Hinc adiumento aequationis [14] et huius $\mathfrak{A}a + 2\mathfrak{B}b + \mathfrak{C}c = m$, facile deducitur (multiplicando primam, secundam, quartam; secundam, tertiam, quintam; quartam, quintam, sextam, resp. per $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$ addendoque producta): $(\alpha\gamma' - \gamma\alpha')\,Umm = maUU$, $(\alpha\delta' + 6\gamma' - \gamma 6' - \delta\alpha')\,Umm = 2mbUU$, $(6\delta' - \delta 6')\,Umm = mcUU$ atque hinc, diuidendo per mU*), $aU = (\alpha\gamma' - \gamma\alpha')m \ldots [19]$; $2bU = (\alpha\delta' + 6\gamma' - \gamma 6' - \delta\alpha')m \ldots [20]$; $cU = (6\delta' - \delta 6')m \ldots [21]$, ex quarum aequationum aliqua U multo facilius quam ex [14] deduci potest. — Simul hinc colligitur, quomodocunque $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$ determinentur (quod infinitis modis diuersis fieri potest), tum T tum U eundem valorem adipisci.

Iam si aequatio 18 multiplicatur per α, 19 per 26, 20 per $-\alpha$, fit per additionem $2\alpha eT + 2(6a - \alpha b)U = 2(\alpha\delta - 6\gamma)\alpha' m = 2e\alpha' m$.

Simili modo fit ex $6[18] + 6[20] - 2\alpha[21]$, $26eT + 2(6b - \alpha c)\,U = 2(\alpha\delta - 6\gamma)\,6'm = 2e6'm$.

Porro ex $\gamma[18] + 2\delta[19] - \gamma[20]$ fit $2\gamma eT + 2(\delta a - \gamma b)U = 2(\alpha\delta - 6\gamma)\gamma' m = 2e\gamma' m$.

Tandem ex $\delta[18] + \delta[20] - 2\gamma[21]$ prodit $2\delta eT + 2(\delta b - \gamma c)U = 2(\alpha\delta - 6\gamma)\delta' m = 2e\delta m$.

*) Hoc non liceret, si esset $U = 0$: tunc vero aequationum [19, 20, 21 veritas statim ex prima, tertia et sexta praecedentium sequeretur.

In quibus formulis, si pro a, b, c valores ex 1, 3, 5 substituuntur, fit

$$\alpha'm = \alpha T - (\alpha B + \gamma C)U$$
$$\beta'm = \beta T - (\beta B + \delta C)U$$
$$\gamma'm = \gamma T + (\alpha A + \gamma B)U$$
$$\delta'm = \delta T + (\beta A + \delta B)U$$

Ex analysi praec. sequitur, nullam transformationem formae F in f propositae similem dari, quae non sit contenta sub formula $X = \frac{1}{m}(\alpha t - (\alpha B + \gamma C)u)x + \frac{1}{m}(\beta t - (\beta B + \delta C)u)y$, $Y = \frac{1}{m}(\gamma t + (\alpha A + \gamma B)u)x + \frac{1}{m}(\delta t + (\beta A + \delta B)u)x)$ . . . (I), designantibus t, u indefinite omnes numeros integros aequationi $tt - Duu = mm$ satisfacientes. Hinc vero concludere nondum possumus, omnes valores ipsorum t, u, aequationi illi satisfacientes, in formula (I) substitutos, transformationes idoneas praebere. At

1. Formam F per substitutionem, e quibusuis ipsorum t, u valoribus ortam, semper in formam f transmutari, per euolutionem confirmari facile potest adiumento aequationem 1, 3, 5 et huius $tt - Duu = mm$. Calculum prolixiorem quam difficiliorem breuitatis gratia supprimimus.

2. Quaeuis transformatio ex formula deducta propositae erit similis. Namque $\frac{1}{m}(\alpha t - (\alpha B + \gamma C)u) \times \frac{1}{m}(\delta t + (\beta A + \delta B)u) - \frac{1}{m}(\beta t - (\beta B + \delta C)u) \times \frac{1}{m}(\gamma t + (\alpha A + \gamma B)u) = \frac{1}{mm}(\alpha\delta - \beta\gamma)(tt - Duu) = \alpha\delta - \beta\gamma$.

3. Si formae F, f determinantes inaequales habent, fieri potest, vt formula (I) pro quibusdam valoribus ipsorum t, u praebeat substitutiones, quae *fractiones* implicent, adeoque reiici debeant, Omnes vero reliquae erunt transformationes idoneae, aliaeque praeter ipsas non dabuntur.

4. Si vero formae F, f eundem determinantem habent adeoque sunt *aequiualentes*, formula (I) nullas transformationes quae fractiones implicent praebebit, adeoque in hoc casu solutionem completam problematis exhibebit. Illud vero ita demonstramus.

Ex theoremate art. praec. sequitur in hocce casu, m simul fore diuisorem communem numerorum A, $2B$, C. Quoniam $tt - Duu = mm$, fit $tt - BBuu = mm - ACuu$, quare $tt - BBuu$ per mm diuisibilis erit: hinc etiam a potiori $4tt - 4BBuu$ adeoque (quia $2B$ per m diuisibilis) etiam $4tt$ per mm et proin $2t$ per m. Hinc $\frac{2}{m}(t + Bu)$, $\frac{2}{m}(t - Bu)$ erunt integri, et quidem, quoniam differentia inter ipsos, $\frac{4}{m}Bu$ est par), aut vterque par, aut vterque impar. Si vterque impar esset, etiam productum impar foret, quod tamquam quadruplum numeri $\frac{1}{mm}(tt - BBuu)$, quem integrum esse modo ostendimus, necessario par: quare hic casus est impossibilis, adeoque $\frac{2}{m}(t + Bu)$, $\frac{2}{m}(t - Bu)$ semper pares, vnde $\frac{1}{m}(t + Bu)$, $\frac{1}{m}(t - Bu)$ erunt integri. Hinc vero nullo negotio deducitur, omnes quatuor coefficientes in (I) semper esse integros. *Q. E. D.*