

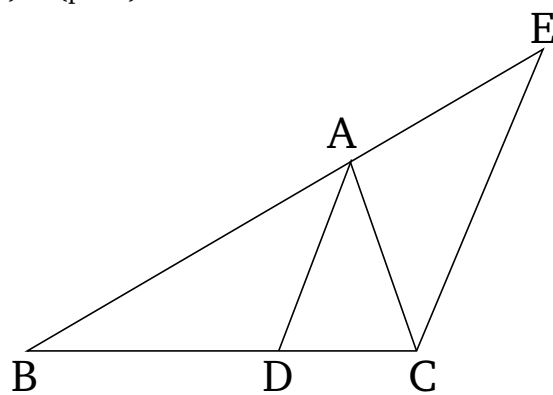
Καὶ ἐπεὶ εἰς παραλλήλους τὰς AD , EG εὐθεῖα ἐνέπεσεν ἡ AG , ἡ ἄρα ὑπὸ AGE γωνία ἴση ἐστὶ τῇ ὑπὸ $ΓAD$. ἀλλ' ἡ ὑπὸ $ΓAD$ τῇ ὑπὸ $BAΔ$ ὑπόκειται ἴση· καὶ ἡ ὑπὸ $BAΔ$ ἄρα τῇ ὑπὸ AGE ἐστὶν ἴση. πάλιν, ἐπεὶ εἰς παραλλήλους τὰς AD , EG εὐθεῖα ἐνέπεσεν ἡ BAE , ἡ ἐκτὸς γωνία ἡ ὑπὸ $BAΔ$ ἴση ἐστὶ τῇ ἐντὸς τῇ ὑπὸ $AEΓ$. ἐδείχθη δὲ καὶ ἡ ὑπὸ AGE τῇ ὑπὸ $BAΔ$ ἴση· καὶ ἡ ὑπὸ AGE ἄρα γωνία τῇ ὑπὸ $AEΓ$ ἐστὶν ἴση· ὥστε καὶ πλευρὰ ἡ AE πλευρᾷ τῇ AG ἐστὶν ἴση. καὶ ἐπεὶ τριγώνου τοῦ BGE παρὰ μίαν τῶν πλευρῶν τὴν EG ἤχται ἡ AD , ἀνάλογον ἄρα ἐστὶν ὡς ἡ BD πρὸς τὴν $ΔΓ$, οὕτως ἡ BA πρὸς τὴν AE . ἴση δὲ ἡ AE τῇ AG · ὡς ἄρα ἡ BD πρὸς τὴν $ΔΓ$, οὕτως ἡ BA πρὸς τὴν AG .

Ἀλλὰ δὴ ἔστω ὡς ἡ BD πρὸς τὴν $ΔΓ$, οὕτως ἡ BA πρὸς τὴν AG , καὶ ἐπεζεύχθω ἡ AD · λέγω, ὅτι δίχα τέμνεται ἡ ὑπὸ $BAΓ$ γωνία ὑπὸ τῆς AD εὐθείας.

Τῶν γὰρ αὐτῶν κατασκευασθέντων, ἐπεὶ ἐστὶν ὡς ἡ BD πρὸς τὴν $ΔΓ$, οὕτως ἡ BA πρὸς τὴν AG , ἀλλὰ καὶ ὡς ἡ BD πρὸς τὴν $ΔΓ$, οὕτως ἐστὶν ἡ BA πρὸς τὴν AE · τριγώνου γὰρ τοῦ BGE παρὰ μίαν τὴν EG ἤχται ἡ AD · καὶ ὡς ἄρα ἡ BA πρὸς τὴν AG , οὕτως ἡ BA πρὸς τὴν AE . ἴση ἄρα ἡ AG τῇ AE · ὥστε καὶ γωνία ἡ ὑπὸ $AEΓ$ τῇ ὑπὸ AGE ἐστὶν ἴση. ἀλλ' ἡ μὲν ὑπὸ $AEΓ$ τῇ ἐκτὸς τῇ ὑπὸ $BAΔ$ [ἐστὶν] ἴση, ἡ δὲ ὑπὸ AGE τῇ ἐναλλάξ τῇ ὑπὸ $ΓAD$ ἐστὶν ἴση· καὶ ἡ ὑπὸ $BAΔ$ ἄρα τῇ ὑπὸ $ΓAD$ ἐστὶν ἴση. ἡ ἄρα ὑπὸ $BAΓ$ γωνία δίχα τέμνεται ὑπὸ τῆς AD εὐθείας.

Ἐὰν ἄρα τριγώνου ἡ γωνία δίχα τμηθῇ, ἡ δὲ τέμνουσα τὴν γωνίαν εὐθεῖα τέμνη καὶ τὴν βάσιν, τὰ τῆς βάσεως τμήματα τὸν αὐτὸν ἔξει λόγον ταῖς λοιπαῖς τοῦ τριγώνου πλευραῖς· καὶ ἐὰν τὰ τῆς βάσεως τμήματα τὸν αὐτὸν ἔχη λόγον ταῖς λοιπαῖς τοῦ τριγώνου πλευραῖς, ἡ ἀπὸ τῆς κορυφῆς ἐπὶ τὴν τομὴν ἐπιζευγνυμένη εὐθεῖα δίχα τέμνει τὴν τοῦ τριγώνου γωνίαν· ὅπερ ἔδει δεῖξαι.

(CE) at (point) E .[†]



And since the straight-line AC falls across the parallel (straight-lines) AD and EC , angle ACE is thus equal to CAD [Prop. 1.29]. But, (angle) CAD is assumed (to be) equal to BAD . Thus, (angle) BAD is also equal to ACE . Again, since the straight-line BAE falls across the parallel (straight-lines) AD and EC , the external angle BAD is equal to the internal (angle) AEC [Prop. 1.29]. And (angle) ACE was also shown (to be) equal to BAD . Thus, angle ACE is also equal to AEC . And, hence, side AE is equal to side AC [Prop. 1.6]. And since AD has been drawn parallel to one of the sides EC of triangle BCE , thus, proportionally, as BD is to DC , so BA (is) to AE [Prop. 6.2]. And AE (is) equal to AC . Thus, as BD (is) to DC , so BA (is) to AC .

And so, let BD be to DC , as BA (is) to AC . And let AD have been joined. I say that angle BAC has been cut in half by the straight-line AD .

For, by the same construction, since as BD is to DC , so BA (is) to AC , then also as BD (is) to DC , so BA is to AE . For AD has been drawn parallel to one (of the sides) EC of triangle BCE [Prop. 6.2]. Thus, also, as BA (is) to AC , so BA (is) to AE [Prop. 5.11]. Thus, AC (is) equal to AE [Prop. 5.9]. And, hence, angle AEC is equal to ACE [Prop. 1.5]. But, AEC [is] equal to the external (angle) BAD , and ACE is equal to the alternate (angle) CAD [Prop. 1.29]. Thus, (angle) BAD is also equal to CAD . Thus, angle BAC has been cut in half by the straight-line AD .

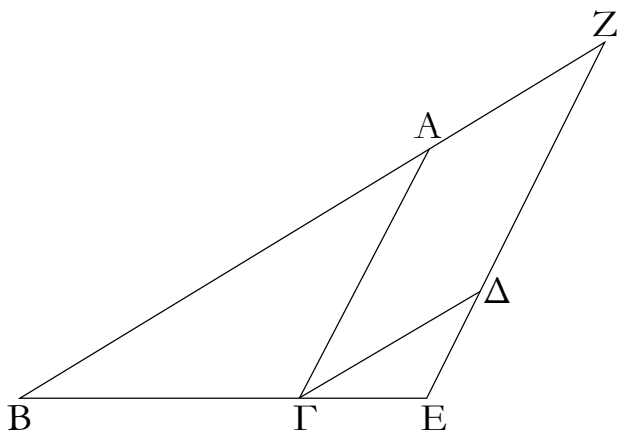
Thus, if an angle of a triangle is cut in half, and the straight-line cutting the angle also cuts the base, then the segments of the base will have the same ratio as the remaining sides of the triangle. And if the segments of the base have the same ratio as the remaining sides of the triangle, then the straight-line joining the vertex to the cutting (point) will cut the angle of the triangle in half. (Which is) the very thing it was required to show.

[†] The fact that the two straight-lines meet follows because the sum of ACE and CAE is less than two right-angles, as can easily be demonstrated.

See Post. 5.

δ'.

Τῶν ἰσογωνίων τριγώνων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας καὶ ὁμόλογοι αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι.



Ἐστω ἰσογώνια τρίγωνα τὰ ABG , DGE ἴσην ἔχοντα τὴν μὲν ὑπὸ ABG γωνίαν τῇ ὑπὸ DGE , τὴν δὲ ὑπὸ BAG τῇ ὑπὸ GDE καὶ ἔτι τὴν ὑπὸ AGB τῇ ὑπὸ GED · λέγω, ὅτι τῶν ABG , DGE τριγώνων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας καὶ ὁμόλογοι αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι.

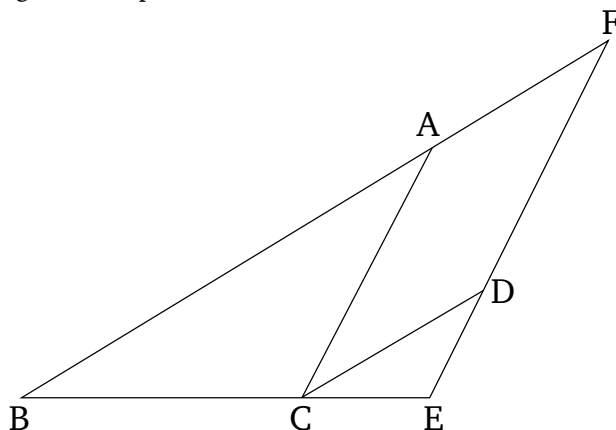
Κεῖσθω γὰρ ἐπ' εὐθείας ἡ BG τῇ GE . καὶ ἐπεὶ αἱ ὑπὸ ABG , AGB γωνίαι δύο ὀρθῶν ἐλάττονές εἰσιν, ἴση δὲ ἡ ὑπὸ AGB τῇ ὑπὸ DEG , αἱ ἄρα ὑπὸ ABG , DEG δύο ὀρθῶν ἐλάττονές εἰσιν· αἱ BA , ED ἄρα ἐκβαλλόμεναι συμπεσοῦνται. ἐκβεβλήσθωσαν καὶ συμπιπτέτωσαν κατὰ τὸ Z .

Καὶ ἐπεὶ ἴση ἐστὶν ἡ ὑπὸ DGE γωνία τῇ ὑπὸ ABG , παράλληλός ἐστιν ἡ BZ τῇ GD . πάλιν, ἐπεὶ ἴση ἐστὶν ἡ ὑπὸ AGB τῇ ὑπὸ DEG , παράλληλός ἐστιν ἡ AG τῇ ZE . παραλληλόγραμμον ἄρα ἐστὶ τὸ $ZAGD$ · ἴση ἄρα ἡ μὲν ZA τῇ GD , ἡ δὲ AG τῇ ZD . καὶ ἐπεὶ τριγώνου τοῦ ZBE παρὰ μίαν τὴν ZE ἤκται ἡ AG , ἐστὶν ἄρα ὡς ἡ BA πρὸς τὴν AZ , οὕτως ἡ BG πρὸς τὴν GE . ἴση δὲ ἡ AZ τῇ GD · ὡς ἄρα ἡ BA πρὸς τὴν GD , οὕτως ἡ BG πρὸς τὴν GE , καὶ ἐναλλάξ ὡς ἡ AB πρὸς τὴν BG , οὕτως ἡ GD πρὸς τὴν GE . πάλιν, ἐπεὶ παράλληλός ἐστιν ἡ GD τῇ BZ , ἐστὶν ἄρα ὡς ἡ BG πρὸς τὴν GE , οὕτως ἡ ZD πρὸς τὴν DE . ἴση δὲ ἡ ZD τῇ AG · ὡς ἄρα ἡ BG πρὸς τὴν GE , οὕτως ἡ AG πρὸς τὴν DE , καὶ ἐναλλάξ ὡς ἡ BG πρὸς τὴν GA , οὕτως ἡ GE πρὸς τὴν ED . ἐπεὶ οὖν ἐδείχθη ὡς μὲν ἡ AB πρὸς τὴν BG , οὕτως ἡ GD πρὸς τὴν GE , ὡς δὲ ἡ BG πρὸς τὴν GA , οὕτως ἡ GE πρὸς τὴν ED , δι' ἴσου ἄρα ὡς ἡ BA πρὸς τὴν AG , οὕτως ἡ GD πρὸς τὴν DE .

Τῶν ἄρα ἰσογωνίων τριγώνων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας καὶ ὁμόλογοι αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι· ὅπερ ἔδει δεῖξαι.

Proposition 4

In equiangular triangles the sides about the equal angles are proportional, and those (sides) subtending equal angles correspond.



Let ABC and DCE be equiangular triangles, having angle ABC equal to DCE , and (angle) BAC to CDE , and, further, (angle) ACB to CED . I say that in triangles ABC and DCE the sides about the equal angles are proportional, and those (sides) subtending equal angles correspond.

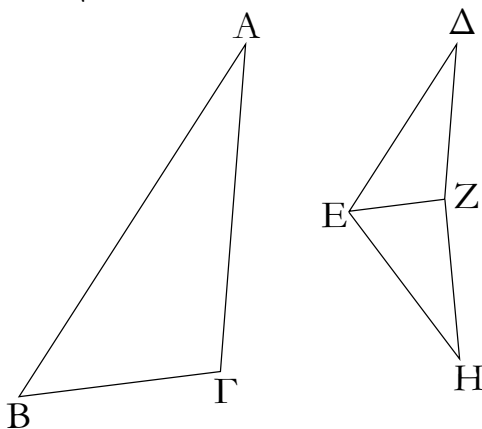
Let BC be placed straight-on to CE . And since angles ABC and ACB are less than two right-angles [Prop 1.17], and ACB (is) equal to DEC , thus ABC and DEC are less than two right-angles. Thus, BA and ED , being produced, will meet [C.N. 5]. Let them have been produced, and let them meet at (point) F .

And since angle DCE is equal to ABC , BF is parallel to CD [Prop. 1.28]. Again, since (angle) ACB is equal to DEC , AC is parallel to FE [Prop. 1.28]. Thus, $FACD$ is a parallelogram. Thus, FA is equal to DC , and AC to FD [Prop. 1.34]. And since AC has been drawn parallel to one (of the sides) FE of triangle FBE , thus as BA is to AF , so BC (is) to CE [Prop. 6.2]. And AF (is) equal to CD . Thus, as BA (is) to CD , so BC (is) to CE , and, alternately, as AB (is) to BC , so DC (is) to CE [Prop. 5.16]. Again, since CD is parallel to BF , thus as BC (is) to CE , so FD (is) to DE [Prop. 6.2]. And FD (is) equal to AC . Thus, as BC is to CE , so AC (is) to DE , and, alternately, as BC (is) to CA , so CE (is) to ED [Prop. 6.2]. Therefore, since it was shown that as AB (is) to BC , so DC (is) to CE , and as BC (is) to CA , so CE (is) to ED , thus, via equality, as BA (is) to AC , so CD (is) to DE [Prop. 5.22].

Thus, in equiangular triangles the sides about the equal angles are proportional, and those (sides) subtend-

ε'.

Ἐάν δύο τρίγωνα τὰς πλευρὰς ἀνάλογον ἔχῃ, ἰσογώνια ἔσται τὰ τρίγωνα καὶ ἴσας ἔξει τὰς γωνίας, ὅφ' ἃς αἱ ὁμόλογοι πλευραὶ ὑποτείνουσιν.



Ἐστω δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ τὰς πλευρὰς ἀνάλογον ἔχοντα, ὡς μὲν τὴν AB πρὸς τὴν $B\Gamma$, οὕτως τὴν ΔE πρὸς τὴν EZ , ὡς δὲ τὴν $B\Gamma$ πρὸς τὴν ΓA , οὕτως τὴν EZ πρὸς τὴν $Z\Delta$, καὶ ἔτι ὡς τὴν BA πρὸς τὴν ΓA , οὕτως τὴν ED πρὸς τὴν ΔZ . λέγω, ὅτι ἰσογώνιον ἔστι τὸ $AB\Gamma$ τρίγωνον τῷ ΔEZ τριγώνῳ καὶ ἴσας ἔξουσιν τὰς γωνίας, ὅφ' ἃς αἱ ὁμόλογοι πλευραὶ ὑποτείνουσιν, τὴν μὲν ὑπὸ $AB\Gamma$ τῇ ὑπὸ ΔEZ , τὴν δὲ ὑπὸ $B\Gamma A$ τῇ ὑπὸ $EZ\Delta$ καὶ ἔτι τὴν ὑπὸ $B\Gamma A$ τῇ ὑπὸ $E\Delta Z$.

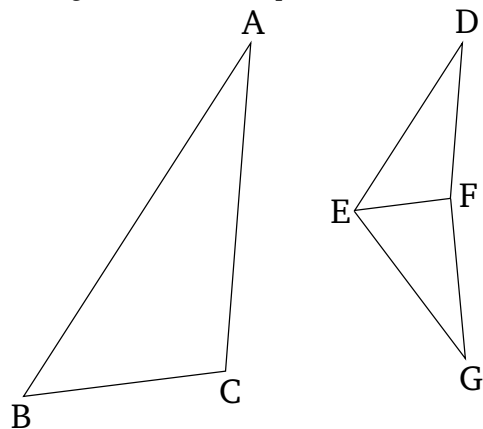
Συνεστάτω γὰρ πρὸς τῇ EZ εὐθείᾳ καὶ τοῖς πρὸς αὐτῇ σημείοις τοῖς E , Z τῇ μὲν ὑπὸ $AB\Gamma$ γωνίᾳ ἴση ἡ ὑπὸ ZEH , τῇ δὲ ὑπὸ $\Gamma B A$ ἴση ἡ ὑπὸ EZH . λοιπὴ ἄρα ἡ πρὸς τῷ A λοιπῇ τῇ πρὸς τῷ H ἔστιν ἴση.

