

ΠΡ ὁποτέρω τῶν MZ, ΝΘ ὁμοιόν τε καὶ ὁμοίως κείμενον εὐθύγραμμον τὸ ΣΡ.

Ἐπεὶ οὖν ἔστιν ὡς ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ EZ πρὸς τὴν ΠΡ, καὶ ἀναγέγραπται ἀπὸ μὲν τῶν AB, ΓΔ ὁμοιά τε καὶ ὁμοίως κείμενα τὰ KAB, ΛΓΔ, ἀπὸ δὲ τῶν EZ, ΠΡ ὁμοιά τε καὶ ὁμοίως κείμενα τὰ MZ, ΣΡ, ἔστιν ἄρα ὡς τὸ KAB πρὸς τὸ ΛΓΔ, οὕτως τὸ MZ πρὸς τὸ ΣΡ. ὑπόκειται δὲ καὶ ὡς τὸ KAB πρὸς τὸ ΛΓΔ, οὕτως τὸ MZ πρὸς τὸ ΝΘ· καὶ ὡς ἄρα τὸ MZ πρὸς τὸ ΣΡ, οὕτως τὸ MZ πρὸς τὸ ΝΘ. τὸ MZ ἄρα πρὸς ἑκάτερον τῶν ΝΘ, ΣΡ τὸν αὐτὸν ἔχει λόγον· ἵσον ἄρα ἔστι τὸ ΝΘ τῷ ΣΡ. ἔστι δὲ αὐτῷ καὶ ὁμοιόν τε καὶ ὁμοίως κείμενον· ἵση ἄρα ἡ ΗΘ τῇ ΠΡ. καὶ ἐπεὶ ἔστιν ὡς ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ EZ πρὸς τὴν ΠΡ, ἵση δὲ ἡ ΠΡ τῇ ΗΘ, ἔστιν ἄρα ὡς ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ EZ πρὸς τὴν ΗΘ.

Ἐὰν ἄρα τέσσαρες εὐθεῖαι ἀνάλογον ὕστειν, καὶ τὰ ἀπὸ αὐτῶν εὐθύγραμμα ὁμοιά τε καὶ ὁμοίως ἀναγέγραμμένα ἀνάλογον ἔσται· καὶ τὰ ἀπὸ αὐτῶν εὐθύγραμμα ὁμοιά τε καὶ ὁμοίως ἀναγέγραμμένα ἀνάλογον ἔσονται· ὅπερ ἔδει δεῖξαι.

(is) to  $QR$  [Prop. 6.12]. And let the rectilinear figure  $SR$ , similar, and similarly laid down, to either of  $MF$  or  $NH$ , have been described on  $QR$  [Props. 6.18, 6.21].

Therefore, since as  $AB$  is to  $CD$ , so  $EF$  (is) to  $QR$ , and the similar, and similarly laid out, (rectilinear figures)  $KAB$  and  $LCD$  have been described on  $AB$  and  $CD$  (respectively), and the similar, and similarly laid out, (rectilinear figures)  $MF$  and  $SR$  on  $EF$  and  $QR$  (respectively), thus as  $KAB$  is to  $LCD$ , so  $MF$  (is) to  $SR$  (see above). And it was also assumed that as  $KAB$  (is) to  $LCD$ , so  $MF$  (is) to  $NH$ . Thus, also, as  $MF$  (is) to  $SR$ , so  $MF$  (is) to  $NH$  [Prop. 5.11]. Thus,  $MF$  has the same ratio to each of  $NH$  and  $SR$ . Thus,  $NH$  is equal to  $SR$  [Prop. 5.9]. And it is also similar, and similarly laid out, to it. Thus,  $GH$  (is) equal to  $QR$ .<sup>†</sup> And since  $AB$  is to  $CD$ , as  $EF$  (is) to  $QR$ , and  $QR$  (is) equal to  $GH$ , thus as  $AB$  is to  $CD$ , so  $EF$  (is) to  $GH$ .

Thus, if four straight-lines are proportional, then similar, and similarly described, rectilinear figures (drawn) on them will also be proportional. And if similar, and similarly described, rectilinear figures (drawn) on them are proportional then the straight-lines themselves will also be proportional. (Which is) the very thing it was required to show.

<sup>†</sup> Here, Euclid assumes, without proof, that if two similar figures are equal then any pair of corresponding sides is also equal.

κγ'.

Τὰ ἵσογώνια παραλληλόγραμμα πρὸς ἄλληλα λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν.

Ἐστω ἵσογώνια παραλληλόγραμμα τὰ ΑΓ, ΓΖ ἵσην ἔχοντα τὴν ὑπὸ ΒΓΔ γωνίαν τῇ ὑπὸ ΕΓΗ· λέγω, ὅτι τὸ ΑΓ παραλληλόγραμμον πρὸς τὸ ΓΖ παραλληλόγραμμον λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν.

Κείσθω γάρ ὕστε ἐπ’ εὐθείας εἶναι τὴν ΒΓ τῇ ΓΗ· ἐπ’ εὐθείας ἄρα ἔστι καὶ ἡ ΔΓ τῇ ΓΕ· καὶ συμπεπληρώσθω τὸ ΔΗ παραλληλόγραμμον, καὶ ἔκκεισθω τις εὐθεῖα ἡ Κ, καὶ γεγονέτω ὡς μὲν ἡ ΒΓ πρὸς τὴν ΓΗ, οὕτως ἡ Κ πρὸς τὴν Λ, ὡς δὲ ἡ ΔΓ πρὸς τὴν ΓΕ, οὕτως ἡ Λ πρὸς τὴν Μ.

Οἱ ἄρα λόγοι τῆς τε Κ πρὸς τὴν Λ καὶ τῆς Λ πρὸς τὴν Μ οἱ αὐτοί εἰσι τοῖς λόγοις τῶν πλευρῶν, τῆς τε ΒΓ πρὸς τὴν ΓΗ καὶ τῆς ΔΓ πρὸς τὴν ΓΕ. ἀλλ’ ὁ τῆς Κ πρὸς Μ λόγος σύγκειται ἐκ τε τοῦ τῆς Κ πρὸς Λ λόγου καὶ τοῦ τῆς Λ πρὸς Μ· ὕστε καὶ ἡ Κ πρὸς τὴν Μ λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν. καὶ ἐπεὶ ἔστιν ὡς ἡ ΒΓ πρὸς τὴν ΓΗ, οὕτως τὸ ΑΓ παραλληλόγραμμον πρὸς τὸ ΓΘ, ἀλλ’ ὡς ἡ ΒΓ πρὸς τὴν ΓΗ, οὕτως ἡ Κ πρὸς τὴν Λ, καὶ ὡς ἄρα ἡ Κ πρὸς τὴν Λ, οὕτως τὸ ΑΓ πρὸς τὸ ΓΘ. πάλιν, ἐπεὶ ἔστιν ὡς ἡ ΔΓ πρὸς τὴν ΓΕ, οὕτως τὸ ΓΘ παραλληλόγραμμον πρὸς τὸ ΓΖ, ἀλλ’ ὡς ἡ ΔΓ πρὸς τὴν ΓΕ,

Equiangular parallelograms have to one another the ratio compounded<sup>†</sup> out of (the ratios of) their sides.

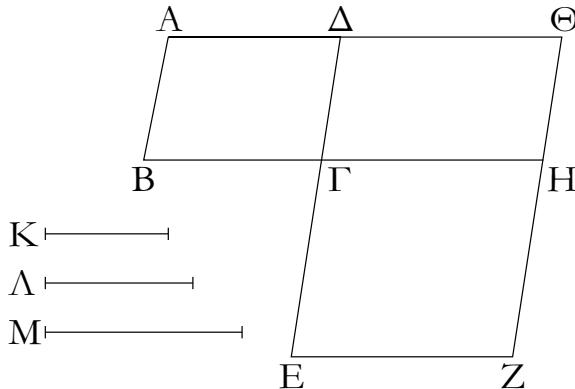
Let  $AC$  and  $CF$  be equiangular parallelograms having angle  $BCD$  equal to  $ECG$ . I say that parallelogram  $AC$  has to parallelogram  $CF$  the ratio compounded out of (the ratios of) their sides.

For let  $BC$  be laid down so as to be straight-on to  $CG$ . Thus,  $DC$  is also straight-on to  $CE$  [Prop. 1.14]. And let the parallelogram  $DG$  have been completed. And let some straight-line  $K$  have been laid down. And let it be contrived that as  $BC$  (is) to  $CG$ , so  $K$  (is) to  $L$ , and as  $DC$  (is) to  $CE$ , so  $L$  (is) to  $M$  [Prop. 6.12].

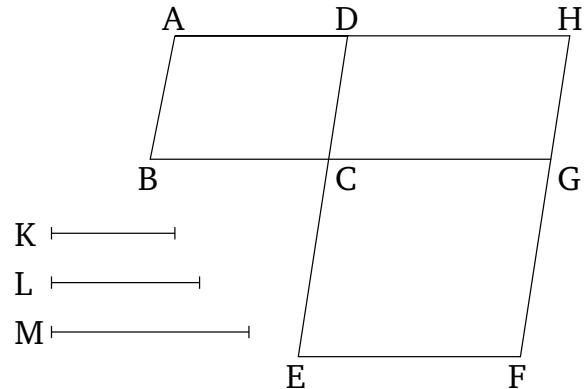
Thus, the ratios of  $K$  to  $L$  and of  $L$  to  $M$  are the same as the ratios of the sides, (namely),  $BC$  to  $CG$  and  $DC$  to  $CE$  (respectively). But, the ratio of  $K$  to  $M$  is compounded out of the ratio of  $K$  to  $L$  and (the ratio) of  $L$  to  $M$ . Hence,  $K$  also has to  $M$  the ratio compounded out of (the ratios of) the sides (of the parallelograms). And since as  $BC$  is to  $CG$ , so parallelogram  $AC$  (is) to  $CH$  [Prop. 6.1], but as  $BC$  (is) to  $CG$ , so  $K$  (is) to  $L$ , thus, also, as  $K$  (is) to  $L$ , so (parallelogram)  $AC$  (is) to  $CH$ . Again, since as  $DC$  (is) to  $CE$ , so parallelogram

οὗτως ἡ Λ πρὸς τὴν Μ, καὶ ὡς ἄρα ἡ Λ πρὸς τὴν Μ, οὗτως τὸ ΓΘ παραλληλόγραμμον πρὸς τὸ ΓΖ παραλληλόγραμμον. ἐπεὶ οὖν ἐδείχθη, ὡς μὲν ἡ Κ πρὸς τὴν Λ, οὗτως τὸ ΑΓ παραλληλόγραμμον πρὸς τὸ ΓΘ παραλληλόγραμμον, ὡς δὲ ἡ Λ πρὸς τὴν Μ, οὗτως τὸ ΓΘ παραλληλόγραμμον πρὸς τὸ ΓΖ παραλληλόγραμμον, διὸ οὖν ἄρα ἐστὶν ὡς ἡ Κ πρὸς τὴν Μ, οὗτως τὸ ΑΓ πρὸς τὸ ΓΖ παραλληλόγραμμον. ἡ δὲ Κ πρὸς τὴν Μ λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν· καὶ τὸ ΑΓ ἄρα πρὸς τὸ ΓΖ λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν.

$CH$  (is) to  $CF$  [Prop. 6.1], but as  $DC$  (is) to  $CE$ , so  $L$  (is) to  $M$ , thus, also, as  $L$  (is) to  $M$ , so parallelogram  $CH$  (is) to parallelogram  $CF$ . Therefore, since it was shown that as  $K$  (is) to  $L$ , so parallelogram  $AC$  (is) to parallelogram  $CH$ , and as  $L$  (is) to  $M$ , so parallelogram  $CH$  (is) to parallelogram  $CF$ , thus, via equality, as  $K$  is to  $M$ , so (parallelogram)  $AC$  (is) to parallelogram  $CF$  [Prop. 5.22]. And  $K$  has to  $M$  the ratio compounded out of (the ratios of) the sides (of the parallelograms). Thus, (parallelogram)  $AC$  also has to (parallelogram)  $CF$  the ratio compounded out of (the ratio of) their sides.



Τὰ ἄρα ισογώνια παραλληλόγραμμα πρὸς ἄλληλα λόγον ἔχει τὸν συγκείμενον ἐκ τῶν πλευρῶν· ὅπερ ἔδει δεῖξαι.



Thus, equiangular parallelograms have to one another the ratio compounded out of (the ratio of) their sides. (Which is) the very thing it was required to show.

<sup>†</sup> In modern terminology, if two ratios are “compounded” then they are multiplied together.

$\chi\delta'$ .

Παντὸς παραλληλογράμμου τὰ περὶ τὴν διάμετρον παραλληλόγραμμα ὅμοιά ἔστι τῷ τε ὅλῳ καὶ ἀλλήλοις.

Ἐστω παραλληλόγραμμον τὸ ΑΒΓΔ, διάμετρος δὲ αὐτοῦ ἡ ΑΓ, περὶ δὲ τὴν ΑΓ παραλληλόγραμμα ἔστω τὰ ΕΗ, ΘΚ· λέγω, ὅτι ἐκάτερον τῶν ΕΗ, ΘΚ παραλληλογράμμων ὅμοιόν ἔστι ὅλῳ τῷ ΑΒΓΔ καὶ ἀλλήλοις.

