aliam huic similem datam esse $X = \alpha'x + \beta'y$, $Y = \gamma'x + \delta'y$, vt quid inde sequatur inuestigemus. Tum positis determinantibus formarum F, f, $= D$, d, atque $\alpha\delta - \beta\gamma = e$, $\alpha'\delta' - \beta'\gamma' = e'$, erit (art. 157), $d = Dee = De'e'$, et quum ex hyp. e, e' eadem signa habeant, $e = e'$, Habebuntur autem sequentes sex aequationes:

$$A\alpha\alpha + 2B\alpha\gamma + C\gamma\gamma = a. \ldots\ldots [1]$$
$$A\alpha'\alpha' + 2B\alpha'\gamma' + C\gamma'\gamma' = a. \ldots\ldots [2]$$
$$A\alpha\beta + B(\alpha\delta + \beta\gamma) + C\gamma\delta = b. \ldots\ldots [3]$$
$$A\alpha'\beta' + B(\alpha'\delta' + \beta'\gamma') + C\gamma'\delta' = b. \ldots\ldots [4]$$
$$A\beta\beta + 2B\beta\delta + C\delta\delta = c. \ldots\ldots [5]$$
$$A\beta'\beta' + 2B\beta'\delta' + C\delta'\delta' = c. \ldots\ldots [6]$$

Si breuitatis gratia numeros $A\alpha\alpha' + B(\alpha\gamma' + \gamma\alpha') + C\gamma\gamma'$, $A(\alpha\beta' + \beta\alpha') + B(\alpha\delta' + \beta\gamma' + \gamma\beta' + \delta\alpha') + C(\gamma\delta' + \delta\gamma')$, $A\beta\beta' + B(\beta\delta' + \delta\beta') + C\delta\delta'$ per a', $2b'$, c' designamus, ex aequ. praecc. sequentes nouas deducemus*):

$$a'a' - D(\alpha\gamma' - \gamma\alpha')^2 = aa. \ldots\ldots [7]$$
$$2a'b' - D(\alpha\gamma' - \gamma\alpha')(\alpha\delta' + \beta\gamma' - \gamma\beta' - \delta\alpha') = 2ab [8]$$
$$4b'b' - D((\alpha\delta' + \beta\gamma' - \gamma\beta' - \delta\alpha')^2 + 2ee') = 2bb + 2ac,$$
vnde fit, addendo $2Dee' = 2d = 2bb - 2ac$,
$$4b'b' - (\alpha\delta' + \beta\gamma' - \gamma\beta' - \delta\alpha')^2 = 4bb. \ldots [9]$$
$$a'c' - D(\alpha\delta' - \gamma\beta')(\beta\gamma' - \delta\alpha') = bb,$$

*) Origo harum aequationum haec est: 7 fit ex 1. 2 (i e. si aequatio (1) in aequationem (2) multiplicatur, siue potius, si illius pars prior in partem priorem huius multiplicatur, illiusque pars posterior in posteriorem huius, productaque aequalia ponuntur); 8 ex 1. 4 + 2. 3; sequens quae non est numerata ex 1. 6 + 2. 5 + 3. 4 + 3. 4; sequens non numerata ex 3. 4; 11 ex 3. 6 + 4. 5; 12 ex 5. 6. Simili designatione etiam in sequentibus semper vtemur. Euolutionem vero lectoribus relinquere debemus.

vnde subtrahendo $D(\alpha\delta - \beta\gamma)(\alpha'\delta' - \beta'\gamma') = bb - ac$, fit

$$a'c' - D(\alpha\gamma' - \gamma\alpha')(\beta\delta' - \delta\beta') = ac \quad \ldots \quad [10]$$
$$2b'c' - D(\alpha\delta' + \beta\gamma' - \gamma\beta' - \delta\alpha')(\beta\delta' - \delta\beta') = 2bc \quad [11]$$
$$c'c' - D(\beta\delta' - \delta\beta')^2 = cc \quad \ldots \quad [12]$$