Ἰσογώνιον ἄρα ἔστι τὸ $AB\Gamma$ τρίγωνον τῷ EZH [τριγώνῳ]. τῶν ἄρα $AB\Gamma$, EZH τριγώνων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας καὶ ὁμόλογοι αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσιν· ἔστιν ἄρα ὡς ἡ AB πρὸς τὴν $B\Gamma$, [οὕτως] ἡ HE πρὸς τὴν EZ . ἀλλ' ὡς ἡ AB πρὸς τὴν $B\Gamma$, οὕτως ὑπόκειται ἡ ΔE πρὸς τὴν EZ · ὡς ἄρα ἡ ΔE πρὸς τὴν EZ , οὕτως ἡ HE πρὸς τὴν EZ . ἑκατέρα ἄρα τῶν ΔE , HE πρὸς τὴν EZ τὸν αὐτὸν ἔχει λόγον· ἴση ἄρα ἔστιν ἡ ΔE τῇ HE . διὰ τὰ αὐτὰ δὲ καὶ ἡ ΔZ τῇ HZ ἔστιν ἴση. ἐπεὶ οὖν ἴση ἔστιν ἡ ΔE τῇ HE , κοινὴ δὲ ἡ EZ , δύο δὲ αἱ ΔE , EZ δυοὶ ταῖς HE , EZ ἴσαι εἰσὶν· καὶ βάσεις ἡ ΔZ βάσει τῇ HZ [ἔστιν] ἴση· γωνία ἄρα ἡ ὑπὸ ΔEZ γωνία τῇ ὑπὸ HEZ ἔστιν ἴση, καὶ τὸ ΔEZ τρίγωνον τῷ HEZ τριγώνῳ ἴσον, καὶ αἱ λοιπαὶ γωνίαι ταῖς λοιπαῖς γωνίαις ἴσαι, ὅφ' ἃς αἱ ἴσαι πλευραὶ ὑποτείνουσιν. ἴση ἄρα ἔστι καὶ ἡ μὲν ὑπὸ ΔZE γωνία τῇ ὑπὸ HZE , ἡ δὲ ὑπὸ $E\Delta Z$ τῇ ὑπὸ EHZ . καὶ ἐπεὶ ἡ μὲν ὑπὸ $Z\Delta E$ τῇ ὑπὸ HEZ ἔστιν ἴση, ἀλλ' ἡ ὑπὸ HEZ τῇ ὑπὸ $AB\Gamma$, καὶ ἡ ὑπὸ

ing equal angles correspond. (Which is) the very thing it was required to show.

Proposition 5

If two triangles have proportional sides then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal.



Let ABC and DEF be two triangles having proportional sides, (so that) as AB (is) to BC , so DE (is) to EF , and as BC (is) to CA , so EF (is) to FD , and, further, as BA (is) to AC , so ED (is) to DF . I say that triangle ABC is equiangular to triangle DEF , and (that the triangles) will have the angles which corresponding sides subtend equal. (That is), (angle) ABC (equal) to DEF , BCA to EFD , and, further, BAC to EDF .

For let (angle) FEG , equal to angle ABC , and (angle) EFG , equal to ACB , have been constructed on the straight-line EF at the points E and F on it (respectively) [Prop. 1.23]. Thus, the remaining (angle) at A is equal to the remaining (angle) at G [Prop. 1.32].

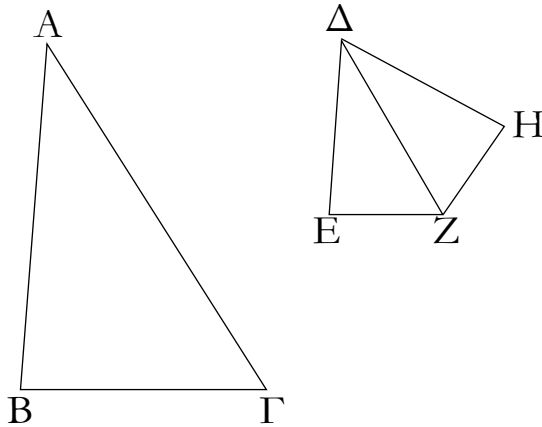
Thus, triangle ABC is equiangular to [triangle] EGF . Thus, for triangles ABC and EGF , the sides about the equal angles are proportional, and (those) sides subtending equal angles correspond [Prop. 6.4]. Thus, as AB is to BC , [so] GE (is) to EF . But, as AB (is) to BC , so, it was assumed, (is) DE to EF . Thus, as DE (is) to EF , so GE (is) to EF [Prop. 5.11]. Thus, DE and GE each have the same ratio to EF . Thus, DE is equal to GE [Prop. 5.9]. So, for the same (reasons), DF is also equal to GF . Therefore, since DE is equal to EG , and EF (is) common, the two (sides) DE , EF are equal to the two (sides) GE , EF (respectively). And base DF [is] equal to base FG . Thus, angle DEF is equal to angle GEF [Prop. 1.8], and triangle DEF (is) equal to triangle GEF , and the remaining angles (are) equal to the remaining angles which the equal sides subtend [Prop. 1.4]. Thus, angle DFE is also equal to GFE , and

ΑΒΓ ἄρα γωνία τῇ ὑπὸ ΔΕΖ ἐστὶν ἴση. διὰ τὰ αὐτὰ δὴ καὶ ἡ ὑπὸ ΑΓΒ τῇ ὑπὸ ΔΖΕ ἐστὶν ἴση, καὶ ἔτι ἡ πρὸς τῷ Α τῇ πρὸς τῷ Δ· ἰσογώνιον ἄρα ἐστὶ τὸ ΑΒΓ τρίγωνον τῷ ΔΕΖ τριγώνῳ.

Ἐάν ἄρα δύο τρίγωνα τὰς πλευρὰς ἀνάλογον ἔχῃ, ἰσογώνια ἔσται τὰ τρίγωνα καὶ ἴσας ἔξει τὰς γωνίας, ὅφ' ἃς αἱ ὁμόλογοι πλευραὶ ὑποτείνουσιν· ὅπερ ἔδει δεῖξαι.

٧'.

Ἐάν δύο τρίγωνα μίαν γωνίαν μιᾶ γωνίᾳ ἴσην ἔχῃ, περὶ δὲ τὰς ἴσας γωνίας τὰς πλευρὰς ἀνάλογον, ἰσογώνια ἔσται τὰ τρίγωνα καὶ ἴσας ἔξει τὰς γωνίας, ὅφ' ἃς αἱ ὁμόλογοι πλευραὶ ὑποτείνουσιν.



Ἐστω δύο τρίγωνα τὰ ΑΒΓ, ΔΕΖ μίαν γωνίαν τὴν ὑπὸ ΒΑΓ μιᾶ γωνίᾳ τῇ ὑπὸ ΕΔΖ ἴσην ἔχοντα, περὶ δὲ τὰς ἴσας γωνίας τὰς πλευρὰς ἀνάλογον, ὥς τὴν ΒΑ πρὸς τὴν ΑΓ, οὕτως τὴν ΕΔ πρὸς τὴν ΔΖ· λέγω, ὅτι ἰσογώνιον ἐστὶ τὸ ΑΒΓ τρίγωνον τῷ ΔΕΖ τριγώνῳ καὶ ἴσην ἔξει τὴν ὑπὸ ΑΒΓ γωνίαν τῇ ὑπὸ ΔΕΖ, τὴν δὲ ὑπὸ ΑΓΒ τῇ ὑπὸ ΔΖΕ.

Συνεστάτω γὰρ πρὸς τῇ ΔΖ εὐθείᾳ καὶ τοῖς πρὸς αὐτῇ σημείοις τοῖς Δ, Ζ ὁποτέρῳ μὲν τῶν ὑπὸ ΒΑΓ, ΕΔΖ ἴση ἡ ὑπὸ ΖΔΗ, τῇ δὲ ὑπὸ ΑΓΒ ἴση ἡ ὑπὸ ΔΖΗ· λοιπὴ ἄρα ἡ πρὸς τῷ Β γωνία λοιπὴ τῇ πρὸς τῷ Η ἴση ἐστίν.

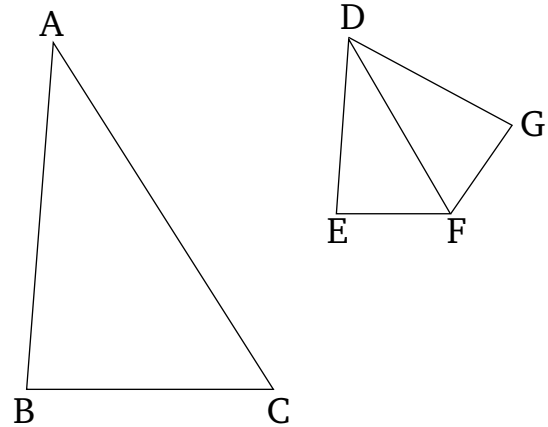
Ἰσογώνιον ἄρα ἐστὶ τὸ ΑΒΓ τρίγωνον τῷ ΔΗΖ τριγώνῳ. ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΒΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΗΔ πρὸς τὴν ΔΖ. ὑπόκειται δὲ καὶ ὡς ἡ ΒΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΖ· καὶ ὡς ἄρα ἡ ΕΔ πρὸς τὴν ΔΖ, οὕτως ἡ ΗΔ πρὸς τὴν ΔΖ. ἴση ἄρα ἡ ΕΔ τῇ ΗΔ· καὶ κοινὴ ἡ ΔΖ· δύο δὴ αἱ ΕΔ, ΔΖ δυσὶ ταῖς ΗΔ, ΔΖ ἴσας εἰσίν· καὶ γωνία ἡ ὑπὸ ΕΔΖ γωνία τῇ ὑπὸ ΗΔΖ [ἐστὶν] ἴση· βάσις ἄρα ἡ ΕΖ βάσει τῇ ΗΖ ἐστὶν ἴση, καὶ τὸ ΔΕΖ τρίγωνον τῷ ΗΔΖ τριγώνῳ ἴσον ἐστίν, καὶ αἱ λοιπαὶ γωνίαι ταῖς λοιπαῖς γωνίαις ἴσας ἔσονται, ὅφ' ἃς ἴσας πλευραὶ ὑποτείνουσιν. ἴση ἄρα ἐστὶν ἡ μὲν ὑπὸ ΔΖΗ τῇ ὑπὸ ΔΖΕ, ἡ δὲ ὑπὸ ΔΗΖ

(angle) EDF to EGF . And since (angle) FED is equal to GEF , and (angle) GEF to ABC , angle ABC is thus also equal to DEF . So, for the same (reasons), (angle) ACB is also equal to DFE , and, further, the (angle) at A to the (angle) at D . Thus, triangle ABC is equiangular to triangle DEF .

Thus, if two triangles have proportional sides then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal. (Which is) the very thing it was required to show.

Proposition 6

If two triangles have one angle equal to one angle, and the sides about the equal angles proportional, then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal.



Let ABC and DEF be two triangles having one angle, BAC , equal to one angle, EDF (respectively), and the sides about the equal angles proportional, (so that) as BA (is) to AC , so ED (is) to DF . I say that triangle ABC is equiangular to triangle DEF , and will have angle ABC equal to DEF , and (angle) ACB to DFE .

For let (angle) FDG , equal to each of BAC and EDF , and (angle) DFG , equal to ACB , have been constructed on the straight-line AF at the points D and F on it (respectively) [Prop. 1.23]. Thus, the remaining angle at B is equal to the remaining angle at G [Prop. 1.32].

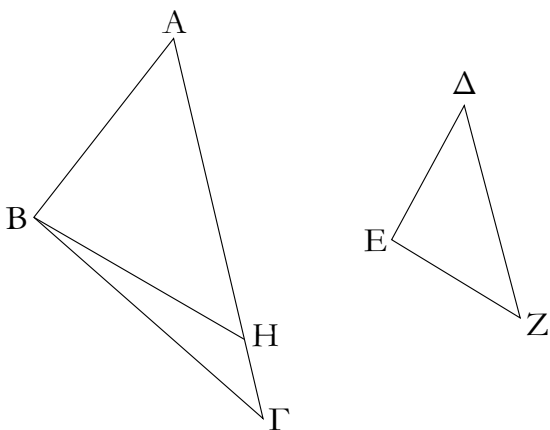
Thus, triangle ABC is equiangular to triangle DGF . Thus, proportionally, as BA (is) to AC , so GD (is) to DF [Prop. 6.4]. And it was also assumed that as BA (is) to AC , so ED (is) to DF . And, thus, as ED (is) to DF , so GD (is) to DF [Prop. 5.11]. Thus, ED (is) equal to DG [Prop. 5.9]. And DF (is) common. So, the two (sides) ED , DF are equal to the two (sides) GD , DF (respectively). And angle EDF [is] equal to angle GDF . Thus, base EF is equal to base GF , and triangle DEF is equal to triangle GDF , and the remaining angles

τῇ ὑπὸ ΔΕΖ. ἀλλ' ἡ ὑπὸ ΔΖΗ τῇ ὑπὸ ΑΓΒ ἐστὶν ἴση· καὶ ἡ ὑπὸ ΑΓΒ ἄρα τῇ ὑπὸ ΔΖΕ ἐστὶν ἴση. ὑπόκειται δὲ καὶ ἡ ὑπὸ ΒΑΓ τῇ ὑπὸ ΕΔΖ ἴση· καὶ λοιπὴ ἄρα ἡ πρὸς τῷ Β λοιπῇ τῇ πρὸς τῷ Ε ἴση ἐστίν· ἰσογώνιον ἄρα ἐστὶ τὸ ΑΒΓ τρίγωνον τῷ ΔΕΖ τριγώνῳ.

Ἐὰν ἄρα δύο τρίγωνα μίαν γωνίαν μιᾶ γωνία ἴσην ἔχῃ, περὶ δὲ τὰς ἴσας γωνίας τὰς πλευρὰς ἀνάλογον, ἰσογώνια ἔσται τὰ τρίγωνα καὶ ἴσας ἔξει τὰς γωνίας, ὅφ' ἂς αἱ ὁμόλογοι πλευραὶ ὑποτείνουσιν· ὅπερ ἔδει δεῖξαι.

ζ'.

Ἐὰν δύο τρίγωνα μίαν γωνίαν μιᾶ γωνία ἴσην ἔχῃ, περὶ δὲ ἄλλας γωνίας τὰς πλευρὰς ἀνάλογον, τῶν δὲ λοιπῶν ἑκατέραν ἅμα ἤτοι ἐλάσσονα ἢ μὴ ἐλάσσονα ὀρθῆς, ἰσογώνια ἔσται τὰ τρίγωνα καὶ ἴσας ἔξει τὰς γωνίας, περὶ ἂς ἀνάλογόν εἰσιν αἱ πλευραί.



Ἐστω δύο τρίγωνα τὰ ΑΒΓ, ΔΕΖ μίαν γωνίαν μιᾶ γωνία ἴσην ἔχοντα τὴν ὑπὸ ΒΑΓ τῇ ὑπὸ ΕΔΖ, περὶ δὲ ἄλλας γωνίας τὰς ὑπὸ ΑΒΓ, ΔΕΖ τὰς πλευρὰς ἀνάλογον, ὥς τὴν ΑΒ πρὸς τὴν ΒΓ, οὕτως τὴν ΔΕ πρὸς τὴν ΕΖ, τῶν δὲ λοιπῶν τῶν πρὸς τοῖς Γ, Ζ πρότερον ἑκατέραν ἅμα ἐλάσσονα ὀρθῆς· λέγω, ὅτι ἰσογώνιον ἐστὶ τὸ ΑΒΓ τρίγωνον τῷ ΔΕΖ τριγώνῳ, καὶ ἴση ἔσται ἡ ὑπὸ ΑΒΓ γωνία τῇ ὑπὸ ΔΕΖ, καὶ λοιπὴ δηλονότι ἡ πρὸς τῷ Γ λοιπῇ τῇ πρὸς τῷ Ζ ἴση.

Εἰ γὰρ ἄνισός ἐστὶν ἡ ὑπὸ ΑΒΓ γωνία τῇ ὑπὸ ΔΕΖ, μία αὐτῶν μείζων ἐστίν. ἔστω μείζων ἡ ὑπὸ ΑΒΓ. καὶ συνεστήτω πρὸς τῇ ΑΒ εὐθείᾳ καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ Β τῇ ὑπὸ ΔΕΖ γωνία ἴση ἡ ὑπὸ ΑΒΗ.

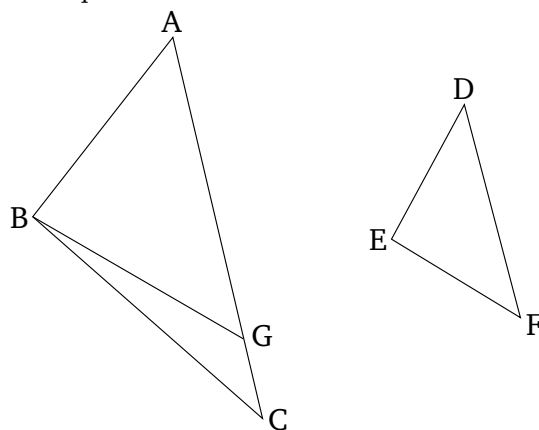
Καὶ ἐπεὶ ἴση ἐστὶν ἡ μὲν Α γωνία τῇ Δ, ἡ δὲ ὑπὸ ΑΒΗ τῇ ὑπὸ ΔΕΖ, λοιπὴ ἄρα ἡ ὑπὸ ΑΗΒ λοιπῇ τῇ ὑπὸ ΔΖΕ ἐστὶν ἴση. ἰσογώνιον ἄρα ἐστὶ τὸ ΑΒΗ τρίγωνον τῷ ΔΕΖ

will be equal to the remaining angles which the equal sides subtend [Prop. 1.4]. Thus, (angle) DFG is equal to DFE , and (angle) DGF to DEF . But, (angle) DFG is equal to ACB . Thus, (angle) ACB is also equal to DFE . And (angle) BAC was also assumed (to be) equal to EDF . Thus, the remaining (angle) at B is equal to the remaining (angle) at E [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle DEF .

