$$0 = q'''an' + qc'n - q'(bn' + b'n)$$
$$0 = q''an' - q'a'n - q(bn' - b'n)$$

II. Iam ponamus, numeros integros $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{A}'$, $\mathfrak{B}'$, $\mathfrak{C}'$, $\mathfrak{N}$, $\mathfrak{N}'$ ita determinatos esse vt fiat $\mathfrak{A}a + 2\mathfrak{B}b + \mathfrak{C}c = m$, $\mathfrak{A}'a' + 2\mathfrak{B}'b' + \mathfrak{C}'c' = m'$, $\mathfrak{N}m'n + \mathfrak{N}'mn' = 1$. Tunc erit $\mathfrak{A}a\mathfrak{N}'n' + 2\mathfrak{B}b\mathfrak{N}'n' + \mathfrak{C}c\mathfrak{N}'n' + \mathfrak{A}'a'\mathfrak{N}n + 2\mathfrak{B}'b'\mathfrak{N}n + \mathfrak{C}'c'\mathfrak{N}n = 1$. Hinc atque ex aequatt. (III) facile confirmatur, si statuatur.

$$-q'\mathfrak{A}\mathfrak{N}' - q''\mathfrak{A}'\mathfrak{N} - q'''(\mathfrak{B}\mathfrak{N}' + \mathfrak{B}'\mathfrak{N}) = q$$
$$q'\mathfrak{A}\mathfrak{N}' - q'''\mathfrak{C}'\mathfrak{N} + q''(\mathfrak{B}\mathfrak{N}' - \mathfrak{B}'\mathfrak{N}) = q'$$
$$-q'''\mathfrak{C}\mathfrak{N}' + q\mathfrak{A}'\mathfrak{N} - q'(\mathfrak{B}\mathfrak{N}' - \mathfrak{B}'\mathfrak{N}) = q''$$
$$q''\mathfrak{C}\mathfrak{N}' + q'\mathfrak{C}'\mathfrak{N} + q(\mathfrak{B}\mathfrak{N}' + \mathfrak{B}'\mathfrak{N}) = q'''$$

fore . . . (IV)

$$q'an' + q''a'n + q'''(bn' + b'n) = q$$
$$-qan' + q'''c'n - q''(bn' - b'n) = q'$$
$$q'''cn' - qa'n + q'(bn' - b'n) = q''$$
$$-q''cn' - q'c'n - q(bn' + b'n) = q'''$$

Quoties $\mu = 1$, hae aequationes non sunt necessariae, sed ipsarum loco aequationes (I), quibus omnino analogae sunt, retineri possunt. Quodsi nunc ex aequatt. II, IV valores ipsorum Ann', $2Bnn'$, Cnn' (i. e. numerorum $q'q'' - qq'''$ etc.) euoluuntur, et quae mutuo se destruunt delentur: inuenietur, singulorum partes esse vel producta ex integris in nn', vel ex integris in $dn'n'$ vel ex integris in $d'nn$, insuperque omnes partes constituentes ipsius $2Bnn'$ implicare factorem 2. Hinc concluditur (quoniam

$dn'n' = d'nn$, et proin $\frac{dn'n'}{nn'} = \frac{d'nn}{nn'} = \sqrt{dd'}$ sunt integri), A, B, C esse numeros integros. *Q. E. P.*

III. Substituendo ex aequatt. (II) valores ipsorum p, p', p'', p''', facile comprobatur adiumento aequatt. (III) et huius $\mathfrak{P}q + \mathfrak{P}'q' + \mathfrak{P}''q'' + \mathfrak{P}'''q''' = 1$, esse $pq' - qp' = an'$, $pq''' - qp''' - p'q'' + q'p'' = 2bn'$, $p''q''' - q''p''' = cn'$, $pq'' - qp'' = a'n$, $pq''' - qp''' + p'q'' - q'p'' = 2b'n$, $p'q''' - q'p''' = c'n$, quae aequationes identicae sunt cum sex prioribus (Ω) art. praec.; tres reliquae autem iam per hyp. locum habent. Quare (*ibid. sub fin.*) forma F transibit in ff' per substitutionem p, p', p'', p'''; q, q', q'', q'''; ipsiusque determinans erit $= D$, siue aequalis diuis. comm. max. numerorum $dm'm'$, $d'mm$, quamobrem per concl. quartam art. praec. F ex f, f' composita erit. *Q. E. S.* Denique facile perspicietur, F ex f, f' *ita* compositam esse vt praescriptum sit, quum signa quantitatum n, n' iam ab initio rite sint determinata.