Ponamus iam, diuisorem communem maximum numerorum a, $2b$, c esse m numerosque $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}$ ita determinatos vt fiat $\mathfrak{A}a + 2\mathfrak{B}b + \mathfrak{C}c = m$ (art. 40); multiplicentur aequationes 7, 8, 9, 10, 11, 12 resp. per $\mathfrak{A}\mathfrak{A}$, $2\mathfrak{A}\mathfrak{B}$, $\mathfrak{B}\mathfrak{B}$, $2\mathfrak{A}\mathfrak{C}$, $2\mathfrak{B}\mathfrak{C}$, $\mathfrak{C}\mathfrak{C}$ summenturque producta. Quodsi iam breuitatis caussa ponimus

$$\mathfrak{A}a' + 2\mathfrak{B}b' + \mathfrak{C}c' = T \quad \ldots \quad [13]$$
$$\mathfrak{A}(\alpha\gamma' - \gamma\alpha') + \mathfrak{B}(\alpha\delta' + \beta\gamma' - \gamma\beta' - \delta\alpha') + \mathfrak{C}(\beta\delta' - \delta\beta') = U \quad \ldots \quad [14]$$

vbi T, U manifesto erunt integri, prodibit:

$$TT - DUU = mm.$$

Deducti itaque sumus ad hanc conclusionem elegantem, *ex binis quibuscunque transformationibus similibus formae F in f sequi solutionem aequationis indeterminatae* $tt - Duu = mm$, *in integris*, scilicet $t = T$, $u = U$. Ceterum quum in ratiociniis nostris non supposuerimus, transformationes esse *diuersas*: *vna* adeo transformatio bis considerata solutionem praebere debet. Tum vero fit propter $\alpha' = \alpha$, $\beta' = \beta$ etc. $a' = a$, $b' = b$, $c' = c$, adeoque $T = m$, $U = 0$, quae solutio per se est obuia.

Iam primam transformationem solutionemque aequationis indeterminatae tamquam cognitas consideremus, et quomodo hinc altera transformatio deduci possit, siue quomodo α', β', γ', δ', ab his $\alpha, \beta, \gamma, \delta$, T, U pendeant, inuestigemus. Ad hunc finem multiplicamus primo aequationem [1] per $\delta\alpha' - \beta\gamma'$, [2] per $\alpha\delta' - \gamma\beta'$, [3] per $\alpha\gamma' - \gamma\alpha'$, [4] per $\gamma\alpha' - \alpha\gamma'$, addimusque producta, vnde prodibit:

$$(e + e')\, a' = (\alpha\delta' - \beta\gamma' - \gamma\beta' + \delta\alpha')\, a \;.\;.\; [15]$$

Simili modo fit ex $(\delta\beta' - \beta\delta')([1] - [2]) + (\alpha\delta' - \beta\gamma' - \gamma\beta' + \delta\alpha')([3] + [4]) + (\alpha\gamma' - \gamma\alpha')([5] - [6])$:

$$2(e + e')\, b' = 2(\alpha\delta' - \beta\gamma' - \gamma\beta' + \delta\alpha')\, b \;.\;.\;.\; [16]$$

Denique ex $(\delta\beta' - \beta\delta')([3] - [4]) + (\alpha\delta' - \gamma\beta')[5] + (\delta\alpha' - \beta\gamma')[6]$ prodit:

$$(e + e')\, c' = (\alpha\delta' - \beta\gamma' - \gamma\beta' + \delta\alpha')\, c \;.\;.\; [17]$$

Substituendo hos valores (15, 16, 17) in 13 fit:

$$(e + e')\, T = (\alpha\delta' - \beta\gamma' - \gamma\beta' + \delta\alpha')(\mathfrak{A}a + 2\mathfrak{B}b + \mathfrak{C}c), \text{ siue } 2eT = (\alpha\delta' - \beta\gamma' - \gamma\beta' + \delta\alpha')m \;.\;.\; [18]$$

vnde T multo facilius deduci potest, quam ex [13]. — Combinando hanc aequationem cum 15, 16, 17 obtinetur $ma' = Ta$, $2mb' = 2Tb$, $mc' = Tc$. Quos valores ipsorum a', $2b'$, c' in aequ. 7—12 substituendo et loco ipsius TT scribendo $mm + DUU$, transeunt illae post mutationes debitas in has:

$$(\alpha\gamma' - \gamma\alpha')^2 mm = aaUU$$
$$(\alpha\gamma' - \gamma\alpha')(\alpha\delta' + \beta\gamma' - \gamma\beta' - \delta\alpha')mm = 2abUU$$
$$(\alpha\delta' + \beta\gamma' - \gamma\beta' - \delta\alpha')^2 mm = 4bbUU$$
$$(\alpha\gamma' - \gamma\alpha')(\beta\delta' - \delta\beta')mm = acUU$$
$$(\alpha\delta' + \beta\gamma' - \gamma\beta' - \delta\alpha')(\beta\delta' - \delta\beta')mm = 2bcUU$$
$$(\beta\delta' - \delta\beta')^2 mm = ccUU$$