8.5 Exercises (page 251)

1. (a) All (x, y)
   (b) All $(x, y) \neq (0, 0)$
   (c) All (x, y) withy $\neq 0$
   (d) All (x, y) with $y \neq 0$ and $\frac{x^2}{y} \neq \frac{\pi}{2} + k\pi \; (k = 0, 1, 2, \ldots)$
   (e) All (x, y) with $x \neq 0$
   (f) All $(x, y) \neq (0, 0)$
   (g) All (x, y) with xy $\neq 1$
   (h) All $(x, y) \neq (0, 0)$
   (i) All $(x, y) \neq (0, 0)$
   (j) All (x, y) with $y \neq 0$ and $0 \leq x \leq y$ or $y \leq x \leq 0$
5. $\lim_{y \to 0} f(x,$ y) does not exist if $x \neq 0$
6. $(1 - m^2)/(1 + m^2)$; *NO*
7. $y = \frac{1}{2}x^2$; f not continuous at $(0, 0)$
8. $f(0, 0) = 1$

8.9 Exercises (page 255)

1. $f'(x; y) = a \cdot y$
2. (a) $f'(x; y) = 4 \, \|x\|^2 \, x \cdot y$
   (b) All points on the line 2x + 3y = $\frac{3}{26}$
   (c) All points on the plane $x + 2y + 3z = 0$
3. $f'(x; y) = x \cdot T(y) + y \cdot T(x)$
4. $\frac{\partial f}{\partial x} = 2x + y^3 \cos(xy)$; $\frac{\partial f}{\partial y} = 2y$ sin (xy) + $xy^2 \cos(xy)$
5. $\frac{\partial f}{\partial x} = x/(x^2 + y^2)^{1/2}$; $\frac{\partial f}{\partial y} =$ y/(x2 + $y^2)^{1/2}$
6. $\frac{af}{\partial x} = y^2/(x^2 + y^2)^{3/2}$; $\frac{\partial f}{\partial y} = -xy/(x^2 + y^2)^{3/2}$
7. $\frac{\partial f}{\partial x} = -2y/(x - y)^2$; $\frac{\partial f}{\partial y} = 2x/(x - y)^2$
8. $D_k f(x) = a_k$, where a = (a,, . . . , a_n)
9. $D_k f(x) = 2 \sum_{j=1}^{n} a_{kj} x_j$
10. $\frac{\partial f}{\partial x} = 4x^3 - 8xy^2$; $\frac{\partial f}{\partial y} = 4y^3 - 8x^2y$
11. $\frac{\partial f}{\partial x} = \frac{2x}{x^2 + y^2}$; $\frac{\partial f}{\partial y} = \frac{2y}{x^2 + y^2}$
12. $\frac{\partial f}{\partial x} = -\frac{2x}{y} \sin(x^2)$; $\frac{\partial f}{\partial y} = -\frac{1}{y^2} \cos(x^2)$
13. $\frac{\partial f}{\partial x} = \frac{2x}{y} \sec^2 \frac{x^2}{y}$; $\frac{\partial f}{\partial y} = -\frac{x^2}{y^2} \sec^2 \frac{x^2}{y}$

14. $\dfrac{\partial f}{\partial x} = -\dfrac{y}{x^2+y^2}$; $\dfrac{\partial f}{\partial y} = \dfrac{1}{x^2+y^2}$

15. $\dfrac{\partial f}{\partial x} = \dfrac{1+y^2}{1+x^2+y^2+x^2y^2}$; $\dfrac{\partial f}{\partial y} = \dfrac{1+x^2}{1+x^2+y^2+x^2y^2}$

16. $\dfrac{\partial f}{\partial x} = y^2x^{y^2-1}$; $\dfrac{\partial f}{\partial y} = 2yx^{y^2}\log x$

17. $\dfrac{\partial f}{\partial x} = -\dfrac{1}{2\sqrt{x(y-x)}}$; $\dfrac{\partial f}{\partial y} = \dfrac{\sqrt{x}}{2y\sqrt{y-x}}$

18. $n = -\frac{3}{2}$

19. $a = b = 1$

22. (b) One example is $f(\boldsymbol{x}) = \boldsymbol{x} \cdot \boldsymbol{y}$, where $\boldsymbol{y}$ is a fixed nonzero vector