Thus, if two triangles have one angle equal to one angle, and the sides about the equal angles proportional, then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal. (Which is) the very thing it was required to show.

Proposition 7

If two triangles have one angle equal to one angle, and the sides about other angles proportional, and the remaining angles either both less than, or both not less than, right-angles, then the triangles will be equiangular, and will have the angles about which the sides are proportional equal.



Let ABC and DEF be two triangles having one angle, BAC , equal to one angle, EDF (respectively), and the sides about (some) other angles, ABC and DEF (respectively), proportional, (so that) as AB (is) to BC , so DE (is) to EF , and the remaining (angles) at C and F , first of all, both less than right-angles. I say that triangle ABC is equiangular to triangle DEF , and (that) angle ABC will be equal to DEF , and (that) the remaining (angle) at C (will be) manifestly equal to the remaining (angle) at F .

For if angle ABC is not equal to (angle) DEF then one of them is greater. Let ABC be greater. And let (angle) ABG , equal to (angle) DEF , have been constructed on the straight-line AB at the point B on it [Prop. 1.23].

And since angle A is equal to (angle) D , and (angle) ABG to DEF , the remaining (angle) AGB is thus equal

τριγώνω. ἔστιν ἄρα ὡς ἡ AB πρὸς τὴν BH , οὕτως ἡ ΔE πρὸς τὴν EZ . ὡς δὲ ἡ ΔE πρὸς τὴν EZ , [οὕτως] ὑπόκειται ἡ AB πρὸς τὴν $B\Gamma$. ἡ AB ἄρα πρὸς ἑκατέραν τῶν $B\Gamma$, BH τὸν αὐτὸν ἔχει λόγον· ἴση ἄρα ἡ $B\Gamma$ τῇ BH . ὥστε καὶ γωνία ἡ πρὸς τῷ Γ γωνία τῇ ὑπὸ $BH\Gamma$ ἐστὶν ἴση. ἐλάττων δὲ ὀρθῆς ὑπόκειται ἡ πρὸς τῷ Γ . ἐλάττων ἄρα ἐστὶν ὀρθῆς καὶ ὑπὸ $BH\Gamma$. ὥστε ἡ ἐφεξῆς αὐτῇ γωνία ἡ ὑπὸ AHB μείζων ἐστὶν ὀρθῆς. καὶ ἐδείχθη ἴση οὕσα τῇ πρὸς τῷ Z . καὶ ἡ πρὸς τῷ Z ἄρα μείζων ἐστὶν ὀρθῆς. ὑπόκειται δὲ ἐλάσσων ὀρθῆς· ὅπερ ἐστὶν ἀτοπον. οὐκ ἄρα ἄνισός ἐστιν ἡ ὑπὸ $AB\Gamma$ γωνία τῇ ὑπὸ ΔEZ . ἴση ἄρα. ἔστι δὲ καὶ ἡ πρὸς τῷ A ἴση τῇ πρὸς τῷ Δ . καὶ λοιπὴ ἄρα ἡ πρὸς τῷ Γ λοιπῇ τῇ πρὸς τῷ Z ἴση ἐστίν. ἰσογώνιον ἄρα ἐστὶ τὸ $AB\Gamma$ τρίγωνον τῷ ΔEZ τριγώνω.

Ἀλλὰ δὴ πάλιν ὑποκείσθω ἑκατέρα τῶν πρὸς τοῖς Γ , Z μὴ ἐλάσσων ὀρθῆς· λέγω πάλιν, ὅτι καὶ οὕτως ἐστὶν ἰσογώνιον τὸ $AB\Gamma$ τρίγωνον τῷ ΔEZ τριγώνω.

Τῶν γὰρ αὐτῶν κατασκευασθέντων ὁμοίως δεῖξομεν, ὅτι ἴση ἐστὶν ἡ $B\Gamma$ τῇ BH . ὥστε καὶ γωνία ἡ πρὸς τῷ Γ τῇ ὑπὸ $BH\Gamma$ ἴση ἐστίν. οὐκ ἐλάττων δὲ ὀρθῆς ἡ πρὸς τῷ Γ . οὐκ ἐλάττων ἄρα ὀρθῆς οὐδὲ ἡ ὑπὸ $BH\Gamma$. τριγώνου δὲ τοῦ $BH\Gamma$ αἱ δύο γωνίαι δύο ὀρθῶν οὐκ εἰσιν ἐλάττονες· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα πάλιν ἄνισός ἐστιν ἡ ὑπὸ $AB\Gamma$ γωνία τῇ ὑπὸ ΔEZ . ἴση ἄρα. ἔστι δὲ καὶ ἡ πρὸς τῷ A τῇ πρὸς τῷ Δ ἴση· λοιπὴ ἄρα ἡ πρὸς τῷ Γ λοιπῇ τῇ πρὸς τῷ Z ἴση ἐστίν. ἰσογώνιον ἄρα ἐστὶ τὸ $AB\Gamma$ τρίγωνον τῷ ΔEZ τριγώνω.

Ἐὰν ἄρα δύο τρίγωνα μίαν γωνίαν μιᾶ γωνία ἴσην ἔχῃ, περὶ δὲ ἄλλας γωνίας τὰς πλευρὰς ἀνάλογον, τῶν δὲ λοιπῶν ἑκατέραν ἅμα ἐλάττονα ἢ μὴ ἐλάττονα ὀρθῆς, ἰσογώνια ἔσται τὰ τρίγωνα καὶ ἴσας ἔξει τὰς γωνίας, περὶ αἷς ἀνάλογόν εἰσιν αἱ πλευραὶ· ὅπερ ἔδει δεῖξαι.

η'.

Ἐὰν ἐν ὀρθογώνιῳ τριγώνῳ ἀπὸ τῆς ὀρθῆς γωνίας ἐπὶ τὴν βάσιν κάθετος ἀχθῇ, τὰ πρὸς τῇ καθέτῳ τρίγωνα ὁμοία ἐστὶ τῷ τε ὅλῳ καὶ ἀλλήλοις.

Ἐστω τρίγωνον ὀρθογώνιον τὸ $AB\Gamma$ ὀρθὴν ἔχον τὴν ὑπὸ $BA\Gamma$ γωνίαν, καὶ ἦχθω ἀπὸ τοῦ A ἐπὶ τὴν $B\Gamma$ κάθετος ἡ AD . λέγω, ὅτι ὁμοῖον ἐστὶν ἑκάτερον τῶν $AB\Delta$, $AD\Gamma$

to the remaining (angle) DFE [Prop. 1.32]. Thus, triangle ABG is equiangular to triangle DEF . Thus, as AB is to BG , so DE (is) to EF [Prop. 6.4]. And as DE (is) to EF , [so] it was assumed (is) AB to BC . Thus, AB has the same ratio to each of BC and BG [Prop. 5.11]. Thus, BC (is) equal to BG [Prop. 5.9]. And, hence, the angle at C is equal to angle BGC [Prop. 1.5]. And the angle at C was assumed (to be) less than a right-angle. Thus, (angle) BGC is also less than a right-angle. Hence, the adjacent angle to it, AGB , is greater than a right-angle [Prop. 1.13]. And (AGB) was shown to be equal to the (angle) at F . Thus, the (angle) at F is also greater than a right-angle. But it was assumed (to be) less than a right-angle. The very thing is absurd. Thus, angle ABC is not unequal to (angle) DEF . Thus, (it is) equal. And the (angle) at A is also equal to the (angle) at D . And thus the remaining (angle) at C is equal to the remaining (angle) at F [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle DEF .

But, again, let each of the (angles) at C and F be assumed (to be) not less than a right-angle. I say, again, that triangle ABC is equiangular to triangle DEF in this case also.

For, with the same construction, we can similarly show that BC is equal to BG . Hence, also, the angle at C is equal to (angle) BGC . And the (angle) at C (is) not less than a right-angle. Thus, BGC (is) not less than a right-angle either. So, in triangle BGC the (sum of) two angles is not less than two right-angles. The very thing is impossible [Prop. 1.17]. Thus, again, angle ABC is not unequal to DEF . Thus, (it is) equal. And the (angle) at A is also equal to the (angle) at D . Thus, the remaining (angle) at C is equal to the remaining (angle) at F [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle DEF .

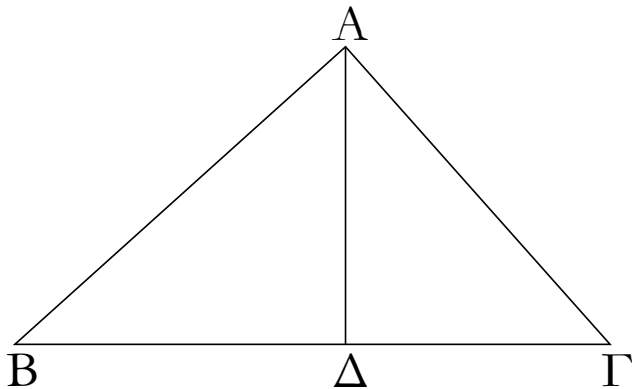
Thus, if two triangles have one angle equal to one angle, and the sides about other angles proportional, and the remaining angles both less than, or both not less than, right-angles, then the triangles will be equiangular, and will have the angles about which the sides (are) proportional equal. (Which is) the very thing it was required to show.

Proposition 8

If, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the triangles around the perpendicular are similar to the whole (triangle), and to one another.

Let ABC be a right-angled triangle having the angle BAC a right-angle, and let AD have been drawn from

τριγώνων ὅλῳ τῷ $AB\Gamma$ καὶ ἔτι ἀλλήλοις.



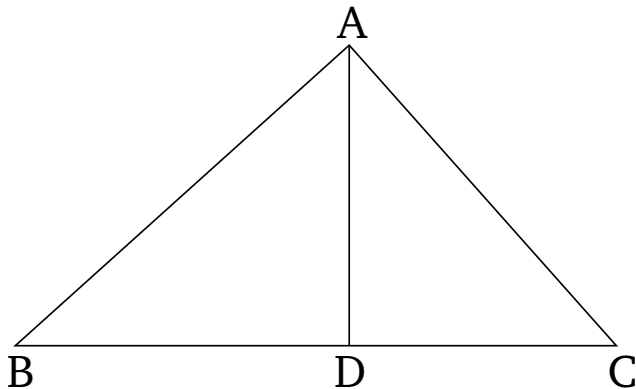
Ἐπεὶ γὰρ ἴση ἐστὶν ἡ ὑπὸ $BA\Gamma$ τῇ ὑπὸ $A\Delta B$ · ὀρθὴ γὰρ ἑκατέρα· καὶ κοινὴ τῶν δύο τριγώνων τοῦ τε $AB\Gamma$ καὶ τοῦ $AB\Delta$ ἡ πρὸς τῷ B , λοιπὴ ἄρα ἡ ὑπὸ AGB λοιπῇ τῇ ὑπὸ $BA\Delta$ ἐστὶν ἴση· ἰσογώνιον ἄρα ἐστὶ τὸ $AB\Gamma$ τρίγωνον τῷ $AB\Delta$ τριγώνῳ. ἔστιν ἄρα ὡς ἡ $B\Gamma$ ὑποτείνουσα τὴν ὀρθὴν τοῦ $AB\Gamma$ τριγώνου πρὸς τὴν BA ὑποτείνουσαν τὴν ὀρθὴν τοῦ $AB\Delta$ τριγώνου, οὕτως αὐτὴ ἡ AB ὑποτείνουσα τὴν πρὸς τῷ Γ γωνίαν τοῦ $AB\Gamma$ τριγώνου πρὸς τὴν $B\Delta$ ὑποτείνουσαν τὴν ἴσην τὴν ὑπὸ $BA\Delta$ τοῦ $AB\Delta$ τριγώνου, καὶ ἔτι ἡ AG πρὸς τὴν $A\Delta$ ὑποτείνουσαν τὴν πρὸς τῷ B γωνίαν κοινὴν τῶν δύο τριγώνων. τὸ $AB\Gamma$ ἄρα τρίγωνον τῷ $AB\Delta$ τριγώνῳ ἰσογώνιον τέ ἐστι καὶ τὰς περὶ τὰς ἴσας γωνίας πλευρὰς ἀνάλογον ἔχει. ὅμοιον ἄρα [ἐστὶ] τὸ $AB\Gamma$ τρίγωνον τῷ $AB\Delta$ τριγώνῳ. ὁμοίως δὲ δείξομεν, ὅτι καὶ τῷ $A\Delta\Gamma$ τριγώνῳ ὅμοιον ἐστὶ τὸ $AB\Gamma$ τρίγωνον· ἑκάτερον ἄρα τῶν $AB\Delta$, $A\Delta\Gamma$ [τριγώνων] ὅμοιον ἐστὶν ὅλῳ τῷ $AB\Gamma$.

Λέγω δὴ, ὅτι καὶ ἀλλήλοις ἐστὶν ὅμοια τὰ $AB\Delta$, $A\Delta\Gamma$ τρίγωνα.

Ἐπεὶ γὰρ ὀρθὴ ἡ ὑπὸ $B\Delta A$ ὀρθὴ τῇ ὑπὸ $A\Delta\Gamma$ ἐστὶν ἴση, ἀλλὰ μὴν καὶ ἡ ὑπὸ $BA\Delta$ τῇ πρὸς τῷ Γ ἐδείχθη ἴση, καὶ λοιπὴ ἄρα ἡ πρὸς τῷ B λοιπῇ τῇ ὑπὸ $\Delta A\Gamma$ ἐστὶν ἴση· ἰσογώνιον ἄρα ἐστὶ τὸ $AB\Delta$ τρίγωνον τῷ $A\Delta\Gamma$ τριγώνῳ. ἔστιν ἄρα ὡς ἡ $B\Delta$ τοῦ $AB\Delta$ τριγώνου ὑποτείνουσα τὴν ὑπὸ $BA\Delta$ πρὸς τὴν ΔA τοῦ $A\Delta\Gamma$ τριγώνου ὑποτείνουσαν τὴν πρὸς τῷ Γ ἴσην τῇ ὑπὸ $BA\Delta$, οὕτως αὐτὴ ἡ $A\Delta$ τοῦ $AB\Delta$ τριγώνου ὑποτείνουσα τὴν πρὸς τῷ B γωνίαν πρὸς τὴν $\Delta\Gamma$ ὑποτείνουσαν τὴν ὑπὸ $\Delta A\Gamma$ τοῦ $A\Delta\Gamma$ τριγώνου ἴσην τῇ πρὸς τῷ B , καὶ ἔτι ἡ BA πρὸς τὴν AG ὑποτείνουσαι τὰς ὀρθὰς· ὅμοιον ἄρα ἐστὶ τὸ $AB\Delta$ τρίγωνον τῷ $A\Delta\Gamma$ τριγώνῳ.

Ἐὰν ἄρα ἐν ὀρθογώνίῳ τριγώνῳ ἀπὸ τῆς ὀρθῆς γωνίας ἐπὶ τὴν βᾶσιν κάθετος ἀχθῇ, τὰ πρὸς τῇ καθέτῳ τρίγωνα ὁμοία ἐστὶ τῷ τε ὅλῳ καὶ ἀλλήλοις [ὅπερ ἔδει δεῖξαι].

A , perpendicular to BC [Prop. 1.12]. I say that triangles ABD and ADC are each similar to the whole (triangle) ABC and, further, to one another.



For since (angle) BAC is equal to ADB —for each (are) right-angles—and the (angle) at B (is) common to the two triangles ABC and ABD , the remaining (angle) ACB is thus equal to the remaining (angle) BAD [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle ABD . Thus, as BC , subtending the right-angle in triangle ABC , is to BA , subtending the right-angle in triangle ABD , so the same AB , subtending the angle at C in triangle ABC , (is) to BD , subtending the equal (angle) BAD in triangle ABD , and, further, (so is) AC to AD , (both) subtending the angle at B common to the two triangles [Prop. 6.4]. Thus, triangle ABC is equiangular to triangle ABD , and has the sides about the equal angles proportional. Thus, triangle ABC [is] similar to triangle ABD [Def. 6.1]. So, similarly, we can show that triangle ABC is also similar to triangle ADC . Thus, [triangles] ABD and ADC are each similar to the whole (triangle) ABC .

So I say that triangles ABD and ADC are also similar to one another.

For since the right-angle BDA is equal to the right-angle ADC , and, indeed, (angle) BAD was also shown (to be) equal to the (angle) at C , thus the remaining (angle) at B is also equal to the remaining (angle) DAC [Prop. 1.32]. Thus, triangle ABD is equiangular to triangle ADC . Thus, as BD , subtending (angle) BAD in triangle ABD , is to DA , subtending the (angle) at C in triangle ADC , (which is) equal to (angle) BAD , so (is) the same AD , subtending the angle at B in triangle ABD , to DC , subtending (angle) DAC in triangle ADC , (which is) equal to the (angle) at B , and, further, (so is) BA to AC , (each) subtending right-angles [Prop. 6.4]. Thus, triangle ABD is similar to triangle ADC [Def. 6.1].

Thus, if, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base

then the triangles around the perpendicular are similar to the whole (triangle), and to one another. [(Which is) the very thing it was required to show.]

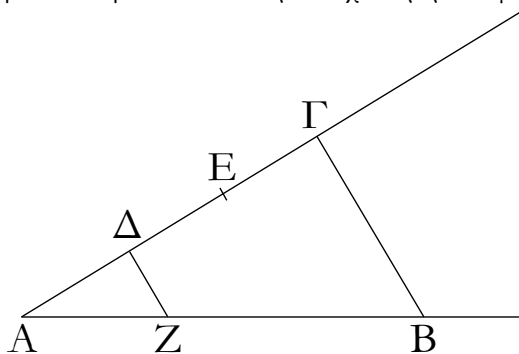
Πόρισμα.

