mae f in ipsas eruatur, eodem modo vt art. 188: *vel* $+\mathfrak{A}$ fore aequalem termino alicui progressionis... $''\alpha, '\alpha, \alpha, \alpha', \alpha''\ldots$, hocque posito $= \alpha^{m}$, $+\mathfrak{B}$ fore $= \beta^{m}$, $+\mathfrak{C} = \gamma^{m}$, $+\mathfrak{D} = \delta^{m}$; *vel* $-\mathfrak{A}$ fore aequalem termino alicui α^{m}, et $-\mathfrak{B}, -\mathfrak{C}, -\mathfrak{D}$ resp. $= \beta^{m}, \gamma^{m}, \delta^{m}$ (vbi m etiam indicem negatiuum designare potest). In vtroque casu F manifesto identica erit cum f^{m}.

Dem. I. Habentur quatuor aequationes, $a\mathfrak{A}\mathfrak{A} + 2b\mathfrak{A}\mathfrak{C} - a'\mathfrak{C}\mathfrak{C} = A \ldots [1]$, $a\mathfrak{A}\mathfrak{B} + b(\mathfrak{A}\mathfrak{D} + \mathfrak{B}\mathfrak{C}) - a'\mathfrak{C}\mathfrak{D} = B \ldots [2]$, $a\mathfrak{B}\mathfrak{B} + 2b\mathfrak{B}\mathfrak{D} - a'\mathfrak{D}\mathfrak{D} = -A' \ldots [3]$; $\mathfrak{A}\mathfrak{D} - \mathfrak{B}\mathfrak{C} = 1 \ldots [4]$. Consideramus autem *primo* casum, vbi aliquis numerorum $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D} = 0$.

1° Si $\mathfrak{A} = 0$, fit ex [4] $\mathfrak{B}\mathfrak{C} = -1$, adeoque $\mathfrak{B} = \pm 1$, $\mathfrak{C} = \mp 1$. Hinc ex [1], $-a' = A$; ex [2], $-b \pm a'\mathfrak{D} = B$ siue $B \equiv -b$ (mod. a' vel A); vnde sequitur formam $(A, B, -A')$ formae $(a, b, -a')$ ab vltima parte contiguam esse. Quoniam vero illa est reducta, necessario cum f' identica erit. Ergo $B = b'$, adeoque ex [2] $b + b' = -a'\mathfrak{C}\mathfrak{D} = \pm a'\mathfrak{D}$; hinc propter $\frac{b+b'}{-a'} = h'$, fit $\mathfrak{D} = \mp h'$. Vnde colligitur, $\mp\mathfrak{A}, \mp\mathfrak{B}, \mp\mathfrak{C}, \mp\mathfrak{D}$ esse resp. $= 0, -1, +1, h'$ siue $= \alpha', \beta', \gamma', \delta'$.

2° Si $\mathfrak{B} = 0$, fit ex [4] $\mathfrak{A} = \pm 1$, $\mathfrak{D} = \pm 1$; ex [3] $a' = A'$; ex [2] $b \mp a'\mathfrak{C} = B$, siue $b \equiv B$ (mod. a). Quoniam vero tum f tum F sunt formae reductae: tum b tum B iacebunt inter $\sqrt{D}$ et $\sqrt{D} \pm a'$ (prout a' pos. vel neg., art. 185, 5). Quare erit necessario $b = B$, et $\mathfrak{C} = 0$. Hinc formae f, F sunt identicae atque $\pm \mathfrak{A}, \pm \mathfrak{B}, \pm \mathfrak{C}, \pm \mathfrak{D} = 1, 0, 0, 1 = \alpha, \beta, \gamma, \delta$ (resp.).

3° Si $\mathfrak{C} = 0$, fit ex [4] $\mathfrak{A} = \pm 1$, $\mathfrak{D} = \pm 1$; ex [1] $a = A$; ex [2] $\pm a\mathfrak{B} + b = B$ siue $b \equiv B$ (mod. a). Quia vero tum b tum B iacent inter $\sqrt{D}$ et $\sqrt{D \mp a}$: erit necessario $B = b$ et $\mathfrak{B} = 0$. Quare casus hic a praecedente non differt.

