quo ipſius y valore in altera æquatione ſubſtituto, emerget CAP. XX.

$$cc = xx - 2fx + ff + (b-g)^2 + \frac{2b(b-g)x}{a} + \frac{bbxx}{aa},$$

ſeu

$$\begin{array}{l} +aa \\ +bb \end{array} xx \begin{array}{l} +2ab(b-g) \\ -2aaf \end{array} x \begin{array}{l} +aa(b-g)^2 \\ +aaff \\ -aacc \end{array} = 0,$$

cujus ergo æquationis radices invenientur per interſectiones Rectæ & Circuli, ita ut, demiſſis ex interſectionibus M & m in Axem perpendiculis MP, mp, valores ipſius x futuri ſint AP & Ap.

490. Quoniam in hac æquatione omnes æquationes quadraticæ continentur, hinc conſtructio generalis æquationum quadraticarum adornari poterit. Sit ſcilicet propoſita hæc æquatio quadratica

$$Axx + Bx + C = 0,$$

quæ ad ſuperiorem formam primum ita reducatur ut primi termini conveniant; multiplicando per $\frac{aa+bb}{A}$,

$$(aa+bb)xx + \frac{B(aa+bb)x}{A} + \frac{C(aa+bb)}{A} = 0.$$

Jam coæquatio reliquorum terminorum dabit

$$2Aab(b-g) - 2Aaaf = B(aa+bb)$$

ideoque fiet

$$af = b(b-g) - \frac{B(aa+bb)}{2Aa}.$$

Unde, cum ſit

$$aa(b-g)^2 + aaff - aacc = \frac{C(aa+bb)}{A},$$

erit

$$(aa+bb)(b-g)^2 - \frac{Bb(b-g)(aa+bb)}{Aa} + \frac{BB(aa+bb)^2}{4A^2a^2} - aacc = \frac{C(aa+bb)}{A}$$

ideoque $(b-$

LIB. II.

$$(b-g)^2 = \frac{Bb(b-g)}{Aa} - \frac{B(aa+bb)}{4A^2a^2} + \frac{aacc}{aa+bb} + \frac{C}{A}$$

ergo

$$b-g = \frac{Bb}{2Aa} \pm \sqrt{\left(\frac{aacc}{aa+bb} + \frac{C}{A} - \frac{B^2}{4AA}\right)}.$$

Manent igitur tres quantitates a, b, & c adhuc indeterminatæ, quas autem ita accipi oportet, ut $\frac{aacc}{aa+bb} + \frac{C}{A} - \frac{B^2}{4AA}$, fiat quantitas affirmativa, quia alioquin $b-g = AB - CD$, hincque CD, fieret quantitas imaginaria.

491. Nihil ergo impedit quominus ponamus $b = 0$, eritque $g = \sqrt{\left(cc - \frac{BB+4AC}{4AA}\right)}$ & $f = \frac{-B}{2A}$. Deinde vero, cum æquatio propoſita $Axx + Bx + C = 0$, radices nullas habeat reales, niſi ſit BB major quam $4AC$, erit hoc caſu $\frac{BB-4AC}{4AA}$ quantitas affirmativa, cui ſi cc ponatur æquale, ut ſit $c = \frac{\sqrt{(BB-4AC)}}{2A}$, fiet quoque $g = 0$, & a prorſus ex calculo excedit. Linea ergo recta EM in ipſum Axem AP incidet, & Centrum Circuli C collocari debebit in puncto D exiſtente $AD = \frac{-B}{2A}$, ex quo Centro ſi Circulus deſcribatur Radio $c = \frac{\sqrt{(BB-4AC)}}{2A}$, hujus interſectiones cum ipſo Axe oſtendent æquationis propoſitæ radices. Ne autem ad hoc conſtructione formulæ irrationalis opus ſit, ponatur $g = c - \frac{k}{2A}$, ut ſit $cc - \frac{2ck}{2A} + \frac{kk}{4AA} = cc - \frac{BB+4AC}{4AA}$, erit $c = \frac{kk+BB-4AC}{4kA}$, & $g = \frac{BB-4AC-kk}{4kA}$. In noſtro ergo arbitrio determinatio quantitatis k relinquitur; qua utcunque aſſumta, quia recta CM in ipſum Axem incidit, Circulus ſequenti modo deſcribi debebit. Sumta $AD = \frac{-B}{2A}$

