geometry, we can express his solution to the 'Delian problem' in the following simple fashion: the parabolas $y = \frac{1}{2}x^2$ and $x = y^2$ intersect at a point whose y coordinate is the cube root of 2. In his 'doubling of the cube', Menaechmus did not stick to straight lines and circles. Indeed, as already mentioned, in 1837, Pierre Wantzel showed that it is not possible to obtain a segment of length equal to the cube root of 2, using only the geometry of straight lines and circles. We shall give a proof of this in Chapter 14.

Exercises

1. Prove that the inscribed regular 2^n-gon takes up more than $1-1/2^{n-1}$ of the area of the circle.

2. If the diameter of a circle is d, prove that the area of the inscribed regular 2^n-gon is $2^{n-3}d^2 \sin(\pi/2^{n-1})$.

3. Prove the theorem of Theaetetus that a natural number has an irrational square root if and only if it is not a perfect square.

4. Using the definition of proportion given by Eudoxus, show that $a : b :: c : d$ if and only if $d : c :: b : a$.

13

Plato and Aristotle on Mathematics

Plato (427–347 BC) believed that the objects in the universe fall into two very different classes, the material and the immaterial. A chair or an ox belongs to the class of material things. A soul or a number belongs to the class of immaterial things. The drawing of a square belongs to the material realm but the square itself belongs to the immaterial realm. Plato says of the students of geometry that they

> make use of the visible forms and talk about them, though they are not thinking of them but of those things of which they are a likeness, pursuing their inquiry for the sake of the square as such and the diagonal as such, and not for the sake of the image of it which they draw (*Republic* 510d).

For Plato, the class of material things is characterized by change, uncertainty, ignorance and imperfection. The drawing of a square can be erased and it is doubtful whether its angles are each exactly 90° or whether its sides are perfectly straight.

On the other hand, the class of immaterial things is characterized by their constancy and perfection and by our certain knowledge of them. The square 'as such' has sides which remain perfectly straight forever. Its properties can be deduced with infallible rigour. We can know with absolute certainty that its diagonals are equal.

Scientists understand change and motion in the universe in terms of unchanging formulas or laws. Plato had a similar outlook (*Timaeus* 52-58). Moreover, he stressed that the formulas or laws have an existence of their own, independent of the material universe.

According to Plato, mathematical objects are not the only immaterial objects. Other immaterial objects are God, goodness, courage and the human soul (*Republic* 380d-383c). However, the best way to begin to know the immaterial realm is to do mathematics. One is to study number theory 'for facilitating the conversion of the soul itself from the world of generation to essence and truth' (*Republic* 525c). One is to study geometry 'to facilitate the apprehension of the idea of good' (*Republic* 525e).

Plato believes that the truths of mathematics are absolute, necessary truths. He believes that, in studying them, we shall be in a better position to know the absolute, necessary truths about what is good and right, and thus be in a better position to become good ourselves.

Platonism as a philosophy of mathematics is the view that at least the most basic mathematical objects (e.g., real numbers, Euclidean squares) actually exist, independently of the human mind which conceives them. Their properties are discovered, not created.

Aristotle (384–322 BC) was a student of Plato, but he disagreed with him about the nature of mathematics. In Book XIII of the *Metaphysics*, Aristotle asserts that

> conclusions contrary alike to truth and to the usual views follow, if one is to suppose the objects of mathematics to exist thus as separate entities (*Metaphysics* 1077a).

For Aristotle, a word like 'two' is not a noun designating an abstract object but rather an adjective describing a concrete object (the *two* yard ladder, a *two* year period).

Whereas Platonism is quite compatible with the view that there are actually infinite lines and sets with an infinite numbers of elements, Aristotle is a staunch finitist. He would have rejected Cantor's 'aleph-null' (*Metaphysics* 1084a); he would have rejected infinitesimals (*Physics* 266b); he did reject infinite sets and infinite magnitudes (*Physics* III). For Aristotle, the geometer can have as much as he needs of an infinite line but he cannot have the whole line in its infinite totality.

Under the influence of Plato, Aristotle formulated a principle according to which every (mathematical) statement is either true or false, but he had his doubts when it came to applying this principle to the everyday temporal world. He wondered whether a statement like 'there will be a sea battle tommorrow' is either true or false (*On Interpretation* 9). How can it be true if the battle may not occur? How can it be false if the fight is a real possibility?

Like the view of the 20th century 'intuitionists', Aristotle's view is human-centered. The reality of numbers has to do, not with some alien heaven, but with the way we describe our surroundings. The infinite must be rejected because we humans work in a finite way. The truth about certain