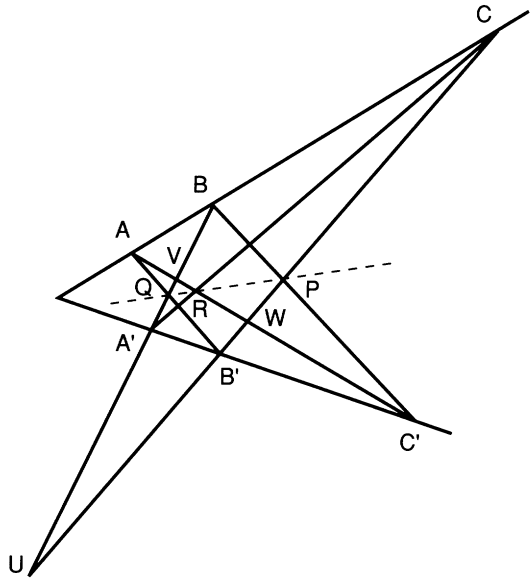

FIGURE 20.1. Pappus's Theorem

$$VA \cdot WC \cdot UB = AW \cdot CU \cdot BV.$$

Multiplying the first three equations and dividing by the product of the last two, we obtain

$$VR \cdot WP \cdot UQ = RW \cdot PU \cdot QV.$$

The result now follows by another application of Menelaus's Theorem. Note that the argument makes use of the commutativity of multiplication.

When CD and EF are parallel, the proof proceeds in a similar fashion. However, one uses, not Menelaus's Theorem, but the theory of similar triangles. We leave the details to the reader.

Among the minor mathematicians of this era was Nicomachus (100 AD) from Palestine. He was a Pythagorean, and he published a book on number theory, which is the basis for many of our speculations about the nature of 'Pythagorean' mathematics.

Hypatia (d. 415) was the daughter of Theon of Alexandria, who had put out an edition of Euclid's *Elements* and a commentary on Ptolemy's *Almagest.* Hypatia wrote commentaries on Apollonius and Diophantus.

According to Socrates Scholasticus (380–450 AD) in Chapter 15 of Book VII of his *History of the Church*, Hypatia was murdered by a mob in the course of an anti-pagan riot. This tragedy is sometimes blamed on the Christian bishop, Cyril, but there is no evidence to support this accusation. The 19th century author C. Kingsley wrote a fascinating historical novel *Hypatia,* which makes this story come to life.

Another minor mathematician at this time was Proclus. He studied in Alexandria and then worked in Athens. He wrote a commentary on the first book of Euclid, which contains valuable information about the history of Greek mathematics.

Boethius studied in Athens but lived in Rome. He is most famous for his *De Consolatione Philosophicae*, which he wrote in prison, about 525 AD His *Arithmetic* and *Geometry* were standard textbooks in the Middle Ages. Unfortunately, they contained much less mathematics than the *Elements.*

In 529 AD, the emperor Justinian closed the pagan schools of philosophy at Athens. The 'Dark Ages' of Europe had begun.

It is interesting that we do not know of many mathematicians of the period (50–500 AD) converting to Christianity. It seems that the Academy at Athens and the University at Alexandria both rejected the new religion. An interesting dialogue between reason and faith might have taken place, but, as it turned out, it was only in the later Middle Ages that thinkers, such as Aquinas (1250 AD), advanced philosophies that were influenced by the *Elements* as well as by the Bible.

When the Arabs conquered Alexandria in 641 AD, there probably was not much left of the famous library. However, according to an often repeated story, dating back to Moslem sources in the 13th century, it was the Arabs who destroyed it. Their commander, Amru, was willing to spare the library, but was dissuaded by Caliph Omar I, who argued thus: 'If the books of the Greeks confirm what is written in the Koran, they are superfluous; if they contradict the Koran, they are dangerous. In either case they should be destroyed.' The story continues, saying that the books served to heat the furnaces of the public bathhouses for six months! An almost identical story is told about another library in Persia. What was the origin of these stories? According to Bernard Lewis, in a Letter to the Editor, *The New York Review of Books* 37, Number 14 (October 27, 1990), they were invented to justify the destruction of a completely different library, a collection of Fatimid books, deemed to be heretical by the orthodox Sunni Sultan Saladin in the 12th century.

Exercises

1. Prove Heron's formula.

2. Give the details of the converse in the proof of the Theorem of Menelaus.

3. Find all the integer solutions of $x^2 + y^2 = z^2$ by using the same technique as Diophantus did in problem 9 of Book II of the *Arithmetica.* (Hint: the given equation has a solution in the integers if and only if $x'^2 + y'^2 = 1^2 + 0^2$ has a corresponding solution in rationals.)

4. Find an infinite family of rational solutions to $x^2 + y^2 = z^2$ (*Arithmetica* IV 1 (Sesiano)).

5. Prove a version of the Theorem of Pappus in case BC' and AB' are parallel.

21
Mathematics in China and India

Not much is known about the development of mathematics in China before contact with the West was established. The 'Arithmetic in Nine Sections' ('Chiu Chang Suan Shu') was written before 200 AD. Like the Rhind Papyrus, it is a list of problems and solutions. Chapter 8 shows how to solve n linear equations in n unknowns, using a method which is essentially the same as Gaussian elimination. One system which is solved is the following:

$$\begin{aligned} 3x + 2y + z &= 39, \\ 2x + 3y + z &= 34, \\ x + 2y + 3z &= 26. \end{aligned}$$

The Chinese interest in systems of linear equations was perhaps linked to their interest in magic squares. The square

$$\begin{array}{ccc} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{array}$$

was supposedly brought to humankind on the back of a tortoise from the River Lo in the days of Emperor Yu. Its 'magic' property is that all rows and columns and the two diagonals have the same sum.

A 'Chinese Remainder Problem' was solved by Sun Tsu (400 AD):

divide by 3, the remainder is 2;
divide by 5, the remainder is 3;
divide by 7, the remainder is 2;
what will be the number?