LIB. II. $z = \frac{y^{\frac{2}{n}} - a^{\frac{2}{n}}}{2y^{\frac{1}{n}}}$. Ponatur $\frac{1}{n} = m$; atque, si in æquatione inter $z$ & $x$ data ubique ponatur $z = \frac{y^{2m} - a^{2m}}{2y^{m}}$, obtinebitur æquatio inter $x$ & $y$ pro Curva quæsita. Cum igitur inter $z$ & $x$ binas invenerimus æquationes; scilicet, vel

$$0 = \alpha + \beta xx + \gamma xz + \delta zz + \epsilon x^4 + \zeta x^3 z + \eta x^2 z^2 + \theta xz^3 + \&c.$$
vel
$$0 = \alpha x + \beta z + \gamma x^3 + \delta x^2 z + \epsilon xz^2 + \zeta z^3 + \eta x^5 + \theta x^4 z + \&c.$$

si in his æquationibus ponatur $z = y^m - \frac{a^{2m}}{y^m}$ ( divisorem 2 negligimus quia pro $Q$ quodcunque multiplum ipsius $z$ sumi potest ), duæ orientur æquationes generales pro Curvis quæsito satisfacientibus.

387. Sit, præter $P$, quoque $R$ Functio par, & præter $Q$ quoque $S$ Functio impar ipsius $x$, ac statuatur pro Curvis quæsitis hæc æquatio $y = \frac{P+Q}{R+S} = PM$: erit ergo $QN = \frac{P-Q}{R-S}$, fietque $\frac{PP-QQ}{RR-SS} = aa$, cui conditioni facillime satisfit ponendo $y = \frac{P+Q}{P-Q}.a$, vel etiam statuendo $y = (\frac{P+Q}{P-Q})^n a$. Hoc modo prius incommodum, quod cuique Abscissæ duæ pluresve Applicatæ respondebant, evitatur, atque ejusmodi Curvæ inveniuntur, ut singulis Abscissis unica tantum Applicata respondeat. Hinc Curva simplicissima satisfaciens erit Linea secundi ordinis hac æquatione $y = \frac{b+x}{b-x}a$ contenta; atque ideo Hyperbola. Hyperbola vero etiam satisfacit æquationi

quationi prius inventæ $y = Q + \sqrt{(aa + QQ)}$, ponendo $Q = nx$: erit enim $yy - 2nxy = aa$. Unde huic problemati duplici modo per Hyperbolam ſatisfieri poteſt. Cap. XVI.

388. His præmiſſis, perſpicuum eſt æquationem pro Curva quæſita ita comparatam eſſe debere, ut ea, ſi loco $x$ ponatur $-x$, & $\frac{aa}{y}$ loco $y$, nullam alterationem patiatur. Hujuſmodi formulæ ſunt $(y^n + \frac{a^{2n}}{y^n})P$, & $(y^n - \frac{a^{2n}}{y^n})Q$; ſi quidem $P$ Functionem parem & $Q$ imparem ipſius $x$ denotet. Quod ſi ergo æquatio formetur, quæ ex quotcunque hujuſmodi formulis fuerit compoſita, ea erit pro Curva quæſtioni ſatisfaciente. Quod ſi ergo $M$, $P$, $R$, $T$, &c., denotent Functiones quaſcunque pares ipſius $x$, atque $N$, $Q$, $S$, $V$, &c. Functiones impares, ſequens æquatio generalis habebitur

$$\begin{aligned} 0 = M &+ \left(\frac{y}{a} + \frac{a}{y}\right)P + \left(\frac{yy}{aa} + \frac{aa}{yy}\right)R + \left(\frac{y^3}{a^3} + \frac{a^3}{y^3}\right)T \text{ \&c.} \\ &+ \left(\frac{y}{a} - \frac{a}{y}\right)Q + \left(\frac{yy}{aa} - \frac{aa}{yy}\right)S + \left(\frac{y^3}{a^3} - \frac{a^3}{y^3}\right)V \text{ \&c.} \end{aligned}$$