Ἐκ δὴ τοῦτου φανερόν, ὅτι ἐὰν ἐν ὀρθογωνίῳ τριγώνῳ ἀπὸ τῆς ὀρθῆς γωνίας ἐπὶ τὴν βάσιν κάθετος ἀχθῇ, ἡ ἀχθεῖσα τῶν τῆς βάσεως τμημάτων μέση ἀνάλογόν ἐστιν ὅπερ ἔδει δεῖξαι.

[†] In other words, the perpendicular is the geometric mean of the pieces.

 ϑ'

Τῆς δοθείσης εὐθείας τὸ προσταχθὲν μέρος ἀφελεῖν.



Ἐστω ἡ δοθεῖσα εὐθεῖα ἡ AB· δεῖ δὴ τῆς AB τὸ προ-
σταχθὲν μέρος ἀφελεῖν.

Ἐπιτετάχθω δὴ τὸ τρίτον. [καί] διηθῶ τις ἀπὸ τοῦ Α εὐθεῖα ἡ ΑΓ γωνίαν περιέχουσα μετὰ τῆς ΑΒ τυχοῦσαν· καὶ εἰληφθῶ τυχὸν σημεῖον ἐπὶ τῆς ΑΓ τὸ Δ, καὶ κείσθωσαν τῇ ΑΔ ἴσαι αἱ ΔΕ, ΕΓ. καὶ ἐπεξεύχθω ἡ ΒΓ, καὶ διὰ τοῦ Δ παράλληλος αὐτῇ ῥιχθῶ ἡ ΔΖ.

Ἐπεὶ οὖν τριγώνου τοῦ ΑΒΓ παρὰ μίαν τῶν πλευρῶν τὴν ΒΓ ἤκται ἡ ΖΔ, ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΓΔ πρὸς τὴν ΔΑ, οὕτως ἡ ΒΖ πρὸς τὴν ΖΑ. διπλῇ δὲ ἡ ΓΔ τῆς ΔΑ· διπλῇ ἄρα καὶ ἡ ΒΖ τῆς ΖΑ· τριπλῇ ἄρα ἡ ΒΑ τῆς ΑΖ.

Τῆς ἄρα δοθείσης εὐθείας τῆς ΑΒ τὸ ἐπιταχθὲν τρίτον μέρος ἀφήρηται τὸ ΑΖ· ὅπερ ἔδει ποιῆσαι.

6.

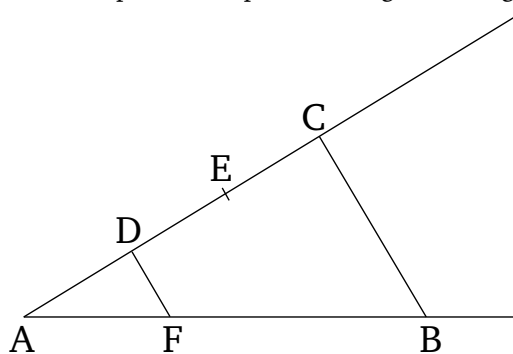
Τὴν δοθεῖσαν εὐθεῖαν ἄτητον τῇ δοθείσῃ τετμημένη
ὁμοίως τεμεῖν.

Corollary

So (it is) clear, from this, that if, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the (straight-line so) drawn is in mean proportion to the pieces of the base.[†] (Which is) the very thing it was required to show.

Proposition 9

To cut off a prescribed part from a given straight-line.



Let AB be the given straight-line. So it is required to cut off a prescribed part from AB .

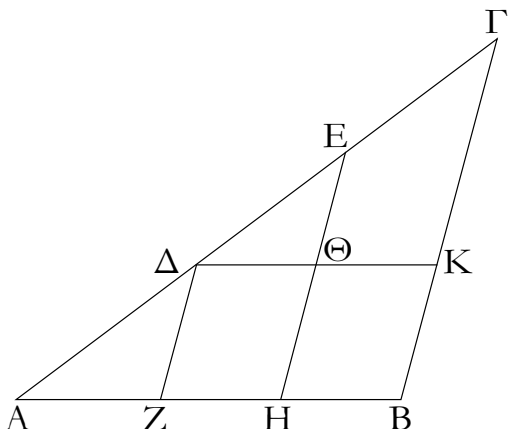
So let a third (part) have been prescribed. [And] let some straight-line AC have been drawn from (point) A , encompassing a random angle with AB . And let a random point D have been taken on AC . And let DE and EC be made equal to AD [Prop. 1.3]. And let BC have been joined. And let DF have been drawn through D parallel to it [Prop. 1.31].

Therefore, since FD has been drawn parallel to one of the sides, BC , of triangle ABC , then, proportionally, as CD is to DA , so BF (is) to FA [Prop. 6.2]. And CD (is) double DA . Thus, BF (is) also double FA . Thus, BA (is) triple AF .

Thus, the prescribed third part, AF , has been cut off from the given straight-line, AB . (Which is) the very thing it was required to do.

Proposition 10

To cut a given uncut straight-line similarly to a given cut (straight-line).



Ἐστω ἡ μὲν δοθεῖσα εὐθεῖα ἀτμητος ἡ AB, ἡ δὲ τετμημένη ἡ AG κατὰ τὰ Δ, Ε σημεία, καὶ κείσθωσαν ὥστε γωνίαν τυχούσαν περιέχειν, καὶ ἐπεζεύχθω ἡ GB, καὶ διὰ τῶν Δ, Ε τῇ BG παράλληλοι ἦχθωσαν αἱ ΔΖ, ΕΗ, διὰ δὲ τοῦ Δ τῇ AB παράλληλος ἦχθω ἡ ΔΘΚ.

Παράλληλόγραμμον ἄρα ἐστὶν ἑκάτερον τῶν ΖΘ, ΘΒ· ἴση ἄρα ἡ μὲν ΔΘ τῇ ΖΗ, ἡ δὲ ΘΚ τῇ ΗΒ. καὶ ἐπεὶ τριγώνου τοῦ ΔΚΓ παρὰ μίαν τῶν πλευρῶν τὴν ΚΓ εὐθεῖα ἦκται ἡ ΘΕ, ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΓΕ πρὸς τὴν ΕΔ, οὕτως ἡ ΚΘ πρὸς τὴν ΘΔ. ἴση δὲ ἡ μὲν ΚΘ τῇ ΒΗ, ἡ δὲ ΘΔ τῇ ΗΖ. ἐστὶν ἄρα ὡς ἡ ΓΕ πρὸς τὴν ΕΔ, οὕτως ἡ ΒΗ πρὸς τὴν ΗΖ. πάλιν, ἐπεὶ τριγώνου τοῦ ΑΗΕ παρὰ μίαν τῶν πλευρῶν τὴν ΗΕ ἦκται ἡ ΖΔ, ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΕΔ πρὸς τὴν ΔΑ, οὕτως ἡ ΗΖ πρὸς τὴν ΖΑ. ἐδείχθη δὲ καὶ ὡς ἡ ΓΕ πρὸς τὴν ΕΔ, οὕτως ἡ ΒΗ πρὸς τὴν ΗΖ· ἐστὶν ἄρα ὡς μὲν ἡ ΓΕ πρὸς τὴν ΕΔ, οὕτως ἡ ΒΗ πρὸς τὴν ΗΖ, ὡς δὲ ἡ ΕΔ πρὸς τὴν ΔΑ, οὕτως ἡ ΗΖ πρὸς τὴν ΖΑ.

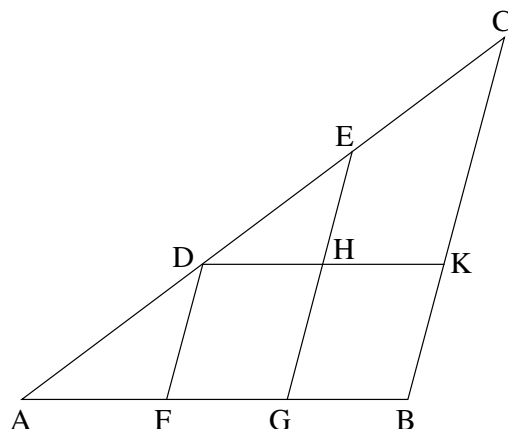
Ἡ ἄρα δοθεῖσα εὐθεῖα ἀτμητος ἡ AB τῇ δοθείσῃ εὐθείᾳ τετμημένῃ τῇ AG ὁμοίως τέτμηται· ὅπερ ἔδει ποιῆσαι.

ια'.

Δύο δοθεισῶν εὐθειῶν τρίτην ἀνάλογον προσευρεῖν.

Ἐστώσαν αἱ δοθεῖσαι [δύο εὐθεῖαι] αἱ BA, AG καὶ κείσθωσαν γωνίαν περιέχουσαι τυχούσαν. δεῖ δὴ τῶν BA, AG τρίτην ἀνάλογον προσευρεῖν. ἐκβεβλήσθωσαν γὰρ ἐπὶ τὰ Δ, Ε σημεία, καὶ κείσθω τῇ AG ἴση ἡ ΒΔ, καὶ ἐπεζεύχθω ἡ BG, καὶ διὰ τοῦ Δ παράλληλος αὐτῇ ἦχθω ἡ ΔΕ.

Ἐπεὶ οὖν τριγώνου τοῦ ΑΔΕ παρὰ μίαν τῶν πλευρῶν τὴν ΔΕ ἦκται ἡ ΒΓ, ἀνάλογόν ἐστιν ὡς ἡ AB πρὸς τὴν ΒΔ, οὕτως ἡ AG πρὸς τὴν ΓΕ. ἴση δὲ ἡ ΒΔ τῇ AG. ἐστὶν ἄρα ὡς ἡ AB πρὸς τὴν AG, οὕτως ἡ AG πρὸς τὴν ΓΕ.



Let AB be the given uncut straight-line, and AC a (straight-line) cut at points D and E, and let (AC) be laid down so as to encompass a random angle (with AB). And let CB have been joined. And let DF and EG have been drawn through (points) D and E (respectively), parallel to BC, and let DHK have been drawn through (point) D, parallel to AB [Prop. 1.31].

Thus, FH and HB are each parallelograms. Thus, DH (is) equal to FG, and HK to GB [Prop. 1.34]. And since the straight-line HE has been drawn parallel to one of the sides, KC, of triangle DKC, thus, proportionally, as CE is to ED, so KH (is) to HD [Prop. 6.2]. And KH (is) equal to BG, and HD to GF. Thus, as CE is to ED, so BG (is) to GF. Again, since FD has been drawn parallel to one of the sides, GE, of triangle AGE, thus, proportionally, as ED is to DA, so GF (is) to FA [Prop. 6.2]. And it was also shown that as CE (is) to ED, so BG (is) to GF. Thus, as CE is to ED, so BG (is) to GF, and as ED (is) to DA, so GF (is) to FA.

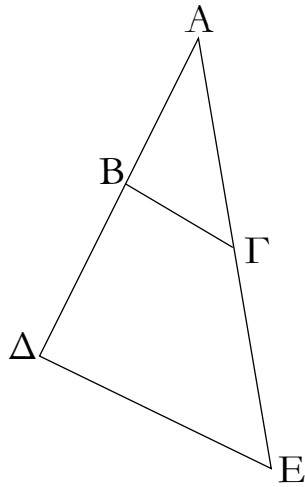
Thus, the given uncut straight-line, AB, has been cut similarly to the given cut straight-line, AC. (Which is) the very thing it was required to do.

Proposition 11

To find a third (straight-line) proportional to two given straight-lines.

Let BA and AC be the [two] given [straight-lines], and let them be laid down encompassing a random angle. So it is required to find a third (straight-line) proportional to BA and AC. For let (BA and AC) have been produced to points D and E (respectively), and let BD be made equal to AC [Prop. 1.3]. And let BC have been joined. And let DE have been drawn through (point) D parallel to it [Prop. 1.31].

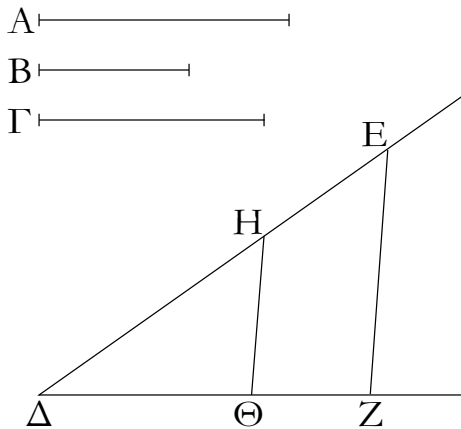
Therefore, since BC has been drawn parallel to one of the sides DE of triangle ADE, proportionally, as AB is to BD, so AC (is) to CE [Prop. 6.2]. And BD (is) equal



Δύο ἄρα δοθεισῶν εὐθειῶν τῶν AB , AG τρίτη ἀνάλογον αὐταῖς προσεύρηται ἡ GE · ὅπερ ἔδει ποιῆσαι.

ιβ'.

Τριῶν δοθεισῶν εὐθειῶν τετάρτην ἀνάλογον προσευρεῖν.



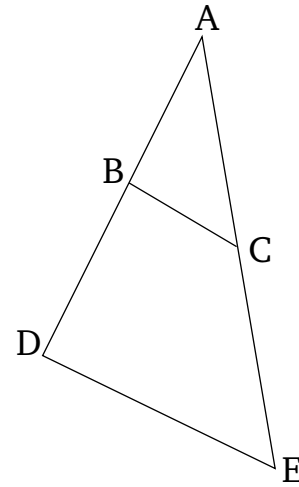
Ἐστωσαν αἱ δοθεῖσαι τρεῖς εὐθεῖαι αἱ A , B , Γ · δεῖ δὴ τῶν A , B , Γ τετάρτην ἀνάλογον προσευρεῖν.

Ἐκκείσθωσαν δύο εὐθεῖαι αἱ DE , DZ γωνίαν περιέχουσai [τυχοῦσαν] τὴν ὑπὸ EDZ · καὶ κείσθω τῇ μὲν A ἴση ἡ ΔH , τῇ δὲ B ἴση ἡ HE , καὶ ἔτι τῇ Γ ἴση ἡ $\Delta\Theta$ · καὶ ἐπιζευχθείσης τῆς $H\Theta$ παράλληλος αὐτῇ ἦχθω διὰ τοῦ E ἡ EZ .

Ἐπεὶ οὖν τριγώνου τοῦ DEZ παρὰ μίαν τὴν EZ ἦκται ἡ $H\Theta$, ἔστιν ἄρα ὡς ἡ ΔH πρὸς τὴν HE , οὕτως ἡ $\Delta\Theta$ πρὸς τὴν ΘZ . ἴση δὲ ἡ μὲν ΔH τῇ A , ἡ δὲ HE τῇ B , ἡ δὲ $\Delta\Theta$ τῇ Γ · ἔστιν ἄρα ὡς ἡ A πρὸς τὴν B , οὕτως ἡ Γ πρὸς τὴν ΘZ .

Τριῶν ἄρα δοθεισῶν εὐθειῶν τῶν A , B , Γ τετάρτη ἀνάλογον προσεύρηται ἡ ΘZ · ὅπερ ἔδει ποιῆσαι.

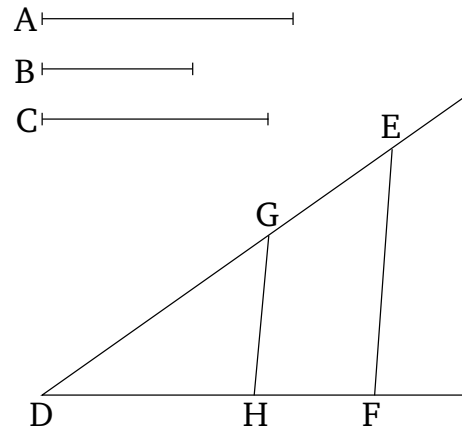
to AC . Thus, as AB is to AC , so AC (is) to CE .



Thus, a third (straight-line), CE , has been found (which is) proportional to the two given straight-lines, AB and AC . (Which is) the very thing it was required to do.

Proposition 12

To find a fourth (straight-line) proportional to three given straight-lines.



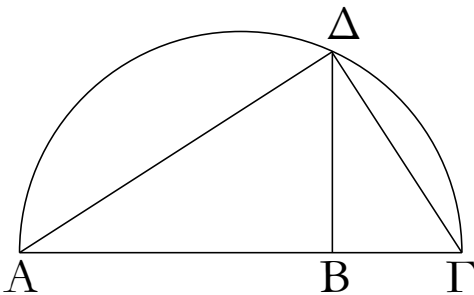
Let A , B , and C be the three given straight-lines. So it is required to find a fourth (straight-line) proportional to A , B , and C .

Let the two straight-lines DE and DF be set out encompassing the [random] angle EDF . And let DG be made equal to A , and GE to B , and, further, DH to C [Prop. 1.3]. And GH being joined, let EF have been drawn through (point) E parallel to it [Prop. 1.31].

Therefore, since GH has been drawn parallel to one of the sides EF of triangle DEF , thus as DG is to GE , so DH (is) to HF [Prop. 6.2]. And DG (is) equal to A , and GE to B , and DH to C . Thus, as A is to B , so C (is)

ιγ'.

Δύο δοθεισῶν εὐθειῶν μέσην ἀνάλογον προσευρεῖν.



Ἐστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι αἱ AB , $BΓ$. δεῖ δὴ τῶν AB , $BΓ$ μέσην ἀνάλογον προσευρεῖν.

Κεῖσθωσαν ἐπ' εὐθείας, καὶ γεγράφθω ἐπὶ τῆς AG ἡμικύκλιον τὸ $ADΓ$, καὶ ἤχθω ἀπὸ τοῦ B σημείου τῇ AG εὐθείᾳ πρὸς ὀρθὰς ἡ BA , καὶ ἐπεζεύχθωσαν αἱ AD , $DΓ$.

Ἐπεὶ ἐν ἡμικυκλίῳ γωνία ἐστὶν ἡ ὑπὸ $ADΓ$, ὀρθή ἐστιν. καὶ ἐπεὶ ἐν ὀρθογωνίῳ τριγώνῳ τῷ $ADΓ$ ἀπὸ τῆς ὀρθῆς γωνίας ἐπὶ τὴν βάσιν κάθετος ἤκται ἡ DB , ἡ DB ἄρα τῶν τῆς βάσεως τμημάτων τῶν AB , $BΓ$ μέση ἀνάλογόν ἐστιν.

