third binomial (straight-line) [Def. 10.7]. And EF (is) rational. And if an area is contained by a rational (straight-line) and a third binomial (straight-line) then the square-root of the area is a second bimedial (straight-line) [Prop. 10.56]. Thus, the square-root of EI—that is to say, of AD—is a second bimedial. And so, let the square on EH be greater than (the square) on HK by the (square) on (some straight-line) incommensurable in length with (EH). And EH and HK are each incommensurable in length with EF. Thus, EK is a sixth binomial (straight-line) [Def. 10.10]. And if an area is contained by a rational (straight-line) and a sixth binomial (straight-line) then the square-root of the area is the square-root of (the sum of) two medial (areas) [Prop. 10.59]. Hence, the square-root of area AD is also the square-root of (the sum of) two medial (areas).

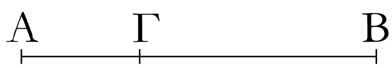
[So, similarly, we can show that, even if AB is less than CD, the square-root of area AD is either a second bimedial or the square-root of (the sum of) two medial (areas).]

Thus, when two medial (areas which are) incommensurable with one another are added together, the remaining two irrational (straight-lines) arise (as the square-roots of the total area)—either a second bimedial, or the square-root of (the sum of) two medial (areas).

A binomial (straight-line), and the (other) irrational (straight-lines) after it, are neither the same as a medial (straight-line) nor (the same) as one another. For the (square) on a medial (straight-line), applied to a rational (straight-line), produces as breadth a rational (straight-line which is) also incommensurable in length with (the straight-line) to which it is applied [Prop. 10.22]. And the (square) on a binomial (straight-line), applied to a rational (straight-line), produces as breadth a first binomial [Prop. 10.60]. And the (square) on a first bimedial (straight-line), applied to a rational (straight-line), produces as breadth a second binomial [Prop. 10.61]. And the (square) on a second bimedial (straight-line), applied to a rational (straight-line), produces as breadth a third binomial [Prop. 10.62]. And the (square) on a major (straight-line), applied to a rational (straight-line), produces as breadth a fourth binomial [Prop. 10.63]. And the (square) on the square-root of a rational plus a medial (area), applied to a rational (straight-line), produces as breadth a fifth binomial [Prop. 10.64]. And the (square) on the square-root of (the sum of) two medial (areas), applied to a rational (straight-line), produces as breadth a sixth binomial [Prop. 10.65]. And the aforementioned breadths differ from the first (breadth), and from one another—from the first, because it is rational—and from one another, because they are not the same in order. Hence, the (previously mentioned) irrational (straight-lines) themselves also differ from one another.

ογʹ.

Ἐὰν ἀπὸ ῥητῆς ῥητὴ ἀφαιρεθῇ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ, ἡ λοιπὴ ἄλογός ἐστιν· καλείσθω δὲ ἀποτομή.

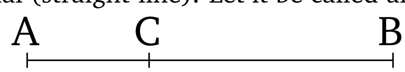


Ἀπὸ γὰρ ῥητῆς τῆς ΑΒ ῥητὴ ἀφῃρήσθω ἡ ΒΓ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ· λέγω, ὅτι ἡ λοιπὴ ἡ ΑΓ ἄλογός ἐστιν ἡ καλουμένη ἀποτομή.

Ἐπεὶ γὰρ ἀσύμμετρός ἐστιν ἡ ΑΒ τῇ ΒΓ μήκει, καί ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως τὸ ἀπὸ τῆς ΑΒ πρὸς τὸ ὑπὸ τῶν ΑΒ, ΒΓ, ἀσύμμετρον ἄρα ἐστὶ τὸ ἀπὸ τῆς ΑΒ τῷ ὑπὸ τῶν ΑΒ, ΒΓ. ἀλλὰ τῷ μὲν ἀπὸ τῆς ΑΒ σύμμετρά ἐστι τὰ ἀπὸ τῶν ΑΒ, ΒΓ τετράγωνα, τῷ δὲ ὑπὸ τῶν ΑΒ, ΒΓ σύμμετρόν ἐστι τὸ δὶς ὑπὸ τῶν ΑΒ, ΒΓ. καὶ ἐπειδήπερ τὰ ἀπὸ τῶν ΑΒ, ΒΓ ἴσα ἐστὶ τῷ δὶς ὑπὸ τῶν ΑΒ, ΒΓ μετὰ τοῦ ἀπὸ ΓΑ, καὶ λοιπῷ ἄρα τῷ ἀπὸ τῆς ΑΓ ἀσύμμετρά ἐστι τὰ ἀπὸ τῶν ΑΒ, ΒΓ. ῥητὰ δὲ τὰ ἀπὸ τῶν ΑΒ, ΒΓ· ἄλογος ἄρα ἐστὶν ἡ ΑΓ· καλείσθω δὲ ἀποτομή. ὅπερ ἔδει δεῖξαι.

Proposition 73

If a rational (straight-line), which is commensurable in square only with the whole, is subtracted from a(nother) rational (straight-line) then the remainder is an irrational (straight-line). Let it be called an apotome.



For let the rational (straight-line) BC, which commensurable in square only with the whole, have been subtracted from the rational (straight-line) AB. I say that the remainder AC is that irrational (straight-line) called an apotome.

For since AB is incommensurable in length with BC, and as AB is to BC, so the (square) on AB (is) to the (rectangle contained) by AB and BC [Prop. 10.21 lem.], the (square) on AB is thus incommensurable with the (rectangle contained) by AB and BC [Prop. 10.11]. But, the (sum of the) squares on AB and BC is commensurable with the (square) on AB [Prop. 10.15], and twice the (rectangle contained) by AB and BC is commensurable with the (rectangle contained) by AB and BC [Prop. 10.6]. And, inasmuch as the (sum of the squares) on AB and BC is equal to twice the (rectangle contained) by AB and BC plus the (square) on CA [Prop. 2.7], the (sum of the squares) on AB and BC is thus also incommensurable with the remaining (square) on AC [Props. 10.13, 10.16]. And the (sum of the squares) on AB and BC is rational. AC is thus an irrational (straight-line) [Def. 10.4]. And let it be called an apotome.[†] (Which is) the very thing it was required to show.

[†] See footnote to Prop. 10.36.

οδʹ.

Ἐὰν ἀπὸ μέσης μέση ἀφαιρεθῇ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ῥητὸν περιέχουσα, ἡ λοιπὴ ἄλογός ἐστιν· καλείσθω δὲ μέσης ἀποτομὴ πρώτη.

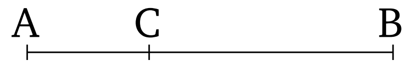
Proposition 74

If a medial (straight-line), which is commensurable in square only with the whole, and which contains a rational (area) with the whole, is subtracted from a(nother) medial (straight-line) then the remainder is an irrational

Ἀπὸ γὰρ μέσης τῆς ΑΒ μέση ἀφῃρήσθω ἡ ΒΓ δυνάμει μόνον σύμμετρος οὖσα τῇ ΑΒ, μετὰ δὲ τῆς ΑΒ ῥητὸν ποιοῦσα τὸ ὑπὸ τῶν ΑΒ, ΒΓ· λέγω, ὅτι ἡ λοιπὴ ἡ ΑΓ ἄλογός ἐστιν· καλείσθω δὲ μέσης ἀποτομὴ πρώτη.

Ἐπεὶ γὰρ αἱ ΑΒ, ΒΓ μέσαι εἰσίν, μέσα ἐστὶ καὶ τὰ ἀπὸ τῶν ΑΒ, ΒΓ. ῥητὸν δὲ τὸ δὶς ὑπὸ τῶν ΑΒ, ΒΓ· ἀσύμμετρα ἄρα τὰ ἀπὸ τῶν ΑΒ, ΒΓ τῷ δὶς ὑπὸ τῶν ΑΒ, ΒΓ· καὶ λοιπῷ ἄρα τῷ ἀπὸ τῆς ΑΓ ἀσύμμετρόν ἐστι τὸ δὶς ὑπὸ τῶν ΑΒ, ΒΓ, ἐπεὶ κἂν τὸ ὅλον ἑνὶ αὐτῶν ἀσύμμετρον ᾖ, καὶ τὰ ἐξ ἀρχῆς μεγέθη ἀσύμμετρα ἔσται. ῥητὸν δὲ τὸ δὶς ὑπὸ τῶν ΑΒ, ΒΓ· ἄλογον ἄρα τὸ ἀπὸ τῆς ΑΓ· ἄλογος ἄρα ἐστὶν ἡ ΑΓ· καλείσθω δὲ μέσης ἀποτομὴ πρώτη.

(straight-line). Let it be called a first apotome of a medial (straight-line).



For let the medial (straight-line) BC, which is commensurable in square only with AB, and which makes with AB the rational (rectangle contained) by AB and BC, have been subtracted from the medial (straight-line) AB [Prop. 10.27]. I say that the remainder AC is an irrational (straight-line). Let it be called the first apotome of a medial (straight-line).

