

the *directrix*. This ratio is called the *eccentricity*. Apollonius did not give this definition and it is doubtful whether he was aware of the eccentricity, which is used to classify conic sections as follows: ellipses have eccentricity  $< 1$  (in particular, a circle has eccentricity 0), parabolas have eccentricity  $= 1$ , and hyperbolas have eccentricity  $> 1$ .

Apollonius also wrote a treatise on 'Tangencies' in which he showed how to give a ruler and compass construction for a circle tangent to three given circles.

Eratosthenes of Cyrene (in North Africa) became the chief librarian at Alexandria. He was interested in many things: philosophy, poetry, history, philology, geography, astronomy and mathematics. We have already mentioned his sieve for constructing the list of primes. Eratosthenes also invented the Julian calendar, with every fourth year containing an extra day, and he calculated the size of the earth. Perhaps the reason his students called him 'beta' (the second letter of the Greek alphabet) was that, although he studied many different things, he never considered himself the leading expert in any one field. It is reported that, in his old age, Eratosthenes went blind and committed suicide by starvation.

Eratosthenes's greatest achievement was the measurement of the circumference of the earth. Eratosthenes correctly assumed that, since the sun is so far from the earth, those of its rays which hit the earth can be regarded as parallel. (Here he used the result of Aristarchus.) Eratosthenes knew that Syene (present day Aswan) is almost exactly on the Tropic of Cancer, that is, at noon on midsummer's day (June 21), the sun is directly overhead, as could be witnessed from the bottom of a well. Eratosthenes observed that at Alexandria, at noon on midsummer's day, the sun was  $360^\circ/50$  from the point directly overhead. He argued that this same angle was subtended at the center of the earth by the arc joining Alexandria to Syene, which is due south of Alexandria. According to Euclid (theorem VI 33), the length of an arc of a circle is proportional to the angle it subtends at the center. So all Eratosthenes had to do was to measure the distance from Alexandria to Syene. This he found to be 5,000 stadia, a stadium being the length of the famous Olympic track. Eratosthenes concluded that the circumference of the earth is to 5,000 stadia as  $360^\circ$  is to  $360^\circ/50$ , and hence the circumference is  $5,000 \times 50 = 250,000$  stadia (Figure 18.2). As the Olympic stadium is about 180 meters, this would make the circumference of the earth about 45,000 kilometers.

Eratosthenes's calculation of the circumference of the earth is remarkably accurate. The correct value is almost exactly 40,000 km; in fact, the *kilometer* was originally defined as  $1/40,000$  of the circumference of the earth. Had Columbus known this, he might never have set out on his journey or called the inhabitants of the New World 'Indians'.

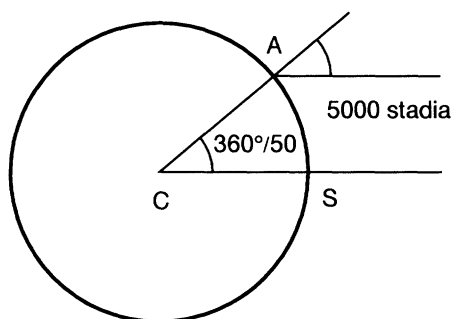


FIGURE 18.2. Circumference of the Earth in stadia

## Exercises

1. Obtain the equation of a conic section with focus  $(a, 0)$ , directrix the  $y$  axis and eccentricity  $e$ .
2. Suppose you know the actual size of the moon. What is a simple way of finding its distance from the earth — without using anything Eratosthenes could not have used?

# 19

## Archimedes

Archimedes (287–212 BC) was the greatest applied mathematician and physicist before Newton. Many stories are told about him. One story relates that, while he was taking a bath, Archimedes suddenly discovered a simple way of determining the ratio of gold to silver in a gold-silver alloy. Elated by his discovery, he leapt from the bath, and ran through the streets of Syracuse, shouting ‘eureka!’ (which means ‘I have found it!’). Unfortunately, he had forgotten to get dressed.

It was no accident that Archimedes made his discovery in a bath. Suppose you have an object made of gold and silver weighing  $m$  ounces. Suppose you wish to determine the number  $x$  of ounces of gold which the goldsmith has put into it. If  $g$  is the density of gold and  $s$  is the density of silver, the volume of the object is  $(x/g) + (m - x)/s$ . What Archimedes realized was that, by immersing the object in a rectangular bath tub, and observing the increase in water level, you can easily determine its volume  $v$ . Solving for  $x$  in the equation

$$v = (x/g) + (m - x)/s,$$

you obtain the mass of gold in the object. Thanks to his mathematics, Archimedes was able to tell his friend, King Hieron of Syracuse, whether the goldsmith had cheated the king by charging him for pure gold, while, in fact, using a certain percentage of silver for his crown. Newton, the first modern physicist to surpass Archimedes, centuries later, was to perform a similar task in unmasking counterfeiters.

When Syracuse was besieged by a Roman army, Archimedes constructed various machines to help defend his city. As well as catapults and cross-