book he wrote on the use of Indian numerals with the title: 'Spoken has al-Khwarizmi ...', or, in the English translation of the Latin translation: 'Spoken has Algoritmi ...'.

Al-Khwarizmi's most important book was the *Hisab al-jabr w'al-muqabalah*, from which we get the word 'algebra'. The word 'al-jabra' means something like 'combining', as in combining the terms to solve an equation. The same root shows up in old Spanish as 'algebrista', a bone-setter, that is, one who joins together the parts of a broken bone.

Al-Khwarizmi's 'Algebra' was based on the work of Brahmagupta. Although it became extremely influential, it is not as interesting from a mathematical point of view as its fame would suggest. It contains nothing that was not known to the ancient Babylonians or ancient Greeks. There are few proofs, and one of them is woefully inadequate. This is al-Khwarizmi's 'proof' of the Theorem of Pythagoras, which only works if the right triangle is isosceles!

Al-Khwarizimi gives three approximations for π. None of them is supported by any reasoning, and al-Khwarizmi does not seem to think it matters which one is used. However, the book is extremely interesting as a source of sociological information. Many of the text problems deal with questions of inheritance according to Muslim religious law. (These problems appear to have been omitted in the Latin translation by Robert of Chester, which popularized al-Khwarizmi's work in Europe.) Here is an example:

> A man, in his illness before his death, makes someone a present of a slave girl, besides whom he has no property. Then he dies. The slave girl is worth 300 dirhams, and her dowry is 100 dirhams. The man to whom she has been presented cohabits with her. What is the legacy?

Here is how one is supposed to solve the problem:

$$300 - x - \frac{100}{300}x = 2x,$$

yielding $x = 90$ dirhams (Rosen [1831]).

Thabit Ibn-Qurra was an extraordinary polymath. He lived in Baghdad and was an active member of a neo-Pythagorean group called the Sabians. He wrote on politics, grammar, symbolism in Plato's *Republic*, smallpox, the anatomy of birds, the beam balance, the salinity of seawater, the sundial, Euclid's Parallel Postulate, cubic equations, the new crescent moon, etc.

Thabit believed there is an actual infinity (as opposed to Aristotle's potential infinity). He did work in spherical trigonometry and in what we now would call calculus. In his *Book on the Determination of Amicable Numbers*, he gave a wholly original rule:

> Let n be a positive integer greater than 1. Let $p = 3 \cdot 2^n - 1$, $q = 3 \cdot 2^{n-1} - 1$ and $r = 9 \cdot 2^{2n-1} - 1$. If p, q and r are primes, then $2^n pq$ and $2^n r$ are 'amicable' (that is, the sum of the proper divisors of each equals the other).

When $n = 2$, we get the amicable pair 220 and 284. When $n = 3$ (or any multiple of 3), r is divisible by 7, and so is not prime. However, when $n = 4$, we have another amicable pair, namely, 17296 and 18416.

Al-Khayyami wrote in Arabic on astronomy and mathematics. He revised the Julian calendar, approaching our Gregorian calendar in accuracy. In a book with the same title as that by al-Khwarizmi, he developed a geometric method for finding the positive real roots of the cubic and quartic equations. The idea was this: to solve the cubic equation

$$x^3 + ax^2 + b^2x + b^2c = 0,$$

one intersects the hyperbola $y = bc/x + b$ with the circle $(x + \frac{1}{2}(a+c))^2 + y^2 = \frac{1}{4}(a-c)^2$ and discards the point $(-c, 0)$.

As Omar Khayyam, he wrote a famous poem in Persian, the *Rubaiyat*. Its 19th century translation by Edward Fitzgerald is still a bestseller. It expounds a rather Epicurean philosophy, claiming that the most important thing is wine and the only sure thing is death:

> Oh, threats of Hell and Hopes of Paradise!
> One thing at least is – *This* Life flies;
> One thing is certain and the rest is Lies;
> The Flower that once has blown for ever dies.

(See LXIII in Appendix 1 of E. FitzGerald's translation of the *Rubaiyat* (London: Bernard Quaritch, 1859).) Omar was not popular with the religious establishment of his day, and even now many Persians prefer the more mystical poetry of his fellow countrymen Hafiz.

One of Omar's contemporaries, Nizam-ul-Mulk, wrote in his autobiography that as a youth he made a mutual assistance pact with two fellow students in Naishapur, namely, Omar Khayyam and Hasan Ben Sabbah. He himself later became vizier to two Sultans of the Seljuk dynasty and was able to carry out his pledge by helping Omar to a yearly pension. Unfortunately, so he recounts, Hasan was not satisfied with the government post offered to him and ultimately became the head of a religious sect, the Ismailis. He surrounded himself by a group of fanatics, called the 'assassins', this word being derived from 'hashish'. The sect exists today, though without the practice of assassination; its leader is the Agha Khan. (Some people deny this story as anti-Ismaili propaganda.)

Exercises

1. Show that 1184 and 1210 is an amicable pair not generated by Thabit's rule.

2. Find another amicable pair which is generated by Thabit's rule.

3. In Thabit's rule, prove that if n is a multiple of 3, then r is a multiple of 7.

4. Prove the following generalization of Thabit's rule: assuming that $p = (2^k + 1)2^{t+k} - 1$, $q = (2^k + 1)2^t - 1$ and $r = (2^k + 1)2^{2t+k} - 1$ are odd primes, then $a = 2^{t+k}pq$ and $b = 2^{t+k}r$ are amicable.

5. Let ABC be a triangle with $\angle A$ obtuse. Let B' and C' be in BC such that $\angle AB'B = \angle AC'C = \angle BAC$. Derive the following theorem of Thabit: $AB^2 + AC^2 = BC(BB' + CC')$.

6. Show that al-Khayyami's method will produce a real solution of the cubic equation in the text.