$(2)(11) - (3)(10) + (4)(9) = 2a\mathfrak{n}'\mathfrak{n}''\mathfrak{B}$, $(2)(3) - (1)(4) = a\mathfrak{n}'\mathfrak{n}''\mathfrak{C}$, $-(9)(16) + (10\,(15) + (11)(14) - (12)(13) = 2b\mathfrak{n}'\mathfrak{n}''\mathfrak{A}$, $(1)(16) - (2)(15) - (3)(14) + (4)(13) + (5)(12) - (6)(11) - (7)(10) + (8)(9) = 4b\mathfrak{n}'\mathfrak{n}''\mathfrak{B}$, $-(1)(8) + (2)(7) + (3)(6) - (4)(5) = 2b\mathfrak{n}'\mathfrak{n}''\mathfrak{C}$, $(14)(15) - (13)(16) = c\mathfrak{n}'\mathfrak{n}''\mathfrak{A}$, $(5)(16) - (6)(15) - (7)(14) + (8)(13) = 2c\mathfrak{n}'\mathfrak{n}''\mathfrak{B}$, $(6)(7) - (5)(8) = c\mathfrak{n}'\mathfrak{n}''\mathfrak{C}$, quas designabimus per Ψ *).

IV. Originem omnium harum 37 aequationum deducere nimis prolixum foret: sufficiet quasdam confirmauisse, ad quarum instar reliquae haud difficulter demonstrari poterunt.

1) Habetur $(1,2) = (1)(10) - (2)(9) = (\mathfrak{p}\mathfrak{q}' - \mathfrak{q}\mathfrak{p}')pp + (\mathfrak{p}\mathfrak{q}''' - \mathfrak{q}\mathfrak{p}''' - \mathfrak{p}'\mathfrak{q}'' + \mathfrak{q}'\mathfrak{p}'')pq + (\mathfrak{p}''\mathfrak{q}''' - \mathfrak{q}''\mathfrak{p}''')qq = \mathfrak{n}''(App + 2Bpq + Cqq) = \mathfrak{n}''aa'$, quae est aequ. prima.

2) Fit $(1,3) = (1)(11) - (3)(9) = (\mathfrak{p}\mathfrak{q}'' - \mathfrak{q}\mathfrak{p}'')(\mathfrak{p}\mathfrak{q}' - \mathfrak{q}\mathfrak{p}') = a''\mathfrak{N}an' = aa''\mathfrak{n}'$, aequ. secunda.

3) Erit $(1,8) = (1)(16) - (8)(9) = (\mathfrak{p}\mathfrak{q}' - \mathfrak{q}\mathfrak{p}')pp''' + (\mathfrak{p}\mathfrak{q}''' - \mathfrak{q}\mathfrak{p}''')pq''' - (\mathfrak{p}'\mathfrak{q}'' - \mathfrak{q}'\mathfrak{p}'')qp''' + (\mathfrak{p}''\mathfrak{q}''' - \mathfrak{q}''\mathfrak{p}''')qq''' = \mathfrak{n}''(App''' + B(pq''' + qp''') + Cqq''') + b''\mathfrak{N}(pq''' - qp''') = \mathfrak{n}''(bb' + \sqrt{dd'}) + b''\mathfrak{N}(bn + b'n')$ **)

*) Obseruare conuenit, 18 alias aequationes his Ψ similes erui posse, in quibus ad dextram loco factorum a, $2b$, c habeantur a', $2b'$, c'; a'', $2b''$, c'': sed has quum ad institutum nostrum non sint necessariae omittimus.

**) Hoc sequitur ex aequ. 10 art. 235 et sqq. Quantitas radicalis $\sqrt{dd'}$ fit $= Dnn' = \mathfrak{D}nn'\mathfrak{N}\mathfrak{N} = \mathfrak{D}\mathfrak{n}\mathfrak{n}'$.

$= \mathfrak{n}''bb' + \mathfrak{n}'bb'' + \mathfrak{n}b'b'' + \mathfrak{D}\mathfrak{n}\mathfrak{n}'\mathfrak{n}''$, aequatio octaua in Φ. Aequationes reliquas lectoribus confirmandas linquimus.