Δύο ἄρα δοθεισῶν εὐθειῶν τῶν AB , $BΓ$ μέση ἀνάλογον προσεύρηται ἡ DB . ὅπερ ἔδει ποιῆσαι.

† In other words, to find the geometric mean of two given straight-lines.

ιδ'.

Τῶν ἴσων τε καὶ ἰσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας· καὶ ὧν ἰσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας, ἴσα ἐστὶν ἑκείνα.

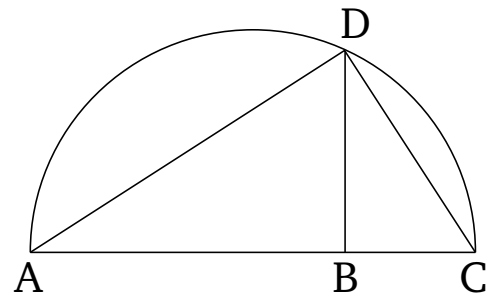
Ἐστω ἴσα τε καὶ ἰσογώνια παραλληλόγραμμα τὰ AB , $BΓ$ ἴσας ἔχοντα τὰς πρὸς τῷ B γωνίας, καὶ κεῖσθωσαν ἐπ' εὐθείας αἱ $ΔB$, BE . ἐπ' εὐθείας ἄρα εἰσὶ καὶ αἱ ZB , BH . λέγω, ὅτι τῶν AB , $BΓ$ ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας, τουτέστιν, ὅτι ἐστὶν ὡς ἡ $ΔB$ πρὸς τὴν BE , οὕτως ἡ HB πρὸς τὴν BZ .

to HF .

Thus, a fourth (straight-line), HF , has been found (which is) proportional to the three given straight-lines, A , B , and C . (Which is) the very thing it was required to do.

Proposition 13

To find the (straight-line) in mean proportion to two given straight-lines.[†]



Let AB and BC be the two given straight-lines. So it is required to find the (straight-line) in mean proportion to AB and BC .

Let (AB and BC) be laid down straight-on (with respect to one another), and let the semi-circle ADC have been drawn on AC [Prop. 1.10]. And let BD have been drawn from (point) B , at right-angles to AC [Prop. 1.11]. And let AD and DC have been joined.

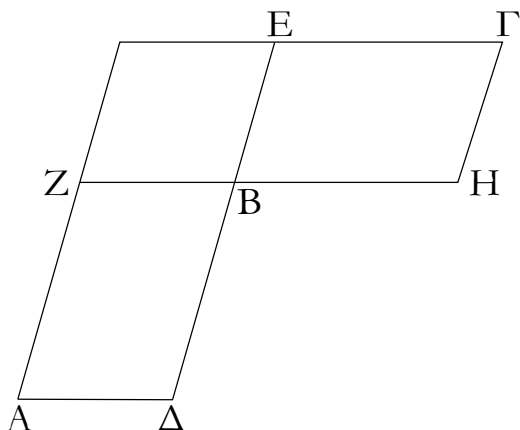
And since ADC is an angle in a semi-circle, it is a right-angle [Prop. 3.31]. And since, in the right-angled triangle ADC , the (straight-line) DB has been drawn from the right-angle perpendicular to the base, DB is thus the mean proportional to the pieces of the base, AB and BC [Prop. 6.8 corr.].

Thus, DB has been found (which is) in mean proportion to the two given straight-lines, AB and BC . (Which is) the very thing it was required to do.

Proposition 14

In equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional. And those equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.

Let AB and BC be equal and equiangular parallelograms having the angles at B equal. And let DB and BE be laid down straight-on (with respect to one another). Thus, FB and BG are also straight-on (with respect to one another) [Prop. 1.14]. I say that the sides of AB and



Συμπεπληρώσθω γὰρ τὸ ZE παραλληλόγραμμον. ἐπεὶ οὖν ἴσον ἐστὶ τὸ AB παραλληλόγραμμον τῷ BG παραλληλόγραμμῳ, ἄλλο δέ τι τὸ ZE, ἔστιν ἄρα ὡς τὸ AB πρὸς τὸ ZE, οὕτως τὸ BG πρὸς τὸ ZE. ἀλλ' ὡς μὲν τὸ AB πρὸς τὸ ZE, οὕτως ἡ ΔB πρὸς τὴν BE, ὡς δὲ τὸ BG πρὸς τὸ ZE, οὕτως ἡ HB πρὸς τὴν BZ· καὶ ὡς ἄρα ἡ ΔB πρὸς τὴν BE, οὕτως ἡ HB πρὸς τὴν BZ. τῶν ἄρα AB, BG παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας.

Ἀλλὰ δὴ ἔστω ὡς ἡ ΔB πρὸς τὴν BE, οὕτως ἡ HB πρὸς τὴν BZ· λέγω, ὅτι ἴσον ἐστὶ τὸ AB παραλληλόγραμμον τῷ BG παραλληλογράμμῳ.

Ἐπεὶ γάρ ἐστιν ὡς ἡ ΔB πρὸς τὴν BE, οὕτως ἡ HB πρὸς τὴν BZ, ἀλλ' ὡς μὲν ἡ ΔB πρὸς τὴν BE, οὕτως τὸ AB παραλληλόγραμμον πρὸς τὸ ZE παραλληλόγραμμον, ὡς δὲ ἡ HB πρὸς τὴν BZ, οὕτως τὸ BG παραλληλόγραμμον πρὸς τὸ ZE παραλληλόγραμμον, καὶ ὡς ἄρα τὸ AB πρὸς τὸ ZE, οὕτως τὸ BG πρὸς τὸ ZE· ἴσον ἄρα ἐστὶ τὸ AB παραλληλόγραμμον τῷ BG παραλληλογράμμῳ.

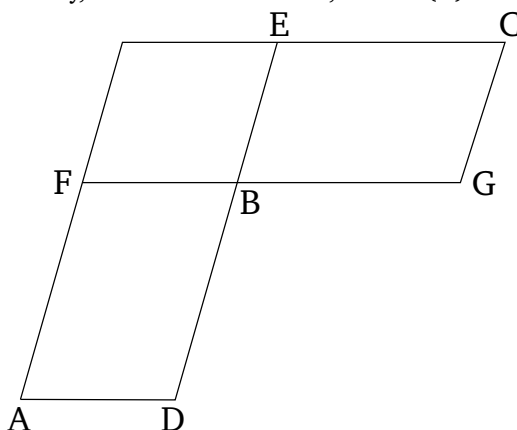
Τῶν ἄρα ἴσων τε καὶ ἰσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας· καὶ ὧν ἰσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας, ἴσα ἐστὶν ἐκεῖνα· ὅπερ ἔδει δεῖξαι.

ιε'.

Τῶν ἴσων καὶ μίαν μὲν ἴσην ἔχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας· καὶ ὧν μίαν μὲν ἴσην ἔχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας, ἴσα ἐστὶν ἐκεῖνα.

Ἐστω ἴσα τρίγωνα τὰ ABΓ, AΔE μίαν μὲν ἴσην ἔχοντα γωνίαν τὴν ὑπὸ BΑΓ τῇ ὑπὸ ΔAE· λέγω, ὅτι τῶν ABΓ, AΔE τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας, τουτέστιν, ὅτι ἐστὶν ὡς ἡ ΓA πρὸς τὴν AΔ, οὕτως

BC about the equal angles are reciprocally proportional, that is to say, that as DB is to BE, so GB (is) to BF.



For let the parallelogram FE have been completed. Therefore, since parallelogram AB is equal to parallelogram BC, and FE (is) some other (parallelogram), thus as (parallelogram) AB is to FE, so (parallelogram) BC (is) to FE [Prop. 5.7]. But, as (parallelogram) AB (is) to FE, so DB (is) to BE, and as (parallelogram) BC (is) to FE, so GB (is) to BF [Prop. 6.1]. Thus, also, as DB (is) to BE, so GB (is) to BF. Thus, in parallelograms AB and BC the sides about the equal angles are reciprocally proportional.

And so, let DB be to BE, as GB (is) to BF. I say that parallelogram AB is equal to parallelogram BC.

For since as DB is to BE, so GB (is) to BF, but as DB (is) to BE, so parallelogram AB (is) to parallelogram FE, and as GB (is) to BF, so parallelogram BC (is) to parallelogram FE [Prop. 6.1], thus, also, as (parallelogram) AB (is) to FE, so (parallelogram) BC (is) to FE [Prop. 5.11]. Thus, parallelogram AB is equal to parallelogram BC [Prop. 5.9].

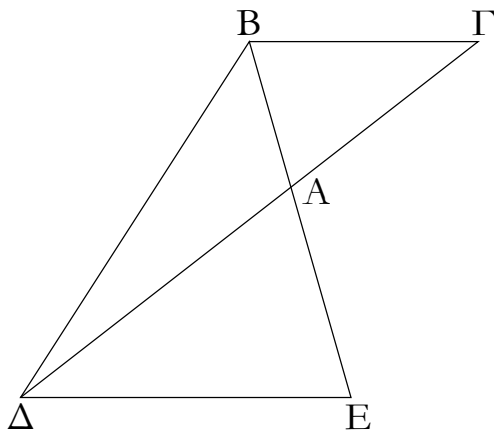
Thus, in equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional. And those equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal. (Which is) the very thing it was required to show.

Proposition 15

In equal triangles also having one angle equal to one (angle) the sides about the equal angles are reciprocally proportional. And those triangles having one angle equal to one angle for which the sides about the equal angles (are) reciprocally proportional are equal.

Let ABC and ADE be equal triangles having one angle equal to one (angle), (namely) BAC (equal) to DAE. I say that, in triangles ABC and ADE, the sides about the

ἡ EA πρὸς τὴν AB .



Κεῖσθω γὰρ ὥστε ἐπ' εὐθείας εἶναι τὴν GA τῇ AD · ἐπ' εὐθείας ἄρα ἐστὶ καὶ ἡ EA τῇ AB . καὶ ἐπεζεύχθω ἡ BD .

Ἐπεὶ οὖν ἴσον ἐστὶ τὸ ABG τρίγωνον τῷ ADE τριγώνῳ, ἄλλο δέ τι τὸ BAD , ἔστιν ἄρα ὡς τὸ GAB τρίγωνον πρὸς τὸ BAD τρίγωνον, οὕτως τὸ EAD τρίγωνον πρὸς τὸ BAD τρίγωνον. ἀλλ' ὡς μὲν τὸ GAB πρὸς τὸ BAD , οὕτως ἡ GA πρὸς τὴν AD , ὡς δὲ τὸ EAD πρὸς τὸ BAD , οὕτως ἡ EA πρὸς τὴν AB . καὶ ὡς ἄρα ἡ GA πρὸς τὴν AD , οὕτως ἡ EA πρὸς τὴν AB . τῶν ABG , ADE ἄρα τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας.

Ἀλλὰ δὴ ἀντιπεπονηθέντων αἱ πλευραὶ τῶν ABG , ADE τριγώνων, καὶ ἔστω ὡς ἡ GA πρὸς τὴν AD , οὕτως ἡ EA πρὸς τὴν AB · λέγω, ὅτι ἴσον ἐστὶ τὸ ABG τρίγωνον τῷ ADE τριγώνῳ.

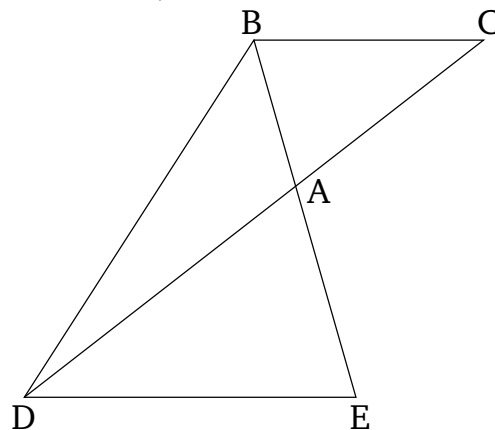
Ἐπιζευχθείσης γὰρ πάλιν τῆς BD , ἐπεὶ ἐστὶν ὡς ἡ GA πρὸς τὴν AD , οὕτως ἡ EA πρὸς τὴν AB , ἀλλ' ὡς μὲν ἡ GA πρὸς τὴν AD , οὕτως τὸ ABG τρίγωνον πρὸς τὸ BAD τρίγωνον, ὡς δὲ ἡ EA πρὸς τὴν AB , οὕτως τὸ EAD τρίγωνον πρὸς τὸ BAD τρίγωνον, ὡς ἄρα τὸ ABG τρίγωνον πρὸς τὸ BAD τρίγωνον, οὕτως τὸ EAD τρίγωνον πρὸς τὸ BAD τρίγωνον. ἐκάτερον ἄρα τῶν ABG , EAD πρὸς τὸ BAD τὸν αὐτὸν ἔχει λόγον. ἴσων ἄρα ἐστὶ τὸ ABG [τρίγωνον] τῷ EAD τριγώνῳ.

Τῶν ἄρα ἴσων καὶ μίαν μιᾷ ἴσην ἐχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας· καὶ ὡς μίαν μιᾷ ἴσην ἐχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας, ἐκεῖνα ἴσα ἐστὶν· ὅπερ ἔδει δεῖξαι.

17'.

Ἐὰν τέσσαρες εὐθεῖαι ἀνάλογον ᾧσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν μέσων περιεχομένῳ ὀρθογώνῳ· καὶ τὸ ὑπὸ τῶν ἄκρων

equal angles are reciprocally proportional, that is to say, that as CA is to AD , so EA (is) to AB .



For let CA be laid down so as to be straight-on (with respect) to AD . Thus, EA is also straight-on (with respect) to AB [Prop. 1.14]. And let BD have been joined.

Therefore, since triangle ABC is equal to triangle ADE , and BAD (is) some other (triangle), thus as triangle CAB is to triangle BAD , so triangle EAD (is) to triangle BAD [Prop. 5.7]. But, as (triangle) CAB (is) to BAD , so CA (is) to AD , and as (triangle) EAD (is) to BAD , so EA (is) to AB [Prop. 6.1]. And thus, as CA (is) to AD , so EA (is) to AB . Thus, in triangles ABC and ADE the sides about the equal angles (are) reciprocally proportional.

And so, let the sides of triangles ABC and ADE be reciprocally proportional, and (thus) let CA be to AD , as EA (is) to AB . I say that triangle ABC is equal to triangle ADE .

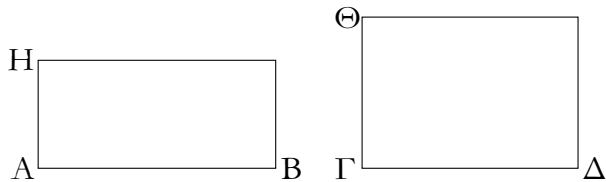
For, BD again being joined, since as CA is to AD , so EA (is) to AB , but as CA (is) to AD , so triangle ABC (is) to triangle BAD , and as EA (is) to AB , so triangle EAD (is) to triangle BAD [Prop. 6.1], thus as triangle ABC (is) to triangle BAD , so triangle EAD (is) to triangle BAD . Thus, (triangles) ABC and EAD each have the same ratio to BAD . Thus, [triangle] ABC is equal to triangle EAD [Prop. 5.9].

Thus, in equal triangles also having one angle equal to one (angle) the sides about the equal angles (are) reciprocally proportional. And those triangles having one angle equal to one angle for which the sides about the equal angles (are) reciprocally proportional are equal. (Which is) the very thing it was required to show.

Proposition 16

If four straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the rectangle contained by the middle (two). And if the rect-

περιεχόμενον ὀρθογώνιον ἴσον ἢ τῷ ὑπὸ τῶν μέσων περιεχομένῳ ὀρθογώνιῳ, αἱ τέσσαρες εὐθεῖαι ἀνάλογον ἔσονται.



Ἐστωσαν τέσσαρες εὐθεῖαι ἀνάλογον αἱ AB, ΓΔ, E, Z, ὥς ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ E πρὸς τὴν Z· λέγω, ὅτι τὸ ὑπὸ τῶν AB, Z περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν ΓΔ, E περιεχομένῳ ὀρθογώνιῳ.

Ἦχθωσαν [γὰρ] ἀπὸ τῶν A, Γ σημείων ταῖς AB, ΓΔ εὐθείαις πρὸς ὀρθὰς αἱ AH, ΓΘ, καὶ κείσθω τῇ μὲν Z ἴση ἡ AH, τῇ δὲ E ἴση ἡ ΓΘ. καὶ συμπληρώσθω τὰ BH, ΔΘ παραλληλόγραμμα.

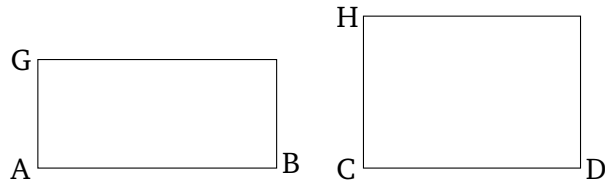
Καὶ ἐπεὶ ἐστὶν ὥς ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ E πρὸς τὴν Z, ἴση δὲ ἡ μὲν E τῇ ΓΘ, ἡ δὲ Z τῇ AH, ἐστὶν ἄρα ὥς ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ ΓΘ πρὸς τὴν AH. τῶν BH, ΔΘ ἄρα παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας. ὦν δὲ ἰσογώνιων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας, ἴσα ἐστὶν ἐκεῖνα· ἴσον ἄρα ἐστὶ τὸ BH παραλληλόγραμμον τῷ ΔΘ παραλληλόγραμμῳ. καὶ ἐστὶ τὸ μὲν BH τὸ ὑπὸ τῶν AB, Z· ἴση γὰρ ἡ AH τῇ Z· τὸ δὲ ΔΘ τὸ ὑπὸ τῶν ΓΔ, E· ἴση γὰρ ἡ E τῇ ΓΘ· τὸ ἄρα ὑπὸ τῶν AB, Z περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν ΓΔ, E περιεχομένῳ ὀρθογώνιῳ.

