Socrates (469–399 BC) was the mentor of Plato. As Plato portrays him in the dialogue *Meno*, Socrates claimed that all knowledge is recollection. In an argument with Meno on the nature of virtue, Socrates bet him that he could make his slave 'remember' a geometric construction and its proof. Asked to double the unit square, the slave, who was completely ignorant of geometry, first offered to double its side, but was soon led to admit his error. Then Socrates got him to look at the figure of a square with the midpoints of its four sides all joined to each other (Figure 12.2) and soon persuaded him to 'remember' that it is the square on the diagonal of the inner square which has double its area.

As a young man, Plato (429–349 BC) was a disciple of Socrates. After the latter's death, Plato travelled to Africa, where he visited Heliopolis, now a suburb of Cairo, and Cyrene in Lybia. There he studied with Theodorus, who had proved the irrationality of the square roots of the nonsquare integers less than 18. Plato also went to Italy and became acquainted with Archytas (428–347 BC), the head of the Pythagorean school. Archytas also had 'doubled the cube', but he did so by going beyond the geometry of straight lines and circles.

Plato returned to Athens in about 380 BC and founded the famous Academy. At the entrance of this school was the inscription: *let no one ignorant of geometry enter here.*

The importance of Plato in the history of mathematics is due not so much to any mathematical contribution of his own as to the influence he exerted on others. It was he who insisted that a 'proper' solution involve no curves other than the circle (*Timaeus* 34a). It was he who emphasized the importance of clear definitions and postulates. Finally, Plato strongly encouraged people to study mathematics because he believed that this study would help them become wise and therefore virtuous. The five Platonic solids, or regular polyhedra, were not discovered by Plato, but he discussed them in the *Timaeus.*

Plato had a brilliant student, Theaetetus, who died in battle in 369 BC, and to whom he dedicated a dialogue. It was Theaetetus who showed that the square root of a natural number is irrational if and only if the natural number is not a square (*Theaetetus* 147c-148b). Theatetus also studied the regular polyhedra, and worked on the theory of proportion. According to van der Waerden [1985], Theaetetus was responsible for Books X and XIII of Euclid's *Elements.*

The most important Athenian mathematician at this time was Eudoxus of Cnidus, another small Greek island near modern Turkey. Eudoxus lived from 408 to 355 BC, and distinguished himself in astronomy, medicine, geography and philosophy – as well as mathematics. Like Plato, he studied astronomy in Heliopolis, and mathematics with Archytas in Tarentum (in what is now southern Italy). As a young man, Eudoxus studied in Plato's

Academy, commuting on foot from Piraeus, the harbour district. Later he engaged in a philosophical controversy with Plato; it seems that Eudoxus anticipated the Epicurean position that humans strive to maximize pleasure minus pain.

In mathematics, Eudoxus was responsible for Books V and XII of Euclid's *Elements*. Book V deals with the theory of proportion. Today, we might define the proportion 'a is to b as c is to d' (written $a : b :: c : d$) as an equation $a/b = c/d$, and we would say that the proportion held just in case $ad = bc$. However, this presupposes our theory of the real number field. It presupposes that we already have some way of understanding what it is to multiply two irrational numbers. Eudoxus was starting from scratch. He could not use multiplication to define proportion because it was in terms of proportion that he defined multiplication. Eudoxus used his theory of proportion to prove the basic laws of multiplication, such as commutativity and associativity. The definition on which he based his development of the number system was the following:

$a : b :: c : d$ if and only if, for all positive integers p and q,

$$pa > qb \text{ if and only if } pc > qd,$$

and likewise with $>$ replaced by $<$.

If we take a/b and c/d to be positive real numbers, this statement asserts:

- the set of rationals above $a/b =$ the set of rationals above c/d
- the set of rationals below $a/b =$ the set of rationals below c/d,

thus anticipating the modern definition of real numbers due to Dedekind.

Actually, Eudoxus assumed that a, b, c and d are geometric quantities. For example, a and b could be arcs of circles and c and d could be angles. This is another reason why he did not write $ad = bc$; for how do you multiply an arc by an angle?

One particular ratio Eudoxus was interested in arose from the following problem; to divide a segment AB by a point H so that $AB/AH = AH/HB$, that is, the whole is to the larger part as the larger part is to the smaller. Taking $AH = x$ and $HB = 1$, we obtain the quadratic equation $x^2 - x - 1 = 0$, so that, upon discarding the negative solution, we find $x = (1 + \sqrt{5})/2$. This, or sometimes its reciprocal $(-1 + \sqrt{5})/2$, is known as the *golden section.*

Eudoxus also gave a proof that the area of a circle is proportional to the square on its diameter (Euclid's *Elements* XII 2), by inscribing regular polygons with 2^n sides in both circles and taking n sufficiently large.

Finally, we should mention Menaechmus (350 BC), another student of Plato's. Menaechmus discovered the conics — the ellipse, hyperbola and parabola — and used them to 'double the cube'. Using modern analytic