

tituatur, & Chorda Ac jungatur: ita ut nunc rectæ MQ & RMS , CAP. V.
ductæ, ut ante, per M lateribus AB & BD parallelæ, latera
quadrilateri $ABDc$ secant in punctis p , Q , R & s ; similis
proprietas locum habebit. Cum enim sit $MP.MQ: BQ \times$
 $DQ = cG. cH: BH. DH$ seu $MP.MQ: MR. MS =$
 $cG. cH: BH. DH$, ob rectam RS ipsi BD parallelam &
æqualem. Triangula vero similia APp , AGc & DSs , cHD ,
præbent has proportionēs $Pp: AP = Gc: AG$; seu, ob $AP:$
 $AG = BQ: BH$, hanc $Pp: BQ = Gc: BH$; altera similitudo
dat hanc $DS (MQ): Ss = cH: DH$, quibus conjunctis fit

$$MQ.Pp: MR.Ss = cG. cH: BH. DH, \text{ ob } BQ = MR.$$

Hæc proportio cum superiori collata præbet

$$MP.MQ: MR.MS = Pp.MQ: MR.Ss,$$

unde addendo antecedentes & consequentes fit

$$MP.MQ: MR.MS = Mp.MQ: MR.Ms,$$

ubique ergo sumantur puncta c & M in Curva, erit semper
ratio $Mp.MQ$ ad $MR.Ms$ eadem, dummodo rectæ MQ
& Rs per M ducantur Chordis AB & BD parallelæ. Ex su-
periore vero proportionē sequitur fore $MP: MS = Mp: Ms$.
Cum igitur, variato puncto c , tantum puncta p & s mutantur,
erit Mp ad Ms in data ratione, utcumque punctum c varietur,
dum punctum M fixum servatur.

99. Quod si quatuor quæcunque puncta A , B , C , D in TAB. VI.
Linea secundi ordinis fuerint data, eaque jungantur rectis, Fig. 23.
ut habeatur trapezium inscriptum $ABDC$, proprietas Sectio-
num conicarum latissime patens ex præcedenti deducitur. Sci-
licet, si ex Curvæ puncto quovis M ad singula trapezii latera
sub datis angulis ducantur rectæ MP , MQ , MR & MS ,
erunt semper rectangula binarum harum linearum ad opposita la-
tera ductarum inter se in data ratione, nempe erit $MP.MQ$
ad

LIB. II. ad $MR.MS$ in data ratione eadem, ubicunque punctum M in Curva capiatur, dummodo anguli ad P, Q, R , & S iidem ferventur. Ad hoc ostendendum ducantur per M duæ rectæ Mq & rs , illa lateri AB hæc lateri BD parallela, ac notentur intersectionum cum lateribus trapezii puncta p, q, r , & s : eritque per prius inventum $Mp.Mq$ ad $Mr.Ms$ in data ratione. Propter omnes autem angulos datos datæ erunt rationes $MP:Mp$, $MQ:Mq$, $MR:Mr$, & $MS:Ms$, ex quibus sequitur fore $MP.MQ$ ad $MR.MS$ in data quoque ratione.

TAB. VI. 100. Quoniam supra vidimus, si Ordinatæ parallelæ MN ,
Fig. 24. mn producantur, quoad tangenti cuiuspiam CPp occurrant in P & p , fore $PM.PN:CP^2 = pm.pn:Cp^2$. Quare, si puncta L & l notentur, ut sit PL media proportionalis inter PM & PN , pariterque pl media proportionalis inter pm & pn , erit $PL^2:CP^2 = pl^2:Cp^2$; ideoque erit $PL:CP = pl:Cp$, unde patet omnia puncta L, l in Linea recta per punctum contactus C transeunte esse sita. Quare, si una Applicata PMN ita secetur in L ut sit $PL^2 = PM.PN$, recta CLD per puncta C & L ducta omnes reliquas Applicatas pmn ita quoque secabit in l ut sit pl media proportionalis inter pm & pn . Vel, si duæ Applicatæ PN & pn ita in punctis L & l secentur, ut sit $PL^2 = PM.PN$ & $pl^2 = pm.pn$ recta per L & l producta per punctum contactus C transibit, atque omnes reliquas Applicatas illis parallelas in eadem ratione secabit.

