

Johann II, was the least distinguished of the three, but he fathered the next generation of Bernoulli mathematicians, Jakob II and Johann III.



Figure 13.6: Johann Bernoulli

Daniel's path to mathematics was very similar to his father's. During his teens he was tutored by his older brother; his father wanted him to go into business, but when that career failed Daniel was permitted to study medicine.

He gained his doctorate in 1721 and made several attempts to win the chair of anatomy and botany in Basel, finally succeeding in 1733. By that time, however, he had drifted into mathematics, with such success that he had been called to the St. Petersburg Academy. During his years there (1725–1733) he conceived his ideas on modes of vibration and produced the first draft of his *Hydrodynamica*. Although he missed finding the basic partial differential equations of hydrodynamics, the *Hydrodynamica* made other important advances. One was the systematic use of a principle of conservation of energy; another was the kinetic theory of gases, including the derivation of Boyle's law that is now standard.



Figure 13.7: Daniel Bernoulli

Unfortunately, publication of the *Hydrodynamica* was delayed until 1738. This left Daniel's priority open to attack, and the one to take advantage of him was his own father. The self-styled Horatius of the priority dispute between Leibniz and Newton attempted the most blatant priority theft in the history of mathematics by publishing a book on hydrodynamics in 1743 and dating it 1732. Daniel was devastated, and wrote to Euler:

Of my entire *Hydrodynamics*, not one iota of which do in fact I owe to my father, I am all at once robbed completely and lose thus in one moment the fruits of the work of ten years. All propositions are taken from my *Hydrodynamics*, and then my father calls his writings *Hydraulics, now for the first time disclosed*, 1732, since my *Hydrodynamics* was printed only in 1738.

[Daniel Bernoulli (1743), in the Truesdell (1960) translation]

The situation was not quite as clear-cut as Daniel claimed [a detailed assessment is in Truesdell (1960)], but at any rate Johann Bernoulli's move backfired. His reputation was so tarnished by the episode that he did not even receive credit for parts of his work that *were* original. Daniel went on to enjoy fame and a long career, becoming professor of physics in 1750 and lecturing to enthusiastic audiences until 1776.

EXERCISES

13.6.1 Use integration by parts to show that

$$\int_0^1 x^n (\log x)^n dx = \frac{(-1)^n n!}{(n+1)^{n+1}}.$$

13.6.2 Deduce that

$$\int_0^1 x^x dx = 1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \dots$$

using a series expansion of $x^x = e^{x \log x}$.