Ἐπεὶ γὰρ τριγώνου τοῦ ΑΒΓ παρὰ μίαν τῶν πλευρῶν τὴν ΒΓ ἤκται ἡ ΕΖ, ἀνάλογόν ἔστιν ὡς ἡ ΒΕ πρὸς τὴν ΕΑ, οὗτως ἡ ΓΖ πρὸς τὴν ΖΑ. πάλιν, ἐπεὶ τριγώνου τοῦ ΑΓΔ παρὰ μίαν τὴν ΓΔ ἤκται ἡ ΖΗ, ἀνάλογόν ἔστιν ὡς ἡ ΓΖ πρὸς τὴν ΖΑ, οὗτως ἡ ΔΗ πρὸς τὴν ΗΑ. ἀλλ' ὡς ἡ ΓΖ πρὸς τὴν ΖΑ, οὗτως ἐδείχθη καὶ ἡ ΒΕ πρὸς τὴν ΕΑ· καὶ ὡς ἄρα ἡ ΒΕ πρὸς τὴν ΕΑ, οὗτως ἡ ΔΗ πρὸς τὴν ΗΑ, καὶ συνθέντι ἄρα ὡς ἡ ΒΑ πρὸς ΑΕ, οὗτως ἡ ΔΑ πρὸς ΑΗ, καὶ ἐναλλάξ ὡς ἡ ΒΑ πρὸς τὴν ΑΔ, οὗτως ἡ ΕΑ πρὸς τὴν ΑΗ. τῶν ἄρα ΑΒΓΔ, ΕΗ παραλληλογράμμων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὴν κοινὴν γωνίαν τὴν ὑπὸ ΒΑΔ· καὶ ἐπεὶ παραλληλός ἔστιν ἡ ΖΗ τῇ ΔΓ, ἵστη ἐστὶν ἡ μὲν ὑπὸ ΑΖΗ γωνία τῇ ὑπὸ ΔΓΑ· καὶ κοινὴ τῶν δύο

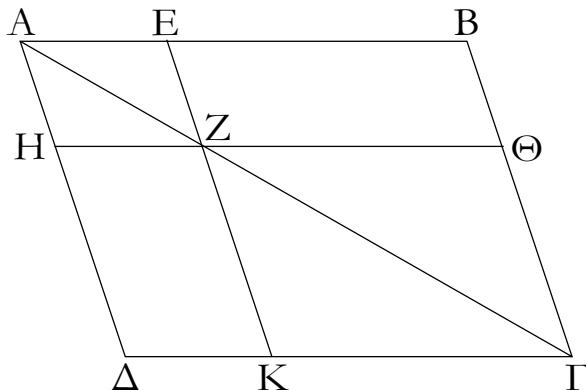
### Proposition 24

In any parallelogram the parallelograms about the diagonal are similar to the whole, and to one another.

Let  $ABCD$  be a parallelogram, and  $AC$  its diagonal. And let  $EG$  and  $HK$  be parallelograms about  $AC$ . I say that the parallelograms  $EG$  and  $HK$  are each similar to the whole (parallelogram)  $ABCD$ , and to one another.

For since  $EF$  has been drawn parallel to one of the sides  $BC$  of triangle  $ABC$ , proportionally, as  $BE$  is to  $EA$ , so  $CF$  (is) to  $FA$  [Prop. 6.2]. Again, since  $FG$  has been drawn parallel to one (of the sides)  $CD$  of triangle  $ACD$ , proportionally, as  $CF$  is to  $FA$ , so  $DG$  (is) to  $GA$  [Prop. 6.2]. But, as  $CF$  (is) to  $FA$ , so it was also shown (is)  $BE$  to  $EA$ . And thus as  $BE$  (is) to  $EA$ , so  $DG$  (is) to  $GA$ . And, thus, compounding, as  $BA$  (is) to  $AE$ , so  $DA$  (is) to  $AG$  [Prop. 5.18]. And, alternately, as  $BA$  (is) to  $AD$ , so  $EA$  (is) to  $AG$  [Prop. 5.16]. Thus, in parallelograms  $ABCD$  and  $EG$  the sides about the common angle  $BAD$  are proportional. And since  $GF$  is parallel to  $DC$ , angle  $AFG$  is equal to  $DCA$  [Prop. 1.29].

τριγώνων τῶν  $A\Delta\Gamma$ ,  $AHZ$  ἡ ὑπὸ  $\Delta AG$  γωνία· ἵσοι γώνιον ἄρα ἐστὶ τὸ  $A\Delta\Gamma$  τρίγωνον τῷ  $AHZ$  τριγώνῳ. διὰ τὰ αὐτὰ δὴ καὶ τὸ  $AG\Gamma$  τρίγωνον ἵσοι γώνιον ἐστὶ τῷ  $AZE$  τριγώνῳ, καὶ ὅλον τὸ  $AB\Gamma\Delta$  παραλληλόγραμμον τῷ  $EH$  παραλληλογράμμῳ ἵσοι γώνιον ἐστιν. ἀνάλογον ἄρα ἐστὶν ὡς ἡ  $AD$  πρὸς τὴν  $\Delta\Gamma$ , οὕτως ἡ  $AH$  πρὸς τὴν  $HZ$ , ὡς δὲ ἡ  $\Delta\Gamma$  πρὸς τὴν  $GA$ , οὕτως ἡ  $HZ$  πρὸς τὴν  $ZA$ , ὡς δὲ ἡ  $AG$  πρὸς τὴν  $\Gamma B$ , οὕτως ἡ  $AZ$  πρὸς τὴν  $ZE$ , καὶ ἔτι ὡς ἡ  $\Gamma B$  πρὸς τὴν  $BA$ , οὕτως ἡ  $ZE$  πρὸς τὴν  $EA$ . καὶ ἐπεὶ ἐδείχθη ὡς μὲν ἡ  $\Delta\Gamma$  πρὸς τὴν  $GA$ , οὕτως ἡ  $HZ$  πρὸς τὴν  $ZA$ , ὡς δὲ ἡ  $AG$  πρὸς τὴν  $\Gamma B$ , οὕτως ἡ  $AZ$  πρὸς τὴν  $ZE$ , δι’ ἵσου ἄρα ἐστὶν ὡς ἡ  $\Delta\Gamma$  πρὸς τὴν  $\Gamma B$ , οὕτως ἡ  $HZ$  πρὸς τὴν  $ZE$ . τῶν ἄρα  $AB\Gamma\Delta$ ,  $EH$  παραλληλογράμμων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας· ὅμοιον ἄρα ἐστὶ τὸ  $AB\Gamma\Delta$  παραλληλογράμμον τῷ  $EH$  παραλληλογράμμῳ. διὰ τὰ αὐτὰ δὴ τὸ  $AB\Gamma\Delta$  παραλληλόγραμμον καὶ τῷ  $K\Theta$  παραλληλογράμμῳ ὅμοιόν ἐστιν· ἐκάτερον ἄρα τῶν  $EH$ ,  $\Theta K$  παραλληλογράμμων τῷ  $AB\Gamma\Delta$  [παραλληλογράμμῳ] ὅμοιόν ἐστιν. τὰ δὲ τῷ οὐτῷ εὐθυγράμμῳ ὅμοια καὶ ἀλλήλοις ἐστὶν ὅμοια· καὶ τὸ  $EH$  ἄρα παραλληλόγραμμον τῷ  $\Theta K$  παραλληλογράμμῳ ὅμοιόν ἐστιν.

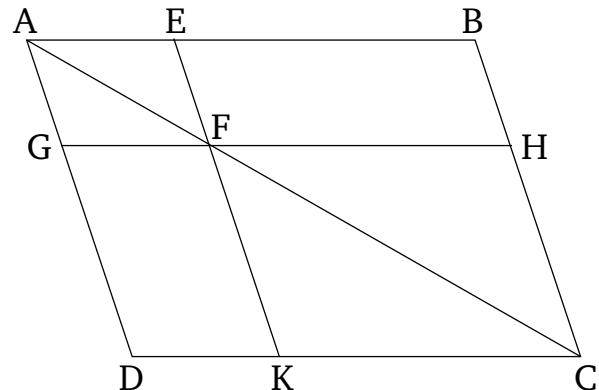


Παντὸς ἄρα παραλληλογράμμου τὰ περὶ τὴν διάμετρον παραλληλόγραμμα ὅμοιά ἐστι τῷ τε ὅλῳ καὶ ἀλλήλοις· ὅπερ ἔδει δεῖξαι.

κε'.

Τῷ δοθέντι εὐθυγράμμῳ ὅμοιον καὶ ἄλλῳ τῷ δοθέντι ἵσον τὸ αὐτὸς συστήσασθαι.

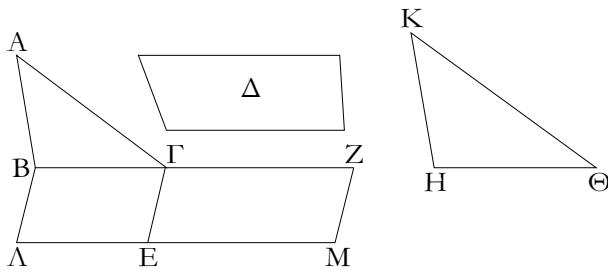
And angle  $DAC$  (is) common to the two triangles  $ADC$  and  $AGF$ . Thus, triangle  $ADC$  is equiangular to triangle  $AGF$  [Prop. 1.32]. So, for the same (reasons), triangle  $ACB$  is equiangular to triangle  $AFE$ , and the whole parallelogram  $ABCD$  is equiangular to parallelogram  $EG$ . Thus, proportionally, as  $AD$  (is) to  $DC$ , so  $AG$  (is) to  $GF$ , and as  $DC$  (is) to  $CA$ , so  $GF$  (is) to  $FA$ , and as  $AC$  (is) to  $CB$ , so  $AF$  (is) to  $FE$ , and, further, as  $CB$  (is) to  $BA$ , so  $FE$  (is) to  $EA$  [Prop. 6.4]. And since it was shown that as  $DC$  is to  $CA$ , so  $GF$  (is) to  $FA$ , and as  $AC$  (is) to  $CB$ , so  $AF$  (is) to  $FE$ , thus, via equality, as  $DC$  is to  $CB$ , so  $GF$  (is) to  $FE$  [Prop. 5.22]. Thus, in parallelograms  $ABCD$  and  $EG$  the sides about the equal angles are proportional. Thus, parallelogram  $ABCD$  is similar to parallelogram  $EG$  [Def. 6.1]. So, for the same (reasons), parallelogram  $ABCD$  is also similar to parallelogram  $KH$ . Thus, parallelograms  $EG$  and  $KH$  are each similar to [parallelogram]  $ABCD$ . And (rectilinear figures) similar to the same rectilinear figure are also similar to one another [Prop. 6.21]. Thus, parallelogram  $EG$  is also similar to parallelogram  $KH$ .



Thus, in any parallelogram the parallelograms about the diagonal are similar to the whole, and to one another. (Which is) the very thing it was required to show.

### Proposition 25

To construct a single (rectilinear figure) similar to a given rectilinear figure, and equal to a different given rectilinear figure.

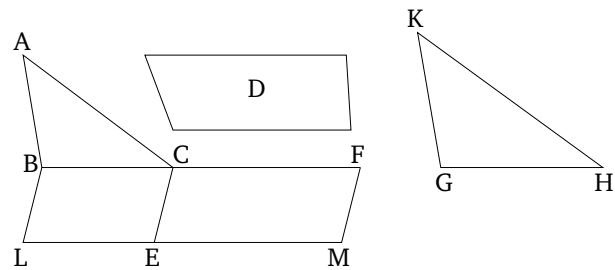


Ἐστω τὸ μὲν δοιθὲν εὐθύγραμμον, ὃ δεῖ ὅμοιον συστήσασθαι, τὸ ΑΒΓ, ὃ δὲ δεῖ ἵσον, τὸ Δ· δεῖ δὴ τῷ μὲν ΑΒΓ ὅμοιον, τῷ δὲ Δ ἵσον τὸ αὐτὸ συστήσασθαι.

Παραβεβλήσθω γὰρ παρὰ μὲν τὴν ΒΓ τῷ ΑΒΓ τριγώνῳ ἵσον παραλληλόγραμμον τὸ ΒΕ, παρὰ δὲ τὴν ΓΕ τῷ Δ ἵσον παραλληλόγραμμον τὸ ΓΜ ἐν γωνίᾳ τῇ ὑπὸ ΖΓΕ, ἡ ἐστιν ἵση τῇ ὑπὸ ΓΒΔ. ἐπ' εὐθείας ἄρα ἐστὶν ἡ μὲν ΒΓ τῇ ΓΖ, ἡ δὲ ΑΕ τῇ ΕΜ. καὶ εἰλήφθω τῶν ΒΓ, ΓΖ μέση ἀνάλογον ἡ ΗΘ, καὶ ἀναγεγράφθω ἀπὸ τῆς ΗΘ τῷ ΑΒΓ ὅμοιον τε καὶ ὁμοίως κείμενον τὸ ΚΗΘ.