For since AB and BC are medial (straight-lines), the (sum of the squares) on AB and BC is also medial. And twice the (rectangle contained) by AB and BC (is) rational. The (sum of the squares) on AB and BC (is) thus incommensurable with twice the (rectangle contained) by AB and BC. Thus, twice the (rectangle contained) by AB and BC is also incommensurable with the remaining (square) on AC [Prop. 2.7], since if the whole is incommensurable with one of the (constituent magnitudes) then the original magnitudes will also be incommensurable (with one another) [Prop. 10.16]. And twice the (rectangle contained) by AB and BC (is) rational. Thus, the (square) on AC is irrational. Thus, AC is an irrational (straight-line) [Def. 10.4]. Let it be called a first apotome of a medial (straight-line).†

† See footnote to Prop. 10.37.

οε´.

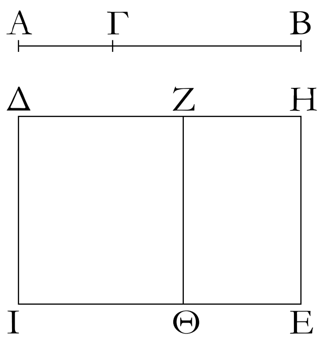
Ἐὰν ἀπὸ μέσης μέση ἀφαιρεθῇ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης μέσον περιέχουσα, ἡ λοιπὴ ἄλογός ἐστιν· καλείσθω δὲ μέσης ἀποτομὴ δευτέρα.

Ἀπὸ γὰρ μέσης τῆς ΑΒ μέση ἀφῃρήσθω ἡ ΓΒ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ τῇ ΑΒ, μετὰ δὲ τῆς ὅλης τῆς ΑΒ μέσον περιέχουσα τὸ ὑπὸ τῶν ΑΒ, ΒΓ· λέγω, ὅτι ἡ λοιπὴ ἡ ΑΓ ἄλογός ἐστιν· καλείσθω δὲ μέσης ἀποτομὴ δευτέρα.

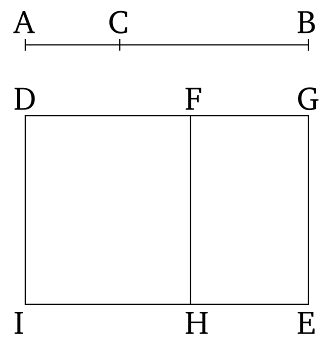
Proposition 75

If a medial (straight-line), which is commensurable in square only with the whole, and which contains a medial (area) with the whole, is subtracted from a(nother) medial (straight-line) then the remainder is an irrational (straight-line). Let it be called a second apotome of a medial (straight-line).

For let the medial (straight-line) CB, which is commensurable in square only with the whole, AB, and which contains with the whole, AB, the medial (rectangle contained) by AB and BC, have been subtracted from the medial (straight-line) AB [Prop. 10.28]. I say that the remainder AC is an irrational (straight-line). Let it be called a second apotome of a medial (straight-line).



Ἐκκείσθω γὰρ ῥητὴ ἡ ΔΙ, καὶ τοῖς μὲν ἀπὸ τῶν ΑΒ, ΒΓ ἴσον παρὰ τὴν ΔΙ παραβεβλήσθω τὸ ΔΕ πλάτος ποιοῦν τὴν ΔΗ, τῷ δὲ δὶς ὑπὸ τῶν ΑΒ, ΒΓ ἴσον παρὰ τὴν ΔΙ παραβεβλήσθω τὸ ΔΘ πλάτος ποιοῦν τὴν ΔΖ· λοιπὸν ἄρα τὸ ΖΕ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΓ. καὶ ἐπεὶ μέσα καὶ σύμμετρά ἐστι τὰ ἀπὸ τῶν ΑΒ, ΒΓ, μέσον ἄρα καὶ τὸ ΔΕ. καὶ παρὰ ῥητὴν τὴν ΔΙ παράκειται πλάτος ποιοῦν τὴν ΔΗ· ῥητὴ ἄρα ἐστὶν ἡ ΔΗ καὶ ἀσύμμετρος τῇ ΔΙ μήκει. πάλιν, ἐπεὶ μέσον ἐστὶ τὸ ὑπὸ τῶν ΑΒ, ΒΓ, καὶ τὸ δὶς ἄρα ὑπὸ τῶν ΑΒ, ΒΓ μέσον ἐστίν. καί ἐστιν ἴσον τῷ ΔΘ· καὶ τὸ ΔΘ ἄρα μέσον ἐστίν. καὶ παρὰ ῥητὴν τὴν ΔΙ παραβέβληται πλάτος ποιοῦν τὴν ΔΖ· ῥητὴ ἄρα ἐστὶν ἡ ΔΖ καὶ ἀσύμμετρος τῇ ΔΙ μήκει. καὶ ἐπεὶ αἱ ΑΒ, ΒΓ δυνάμει μόνον σύμμετροί εἰσιν, ἀσύμμετρος ἄρα ἐστὶν ἡ ΑΒ τῇ ΒΓ μήκει· ἀσύμμετρον ἄρα καὶ τὸ ἀπὸ τῆς ΑΒ τετράγωνον τῷ ὑπὸ τῶν ΑΒ, ΒΓ. ἀλλὰ τῷ μὲν ἀπὸ τῆς ΑΒ σύμμετρά ἐστι τὰ ἀπὸ τῶν ΑΒ, ΒΓ, τῷ δὲ ὑπὸ τῶν ΑΒ, ΒΓ σύμμετρόν ἐστι τὸ δὶς ὑπὸ τῶν ΑΒ, ΒΓ· ἀσύμμετρον ἄρα ἐστὶ τὸ δὶς ὑπὸ τῶν ΑΒ, ΒΓ τοῖς ἀπὸ τῶν ΑΒ, ΒΓ. ἴσον δὲ τοῖς μὲν ἀπὸ τῶν ΑΒ, ΒΓ τὸ ΔΕ, τῷ δὲ δὶς ὑπὸ τῶν ΑΒ, ΒΓ τὸ ΔΘ· ἀσύμμετρον ἄρα [ἐστὶ] τὸ ΔΕ τῷ ΔΘ. ὡς δὲ τὸ ΔΕ πρὸς τὸ ΔΘ, οὕτως ἡ ΗΔ πρὸς τὴν ΔΖ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΗΔ τῇ ΔΖ. καί εἰσιν ἀμφότεραι ῥηταί· αἱ ἄρα ΗΔ, ΔΖ ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἡ ΖΗ ἄρα ἀποτομή ἐστιν. ῥητὴ δὲ ἡ ΔΙ· τὸ δὲ ὑπὸ ῥητῆς καὶ ἀλόγου περιεχόμενον ἄλογόν ἐστιν, καὶ ἡ δυναμένη αὐτὸ ἄλογός ἐστιν. καὶ δύναται τὸ ΖΕ ἡ ΑΓ· ἡ ΑΓ ἄρα ἄλογός ἐστιν· καλείσθω δὲ μέσης ἀποτομὴ δευτέρα. ὅπερ ἔδει δεῖξαι.



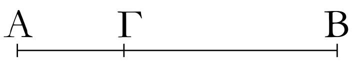
For let the rational (straight-line) DI be laid down. And let DE, equal to the (sum of the squares) on AB and BC, have been applied to DI, producing DG as breadth. And let DH, equal to twice the (rectangle contained) by AB and BC, have been applied to DI, producing DF as breadth. The remainder FE is thus equal to the (square) on AC [Prop. 2.7]. And since the (squares) on AB and BC are medial and commensurable (with one another), DE (is) thus also medial [Props. 10.15, 10.23 corr.]. And it is applied to the rational (straight-line) DI, producing DG as breadth. Thus, DG is rational, and incommensurable in length with DI [Prop. 10.22]. Again, since the (rectangle contained) by AB and BC is medial, twice the (rectangle contained) by AB and BC is thus also medial [Prop. 10.23 corr.]. And it is equal to DH. Thus, DH is also medial. And it has been applied to the rational (straight-line) DI, producing DF as breadth. DF is thus rational, and incommensurable in length with DI [Prop. 10.22]. And since AB and BC are commensurable in square only, AB is thus incommensurable in length with BC. Thus, the square on AB (is) also incommensurable with the (rectangle contained) by AB and BC [Props. 10.21 lem., 10.11]. But, the (sum of the squares) on AB and BC is commensurable with the (square) on AB [Prop. 10.15], and twice the (rectangle contained) by AB and BC is commensurable with the (rectangle contained) by AB and BC [Prop. 10.6]. Thus, twice the (rectangle contained) by AB and BC is incommensurable with the (sum of the squares) on AB and BC [Prop. 10.13]. And DE is equal to the (sum of the squares) on AB and BC, and DH to twice the (rectangle contained) by AB and BC. Thus, DE [is] incommensurable with DH. And as DE (is) to DH, so GD (is) to DF [Prop. 6.1]. Thus, GD is incommensurable with DF [Prop. 10.11]. And they are both rational (straight-lines). Thus, GD and DF are rational (straight-lines which are) commensurable in square only. Thus, FG is an apotome [Prop. 10.73]. And DI (is) rational. And the (area) contained by a rational and an irrational (straight-line) is irrational [Prop. 10.20], and its square-root is irrational.

And AC is the square-root of FE. Thus, AC is an irrational (straight-line) [Def. 10.4]. And let it be called the second apotome of a medial (straight-line).† (Which is) the very thing it was required to show.