Ἀλλὰ δὴ τὸ ὑπὸ τῶν AB, Z περιεχόμενον ὀρθογώνιον ἴσον ἔστω τῷ ὑπὸ τῶν ΓΔ, E περιεχομένῳ ὀρθογώνιῳ. λέγω, ὅτι αἱ τέσσαρες εὐθεῖαι ἀνάλογον ἔσονται, ὥς ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ E πρὸς τὴν Z.

Τῶν γὰρ αὐτῶν κατασκευασθέντων, ἐπεὶ τὸ ὑπὸ τῶν AB, Z ἴσον ἐστὶ τῷ ὑπὸ τῶν ΓΔ, E, καὶ ἐστὶ τὸ μὲν ὑπὸ τῶν AB, Z τὸ BH· ἴση γὰρ ἐστὶν ἡ AH τῇ Z· τὸ δὲ ὑπὸ τῶν ΓΔ, E τὸ ΔΘ· ἴση γὰρ ἡ ΓΘ τῇ E· τὸ ἄρα BH ἴσον ἐστὶ τῷ ΔΘ. καὶ ἐστὶν ἰσογώνια. τῶν δὲ ἴσων καὶ ἰσογώνιων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας. ἐστὶν ἄρα ὥς ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ ΓΘ πρὸς τὴν AH. ἴση δὲ ἡ μὲν ΓΘ τῇ E, ἡ δὲ AH τῇ Z· ἐστὶν ἄρα ὥς ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ E πρὸς τὴν Z.

Ἐὰν ἄρα τέσσαρες εὐθεῖαι ἀνάλογον ᾖσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν μέσων περιεχομένῳ ὀρθογώνιῳ· καὶ τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὀρθογώνιον ἴσον ἢ τῷ ὑπὸ τῶν μέσων περιεχομένῳ ὀρθογώνιῳ, αἱ τέσσαρες εὐθεῖαι ἀνάλογον ἔσονται· ὅπερ εἶδει δεῖξαι.

angle contained by the (two) outermost is equal to the rectangle contained by the middle (two) then the four straight-lines will be proportional.



Let AB, CD, E, and F be four proportional straight-lines, (such that) as AB (is) to CD, so E (is) to F. I say that the rectangle contained by AB and F is equal to the rectangle contained by CD and E.

[For] let AG and CH have been drawn from points A and C at right-angles to the straight-lines AB and CD (respectively) [Prop. 1.11]. And let AG be made equal to F, and CH to E [Prop. 1.3]. And let the parallelograms BG and DH have been completed.

And since as AB is to CD, so E (is) to F, and E (is) equal CH, and F to AG, thus as AB is to CD, so CH (is) to AG. Thus, in the parallelograms BG and DH the sides about the equal angles are reciprocally proportional. And those equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal [Prop. 6.14]. Thus, parallelogram BG is equal to parallelogram DH. And BG is the (rectangle contained) by AB and F. For AG (is) equal to F. And DH (is) the (rectangle contained) by CD and E. For E (is) equal to CH. Thus, the rectangle contained by AB and F is equal to the rectangle contained by CD and E.

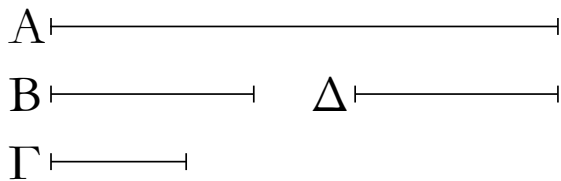
And so, let the rectangle contained by AB and F be equal to the rectangle contained by CD and E. I say that the four straight-lines will be proportional, (so that) as AB (is) to CD, so E (is) to F.

For, with the same construction, since the (rectangle contained) by AB and F is equal to the (rectangle contained) by CD and E. And BG is the (rectangle contained) by AB and F. For AG is equal to F. And DH (is) the (rectangle contained) by CD and E. For CH (is) equal to E. BG is thus equal to DH. And they are equiangular. And in equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional [Prop. 6.14]. Thus, as AB is to CD, so CH (is) to AG. And CH (is) equal to E, and AG to F. Thus, as AB is to CD, so E (is) to F.

Thus, if four straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the rectangle contained by the middle (two). And if the rectangle contained by the (two) outermost is equal to

ιζ'.

Ἐάν τρεῖς εὐθεῖαι ἀνάλογον ὦσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ἀπὸ τῆς μέσης τετραγώνῳ· καὶ τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὀρθογώνιον ἴσον ἢ τῷ ἀπὸ τῆς μέσης τετραγώνῳ, αἱ τρεῖς εὐθεῖαι ἀνάλογον ἔσσονται.



Ἐστωσαν τρεῖς εὐθεῖαι ἀνάλογον αἱ A, B, Γ , ὡς ἡ A πρὸς τὴν B , οὕτως ἡ B πρὸς τὴν Γ . λέγω, ὅτι τὸ ὑπὸ τῶν A, Γ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ἀπὸ τῆς B τετραγώνῳ.

Κείσθω τῇ B ἴση ἡ Δ .

Καὶ ἐπεὶ ἐστὶν ὡς ἡ A πρὸς τὴν B , οὕτως ἡ B πρὸς τὴν Γ , ἴση δὲ ἡ B τῇ Δ , ἔστιν ἄρα ὡς ἡ A πρὸς τὴν B , ἡ Δ πρὸς τὴν Γ . ἐὰν δὲ τέσσαρες εὐθεῖαι ἀνάλογον ὦσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον [ὀρθογώνιον] ἴσον ἐστὶ τῷ ὑπὸ τῶν μέσων περιεχομένῳ ὀρθογώνιῳ. τὸ ἄρα ὑπὸ τῶν A, Γ ἴσον ἐστὶ τῷ ὑπὸ τῶν B, Δ . ἀλλὰ τὸ ὑπὸ τῶν B, Δ τὸ ἀπὸ τῆς B ἐστίν· ἴση γὰρ ἡ B τῇ Δ . τὸ ἄρα ὑπὸ τῶν A, Γ περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ἀπὸ τῆς B τετραγώνῳ.

Ἀλλὰ δὴ τὸ ὑπὸ τῶν A, Γ ἴσον ἔστω τῷ ἀπὸ τῆς B . λέγω, ὅτι ἐστὶν ὡς ἡ A πρὸς τὴν B , οὕτως ἡ B πρὸς τὴν Γ .

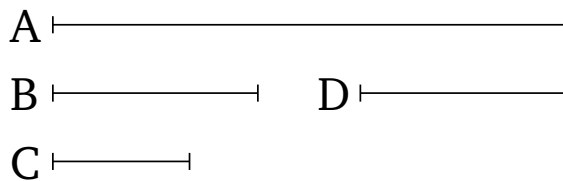
Τῶν γὰρ αὐτῶν κατασκευασθέντων, ἐπεὶ τὸ ὑπὸ τῶν A, Γ ἴσον ἐστὶ τῷ ἀπὸ τῆς B , ἀλλὰ τὸ ἀπὸ τῆς B τὸ ὑπὸ τῶν B, Δ ἐστίν· ἴση γὰρ ἡ B τῇ Δ . τὸ ἄρα ὑπὸ τῶν A, Γ ἴσον ἐστὶ τῷ ὑπὸ τῶν B, Δ . ἐὰν δὲ τὸ ὑπὸ τῶν ἄκρων ἴσον ἢ τῷ ὑπὸ τῶν μέσων, αἱ τέσσαρες εὐθεῖαι ἀνάλογόν εἰσιν. ἔστιν ἄρα ὡς ἡ A πρὸς τὴν B , οὕτως ἡ Δ πρὸς τὴν Γ . ἴση δὲ ἡ B τῇ Δ . ὡς ἄρα ἡ A πρὸς τὴν B , οὕτως ἡ B πρὸς τὴν Γ .

Ἐὰν ἄρα τρεῖς εὐθεῖαι ἀνάλογον ὦσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ἀπὸ τῆς μέσης τετραγώνῳ· καὶ τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὀρθογώνιον ἴσον ἢ τῷ ἀπὸ τῆς μέσης τετραγώνῳ, αἱ τρεῖς εὐθεῖαι ἀνάλογον ἔσσονται· ὅπερ ἔδει δεῖξαι.

the rectangle contained by the middle (two) then the four straight-lines will be proportional. (Which is) the very thing it was required to show.

Proposition 17

If three straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the square on the middle (one). And if the rectangle contained by the (two) outermost is equal to the square on the middle (one) then the three straight-lines will be proportional.



Let A, B and C be three proportional straight-lines, (such that) as A (is) to B , so B (is) to C . I say that the rectangle contained by A and C is equal to the square on B .

Let D be made equal to B [Prop. 1.3].

And since as A is to B , so B (is) to C , and B (is) equal to D , thus as A is to B , (so) D (is) to C . And if four straight-lines are proportional then the [rectangle] contained by the (two) outermost is equal to the rectangle contained by the middle (two) [Prop. 6.16]. Thus, the (rectangle contained) by A and C is equal to the (rectangle contained) by B and D . But, the (rectangle contained) by B and D is the (square) on B . For B (is) equal to D . Thus, the rectangle contained by A and C is equal to the square on B .

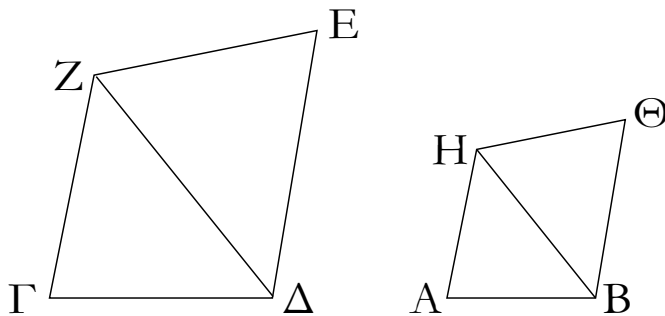
And so, let the (rectangle contained) by A and C be equal to the (square) on B . I say that as A is to B , so B (is) to C .

For, with the same construction, since the (rectangle contained) by A and C is equal to the (square) on B . But, the (square) on B is the (rectangle contained) by B and D . For B (is) equal to D . The (rectangle contained) by A and C is thus equal to the (rectangle contained) by B and D . And if the (rectangle contained) by the (two) outermost is equal to the (rectangle contained) by the middle (two) then the four straight-lines are proportional [Prop. 6.16]. Thus, as A is to B , so D (is) to C . And B (is) equal to D . Thus, as A (is) to B , so B (is) to C .

Thus, if three straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the square on the middle (one). And if the rectangle contained by the (two) outermost is equal to the square on the middle (one) then the three straight-lines will be proportional. (Which is) the very thing it was required to

ιη'.

Ἀπὸ τῆς δοθείσης εὐθείας τῷ δοθέντι εὐθυγράμμῳ ὁμοῖόν τε καὶ ὁμοίως κείμενον εὐθυγράμμον ἀναγράψαι.



Ἐστω ἡ μὲν δοθεῖσα εὐθεῖα ἡ ΑΒ, τὸ δὲ δοθὲν εὐθυγράμμον τὸ ΓΕ· δεῖ δὲ ἀπὸ τῆς ΑΒ εὐθείας τῷ ΓΕ εὐθυγράμμῳ ὁμοῖόν τε καὶ ὁμοίως κείμενον εὐθυγράμμον ἀναγράψαι.

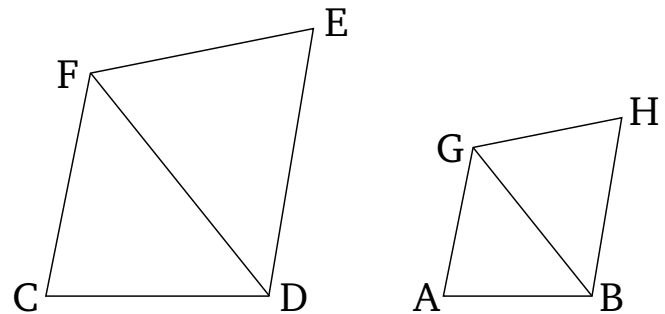
Ἐπεζεύχθω ἡ ΔΖ, καὶ συνεστάτω πρὸς τῇ ΑΒ εὐθείᾳ καὶ τοῖς πρὸς αὐτῇ σημείοις τοῖς Α, Β τῇ μὲν πρὸς τῷ Γ γωνία ἴση ἢ ὑπὸ ΗΑΒ, τῇ δὲ ὑπὸ ΓΔΖ ἴση ἢ ὑπὸ ΑΒΗ. λοιπὴ ἄρα ἡ ὑπὸ ΓΖΔ τῇ ὑπὸ ΑΗΒ ἐστὶν ἴση· ἰσογώνιον ἄρα ἐστὶ τὸ ΖΓΔ τρίγωνον τῷ ΗΑΒ τριγώνῳ. ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΖΔ πρὸς τὴν ΗΒ, οὕτως ἡ ΖΓ πρὸς τὴν ΗΑ, καὶ ἡ ΓΔ πρὸς τὴν ΑΒ. πάλιν συνεστάτω πρὸς τῇ ΒΗ εὐθείᾳ καὶ τοῖς πρὸς αὐτῇ σημείοις τοῖς Β, Η τῇ μὲν ὑπὸ ΔΖΕ γωνία ἴση ἢ ὑπὸ ΒΗΘ, τῇ δὲ ὑπὸ ΖΔΕ ἴση ἢ ὑπὸ ΗΒΘ. λοιπὴ ἄρα ἡ πρὸς τῷ Ε λοιπῇ τῇ πρὸς τῷ Θ ἐστὶν ἴση· ἰσογώνιον ἄρα ἐστὶ τὸ ΖΔΕ τρίγωνον τῷ ΗΘΒ τριγώνῳ. ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΖΔ πρὸς τὴν ΗΒ, οὕτως ἡ ΖΕ πρὸς τὴν ΗΘ καὶ ἡ ΕΔ πρὸς τὴν ΘΒ. ἐδείχθη δὲ καὶ ὡς ἡ ΖΔ πρὸς τὴν ΗΒ, οὕτως ἡ ΖΓ πρὸς τὴν ΗΑ καὶ ἡ ΓΔ πρὸς τὴν ΑΒ· καὶ ὡς ἄρα ἡ ΖΓ πρὸς τὴν ΑΗ, οὕτως ἡ τε ΓΔ πρὸς τὴν ΑΒ καὶ ἡ ΖΕ πρὸς τὴν ΗΘ καὶ ἔτι ἡ ΕΑ πρὸς τὴν ΘΒ. καὶ ἐπεὶ ἴση ἐστὶν ἡ μὲν ὑπὸ ΓΖΔ γωνία τῇ ὑπὸ ΑΗΒ, ἡ δὲ ὑπὸ ΔΖΕ τῇ ὑπὸ ΒΗΘ, ὅλη ἄρα ἡ ὑπὸ ΓΖΕ ὅλη τῇ ὑπὸ ΑΗΘ ἐστὶν ἴση. διὰ τὰ αὐτὰ δὲ καὶ ἡ ὑπὸ ΓΔΕ τῇ ὑπὸ ΑΒΘ ἐστὶν ἴση. ἔστι δὲ καὶ ἡ μὲν πρὸς τῷ Γ τῇ πρὸς τῷ Α ἴση, ἡ δὲ πρὸς τῷ Ε τῇ πρὸς τῷ Θ. ἰσογώνιον ἄρα ἐστὶ τὸ ΑΘ τῷ ΓΕ· καὶ τὰς περὶ τὰς ἴσας γωνίας αὐτῶν πλευρὰς ἀνάλογον ἔχει· ὁμοῖον ἄρα ἐστὶ τὸ ΑΘ εὐθυγράμμον τῷ ΓΕ εὐθυγράμμῳ.

Ἀπὸ τῆς δοθείσης ἄρα εὐθείας τῆς ΑΒ τῷ δοθέντι εὐθυγράμμῳ τῷ ΓΕ ὁμοῖόν τε καὶ ὁμοίως κείμενον εὐθυγράμμον ἀναγράφεται τὸ ΑΘ· ὅπερ ἔδει ποιῆσαι.

show.

Proposition 18

To describe a rectilinear figure similar, and similarly laid down, to a given rectilinear figure on a given straight-line.



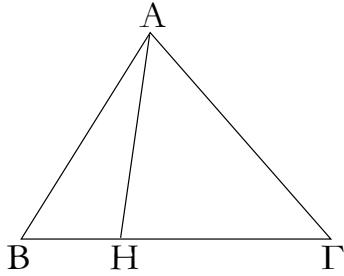
Let ΑΒ be the given straight-line, and CΕ the given rectilinear figure. So it is required to describe a rectilinear figure similar, and similarly laid down, to the rectilinear figure CΕ on the straight-line ΑΒ.

Let DF have been joined, and let GAB, equal to the angle at C, and ABG, equal to (angle) CDF, have been constructed on the straight-line ΑΒ at the points Α and Β on it (respectively) [Prop. 1.23]. Thus, the remaining (angle) CFD is equal to AGB [Prop. 1.32]. Thus, triangle FCD is equiangular to triangle GAB. Thus, proportionally, as FD is to GB, so FC (is) to GA, and CD to AB [Prop. 6.4]. Again, let BGH, equal to angle DFE, and GBH equal to (angle) FDE, have been constructed on the straight-line BG at the points G and B on it (respectively) [Prop. 1.23]. Thus, the remaining (angle) at Ε is equal to the remaining (angle) at Η [Prop. 1.32]. Thus, triangle FDE is equiangular to triangle GHB. Thus, proportionally, as FD is to GB, so FE (is) to GH, and ED to HB [Prop. 6.4]. And it was also shown (that) as FD (is) to GB, so FC (is) to GA, and CD to AB. Thus, also, as FC (is) to AG, so CD (is) to AB, and FE to GH, and, further, ED to HB. And since angle CFD is equal to AGB, and DFE to BGH, thus the whole (angle) CFE is equal to the whole (angle) AGH. So, for the same (reasons), (angle) CDE is also equal to ABH. And the (angle) at C is also equal to the (angle) at Α, and the (angle) at Ε to the (angle) at Η. Thus, (figure) AH is equiangular to CΕ. And (the two figures) have the sides about their equal angles proportional. Thus, the rectilinear figure AH is similar to the rectilinear figure CΕ [Def. 6.1].