Καὶ ἐπεὶ ἐστιν ὡς ἡ ΒΓ πρὸς τὴν ΗΘ, οὕτως ἡ ΗΘ πρὸς τὴν ΓΖ, ἐὰν δὲ τρεῖς εὐθεῖαι ἀνάλογον ὁσιν, ἐστιν ὡς ἡ πρώτη πρὸς τὴν τρίτην, οὕτως τὸ ἀπὸ τῆς πρώτης εἴδος πρὸς τὸ ἀπὸ τῆς δευτέρας τὸ ὅμοιον καὶ ὁμοίως ἀναγραφόμενον, ἐστιν ἄρα ὡς ἡ ΒΓ πρὸς τὴν ΓΖ, οὕτως τὸ ΑΒΓ τριγώνον πρὸς τὸ ΚΗΘ τριγώνον. ὅλλα καὶ ὡς ἡ ΒΓ πρὸς τὴν ΓΖ, οὕτως τὸ ΒΕ παραλληλόγραμμον πρὸς τὸ ΕΖ παραλληλόγραμμον. καὶ ὡς ἄρα τὸ ΑΒΓ τριγώνον πρὸς τὸ ΚΗΘ τριγώνον, οὕτως τὸ ΒΕ παραλληλόγραμμον πρὸς τὸ ΕΖ παραλληλόγραμμον· ἐναλλὰξ ἄρα ὡς τὸ ΑΒΓ τριγώνον πρὸς τὸ ΒΕ παραλληλόγραμμον, οὕτως τὸ ΚΗΘ τριγώνον πρὸς τὸ ΕΖ παραλληλόγραμμον. ἵσον δὲ τὸ ΑΒΓ τριγώνον τῷ ΒΕ παραλληλογράμμῳ· ἵσον ἄρα καὶ τὸ ΚΗΘ τριγώνον τῷ ΕΖ παραλληλογράμμῳ. ὅλλα τὸ ΕΖ παραλληλόγραμμον τῷ Δ ἐστιν ἵσον· καὶ τὸ ΚΗΘ ἄρα τῷ Δ ἐστιν ἵσον. ἐστι δὲ τὸ ΚΗΘ καὶ τῷ ΑΒΓ ὅμοιον.

Τῷ ἄρα δοιθέντι εὐθύγράμμῳ τῷ ΑΒΓ ὅμοιον καὶ ὁλῶ τῷ δοιθέντι τῷ Δ ἵσον τὸ αὐτὸ συνέσταται τὸ ΚΗΘ· ὅπερ ἔδει ποιῆσαι.



Let  $ABC$  be the given rectilinear figure to which it is required to construct a similar (rectilinear figure), and  $D$  the (rectilinear figure) to which (the constructed figure) is required (to be) equal. So it is required to construct a single (rectilinear figure) similar to  $ABC$ , and equal to  $D$ .

For let the parallelogram  $BE$ , equal to triangle  $ABC$ , have been applied to (the straight-line)  $BC$  [Prop. 1.44], and the parallelogram  $CM$ , equal to  $D$ , (have been applied) to (the straight-line)  $CE$ , in the angle  $FCE$ , which is equal to  $CBL$  [Prop. 1.45]. Thus,  $BC$  is straight-on to  $CF$ , and  $LE$  to  $EM$  [Prop. 1.14]. And let the mean proportion  $GH$  have been taken of  $BC$  and  $CF$  [Prop. 6.13]. And let  $KGH$ , similar, and similarly laid out, to  $ABC$  have been described on  $GH$  [Prop. 6.18].

And since as  $BC$  is to  $GH$ , so  $GH$  (is) to  $CF$ , and if three straight-lines are proportional then as the first is to the third, so the figure (described) on the first (is) to the similar, and similarly described, (figure) on the second [Prop. 6.19 corr.], thus as  $BC$  is to  $CF$ , so triangle  $ABC$  (is) to triangle  $KGH$ . But, also, as  $BC$  (is) to  $CF$ , so parallelogram  $BE$  (is) to parallelogram  $EF$  [Prop. 6.1]. And, thus, as triangle  $ABC$  (is) to triangle  $KGH$ , so parallelogram  $BE$  (is) to parallelogram  $EF$ . Thus, alternately, as triangle  $ABC$  (is) to parallelogram  $BE$ , so triangle  $KGH$  (is) to parallelogram  $EF$  [Prop. 5.16]. And triangle  $ABC$  (is) equal to parallelogram  $BE$ . Thus, triangle  $KGH$  (is) also equal to parallelogram  $EF$ . But, parallelogram  $EF$  is equal to  $D$ . Thus,  $KGH$  is also equal to  $D$ . And  $KGH$  is also similar to  $ABC$ .

Thus, a single (rectilinear figure)  $KGH$  has been constructed (which is) similar to the given rectilinear figure  $ABC$ , and equal to a different given (rectilinear figure)  $D$ . (Which is) the very thing it was required to do.

χτ'.

Ἐὰν ἀπὸ παραλληλογράμμου παραλληλόγραμμον ἀφαιρεθῇ ὅμοιον τε τῷ ὅλῳ καὶ ὁμοίως κείμενον κοινὴν γωνίαν ἔχον αὐτῷ, περὶ τὴν αὐτὴν διάμετρόν ἐστι τῷ ὅλῳ.

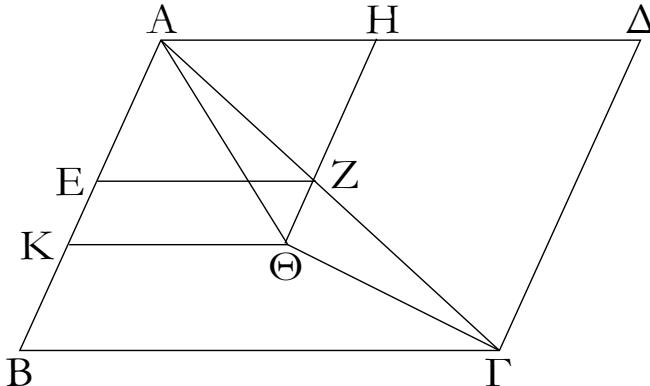
Ἀπὸ γὰρ παραλληλογράμμου τοῦ ΑΒΓΔ παραλληλόγραμμον ἀφηρήσθω τὸ ΑΖ ὅμοιον τῷ ΑΒΓΔ καὶ ὁμοίως κείμενον κοινὴν γωνίαν ἔχον αὐτῷ τὴν ὑπὸ ΔΑΒ· λέγω,

### Proposition 26

If from a parallelogram a(nother) parallelogram is subtracted (which is) similar, and similarly laid out, to the whole, having a common angle with it, then (the subtracted parallelogram) is about the same diagonal as the whole.

For, from parallelogram  $ABCD$ , let (parallelogram)

ὅτι περὶ τὴν αὐτὴν διάμετρόν ἐστι τὸ ΑΒΓΔ τῷ ΑΖ.

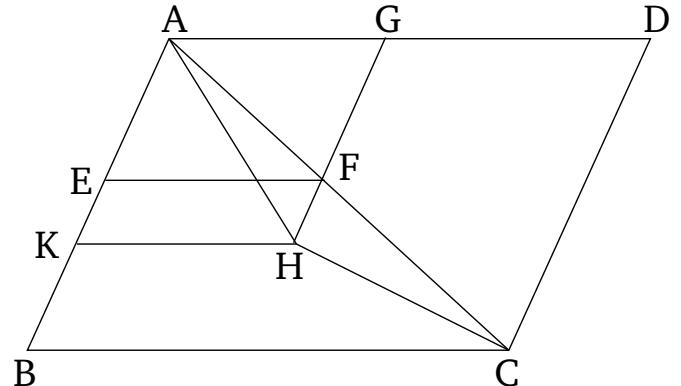


Μὴ γάρ, ἀλλ᾽ εἰ δυνατόν, ἔστω [αὐτῶν] διάμετρος ἡ ΑΘΓ, καὶ ἐκβληθεῖσα ἡ ΗΖ διήχθω ἐπὶ τὸ Θ, καὶ ἤχθω διὰ τοῦ Θ ὁπορέρα τῶν ΑΔ, ΒΓ παράλληλος ἡ ΘΚ.

Ἐπεὶ οὖν περὶ τὴν αὐτὴν διάμετρόν ἐστι τὸ ΑΒΓΔ τῷ ΚΗ, ἔστιν ἄρα ὡς ἡ ΔΑ πρὸς τὴν ΑΒ, οὕτως ἡ ΗΑ πρὸς τὴν ΑΚ. ἔστι δὲ καὶ διὰ τὴν ὁμοιότητα τῶν ΑΒΓΔ, ΕΗ καὶ ὡς ἡ ΔΑ πρὸς τὴν ΑΒ, οὕτως ἡ ΗΑ πρὸς τὴν ΑΕ· καὶ ὡς ἄρα ἡ ΗΑ πρὸς τὴν ΑΚ, οὕτως ἡ ΗΑ πρὸς τὴν ΑΕ. ἡ ΗΑ ἄρα πρὸς τὴν ΑΚ ἐκατέρων τῶν ΑΚ, ΑΕ τὸν αὐτὸν ἔχει λόγον. Ἰση ἄρα ἔστιν ἡ ΑΕ τῇ ΑΚ ἡ ἐλάττων τῇ μείζονι· ὅπερ ἔστιν ἀδύνατον. οὐκ ἄρα οὔτι περὶ τὴν αὐτὴν διάμετρον τὸ ΑΒΓΔ τῷ ΑΖ· περὶ τὴν αὐτὴν ἄρα ἔστι διάμετρον τὸ ΑΒΓΔ παραλληλόγραμμον τῷ ΑΖ παραλληλογράμμῳ.

Ἐὰν ἄρα ἀπὸ παραλληλογράμμου παραλληλόγραμμον ἀφαιρεθῇ ὁμοίων τε τῷ ὅλῳ καὶ ὁμοίως κείμενον κοινὴν γωνίαν ἔχον αὐτῷ, περὶ τὴν αὐτὴν διάμετρόν ἐστι τῷ ὅλῳ ὅπερ ἔδει δεῖξαι.

*AF* have been subtracted (which is) similar, and similarly laid out, to *ABCD*, having the common angle *DAB* with it. I say that *ABCD* is about the same diagonal as *AF*.



For (if) not, then, if possible, let *AHC* be [*ABCD*'s] diagonal. And producing *GF*, let it have been drawn through to (point) *H*. And let *HK* have been drawn through (point) *H*, parallel to either of *AD* or *BC* [Prop. 1.31].

Therefore, since *ABCD* is about the same diagonal as *KG*, thus as *DA* is to *AB*, so *GA* (is) to *AK* [Prop. 6.24]. And, on account of the similarity of *ABCD* and *EG*, also, as *DA* (is) to *AB*, so *GA* (is) to *AE*. Thus, also, as *GA* (is) to *AK*, so *GA* (is) to *AE*. Thus, *GA* has the same ratio to each of *AK* and *AE*. Thus, *AE* is equal to *AK* [Prop. 5.9], the lesser to the greater. The very thing is impossible. Thus, *ABCD* is not not about the same diagonal as *AF*. Thus, parallelogram *ABCD* is about the same diagonal as parallelogram *AF*.

Thus, if from a parallelogram a(nother) parallelogram is subtracted (which is) similar, and similarly laid out, to the whole, having a common angle with it, then (the subtracted parallelogram) is about the same diagonal as the whole. (Which is) the very thing it was required to show.

### κζ'.

Πάντων τῶν παρὰ τὴν αὐτὴν εὐθεῖαν παραβαλλομένων παραλληλογράμμων καὶ ἐλλειπόντων εἰδεσι παραλληλογράμμοις ὁμοίοις τε καὶ ὁμοίως κειμένοις τῷ ἀπὸ τῆς ἡμισείας ἀναγραφομένῳ μέγιστόν ἐστι τὸ ἀπὸ τῆς ἡμισείας παραβαλλόμενον [παραλληλόγραμμον] ὁμοιον δὲν τῷ ἐλλείμμαντι.

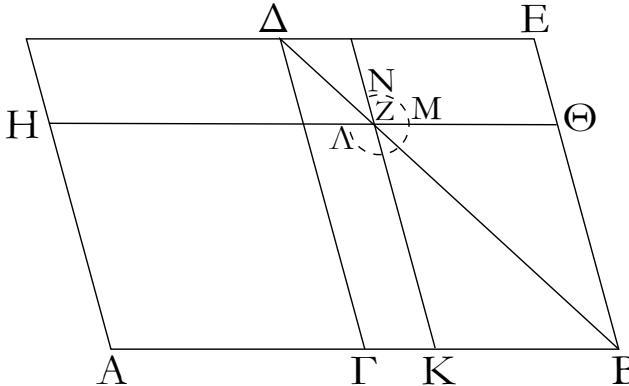
Ἐστω εὐθεῖα ἡ ΑΒ καὶ τετμήσθω δίχα κατὰ τὸ Γ, καὶ παραβεβλήσθω παρὰ τὴν ΑΒ εὐθεῖαν τὸ ΑΔ παραλληλόγραμμον ἐλλεῖπον εἰδει παραλληλογράμμῳ τῷ ΔΒ ἀναγραφέντι ἀπὸ τῆς ἡμισείας τῆς ΑΒ, τουτέστι τῆς ΓΒ· λέγω, ὅτι πάντων τῶν παρὰ τὴν ΑΒ παραβαλλομένων παραλληλογράμμων καὶ ἐλλειπόντων εἰδεσι [παραλληλογράμμοις] ὁμοίοις τε καὶ ὁμοίως κειμένοις τῷ ΔΒ μέγιστόν ἐστι τὸ

### Proposition 27

Of all the parallelograms applied to the same straight-line, and falling short by parallelogrammic figures similar, and similarly laid out, to the (parallelogram) described on half (the straight-line), the greatest is the [parallelogram] applied to half (the straight-line) which (is) similar to (that parallelogram) by which it falls short.

