LIB. II. driformem. Verum ex æquatione $yy - Py + Q = 0$, elicitur $y = \frac{1}{2}P \pm \surd(-\frac{3}{4}PP \pm \surd(\frac{1}{2}P^4 + \frac{1}{2}a^4))$, unde patet Applicatam y realem esse non posse nisi $\surd(\frac{1}{2}P^4 + \frac{1}{2}a^4)$ affirmative sumatur; quare, non obstante Functionis Q biformitate, Applicata y nunquam plures duobus valores habebit, quorum biquadratorum summa erit constans, sicut natura quæstionis requirit.

371. Quod si porro ejusmodi requiratur Curva, ut binorum ipsius y valorum cuique Abscissæ x respondentium potestates quintæ summam constantem constituant, seu ut sit $PM^5 + PN^5 = a^5$, debebit esse $P^5 - 5P^3Q + 5PQ^2 = a^5$. Cum igitur ex æquatione pro Curva $yy - Py + Q = 0$, sit $Q = -yy + Py$, erit $P^5 - 5P^4y + 10P^3yy - 10P^2y^3 + 5Py^4 = a^5$, seu $(P - y)^5 + y^5 = a^5$. Eodem modo reperietur, si debeat esse $PM^6 + PN^6 = a^6$ hæc æquatio $(P - y)^6 + y^6 = a^6$. Atque generaliter si quæratur Curva in qua sit $PM^n + PN^n = a^n$, obtinebitur ista æquatio $(P - y)^n + y^n = a^n$: ubi pro P Functio quæcunque uniformis ipsius x pro lubitu accipi potest. Ratio autem hujus æquationis in promtu est: cum enim summa ambarum Applicatarum sit $= P$, si altera sit y, altera erit $= P - y$, unde statim fit $(P - y)^n + y^n = a^n$.

372. Quod si autem loco Q eliminetur P, ponendo in æquationibus, quibus relatio inter P & Q continetur, $P = \frac{yy + Q}{y}$, orietur pro $PM^n + PN^n = a^n$ hæc æquatio $y^n + \frac{Q^n}{y^n} = a^n$ Cum enim Applicatarum productum sit $= Q$, si una ponatur $= y$, erit altera $= \frac{Q}{y}$: unde æquatio inventa

venta ſtatim fluit. Pro Curvis ergo, in quibus ſit $PM^n + PN^n = a^n$, duas nacti ſumus æquationes generales, alteram $(P - y)^n + y^n = a^n$, alteram $y^n + \frac{Q^n}{y^n} = a^n$: ex quarum poſteriori emergit $y^{2n} = a^n y^n - Q^n$, & $y^n = \frac{1}{2} a^n \pm \surd(\frac{1}{4} a^{2n} - Q^n)$, ita ut ſit $y = \sqrt[n]{}(\frac{1}{2} a^n \pm \surd(\frac{1}{4} a^{2n} - Q^n))$, quæ eſt Functio tantum biformis, atque pro quavis Abſciſſa plures duabus Applicatas non exhibet, dummodo Q^n fuerit Functio rationalis ſeu uniformis, ipſius x. Prior autem æquatio $y^n + (P - y)^n = a^n$ hac gaudet prærogativa ut numerus dimenſionum ſit minor. CAP. XVI.

373. Neque vero hæ æquationes ſolum quæſtionem ſolvunt ſi n ſit numerus integer affirmativus, ſed etiam ſi ſit vel negativus vel fractus. Sic

ſi debeat eſſe	habebitur hæc æquatio
$\frac{1}{PM} + \frac{1}{PN} = \frac{1}{a}$	$aP = Py - yy$ ſeu $aQ + ayy = Qy$
$\frac{1}{PM^2} + \frac{1}{PN^2} = \frac{1}{a^2}$	$a^2y^2 + a^2(P - y)^2 = y^2(P - y)^2$ ſeu $a^2Q^2 + a^2y^4 = Q^2y^2$
$\frac{1}{PM^3} + \frac{1}{PN^3} = \frac{1}{a^3}$	$a^3y^3 + a^3(P - y)^3 = y^3(P - y)^3$ ſeu $a^3Q^3 + a^3y^6 = Q^3y^3$

&c.

Pro

LIB. II. Pro exponentibus autem fractis ita res ſe habebit:

ſi debeat eſſe	habebitur hæc æquatio
$\sqrt{PM} + \sqrt{PN} = \sqrt{a}$	$\sqrt{y} + \sqrt{(P - y)} = \sqrt{a}$ ſeu $y = \sqrt{ay} - \sqrt{Q}$ quæ ad rationalitatem reductæ præbent $yy - Py + \frac{1}{4}(a - P)^2 = 0$ ſeu $yy - (a - 2\sqrt{Q})y + Q = 0$
$\sqrt[3]{PM} + \sqrt[3]{PN} = \sqrt[3]{a}$	$\sqrt[3]{y} + \sqrt[3]{(P - y)} = \sqrt[3]{a}$ vel $yy - Py + \frac{1}{27a}(a - P)^3 = 0$ ſeu $\sqrt[3]{y} + \sqrt[3]{\frac{Q}{y}} = \sqrt[3]{a}$ vel $yy - (a - 3\sqrt[3]{aQ})y + Q = 0$ &c.

Hoc igitur modo omnes Curvæ algebraïcæ, in quibus ubique ſit $PM^n + PN^n = a^n$, una æquatione generali comprehendi poſſunt, ſive n ſit numerus integer affirmativus, ſive negativus, ſive fractus.

