rational inquiry that, it was hoped, would allow the proper organization of all knowledge and the proper conduct of all human affairs. The movement to reorganize knowledge and conduct along rational lines became known as the Enlightenment, and it was particularly strong in France, where philosophers saw it as a means to overthrow existing institutions, particularly the church. Around 1745 d'Alembert became immersed in the ferment of the Enlightenment, then bubbling in the salons and cafés of Paris. He made friends with the leading lights—Diderot, Condillac, Rousseau—and was in demand at the most fashionable salons for his wit and gift for mimicry.

The Enlightenment was not all talk, however, and one of its most solid achievements was the 17-volume *Encyclopédie*, edited by Diderot between 1745 and 1772. D'Alembert wrote the introduction to the *Encyclopédie*, the *Discours préliminaire*, and in it summed up his views on the unity of all knowledge. It contributed greatly to the success of the project, and was the main reason for his election to the Académie Française. D'Alembert was also scientific editor and wrote many of the mathematics articles. Eventually a split developed among the encyclopedists between the extreme materialists, led by Diderot, and the more moderate faction of Voltaire. Diderot leaned toward biology, for which he conjectured an absurd pseudomathematical basis, while deploring the "impracticality" of ordinary mathematics. D'Alembert sided with Voltaire and cut his ties with the *Encyclopédie* in 1758.

Nevertheless, intellectual fashion was moving away from mathematics, and in the 1760s d'Alembert found himself with only one philosopher friend still interested in it, the probability theorist Condorcet. At about this time d'Alembert met the one love of his life, Julie de Lespinasse. Julie was the cousin of Madame du Deffand, whose salon d'Alembert attended. After a quarrel over poaching the salon's members, Julie set up a salon of her own, with d'Alembert's help. When Julie became ill with smallpox, d'Alembert nursed her back to health; when he himself fell sick, she persuaded him to move in with her. This was in 1765, when he finally left his foster home. For the next ten years his life revolved around Julie's salon, and her death in 1776 came as a cruel blow. Humiliation was added to sorrow when he discovered from her letters that she had been passionately involved with other men throughout their time together.

D'Alembert spent his last seven years in a small apartment in the Louvre, to which he was entitled as permanent secretary of the Académie Française. He found himself unable to work in mathematics, although it was the only thing that still interested him, and he became gloomy about the future of mathematics itself. Despite his gloom, he did what he could to support and encourage young mathematicians. Perhaps the finest achievement of d'Alembert's later years was to launch the careers of Lagrange and Laplace, whose work in mechanics ultimately completed much of his own. It must have given him some satisfaction to anticipate the future successes of his gifted protégés, even though they effectively ended the theory of mechanics as d'Alembert knew it. What he could not have anticipated was that a minor element of his work, the use of complex numbers, would blossom in the next century (see Sections 16.1 and 16.2) and that mathematics would break out of the bounds set by eighteenth-century thinking.

15

Complex Numbers and Curves

15.1 Roots and Intersections

There is a close connection between intersections of algebraic curves and roots of polynomial equations, going back as far as Menaechmus' construction of $\sqrt[3]{2}$ (a root of the equation $x^3 = 2$) by intersecting a parabola and a hyperbola (Section 2.4). The most direct connection, of course, occurs in the case of a polynomial curve

$$y = p(x) \tag{1}$$

whose intersections with the axis $y = 0$ are just the real roots of the equation

$$p(x) = 0. \tag{2}$$

If (2) has k real roots, then the curve (1) has k intersections with the axis $y = 0$. Here we must count intersections the same way we count roots, according to *multiplicity*. A root r of (2) has multiplicity μ if the factor $(x - r)$ occurs μ times in $p(x)$, and the root r is then counted μ times.

This way of counting is also geometrically natural because if, for example, the curve $y = p(x)$ meets the axis $y = 0$ with multiplicity 2 at 0, then a line $y = \varepsilon x$ "close" to the axis meets the curve twice—once near the intersection with the axis and once precisely there. The intersection of $y = x^2$ with $y = 0$ (Figure 15.1) can therefore be considered as two *coincident points* to which the distinct intersections with $y = \varepsilon x$ tend as $\varepsilon \to 0$.

J. Stillwell, *Mathematics and Its History*