Let *AB* be a straight-line, and let it have been cut in half at (point) *C* [Prop. 1.10]. And let the parallelogram *AD* have been applied to the straight-line *AB*, falling short by the parallelogrammic figure *DB* (which is) applied to half of *AB*—that is to say, *CB*. I say that of all the parallelograms applied to *AB*, and falling short by

ΑΔ. παραβεβλήσθω γάρ παρὰ τὴν  $AB$  εὐθεῖαν τὸ  $AZ$  παραλληλόγραμμον ἐλλεῖπον εἴδει παραλληλογράμμῳ τῷ  $ZB$  ὁμοίῳ τε καὶ ὁμοίως κειμένῳ τῷ  $ΔB$ · λέγω, ὅτι μεῖζόν ἐστι τὸ  $AΔ$  τοῦ  $AZ$ .

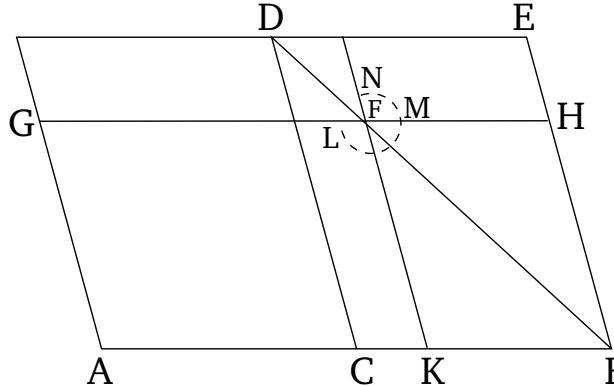


Ἐπεὶ γάρ ὁμοίόν ἐστι τὸ  $ΔB$  παραλληλόγραμμον τῷ  $ZB$  παραλληλογράμμῳ, περὶ τὴν αὐτήν εἰσὶ διάμετρον. ἦχθω αὐτῶν διάμετρος ἡ  $ΔB$ , καὶ καταγεγράψθω τὸ σχῆμα.

Ἐπεὶ οὖν ἵστηται τὸ  $ΓZ$  τῷ  $ZE$ , κοινὸν δὲ τὸ  $ZB$ , ὅλον ἄρα τὸ  $ΓΘ$  ὅλῳ τῷ  $KE$  ἐστιν ἵστηται. ἀλλὰ τὸ  $ΓΘ$  τῷ  $ΓΗ$  ἐστιν ἵστηται, ἐπεὶ καὶ ἡ  $ΑΓ$  τῇ  $ΓΒ$ . καὶ τὸ  $ΗΓ$  ἄρα τῷ  $EK$  ἐστιν ἵστηται. κοινὸν προσκείσθω τὸ  $ΓZ$ · ὅλον ἄρα τὸ  $AZ$  τῷ  $ΔMN$  γνώμονί ἐστιν ἵστηται ὥστε τὸ  $ΔB$  παραλληλόγραμμον, τουτέστι τὸ  $AΔ$ , τοῦ  $AZ$  παραλληλογράμμου μεῖζόν ἐστιν.

Πάντων ἄρα τῶν παρὰ τὴν αὐτήν εὐθεῖαν παραβαλλομένων παραλληλογράμμων καὶ ἐλλειπόντων εἴδεσι παραλληλογράμμους ὁμοίους τε καὶ ὁμοίως κειμένους τῷ ἀπὸ τῆς ἡμισείας ἀναγραφομένῳ μέγιστον ἐστι τὸ ἀπὸ τῆς ἡμισείας παραβληθέν· ὅπερ ἔδει δεῖξαι.

[parallelogrammic] figures similar, and similarly laid out, to  $DB$ , the greatest is  $AD$ . For let the parallelogram  $AF$  have been applied to the straight-line  $AB$ , falling short by the parallelogrammic figure  $FB$  (which is) similar, and similarly laid out, to  $DB$ . I say that  $AD$  is greater than  $AF$ .



For since parallelogram  $DB$  is similar to parallelogram  $FB$ , they are about the same diagonal [Prop. 6.26]. Let their (common) diagonal  $DB$  have been drawn, and let the (rest of the) figure have been described.

Therefore, since (complement)  $CF$  is equal to (complement)  $FE$  [Prop. 1.43], and (parallelogram)  $FB$  is common, the whole (parallelogram)  $CH$  is thus equal to the whole (parallelogram)  $KE$ . But, (parallelogram)  $CH$  is equal to  $CG$ , since  $AC$  (is) also (equal) to  $CB$  [Prop. 6.1]. Thus, (parallelogram)  $GC$  is also equal to  $EK$ . Let (parallelogram)  $CF$  have been added to both. Thus, the whole (parallelogram)  $AF$  is equal to the gnomon  $LMN$ . Hence, parallelogram  $DB$ —that is to say,  $AD$ —is greater than parallelogram  $AF$ .

Thus, for all parallelograms applied to the same straight-line, and falling short by a parallelogrammic figure similar, and similarly laid out, to the (parallelogram) described on half (the straight-line), the greatest is the [parallelogram] applied to half (the straight-line). (Which is) the very thing it was required to show.

καὶ.

Παρὰ τὴν δοθεῖσαν εὐθεῖαν τῷ δοθέντι εὐθυγράμμῳ ἵστηται παραλληλόγραμμον παραβαλεῖν ἐλλεῖπον εἴδει παραλληλογράμμῳ ὁμοίῳ τῷ δοθέντι· δεῖ δὲ τὸ διδόμενον εὐθυγράμμῳ [ὅς δεῖ ἵστηται παραβαλεῖν] μὴ μεῖζον εῖναι τοῦ ἀπὸ τῆς ἡμισείας ἀναγραφομένου ὁμοίου τῷ ἐλλείμματι [τοῦ τε ἀπὸ τῆς ἡμισείας καὶ ὃ δεῖ ὁμοίον ἐλλείπειν].

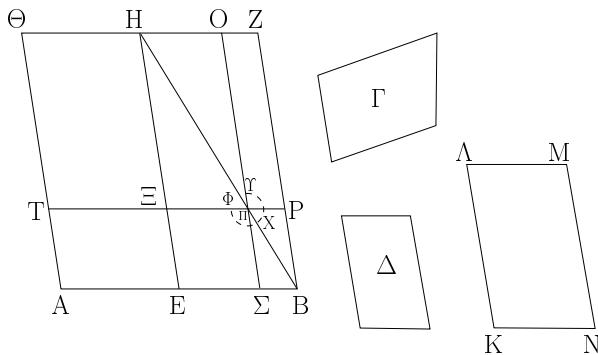
Ἐστω ἡ μὲν δοθεῖσα εὐθεῖα ἡ  $AB$ , τὸ δὲ δοθὲν εὐθυγράμμον, ὃ δεῖ ἵστηται παρὰ τὴν  $AB$  παραβαλεῖν, τὸ  $Γ$  μὴ μεῖζον [δοθὲν] τοῦ ἀπὸ τῆς ἡμισείας τῆς  $AB$  ἀναγραφομένου ὁμοίου τῷ ἐλλείμματι, ὃ δὲ δεῖ ὁμοίον ἐλλείπειν, τὸ  $Δ$ . δεῖ δὴ

### Proposition 28<sup>†</sup>

To apply a parallelogram, equal to a given rectilinear figure, to a given straight-line, (the applied parallelogram) falling short by a parallelogrammic figure similar to a given (parallelogram). It is necessary for the given rectilinear figure [to which it is required to apply an equal (parallelogram)] not to be greater than the (parallelogram) described on half (of the straight-line) and similar to the deficit.

Let  $AB$  be the given straight-line, and  $C$  the given rectilinear figure to which the (parallelogram) applied to

παρὰ τὴν δοιθεῖσαν εὐθεῖαν τὴν  $AB$  τῷ δοιθέντι εὐθυγράμμῳ τῷ  $\Gamma$  ἵσον παραλληλόγραμμον παραβαλεῖν ἐλλεῖπον εἰδει παραλληλογράμμῳ ὁμοίῳ ὅντι τῷ  $\Delta$ .



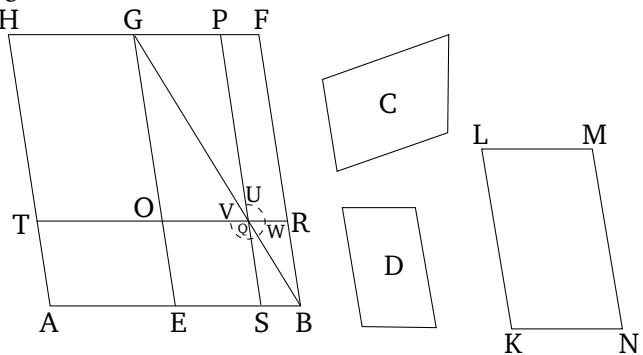
Τετμήσθω ἡ  $AB$  δίχα κατὰ τὸ  $E$  σημεῖον, καὶ ἀναγεγράφω ἀπὸ τῆς  $EB$  τῷ  $\Delta$  ὁμοιον καὶ ὁμοίως κείμενον τὸ  $EBZH$ , καὶ συμπεπληρώσθω τὸ  $AH$  παραλληλόγραμμον.

Εἰ μὲν οὖν ἵσον ἔστι τὸ  $AH$  τῷ  $\Gamma$ , γεγονός ἂν εἴη τὸ ἐπιταχθέν· παραβέβληται γάρ παρὰ τὴν δοιθεῖσαν εὐθεῖαν τὴν  $AB$  τῷ δοιθέντι εὐθυγράμμῳ τῷ  $\Gamma$  ἵσον παραλληλόγραμμον τὸ  $AH$  ἐλλεῖπον εἰδει παραλληλογράμμῳ τῷ  $HB$  ὁμοίῳ ὅντι τῷ  $\Delta$ . εἰ δὲ οὕ, μεῖζόν ἔστω τὸ  $\Theta E$  τοῦ  $\Gamma$ . ἵσον δὲ τὸ  $\Theta E$  τῷ  $HB$ · μεῖζον ἄρα καὶ τὸ  $HB$  τοῦ  $\Gamma$ . φ δὴ μεῖζόν ἔστι τὸ  $HB$  τοῦ  $\Gamma$ , ταῦτη τῇ ὑπεροχῇ ἵσον, τῷ δὲ  $\Delta$  ὁμοιον καὶ ὁμοίως κείμενον τὸ αὐτὸ συνεστάτω τὸ  $KLMN$ . ἀλλὰ τὸ  $\Delta$  τῷ  $HB$  [ἔστιν] ὁμοιον· καὶ τὸ  $KM$  ἄρα τῷ  $HB$  ἔστιν ὁμοιον. ἔστω οὖν ὁμόλογος ἡ μὲν  $KL$  τῇ  $HE$ , ἡ δὲ  $LM$  τῇ  $HZ$ . καὶ ἐπεὶ ἵσον ἔστι τὸ  $HB$  τοῖς  $\Gamma$ ,  $KM$ , μεῖζον ἄρα ἔστι τὸ  $HB$  τοῦ  $KM$ · μεῖζων ἄρα ἔστι καὶ ἡ μὲν  $HE$  τῆς  $KL$ , ἡ δὲ  $HZ$  τῆς  $LM$ . κείσθω τῇ μὲν  $KL$  ἵση ἡ  $HE$ , τῇ δὲ  $LM$  ἵση ἡ  $HO$ , καὶ συμπεπληρώσθω τὸ  $\XiHO\Gamma$  παραλληλόγραμμον· ἵσον ἄρα καὶ ὁμοιον ἔστι [τὸ  $H\Gamma$ ] τῷ  $KM$  [ἀλλὰ τὸ  $KM$  τῷ  $HB$  ὁμοιόν ἔστιν]. καὶ τὸ  $H\Gamma$  ἄρα τῷ  $HB$  ὁμοιόν ἔστιν· περὶ τὴν αὐτὴν ἄρα διάμετρόν ἔστι τὸ  $H\Gamma$  τῷ  $HB$ . ἔστω αὐτῶν διάμετρος ἡ  $H\Gamma$ , καὶ καταγεγράφω τὸ σχῆμα.