$\frac{-B}{2A}$, capiatur perpendiculum $CD = \frac{BB - 4AC - kk}{4Ak}$, & Centro C deſcribatur Circulus cujus Radius $= \frac{BB - 4AC + kk}{4Ak}$; hujuſque interſectiones cum Axe oſtendent radices æquationis propoſitæ. Quod ſi ergo ſtatuatur $k = -B$, ſumta $AD = \frac{-B}{2A}$, capiatur $CD = \frac{C}{B}$, & Circuli Centro C deſcribendi Radius erit $= \frac{-BB + 2AC}{2AB} = \frac{-B}{2A} + \frac{C}{B}$, ex quo Radius Circuli erit $= AD + CD$; quæ conſtructio pro praxi commodiſſima videtur. CAP. XX.

492. Conſideremus jam duos Circulos ſe interſecantes: ſitque pro primo $AD = a$, $CD = b$, & ejus Radius $CM = c$; eritque, poſitis $AP = x$ & $PM = y$, $DP = a - x$, $CD - PM = b - y$; ideoque, ex natura Circuli, habebitur TAB. XXIV. Fig. 99.

$$xx - 2ax + aa + yy - 2by + bb = cc.$$

Simili modo pro altero Circulo ſit $Ad = f$, $dc = g$, ejuſque Radius $cM = h$, eritque

$$xx - 2fx + ff + yy + 2gy + gg = hh,$$

quibus æquationibus a ſe invicem ſubtractis, remanebit

$$2(f-a)x + aa - ff - 2(b+g)y + bb - gg = cc - hh,$$

ergo

$$y = \frac{aa + bb - ff - gg - cc + hh - 2(a-f)x}{2(b+g)};$$

hincque

$$b - y = \frac{bb + 2bg - aa + ff + gg + cc - hh + 2(a-f)x}{2(b+g)},$$

&

$$a - x = \frac{2a(b+g) - 2(b+g)x}{2(b+g)}.$$

Cum igitur ſit $(a-x)^2 + (b-y)^2 = cc$, erit, facta ſubſtitutione,

LIB. II.

$$\begin{matrix} +4(a-f)^2 \\ +4(b+g)^2 \end{matrix}\, xx \begin{matrix} -4(a+f)(b+g)^2 \\ -4(a-f)(aa-ff) \\ +4(a-f)(cc-hh) \end{matrix}\, x \begin{matrix} +(b+g)^4 \\ +2(aa-cc)(b+g)^2 \\ +2(ff-hh)(b+g)^2 \\ +(aa-cc-ff+hh)^2 \end{matrix} = 0.$$

Hujus ergo æquationis ope infinitis modis construi poterit æquatio $Axx + Bx + C = 0$; simul vero intelligitur æquationem quadratica altiorem per intersectionem duorum Circulorum construi non posse, propterea quod duo Circuli se mutuo in pluribus quam duobus punctis intersecare nequeunt. Cum igitur eadem æquatio quadratica construi possit per intersectionem Rectæ & Circuli, hæc constructio illi, quæ duos Circulos requirit, merito præfertur, nisi forte in casibus quibusdam sigularibus facilis Linearum a, b, f, g, c & h determinatio sponte se prodat.

TAB. XXIV. Fig. 100.

493. Intersecetur nunc Circulus a Parabola: sit scilicet, demisso ex Centro Circuli C in Axem AP perpendiculo CD, $AD = a$, $CD = b$, & Radius Circuli $CM = c$, erit inter Coordinatas orthogonales $AP = x$, $PM = y$, æquatio pro Circulo $(x-a)^2 + (y-b)^2 = cc$. Parabolæ vero Axis FB statuatur ad Axem hic assumtum AP normalis: sitque $AE = f$, $EF = g$, & Parameter Parabolæ $= 2h$; erit, ex natura Parabolæ, $EP^2 = 2h(EF + PM)$, seu in symbolis $(x-f)^2 = 2h(g+y)$, unde erit $y = \frac{(x-f)^2}{2h} - g$ & $y - b = \frac{(x-f)^2}{2h} - (b+g)$. Qui valor si in priori æquatione substituatur, eliminabitur y, eritque

$$\frac{(x-f)^4}{4hh} - \frac{(b+g)(x-f)^2}{h} + (b+g)^2 + (x-a)^2 = cc$$

sive

$$x^4 - 4fx^3 \begin{matrix} +6ff \\ -4h(b+g) \\ +4hh \end{matrix}\, x^2 + \begin{matrix} -4f^3 \\ 4fh(b+g) \\ -8ahh \end{matrix}\, x \begin{matrix} +f^4 \\ -4ffh(b+g) \\ +4hh(b+g)^2 \\ +4aahh \\ -4cchh \end{matrix} = 0$$

cujus

cujus æquationis radices erunt Abſciſſæ AP, Ap, Ap, Ap, unde Applicatæ per interſectionum puncta M, m, m, m, tranſeunt. CAP. XX.