quæ ſi multiplicetur per Functionem imparem ipſius $x$, Functiones pares in impares & viciſſim permutabuntur: unde etiam hujuſmodi æquatio ſatisfaciet

$$\begin{aligned} 0 = N &+ \left(\frac{y}{a} + \frac{a}{y}\right)Q + \left(\frac{yy}{aa} + \frac{aa}{yy}\right)S + \left(\frac{y^3}{a^3} + \frac{a^3}{y^3}\right)V \text{ \&c.} \\ &+ \left(\frac{y}{a} - \frac{a}{y}\right)P + \left(\frac{yy}{aa} - \frac{aa}{yy}\right)R + \left(\frac{y^3}{a^3} - \frac{a^3}{y^3}\right)T \text{ \&c.} \end{aligned}$$

quæ æquationes a fractionibus liberatæ dabunt has æquationes rationales ordinis indefiniti $n$

Lib. II.

I.

$$\begin{aligned} 0 = {} & a^n y^n M + a^{n-1} y^{n+1}(P+Q) + a^{n-2} y^{n+2}(R+S) + a^{n-3} y^{n+3}(T+V) \&c. \\ & + a^{n+1} y^{n-1}(P-Q) + a^{n+2} y^{n-2}(R-S) + a^{n+3} y^{n-3}(T-V) \&c. \end{aligned}$$

II.

$$\begin{aligned} 0 = {} & a^n y^n N + a^{n-1} y^{n+1}(P+Q) + a^{n-2} y^{n+2}(R+S) + a^{n-3} y^{n+3}(T+V) \&c. \\ & - a^{n+1} y^{n-1}(P-Q) - a^{n+2} y^{n-2}(R-S) - a^{n+3} y^{n-3}(T-V) \&c. \end{aligned}$$

389. In formulis vero $(y^n + \frac{a^{2n}}{y^n})P$, & $(y^n - \frac{a^{2n}}{y^n})Q$ loco $n$ quoque numeros fractos scribere licet. Quare, si pro $n$ scribantur numeri $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, &c. ex æquationibus generalibus hinc oriundis irrationalitas sponte evanescet : habebitur enim

$$\begin{aligned} 0 = {} & \frac{y+a}{\sqrt{ay}} P + \frac{y^3+a^3}{ay\sqrt{ay}} R + \frac{y^5+a^5}{a^2y^2\sqrt{ay}} T + \&c. \\ & + \frac{y-a}{\sqrt{ay}} Q + \frac{y^3-a^3}{ay\sqrt{ay}} S + \frac{y^5-a^5}{a^2y^2\sqrt{ay}} V + \&c. \end{aligned}$$

vel hæc æquatio

$$\begin{aligned} 0 = {} & + \frac{y+a}{\sqrt{ay}} Q + \frac{y^3+a^3}{ay\sqrt{ay}} S + \frac{y^5+a^5}{a^2y^2\sqrt{ay}} V + \&c. \\ & + \frac{y-a}{\sqrt{ay}} P + \frac{y^3-a^3}{ay\sqrt{ay}} R + \frac{y^5-a^5}{a^2y^2\sqrt{ay}} T + \&c. \end{aligned}$$

quæ a fractionibus liberatæ abeunt in has

$$\begin{aligned} 0 = {} & + a^n y^{n+1}(P+Q + a^{n-1} y^{n+2}(R+S) + a^{n-2} y^{n+3}(T+V) \&c. \\ & + a^{n+1} y^n(P-Q) + a^{n+2} y^{n-1}(R-S) + a^{n+3} y^{n-2}(T-V) \&c. \end{aligned}$$