† See footnote to Prop. 10.38.

οϛʹ.

Ἐὰν ἀπὸ εὐθείας εὐθεῖα ἀφαιρεθῇ δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ποιοῦσα τὰ μὲν ἀπ᾿ αὐτῶν ἅμα ῥητόν, τὸ δ᾿ ὑπ᾿ αὐτῶν μέσον, ἡ λοιπὴ ἄλογός ἐστιν· καλείσθω δὲ ἐλάσσων.



Ἀπὸ γὰρ εὐθείας τῆς ΑΒ εὐθεῖα ἀφῃρήσθω ἡ ΒΓ δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ ποιοῦσα τὰ προκείμενα. λέγω, ὅτι ἡ λοιπὴ ἡ ΑΓ ἄλογός ἐστιν ἡ καλουμένη ἐλάσσων.

Ἐπεὶ γὰρ τὸ μὲν συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΒ, ΒΓ τετραγώνων ῥητόν ἐστιν, τὸ δὲ δὶς ὑπὸ τῶν ΑΒ, ΒΓ μέσον, ἀσύμμετρα ἄρα ἐστὶ τὰ ἀπὸ τῶν ΑΒ, ΒΓ τῷ δὶς ὑπὸ τῶν ΑΒ, ΒΓ· καὶ ἀναστρέψαντι λοιπῷ τῷ ἀπὸ τῆς ΑΓ ἀσύμμετρά ἐστι τὰ ἀπὸ τῶν ΑΒ, ΒΓ. ῥητὰ δὲ τὰ ἀπὸ τῶν ΑΒ, ΒΓ· ἄλογον ἄρα τὸ ἀπὸ τῆς ΑΓ· ἄλογος ἄρα ἡ ΑΓ· καλείσθω δὲ ἐλάσσων. ὅπερ ἔδει δεῖξαι.

Proposition 76

If a straight-line, which is incommensurable in square with the whole, and with the whole makes the (squares) on them (added) together rational, and the (rectangle contained) by them medial, is subtracted from a(nother) straight-line then the remainder is an irrational (straight-line). Let it be called a minor (straight-line).

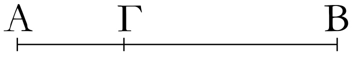
For let the straight-line BC, which is incommensurable in square with the whole, and fulfils the (other) prescribed (conditions), have been subtracted from the straight-line AB [Prop. 10.33]. I say that the remainder AC is that irrational (straight-line) called minor.

For since the sum of the squares on AB and BC is rational, and twice the (rectangle contained) by AB and BC (is) medial, the (sum of the squares) on AB and BC is thus incommensurable with twice the (rectangle contained) by AB and BC. And, via conversion, the (sum of the squares) on AB and BC is incommensurable with the remaining (square) on AC [Props. 2.7, 10.16]. And the (sum of the squares) on AB and BC (is) rational. The (square) on AC (is) thus irrational. Thus, AC (is) an irrational (straight-line) [Def. 10.4]. Let it be called a minor (straight-line).† (Which is) the very thing it was required to show.

† See footnote to Prop. 10.39.

οζʹ.

Ἐὰν ἀπὸ εὐθείας εὐθεῖα ἀφαιρεθῇ δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ποιοῦσα τὸ μὲν συγκείμενον ἐκ τῶν ἀπ᾿ αὐτῶν τετραγώνων μέσον, τὸ δὲ δὶς ὑπ᾿ αὐτῶν ῥητόν, ἡ λοιπὴ ἄλογός ἐστιν· καλείσθω δὲ ἡ μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσα.

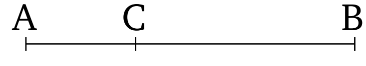


Ἀπὸ γὰρ εὐθείας τῆς ΑΒ εὐθεῖα ἀφῃρήσθω ἡ ΒΓ δυνάμει ἀσύμμετος οὖσα τῇ ΑΒ ποιοῦσα τὰ προκείμενα· λέγω, ὅτι ἡ λοιπὴ ἡ ΑΓ ἄλογός ἐστιν ἡ προειρημένη.

Ἐπεὶ γὰρ τὸ μὲν συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΒ, ΒΓ

Proposition 77

If a straight-line, which is incommensurable in square with the whole, and with the whole makes the sum of the squares on them medial, and twice the (rectangle contained) by them rational, is subtracted from a(nother) straight-line then the remainder is an irrational (straight-line). Let it be called that which makes with a rational (area) a medial whole.



For let the straight-line BC, which is incommensurable in square with AB, and fulfils the (other) prescribed (conditions), have been subtracted from the straight-line AB [Prop. 10.34]. I say that the remainder AC is the τετραγώνων μέσον ἐστίν, τὸ δὲ δὶς ὑπὸ τῶν ΑΒ, ΒΓ ῥητόν, ἀσύμμετρα ἄρα ἐστὶ τὰ ἀπὸ τῶν ΑΒ, ΒΓ τῷ δὶς ὑπὸ τῶν ΑΒ, ΒΓ· καὶ λοιπὸν ἄρα τὸ ἀπὸ τῆς ΑΓ ἀσύμμετρόν ἐστι τῷ δὶς ὑπὸ τῶν ΑΒ, ΒΓ. καί ἐστι τὸ δὶς ὑπὸ τῶν ΑΒ, ΒΓ ῥητόν· τὸ ἄρα ἀπὸ τῆς ΑΓ ἄλογόν ἐστιν· ἄλογος ἄρα ἐστὶν ἡ ΑΓ· καλείσθω δὲ ἡ μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσα. ὅπερ ἔδει δεῖξαι.

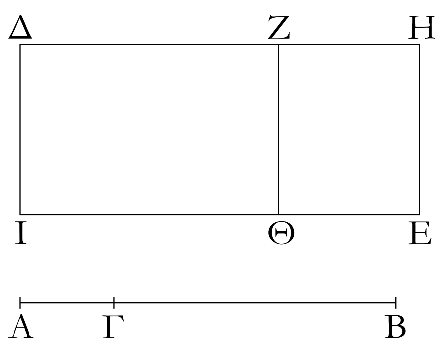
aforementioned irrational (straight-line).

For since the sum of the squares on AB and BC is medial, and twice the (rectangle contained) by AB and BC rational, the (sum of the squares) on AB and BC is thus incommensurable with twice the (rectangle contained) by AB and BC. Thus, the remaining (square) on AC is also incommensurable with twice the (rectangle contained) by AB and BC [Props. 2.7, 10.16]. And twice the (rectangle contained) by AB and BC is rational. Thus, the (square) on AC is irrational. Thus, AC is an irrational (straight-line) [Def. 10.4]. And let it be called that which makes with a rational (area) a medial whole.† (Which is) the very thing it was required to show.

† See footnote to Prop. 10.40.

οηʹ.

Ἐὰν ἀπὸ εὐθείας εὐθεῖα ἀφαιρεθῇ δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ποιοῦσα τό τε συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων μέσον τό τε δὶς ὑπ᾽ αὐτῶν μέσον καὶ ἔτι τὰ ἀπ᾽ αὐτῶν τετράγωνα ἀσύμμετρα τῷ δὶς ὑπ᾽ αὐτῶν, ἡ λοιπὴ ἄλογός ἐστιν· καλείσθω δὲ ἡ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσα.

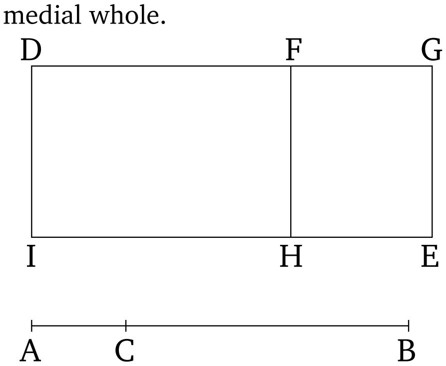


Ἀπὸ γὰρ εὐθείας τῆς ΑΒ εὐθεῖα ἀφῃρήσθω ἡ ΒΓ δυνάμει ἀσύμμετρος οὖσα τῇ ΑΒ ποιοῦσα τὰ προκείμενα· λέγω, ὅτι ἡ λοιπὴ ἡ ΑΓ ἄλογός ἐστιν ἡ καλουμένη ἡ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσα.

