

where $s = (a + b + c + d)/2$, thus generalizing Heron's formula. He did not give a proof for this formula, and it seems that a careless copyist omitted the word 'cyclic'.

Following Brahmagupta, Mahavira discusses the properties of zero, stating that

$$a + 0 = a, \quad a - 0 = a, \quad a \cdot 0 = 0.$$

He does not quite know what to do with $a/0$, apparently believing that it equals a , unless we again blame a copyist. He also talks about negative numbers and asserts that they are not squares. He tries to give some social significance to his equations. When posing the equation

$$x/4 + 2\sqrt{x} + 15 = x,$$

he writes:

One fourth of a herd of camels was seen in a forest. Twice the square root of that herd had moved on to the mountain slope. Three times five camels, however, were found to remain on the river bank. What is the number of that herd of camels?

Bhaskara (1114–ca. 1185) named one of his mathematics books after his daughter Lilavati. This book deals with weights and measures, the decimal notation, the operations of arithmetic, reduction of fractions to common denominators, linear and quadratic equations, arithmetic and geometric progressions, interest and discount, triangles and quadrilaterals, approximations to π , trigonometric formulas, volumes of solids, indeterminate linear equations, and combinations. It contains the earliest extant exposition of the decimal notation using the sign for zero. Concerning calculations with zero, Bhaskara gives the following rules:

$$a \pm 0 = a, \quad 0^2 = 0, \quad \sqrt{0} = 0, \quad a/0 = \infty.$$

Some of his problems throw light on economic and social history. A female slave of age 16 is said to be worth eight oxen who have worked for two years; her price declines as she grows older.

In the first volume of his *History of Mathematics*, D.E. Smith relates that the *Lilavati* was translated into Persian in 1587 by someone called Fyzi. Fyzi claims that Bhaskara dedicated the book to his daughter in order to console her for remaining single. According to astrologers, there was but one lucky moment at which Lilavati might marry. Unfortunately, one of Lilavati's pearls fell into the water clock, and it stopped without anyone noticing. The lucky moment passed, and Lilavati missed her chance of an astrologically sound marriage. Bhaskara, accepting the advice of the astrologers, decided to give Lilavati a book of mathematics instead of a husband.

Perhaps the greatest Indian mathematician in the 20th century was Srinivasa Ramanujan (1887–1920), who was essentially self-taught and discovered many beautiful and ingenious formulas.

Exercises

1. Solve the system of three equations in three unknowns given in the Chiu Chang Suan Shu.
2. Find a 4 by 4 magic square, using the numbers from 1 to 16.
3. Find all the solutions to Sun Tsu's Chinese Remainder Problem.
4. Show that Ch'in Chiu Shao's polynomial had four linear factors.
5. Find a solution of De Gang Ma's equation (first posed by E. Lucas in 1875) with $x > 1$.
6. How many camels were there in Brahmagupta's problem?
7. How many bees were there in Bhaskara's problem?
8. If you know the sides of a cyclic quadrilateral, can you determine the radius of the circumscribing circle? Why? How?

22

Mathematics in Islamic Countries

When we speak of Arabic mathematicians, we must remember that Arabic was the common language of intellectuals in the Islamic world, just as Latin was in medieval Europe. In fact, the mathematicians may have been Turks or Persians.

During the period we are concerned with here, the intellectual center of the Arab world was Baghdad, which was founded in the 8th century and was the seat of the eastern Caliphs. It was visited by Greek and Jewish physicians from the West and by Indian and Persian scholars from the East.

Caliph al-Mamun established a ‘House of Wisdom’, or university, at Baghdad at the beginning of the 9th century. He ordered the translations of many Greek manuscripts, and it was thanks to his policy that many of these works were preserved. Three mathematicians associated with the House of Wisdom are

- Al-Khwarizmi, about 830,
- Thabit Ibn-Qurra (836–901),
- Al-Khayyami (Omar Khayyam) (ca. 1050–1123).

Muhammed ibn-Musa al-Khwarizmi probably came from Khorezm, which corresponds to the modern Khiva and the district surrounding it, south of the Aral Sea in central Asia. But some people deny this and say that he was merely a member of the Khwarizmi tribe, which in fact came to rule the Turko-Persian empire in 1194 AD. Our word ‘algorithm’ comes from a