TAB. VI. 101. His Linearum secundi ordinis proprietatibus, quæ ex
Fig. 25. forma æquationis immediate consequuntur, expositis; progrediamur ad alias magis reconditas investigandas. Sit igitur posita æquatio pro his Lineis secundi ordinis generalis

$$yy + \frac{(\epsilon x + \gamma)}{\zeta} y + \frac{\delta xx + \epsilon x + \alpha}{\zeta} = 0,$$

ex qua cum cuivis Abscissæ $AP = x$, duplex Applicata y
nempe

nempe PM & PN respondeat, positio Diametri omnes Or- CAP. V.
 dinatas MN bifariam secantis definiri potest. Sit enim IG —
 ista Diameter, quæ Ordinatam MN secabit in puncto medio
 L , quod ergo punctum est in Diametro. Ponatur $PL = z$;
 & cum sit $z = \frac{1}{2} PM + \frac{1}{2} PN$, erit $z = \frac{\overline{\varepsilon x} \gamma}{2\zeta}$,
 seu $2\zeta z + \varepsilon x + \gamma = 0$, quæ est æquatio positionem Diametri
 IG præbens.

102. Hinc porro longitudo Diametri IG definiri poterit,
 quæ dat loca bina in Curva, ubi puncta M & N coincidunt,
 seu ubi fit $PM = PN$. Ex æquatione vero dantur $PM +$
 $PN = \frac{\overline{\varepsilon x} \gamma}{\zeta}$ & $PM \cdot PN = \frac{\delta x x + \zeta x + \alpha}{\zeta}$, unde fit
 $(PM - PN)^2 = (PM + PN)^2 - 4PM \cdot PN =$
 $\frac{(\varepsilon\varepsilon - 4\delta\zeta^2)xx + 2(\varepsilon\gamma - 2\zeta\zeta)x + (\gamma\gamma - 4\alpha\zeta^2)}{\zeta\zeta} = 0$, seu
 $xx - \frac{2(2\zeta\zeta - \varepsilon\gamma)}{\varepsilon\varepsilon - 4\delta\zeta^2} x + \frac{\gamma\gamma - 4\alpha\zeta^2}{\varepsilon\varepsilon - 4\delta\zeta^2} = 0$, cujus æquatio-
 nis propterea radices sunt AK & AH ita ut sit $AK + AH =$
 $\frac{4\zeta\zeta - 2\varepsilon\gamma}{\varepsilon\varepsilon - 4\delta\zeta^2}$ & $AK \cdot AH = \frac{\gamma\gamma - 4\alpha\zeta^2}{\varepsilon\varepsilon - 4\delta\zeta^2}$: hinc fit $(AH -$
 $AK)^2 = KH^2 = \frac{4(2\zeta\zeta - \varepsilon\gamma)^2 - 4(\varepsilon\varepsilon - 4\delta\zeta^2)(\gamma\gamma - 4\alpha\zeta^2)}{(\varepsilon\varepsilon - 4\delta\zeta^2)^2}$.
 At est $IG^2 = \frac{\varepsilon\varepsilon + 4\zeta\zeta}{4\zeta\zeta} KH^2$, si quidem Applicatæ ad A-
 xem normales statuuntur.

103. Sint istæ Applicatæ, quas hic sumus contemplati, nor-
 males ad Axem AH ; nunc vero hinc quæramus æquationem
 pro Applicatis obliquangulis. Ducatur ergo ex quovis Curvæ
 puncto M ad Axem Applicata obliquangula Mp faciens cum
 Axe angulum MpH , cujus Sinus sit $= \mu$ & Cofinus $= \nu$.
 Sit nova Abscissa $Ap = t$, Applicata $pM = u$, erit $\frac{y}{u} = \mu$
 & $\frac{Pp}{u} = \nu$, unde erit $y = \mu u$ & $x = t + \nu u$, qui valores

LIB. II. in æquatione inter x & y , quæ erat $0 = \alpha + \epsilon x + \gamma y +$
 $\delta x x + \epsilon x y + \zeta y y$, substituti præbent

$$0 = \alpha + \epsilon t + \nu \epsilon u + \delta t t + 2 \nu \delta t u + \nu \nu \delta u u \\ + \mu \gamma u \quad + \mu \epsilon t u \quad + \mu \nu \epsilon u u \\ + \mu \mu \zeta u u$$

feu

$$u u + \frac{(\mu \epsilon + 2 \nu \delta) t + \mu \gamma + \nu \epsilon) u + \delta t t + \epsilon t + \alpha}{\mu \mu \zeta + \mu \nu \epsilon + \nu \nu \delta} = 0.$$