374. Quæ hic de conditione duarum Applicatarum unicuique Abſciſſæ x reſpondentium ſunt expoſita, eadem methodo transferri poſſunt ad ternas Applicatas ſingulis Abſciſſis reſpondentes. Æquatio autem generalis pro Curvis, quas ſingulæ Applicatæ in tribus punctis ſecant eſt hæc

$$y^3 - Py^2 + Qy - R = 0,$$

denotantibus litteris P, Q, & R Functiones quaſcunque uniformes

formes ipſius x. Sint p, q, r tres Applicatæ Abſciſſæ x reſpondentes, quarum una quidem ſemper eſt realis, verum hic ad ea potiſſimum Curvæ loca ſpectamus, in quibus omnes tres Applicatæ ſint reales. Erit autem ex natura æquationum $P = p + q + r$; $Q = pq + pr + qr$; & $R = pqr$. Quare, ſi Curva deſideretur, in qua ſit vel $p + q + r$ vel $pq + pr + qr$, vel pqr quantitas conſtans, nil aliud eſt faciendum niſi ut vel P, vel Q vel R quantitas conſtituatur conſtans, binis reliquis manentibus arbitrariis. CAP. XVI.

375. Hinc quoque Curvæ inveniri poterunt, in quibus ſit $p^n + q^n + r^n$, quantitas conſtans ubique; eſt enim, per ea quæ in ſuperiori libro ſunt tradita,

$$\begin{aligned}
p + q + r &= P \\
p^2 + q^2 + r^2 &= P^2 - 2Q \\
p^3 + q^3 + r^3 &= P^3 - 3PQ + 3R \\
p^4 + q^4 + r^4 &= P^4 - 4P^2Q + 2QQ + 4PR \\
p^5 + q^5 + r^5 &= P^5 - 5P^3Q + 5PQQ + 5PPR - 5QR
\end{aligned}$$

&c.

Deinde, ſi n ſit numerus negativus, ponatur $z = \frac{1}{y}$; erit $z^3 - \frac{Qzz}{R} + \frac{Pz}{R} - \frac{1}{R} = 0$, & hujus æquationis tres radices ſunt $\frac{1}{p}$, $\frac{1}{q}$, $\frac{1}{r}$. Hinc ſimili modo erit

$$\begin{aligned}
\frac{1}{p} + \frac{1}{q} + \frac{1}{r} &= \frac{Q}{R} \\
\frac{1}{p^2} + \frac{1}{q^2} + \frac{1}{r^2} &= \frac{Q^2 - 2PR}{RR} \\
\frac{1}{p^3} + \frac{1}{q^3} + \frac{1}{r^3} &= \frac{Q^3 - 3PQR + 3RR}{R^3} \\
\frac{1}{p^4} + \frac{1}{q^4} + \frac{1}{r^4} &= \frac{Q^4 - 4PQ^2R + 4QRR + 2P^2R^2}{R^4}
\end{aligned}$$

&c.

LIB. II. Hujusmodi ergo expressio quantitati constanti æqualis posita præbebit relationem idoneam inter Functiones P, Q & R. Atque, si hujus æquationis ope, ex æquatione $y^3 - Py^2 + Qy - R = 0$, una harum Functionum P, Q, vel R eliminetur, habebitur æquatio pro Curva quæsita. Sic, si quæratur Curva in qua sit $p^3 + q^3 + r^3 = a^3$, fiet $P^3 - 3PQ + 3R = a^3$; &, ob $R = y^3 - Py^2 + Qy$, habebitur hæc æquatio $3y^3 - 3Py^2 + 3Qy + P^3 - 3PQ = a^3$ pro Curvis quæsito satisfacientibus.

376. Sive igitur n sit numerus affirmativus sive negativus integer, solutio per datas formulas facile expedietur; at major difficultas occurrit si n fuerit numerus fractus. Proponatur quærenda Linea curva, in qua sit $\sqrt{p} + \sqrt{q} + \sqrt{r} = \sqrt{a}$. Sumantur utrinque quadrata: atque, ob $p + q + r = P$, habebitur $P + 2\sqrt{pq} + 2\sqrt{pr} + 2\sqrt{qr} = a$, seu $\frac{a - P}{2} = \sqrt{pq} + \sqrt{pr} + \sqrt{qr}$. Sumantur denuo quadrata; atque, ob $pq + pr + qr = Q$, erit $\frac{(a - P)^2}{4} = Q + 2\sqrt{p^2qr} + 2\sqrt{pq^2r} + 2\sqrt{pqr^2} = Q + 2(\sqrt{p} + \sqrt{q} + \sqrt{r})\sqrt{pqr} = 2\sqrt{aR} + Q$: unde oritur $(a - P)^2 = 4Q + 8\sqrt{aR}$, seu $Q = \frac{(a - P)^2}{4} - 2\sqrt{aR}$. Quare, Curvæ quæsitæ continebuntur in hac æquatione $y^3 - Pyy + (\frac{1}{4}(a - P)^2 - 2\sqrt{aR})y - R = 0$; seu, (sublata irrationalitate, ob $R = \frac{(aa - 2aP + PP - 4Q)^2}{64a}$,) in hac æquatione $y^3 - Pyy + Qy - \frac{(aa - 2aP + PP - 4Q)^2}{64a} = 0$.

377. Hæc autem operatio nimis fit molesta, si radices altiorum potestatum proponantur: alia ergo via erit ineunda, quæ ex hoc exemplo perspicietur. Quæratur nempe Curva in qua sit $\sqrt[3]{p} + \sqrt[3]{q} + \sqrt[3]{r} = \sqrt[3]{a}$. Ponatur $\sqrt[3]{pq} + \sqrt[3]{pr} + \sqrt[3]{qr} = v$: &, cum sit $\sqrt[3]{pqr} = \sqrt[3]{R}$, fiet $\sqrt[3]{p^2} + \sqrt[3]{q^2} + \sqrt[3]{r^2} = \sqrt[3]{aa} - 2v$;