Ἐκκείσθω γὰρ ῥητὴ ἡ ΔΙ, καὶ τοῖς μὲν ἀπὸ τῶν ΑΒ, ΒΓ ἴσον παρὰ τὴν ΔΙ παραβεβλήσθω τὸ ΔΕ πλάτος ποιοῦν τὴν ΔΗ, τῷ δὲ δὶς ὑπὸ τῶν ΑΒ, ΒΓ ἴσον ἀφῃρήσθω τὸ ΔΘ [πλάτος ποιοῦν τὴν ΔΖ]. λοιπὸν ἄρα τὸ ΖΕ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΓ· ὥστε ἡ ΑΓ δύναται τὸ ΖΕ. καὶ ἐπεὶ τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΒ, ΒΓ τετραγώνων μέσον ἐστὶ καί ἐστιν ἴσον τῷ ΔΕ, μέσον ἄρα [ἐστὶ] τὸ ΔΕ. καὶ παρὰ ῥητὴν τὴν ΔΙ παράκειται πλάτος ποιοῦν τὴν ΔΗ· ῥητὴ ἄρα ἐστὶν ἡ ΔΗ καὶ ἀσύμμετρος τῇ ΔΙ μήκει. πάλιν, ἐπεὶ τὸ δὶς ὑπὸ τῶν ΑΒ, ΒΓ μέσον ἐστὶ καί ἐστιν ἴσον τῷ ΔΘ, τὸ ἄρα

Proposition 78

If a straight-line, which is incommensurable in square with the whole, and with the whole makes the sum of the squares on them medial, and twice the (rectangle contained) by them medial, and, moreover, the (sum of the) squares on them incommensurable with twice the (rectangle contained) by them, is subtracted from a(nother) straight-line then the remainder is an irrational (straight-line). Let it be called that which makes with a medial (area) a medial whole.



For let the straight-line BC, which is incommensurable in square AB, and fulfils the (other) prescribed (conditions), have been subtracted from the (straight-line) AB [Prop. 10.35]. I say that the remainder AC is the irrational (straight-line) called that which makes with a medial (area) a medial whole.

For let the rational (straight-line) DI be laid down. And let DE, equal to the (sum of the squares) on AB and BC, have been applied to DI, producing DG as breadth. And let DH, equal to twice the (rectangle contained) by AB and BC, have been subtracted (from DE) [producing DF as breadth]. Thus, the remainder FE is equal to the (square) on AC [Prop. 2.7]. Hence, AC is the square-root of FE. And since the sum of the squares on

ΔΘ μέσον ἐστίν. καὶ παρὰ ῥητὴν τὴν ΔΙ παράκειται πλάτος ποιοῦν τὴν ΔΖ· ῥητὴ ἄρα ἐστὶ καὶ ἡ ΔΖ καὶ ἀσύμμετρος τῇ ΔΙ μήκει. καὶ ἐπεὶ ἀσύμμετρά ἐστι τὰ ἀπὸ τῶν ΑΒ, ΒΓ τῷ δὶς ὑπὸ τῶν ΑΒ, ΒΓ, ἀσύμμετρον ἄρα καὶ τὸ ΔΕ τῷ ΔΘ. ὡς δὲ τὸ ΔΕ πρὸς τὸ ΔΘ, οὕτως ἐστὶ καὶ ἡ ΔΗ πρὸς τὴν ΔΖ· ἀσύμμετρος ἄρα ἡ ΔΗ τῇ ΔΖ. καί εἰσιν ἀμφότεραι ῥηταί· αἱ ΗΔ, ΔΖ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι. ἀποτομὴ ἄρα ἐστίν ἡ ΖΗ· ῥητὴ δὲ ἡ ΖΘ. τὸ δὲ ὑπὸ ῥητῆς καὶ ἀποτομῆς περιεχόμενον [ὀρθογώνιον] ἄλογόν ἐστιν, καὶ ἡ δυναμένη αὐτὸ ἄλογός ἐστιν· καὶ δύναται τὸ ΖΕ ἡ ΑΓ· ἡ ΑΓ ἄρα ἄλογός ἐστιν· καλείσθω δὲ ἡ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσα. ὅπερ ἔδει δεῖξαι.

AB and BC is medial, and is equal to DE, DE [is] thus medial. And it is applied to the rational (straight-line) DI, producing DG as breadth. Thus, DG is rational, and incommensurable in length with DI [Prop 10.22]. Again, since twice the (rectangle contained) by AB and BC is medial, and is equal to DH, DH is thus medial. And it is applied to the rational (straight-line) DI, producing DF as breadth. Thus, DF is also rational, and incommensurable in length with DI [Prop. 10.22]. And since the (sum of the squares) on AB and BC is incommensurable with twice the (rectangle contained) by AB and BC, DE (is) also incommensurable with DH. And as DE (is) to DH, so DG also is to DF [Prop. 6.1]. Thus, DG (is) incommensurable (in length) with DF [Prop. 10.11]. And they are both rational. Thus, GD and DF are rational (straight-lines which are) commensurable in square only. Thus, FG is an apotome [Prop. 10.73]. And FH (is) rational. And the [rectangle] contained by a rational (straight-line) and an apotome is irrational [Prop. 10.20], and its square-root is irrational. And AC is the square-root of FE. Thus, AC is irrational. Let it be called that which makes with a medial (area) a medial whole.[†] (Which is) the very thing it was required to show.

[†] See footnote to Prop. 10.41.

οθ΄.

Τῇ ἀποτομῇ μία [μόνον] προσαρμόζει εὐθεῖα ῥητὴ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ.

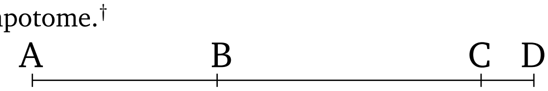
Ἔστω ἀποτομὴ ἡ ΑΒ, προσαρμόζουσα δὲ αὐτῇ ἡ ΒΓ· αἱ ΑΓ, ΓΒ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι· λέγω, ὅτι τῇ ΑΒ ἑτέρα οὐ προσαρμόζει ῥητὴ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ.

Εἰ γὰρ δυνατόν, προσαρμοζέτω ἡ ΒΔ· καὶ αἱ ΑΔ, ΔΒ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι. καὶ ἐπεί, ᾧ ὑπερέχει τὰ ἀπὸ τῶν ΑΔ, ΔΒ τοῦ δὶς ὑπὸ τῶν ΑΔ, ΔΒ, τούτῳ ὑπερέχει καὶ τὰ ἀπὸ τῶν ΑΓ, ΓΒ τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ· τῷ γὰρ αὐτῷ τῷ ἀπὸ τῆς ΑΒ ἀμφότερα ὑπερέχει· ἐναλλὰξ ἄρα, ᾧ ὑπερέχει τὰ ἀπὸ τῶν ΑΔ, ΔΒ τῶν ἀπὸ τῶν ΑΓ, ΓΒ, τούτῳ ὑπερέχει [καὶ] τὸ δὶς ὑπὸ τῶν ΑΔ, ΔΒ τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ. τὰ δὲ ἀπὸ τῶν ΑΔ, ΔΒ τῶν ἀπὸ τῶν ΑΓ, ΓΒ ὑπερέχει ῥητῷ· ῥητὰ γὰρ ἀμφότερα. καὶ τὸ δὶς ἄρα ὑπὸ τῶν ΑΔ, ΔΒ τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ ὑπερέχει ῥητῷ· ὅπερ ἐστὶν ἀδύνατον· μέσα γὰρ ἀμφότερα, μέσον δὲ μέσου οὐχ ὑπερέχει ῥητῷ. τῇ ἄρα ΑΒ ἑτέρα οὐ προσαρμόζει ῥητὴ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ.

Μία ἄρα μόνη τῇ ἀποτομῇ προσαρμόζει ῥητὴ δυνάμει

Proposition 79

[Only] one rational straight-line, which is commensurable in square only with the whole, can be attached to an apotome.[†]



Let AB be an apotome, with BC (so) attached to it. AC and CB are thus rational (straight-lines which are) commensurable in square only [Prop. 10.73]. I say that another rational (straight-line), which is commensurable in square only with the whole, cannot be attached to AB.

For, if possible, let BD be (so) attached (to AB). Thus, AD and DB are also rational (straight-lines which are) commensurable in square only [Prop. 10.73]. And since by whatever (area) the (sum of the squares) on AD and DB exceeds twice the (rectangle contained) by AD and DB, the (sum of the squares) on AC and CB also exceeds twice the (rectangle contained) by AC and CB by this (same area). For both exceed by the same (area)—(namely), the (square) on AB [Prop. 2.7]. Thus, alternately, by whatever (area) the (sum of the squares) on AD and DB exceeds the (sum of the squares) on AC and CB, twice the (rectangle contained) by AD and DB [also] exceeds twice the (rectangle contained) by AC and μόνον σύμμετρος οὖσα τῇ ὅλῃ· ὅπερ ἔδει δεῖξαι.

CB by this (same area). And the (sum of the squares) on AD and DB exceeds the (sum of the squares) on AC and CB by a rational (area). For both (are) rational (areas). Thus, twice the (rectangle contained) by AD and DB also exceeds twice the (rectangle contained) by AC and CB by a rational (area). The very thing is impossible. For both are medial (areas) [Prop. 10.21], and a medial (area) cannot exceed a(nother) medial (area) by a rational (area) [Prop. 10.26]. Thus, another rational (straight-line), which is commensurable in square only with the whole, cannot be attached to AB.

Thus, only one rational (straight-line), which is commensurable in square only with the whole, can be attached to an apotome. (Which is) the very thing it was required to show.

† This proposition is equivalent to Prop. 10.42, with minus signs instead of plus signs.

π′.

Τῇ μέσης ἀποτομῇ πρώτῃ μία μόνον προσαρμόζει εὐθεῖα μέση δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ῥητὸν περιέχουσα.