Ἐπεὶ οὖν ἵσον ἔστι τὸ  $BH$  τοῖς  $\Gamma$ ,  $KM$ , ὃν τὸ  $H\Gamma$  τῷ  $KM$  ἔστιν ἵσον, λοιπὸς ἄρα ὁ  $\Gamma X\Phi$  γνόμων λοιπῷ τῷ  $\Gamma$  ἵσος ἔστιν. καὶ ἐπεὶ ἵσον ἔστι τὸ  $OP$  τῷ  $\Xi\Sigma$ , κοινὸν προσκείσθω τὸ  $PB$ · ὅλον ἄρα τὸ  $OB$  ὅλῳ τῷ  $\Xi B$  ἵσον ἔστιν. ἀλλὰ τὸ  $\Xi B$  τῷ  $TE$  ἔστιν ἵσον, ἐπεὶ καὶ πλευρὰ ἡ  $AE$  πλευρᾷ τῇ  $EB$  ἔστιν ἵση· καὶ τὸ  $TE$  ἄρα τῷ  $OB$  ἔστιν ἵσον. κοινὸν προσκείσθω τὸ  $\Xi\Sigma$ · ὅλον ἄρα τὸ  $T\Sigma$  ὅλῳ τῷ  $\Phi X\Upsilon$  γνόμων ἔστιν ἵσον. ἀλλ᾽ ὁ  $\Phi X\Upsilon$  γνόμων τῷ  $\Gamma$  ἐδείχθη ἵσος· καὶ τὸ  $T\Sigma$  ἄρα τῷ  $\Gamma$  ἔστιν ἵσον.

Παρὰ τὴν δοιθεῖσαν ἄρα εὐθεῖαν τὴν  $AB$  τῷ δοιθέντι εὐθυγράμμῳ τῷ  $\Gamma$  ἵσον παραλληλόγραμμον παραβέβληται τὸ  $\Sigma T$  ἐλλεῖπον εἰδει παραλληλογράμμῳ τῷ  $PB$  ὁμοίῳ ὅντι

$AB$  is required (to be) equal, [being] not greater than the (parallelogram) described on half of  $AB$  and similar to the deficit, and  $D$  the (parallelogram) to which the deficit is required (to be) similar. So it is required to apply a parallelogram, equal to the given rectilinear figure  $C$ , to the straight-line  $AB$ , falling short by a parallelogrammic figure which is similar to  $D$ .



Let  $AB$  have been cut in half at point  $E$  [Prop. 1.10], and let (parallelogram)  $EBFG$ , (which is) similar, and similarly laid out, to (parallelogram)  $D$ , have been described on  $EB$  [Prop. 6.18]. And let parallelogram  $AG$  have been completed.

Therefore, if  $AG$  is equal to  $C$  then the thing prescribed has happened. For a parallelogram  $AG$ , equal to the given rectilinear figure  $C$ , has been applied to the given straight-line  $AB$ , falling short by a parallelogrammic figure  $GB$  which is similar to  $D$ . And if not, let  $HE$  be greater than  $C$ . And  $HE$  (is) equal to  $GB$  [Prop. 6.1]. Thus,  $GB$  (is) also greater than  $C$ . So, let (parallelogram)  $KLMN$  have been constructed (so as to be) both similar, and similarly laid out, to  $D$ , and equal to the excess by which  $GB$  is greater than  $C$  [Prop. 6.25]. But,  $GB$  [is] similar to  $D$ . Thus,  $KM$  is also similar to  $GB$  [Prop. 6.21]. Therefore, let  $KL$  correspond to  $GE$ , and  $LM$  to  $GF$ . And since (parallelogram)  $GB$  is equal to (figure)  $C$  and (parallelogram)  $KM$ ,  $GB$  is thus greater than  $KM$ . Thus,  $GE$  is also greater than  $KL$ , and  $GF$  than  $LM$ . Let  $GO$  be made equal to  $KL$ , and  $GP$  to  $LM$  [Prop. 1.3]. And let the parallelogram  $OGPQ$  have been completed. Thus,  $[GQ]$  is equal and similar to  $KM$  [but,  $KM$  is similar to  $GB$ ]. Thus,  $GQ$  is also similar to  $GB$  [Prop. 6.21]. Thus,  $GQ$  and  $GB$  are about the same diagonal [Prop. 6.26]. Let  $GQB$  be their (common) diagonal, and let the (remainder of the) figure have been described.

Therefore, since  $BG$  is equal to  $C$  and  $KM$ , of which  $GQ$  is equal to  $KM$ , the remaining gnomon  $UWV$  is thus equal to the remainder  $C$ . And since (the complement)  $PR$  is equal to (the complement)  $OS$  [Prop. 1.43], let (parallelogram)  $QB$  have been added to both. Thus, the whole (parallelogram)  $PB$  is equal to the whole (par-

τῷ Δ [ἐπειδήπερ τὸ ΠΒ τῷ ΗΠ ὄμοιόν ἐστιν]. ὅπερ ἔδει ποιῆσαι.

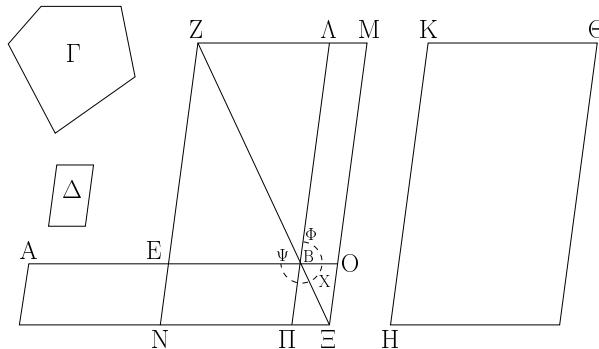
parallelogram)  $OB$ . But,  $OB$  is equal to  $TE$ , since side  $AE$  is equal to side  $EB$  [Prop. 6.1]. Thus,  $TE$  is also equal to  $PB$ . Let (parallelogram)  $OS$  have been added to both. Thus, the whole (parallelogram)  $TS$  is equal to the gnomon  $VWU$ . But, gnomon  $VWU$  was shown (to be) equal to  $C$ . Therefore, (parallelogram)  $TS$  is also equal to (figure)  $C$ .

Thus, the parallelogram  $ST$ , equal to the given rectilinear figure  $C$ , has been applied to the given straight-line  $AB$ , falling short by the parallelogrammic figure  $QB$ , which is similar to  $D$  [inasmuch as  $QB$  is similar to  $GQ$  [Prop. 6.24] ]. (Which is) the very thing it was required to do.

<sup>†</sup> This proposition is a geometric solution of the quadratic equation  $x^2 - \alpha x + \beta = 0$ . Here,  $x$  is the ratio of a side of the deficit to the corresponding side of figure  $D$ ,  $\alpha$  is the ratio of the length of  $AB$  to the length of that side of figure  $D$  which corresponds to the side of the deficit running along  $AB$ , and  $\beta$  is the ratio of the areas of figures  $C$  and  $D$ . The constraint corresponds to the condition  $\beta < \alpha^2/4$  for the equation to have real roots. Only the smaller root of the equation is found. The larger root can be found by a similar method.

καὶ.

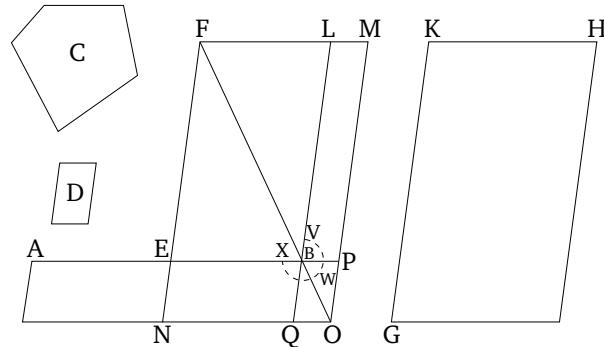
Παρὰ τὴν δοιθεῖσαν εὐθεῖαν τῷ δοιθέντι εὐθυγράμμῳ ἵσον παραληλόγραμμον παραβαλεῖν ὑπερβάλλον εἰδει παραληλογράμμῳ ὄμοιῷ τῷ δοιθέντι.



Ἐστω ἡ μὲν δοιθεῖσα εὐθεῖα ἡ ΑΒ, τὸ δὲ δοιθὲν εὐθυγράμμῳ, ὡς δεῖ ἵσον παρὰ τὴν ΑΒ παραβαλεῖν, τὸ Γ, τῷ δὲ δεῖ ὄμοιον ὑπερβάλλειν, τὸ Δ· δεῖ δὴ παρὰ τὴν ΑΒ εὐθεῖαν τῷ Γ εὐθυγράμμῳ ἵσον παραληλόγραμμον παραβαλεῖν ὑπερβάλλον εἰδει παραληλογράμμῳ ὄμοιῷ τῷ Δ.

Τετμήσθω ἡ ΑΒ δίχα κατὰ τὸ Ε, καὶ ἀναγεγράνω ἀπὸ τῆς ΕΒ τῷ Δ ὄμοιον καὶ ὄμοιώς κείμενον παραληλόγραμμον τὸ ΒΖ, καὶ συναμφοτέροις μὲν τοῖς ΒΖ, Γ ἵσον, τῷ δὲ Δ ὄμοιον καὶ ὄμοιώς κείμενον τὸ αὐτὸ συνεστάτω τὸ ΗΘ. ὄμολογος δὲ ἔστω ἡ μὲν ΚΘ τῇ ΖΛ, ἡ δὲ ΚΗ τῇ ΖΕ. καὶ ἐπεὶ μεῖζόν ἐστι τὸ ΗΘ τοῦ ΖΒ, μεῖζων ἄρα ἐστὶ καὶ ἡ μὲν ΚΘ τῆς ΖΛ, ἡ δὲ ΚΗ τῆς ΖΕ. ἐκβεβλήσθωσαν αἱ ΖΛ, ΖΕ, καὶ τῇ μὲν ΚΘ ἵση ἔστω ἡ ΖΛΜ, τῇ δὲ ΚΗ ἵση ἡ ΖΕΝ, καὶ συμπεπληρώσθω τὸ ΜΝ· τὸ ΜΝ ἄρα τῷ ΗΘ ἵσον τέ ἐστι καὶ ὄμοιον. ἀλλὰ τὸ ΗΘ τῷ ΕΛ ἐστιν ὄμοιον.

To apply a parallelogram, equal to a given rectilinear figure, to a given straight-line, (the applied parallelogram) overshooting by a parallelogrammic figure similar to a given (parallelogram).



Let  $AB$  be the given straight-line, and  $C$  the given rectilinear figure to which the (parallelogram) applied to  $AB$  is required (to be) equal, and  $D$  the (parallelogram) to which the excess is required (to be) similar. So it is required to apply a parallelogram, equal to the given rectilinear figure  $C$ , to the given straight-line  $AB$ , overshooting by a parallelogrammic figure similar to  $D$ .

Let  $AB$  have been cut in half at (point)  $E$  [Prop. 1.10], and let the parallelogram  $BF$ , (which is) similar, and similarly laid out, to  $D$ , have been described on  $EB$  [Prop. 6.18]. And let (parallelogram)  $GH$  have been constructed (so as to be) both similar, and similarly laid out, to  $D$ , and equal to the sum of  $BF$  and  $C$  [Prop. 6.25]. And let  $KG$  correspond to  $FL$ , and  $KG$  to  $FE$ . And since (parallelogram)  $GH$  is greater than (parallelogram)  $FB$ ,

καὶ τὸ MN ἄρα τῷ EL ὅμοιόν ἐστιν· περὶ τὴν αὐτὴν ἄρα διάμετρόν ἐστι τὸ EL τῷ MN. ἥχθω αὐτῶν διάμετρος ἡ ZΞ, καὶ καταγεγράψω τὸ σχῆμα.

Ἐπεὶ οὖν ἐστὶ τὸ ΗΘ τοῖς EL, Γ, ἀλλὰ τὸ ΗΘ τῷ MN οὖν ἐστίν, καὶ τὸ MN ἄρα τοῖς EL, Γ οὖν ἐστίν. κοινὸν ἀφηρήσθω τὸ EL· λοιπὸς ἄρα ὁ ΨΧΦ γνώμων τῷ Γ ἐστιν οὗσος. καὶ ἐπεὶ οἴστην ἡ AE τῇ EB, οὖν ἐστὶ καὶ τὸ AN τῷ NB, τουτέστι τῷ ΛΟ. κοινὸν προσκείσθω τὸ ΕΞ· ὅλον ἄρα τὸ AE οἴστην ἐστὶ τῷ ΦΧΨ γνώμονι. ἀλλὰ ὁ ΦΧΨ γνώμων τῷ Γ οἴστην ἐστίν· καὶ τὸ AE ἄρα τῷ Γ οὖν ἐστίν.

Παρὰ τὴν δοιθεῖσαν ἄρα εὐθεῖαν τὴν AB τῷ δοιθέντι εὐθυγράμμῳ τῷ Γ οὖν παραλληλόγραμμον παραβέβληται τὸ AE ὑπερβάλλον εἴδει παραλληλογράμμῳ τῷ ΠΟ ὅμοιῷ ὅντι τῷ Δ, ἐπεὶ καὶ τῷ EL ἐστιν ὅμοιον τὸ ΟΠ· ὅπερ ἔδει ποιῆσαι.

