AWYVPQCTBLWYLPASQJWUPGBUSHFACELDLLDLWLBWAFAHS
EBYJXXACELWCJTQMARKDDLWCSXBUDLKDPLXSEQCJTNWPR
WSRGBCLWPGJEZIFWIMJDLLDAGCQMAYLTGLPPJXTWSGFRM
VTLGUYUXJAIGWHCPXQLTBXDPVTAGSGFVRZTWTGMMVFLXR
LDKWPRLWCSXPHDPLPKSHQGULMBZWGQAPQCTBAURZTWSHQ
MBCVXAGJJVGCSSGLIFWNQSXBFDGSHIWSFGLRZTWEPLSVC
VIFWNQSXBOWCFHMETRZXLYPPJXTWSGFRMVTRZTWHWMFTB
OPQZXLYIMFPLVWYVIFWDPAVGFPJETQKPEWGCSSRGIFWB

2. Find the inverses of the following matrices mod N. Write the entries in the inverse matrix as nonnegative integers less than N.

(a) $\begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix}$ mod 5 (b) $\begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix}$ mod 29 (c) $\begin{pmatrix} 15 & 17 \\ 4 & 9 \end{pmatrix}$ mod 26

(d) $\begin{pmatrix} 40 & 0 \\ 0 & 21 \end{pmatrix}$ mod 841 (e) $\begin{pmatrix} 197 & 62 \\ 603 & 271 \end{pmatrix}$ mod 841

In Exercises 3—5, find all solutions $\binom{x}{y}$ modulo N, writing x and y as nonnegative integers less than N.

3.

(a) $\begin{aligned} x + 4y &\equiv 1 \; mod \; 9 \\ 5x + 7y &\equiv 1 \; mod \; 9 \end{aligned}$ (b) $\begin{aligned} x + 4y &\equiv 1 \; mod \; 9 \\ 5x + 8y &\equiv 1 \; mod \; 9 \end{aligned}$

(c) $\begin{aligned} x + 4y &\equiv 1 \; mod \; 9 \\ 5x + 8y &\equiv 2 \; mod \; 9 \end{aligned}$ (d) $\begin{aligned} x + 4y &\equiv 0 \; mod \; 9 \\ 5x + 8y &\equiv 0 \; mod \; 9 \end{aligned}$

4.

(a) $\begin{aligned} 17x + 11y &\equiv 7 \; mod \; 29 \\ 13x + 10y &\equiv 8 \; mod \; 29 \end{aligned}$ (b) $\begin{aligned} 17x + 11y &\equiv 0 \; mod \; 29 \\ 13x + 10y &\equiv 0 \; mod \; 29 \end{aligned}$

(c) $\begin{aligned} 9x + 20y &\equiv 0 \; mod \; 29 \\ 16x + 13y &\equiv 0 \; mod \; 29 \end{aligned}$ (d) $\begin{aligned} 9x + 20y &\equiv 10 \; mod \; 29 \\ 16x + 13y &\equiv 21 \; mod \; 29 \end{aligned}$

(e) $\begin{aligned} 9x + 20y &\equiv 1 \; mod \; 29 \\ 16x + 13y &\equiv 2 \; mod \; 29 \end{aligned}$

5.

(a) $\begin{aligned} 480x + 971y &\equiv 416 \; mod \; 1111 \\ 297x + 398y &\equiv 319 \; mod \; 1111 \end{aligned}$ (b) $\begin{aligned} 480x + 971y &\equiv 109 \; mod \; 1111 \\ 297x + 398y &\equiv 906 \; mod \; 1111 \end{aligned}$

(c) $\begin{aligned} 480x + 971y &\equiv 0 \; mod \; 1111 \\ 297x + 398y &\equiv 0 \; mod \; 1111 \end{aligned}$ (d) $\begin{aligned} 480x + 971y &\equiv 0 \; mod \; 1111 \\ 298x + 398y &\equiv 0 \; mod \; 1111 \end{aligned}$

(e) $\begin{aligned} 480x + 971y &\equiv 648 \; mod \; 1111 \\ 298x + 398y &\equiv 1004 \; mod \; 1111 \end{aligned}$

6. The *Fibonacci* numbers can be defined by the rule $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_{n+1} = f_n + f_{n-1}$ for $n > 1$, or, equivalently, by means of the matrix equation