494. In hac æquatione ſex inſunt conſtantes a, b, c, f, g, & h; quarum vero binæ $b+g$ pro una ſunt reputandæ, ita ut quinque ſolum, ponendo $b+g=k$, ineſſe cenſendæ ſint. Poſito ſcilicet $CD+EF=b+g=k$, ſequens habebitur æquatio

$$x^4 - 4fx^3 \begin{array}{l} +6ff \\ -4hk\,xx \\ +4hh \end{array} \begin{array}{l} -4f^3 \\ +4fhk\,x \\ -8ahh \end{array} \begin{array}{l} +f^4 \\ -4ffhk \\ +4hhkk \\ +4aahh \\ -4cchh \end{array} = 0.$$

Ad hanc autem formam omnis æquatio biquadratica revocari poteſt; ſit enim propoſita hæc æquatio

$$x^4 - Ax^3 + Bxx - Cx + D = 0$$

erit, comparatione inſtituta,

$$4f = A \quad \text{ſeu} \quad f = \frac{1}{4}A$$

$$6ff - 4hk + 4hh = B \quad \text{ſeu} \quad \frac{3}{8}AA - 4hk + 4hh = B,$$

unde fit

$$k = \frac{3AA}{32h} + h - \frac{B}{4h},$$

$$4f^3 - 4fhk + 8ahh = C$$

ſive

$$\frac{1}{16}A^3 - \frac{3}{32}A^3 - Ahh + \frac{1}{4}AB + 8ahh = C$$

ergo

$$a = \frac{A^3}{256hh} + \frac{A}{8} - \frac{AB}{32hh} + \frac{C}{8hh}.$$

Denique eſt

$$(ff - 2hk)^2 + 4aahh - 4cchh = D.$$

At eſt

 $ff-$

LIB. II.

$$ff - 2hk = \frac{B}{2} - 2hh - \frac{AA}{16},$$

&

$2ah = \frac{A^3}{128h} + \frac{Ah}{4} - \frac{AB}{16h} + \frac{C}{4h}$, quibus valoribus substitutis emerget æquatio c & h involvens, quas propterea convenientissime inde definiri oportet, ita scilicet ut utraque valorem obtineat realem.

495. Quoniam vero in omni æquatione biquadratica secundus terminus facile tolli potest; ponamus ipsum jam esse sublatum, ideoque construendam esse hanc æquationem

$$x^4 * + Bxx - Cx + D = 0.$$

Erit ergo primum, $f = 0$; secundo $k = h - \frac{B}{4h}$; tertio $a = \frac{C}{8hh}$; atque, ob $2hk - ff = 2hh - \frac{B}{2}$, & $2ah = \frac{C}{4h}$, quarto $4h^4 - 2Bhh + \frac{1}{4}BB + \frac{CC}{16hh} - 4cchh = D$, unde fit $64cch^4 = CC + 4BBhh - 32Bh^4 + 64h^6 - 16Dhh$; ideoque $8chh = \sqrt{(4hh(B - 4hh)^2 + CC - 16Dhh)}$. Quoniam vero hoc imprimis est efficiendum ut tam c quam h obtineant valores reales, ponatur $c = h - \frac{B+q}{4h}$, eritque

$$CC - 16Dhh + 8Bhhq - 32h^4q - 4hhqq = 0.$$

Quo igitur quæsito satisfaciamus, duo casus sunt distinguendi, alter quo D est quantitas negativa, alter quo D est quantitas affirmativa. Sit igitur

I.

D quantitas affirmativa $= + EE$, ita ut construi debeat hæc æquatio

$$x^4 * + Bx^2 - Cx + EE = 0,$$

ponatur ad hoc $q = 0$, ut sit $c = \frac{4hh - B}{4h}$, fietque $hh =$

CC