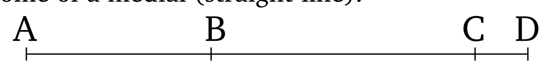
Ἔστω γὰρ μέσης ἀποτομὴ πρώτη ἡ ΑΒ, καὶ τῇ ΑΒ προσαρμοζέτω ἡ ΒΓ· αἱ ΑΓ, ΓΒ ἄρα μέσαι εἰσὶ δυνάμει μόνον σύμμετροι ῥητὸν περιέχουσαι τὸ ὑπὸ τῶν ΑΓ, ΓΒ· λέγω, ὅτι τῇ ΑΒ ἑτέρα οὐ προσαρμόζει μέση δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ῥητὸν περιέχουσα.

Εἰ γὰρ δυνατόν, προσαρμοζέτω καὶ ἡ ΔΒ· αἱ ἄρα ΑΔ, ΔΒ μέσαι εἰσὶ δυνάμει μόνον σύμμετροι ῥητὸν περιέχουσαι τὸ ὑπὸ τῶν ΑΔ, ΔΒ. καὶ ἐπεί, ᾧ ὑπερέχει τὰ ἀπὸ τῶν ΑΔ, ΔΒ τοῦ δὶς ὑπὸ τῶν ΑΔ, ΔΒ, τούτῳ ὑπερέχει καὶ τὰ ἀπὸ τῶν ΑΓ, ΓΒ τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ· τῷ γὰρ αὐτῷ [πάλιν] ὑπερέχουσι τῷ ἀπὸ τῆς ΑΒ· ἐναλλὰξ ἄρα, ᾧ ὑπερέχει τὰ ἀπὸ τῶν ΑΔ, ΔΒ τῶν ἀπὸ τῶν ΑΓ, ΓΒ, τούτῳ ὑπερέχει καὶ τὸ δὶς ὑπὸ τῶν ΑΔ, ΔΒ τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ. τὸ δὲ δὶς ὑπὸ τῶν ΑΔ, ΔΒ τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ ὑπερέχει ῥητῷ· ῥητὰ γὰρ ἀμφότερα. καὶ τὰ ἀπὸ τῶν ΑΔ, ΔΒ ἄρα τῶν ἀπὸ τῶν ΑΓ, ΓΒ [τετραγώνων] ὑπερέχει ῥητῷ· ὅπερ ἐστὶν ἀδύνατον· μέσα γάρ ἐστιν ἀμφότερα, μέσον δὲ μέσου οὐχ ὑπερέχει ῥητῷ.

Τῇ ἄρα μέσης ἀποτομῇ πρώτῃ μία μόνον προσαρμόζει εὐθεῖα μέση δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ῥητὸν περιέχουσα· ὅπερ ἔδει δεῖξαι.

Proposition 80

Only one medial straight-line, which is commensurable in square only with the whole, and contains a rational (area) with the whole, can be attached to a first apotome of a medial (straight-line).†



For let AB be a first apotome of a medial (straight-line), and let BC be (so) attached to AB. Thus, AC and CB are medial (straight-lines which are) commensurable in square only, containing a rational (area)—(namely, that contained) by AC and CB [Prop. 10.74]. I say that a(nother) medial (straight-line), which is commensurable in square only with the whole, and contains a rational (area) with the whole, cannot be attached to AB.

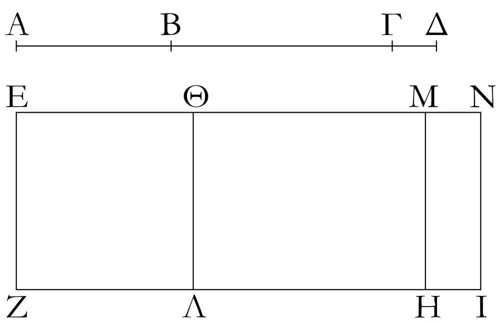
For, if possible, let DB also be (so) attached to AB. Thus, AD and DB are medial (straight-lines which are) commensurable in square only, containing a rational (area)—(namely, that) contained by AD and DB [Prop. 10.74]. And since by whatever (area) the (sum of the squares) on AD and DB exceeds twice the (rectangle contained) by AD and DB, the (sum of the squares) on AC and CB also exceeds twice the (rectangle contained) by AC and CB by this (same area). For [again] both exceed by the same (area)—(namely), the (square) on AB [Prop. 2.7]. Thus, alternately, by whatever (area) the (sum of the squares) on AD and DB exceeds the (sum of the squares) on AC and CB, twice the (rectangle contained) by AD and DB also exceeds twice the (rectangle contained) by AC and CB by this (same area). And twice the (rectangle contained) by AD and DB exceeds twice the (rectangle contained) by AC and CB by a rational (area). For both (are) rational (areas). Thus, the (sum of the squares) on AD and DB also exceeds the (sum of the) [squares] on AC and CB by a rational (area). The very thing is impossible. For both are medial (areas) [Props. 10.15, 10.23 corr.], and a medial (area) cannot exceed a(nother) medial (area) by a rational (area) [Prop. 10.26].

Thus, only one medial (straight-line), which is commensurable in square only with the whole, and contains a rational (area) with the whole, can be attached to a first apotome of a medial (straight-line). (Which is) the very thing it was required to show.

† This proposition is equivalent to Prop. 10.43, with minus signs instead of plus signs.

πα΄.

Τῇ μέσης ἀποτομῇ δευτέρᾳ μία μόνον προσαρμόζει εὐθεῖα μέση δυνάμει μόνον σύμμετρος τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης μέσον περιέχουσα.

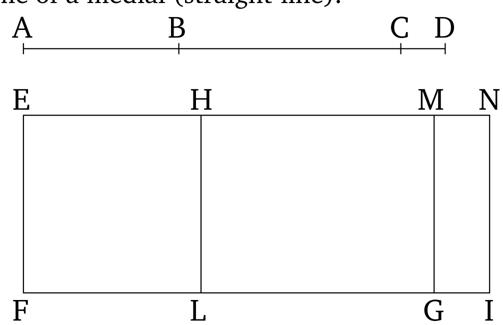


Ἔστω μέσης ἀποτομὴ δευτέρα ἡ ΑΒ καὶ τῇ ΑΒ προσαρμόζουσα ἡ ΒΓ· αἱ ἄρα ΑΓ, ΓΒ μέσαι εἰσὶ δυνάμει μόνον σύμμετροι μέσον περιέχουσαι τὸ ὑπὸ τῶν ΑΓ, ΓΒ· λέγω, ὅτι τῇ ΑΒ ἑτέρα οὐ προσαρμόσει εὐθεῖα μέση δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης μέσον περιέχουσα.

Εἰ γὰρ δυνατόν, προσαρμοζέτω ἡ ΒΔ· καὶ αἱ ΑΔ, ΔΒ ἄρα μέσαι εἰσὶ δυνάμει μόνον σύμμετροι μέσον περιέχουσαι τὸ ὑπὸ τῶν ΑΔ, ΔΒ. καὶ ἐκκείσθω ῥητὴ ἡ ΕΖ, καὶ τοῖς μὲν ἀπὸ τῶν ΑΓ, ΓΒ ἴσον παρὰ τὴν ΕΖ παραβεβλήσθω τὸ ΕΗ πλάτος ποιοῦν τὴν ΕΜ· τῷ δὲ δὶς ὑπὸ τῶν ΑΓ, ΓΒ ἴσον ἀφῃρήσθω τὸ ΘΗ πλάτος ποιοῦν τὴν ΘΜ· λοιπὸν ἄρα τὸ ΕΛ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΒ· ὥστε ἡ ΑΒ δύναται τὸ ΕΛ. πάλιν δὴ τοῖς ἀπὸ τῶν ΑΔ, ΔΒ ἴσον παρὰ τὴν ΕΖ παραβεβλήσθω τὸ ΕΙ πλάτος ποιοῦν τὴν ΕΝ· ἔστι δὲ καὶ τὸ ΕΛ ἴσον τῷ ἀπὸ τῆς ΑΒ τετραγώνῳ· λοιπὸν ἄρα τὸ ΘΙ ἴσον ἐστὶ τῷ δὶς ὑπὸ τῶν ΑΔ, ΔΒ. καὶ ἐπεὶ μέσαι εἰσὶν αἱ ΑΓ, ΓΒ, μέσα ἄρα ἐστὶ καὶ τὰ ἀπὸ τῶν ΑΓ, ΓΒ. καί ἐστιν ἴσα τῷ ΕΗ· μέσον ἄρα καὶ τὸ ΕΗ. καὶ παρὰ ῥητὴν τὴν ΕΖ παράκειται πλάτος ποιοῦν

Proposition 81

Only one medial straight-line, which is commensurable in square only with the whole, and contains a medial (area) with the whole, can be attached to a second apotome of a medial (straight-line).†



Let AB be a second apotome of a medial (straight-line), with BC (so) attached to AB. Thus, AC and CB are medial (straight-lines which are) commensurable in square only, containing a medial (area)—(namely, that contained) by AC and CB [Prop. 10.75]. I say that a(nother) medial straight-line, which is commensurable in square only with the whole, and contains a medial (area) with the whole, cannot be attached to AB.

