(The *circumcenter, orthocenter* and *centroid* of a triangle are the meeting points of the right bisectors, altitudes and medians, respectively.)

7. Fermat was wrong when he conjectured that all natural numbers of the form $2^{2^n} + 1$ are primes.

Euler recognized the importance of convergence in dealing with infinite series, but he did not always pay attention to it. For example, he would write

$$1/(x-1) = 1/x + 1/x^2 + 1/x^3 + \cdots$$

(which is correct when $|x| > 1$) and put $x = 1/2$ to obtain $-2 = 2 + 4 + 8 + \cdots$. He also showed a lack of rigour in employing his principle of 'conservation of form', according to which a theorem true for natural number exponents also holds for any real exponent. In this way, he obtained facile 'proofs' of the generalized Binomial Theorem, and the generalized de Moivre's Theorem.

In addition to his numerous discoveries in pure mathematics, by no means all of which have been discussed here, Euler also made important contributions to mechanics. He elaborated the Principle of Least Action. Finally, he worked out a theory of lunar motion. His collected works run to about 75 volumes.

The following proof of Euler's formula $V + F - E = 2$ was suggested by H. S. M. Coxeter.

Let O be a point in the interior of the convex polyhedron. About O as center describe a sphere which contains the polyhedron. Now imagine a source of light placed at O. Rays emanating from O will project the polyhedron onto the surface of the sphere, mapping each flat polygon onto a spherical polygon whose sides are arcs of great circles. (This idea is said to be due to the Arabic mathematician Abu'l Wafa.) Choose a point in the interior of each spherical polygon and join it to the vertices by arcs of great circles, thus dividing each spherical polygon into as many spherical triangles as it has sides. Then

$$\begin{aligned}
720^\circ &= \text{area of sphere} \\
&= \text{sum of areas of spherical triangles} \\
&= \text{sum of angles of spherical triangles} - 180^\circ \times \text{number of triangles} \\
&= \text{sum of angles at interior points} + \\
&\quad\ \text{sum of angles at vertices} - 180^\circ \times 2E \\
&= 360^\circ \times F + 360^\circ \times V - 360^\circ \times E.
\end{aligned}$$

Dividing by 360°, we obtain Euler's formula.

Joseph Lagrange was born in Italy of mixed French and Italian parentage. His father lost the family fortune through speculation, but Lagrange later commented that, if it had not been for this, he might never have turned to mathematics. He was converted to mathematics through an essay by Halley.

At age 23, Lagrange was able to explain, on the basis of Newton's theory of gravitation, why the moon always shows the same face to the earth.

Having acquired an early fame, Lagrange spent 25 years in Prussia at the invitation of Frederick II. After Frederick's death, Lagrange moved to Paris, where he became a favourite of Marie Antoinette. He had mixed feelings about the Revolution, especially when his friend, the chemist Lavoisier, was guillotined, but he stuck it out. He was involved in the introduction of the decimal system for weights and measures. When people pleaded the advantages of the base 12, he would ironically defend the base 11. He became professor of mathematics at the Ecole Polytechnique.

Lagrange was a universal mathematical genius, his interests ranging from number theory to physics. Among his achievements are the following:

1. The first proof of Wilson's Theorem that, if p is a prime number, then it is a factor of $(p-1)!+1$;

2. The first complete solution of the Diophantine equation $x^2 - Ry^2 = 1$, where R is a given nonsquare positive integer; Lagrange generalized this to give a complete treatment of Diophantine equations of the form
$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$
where A, B, C, D, E, and F are given integers;

3. The first proof that every natural number is a sum of four squares of natural numbers (e.g., $7 = 2^2+1^2+1^2+1^2$ and $9 = 3^2+0^2+0^2+0^2$);

4. A systematic theory of differential equations;

5. The *Mécanique*, which he conceived at the age of 19 but only published at 52, in which he expressed the dynamics of a rigid system by the equations
$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\theta}} - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = 0 \, ,$$
where T is the total kinetic energy, V is the potential energy, t is the time, θ is any coordinate, and $\dot{\theta} = d\theta/dt$; Lagrange observed that his equations expressed the fact that the total *action* $\int_a^b (T-V)dt$ was minimal; to justify this observation, he had to invent the calculus of variations.