One of the few personal documents preserved by Harriot is his summary of the speech Raleigh made at the scene of his execution in 1618 [see Shirley (1983, p. 447)].

A month after Raleigh's death, a bright comet appeared in the skies, and Harriot's observations of it were his last major scientific endeavor. He had been suffering for some years from a painful cancer of the nose and finally succumbed to it on a visit to London in 1621. He was buried at St. Christopher's Church in Threadneedle Street, later destroyed in the Great Fire of 1666. The site is now part of the Bank of England, where a replica of Harriot's original monument was installed on July 2, 1971, the three hundred and fiftieth anniversary of his death.

Carl Friedrich Gauss was born in Brunswick (Braunschweig) in 1777 and died in Göttingen in 1855. He was the only child of Gebhard Gauss and Dorothea Benze, though his father had another son from a previous marriage. Gebhard earned his living mainly from manual labor, but he also did a little accounting, and Gauss is said to have corrected an error in his father's arithmetic at the age of three. (It should be borne in mind here that stories about Gauss' youth were told by Gauss himself in old age, and in a few cases there is evidence that he was prone to exaggerate his own precocity.) Gauss was not close to his father and believed that his genius was inherited from his mother. He started school in 1784 and his teacher, Büttner, soon recognized his ability and obtained advanced books for him. Büttner's assistant, Martin Bartels (1769–1836), also gave Gauss special attention. Bartels was himself a beginning mathematician who later became professor at the University of Kazan and the teacher of Lobachevsky (see next chapter).

Gauss entered secondary school in 1788, and in 1791 he won an annual grant from the Duke of Brunswick, something like a government scholarship. He was also selected to enter the Collegium Carolinum, a new scientific academy for outstanding secondary students. In his years there, 1792–1795, Gauss studied the works of Newton, Euler, and Lagrange and began investigations of his own, mainly numerical experiments on such things as the arithmetic-geometric mean. In 1795 he left Brunswick to study at Göttingen in the adjoining state of Hannover, which was then ruled by George III of England. The duke would have preferred Gauss to remain in Brunswick, and the local university of Helmstedt, but continued his financial support nevertheless. Gauss actually chose Göttingen because of its better library and later spoke very disparagingly of its mathematics professor, Kaestner. It is true that Gauss' student achievements, which began with his construction of the regular 17-gon (Section 2.3) and culminated in his proof of the fundamental theorem of algebra (Section 14.7), dwarfed those of his teachers. Still, one wonders whether Kaestner's definition of curvature (Section 17.2) might not have been useful to Gauss when he took up differential geometry later.

Figure 17.12: Carl Friedrich Gauss

Gauss returned to Brunswick in 1798 and lived there until 1807. Figure 17.12 is a portrait of him from this period, which was the happiest and most productive of his life. Gauss published his great work on number theory, the *Disquisitiones arithmeticae*, in 1801, made a spectacular entry into astronomy in the same year by predicting the position of the asteroid Ceres, and married Johanna Osthoff in 1805. Writing to his friend Farkas

Bolyai in 1804, Gauss was uncharacteristically warm and open when it came to Johanna:

> The beautiful face of a madonna, a mirror of peace of mind and health, tender, somewhat fanciful eyes, a blameless figure—this is one thing; a bright mind and an educated language—this is another; but the quiet, serene, modest and chaste soul of an angel who can do no harm to any creature—that is the best.
>
> [Translation from Kaufmann-Bühler (1981), p. 49]

If only Johanna had lived longer, Gauss might have become a very different man. But in 1809 she died, shortly after giving birth to their third child. Gauss was devastated by the blow and never quite recovered his equilibrium.

Less than a year after Johanna's death, he married Minna Waldeck, the daughter of a Göttingen professor. Unlike Johanna, who was a tanner's daughter, Minna had social status and pretensions that caused Gauss uneasiness and embarrassment. Soon after their engagement, for example, he had to tell Minna not to write to his mother, as his mother could not read. Minna also suffered from poor health, and after the couple had had three children between 1811 to 1816 she became virtually a permanent invalid. Gauss found this burden difficult to bear and compounded his problems by unsympathetic treatment of his children. The family drama came to a head in 1830, when their eldest son, Eugen, emigrated to America after a row with his father. The following year Minna died of tuberculosis.

During this unhappy period Gauss was less productive mathematically, but this was not due to family troubles so much as his choice of career. He had become director of the Göttingen observatory in 1807 and in 1817 substituted geodesy for some of his astronomical duties, doing arduous field work every summer from 1818 to 1825 for the geodetic survey of Hannover. Gauss appears to have seldom regretted this choice of career—he disliked teaching and thought that other mathematicians had little to teach him—but it cannot be said that his contributions to astronomy and geodesy were as great as his contributions to mathematics. Indeed, the best things to come out of his geodetic work were his theory on conformal mapping and complex functions (Section 16.2) and his intrinsic notion of curvature (Section 17.3).

In the 1830s Gauss experienced something of a rebirth with the arrival of the young physicist Wilhelm Weber in Göttingen. The two collaborated enthusiastically in the investigation of magnetism, with Gauss making contributions to both the theory and practice (the electric telegraph). However their partnership was broken in 1837 when Weber was fired for his courageous refusal to swear an oath of allegiance to the new king of Hannover.

Among the few bright spots of Gauss' later years were his students Eisenstein and Dedekind, as well as Riemann's lecture on the foundations of geometry in 1854. After spending most of his life aloof from other mathematicians, Gauss must have been comforted to find at last that there *were* students capable of understanding his ideas and carrying them further. We can only wonder what might have been if he had made this discovery earlier.

18

Noneuclidean Geometry

18.1 The Parallel Axiom

Until the nineteenth century, Euclid's geometry enjoyed absolute authority, both as an axiomatic system and as a description of physical space. Euclid's proofs were regarded as models of logical rigor, and his axioms were accepted as correct statements about physical space. Even today, Euclidean geometry is the simplest type of geometry, and it furnishes the simplest description of physical space for everyday purposes. Beyond the everyday world, however, lies a vast universe that can be understood only with the help of an expanded geometry. The expansion of geometric concepts initially grew from dissatisfaction with one of Euclid's axioms, the *parallel axiom*.

For our purposes, the most convenient statement of the parallel axiom is as follows:

Axiom P_1. For each straight line L and point P outside L there is exactly one line through P that does not meet L.

There are many other equivalent statements of Axiom P_1, some obviously fairly close to it, for example, Euclid's own:

> That if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.
>
> [Heath (1925), p. 202]