104. Hic ergo iterum quævis Applicata duplicem habebit
 valorem, nempe pM & pn : quare Ordinarum Mn Diameter
ig pari modo ut ante definitur. Scilicet, bisecta Ordinata
 Mn in l erit l , punctum in Diametro. Ponatur ergo $pl = v$,
 erit $v = \frac{pM + pn}{2} = \frac{(\mu \epsilon + 2 \nu \delta) t - \mu \gamma - \nu \epsilon}{2(\mu \mu \zeta + \mu \nu \epsilon + \nu \nu \delta)}$. Demitta-
 tur ex l in Axem AH perpendiculum lq , ac ponatur $Aq = p$,
 $ql = q$, erit $\mu = \frac{q}{v}$ & $\nu = \frac{p q}{v} = \frac{p - t}{v}$, unde fit $v =$
 $\frac{q}{\mu}$, & $t = p - \nu v = p - \frac{\nu q}{\mu}$. Substituantur hi valores

$$\text{in æquatione inter } t \text{ \& } v \text{ ante inventa, \& prodibit } \frac{q}{\mu} = \\ \frac{\mu \epsilon p - 2 \nu \delta p + \nu \epsilon q + 2 \nu \nu \delta q : \mu - \mu \gamma - \nu \epsilon}{2 \mu \mu \zeta + 2 \mu \nu \epsilon + 2 \nu \nu \delta}$$

feu

$$(2 \mu \mu \zeta + \mu \nu \epsilon) q + (\mu \mu \epsilon + 2 \mu \nu \delta) p + \mu \mu \gamma + \mu \nu \epsilon = 0,$$

feu

$$(2 \mu \zeta + \nu \epsilon) q + (\mu \epsilon + 2 \nu \delta) p + \gamma \mu + \nu \epsilon = 0,$$

qua æquatione positio Diametri *ig* definitur.

105. Prior Diameter IG , cujus positio per hanc æquatio-
 nem determinabatur $2 \zeta z + \epsilon x + \gamma = 0$, producta cum Axem
 concurrat in O , eritque $AO = \frac{\gamma}{\epsilon}$; hinc fit $PO =$

$$\frac{\gamma}{\epsilon} - x, \text{ \& anguli } LOP \text{ tangens erit } = \frac{z}{PO} = \frac{\epsilon z}{\epsilon x + \gamma}$$

$= \frac{\varepsilon}{2\zeta}$, & tangens anguli MLG , sub quo Diameter IG CAP. V.
 Ordinatas MN bifecat erit $= \frac{2\zeta}{\varepsilon}$. Altera vero Diameter ig
 producta Axi occurrat in o , eritque $Ao = \frac{\mu\gamma - \nu\zeta}{\mu\varepsilon + 2\nu\delta}$, &
 anguli Aol tangens erit $= \frac{\mu\varepsilon + 2\nu\delta}{2\mu\zeta + \nu\varepsilon}$. Cum jam anguli AOL
 tangens sit $= \frac{\varepsilon}{2\zeta}$, ambæ Diametri se mutuo interfecabunt in
 puncto quodam C , facientque angulum $OCo = Aol - AOL$,
 cujus propterea tangens est $= \frac{4\nu\delta\zeta - \nu\varepsilon\varepsilon}{4\mu\zeta^2 + 2\nu\delta\varepsilon + 2\nu\varepsilon\zeta + \mu\varepsilon\varepsilon}$. An-
 gulus autem, sub quo hæc altera Diameter suas Ordinatas bi-
 fecat, est $Mlo = 180^\circ - lpo - Aol$: hujus propterea tan-
 gens est $= \frac{2\mu\mu\zeta^2 + 2\mu\nu\varepsilon + 2\nu\nu\delta}{\mu\mu\varepsilon + 2\mu\nu\delta - 2\mu\nu\zeta - \nu\nu\varepsilon}$.

106. Inquiramus autem in punctum C , ubi hæ duæ Dia-
 metri se mutuo interfecant: ex quo ad Axem perpendiculum
 CD demittatur, ac vocetur $AD = g$, $CD = h$; eritque
 primo, quod C in Diametro IG extat, $2\zeta h + \varepsilon g + \gamma = 0$.
 Deinde, quia C quoque in Diametro ig reperitur, erit

$$(2\mu\zeta + \nu\varepsilon)h + (\mu\varepsilon + 2\nu\delta)g + \mu\gamma + \nu\zeta = 0.$$

Subtrahatur hinc prior æquatio per μ multiplicata, ac remanebit

$$\nu\varepsilon h + 2\nu\delta g + \nu\zeta = 0, \text{ seu } \varepsilon h + 2\delta g + \zeta = 0.$$

Ex his fit $h = \frac{\varepsilon\delta - \gamma}{2\zeta} = \frac{2\delta g - \zeta}{\varepsilon}$, ideoque

$(\varepsilon\varepsilon - 4\delta\zeta)g = 2\zeta^2 - \gamma\varepsilon$, & $g = \frac{2\zeta^2 - \gamma\varepsilon}{\varepsilon\varepsilon - 4\delta\zeta}$ & $h = \frac{2\gamma\delta - \zeta\varepsilon}{\varepsilon\varepsilon - 4\delta\zeta}$. In quibus determinationibus cum non infint
 quantitates μ & ν , a quibus obliquitas Applicatarum $p Mn$
 pendet, manifestum est punctum C idem manere, utcunque
 obliquitas varietur.