For, if possible, let BD be (so) attached. Thus, AD and DB are also medial (straight-lines which are) commensurable in square only, containing a medial (area)—(namely, that contained) by AD and DB [Prop. 10.75]. And let the rational (straight-line) EF be laid down. And let EG, equal to the (sum of the squares) on AC and CB, have been applied to EF, producing EM as breadth. And let HG, equal to twice the (rectangle contained) by AC and CB, have been subtracted (from EG), producing HM as breadth. The remainder EL is thus equal to the (square) on AB [Prop. 2.7]. Hence, AB is the τὴν ΕΜ· ῥητὴ ἄρα ἐστὶν ἡ ΕΜ καὶ ἀσύμμετρος τῇ ΕΖ μήκει. πάλιν, ἐπεὶ μέσον ἐστὶ τὸ ὑπὸ τῶν ΑΓ, ΓΒ, καὶ τὸ δὶς ὑπὸ τῶν ΑΓ, ΓΒ μέσον ἐστίν. καί ἐστιν ἴσον τῷ ΘΗ· καὶ τὸ ΘΗ ἄρα μέσον ἐστίν. καὶ παρὰ ῥητὴν τὴν ΕΖ παράκειται πλάτος ποιοῦν τὴν ΘΜ· ῥητὴ ἄρα ἐστὶ καὶ ἡ ΘΜ καὶ ἀσύμμετρος τῇ ΕΖ μήκει. καὶ ἐπεὶ αἱ ΑΓ, ΓΒ δυνάμει μόνον σύμμετροί εἰσιν, ἀσύμμετρος ἄρα ἐστὶν ἡ ΑΓ τῇ ΓΒ μήκει. ὡς δὲ ἡ ΑΓ πρὸς τὴν ΓΒ, οὕτως ἐστὶ τὸ ἀπὸ τῆς ΑΓ πρὸς τὸ ὑπὸ τῶν ΑΓ, ΓΒ· ἀσύμμετρον ἄρα ἐστὶ τὸ ἀπὸ τῆς ΑΓ τῷ ὑπὸ τῶν ΑΓ, ΓΒ. ἀλλὰ τῷ μὲν ἀπὸ τῆς ΑΓ σύμμετρά ἐστι τὰ ἀπὸ τῶν ΑΓ, ΓΒ, τῷ δὲ ὑπὸ τῶν ΑΓ, ΓΒ σύμμετρόν ἐστι τὸ δὶς ὑπὸ τῶν ΑΓ, ΓΒ· ἀσύμμετρα ἄρα ἐστὶ τὰ ἀπὸ τῶν ΑΓ, ΓΒ τῷ δὶς ὑπὸ τῶν ΑΓ, ΓΒ. καί ἐστι τοῖς μὲν ἀπὸ τῶν ΑΓ, ΓΒ ἴσον τὸ ΕΗ, τῷ δὲ δὶς ὑπὸ τῶν ΑΓ, ΓΒ ἴσον τὸ ΗΘ· ἀσύμμετρον ἄρα ἐστὶ τὸ ΕΗ τῷ ΘΗ. ὡς δὲ τὸ ΕΗ πρὸς τὸ ΘΗ, οὕτως ἐστὶν ἡ ΕΜ πρὸς τὴν ΘΜ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΕΜ τῇ ΜΘ μήκει. καί εἰσιν ἀμφότεραι ῥηταί· αἱ ΕΜ, ΜΘ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΕΘ, προσαρμόζουσα δὲ αὐτῇ ἡ ΘΜ. ὁμοίως δὴ δείξομεν, ὅτι καὶ ἡ ΘΝ αὐτῇ προσαρμόζει· τῇ ἄρα ἀποτομῇ ἄλλη καὶ ἄλλη προσαρμόζει εὐθεῖα δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ· ὅπερ ἐστὶν ἀδύνατον.

Τῇ ἄρα μέσης ἀποτομῇ δευτέρᾳ μία μόνον προσαρμόζει εὐθεῖα μέση δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης μέσον περιέχουσα· ὅπερ ἔδει δεῖξαι.

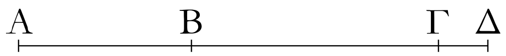
square-root of EL. So, again, let EI, equal to the (sum of the squares) on AD and DB have been applied to EF, producing EN as breadth. And EL is also equal to the square on AB. Thus, the remainder HI is equal to twice the (rectangle contained) by AD and DB [Prop. 2.7]. And since AC and CB are (both) medial (straight-lines), the (sum of the squares) on AC and CB is also medial. And it is equal to EG. Thus, EG is also medial [Props. 10.15, 10.23 corr.]. And it is applied to the rational (straight-line) EF, producing EM as breadth. Thus, EM is rational, and incommensurable in length with EF [Prop. 10.22]. Again, since the (rectangle contained) by AC and CB is medial, twice the (rectangle contained) by AC and CB is also medial [Prop. 10.23 corr.]. And it is equal to HG. Thus, HG is also medial. And it is applied to the rational (straight-line) EF, producing HM as breadth. Thus, HM is also rational, and incommensurable in length with EF [Prop. 10.22]. And since AC and CB are commensurable in square only, AC is thus incommensurable in length with CB. And as AC (is) to CB, so the (square) on AC is to the (rectangle contained) by AC and CB [Prop. 10.21 corr.]. Thus, the (square) on AC is incommensurable with the (rectangle contained) by AC and CB [Prop. 10.11]. But, the (sum of the squares) on AC and CB is commensurable with the (square) on AC, and twice the (rectangle contained) by AC and CB is commensurable with the (rectangle contained) by AC and CB [Prop. 10.6]. Thus, the (sum of the squares) on AC and CB is incommensurable with twice the (rectangle contained) by AC and CB [Prop. 10.13]. And EG is equal to the (sum of the squares) on AC and CB. And GH is equal to twice the (rectangle contained) by AC and CB. Thus, EG is incommensurable with HG. And as EG (is) to HG, so EM is to HM [Prop. 6.1]. Thus, EM is incommensurable in length with MH [Prop. 10.11]. And they are both rational (straight-lines). Thus, EM and MH are rational (straight-lines which are) commensurable in square only. Thus, EH is an apotome [Prop. 10.73], and HM (is) attached to it. So, similarly, we can show that HN (is) also (commensurable in square only with EN and is) attached to (EH). Thus, different straight-lines, which are commensurable in square only with the whole, are attached to an apotome. The very thing is impossible [Prop. 10.79].

Thus, only one medial straight-line, which is commensurable in square only with the whole, and contains a medial (area) with the whole, can be attached to a second apotome of a medial (straight-line). (Which is) the very thing it was required to show.

† This proposition is equivalent to Prop. 10.44, with minus signs instead of plus signs.

πβ´.

Τῇ ἐλάσσονι μία μόνον προσαρμόζει εὐθεῖα δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ ποιοῦσα μετὰ τῆς ὅλης τὸ μὲν ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων ῥητόν, τὸ δὲ δὶς ὑπ᾽ αὐτῶν μέσον.



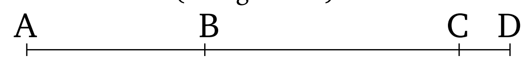
Ἔστω ἡ ἐλάσσων ἡ ΑΒ, καὶ τῇ ΑΒ προσαρμόζουσα ἔστω ἡ ΒΓ· αἱ ἄρα ΑΓ, ΓΒ δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὸ μὲν συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων ῥητόν, τὸ δὲ δὶς ὑπ᾽ αὐτῶν μέσον· λέγω, ὅτι τῇ ΑΒ ἑτέρα εὐθεῖα οὐ προσαρμόσει τὰ αὐτὰ ποιοῦσα.

Εἰ γὰρ δυνατόν, προσαρμοζέτω ἡ ΒΔ· καὶ αἱ ΑΔ, ΔΒ ἄρα δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὰ προειρημένα. καὶ ἐπεί, ᾧ ὑπερέχει τὰ ἀπὸ τῶν ΑΔ, ΔΒ τῶν ἀπὸ τῶν ΑΓ, ΓΒ, τούτῳ ὑπερέχει καὶ τὸ δὶς ὑπὸ τῶν ΑΔ, ΔΒ τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ, τὰ δὲ ἀπὸ τῶν ΑΔ, ΔΒ τετράγωνα τῶν ἀπὸ τῶν ΑΓ, ΓΒ τετραγώνων ὑπερέχει ῥητῷ· ῥητὰ γάρ ἐστιν ἀμφότερα· καὶ τὸ δὶς ὑπὸ τῶν ΑΔ, ΔΒ ἄρα τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ ὑπερέχει ῥητῷ· ὅπερ ἐστὶν ἀδύνατον· μέσα γάρ ἐστιν ἀμφότερα.

Τῇ ἄρα ἐλάσσονι μία μόνον προσαρμόζει εὐθεῖα δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ καὶ ποιοῦσα τὰ μὲν ἀπ᾽ αὐτῶν τετράγωνα ἅμα ῥητόν, τὸ δὲ δὶς ὑπ᾽ αὐτῶν μέσον· ὅπερ ἔδει δεῖξαι.

Proposition 82

Only one straight-line, which is incommensurable in square with the whole, and (together) with the whole makes the (sum of the) squares on them rational, and twice the (rectangle contained) by them medial, can be attached to a minor (straight-line).



