the height of pyramids. To impress his business minded fellow citizens, he once cornered the market in olive oil and, incidentally, made himself rich.

Thales's name is associated with a number of elementary theorems in geometry:

1. a circle is bisected by a diameter;
2. the base angles of an isosceles triangle are equal;
3. when two lines intersect, vertically opposite angles are equal;
4. the angle-angle-side congruence theorem;
5. the angle subtended by a diameter of a circle is a right angle (that is, if A, B, C are points on a circle and AC is a diameter, then $\angle ABC$ is a right angle).

Theorem 5 is called *Thales's theorem.* To prove this he also had to know the following:

6. the sum of the angles in a triangle is equal to two right angles (or, as we now say, in slavish imitation of the Babylonians, 180°).

All of these theorems must have been known empirically by the Egyptians and Babylonians. The reason they are associated with Thales is not that he discovered them, but that he was the first to prove them. This was the essential difference between pre-Greek and Greek mathematics: the Greeks established the logical connections among their results; they gave the first abstract proofs in mathematics.

As a philosopher, Thales is known for his statement that everything is made of water. How should we interpret this, and what is its relevance to mathematics?

As we look around us, we observe that there are two kinds of things: those that can be counted, such as pebbles and cows, and those that can only be measured, such as butter and water. This physical distinction between 'discrete' and 'continuous' is reflected on a linguistic level: it is perfectly correct to say 'one cow, two cows' but it sounds rather odd to say 'one butter, two butters', to put it mildly (however, see Exercise 5). We call the former sort of nouns *count nouns*, and the latter *mass nouns.* To some extent, this distinction is a convention. For example, in modern English, we can count peas but not rice, while a hundred years ago, 'pease' was not a plural but a mass noun. (A hundred years from now, 'rice' may be the plural of 'rouse'.)

A question which physicists are still working on is this: is the material universe ultimately countable — consisting of discrete, unconnected fragments — or is the material universe ultimately continuous — that is, should it be understood in terms of a connected continuum? If the first, how do we explain the unity of nature, how do we understand the continuity of change and motion? If the second, how do we explain the diversity of nature, how do we understand the individuality of distinct, single objects?

This issue was addressed by more than one Greek thinker. As we shall see, Pythagoras and Democritus took the view that reality is basically discrete. They then tried to understand apparently continuous entities in terms of discrete entities (e.g., lengths as ratios of whole numbers). Thales, on the other hand, took the view that 'all is water'. In other words, the material universe is best understood in terms of a single substance, namely, water. (Here we are using the word 'substance' not in the Aristotelian sense of 'individual entity', but in the more common sense of 'material having uniform properties'.) Thales had undoubtedly noticed that ice and steam are both forms of water, but we do not know why he picked water as the fundamental substance. (It has been suggested by Marxist historians that this was so because his city Miletus was a *maritime* power.) What is important is not that Thales overlooked the possibility of, say, there being 90 different substances, but that he raised a fascinating problem about the universe, which has not been resolved to this day.

Other Ionian philosophers agreed with Thales that there was a single substance, but not that it was water. Anaximenes of Miletus (550 BC) identified the primal substance as air. Heraclitus of Ephesus (500 BC) held that everything is made of fire. Anaximander was a follower and compatriot of Thales, who like Thales, took the view that the universe is best understood in terms of a single substance. Unlike Thales, he did not think this substance was water. He thought it was something he called the *Infinite.* The Infinite could take on the forms of earth, water, air, and fire. Today we might refer to solid, liquid, gas and energy, respectively.

Exercises

1. Let ABC be a triangle, and let d be a straight line through A parallel to BC. Assuming that the 'alternate angles are equal', prove that the sum of the angles of ABC equals two right angles.

2. Prove the Theorem of Thales, using Exercise 1 and the theorem that the base angles of an isosceles triangle are equal.

3. Prove the converse of Thales's Theorem: if A, B and C are points on a circle and $\angle ABC$ is a right angle then AC is a diameter.

4. How would you measure the height of a pyramid (or tree, for that matter), using similar triangles?

5. Some nouns like 'rice' are definitely mass nouns. We cannot say 'two rices'. Other nouns are more problematic. 'Whisky' is normally a mass noun, but 'two whiskies, please' is perfectly good English (meaning two glasses of whisky). Some languages have many fewer count nouns than English. For example, in Indonesian it is incorrect to say 'two

cows'; you have to say 'two tails of cow', as we might say 'two head of cattle'. (It is amusing to note that our mass noun 'cattle' is itself ultimately derived from the Latin word 'caput' meaning 'head'.) Write an essay on the distinction between count nouns and mass nouns and its relevance to mathematics.