*KH* is thus also greater than *FL*, and *KG* than *FE*. Let *FL* and *FE* have been produced, and let *FLM* be (made) equal to *KH*, and *FEN* to *KG* [Prop. 1.3]. And let (parallelogram) *MN* have been completed. Thus, *MN* is equal and similar to *GH*. But, *GH* is similar to *EL*. Thus, *MN* is also similar to *EL* [Prop. 6.21]. *EL* is thus about the same diagonal as *MN* [Prop. 6.26]. Let their (common) diagonal *FO* have been drawn, and let the (remainder of the) figure have been described.

And since (parallelogram) *GH* is equal to (parallelogram) *EL* and (figure) *C*, but *GH* is equal to (parallelogram) *MN*, *MN* is thus also equal to *EL* and *C*. Let *EL* have been subtracted from both. Thus, the remaining gnomon *XWV* is equal to (figure) *C*. And since *AE* is equal to *EB*, (parallelogram) *AN* is also equal to (parallelogram) *NB* [Prop. 6.1], that is to say, (parallelogram) *LP* [Prop. 1.43]. Let (parallelogram) *EO* have been added to both. Thus, the whole (parallelogram) *AO* is equal to the gnomon *VWX*. But, the gnomon *VWX* is equal to (figure) *C*. Thus, (parallelogram) *AO* is also equal to (figure) *C*.

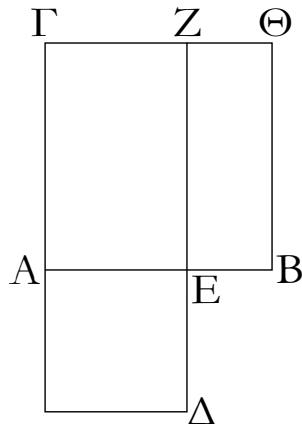
Thus, the parallelogram *AO*, equal to the given rectilinear figure *C*, has been applied to the given straight-line *AB*, overshooting by the parallelogrammic figure *QP* which is similar to *D*, since *PQ* is also similar to *EL* [Prop. 6.24]. (Which is) the very thing it was required to do.

<sup>†</sup> This proposition is a geometric solution of the quadratic equation  $x^2 + \alpha x - \beta = 0$ . Here,  $x$  is the ratio of a side of the excess to the corresponding side of figure *D*,  $\alpha$  is the ratio of the length of *AB* to the length of that side of figure *D* which corresponds to the side of the excess running along *AB*, and  $\beta$  is the ratio of the areas of figures *C* and *D*. Only the positive root of the equation is found.

λ'.

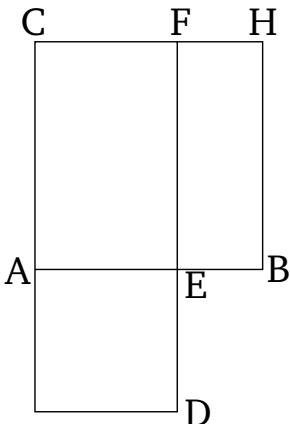
Proposition 30<sup>†</sup>

Τὴν δοιθεῖσαν εὐθεῖαν πεπερασμένην ἄκρον καὶ μέσον λόγον τεμεῖν.



Ἐστω ἡ δοιθεῖσα εὐθεῖα πεπερασμένη ἡ AB· δεῖ δὴ τὴν AB εὐθεῖαν ἄκρον καὶ μέσον λόγον τεμεῖν.

To cut a given finite straight-line in extreme and mean ratio.



Let *AB* be the given finite straight-line. So it is required to cut the straight-line *AB* in extreme and mean

Ἄναγεγράφθω ἀπὸ τῆς  $AB$  τετράγωνον τὸ  $BG$ , καὶ παραβεβλήσθω παρὰ τὴν  $AG$  τῷ  $BG$  ἵσον παραλληλόγραμμον τὸ  $ΓΔ$  ὑπερβάλλον εἴδει τῷ  $AD$  ὁμοίῳ τῷ  $BG$ .

Τετράγωνον δέ ἔστι τὸ  $BG$ · τετράγωνον ἄρα ἔστι καὶ τὸ  $AD$ . καὶ ἐπεὶ ἵσον ἔστι τὸ  $BG$  τῷ  $ΓΔ$ , κοινὸν ἀφηρήσθω τὸ  $ΓE$ · λοιπὸν ἄρα τὸ  $BZ$  λοιπῷ τῷ  $AD$  ἔστιν ἵσον. ἔστι δὲ αὐτῷ καὶ ἵσογώνιον τῶν  $BZ$ ,  $AD$  ἄρα ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας· ἔστιν ἄρα ὡς ἡ  $ZE$  πρὸς τὴν  $ED$ , οὕτως ἡ  $AE$  πρὸς τὴν  $EB$ . ἵση δὲ ἡ μὲν  $ZE$  τῇ  $AB$ , ἡ δὲ  $ED$  τῇ  $AE$ . ἔστιν ἄρα ὡς ἡ  $BA$  πρὸς τὴν  $AE$ , οὕτως ἡ  $AE$  πρὸς τὴν  $EB$ . μείζων δὲ ἡ  $AB$  τῆς  $AE$ · μείζων ἄρα καὶ ἡ  $AE$  τῆς  $EB$ .

Ἡ ἄρα  $AB$  εὐθεῖα ἄκρον καὶ μέσον λόγον τέτμηται κατὰ τὸ  $E$ , καὶ τὸ μείζον αὐτῆς τμῆμα ἔστι τὸ  $AE$ · ὥπερ ἔδει ποιῆσαι.

ratio.

Let the square  $BC$  have been described on  $AB$  [Prop. 1.46], and let the parallelogram  $CD$ , equal to  $BC$ , have been applied to  $AC$ , overshooting by the figure  $AD$  (which is) similar to  $BC$  [Prop. 6.29].

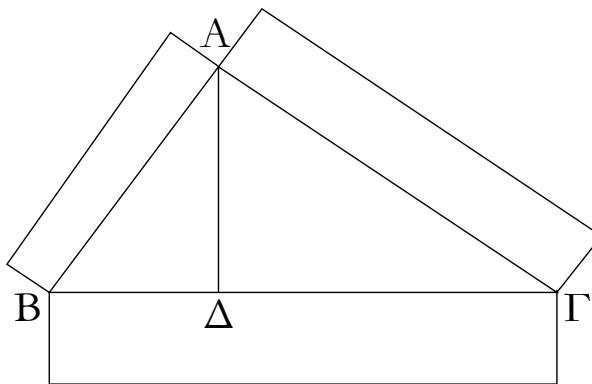
And  $BC$  is a square. Thus,  $AD$  is also a square. And since  $BC$  is equal to  $CD$ , let (rectangle)  $CE$  have been subtracted from both. Thus, the remaining (rectangle)  $BF$  is equal to the remaining (square)  $AD$ . And it is also equiangular to it. Thus, the sides of  $BF$  and  $AD$  about the equal angles are reciprocally proportional [Prop. 6.14]. Thus, as  $FE$  is to  $ED$ , so  $AE$  (is) to  $EB$ . And  $FE$  (is) equal to  $AB$ , and  $ED$  to  $AE$ . Thus, as  $BA$  is to  $AE$ , so  $AE$  (is) to  $EB$ . And  $AB$  (is) greater than  $AE$ . Thus,  $AE$  (is) also greater than  $EB$  [Prop. 5.14].

Thus, the straight-line  $AB$  has been cut in extreme and mean ratio at  $E$ , and  $AE$  is its greater piece. (Which is) the very thing it was required to do.

† This method of cutting a straight-line is sometimes called the “Golden Section”—see Prop. 2.11.

$\lambda\alpha'$ .

Ἐν τοῖς ὀρθογωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτεινούσης πλευρᾶς εἴδος ἵσον ἔστι τοῖς ἀπὸ τῶν τὴν ὀρθὴν γωνίαν περιεχουσῶν πλευρῶν εἴδεσι τοῖς ὁμοίοις τε καὶ ὁμοίως ἀναγραφομένοις.



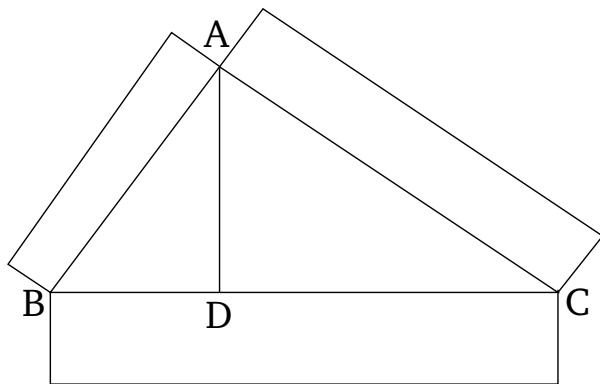
Ἐστω τρίγωνον ὀρθογωνίον τὸ  $ABC$  ὀρθὴν ἔχον τὴν ὑπὸ  $BAG$  γωνίαν λέγω, ὅτι τὸ ἀπὸ τῆς  $BG$  εἴδος ἵσον ἔστι τοῖς ἀπὸ τῶν  $BA$ ,  $AG$  εἴδεσι τοῖς ὁμοίοις τε καὶ ὁμοίως ἀναγραφομένοις.

Ἔχων κάθετος ἡ  $AD$ .

Ἐπεὶ οὖν ἐν ὀρθογωνίῳ τριγώνῳ τῷ  $ABG$  ἀπὸ τῆς πρὸς τῷ  $A$  ὀρθῆς γωνίας ἐπὶ τὴν  $BG$  βάσιν κάθετος ἔχει τὸ  $AD$ , τὰ  $ABΔ$ ,  $ΔΔΓ$  πρὸς τῇ καθέτῳ τρίγωνα ὁμοιά ἔστι τῷ τε ὅλῳ τῷ  $ABG$  καὶ ἀλλήλοις. καὶ ἐπεὶ ὁμοιόν ἔστι τὸ  $ABG$  τῷ  $ABΔ$ , ἔστιν ἄρα ὡς ἡ  $ΓB$  πρὸς τὴν  $BA$ , οὕτως ἡ  $AB$  πρὸς τὴν  $BΔ$ . καὶ ἐπεὶ τρεῖς εὐθεῖαι ἀνάλογόν εἰσιν, ἔστιν ὡς ἡ πρώτη πρὸς τὴν τρίτην, οὕτως τὸ ἀπὸ τῆς πρώτης εἴδος πρὸς

### Proposition 31

In right-angled triangles, the figure (drawn) on the side subtending the right-angle is equal to the (sum of the) similar, and similarly described, figures on the sides surrounding the right-angle.



Let  $ABC$  be a right-angled triangle having the angle  $BAC$  a right-angle. I say that the figure (drawn) on  $BC$  is equal to the (sum of the) similar, and similarly described, figures on  $BA$  and  $AC$ .

Let the perpendicular  $AD$  have been drawn [Prop. 1.12].

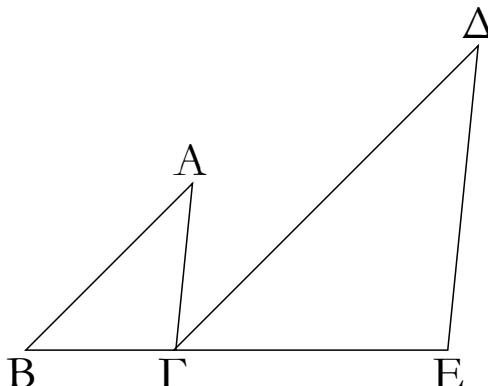
Therefore, since, in the right-angled triangle  $ABC$ , the (straight-line)  $AD$  has been drawn from the right-angle at  $A$  perpendicular to the base  $BC$ , the triangles  $ABD$  and  $ADC$  about the perpendicular are similar to the whole (triangle)  $ABC$ , and to one another [Prop. 6.8]. And since  $ABC$  is similar to  $ABD$ , thus

τὸ ἀπὸ τῆς δευτέρας τὸ ὅμοιον καὶ ὁμοίως ἀναγραφόμενον. ὡς ἄρα ἡ ΓΒ πρὸς τὴν ΒΔ, οὕτως τὸ ἀπὸ τῆς ΓΒ εἶδος πρὸς τὸ ἀπὸ τῆς ΒΑ τὸ ὅμοιον καὶ ὁμοίως ἀναγραφόμενον. διὰ τὰ αὐτὰ δὴ καὶ ὡς ἡ ΒΓ πρὸς τὴν ΓΔ, οὕτως τὸ ἀπὸ τῆς ΒΓ εἶδος πρὸς τὸ ἀπὸ τῆς ΓΑ. ὥστε καὶ ὡς ἡ ΒΓ πρὸς τὰς ΒΔ, ΔΓ, οὕτως τὸ ἀπὸ τῆς ΒΓ εἶδος πρὸς τὰ ἀπὸ τῶν ΒΑ, ΑΓ τὰ ὅμοια καὶ ὁμοίως ἀναγραφόμενα. ἵση δὲ ἡ ΒΓ τὰς ΒΔ, ΔΓ· ἵσον ἄρα καὶ τὸ ἀπὸ τῆς ΒΓ εἶδος τοῖς ἀπὸ τῶν ΒΑ, ΑΓ εἶδεσι τοῖς ὅμοιοις τε καὶ ὁμοίως ἀναγραφομένοις.