Let AB be a minor (straight-line), and let BC be (so) attached to AB. Thus, AC and CB are (straight-lines which are) incommensurable in square, making the sum of the squares on them rational, and twice the (rectangle contained) by them medial [Prop. 10.76]. I say that another another straight-line fulfilling the same (conditions) cannot be attached to AB.

For, if possible, let BD be (so) attached (to AB). Thus, AD and DB are also (straight-lines which are) incommensurable in square, fulfilling the (other) aforementioned (conditions) [Prop. 10.76]. And since by whatever (area) the (sum of the squares) on AD and DB exceeds the (sum of the squares) on AC and CB, twice the (rectangle contained) by AD and DB also exceeds twice the (rectangle contained) by AC and CB by this (same area) [Prop. 2.7]. And the (sum of the) squares on AD and DB exceeds the (sum of the) squares on AC and CB by a rational (area). For both are rational (areas). Thus, twice the (rectangle contained) by AD and DB also exceeds twice the (rectangle contained) by AC and CB by a rational (area). The very thing is impossible. For both are medial (areas) [Prop. 10.26].

Thus, only one straight-line, which is incommensurable in square with the whole, and (with the whole) makes the squares on them (added) together rational, and twice the (rectangle contained) by them medial, can be attached to a minor (straight-line). (Which is) the very thing it was required to show.

† This proposition is equivalent to Prop. 10.45, with minus signs instead of plus signs.

πγ´.

Τῇ μετὰ ῥητοῦ μέσον τὸ ὅλον ποιούσῃ μία μόνον προσαρμόζει εὐθεῖα δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ποιοῦσα τὸ μὲν συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων μέσον, τὸ δὲ δὶς ὑπ᾽ αὐτῶν ῥητόν.

Proposition 83

Only one straight-line, which is incommensurable in square with the whole, and (together) with the whole makes the sum of the squares on them medial, and twice the (rectangle contained) by them rational, can be attached to that (straight-line) which with a rational (area) makes a medial whole.†

Ἔστω ἡ μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσα ἡ ΑΒ, καὶ τῇ ΑΒ προσαρμοζέτω ἡ ΒΓ· αἱ ἄρα ΑΓ, ΓΒ δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὰ προκείμενα· λέγω, ὅτι τῇ ΑΒ ἑτέρα οὐ προσαρμόσει τὰ αὐτὰ ποιοῦσα.

Εἰ γὰρ δυνατόν, προσαρμοζέτω ἡ ΒΔ· καὶ αἱ ΑΔ, ΔΒ ἄρα εὐθεῖαι δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὰ προκείμενα. ἐπεὶ οὖν, ᾧ ὑπερέχει τὰ ἀπὸ τῶν ΑΔ, ΔΒ τῶν ἀπὸ τῶν ΑΓ, ΓΒ, τούτῳ ὑπερέχει καὶ τὸ δὶς ὑπὸ τῶν ΑΔ, ΔΒ τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ ἀκολούθως τοῖς πρὸ αὐτοῦ, τὸ δὲ δὶς ὑπὸ τῶν ΑΔ, ΔΒ τοῦ δὶς ὑπὸ τῶν ΑΓ, ΓΒ ὑπερέχει ῥητῷ· ῥητὰ γάρ ἐστιν ἀμφότερα· καὶ τὰ ἀπὸ τῶν ΑΔ, ΔΒ ἄρα τῶν ἀπὸ τῶν ΑΓ, ΓΒ ὑπερέχει ῥητῷ· ὅπερ ἐστὶν ἀδύνατον· μέσα γάρ ἐστιν ἀμφότερα.

Οὐκ ἄρα τῇ ΑΒ ἑτέρα προσαρμόσει εὐθεῖα δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ποιοῦσα τὰ προειρημένα· μία ἄρα μόνον προσαρμόσει· ὅπερ ἔδει δεῖξαι.

Let AB be a (straight-line) which with a rational (area) makes a medial whole, and let BC be (so) attached to AB. Thus, AC and CB are (straight-lines which are) incommensurable in square, fulfilling the (other) proscribed (conditions) [Prop. 10.77]. I say that another (straight-line) fulfilling the same (conditions) cannot be attached to AB.

For, if possible, let BD be (so) attached (to AB). Thus, AD and DB are also straight-lines (which are) incommensurable in square, fulfilling the (other) prescribed (conditions) [Prop. 10.77]. Therefore, analogously to the (propositions) before this, since by whatever (area) the (sum of the squares) on AD and DB exceeds the (sum of the squares) on AC and CB, twice the (rectangle contained) by AD and DB also exceeds twice the (rectangle contained) by AC and CB by this (same area). And twice the (rectangle contained) by AD and DB exceeds twice the (rectangle contained) by AC and CB by a rational (area). For they are (both) rational (areas). Thus, the (sum of the squares) on AD and DB also exceeds the (sum of the squares) on AC and CB by a rational (area). The very thing is impossible. For both are medial (areas) [Prop. 10.26].

Thus, another straight-line cannot be attached to AB, which is incommensurable in square with the whole, and fulfills the (other) aforementioned (conditions) with the whole. Thus, only one (such straight-line) can be (so) attached. (Which is) the very thing it was required to show.

† This proposition is equivalent to Prop. 10.46, with minus signs instead of plus signs.

πδ΄.

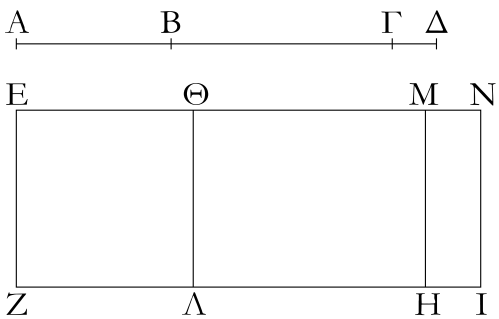
Τῇ μετὰ μέσου μέσον τὸ ὅλον ποιούσῃ μία μόνη προσαρμόζει εὐθεῖα δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ποιοῦσα τό τε συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων μέσον τό τε δὶς ὑπ᾽ αὐτῶν μέσον καὶ ἔτι ἀσύμμετρον τῷ συγκειμένῳ ἐκ τῶν ἀπ᾽ αὐτῶν.

Ἔστω ἡ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσα ἡ ΑΒ, προσαρμόζουσα δὲ αὐτῇ ἡ ΒΓ· αἱ ἄρα ΑΓ, ΓΒ δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὰ προειρημένα. λέγω, ὅτι τῇ ΑΒ ἑτέρα οὐ προσαρμόσει ποιοῦσα προειρημένα.

Proposition 84

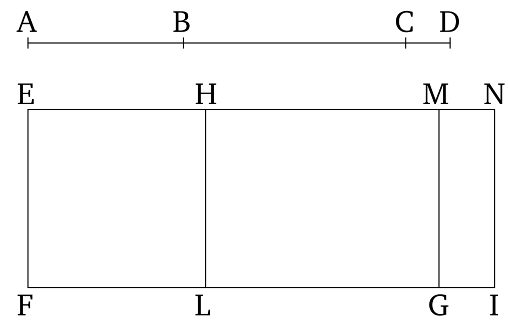
Only one straight-line, which is incommensurable in square with the whole, and (together) with the whole makes the sum of the squares on them medial, and twice the (rectangle contained) by them medial, and, moreover, incommensurable with the sum of the (squares) on them, can be attached to that (straight-line) which with a medial (area) makes a medial whole.†

Let AB be a (straight-line) which with a medial (area) makes a medial whole, BC being (so) attached to it. Thus, AC and CB are incommensurable in square, fulfilling the (other) aforementioned (conditions) [Prop. 10.78]. I say that a(nother) (straight-line) fulfilling the aforementioned (conditions) cannot be attached to AB.