237. Theorema. *Si forma F in productum e duabus formis f, f' est transformabilis, atque forma f' formam f'' implicat: F etiam in productum e formis f, f'' transformabilis erit.*

Dem. Retineantur pro formis F, f, f' omnia signa art. 235; forma f'' sit $= (a'', b'', c'')$, transeatque f' in f'' per substitutionem α, β, γ, δ. Tunc nullo negotio perspicietur, F transire in ff'' per substitutionem $\alpha p + \gamma p'$, $\beta p + \delta p'$, $\alpha p'' +$

$\gamma p'''$, $\beta p'' + \delta p'''$; $\alpha q + \gamma q'$, $\beta q + \delta q'$; $\alpha q'' + \gamma q'''$, $\beta q'' + \delta q'''$. *Q. E. D.*

Positis breuitatis caussa coefficientibus $\alpha p + \gamma p'$, $\beta p + \delta p'$ etc., $= \mathfrak{P}, \mathfrak{P}', \mathfrak{P}'', \mathfrak{P}'''$; $\mathfrak{Q}, \mathfrak{Q}', \mathfrak{Q}'', \mathfrak{Q}'''$; numeroque $\alpha\delta - \beta\gamma = e$: ex aequatt. Ω art. 235 facile confirmatur, esse $\mathfrak{P}\mathfrak{Q}' - \mathfrak{Q}\mathfrak{P}' = an'e$, $\mathfrak{P}\mathfrak{Q}''' - \mathfrak{Q}\mathfrak{P}''' - \mathfrak{P}'\mathfrak{Q}'' + \mathfrak{Q}'\mathfrak{P}'' = 2bn'e$, $\mathfrak{P}''\mathfrak{Q}''' - \mathfrak{Q}''\mathfrak{P}''' = cn'e$; $\mathfrak{P}\mathfrak{Q}'' - \mathfrak{Q}\mathfrak{P}'' = \alpha\alpha a'n + 2\alpha\gamma b'n + \gamma\gamma c'n = a''n$, $\mathfrak{P}\mathfrak{Q}''' - \mathfrak{Q}\mathfrak{P}''' + \mathfrak{P}'\mathfrak{Q}'' - \mathfrak{Q}'\mathfrak{P}'' = 2b''n$, $\mathfrak{P}'\mathfrak{Q}''' - \mathfrak{Q}'\mathfrak{P}''' = c''n$; $\mathfrak{Q}'\mathfrak{Q}'' - \mathfrak{Q}\mathfrak{Q}''' = Ann'e$, $\mathfrak{P}\mathfrak{Q}''' + \mathfrak{Q}\mathfrak{P}''' - \mathfrak{P}'\mathfrak{Q}'' - \mathfrak{Q}'\mathfrak{P}'' = 2Bnn'e$, $\mathfrak{P}'\mathfrak{P}'' - \mathfrak{P}\mathfrak{P}''' = Cnn'e$. Iam designato determinante formae f'' per d'', erit e radix quadrata ex $\frac{d''}{d'}$, et quidem positiua vel negatiua, prout forma f' formam f'' vel proprie vel improprie implicat. Quare $n'e$ erit radix quadrata ex $\frac{d''}{D}$; vnde patet, nouem aequationes praecedentes aequationibus Ω art. 235 prorsus analogas esse, formamque f in transformatione formae F in ff'' eodem modo accipi, vt in transformatione formae F in ff'; formam f'' vero in illa vel eodem modo vt f' in hac, vel opposito, prout f' ipsam f'' proprie implicet vel improprie.

238. Theorema. *Si forma F sub forma F' est contenta atque in productum e formis f, f' transformabilis: etiam forma F' in idem productum transformabilis erit.*

Dem. Retentis pro formis F, f, f' iisdem signis vt supra et supponendo formam F' trans-