Ἐν ἄρα τοῖς ὄρθιογωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ὄρθην γωνίαν ὑποτεινούσης πλευρᾶς εἶδος ἵσον ἐστὶ τοῖς ἀπὸ τῶν τὴν ὄρθην γωνίαν περιεχουσῶν πλευρῶν εἶδεσι τοῖς ὅμοιοις τε καὶ ὁμοίως ἀναγραφομένοις· ὅπερ ἔδει δεῖξαι.

λβ'.

Ἐδών δύο τρίγωνα συντεθῆ κατὰ μίαν γωνίαν τὰς δύο πλευρὰς ταῖς δυσὶ πλευραῖς ἀνάλογον ἔχοντα ὥστε τὰς ὁμολόγους αὐτῶν πλευρὰς καὶ παραλλήλους εἶναι, αἱ λοιπαὶ τῶν τριγώνων πλευραὶ ἐπ' εὐθείας ἔσονται.



Ἐστω δύο τρίγωνα τὰ ΑΒΓ, ΔΓΕ τὰς δύο πλευρὰς τὰς ΒΑ, ΑΓ ταῖς δυσὶ πλευραῖς ταῖς ΔΓ, ΔΕ ἀνάλογον ἔχοντα, ὡς μὲν τὴν ΑΒ πρὸς τὴν ΑΓ, οὕτως τὴν ΔΓ πρὸς τὴν ΔΕ, παράλληλον δὲ τὴν μὲν ΑΒ τῇ ΔΓ, τὴν δὲ ΑΓ τῇ ΔΕ· λέγω, ὅτι ἐπ' εὐθείας ἔστιν ἡ ΒΓ τῇ ΓΕ.

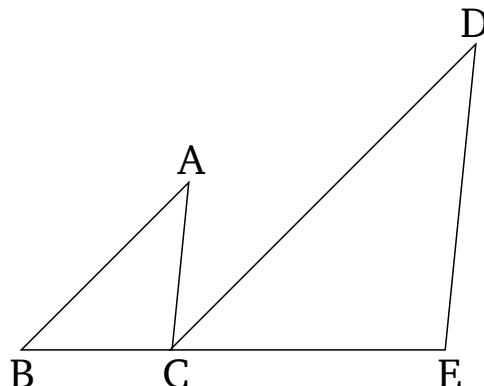
Ἐπεὶ γὰρ παράλληλός ἔστιν ἡ ΑΒ τῇ ΔΓ, καὶ εἰς αὐτὰς ἐμπέπτωκεν εὐθεῖα ἡ ΑΓ, αἱ ἐναλλὰξ γωνίαι αἱ ὑπὸ ΒΑΓ, ΑΓΔ ἵσαι ἀλλήλαις εἰσὶν. διὰ τὰ αὐτὰ δὴ καὶ ἡ ὑπὸ ΓΔΕ τῇ ὑπὸ ΑΓΔ ἵση ἔστιν. ὥστε καὶ ἡ ὑπὸ ΒΑΓ τῇ ὑπὸ ΓΔΕ ἔστιν ἵση. καὶ ἐπεὶ δύο τρίγωνά ἔστι τὰ ΑΒΓ, ΔΓΕ μίαν γωνίαν τὴν πρὸς τῷ Α μιᾷ γωνίᾳ τῇ πρὸς τῷ Δ ἵσην ἔχοντα, περὶ

as  $CB$  is to  $BA$ , so  $AB$  (is) to  $BD$  [Def. 6.1]. And since three straight-lines are proportional, as the first is to the third, so the figure (drawn) on the first is to the similar, and similarly described, (figure) on the second [Prop. 6.19 corr.]. Thus, as  $CB$  (is) to  $BD$ , so the figure (drawn) on  $CB$  (is) to the similar, and similarly described, (figure) on  $BA$ . And so, for the same (reasons), as  $BC$  (is) to  $CD$ , so the figure (drawn) on  $BC$  (is) to the (figure) on  $CA$ . Hence, also, as  $BC$  (is) to  $BD$  and  $DC$ , so the figure (drawn) on  $BC$  (is) to the (sum of the) similar, and similarly described, (figures) on  $BA$  and  $AC$  [Prop. 5.24]. And  $BC$  is equal to  $BD$  and  $DC$ . Thus, the figure (drawn) on  $BC$  (is) also equal to the (sum of the) similar, and similarly described, figures on  $BA$  and  $AC$  [Prop. 5.9].

Thus, in right-angled triangles, the figure (drawn) on the side subtending the right-angle is equal to the (sum of the) similar, and similarly described, figures on the sides surrounding the right-angle. (Which is) the very thing it was required to show.

### Proposition 32

If two triangles, having two sides proportional to two sides, are placed together at a single angle such that the corresponding sides are also parallel, then the remaining sides of the triangles will be straight-on (with respect to one another).



Let  $ABC$  and  $DCE$  be two triangles having the two sides  $BA$  and  $AC$  proportional to the two sides  $DC$  and  $DE$ —so that as  $AB$  (is) to  $AC$ , so  $DC$  (is) to  $DE$ —and (having side)  $AB$  parallel to  $DC$ , and  $AC$  to  $DE$ . I say that (side)  $BC$  is straight-on to  $CE$ .

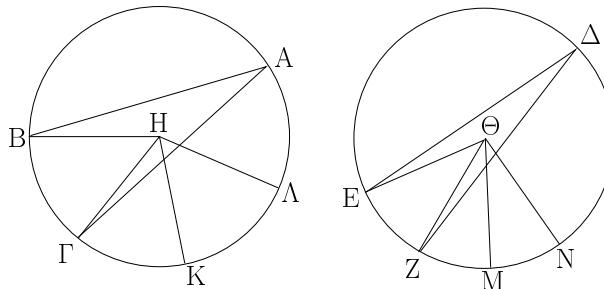
For since  $AB$  is parallel to  $DC$ , and the straight-line  $AC$  has fallen across them, the alternate angles  $BAC$  and  $ACD$  are equal to one another [Prop. 1.29]. So, for the same (reasons),  $CDE$  is also equal to  $ACD$ . And, hence,  $BAC$  is equal to  $CDE$ . And since  $ABC$  and  $DCE$  are two triangles having the one angle at  $A$  equal to the one

δὲ τὰς ἵσας γωνίας τὰς πλευρὰς ἀνάλογον, ὡς τὴν  $BA$  πρὸς τὴν  $ΑΓ$ , οὕτως τὴν  $ΓΔ$  πρὸς τὴν  $ΔΕ$ , ἵσογώνιον ἄρα ἐστὶ τὸ  $ΑΒΓ$  τρίγωνον τῷ  $ΔΓΕ$  τριγώνῳ ἵση ἄρα ἡ ὑπὸ  $ΑΒΓ$  γωνία τῇ ὑπὸ  $ΔΓΕ$ . ἐδείχθη δὲ καὶ ἡ ὑπὸ  $ΑΓΔ$  τῇ ὑπὸ  $ΒΑΓ$  ἵση· ὅλη ἄρα ἡ ὑπὸ  $ΑΓΕ$  δυσὶ ταῖς ὑπὸ  $ΑΒΓ$ ,  $ΒΑΓ$  ἵση ἐστίν. κοινὴ προσκείσθω ἡ ὑπὸ  $ΑΓΒ$ · αἱ ἄρα ὑπὸ  $ΑΓΕ$ ,  $ΑΓΒ$  ταῖς ὑπὸ  $ΒΑΓ$ ,  $ΑΓΒ$ ,  $ΓΒΑ$  ἵσαι εἰσίν. ἀλλ᾽ αἱ ὑπὸ  $ΒΑΓ$ ,  $ΑΒΓ$ ,  $ΑΓΒ$  δυσὶν ὄρθαις ἵσαι εἰσίν· καὶ αἱ ὑπὸ  $ΑΓΕ$ ,  $ΑΓΒ$  ἄρα δυσὶν ὄρθαις ἵσαι εἰσίν. πρὸς δή τινι εὐθείᾳ τῇ  $ΑΓ$  καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ  $Γ$  δύο εὐθεῖαι αἱ  $ΒΓ$ ,  $ΓΕ$  μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι τὰς ἐφεξῆς γωνάις τὰς ὑπὸ  $ΑΓΕ$ ,  $ΑΓΒ$  δυσὶν ὄρθαις ἵσας ποιοῦσιν ἐπ' εὐθείας ἄρα ἐστὶν ἡ  $ΒΓ$  τῇ  $ΓΕ$ .

Ἐὰν ἄρα δύο τρίγωνα συντεθῆ κατὰ μίαν γωνίαν τὰς δύο πλευρὰς ταῖς δυσὶ πλευραῖς ἀνάλογον ἔχοντα ὥστε τὰς ὁμολόγους αὐτῶν πλευρὰς καὶ παραλλήλους εἶναι, αἱ λοιπαὶ τῶν τριγώνων πλευραὶ ἐπ' εὐθείας ἔσονται· ὅπερ ἔδει δεῖξαι.

λγ'.

Ἐν τοῖς ἵσοις κύκλοις αἱ γωνίαι τὸν αὐτὸν ἔχουσι λόγον ταῖς περιφερείαις, ἐφ' ὃν βεβήκασιν, ἐάν τε πρὸς τοῖς κέντροις ἔάν τε πρὸς ταῖς περιφερείαις ὡσι βεβηκύται.



Ἐστωσαν ἵσοι κύκλοι οἱ  $ΑΒΓ$ ,  $ΔΕΖ$ , καὶ πρὸς μὲν τοῖς κέντροις αὐτῶν τοῖς  $H$ ,  $Θ$  γωνίαι ἔστωσαν αἱ ὑπὸ  $ΒΗΓ$ ,  $ΕΘΖ$ , πρὸς δὲ ταῖς περιφερείαις αἱ ὑπὸ  $ΒΑΓ$ ,  $ΕΔΖ$  λέγω, ὅτι ἐστὶν ὡς ἡ  $ΒΓ$  περιφέρεια πρὸς τὴν  $ΕΖ$  περιφέρειαν, οὕτως ἡ τε ὑπὸ  $ΒΗΓ$  γωνία πρὸς τὴν ὑπὸ  $ΕΘΖ$  καὶ ἡ ὑπὸ  $ΒΑΓ$  πρὸς τὴν ὑπὸ  $ΕΔΖ$ .

Κείσθωσαν γάρ τῇ μὲν  $ΒΓ$  περιφερείᾳ ἵσαι κατὰ τὸ ἐξῆς ὀσαιδηποτοῦν αἱ  $ΓΚ$ ,  $ΚΛ$ , τῇ δὲ  $ΕΖ$  περιφερείᾳ ἵσαι ὀσαιδηποτοῦν αἱ  $ΖΜ$ ,  $ΜΝ$ , καὶ ἐπεζεύχθωσαν αἱ  $ΗΚ$ ,  $ΗΛ$ ,  $ΘΜ$ ,  $ΘΝ$ .

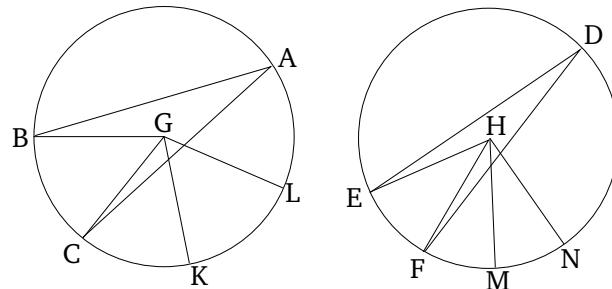
Ἐπεὶ οὖν ἵσαι εἰσὶν αἱ  $ΒΓ$ ,  $ΓΚ$ ,  $ΚΛ$  περιφέρειαι ἀλλήλαις, ἵσαι εἰσὶν καὶ αἱ ὑπὸ  $ΒΗΓ$ ,  $ΓΗΚ$ ,  $ΚΗΛ$  γωνίαι ἀλλήλαις· ὀσαιπλασίων ἄρα ἐστὶν ἡ  $ΒΛ$  περιφέρεια τῆς  $ΒΓ$ , τοσαυταπλασίων ἐστὶ καὶ ἡ ὑπὸ  $ΒΗΛ$  γωνία τῆς ὑπὸ  $ΒΗΓ$ . διὰ τὰ

angle at  $D$ , and the sides about the equal angles proportional, (so that) as  $BA$  (is) to  $AC$ , so  $CD$  (is) to  $DE$ , triangle  $ABC$  is thus equiangular to triangle  $DCE$  [Prop. 6.6]. Thus, angle  $ABC$  is equal to  $DCE$ . And (angle)  $ACD$  was also shown (to be) equal to  $BAC$ . Thus, the whole (angle)  $ACE$  is equal to the two (angles)  $ABC$  and  $BAC$ . Let  $ACB$  have been added to both. Thus,  $ACE$  and  $ACB$  are equal to  $BAC$ ,  $ACB$ , and  $CBA$ . But,  $BAC$ ,  $ABC$ , and  $ACB$  are equal to two right-angles [Prop. 1.32]. Thus,  $ACE$  and  $ACB$  are also equal to two right-angles. Thus, the two straight-lines  $BC$  and  $CE$ , not lying on the same side, make adjacent angles  $ACE$  and  $ACB$  (whose sum is) equal to two right-angles with some straight-line  $AC$ , at the point  $C$  on it. Thus,  $BC$  is straight-on to  $CE$  [Prop. 1.14].

