militer affecta vt in art. praec., puta in $F = \varphi(t, u, v \ldots)$ pro indeterminatis t, u, v etc. substituantur aggregata (γ, λ), (γ, λ'), (γ, λ'') etc. resp., eius valorem per praecepta art. praec. reduci ad $A + a(\gamma, 1) + a'(\gamma, g) \ldots + a^{\zeta}(\gamma, g^{\alpha\delta - \alpha}) \ldots + a^{\theta}(\lambda, g^{\alpha\beta - 1}) = W$. Tum dico, si F sit functio inuariabilis, eas periodos in W, quae sub eadem periodo $\beta\gamma$ terminorum contentae sint, i. e. generaliter tales (γ, g^{μ}) et $(\gamma, g^{\alpha\nu + \mu})$ designante ν integrum quemcunque, coëfficientes eosdem habituras esse.

Dem. Quum periodus $(\beta\gamma, \lambda g^{\alpha})$ identica sit cum hac $(\beta\gamma, \lambda)$, minores hae $(\gamma, \lambda g^{\alpha})$, $(\gamma, \lambda' g^{\alpha})$, $(\gamma, \lambda'' g^{\alpha})$ etc., e quibus manifesto prior constat, necessario cum iis conuenient e quibus posterior constat, etsi alio ordine. Quodsi itaque, illis pro t, u, v etc. resp. substitutis, F in W' transire supponitur, W' coincidet cum W. At per art. 347 erit $W' = A + a(\gamma, g^{\alpha}) + a'(\gamma, g^{\alpha + 1}) \ldots + a^{\zeta}(\gamma, g^{\alpha\delta}) \ldots + a^{\theta}(\gamma, g^{\alpha\beta + \alpha - 1}) = A + a(\gamma, g^{\alpha}) + a'(\gamma, g^{\alpha + 1}) \ldots + a^{\zeta}(\gamma, 1) \ldots + a^{\theta}(\gamma, g^{\alpha - 1})$; quare quum haec expressio cum W conuenire debeat, coëfficiens primus, secundus, tertius etc. in W (incipiendo ab a) necessario conueniet cum $\alpha + 1^{\text{to}}$, $\alpha + 2^{\text{to}}$, $\alpha + 3^{\text{to}}$ etc., vnde nullo negotio concluditur, generaliter coëfficientes periodorum (γ, g^{μ}), $(\gamma, g^{\alpha + \mu})$, $(\gamma, g^{2\alpha + \mu}) \ldots (\gamma, g^{\nu\alpha + \mu})$, qui sunt $\mu + 1^{\text{tus}}$, $\alpha + \mu + 1^{\text{tus}}$, $2\alpha + \mu + 1^{\text{tus}} \ldots \nu\alpha + \mu + 1^{\text{tus}}$, inter se conuenire debere. *Q. E. D.*

Hinc manifestum est, W reduci posse ad formam $A + a(\beta\gamma, 1) + a'(\beta\gamma, g) \ldots + a^{\varepsilon}(\beta\gamma, g^{\alpha - 1})$, vbi omnes coëfficientes A, a etc. integri erunt, si omnes coëfficientes determinati in F sunt integri. Porro facile perspicietur, si postea pro indeterminatis in F substituantur β periodi γ terminorum in alia periodo $\beta\gamma$ terminorum puta in $(\beta\gamma, \lambda k)$ contentae, quae manifesto erunt $(\gamma, \lambda k)$, $(\gamma, \lambda' k)$, $(\gamma, \lambda'' k)$ etc., valorem inde prodeuntem fore $A + a(\beta\gamma, k) + a'(\beta\gamma, gk) \ldots + a^{\varsigma}(\beta\gamma, g^{\alpha-1}k)$.

Ceterum patet, theorema ad eum quoque casum extendi posse, vbi $\alpha = 1$, siue $\beta\gamma = n - 1$; scilicet hic *omnes* coëfficientes in W aequales erunt, vnde W reducetur sub formam $A + a(\beta\gamma, 1)$.

351. Retentis itaque omnibus signis art. praec., manifestum est, singulos coëfficientes aequationis, cuius radices sunt β aggregata (γ, λ), (γ, λ'), (γ, λ'') etc., sub formam talem $A + a(\beta\gamma, 1) + a'(\beta\gamma, g) \ldots + a^{\varsigma}(\beta\gamma, g^{\alpha-1})$ reduci posse, atque numeros A, a etc. omnes fieri integros; aequationem autem, cuius radices sint β periodi γ terminorum in alia periodo $(\beta\gamma, k\lambda)$ contentae, ex illa deriuari, si vbique in coëfficientibus pro qualibet periodo $(\beta\gamma, \mu)$ substituatur $(\beta\gamma, k\mu)$. Si igitur $\alpha = 1$, omnes β periodi γ terminorum determinabuntur per aequationem β^{ti} gradus, cuius singuli coëfficientes sub formam $A + a(\beta\gamma, 1)$ rediguntur, *adeoque sunt quantitates cognitae*, quoniam $(\beta\gamma, 1) = (n - 1, 1) = -1$. Si vero $\alpha > 1$, coëfficientes aequationis, cuius radices sunt omnes periodi γ terminorum in aliqua periodo data $\beta\gamma$ terminorum contentae, quantitates cognitae erunt,

simulac valores numerici omnium α periodorum $\beta\gamma$ terminorum innotuerunt. — Ceterum calculus coëfficientium harum aequationum saepe commodius instituitur, praesertim quando β non est valde paruus, si primo summae potestatum radicum eruuntur, ac dein ex his per theorema Newtonianum coëfficientes deducuntur, simili modo vt supra art. 349.

Ex. I. Quaeritur pro $n = 19$ aequatio cuius radices sint aggregata (6, 1), (6, 2), (6, 4). Designando has radices per p, p', p'' resp., et aequationem quaesitam per $x^3 - Axx + Bx - C = 0$, fit $A = p + p' + p''$, $B = pp' + pp'' + p'p''$, $C = pp'p''$. Hinc $A = (18, 1) = -1$; porro habetur $pp' = p + 2p' + 3p''$, $pp'' = 2p + 3p' + p''$, $p'p'' = 3p + p' + 2p''$, vnde $B = 6(p + p' + p'') = 6(18, 1) = -6$; denique fit $C = (p + 2p' + 3p'')p'' = 3(6, 0) + 11(p + p' + p'') = 18 - 11 = 7$; quare aequatio quaesita $x^3 + xx - 6x - 7 = 0$. — Vtendo methodo altera habemus $p + p' + p'' = -1$; $pp = 6 + 2p + p' + 2p''$, $p'p' = 6 + 2p' + p'' + 2p$, $p''p'' = 6 + 2p'' + p + 2p'$, vnde $pp + p'p' + p''p'' = 18 + 5(p + p' + p'') = 13$; similiterque $p^3 + p'^3 + p''^3 = 36 + 34(p + p' + p'') = 2$; hinc per theorema Newtonianum eadem aequatio deriuatur vt ante.

II. Quaeritur pro $n = 19$ aequatio cuius radices sint aggregata (2, 1), (2, 7), (2, 8). Quibus resp. per q, q', q'' designatis, inuenitur $q + q' + q'' = (6, 1)$, $qq' + qq'' + q'q'' = (6, 1) + (6, 4)$, $qq'q'' = 2 + (6, 2)$, vnde,