&

$$\begin{aligned} 0 = {} & + a^n y^{n+1}(P+Q + a^{n-1} y^{n+2}(R+S) + a^{n-2} y^{n+3}(T+V) \&c. \\ & - a^{n+1} y^n(P-Q) - a^{n+2} y^{n-1}(R-S) - a^{n+3} y^{n-2}(T-V) \&c. \end{aligned}$$

390. Ex

390. Ex his quatuor æquationibus jam ex ſingulis Linearum ordinibus eæ, quæ problema reſolvant, facile invenientur. Ac primo quidem, ex primo ordine ſatisfacit Linea recta Axi $AP$ parallela ac per punctum $B$ tranſiens. Ex ordine ſecundo binæ æquationes priores, faciendo $n = 1$, dant $\alpha a x y + yy - aa = 0$, quæ ex ſecunda naſcitur, ponendo $N = \alpha x$, & $P = 1$, & $Q = 0$. Prima enim nullam dat Lineam curvam; binæ poſteriores æquationes dant, faciendo $n = 0$, $y(\alpha + \beta x) \pm a(\alpha - \beta x) = 0$. Ex ordine tertio binæ æquationes priores dant, faciendo $n = 1$. Cap. XVI.

$$0 = ay(\alpha + \beta xx) + yy(\gamma + \delta x) + aa(\gamma - \delta x)$$

&

$$0 = \alpha a y x + yy(\gamma + \delta x) - aa(\gamma - \delta x)$$

binæ autem æquationes poſteriores dant, ponendo $n = 0$, & $n = 1$

$$0 = y(\alpha + \beta x + \gamma xx) \pm a(\alpha - \beta x + \gamma xx)$$

&

$$0 = ay^2(\alpha + \beta x) + y^3 \pm a^2 y(\alpha - \beta x) \pm a^3$$

ſimilique modo ex ſequentibus ordinibus omnes Lineæ quæſito ſatisfacientes reperientur.

Lib. II.

## CAPUT XVII.

### *De inventione Curvarum ex aliis proprietatibus.*

391. Quæstiones, quas in præcedente Capite resolvimus, ita erant comparatæ, ut ad æquationem inter Coordinatas, sive rectangulas sive obliquangulas, facile revocari possent. Nunc igitur ejusmodi proprietates contemplemur, quæ non immediate Applicatas inter se parallelas respiciant; veluti, si rectarum ex dato quodam puncto ad Curvam

Tab. XX. Fig. 81.

eductarum indoles quæpiam proponatur. Sit $C$ punctum, unde rectæ ad Curvam educantur $CM$, $CN$, atque proprietas quæpiam has rectas respiciens fuerit proposita: conveniet a modo hactenus usitato naturam Curvarum per Coordinatas exprimendi, ita recedere, ut istæ rectæ in æquationem introducantur.

392. Cum igitur pluribus aliis modis naturæ Linearum æquationibus comprehendi queant, quæ inter duas variabiles formentur, in præsenti negotio quantitas rectæ $CM$ ex dato puncto $C$ ad Curvam eductæ alterius variabilis locum sustineat. Tum vero alia opus erit variabili, qua situs rectæ $CM$ definiatur; hunc in finem assumatur recta quæpiam $CA$ per punctum $C$ ducta pro Axe, atque angulus $ACM$, seu quantitas ab hoc angulo pendens, commodissime vicem alterius variabilis tenebit. Sit ergo recta $CM = z$, & angulus $ACM = \phi$, cujus sinus, tangensve in æquationem ingrediatur; atque manifestum est, si detur æquatio quæcunque inter $z$ & $\sin.\phi$, seu $\tang.\phi$, per eam Curvæ $AMN$ naturam determinari, pro quovis enim angulo $ACM$, definitur longitudo rectæ $CM$ sicque punctum Curvæ $M$ determinatur.

393. Diligentius autem perpendamus hunc Lineas curvas exprimendi modum. Ac primo quidem æquetur distantia $z$ Functioni cuicunque sinus anguli $\phi$; quæ Functio si fuerit uniformis,