

thirty cubits long gave the measurement of its girth. Under its rim and completely encircling it were gourds; they went around the basin over a length of thirty cubits.

But perhaps the basin was hexagonal and not circular!

## Exercises

1. Consider the simultaneous equations  $xy + x - y = a$  and  $x + y = b$ , where  $a$  and  $b$  are given integers. What is a necessary and sufficient condition on  $a$  and  $b$  so that  $x$  and  $y$  will be integers?
2. Solve the simultaneous pair (\*) (from a Susa tablet).
3. Prove by mathematical induction that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = n(n+1)(2n+1)/6.$$

4. Rabbi Nehemiah (150 AD) was unhappy with the idea that the Bible had used a very inaccurate value for  $\pi$  and he suggested that ‘the diameter of 10 cubits included the walls of the basin, while the circumference excluded them.’ Assuming that he was right, and assuming that the Bible used a perfectly accurate value for  $\pi$ , how wide was the wall (or rim) of the basin?
5. Prove that if a triangle has sides of lengths  $a, b$  and  $c$ , and if  $a^2 + b^2 = c^2$ , then the triangle is right angled.
6. Prove the Babylonian theorem for Pythagorean triangles.
7. Prove the converse of the Babylonian theorem for Pythagorean triangles.
8. In 1901, L. Kronecker gave the first proof that all positive integer solutions of  $a^2 + b^2 = c^2$  are given without duplication by  $a = 2uvk$ ,  $b = (u^2 - v^2)k$ ,  $c = (u^2 + v^2)k$ , where  $u, v$  and  $k$  are positive integers such that  $u > v$ ,  $u$  and  $v$  are not both odd, and  $u$  and  $v$  are relatively prime. Prove Kronecker’s theorem.
9. The 15 Pythagorean triangles in Plimpton 322 have angles which approximate the 15 whole number angles from  $44^\circ$  to  $58^\circ$  inclusive. Find a Pythagorean triangle, with relatively prime sides, one of whose angles is within  $2/5^\circ$  of  $47^\circ$ . (Hint: see Anglin [1988].)

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## The Dawn of Greek Mathematics

The ancient Greek world was not confined to what we now call Greece, but extended to Ionia (western Turkey) in the east, southern Italy and Sicily in the west, and later to Alexandria (Egypt) in the south. Not surprisingly, Greek philosophy and mathematics began in Ionia, where the influence of the older civilizations of the east (e.g., Babylon) was greatest. Later, political events caused many Greeks to emigrate from Ionia to Italy, and this became the center of intellectual life for a while. After the war between a Greek coalition and the Persians ended in the defeat of the latter (490 BC), philosophy and mathematics flourished in Athens. Ultimately, after the founding of Alexandria (331 BC), it was there that most of the major scientific developments took place until about 500 AD.

The first Greek mathematician and philosopher is Thales of Miletus (600 BC). According to Proclus, Thales visited Egypt and brought back the knowledge of geometry from there. He may also have been influenced by Indian thought via Persia. He is said to have predicted the solar eclipse which occurred over the Near East in May of 585 BC. To do this, he may have made use of observations which the Babylonians had accumulated over many centuries.

Plato repeats a story about Thales being an absent-minded professor, who was so preoccupied with celestial matters that he did not observe what was in front of his feet and once fell into a well (*Theaetetus* 174a). According to other anecdotes, however, Thales could turn his mind to practical matters when necessary. He constructed an almanac and he used the theory of similar triangles to calculate the distance of ships from shore and