V. Ex aequatt. Φ sequitur, viginti octo numeros $(1, 2)$, $(1, 3)$ etc. nullum diuisorem communem habere, sequenti modo. Primo obseruamus, viginti septem producta e ternis factoribus, quorum primus vel $\mathfrak{n}$, secundus aliquis numerorum a', $2b'$, c', tertiusque aliquis numerorum a'', $2b''$, c''; vel primus $\mathfrak{n}'$, secundus aliquis e numeris a, $2b$, c, tertius aliquis numerorum a'', $2b''$, c''; vel denique primus $\mathfrak{n}''$, secundus aliquis numerorum a, $2b$, c tertiusque aliquis e numeris a', $2b'$, c' — singula haec viginti septem producta propter aequatt. Φ aequalia esse vel alicui ex viginti octo numeris $(1, 2)$, $(1, 3)$ etc. vel plurium summae aut differentiae, (e. g. $\mathfrak{n}a'a'' = (1, 5)$, $2\mathfrak{n}a'b'' = (1, 6) + (2, 5)$, $4\mathfrak{n}b'b'' = (1, 8) + (2, 7) + (3, 6) + (4, 5)$, et sic de reliquis); quamobrem si hi numeri diuisorem communem haberent, hic necessario etiam omnia illa producta metiri deberet. Hinc vero facile deducitur adiumento art. 40 et per methodum saepius in praecedentibus adhibitam, eundem diuisorem etiam numeros $\mathfrak{n}m'm''$, $\mathfrak{n}'mm''$, $\mathfrak{n}''mm'$ metiri debere, adeoque horum quadrata quae sunt $\frac{dm'm'm''m''}{\mathfrak{D}}$, $\frac{d'mmm''m''}{\mathfrak{D}}$, $\frac{d''mmm'm'}{\mathfrak{D}}$ per illius quadratum diuisibilia esse, *Q. E. A.*, quoniam per I trium numeratorum diuisor communis maximus est $\mathfrak{D}$, adeoque quadrata ipsa diuisorem communem habere nequeunt.

VI. Haec omnia pertinent ad transformationem formae f in $ff'f''$; et ex transformationibus formae F in ff' formaeque $\mathfrak{F}$ in Ff'' deducta sunt. Sed prorsus simili modo e transformationibus formae F' in ff'' formaeque $\mathfrak{F}$ in $F'f'$ deriuabitur transformatio formae $\mathfrak{F}'$ in $ff'f''$ talis: $\mathfrak{X}' = (1)'xx'x'' + (2)'xx'y'' + (3)'xy'x'' +$ etc., $\mathfrak{Y}' = (9)'xx'x'' + (10)'xx'y'' +$ etc. (designando omnes coefficientes similiter vt in transformatione formae $\mathfrak{F}$ in $ff'f''$, singulisque distinctionis caussa lineolam affigendo), ex qua perinde vt ante 28 aequationes ipsis Φ analogae deducentur, quas per Φ' designabimus, nouemque aliae ipsis Ψ analogae, quas exprimemus per Ψ'. Scilicet denotando $(1)'(10)' - (2)'(9)'$ per $(1,2)'$, $(1)'(11)' - (3)'(9)'$ per $(1,3)'$ etc., aequationes Φ' erunt $(1,2)' = aa'\mathfrak{n}''$, $(1,3)' = ab'\mathfrak{n}'' + ab''\mathfrak{n}'$ etc.; aequationes Ψ' autem $(10)'(11)' - (9)'(12)' = a\mathfrak{n}'\mathfrak{n}''\mathfrak{A}'$ etc. (Euolutionem vberiorem breuitatis gratia lectoribus relinquimus; ceterum periti nouum calculum ne necessarium quidem esse, sed analysin primam per analogiam facile huc transferri posse inuenient). Quibus ita factis, ex Φ et Φ' statim sequitur $(1,2) = (1,2)'$, $(1,3) = (1,3)'$, $(1,4) = (1,4)'$, $(2,3) = (2,3)'$ etc.; hinc vero et inde quod omnes $(1,2)$, $(1,3)$, $(2,3)$ etc. diuisorem communem (per V) non habent, adiumento lemmatis art. 234 concluditur, quatuor numeros integros α, β, γ, δ ita determinari posse, vt fiat $\alpha(1)' + \beta(9)' = (1)$, $\alpha(2)' + \beta(10)' = (2)$, $\alpha(3)' + \beta(11)' = (3)$ etc.; $\gamma(1)' + \delta(9)' = (9)$, $\gamma(2)' + \delta(10)' = (10)$ etc., atque $\alpha\delta - \beta\gamma = 1$.

VII. Hinc atque substituendo ex tribus aequatt. primis Ψ valores ipsorum $a\mathfrak{A}$, $a\mathfrak{B}$, $a\mathfrak{C}$,