Thus, if two triangles, having two sides proportional to two sides, are placed together at a single angle such that the corresponding sides are also parallel, then the remaining sides of the triangles will be straight-on (with respect to one another). (Which is) the very thing it was required to show.

### Proposition 33

In equal circles, angles have the same ratio as the (ratio of the) circumferences on which they stand, whether they are standing at the centers (of the circles) or at the circumferences.



Let  $ABC$  and  $DEF$  be equal circles, and let  $BGC$  and  $EHF$  be angles at their centers,  $G$  and  $H$  (respectively), and  $BAC$  and  $EDF$  (angles) at their circumferences. I say that as circumference  $BC$  is to circumference  $EF$ , so angle  $BGC$  (is) to  $EHF$ , and (angle)  $BAC$  to  $EDF$ .

For let any number whatsoever of consecutive (circumferences),  $CK$  and  $KL$ , be made equal to circumference  $BC$ , and any number whatsoever,  $FM$  and  $MN$ , to circumference  $EF$ . And let  $GK$ ,  $GL$ ,  $HM$ , and  $HN$  have been joined.

Therefore, since circumferences  $BC$ ,  $CK$ , and  $KL$  are equal to one another, angles  $BGC$ ,  $CGK$ , and  $KGL$  are also equal to one another [Prop. 3.27]. Thus, as many times as circumference  $BL$  is (divisible) by  $BC$ , so many

αὐτὰ δὴ καὶ ὁσαπλασίων ἐστὶν ἡ NE περιφέρεια τῆς EZ, το-  
σαυταπλασίων ἐστὶ καὶ ἡ ὑπὸ NΘΕ γωνία τῆς ὑπὸ EΘZ. εἰ  
ἄρα ἵση ἐστὶν ἡ BL περιφέρεια τῇ EN περιφερείᾳ, ἵση ἐστὶ  
καὶ γωνία ἡ ὑπὸ BΗL τῇ ὑπὸ EΘN, καὶ εἰ μείζων ἐστὶν ἡ BL  
περιφέρεια τῆς EN περιφερείας, μείζων ἐστὶ καὶ ἡ ὑπὸ BΗL  
γωνία τῆς ὑπὸ EΘN, καὶ εἰ ἐλάσσων, ἐλάσσων. τεσσάρων  
δὴ ὅντων μεγεθῶν, δύο μὲν περιφερειῶν τῶν BΓ, EZ, δύο  
δὲ γωνιῶν τῶν ὑπὸ BΗΓ, EΘZ, εἴληπται τῆς μὲν BΓ περι-  
φερείας καὶ τῆς ὑπὸ BΗΓ γωνίας ἵσακις πολλαπλασίων ἡ τε  
BL περιφέρεια καὶ ἡ ὑπὸ BΗL γωνία, τῆς δὲ EZ περιφερείας  
καὶ τῆς ὑπὸ EΘZ γωνίας ἡ τε EN περιφέρεια καὶ ἡ ὑπὸ EΘN  
γωνία. καὶ δέδεικται, ὅτι εἰ ὑπερέχει ἡ BL περιφέρεια τῆς  
EN περιφερείας, ὑπερέχει καὶ ἡ ὑπὸ BΗL γωνία τῆς ὑπὸ<sup>1</sup>  
EΘN γωνίας, καὶ εἰ ἵση, ἵση, καὶ εἰ ἐλάσσων, ἐλάσσων.  
ἐστιν ἄρα, ὡς ἡ BΓ περιφέρεια πρὸς τὴν EZ, οὕτως ἡ ὑπὸ<sup>1</sup>  
BΗΓ γωνία πρὸς τὴν ὑπὸ EΘZ. ἀλλ᾽ ὡς ἡ ὑπὸ BΗΓ γωνία  
πρὸς τὴν ὑπὸ EΘZ, οὕτως ἡ ὑπὸ BΑΓ πρὸς τὴν ὑπὸ EΔZ.  
διπλασία γὰρ ἐκατέρα ἐκατέρας. καὶ ὡς ἄρα ἡ BΓ περιφέρεια  
πρὸς τὴν EZ περιφερειῶν, οὕτως ἡ τε ὑπὸ BΗΓ γωνία πρὸς  
τὴν ὑπὸ EΘZ καὶ ἡ ὑπὸ BΑΓ πρὸς τὴν ὑπὸ EΔZ.

Ἐν ἄρα τοῖς ἵσοις κύκλοις αἱ γωνίαι τὸν αὐτὸν ἔχουσι  
λόγον ταῖς περιφερείαις, ἐφ' ὃν βεβήκασιν, ἐάν τε πρὸς τοῖς  
κέντροις ἔάν τε πρὸς ταῖς περιφερείαις ὅσι βεβηκύιαι. ὅπερ  
ἔδει δεῖξαι.

times is angle  $BGL$  also (divisible) by  $BGC$ . And so, for the same (reasons), as many times as circumference  $NE$  is (divisible) by  $EF$ , so many times is angle  $NHE$  also (divisible) by  $EHF$ . Thus, if circumference  $BL$  is equal to circumference  $EN$  then angle  $BGL$  is also equal to  $EHN$  [Prop. 3.27], and if circumference  $BL$  is greater than circumference  $EN$  then angle  $BGL$  is also greater than  $EHN$ ,<sup>†</sup> and if ( $BL$  is) less (than  $EN$  then  $BGL$  is also) less (than  $EHN$ ). So there are four magnitudes, two circumferences  $BC$  and  $EF$ , and two angles  $BGC$  and  $EHF$ . And equal multiples have been taken of circumference  $BC$  and angle  $BGC$ , (namely) circumference  $BL$  and angle  $BGL$ , and of circumference  $EF$  and angle  $EHF$ , (namely) circumference  $EN$  and angle  $EHN$ . And it has been shown that if circumference  $BL$  exceeds circumference  $EN$  then angle  $BGL$  also exceeds angle  $EHN$ , and if ( $BL$  is) equal (to  $EN$  then  $BGL$  is also) equal (to  $EHN$ ), and if ( $BL$  is) less (than  $EN$  then  $BGL$  is also) less (than  $EHN$ ). Thus, as circumference  $BC$  (is) to  $EF$ , so angle  $BGC$  (is) to  $EHF$  [Def. 5.5]. But as angle  $BGC$  (is) to  $EHF$ , so (angle)  $BAC$  (is) to  $EDF$  [Prop. 5.15]. For the former (are) double the latter (respectively) [Prop. 3.20]. Thus, also, as circumference  $BC$  (is) to circumference  $EF$ , so angle  $BGC$  (is) to  $EHF$ , and  $BAC$  to  $EDF$ .

Thus, in equal circles, angles have the same ratio as the (ratio of the) circumferences on which they stand, whether they are standing at the centers (of the circles) or at the circumferences. (Which is) the very thing it was required to show.

<sup>†</sup> This is a straight-forward generalization of Prop. 3.27

# ELEMENTS BOOK 7

*Elementary Number Theory*<sup>†</sup>

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<sup>†</sup>The propositions contained in Books 7–9 are generally attributed to the school of Pythagoras.

Ὀροι.

α'. Μονάς ἔστιν, καθ' ἣν ἔκαστον τῶν ὄντων ἐν λέγεται.

β'. Ἀριθμὸς δὲ τὸ ἐκ μονάδων συγκείμενον πλῆθος.

γ'. Μέρος ἔστιν ἀριθμὸς ἀριθμοῦ ὁ ἐλάσσον τοῦ μείζονος, ὅταν καταμετρῇ τὸν μείζονα.

δ'. Μέρη δέ, ὅταν μὴ καταμετρῇ.

ε'. Πολλαπλάσιος δὲ ὁ μείζων τοῦ ἐλάσσονος, ὅταν καταμετρήται ὑπὸ τοῦ ἐλάσσονος.

Ϛ'. Ἀρτιος ἀριθμός ἔστιν ὁ δίχα διαιρούμενος.

ζ'. Περισσὸς δὲ ὁ μὴ διαιρούμενος δίχα ἢ [ό] μονάδι διαιρέρων ἀρτίου ἀριθμοῦ.

η'. Ἀρτιάκις ἀρτιος ἀριθμός ἔστιν ὁ ὑπὸ ἀρτίου ἀριθμοῦ μετρούμενος κατὰ ἀρτίον ἀριθμόν.

θ'. Ἀρτιάκις δὲ περισσός ἔστιν ὁ ὑπὸ ἀρτίου ἀριθμοῦ μετρούμενος κατὰ περισσὸν ἀριθμόν.

ι'. Περισσάκις δὲ περισσὸς ἀριθμός ἔστιν ὁ ὑπὸ περισσοῦ ἀριθμοῦ μετρούμενος κατὰ περισσὸν ἀριθμόν.

ια'. Πρῶτος ἀριθμός ἔστιν ὁ μονάδι μόνη μετρούμενος.

ιβ'. Πρῶτοι πρὸς ἀλλήλους ἀριθμοί εἰσιν οἱ μονάδι μόνη μετρούμενοι κοινῷ μέτρῳ.

ιγ'. Σύνθετος ἀριθμός ἔστιν ὁ ἀριθμῷ τινι μετρούμενος.

ιδ'. Σύνθετοι δὲ πρὸς ἀλλήλους ἀριθμοί εἰσιν οἱ ἀριθμῷ τινι μετρούμενοι κοινῷ μέτρῳ.

ιε'. Ἀριθμὸς ἀριθμὸν πολλαπλασιάζειν λέγεται, ὅταν, ὅσαι εἰσὶν ἐν αὐτῷ μονάδες, τοσαυτάκις συντεθῇ ὁ πολλαπλασιάζομενος, καὶ γένηται τις.

ιϚ'. Ὄταν δὲ δύο ἀριθμοὶ πολλαπλασιάσαντες ἀλλήλους ποιῶσι τινα, ὁ γενόμενος ἐπίπεδος καλεῖται, πλευραὶ δὲ αὐτοῦ οἱ πολλαπλασιάσαντες ἀλλήλους ἀριθμοί.

ιζ'. Ὄταν δὲ τρεῖς ἀριθμοὶ πολλαπλασιάσαντες ἀλλήλους ποιῶσι τινα, ὁ γενόμενος στερεός ἔστιν, πλευραὶ δὲ αὐτοῦ οἱ πολλαπλασιάσαντες ἀλλήλους ἀριθμοί.

ιη'. Τετράγωνος ἀριθμός ἔστιν ὁ ἴσακις ἵσος ἢ [ό] ὑπὸ δύο ἵσων ἀριθμῶν περιεχόμενος.

ιθ'. Κύβος δὲ ὁ ἴσακις ἵσος ἴσακις ἢ [ό] ὑπὸ τριῶν ἵσων ἀριθμῶν περιεχόμενος.

ιχ'. Ἀριθμοὶ ἀνάλογον εἰσιν, ὅταν ὁ πρῶτος τοῦ δευτέρου καὶ ὁ τρίτος τοῦ τετάρτου ἴσακις ἢ πολλαπλάσιος ἢ τὸ αὐτὸ μέρος ἢ τὰ αὐτὰ μέρη ὢσιν.

ια'. Ὁμοιοι ἐπίπεδοι καὶ στερεοὶ ἀριθμοί εἰσιν οἱ ανάλογον ἔχοντες τὰς πλευράς.

ιβ'. Τέλειος ἀριθμός ἔστιν ὁ τοῖς ἔαυτοῦ μέρεσιν ἵσος ὥν.

### Definitions

1. A unit is (that) according to which each existing (thing) is said (to be) one.

2. And a number (is) a multitude composed of units.<sup>†</sup>

3. A number is part of a(nother) number, the lesser of the greater, when it measures the greater.<sup>‡</sup>

4. But (the lesser is) parts (of the greater) when it does not measure it.<sup>§</sup>

5. And the greater (number is) a multiple of the lesser when it is measured by the lesser.

6. An even number is one (which can be) divided in half.

7. And an odd number is one (which can)not (be) divided in half, or which differs from an even number by a unit.

8. An even-times-even number is one (which is) measured by an even number according to an even number.<sup>¶</sup>

9. And an even-times-odd number is one (which is) measured by an even number according to an odd number.\*

10. And an odd-times-odd number is one (which is) measured by an odd number according to an odd number.<sup>§</sup>

11. A prime<sup>||</sup> number is one (which is) measured by a unit alone.

12. Numbers prime to one another are those (which are) measured by a unit alone as a common measure.

13. A composite number is one (which is) measured by some number.

14. And numbers composite to one another are those (which are) measured by some number as a common measure.

15. A number is said to multiply a(nother) number when the (number being) multiplied is added (to itself) as many times as there are units in the former (number), and (thereby) some (other number) is produced.

16. And when two numbers multiplying one another make some (other number) then the (number so) created is called plane, and its sides (are) the numbers which multiply one another.

17. And when three numbers multiplying one another make some (other number) then the (number so) created is (called) solid, and its sides (are) the numbers which multiply one another.

18. A square number is an equal times an equal, or (a plane number) contained by two equal numbers.

19. And a cube (number) is an equal times an equal times an equal, or (a solid number) contained by three equal numbers.