4° Si $\mathfrak{D} = 0$, fit ex [4] $\mathfrak{B} = \pm 1$, $\mathfrak{C} = \mp 1$; ex [3] $a = -A'$; ex [2] $\pm a\mathfrak{A} - b = B$ siue $B \equiv -b$ (mod. a). Hinc forma F formae f a parte prima contigua erit, et proin cum forma $'f$ identica. Quare propter $\frac{'b + b}{a} = h$, et $B = 'b$, erit $\pm \mathfrak{A} = h$. Vnde colligitur $\pm \mathfrak{A}, \pm \mathfrak{B}, \pm \mathfrak{C}, \pm \mathfrak{D}$ resp. esse $= h, 1, -1, 0, = '\alpha, '\beta, '\gamma, '\delta$.

Superest itaque casus vbi nullus numerorum $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D} = 0$. Hic per Lemma art. 190 quantitates $\frac{\mathfrak{A}}{\mathfrak{C}}, \frac{\mathfrak{B}}{\mathfrak{D}}, \frac{\mathfrak{C}}{\mathfrak{A}}, \frac{\mathfrak{D}}{\mathfrak{B}}$ idem signum habebunt, oriunturque inde duo casus, quum signum hoc vel cum signo ipsorum a, a' conuenire vel ipsi oppositum esse possit.

II. Si $\frac{\mathfrak{A}}{\mathfrak{C}}, \frac{\mathfrak{B}}{\mathfrak{D}}$ idem signum habent vt a: quantitas $\frac{\sqrt{D} - b}{a}$ (quam designabimus per L)

inter has fractiones sita erit (art. 191). Demonstrabimus iam, $\frac{\mathfrak{A}}{\mathfrak{B}}$ aequalem fore alicui fractionum $\frac{\alpha''}{\gamma''}$, $\frac{\alpha'''}{\gamma'''}$, $\frac{\alpha^{\text{IV}}}{\gamma^{\text{IV}}}$ etc., atque $\frac{\mathfrak{B}}{\mathfrak{D}}$ proxime sequenti, scilicet si $\frac{\mathfrak{A}}{\mathfrak{C}}$ fuerit $= \frac{\alpha^{\text{m}}}{\gamma^{\text{m}}}$, $\frac{\mathfrak{B}}{\mathfrak{D}}$ fore $= \frac{\alpha^{\text{m}+1}}{\gamma^{\text{m}+1}}$. In art. praec. ostendimus, quantitates $\frac{\alpha'}{\gamma'}$, $\frac{\alpha''}{\gamma''}$, $\frac{\alpha'''}{\gamma'''}$ etc., (quas breuitatis gratia per (1), (2), (3) etc. denotabimus) atque L, hunc ordinem (I): obseruare (1), (3), (5)... L... (6), (4), (2); prima harum quantitatum est $= 0$ (propter $\alpha' = 0$), reliquae omnes idem signum habent vt L siue a. Quoniam vero per hyp. $\frac{\mathfrak{A}}{\mathfrak{C}}$, $\frac{\mathfrak{B}}{\mathfrak{D}}$ (pro quibus scribemus $\mathfrak{M}$, $\mathfrak{N}$) idem signum habent: patet has quantitates ipsi (1) a dextra iacere (aut si mauis ab eadem parte a qua L), et quidem, quum L iaceat inter ipsas, alteram ipsi L a dextra, alteram a laeua. Facile vero ostendi potest, $\mathfrak{M}$ ipsi (2) a dextra iacere non posse alioquin enim $\mathfrak{N}$ iaceret inter (1) et L, vnde sequeretur *primo* (2) iacere inter $\mathfrak{M}$ et $\mathfrak{N}$, adeoque denominatorem fractionis (2) maiorem esse denominatore fractionis $\mathfrak{N}$ (art. 190), *secundo* $\mathfrak{M}$ iacere inter (1) et (2), adeoque denom. fractionis $\mathfrak{N}$ esse maiorem quam denom. fractionis (2), *Q. E. A.*

Supponamus $\mathfrak{M}$ nulli fractionum (2), (3), (4) etc. aequalem esse, vt, quid inde sequatur, videamus. Tum manifestum est, si fractio $\mathfrak{M}$ ips