Thus, the rectilinear figure AH, similar, and similarly laid down, to the given rectilinear figure CΕ has been constructed on the given straight-line ΑΒ. (Which is) the

10'.

Τὰ ὅμοια τρίγωνα πρὸς ἀλλήλα ἐν διπλασίονι λόγῳ ἐστὶ τῶν ὁμολόγων πλευρῶν.



Ἐστω ὅμοια τρίγωνα τὰ ABG , ΔEZ ἴσην ἔχοντα τὴν πρὸς τῷ B γωνίαν τῇ πρὸς τῷ E , ὡς δὲ τὴν AB πρὸς τὴν BG , οὕτως τὴν ΔE πρὸς τὴν EZ , ὥστε ὁμόλογον εἶναι τὴν BG τῇ EZ : λέγω, ὅτι τὸ ABG τρίγωνον πρὸς τὸ ΔEZ τρίγωνον διπλασίονα λόγον ἔχει ἢ πρὸς τὴν EZ .

Εἰλήφθω γὰρ τῶν BG , EZ τρίτη ἀνάλογον ἡ BH , ὥστε εἶναι ὡς τὴν BG πρὸς τὴν EZ , οὕτως τὴν EZ πρὸς τὴν BH : καὶ ἐπεζεύχθω ἡ AH .

Ἐπεὶ οὖν ἐστὶν ὡς ἡ AB πρὸς τὴν BG , οὕτως ἡ ΔE πρὸς τὴν EZ , ἐναλλάξ ἄρα ἐστὶν ὡς ἡ AB πρὸς τὴν ΔE , οὕτως ἡ BG πρὸς τὴν EZ . ἀλλ' ὡς ἡ BG πρὸς EZ , οὕτως ἐστὶν ἡ EZ πρὸς BH . καὶ ὡς ἄρα ἡ AB πρὸς ΔE , οὕτως ἡ EZ πρὸς BH : τῶν ABH , ΔEZ ἄρα τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωναίας. ὦν δὲ μίαν μὲν ἴσην ἔχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωναίας, ἴσα ἐστὶν ἕκαστα. ἴσον ἄρα ἐστὶ τὸ ABH τρίγωνον τῷ ΔEZ τριγώνῳ. καὶ ἐπεὶ ἐστὶν ὡς ἡ BG πρὸς τὴν EZ , οὕτως ἡ EZ πρὸς τὴν BH , ἐὰν δὲ τρεῖς εὐθεῖαι ἀνάλογον ὦσιν, ἡ πρώτη πρὸς τὴν τρίτην διπλασίονα λόγον ἔχει ἢ πρὸς τὴν δευτέραν, ἡ BG ἄρα πρὸς τὴν BH διπλασίονα λόγον ἔχει ἢ πρὸς τὴν EZ . ὡς δὲ ἡ BG πρὸς τὴν BH , οὕτως τὸ ABG τρίγωνον πρὸς τὸ ABH τρίγωνον: καὶ τὸ ABG ἄρα τρίγωνον πρὸς τὸ ΔEZ τρίγωνον διπλασίονα λόγον ἔχει ἢ πρὸς τὴν EZ .

Τὰ ἄρα ὅμοια τρίγωνα πρὸς ἀλλήλα ἐν διπλασίονι λόγῳ ἐστὶ τῶν ὁμολόγων πλευρῶν. [ὅπερ ἔδει δεῖξαι.]

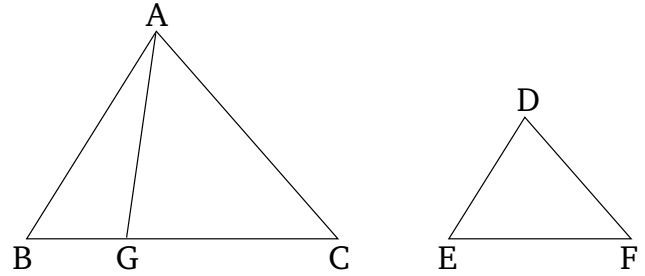
Πόρισμα.

Ἐκ δὲ τούτου φανερόν, ὅτι, ἐὰν τρεῖς εὐθεῖαι ἀνάλογον ὦσιν, ἐστὶν ὡς ἡ πρώτη πρὸς τὴν τρίτην, οὕτως τὸ ἀπὸ

very thing it was required to do.

Proposition 19

Similar triangles are to one another in the squared[†] ratio of (their) corresponding sides.



Let ABC and DEF be similar triangles having the angle at B equal to the (angle) at E , and AB to BC , as DE (is) to EF , such that BC corresponds to EF . I say that triangle ABC has a squared ratio to triangle DEF with respect to (that side) BC (has) to EF .

For let a third (straight-line), BG , have been taken (which is) proportional to BC and EF , so that as BC (is) to EF , so EF (is) to BG [Prop. 6.11]. And let AG have been joined.

Therefore, since as AB is to BC , so DE (is) to EF , thus, alternately, as AB is to DE , so BC (is) to EF [Prop. 5.16]. But, as BC (is) to EF , so EF is to BG . And, thus, as AB (is) to DE , so EF (is) to BG . Thus, for triangles ABG and DEF , the sides about the equal angles are reciprocally proportional. And those triangles having one (angle) equal to one (angle) for which the sides about the equal angles are reciprocally proportional are equal [Prop. 6.15]. Thus, triangle ABG is equal to triangle DEF . And since as BC (is) to EF , so EF (is) to BG , and if three straight-lines are proportional then the first has a squared ratio to the third with respect to the second [Def. 5.9], BC thus has a squared ratio to BG with respect to (that) CB (has) to EF . And as CB (is) to BG , so triangle ABC (is) to triangle ABG [Prop. 6.1]. Thus, triangle ABC also has a squared ratio to (triangle) ABG with respect to (that side) BC (has) to EF . And triangle ABG (is) equal to triangle DEF . Thus, triangle ABC also has a squared ratio to triangle DEF with respect to (that side) BC (has) to EF .

Thus, similar triangles are to one another in the squared ratio of (their) corresponding sides. [(Which is) the very thing it was required to show].

Corollary

So it is clear, from this, that if three straight-lines are proportional, then as the first is to the third, so the figure

τῆς πρώτης εἶδος πρὸς τὸ ἀπὸ τῆς δευτέρας τὸ ὅμοιον καὶ ὁμοίως ἀναγραφόμενον. ὅπερ ἔδει δεῖξαι.

(described) on the first (is) to the similar, and similarly described, (figure) on the second. (Which is) the very thing it was required to show.

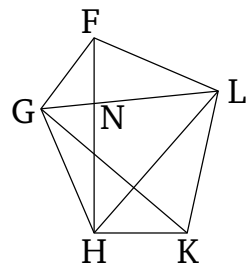
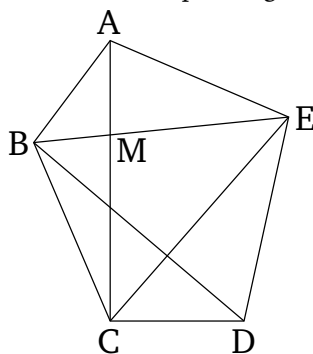
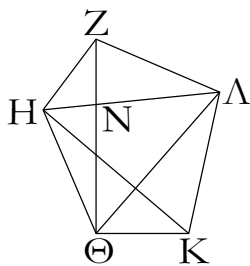
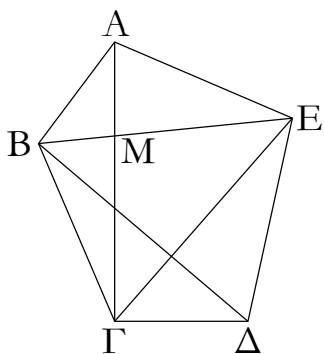
† Literally, “double”.

κ'.

Proposition 20

Τὰ ὅμοια πολύγωνα εἰς τε ὅμοια τρίγωνα διαιρεῖται καὶ εἰς ἴσα τὸ πλῆθος καὶ ὁμόλογα τοῖς ὅλοις, καὶ τὸ πολύγωνον πρὸς τὸ πολύγωνον διπλασίονα λόγον ἔχει ἥπερ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν.

Similar polygons can be divided into equal numbers of similar triangles corresponding (in proportion) to the wholes, and one polygon has to the (other) polygon a squared ratio with respect to (that) a corresponding side (has) to a corresponding side.



Ἐστω ὅμοια πολύγωνα τὰ ABΓΔΕ, ΖΗΘΚΛ, ὁμόλογος δὲ ἔστω ἡ AB τῇ ΖΗ· λέγω, ὅτι τὰ ABΓΔΕ, ΖΗΘΚΛ πολύγωνα εἰς τε ὅμοια τρίγωνα διαιρεῖται καὶ εἰς ἴσα τὸ πλῆθος καὶ ὁμόλογα τοῖς ὅλοις, καὶ τὸ ABΓΔΕ πολύγωνον πρὸς τὸ ΖΗΘΚΛ πολύγωνον διπλασίονα λόγον ἔχει ἥπερ ἡ AB πρὸς τὴν ΖΗ.

Ἐπεξεύχθωσαν αἱ BE, EΓ, ΗΛ, ΛΘ.

Καὶ ἐπεὶ ὁμοίον ἐστὶ τὸ ABΓΔΕ πολύγωνον τῷ ΖΗΘΚΛ πολυγώνῳ, ἴση ἐστὶν ἡ ὑπὸ BAE γωνία τῇ ὑπὸ HZΛ. καὶ ἐστὶν ὡς ἡ BA πρὸς AE, οὕτως ἡ HZ πρὸς ZΛ. ἐπεὶ οὖν δύο τρίγωνα ἐστὶ τὰ ABE, ZHL μίαν γωνίαν μὲν γωνίᾳ ἴσην ἔχοντα, περὶ δὲ τὰς ἴσας γωνίας τὰς πλευρὰς ἀνάλογον, ἰσογώνιον ἄρα ἐστὶ τὸ ABE τρίγωνον τῷ ZHL τριγώνῳ· ὥστε καὶ ὁμοίον· ἴση ἄρα ἐστὶν ἡ ὑπὸ ABE γωνία τῇ ὑπὸ ZHL. ἔστι δὲ καὶ ὅλη ἡ ὑπὸ ABΓ ὅλη τῇ ὑπὸ ZHΘ ἴση διὰ τὴν ὁμοιότητα τῶν πολυγώνων· λοιπὴ ἄρα ἡ ὑπὸ EBF γωνία τῇ ὑπὸ ΛΗΘ ἐστὶν ἴση. καὶ ἐπεὶ διὰ τὴν ὁμοιότητα τῶν ABE, ZHL τριγώνων ἐστὶν ὡς ἡ EB πρὸς BA, οὕτως ἡ ΛΗ πρὸς HZ, ἀλλὰ μὴν καὶ διὰ τὴν ὁμοιότητα τῶν πολυγώνων ἐστὶν ὡς ἡ AB πρὸς BΓ, οὕτως ἡ ΖΗ πρὸς ΗΘ, δι' ἴσου ἄρα ἐστὶν ὡς ἡ EB πρὸς BΓ, οὕτως ἡ ΛΗ πρὸς ΗΘ, καὶ περὶ τὰς ἴσας γωνίας τὰς ὑπὸ EBF, ΛΗΘ αἱ πλευραὶ ἀνάλογόν εἰσιν· ἰσογώνιον ἄρα ἐστὶ τὸ EBF τρίγωνον τῷ ΛΗΘ τριγώνῳ· ὥστε καὶ ὁμοίον ἐστὶ τὸ EBF τρίγωνον τῷ ΛΗΘ τριγώνῳ. διὰ τὰ αὐτὰ δὲ καὶ τὸ EΓΔ τρίγωνον ὁμοίον ἐστὶ τῷ ΛΘΚ τριγώνῳ. τὰ ἄρα ὅμοια πολύγωνα τὰ ABΓΔΕ, ΖΗΘΚΛ εἰς τε ὅμοια τρίγωνα διήρηται καὶ εἰς ἴσα

Let $ABCDE$ and $FGHKL$ be similar polygons, and let AB correspond to FG . I say that polygons $ABCDE$ and $FGHKL$ can be divided into equal numbers of similar triangles corresponding (in proportion) to the wholes, and (that) polygon $ABCDE$ has a squared ratio to polygon $FGHKL$ with respect to that AB (has) to FG .

Let BE , EC , GL , and LH have been joined.

And since polygon $ABCDE$ is similar to polygon $FGHKL$, angle BAE is equal to angle GFL , and as BA is to AE , so GF (is) to FL [Def. 6.1]. Therefore, since ABE and FGL are two triangles having one angle equal to one angle and the sides about the equal angles proportional, triangle ABE is thus equiangular to triangle FGL [Prop. 6.6]. Hence, (they are) also similar [Prop. 6.4, Def. 6.1]. Thus, angle ABE is equal to (angle) FGL . And the whole (angle) ABC is equal to the whole (angle) FGH , on account of the similarity of the polygons. Thus, the remaining angle EBC is equal to LGH . And since, on account of the similarity of triangles ABE and FGL , as EB is to BA , so LG (is) to GF , but also, on account of the similarity of the polygons, as AB is to BC , so FG (is) to GH , thus, via equality, as EB is to BC , so LG (is) to GH [Prop. 5.22], and the sides about the equal angles, EBC and LGH , are proportional. Thus, triangle EBC is equiangular to triangle LGH [Prop. 6.6]. Hence, triangle EBC is also similar to triangle LGH [Prop. 6.4, Def. 6.1]. So, for the same (reasons), triangle ECD is also similar

τὸ πλῆθος.

Λέγω, ὅτι καὶ ὁμόλογα τοῖς ὅλοις, τούτέστιν ὥστε ἀνάλογον εἶναι τὰ τρίγωνα, καὶ ἡγούμενα μὲν εἶναι τὰ ABE , $EBΓ$, $ΕΓΔ$, ἐπόμενα δὲ αὐτῶν τὰ ZHA , $ΛΗΘ$, $ΛΘΚ$, καὶ ὅτι τὸ $ABΓΔΕ$ πολύγωνον πρὸς τὸ $ZHΘΚΛ$ πολύγωνον διπλασίονα λόγον ἔχει ἥπερ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν, τούτέστιν ἡ AB πρὸς τὴν ZH .

Ἐπεξεύχθησαν γάρ αἱ $ΑΓ$, $ΖΘ$. καὶ ἐπεὶ διὰ τὴν ὁμοιότητα τῶν πολυγώνων ἴση ἐστὶν ἡ ὑπὸ $ABΓ$ γωνία τῇ ὑπὸ $ZHΘ$, καὶ ἐστὶν ὡς ἡ AB πρὸς $BΓ$, οὕτως ἡ ZH πρὸς $HΘ$, ἰσογώνιον ἐστὶ τὸ $ABΓ$ τρίγωνον τῷ $ZHΘ$ τριγώνῳ· ἴση ἄρα ἐστὶν ἡ μὲν ὑπὸ $BAΓ$ γωνία τῇ ὑπὸ $HΖΘ$, ἡ δὲ ὑπὸ $BΓΑ$ τῇ ὑπὸ $HΘΖ$. καὶ ἐπεὶ ἴση ἐστὶν ἡ ὑπὸ BAM γωνία τῇ ὑπὸ HZN , ἐστὶ δὲ καὶ ἡ ὑπὸ ABM τῇ ὑπὸ ZHN ἴση, καὶ λοιπὴ ἄρα ἡ ὑπὸ AMB λοιπὴ τῇ ὑπὸ ZNH ἴση ἐστὶν· ἰσογώνιον ἄρα ἐστὶ τὸ ABM τρίγωνον τῷ ZHN τριγώνῳ. ὁμοίως δὲ δεῖξομεν, ὅτι καὶ τὸ $BMΓ$ τρίγωνον ἰσογώνιον ἐστὶ τῷ $HNΘ$ τριγώνῳ. ἀνάλογον ἄρα ἐστὶν, ὡς μὲν ἡ AM πρὸς MB , οὕτως ἡ ZN πρὸς NH , ὡς δὲ ἡ BM πρὸς $ΜΓ$, οὕτως ἡ HN πρὸς $NΘ$. ὥστε καὶ δι' ἴσου, ὡς ἡ AM πρὸς $ΜΓ$, οὕτως ἡ ZN πρὸς $NΘ$. ἀλλ' ὡς ἡ AM πρὸς $ΜΓ$, οὕτως τὸ ABM [τρίγωνον] πρὸς τὸ $MBΓ$, καὶ τὸ AME πρὸς τὸ $EMΓ$. πρὸς ἀλλήλα γάρ εἰσιν ὡς αἱ βάσεις. καὶ ὡς ἄρα ἐν τῶν ἡγουμένων πρὸς ἐν τῶν ἐπομένων, οὕτως ἅπαντα τὰ ἡγούμενα πρὸς ἅπαντα τὰ ἐπόμενα· ὡς ἄρα τὸ AMB τρίγωνον πρὸς τὸ $BMΓ$, οὕτως τὸ ABE πρὸς τὸ $ΓBE$. ἀλλ' ὡς τὸ AMB πρὸς τὸ $BMΓ$, οὕτως ἡ AM πρὸς $ΜΓ$. καὶ ὡς ἄρα ἡ AM πρὸς $ΜΓ$, οὕτως τὸ ABE τρίγωνον πρὸς τὸ $EBΓ$ τρίγωνον. διὰ τὰ αὐτὰ δὲ καὶ ὡς ἡ ZN πρὸς $NΘ$, οὕτως τὸ ZHA τρίγωνον πρὸς τὸ $ΗΛΘ$ τρίγωνον. καὶ ἐστὶν ὡς ἡ AM πρὸς $ΜΓ$, οὕτως ἡ ZN πρὸς $NΘ$. καὶ ὡς ἄρα τὸ ABE τρίγωνον πρὸς τὸ $BEΓ$ τρίγωνον, οὕτως τὸ ZHA τρίγωνον πρὸς τὸ $ΗΛΘ$ τρίγωνον, καὶ ἐναλλάξ ὡς τὸ ABE τρίγωνον πρὸς τὸ ZHA τρίγωνον, οὕτως τὸ $BEΓ$ τρίγωνον πρὸς τὸ $ΗΛΘ$ τρίγωνον. ὁμοίως δὲ δεῖξομεν ἐπιzeugηθεῖσιν τῶν $BΔ$, HK , ὅτι καὶ ὡς τὸ $BEΓ$ τρίγωνον πρὸς τὸ $ΛΗΘ$ τρίγωνον, οὕτως τὸ $ΕΓΔ$ τρίγωνον πρὸς τὸ $ΛΘΚ$ τρίγωνον. καὶ ἐπεὶ ἐστὶν ὡς τὸ ABE τρίγωνον πρὸς τὸ ZHA τρίγωνον, οὕτως τὸ $EBΓ$ πρὸς τὸ $ΛΗΘ$, καὶ ἔτι τὸ $ΕΓΔ$ πρὸς τὸ $ΛΘΚ$, καὶ ὡς ἄρα ἐν τῶν ἡγουμένων πρὸς ἐν τῶν ἐπομένων, οὕτως ἅπαντα τὰ ἡγούμενα πρὸς ἅπαντα τὰ ἐπόμενα· ἐστὶν ἄρα ὡς τὸ ABE τρίγωνον πρὸς τὸ ZHA τρίγωνον, οὕτως τὸ $ABΓΔΕ$ πολύγωνον πρὸς τὸ $ZHΘΚΛ$ πολύγωνον. ἀλλὰ τὸ ABE τρίγωνον πρὸς τὸ ZHA τρίγωνον διπλασίονα λόγον ἔχει ἥπερ ἡ AB ὁμόλογος πλευρὰ πρὸς τὴν ZH ὁμόλογον πλευράν· τὰ γὰρ ὅμοια τρίγωνα ἐν διπλασίονι λόγῳ ἐστὶ τῶν ὁμολόγων πλευρῶν. καὶ τὸ $ABΓΔΕ$ ἄρα πολύγωνον πρὸς τὸ $ZHΘΚΛ$ πολύγωνον διπλασίονα λόγον ἔχει ἥπερ ἡ AB ὁμόλογος πλευρὰ πρὸς τὴν ZH ὁμόλογον πλευράν.