Εἰ γὰρ δυνατόν, προσαρμοζέτω ἡ ΒΔ, ὥστε καὶ τὰς ΑΔ, ΔΒ δυνάμει ἀσυμμέτρους εἶναι ποιούσας τά τε ἀπὸ τῶν ΑΔ, ΔΒ τετράγωνα ἅμα μέσον καὶ τὸ δὶς ὑπὸ τῶν ΑΔ, ΔΒ μέσον καὶ ἔτι τὰ ἀπὸ τῶν ΑΔ, ΔΒ ἀσύμμετρα τῷ δὶς ὑπὸ τῶν ΑΔ, ΔΒ· καὶ ἐκκείσθω ῥητὴ ἡ ΕΖ, καὶ τοῖς μὲν ἀπὸ τῶν ΑΓ, ΓΒ ἴσον παρὰ τὴν ΕΖ παραβεβλήσθω τὸ ΕΗ πλάτος ποιοῦν τὴν ΕΜ, τῷ δὲ δὶς ὑπὸ τῶν ΑΓ, ΓΒ ἴσον παρὰ τὴν ΕΖ παραβεβλήσθω τὸ ΘΗ πλάτος ποιοῦν τὴν ΘΜ· λοιπὸν ἄρα τὸ ἀπὸ τῆς ΑΒ ἴσον ἐστὶ τῷ ΕΛ· ἡ ἄρα ΑΒ δύναται τὸ ΕΛ. πάλιν τοῖς ἀπὸ τῶν ΑΔ, ΔΒ ἴσον παρὰ τὴν ΕΖ παραβεβλήσθω τὸ ΕΙ πλάτος ποιοῦν τὴν ΕΝ. ἔστι δὲ καὶ τὸ ἀπὸ τῆς ΑΒ ἴσον τῷ ΕΛ· λοιπὸν ἄρα τὸ δὶς ὑπὸ τῶν ΑΔ, ΔΒ ἴσον [ἐστὶ] τῷ ΘΙ. καὶ ἐπεὶ μέσον ἐστὶ τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΓ, ΓΒ καί ἐστιν ἴσον τῷ ΕΗ, μέσον ἄρα ἐστὶ καὶ τὸ ΕΗ. καὶ παρὰ ῥητὴν τὴν ΕΖ παράκειται πλάτος ποιοῦν τὴν ΕΜ· ῥητὴ ἄρα ἐστὶν ἡ ΕΜ καὶ ἀσύμμετρος τῇ ΕΖ μήκει. πάλιν, ἐπεὶ μέσον ἐστὶ τὸ δὶς ὑπὸ τῶν ΑΓ, ΓΒ καί ἐστιν ἴσον τῷ ΘΗ, μέσον ἄρα καὶ τὸ ΘΗ. καὶ παρὰ ῥητὴν τὴν ΕΖ παράκειται πλάτος ποιοῦν τὴν ΘΜ· ῥητὴ ἄρα ἐστὶν ἡ ΘΜ καὶ ἀσύμμετρος τῇ ΕΖ μήκει. καὶ ἐπεὶ ἀσύμμετρά ἐστι τὰ ἀπὸ τῶν ΑΓ, ΓΒ τῷ δὶς ὑπὸ τῶν ΑΓ, ΓΒ, ἀσύμμετρόν ἐστι καὶ τὸ ΕΗ τῷ ΘΗ· ἀσύμμετρος ἄρα ἐστὶ καὶ ἡ ΕΜ τῇ ΜΘ μήκει. καί εἰσιν ἀμφότεραι ῥηταί· αἱ ἄρα ΕΜ, ΜΘ ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΕΘ, προσαρμόζουσα δὲ αὐτῇ ἡ ΘΜ. ὁμοίως δὴ δείξομεν, ὅτι ἡ ΕΘ πάλιν ἀποτομή ἐστιν, προσαρμόζουσα δὲ αὐτῇ ἡ ΘΝ. τῇ ἄρα ἀποτομῇ ἄλλη καὶ ἄλλη προσαρμόζει ῥητὴ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ· ὅπερ ἐδείχθη ἀδύνατον. οὐκ ἄρα τῇ ΑΒ ἑτέρα προσαρμόσει εὐθεῖα.

Τῇ ἄρα ΑΒ μία μόνον προσαρμόζει εὐθεῖα δυνάμει ἀσύμμετρος οὖσα τῇ ὅλῃ, μετὰ δὲ τῆς ὅλης ποιοῦσα τά τε ἀπ᾽ αὐτῶν τετράγωνα ἅμα μέσον καὶ τὸ δὶς ὑπ᾽ αὐτῶν μέσον καὶ ἔτι τὰ ἀπ᾽ αὐτῶν τετράγωνα ἀσύμμετρα τῷ δὶς ὑπ᾽ αὐτῶν· ὅπερ ἔδει δεῖξαι.



For, if possible, let BD be (so) attached. Hence, AD and DB are also (straight-lines which are) incommensurable in square, making the squares on AD and DB (added) together medial, and twice the (rectangle contained) by AD and DB medial, and, moreover, the (sum of the squares) on AD and DB incommensurable with twice the (rectangle contained) by AD and DB [Prop. 10.78]. And let the rational (straight-line) EF be laid down. And let EG, equal to the (sum of the squares) on AC and CB, have been applied to EF, producing EM as breadth. And let HG, equal to twice the (rectangle contained) by AC and CB, have been applied to EF, producing HM as breadth. Thus, the remaining (square) on AB is equal to EL [Prop. 2.7]. Thus, AB is the square-root of EL. Again, let EI, equal to the (sum of the squares) on AD and DB, have been applied to EF, producing EN as breadth. And the (square) on AB is also equal to EL. Thus, the remaining twice the (rectangle contained) by AD and DB [is] equal to HI [Prop. 2.7]. And since the sum of the (squares) on AC and CB is medial, and is equal to EG, EG is thus also medial. And it is applied to the rational (straight-line) EF, producing EM as breadth. EM is thus rational, and incommensurable in length with EF [Prop. 10.22]. Again, since twice the (rectangle contained) by AC and CB is medial, and is equal to HG, HG is thus also medial. And it is applied to the rational (straight-line) EF, producing HM as breadth. HM is thus rational, and incommensurable in length with EF [Prop. 10.22]. And since the (sum of the squares) on AC and CB is incommensurable with twice the (rectangle contained) by AC and CB, EG is also incommensurable with HG. Thus, EM is also incommensurable in length with MH [Props. 6.1, 10.11]. And they are both rational (straight-lines). Thus, EM and MH are rational (straight-lines which are) commensurable in square only. Thus, EH is an apotome [Prop. 10.73], with HM attached to it. So, similarly, we can show that EH is again an apotome, with HN attached to it. Thus, different rational (straight-lines), which are commensurable in square only with the whole, are attached to an apotome. The very thing was shown (to be) impossible [Prop. 10.79]. Thus, another straight-line cannot be (so) attached to AB.

Thus, only one straight-line, which is incommensurable in square with the whole, and (together) with the whole makes the squares on them (added) together medial, and twice the (rectangle contained) by them medial, and, moreover, the (sum of the) squares on them incommensurable with the (rectangle contained) by them, can be attached to AB. (Which is) the very thing it was required to show.

† This proposition is equivalent to Prop. 10.47, with minus signs instead of plus signs.

Ὅροι τρίτοι.

ια´. Ὑποκειμένης ῥητῆς καὶ ἀποτομῆς, ἐὰν μὲν ἡ ὅλη τῆς προσαρμοζούσης μεῖζον δύνηται τῷ ἀπὸ συμμέτρου ἑαυτῇ μήκει, καὶ ἡ ὅλη σύμμετρος ᾖ τῇ ἐκκειμένῃ ῥητῇ μήκει, καλείσθω ἀποτομὴ πρώτη.

ιβ´. Ἐὰν δὲ ἡ προσαρμόζουσα σύμμετρος ᾖ τῇ ἐκκειμένῃ ῥητῇ μήκει, καὶ ἡ ὅλη τῆς προσαρμοζούσης μεῖζον δύνηται τῷ ἀπὸ συμμέτρου ἑαυτῇ, καλείσθω ἀποτομὴ δευτέρα.

ιγ´. Ἐὰν δὲ μηδετέρα σύμμετρος ᾖ τῇ ἐκκειμένῃ ῥητῇ μήκει, ἡ δὲ ὅλη τῆς προσαρμοζούσης μεῖζον δύνηται τῷ ἀπὸ συμμέτρου ἑαυτῇ, καλείσθω ἀποτομὴ τρίτη.

ιδ´. Πάλιν, ἐὰν ἡ ὅλη τῆς προσαρμοζούσης μεῖζον δύνηται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ [μήκει], ἐὰν μὲν ἡ ὅλη σύμμετρος ᾖ τῇ ἐκκειμένῃ ῥητῇ μήκει, καλείσθω ἀποτομὴ τετάρτη.

ιε´. Ἐὰν δὲ ἡ προσαρμόζουσα, πέμπτη.

ιϛ´. Ἐὰν δὲ μηδετέρα, ἕκτη.

Definitions III

11. Given a rational (straight-line) and an apotome, if the square on the whole is greater than the (square on a straight-line) attached (to the apotome) by the (square) on (some straight-line) commensurable in length with (the whole), and the whole is commensurable in length with the (previously) laid down rational (straight-line), then let the (apotome) be called a first apotome.

12. And if the attached (straight-line) is commensurable in length with the (previously) laid down rational (straight-line), and the square on the whole is greater than (the square on) the attached (straight-line) by the (square) on (some straight-line) commensurable (in length) with (the whole), then let the (apotome) be called a second apotome.

13. And if neither of (the whole or the attached straight-line) is commensurable in length with the (previously) laid down rational (straight-line), and the square on the whole is greater than (the square on) the attached (straight-line) by the (square) on (some straight-line) commensurable (in length) with (the whole), then let the (apotome) be called a third apotome.

14. Again, if the square on the whole is greater than (the square on) the attached (straight-line) by the (square) on (some straight-line) incommensurable [in length] with (the whole), and the whole is commensurable in length with the (previously) laid down rational (straight-line), then let the (apotome) be called a fourth apotome.

15. And if the attached (straight-line is commensurable), a fifth (apotome).

16. And if neither (the whole nor the attached straight-line is commensurable), a sixth (apotome).

πε´.

Εὑρεῖν τὴν πρώτην ἀποτομήν.

Proposition 85

To find a first apotome.