Only one of the infinitely many solutions is given, but Sun Tsu's method allowed him to give as many others as he wanted.

Tsu Ch'ung Chih (475 AD) gave upper and lower bounds for π. He used the method of Archimedes, but he obtained sharper estimates. In the 'Nine Sections of Mathematics' (1247), Ch'in Chiu Shao (Qin Jiushao) found a root of

$$x^4 - 763200x^2 + 40642560000,$$

using what is in effect Horner's method (rediscovered by William Horner in 1819). In the 'Precious Mirror of the Four Elements' (1303), Chu Shih Chieh (Zhu Shijie) gives 'Pascal's Triangle' (also known in India and later rediscovered by Pascal in 1653).

In the 16th and 17th centuries, Christian missionaries from Europe entered China, introducing Western mathematics (e.g., the theory of logarithms). Today, the Chinese contribute to all branches of mathematics. They have retained, however, a strong interest in the theory of numbers. For example, in 1985, De Gang Ma gave the first elementary solution of the Diophantine equation

$$x(x+1)(2x+1) = 6y^2.$$

(See Anglin [1995], Section 4.4.)

We have already heard about Chen Jing-run and the progress he made on the Goldbach conjecture.

The earliest Indian mathematical texts we have are the *Sulvasutras* or 'rules of the cord'. They were written sometime between 500 BC and 200 AD. They contain some elementary geometry related to the construction of altars. It is observed that $12^2 + 35^2 = 37^2$ and, presumably, the general method for constructing such 'Pythagorean triples' was known.

Mathematics in India turns up in unexpected places. Around 800 BC a certain Pingala wrote a book on the *Science of verse meters.* Syllables in Vedic poetry are distinguished to be either 'light' or 'heavy'; the former are assigned the binary digit 1, the latter the binary digit 0. Thus, with each line of verse is asssociated a number written in binary scale, which is then converted into decimal scale. Conversion in the converse direction is also discussed. (See van Nooten [1993].)

Four noteworthy Indian mathematicians were:

- Aryabhata the Elder, born in 476 AD,
- Brahmagupta, flourished in 628 AD,
- Mahavira, lived about 850 AD,
- Bhaskara, 1114–1185 AD.

In their work, ancient Indian traditions and Alexandrian mathematics seem to flow together. Yet, unlike the ancient Greeks, these mathematicians did not include many proofs in their books. Also, unlike the ancient Greeks, they wrote their mathematics books in verse! For example, Bhaskara gives the following problem:

> The square root of half the number of bees in a swarm
> Has flown out upon a jasmine bush;
> Eight ninths of the swarm has remained behind;
> And a female bee flies about a male who is buzzing inside a lotus flower;
> In the night, allured by the flower's sweet odour, he went inside it
> And now he is trapped!
> Tell me, most enchanting lady, the number of bees.

This is certainly a poetic way of asking for the solution of

$$\sqrt{x/2} + 8x/9 + 2 = x.$$

Aryabhata wrote on arithmetic and algebra, on plane and spherical trigonometry, and on astronomy. He knew the formulas for the sum $1^k + 2^k + \cdots + n^k$ for $k = 1, 2$ and 3. He solved quadratic and indeterminate equations. He was one of the first to make use of the sine function, defining $\sin A$ as (chord $2A)/2$, thus helping to simplify the addition formulas and giving trigonometry its modern form. He also produced a quite accurate sine table.

Brahmagupta (b. 598) studied the Diophantine equation $x^2 - Ry^2 = 1$ (where R is a given, positive nonsquare integer). This equation is mistakenly called 'Pell's equation', after John Pell (1610–1685), who actually had nothing to do with it. A complete solution to this equation was given by Lagrange in 1767.

Brahmagupta calculated the surface and volume of pyramids and cones, using $\sqrt{10}$ as an approximation for π. Most importantly, he was the first person to give a systematic presentation of the rules for working with zero and negative numbers, and he may have been the first to introduce negative numbers into the number system. According to *The Treasury of Mathematics,* edited by Henrietta O. Midonick, Brahmagupta's *Brahmasphuita Siddhanta* tells us that positive divided by positive, or negative divided by negative, is positive, whereas positive divided by negative, or negative divided by positive is negative. It seems, however, that Brahmagupta had some trouble dividing by zero.

One of Brahmagupta's more technical achievements was his discovery of the formula for the area of a cyclic quadrilateral with sides a, b, c and d:

$$\sqrt{(s-a)(s-b)(s-c)(s-d)},$$