Τὰ ἄρα ὅμοια πολύγωνα εἰς τε ὅμοια τρίγωνα διαιρεῖται καὶ εἰς ἴσα τὸ πλῆθος καὶ ὁμόλογα τοῖς ὅλοις, καὶ τὸ

to triangle LHK . Thus, the similar polygons $ABCDE$ and $FGHKL$ have been divided into equal numbers of similar triangles.

I also say that (the triangles) correspond (in proportion) to the wholes. That is to say, the triangles are proportional: ABE , EBC , and ECD are the leading (magnitudes), and their (associated) following (magnitudes are) FGL , LGH , and LHK (respectively). (I) also (say) that polygon $ABCDE$ has a squared ratio to polygon $FGHKL$ with respect to (that) a corresponding side (has) to a corresponding side—that is to say, (side) AB to FG .

For let AC and FH have been joined. And since angle ABC is equal to FGH , and as AB is to BC , so FG (is) to GH , on account of the similarity of the polygons, triangle ABC is equiangular to triangle FGH [Prop. 6.6]. Thus, angle BAC is equal to GFH , and (angle) BCA to GHF . And since angle BAM is equal to GFN , and (angle) ABM is also equal to FGN (see earlier), the remaining (angle) AMB is thus also equal to the remaining (angle) FNG [Prop. 1.32]. Thus, triangle ABM is equiangular to triangle FGN . So, similarly, we can show that triangle BMC is also equiangular to triangle GNH . Thus, proportionally, as AM is to MB , so FN (is) to NG , and as BM (is) to MC , so GN (is) to NH [Prop. 6.4]. Hence, also, via equality, as AM (is) to MC , so FN (is) to NH [Prop. 5.22]. But, as AM (is) to MC , so [triangle] ABM is to MBC , and AME to EMC . For they are to one another as their bases [Prop. 6.1]. And as one of the leading (magnitudes) is to one of the following (magnitudes), so (the sum of) all the leading (magnitudes) is to (the sum of) all the following (magnitudes) [Prop. 5.12]. Thus, as triangle AMB (is) to BMC , so (triangle) ABE (is) to CBE . But, as (triangle) AMB (is) to BMC , so AM (is) to MC . Thus, also, as AM (is) to MC , so triangle ABE (is) to triangle EBC . And so, for the same (reasons), as FN (is) to NH , so triangle FGL (is) to triangle GLH . And as AM is to MC , so FN (is) to NH . Thus, also, as triangle ABE (is) to triangle BEC , so triangle FGL (is) to triangle GLH , and, alternately, as triangle ABE (is) to triangle FGL , so triangle BEC (is) to triangle GLH [Prop. 5.16]. So, similarly, we can also show, by joining BD and GK , that as triangle BEC (is) to triangle LGH , so triangle ECD (is) to triangle LHK . And since as triangle ABE is to triangle FGL , so (triangle) EBC (is) to LGH , and, further, (triangle) ECD to LHK , and also as one of the leading (magnitudes) is to one of the following, so (the sum of) all the leading (magnitudes) is to (the sum of) all the following [Prop. 5.12], thus as triangle ABE is to triangle FGL , so polygon $ABCDE$ (is) to polygon $FGHKL$. But, triangle ABE has a squared ratio

πολύγωνον πρὸς τὸ πολύγωνον διπλασίονα λόγον ἔχει ἢ περ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν [ὅπερ ἔδει δεῖξαι].

to triangle FGL with respect to (that) the corresponding side AB (has) to the corresponding side FG . For, similar triangles are in the squared ratio of corresponding sides [Prop. 6.14]. Thus, polygon $ABCDE$ also has a squared ratio to polygon $FGHKL$ with respect to (that) the corresponding side AB (has) to the corresponding side FG .

Thus, similar polygons can be divided into equal numbers of similar triangles corresponding (in proportion) to the wholes, and one polygon has to the (other) polygon a squared ratio with respect to (that) a corresponding side (has) to a corresponding side. [(Which is) the very thing it was required to show].

Πόρισμα.

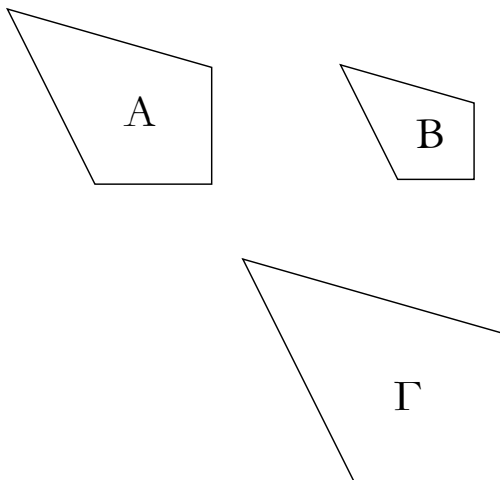
Ὡσαύτως δὲ καὶ ἐπὶ τῶν [ὁμοίων] τετραπλεύρων δειχθήσεται, ὅτι ἐν διπλασίονι λόγῳ εἰσὶ τῶν ὁμολόγων πλευρῶν. ἐδείχθη δὲ καὶ ἐπὶ τῶν τριγώνων· ὥστε καὶ καθόλου τὰ ὅμοια εὐθύγραμμα σχήματα πρὸς ἄλληλα ἐν διπλασίονι λόγῳ εἰσὶ τῶν ὁμολόγων πλευρῶν. ὅπερ ἔδει δεῖξαι.

Corollary

And, in the same manner, it can also be shown for [similar] quadrilaterals that they are in the squared ratio of (their) corresponding sides. And it was also shown for triangles. Hence, in general, similar rectilinear figures are also to one another in the squared ratio of (their) corresponding sides. (Which is) the very thing it was required to show.

κα'.

Τὰ τῶ αὐτῶ εὐθυγράμμω ὅμοια καὶ ἀλλήλοις ἐστὶν ὅμοια.

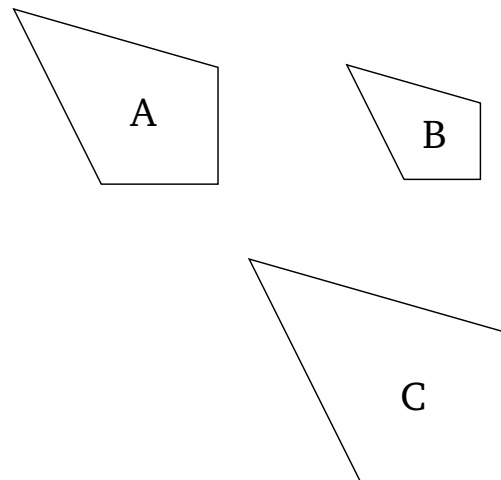


Ἐστω γὰρ ἑκάτερον τῶν A , B εὐθυγράμμων τῶ Γ ὁμοιον· λέγω, ὅτι καὶ τὸ A τῶ B ἐστὶν ὁμοιον.

Ἐπεὶ γὰρ ὁμοιον ἐστὶ τὸ A τῶ Γ , ἰσογώνιον τέ ἐστὶν αὐτῶ καὶ τὰς περὶ τὰς ἴσας γωνίας πλευρὰς ἀνάλογον ἔχει. πάλιν, ἐπεὶ ὁμοιον ἐστὶ τὸ B τῶ Γ , ἰσογώνιον τέ ἐστὶν αὐτῶ καὶ τὰς περὶ τὰς ἴσας γωνίας πλευρὰς ἀνάλογον ἔχει. ἑκάτερον ἄρα τῶν A , B τῶ Γ ἰσογώνιον τέ ἐστὶ καὶ τὰς περὶ τὰς ἴσας γωνίας πλευρὰς ἀνάλογον ἔχει [ὥστε καὶ τὸ A τῶ B ἰσογώνιον τέ ἐστὶ καὶ τὰς περὶ τὰς ἴσας γωνίας

Proposition 21

(Rectilinear figures) similar to the same rectilinear figure are also similar to one another.



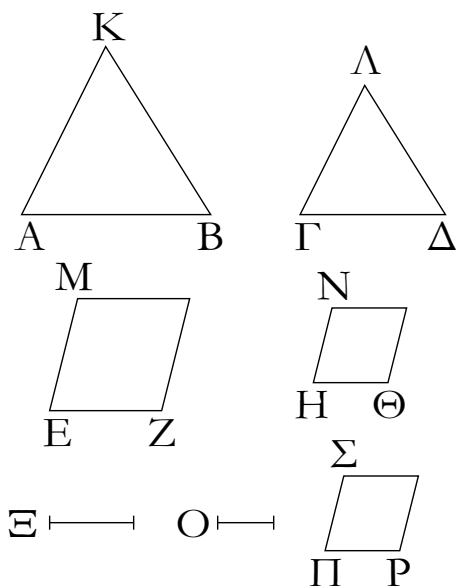
Let each of the rectilinear figures A and B be similar to (the rectilinear figure) C . I say that A is also similar to B .

For since A is similar to C , (A) is equiangular to (C), and has the sides about the equal angles proportional [Def. 6.1]. Again, since B is similar to C , (B) is equiangular to (C), and has the sides about the equal angles proportional [Def. 6.1]. Thus, A and B are each equiangular to C , and have the sides about the equal angles

πλευρὰς ἀνάλογον ἔχει]. ὁμοιον ἄρα ἐστὶ τὸ A τῷ B · ὅπερ ἔδει δεῖξαι.

κβ'.

Ἐάν τέσσαρες εὐθεῖαι ἀνάλογον ᾧσιν, καὶ τὰ ἀπ' αὐτῶν εὐθύγραμμα ὁμοία τε καὶ ὁμοίως ἀναγεγραμμένα ἀνάλογον ἔσται· καὶ τὰ ἀπ' αὐτῶν εὐθύγραμμα ὁμοία τε καὶ ὁμοίως ἀναγεγραμμένα ἀνάλογον ἦ, καὶ αὐτὰ αἱ εὐθεῖαι ἀνάλογον ἔσονται.



Ἐστωσαν τέσσαρες εὐθεῖαι ἀνάλογον αἱ AB , $\Gamma\Delta$, EZ , $H\Theta$, ὡς ἡ AB πρὸς τὴν $\Gamma\Delta$, οὕτως ἡ EZ πρὸς τὴν $H\Theta$, καὶ ἀναγεγράφθωσαν ἀπὸ μὲν τῶν AB , $\Gamma\Delta$ ὁμοία τε καὶ ὁμοίως κείμενα εὐθύγραμμα τὰ KAB , $\Lambda\Gamma\Delta$, ἀπὸ δὲ τῶν EZ , $H\Theta$ ὁμοία τε καὶ ὁμοίως κείμενα εὐθύγραμμα τὰ MZ , $N\Theta$ · λέγω, ὅτι ἐστὶν ὡς τὸ KAB πρὸς τὸ $\Lambda\Gamma\Delta$, οὕτως τὸ MZ πρὸς τὸ $N\Theta$.

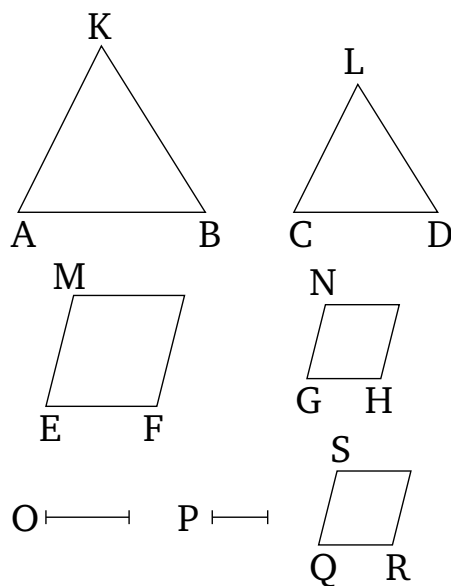
Εἰλήφθω γὰρ τῶν μὲν AB , $\Gamma\Delta$ τρίτη ἀνάλογον ἡ Ξ , τῶν δὲ EZ , $H\Theta$ τρίτη ἀνάλογον ἡ O . καὶ ἐπεὶ ἐστὶν ὡς μὲν ἡ AB πρὸς τὴν $\Gamma\Delta$, οὕτως ἡ EZ πρὸς τὴν $H\Theta$, ὡς δὲ ἡ $\Gamma\Delta$ πρὸς τὴν Ξ , οὕτως ἡ $H\Theta$ πρὸς τὴν O , δι' ἴσου ἄρα ἐστὶν ὡς ἡ AB πρὸς τὴν Ξ , οὕτως ἡ EZ πρὸς τὴν O . ἀλλ' ὡς μὲν ἡ AB πρὸς τὴν Ξ , οὕτως [καὶ] τὸ KAB πρὸς τὸ $\Lambda\Gamma\Delta$, ὡς δὲ ἡ EZ πρὸς τὴν O , οὕτως τὸ MZ πρὸς τὸ $N\Theta$ · καὶ ὡς ἄρα τὸ KAB πρὸς τὸ $\Lambda\Gamma\Delta$, οὕτως τὸ MZ πρὸς τὸ $N\Theta$.

Ἀλλὰ δὴ ἔστω ὡς τὸ KAB πρὸς τὸ $\Lambda\Gamma\Delta$, οὕτως τὸ MZ πρὸς τὸ $N\Theta$ · λέγω, ὅτι ἐστὶ καὶ ὡς ἡ AB πρὸς τὴν $\Gamma\Delta$, οὕτως ἡ EZ πρὸς τὴν $H\Theta$. εἰ γὰρ μὴ ἐστὶν, ὡς ἡ AB πρὸς τὴν $\Gamma\Delta$, οὕτως ἡ EZ πρὸς τὴν $H\Theta$, ἔστω ὡς ἡ AB πρὸς τὴν $\Gamma\Delta$, οὕτως ἡ EZ πρὸς τὴν $\Pi\Gamma$, καὶ ἀναγεγράφθω ἀπὸ τῆς

proportional [hence, A is also equiangular to B , and has the sides about the equal angles proportional]. Thus, A is similar to B [Def. 6.1]. (Which is) the very thing it was required to show.

Proposition 22

If four straight-lines are proportional then similar, and similarly described, rectilinear figures (drawn) on them will also be proportional. And if similar, and similarly described, rectilinear figures (drawn) on them are proportional then the straight-lines themselves will also be proportional.



Let AB , CD , EF , and GH be four proportional straight-lines, (such that) as AB (is) to CD , so EF (is) to GH . And let the similar, and similarly laid out, rectilinear figures KAB and LCD have been described on AB and CD (respectively), and the similar, and similarly laid out, rectilinear figures MF and NH on EF and GH (respectively). I say that as KAB is to LCD , so MF (is) to NH .

For let a third (straight-line) O have been taken (which is) proportional to AB and CD , and a third (straight-line) P proportional to EF and GH [Prop. 6.11]. And since as AB is to CD , so EF (is) to GH , and as CD (is) to O , so GH (is) to P , thus, via equality, as AB is to O , so EF (is) to P [Prop. 5.22]. But, as AB (is) to O , so [also] KAB (is) to LCD , and as EF (is) to P , so MF (is) to NH [Prop. 5.19 corr.]. And, thus, as KAB (is) to LCD , so MF (is) to NH .

And so let KAB be to LCD , as MF (is) to NH . I say also that as AB is to CD , so EF (is) to GH . For if as AB is to CD , so EF (is) not to GH , let AB be to CD , as EF