

9. The term from the rho method becomes  $3.2 \times 10^{12}$  times as great, while the term from the factor base method becomes  $2.6 \times 10^6$  times as great.
10. (a) For  $s < s_0$ , we have  $h(s) \geq f(s) > f(s_0) = \frac{1}{2}h(s_0)$ , and for  $s > s_0$ , we have  $h(s) \geq g(s) > g(s_0) = \frac{1}{2}h(s_0)$ . (b) Apply part (a) to  $\log(f(s))$  and  $\log(g(s))$ .

#### § V.4.

1. (a)  $\frac{1}{1+} \frac{1}{1+} \frac{1}{4+}$ ; (b)  $\frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{1+}$ ; (c)  $1 + \frac{1}{7+} \frac{1}{1+} \frac{1}{2+} \frac{1}{4+}$ .
2. (a) Since  $a + \frac{1}{x} = x$ , it follows that  $x$  is the positive root of  $x^2 - ax - 1 = 0$ , i.e.,  $x = (a + \sqrt{a^2 + 4})/2$ . (b) Since the  $a_i$ 's are 1, the recurrence relation for the numerators and denominators of the convergents are the same as for the Fibonacci numbers.
3.  $2 + \frac{1}{1+} \frac{1}{2+} \frac{1}{1+} \frac{1}{1+} \frac{1}{4+} \frac{1}{1+} \frac{1}{1+} \frac{1}{6+} \dots$ ; it is possible to show that the  $a_i$ 's for  $i \equiv 2 \pmod{3}$  are the successive even integers, and all other  $a_i$ 's are 1.
4. For each  $b_i$  you have  $b_i^2 - c_i^2 n$  is the least absolute residue of  $b_i^2$  modulo  $n$ . If  $p$  divides this least absolute residue, then  $b_i^2 \equiv c_i^2 n \pmod{p}$ , and this means that  $n$  is a quadratic residue modulo  $p$ .
5. The tables below go through the first value of  $i$  such that the least absolute residues of  $b_0^2, \dots, b_i^2$  give a factorization of  $n$ . In four cases (parts (g), (i), (j), (k)) there is an earlier value of  $i$  such that some subset of these residues have corresponding vectors  $\vec{e}_i$  which sum to zero; however, in those cases we end up with  $b \equiv \pm c \pmod{n}$ .

(a)

$i$	0	1	2	3
$a_i$	97	1	1	17
$b_i$	97	98	195	3413
$b_i^2 \pmod{n}$	-100	95	-11	44

$$B = \{-1, 2, 5, 11\}, b = 97 \cdot 195 \cdot 3413, c = 2^2 \cdot 5 \cdot 11, g.c.d.(b+c, n) = 257.$$

(b)

$i$	0	1	2	3
$a_i$	116	2	4	1
$b_i$	116	233	1048	1281
$b_i^2 \pmod{n}$	-105	45	-137	80

$$B = \{2, 3, 5\}, b = 233 \cdot 1281, c = 2^2 \cdot 3 \cdot 5, g.c.d.(b+c, n) = 191.$$

(c)

$i$	0	1	2
$a_i$	93	1	2
$b_i$	93	94	281
$b_i^2 \pmod{n}$	-128	59	-32

$$B = \{-1, 2\}, b = 93 \cdot 281, c = 2^6, g.c.d.(b+c, n) = 67.$$