$= d$, $md = M$, $m' = M'd$, $m'' = M''d$, $n = Nd$, $n' = N'$, $n'' = N''$, patetque $f$ transire per substitutionem $(S)$

$$\begin{matrix} \alpha d, & \alpha', & \alpha'' \\ \beta d, & \beta', & \beta'' \\ \gamma d, & \gamma', & \gamma'' \end{matrix}$$

in formam ternariam $\begin{pmatrix} Md, & M'd, & M''d \\ Nd, & N'd, & N''d \end{pmatrix} = g'$, determinantis $d^3$, quae itaque sub $f$ contenta erit. Iam dico, huic formae $g'$ necessario aequiualere hanc $\begin{pmatrix} d, & 0, & 0 \\ d, & 0, & 0 \end{pmatrix} = g''$. Patet enim, $\begin{pmatrix} M, & M', & M'' \\ N, & N', & N'' \end{pmatrix} = g'''$ fore formam ternariam determinantis 1; porro quum per hyp. $a$, $b$, $c$ eadem signa non habeant, $f$ erit forma indefinita, vnde facile concluditur etiam $g'$ et $g'''$ indefinitas esse debere; quare $g'''$ aequiualebit formae $\begin{pmatrix} 1, & 0, & 0 \\ 1, & 0, & 0 \end{pmatrix}$, (art. 277) poteritque transformatio $(S')$ illius in hanc inueniri; manifesto autem per $(S')$ forma $g'$ transibit in $g''$. Hinc etiam $g''$ sub $f$ contenta erit, et ex combinatione substitutionem $(S)$, $(S')$ deducetur transformatio formae $f$ in $g''$. Quae si fuerit

$$\begin{matrix} \delta, & \delta', & \delta'' \\ \varepsilon, & \varepsilon', & \varepsilon'' \\ \zeta, & \zeta', & \zeta'' \end{matrix}$$

manifestum est, duplicem solutionem aequationis ( ) haberi, puta $x = \delta'$, $y = \varepsilon'$, $z = \zeta'$, et $x = \delta''$, $y = \varepsilon''$, $z = \zeta''$; simul patet, neutros valores simul $= 0$ euadere posse, quum necessario fiat $\delta\varepsilon'\zeta'' + \delta'\varepsilon''\zeta + \delta''\varepsilon\zeta' - \delta\varepsilon''\zeta' - \delta'\varepsilon\zeta'' - \delta''\varepsilon'\zeta = d$. *Q. E. S.*

*Exemplum.* Sit aequatio proposita $7xx - 15yy + 23zz = 0$, quae resolubilis est quia $345R7$, $-161R15$, $105R23$. Habentur hic valores ipsorum $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$ hi $3$, $7$, $6$; faciendoque $\mathfrak{a} = \mathfrak{b} = \mathfrak{c} = 1$ inuenitur $A = 98$, $B = -39$, $C = -8$. Hinc eruitur substitutio

$$\begin{matrix} 3, & 5, & 22 \\ -1, & 2, & -28 \\ 8, & 25, & -7 \end{matrix}$$

per quam $f$ transit in $\begin{pmatrix} 1520, & 14490, & -7245 \\ -2415, & -1246, & 4735 \end{pmatrix} = g$. Hinc fit

$$(S) = \begin{cases} 7245, & 5, & 22 \\ -2415, & 2, & -28 \\ 19320, & 25, & -7 \end{cases}$$

$$g''' = \begin{pmatrix} 3670800, & 6, & -3 \\ -1, & -1246, & 4735 \end{pmatrix}$$

Forma $g'''$ transire inuenitur in $\begin{pmatrix} 1, & 0, & 0 \\ 1, & 0, & 0 \end{pmatrix}$ per substitutionem

$$\left.\begin{matrix} 3, & 5, & 1 \\ -2440, & -4066, & -813 \\ -433, & -722, & -144 \end{matrix}\right\} \ldots (S')$$

qua cum $(S)$ combinata prodit haec:

$$\begin{matrix} 9, & 11, & 12 \\ -1, & 9, & -9 \\ -9, & 4, & 3 \end{matrix}$$

per quam $f$ transit in $g''$. Habemus itaque duplicem aequationis propositae solutionem $x = 11$, $y = 9$, $z = 4$, et $x = 12$, $y = -9$, $z = 3$; posterior simplicior redditur diuidendo valores per diuisorem communem 3, vnde $x = 4$, $y = -3$, $z = 1$.

295. Pars posterior theorematis art. praec. etiam sequenti modo absolui potest. Quaeratur integer $h$ talis vt sit $ah \equiv \mathfrak{C}$ (mod. $c$), (characteres $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$ eadem significatione accipimus vt in art. praec.), fiatque $ahh + b = ci$. Tunc facile perspicitur, $i$ fieri integrum, numerumque $-ab$ esse determinantem formae binariae $(ac, ah, i) \ldots \phi$. Haec forma certo non erit positiua (quum enim per hyp. $a$, $b$, $c$ eadem signa non habeant, $ab$ et $ac$ simul positiui esse nequeunt); porro habebit numerum characteristicum $-1$, quod synthetice ita demonstramus: Determinentur integri $e$, $e'$ ita vt sit $e \equiv 0$ (mod. $a$) et $\equiv \mathfrak{B}$ (mod. $b$); $ce' \equiv \mathfrak{A}$ (mod. $a$) et $\equiv h\mathfrak{B}$ (mod. $b$), eritque $(e, e')$ valor expr. $\sqrt{-(ac, ah, i)}$. Nam secundum modulum $a$ erit $ee \equiv 0 \equiv -ac$, $ee' \equiv 0 \equiv -ah$, $cce'e' \equiv \mathfrak{A}\mathfrak{A} \equiv -bc \equiv -cci$ adeoque $e'e' \equiv -i$; secundum modulum $b$ autem erit $ee \equiv \mathfrak{B}\mathfrak{B} \equiv -ac$, $cee' \equiv h\mathfrak{B}\mathfrak{B} \equiv -ach$ adeoque $ee' \equiv -ah$, $cce'e' \equiv hh\mathfrak{B}\mathfrak{B} \equiv -achh \equiv -cci$ adeoque $e'e' \equiv -i$; eaedem vero tres congruentiae quae secundum vtrumque