8.14 Exercises (page 262)

1. (a) $(2x + y^3 \cos xy)\boldsymbol{i} + (2y \sin xy + xy^2 \cos xy)\boldsymbol{j}$
 (b) $e^x \cos y\,\boldsymbol{i} - e^x \sin y\,\boldsymbol{j}$
 (c) $2xy^3z^4\boldsymbol{i} + 3x^2y^2z^4\boldsymbol{j} + 4x^2y^3z^3\boldsymbol{k}$
 (d) $2x\boldsymbol{i} - 2y\boldsymbol{j} + 4z\boldsymbol{k}$
 (e) $\dfrac{2x}{x^2+2y^2-3z^2}\boldsymbol{i} + \dfrac{4y}{x^2+2y^2-3z^2}\boldsymbol{j} - \dfrac{6z}{x^2+2y^2-3z^2}\boldsymbol{k}$
 (f) $y^zx^{y^z-1}\boldsymbol{i} + zy^{z-1}x^{y^z}\log x\,\boldsymbol{j} + y^zx^{y^z}\log x \log y\,\boldsymbol{k}$
2. (a) $-2/\sqrt{6}$
 (b) $1/\sqrt{6}$
3. (1, 0), in the direction of $\boldsymbol{i}$; (-1, 0), in the direction of $-\boldsymbol{i}$
4. $2\boldsymbol{i} + 2\boldsymbol{j}$; $\frac{14}{5}$
5. $(a, b, c) = (6, 24, -8)$ or $(-6, -24, 8)$
6. The set of points (x, y) on the line $5x - 3y = 6$; $\nabla f(\boldsymbol{a}) = 5\boldsymbol{i} - 3\boldsymbol{j}$
8. (c) Yes
 (d) $f(x, y, z) = \frac{1}{2}(x^2 + y^2 + z^2)$
11. (b) implies (a) and (c); (d) implies (a), (b), and (c); (f) implies (a)

8.17 Exercises (page 268)

1. (b) $F''(t) = \dfrac{\partial^2 f}{\partial x^2}[X'(t)]^2 + 2\dfrac{\partial^2 f}{\partial x\,\partial y}X'(t)\,Y'(t) + \dfrac{\partial^2 f}{\partial y^2}[Y'(t)]^2 + \dfrac{\partial f}{\partial x}X''(t) + \dfrac{\partial f}{\partial y}Y''(t)$

2. (a) $F'(t) = 4t^3 + 2t$; $F''(t) = 12t^2 + 2$
 (b) $F'(t) = (2\cos^2 t - 1)e^{\cos t \sin t}\cos(\cos t \sin^2 t) + (3\sin^3 t - 2\sin t)e^{\cos t \sin t}\sin(\cos t \sin^2 t)$;
 $F''(t) = (5\cos^6 t - 3\cos^4 t - 4\cos^3 t - \cos^2 t - 4\cos t)e^{\cos t \sin t}\cos(\cos t \sin^2 t)$
 $\quad + (14\sin^3 t - 12\sin^5 t - 4\sin t + 7\cos t - 9\cos^3 t)e^{\cos t \sin t}\sin(\cos t \sin^2 t)$
 (c) $F'(t) = \dfrac{2e^{2t}\exp(e^{2t})}{1+\exp(e^{2t})} + \dfrac{2e^{-2t}\exp(e^{-2t})}{1+\exp(e^{-2t})}$, where $\exp(u) = e^u$;

$$F''(t) = \frac{4[1 + e^{2t} + \exp(e^{2t})]e^{2t}\exp(e^{2t})}{[1+\exp(e^{2t})]^2} - \frac{4[1 + e^{-2t} + \exp(e^{-2t})]e^{-2t}\exp(e^{-2t})}{[1+\exp(e^{-2t})]^2}$$

3. (a) $-\frac{2}{3}$
(b) $x^2 - y^2$
(c) 0
4. (a) $(1 + 3x^2 + 3y^2)(x\boldsymbol{i} + y\boldsymbol{j}) - (x^2 + y^2)^{1/2}\boldsymbol{k}$, or any scalar multiple thereof
(b) $\cos\theta = -[1 + (1 + 3(x^2 + y^2))^2]^{-1/2}$; $\cos\theta \to -\frac{1}{2}\sqrt{2}$ as $(x, y, z) \to (0, 0, 0)$
5. $U(x, y) = \frac{1}{2}\log(x^2 + y^2)$; $V(x, y) = \arctan(y/x)$
6. (b) No
8. $x/x_0 + y/y_0 + z/z_0 = 3$
9. $x + y + 2z = 4,\ x - y - z = -1$
10. $c = \pm\sqrt{3}$

8.22 Exercises (page 275)

1. (b) $\dfrac{\partial f}{\partial x} = -2x\sin(x^2 + y^2)\cos[\cos(x^2 + y^2)]e^{\sin[\cos(x^2 + y^2)]}$

2. $\dfrac{\partial F}{\partial x} = \dfrac{1}{2}\dfrac{\partial f}{\partial u} + \dfrac{1}{2}\dfrac{\partial f}{\partial v}$; $\dfrac{\partial F}{\partial y} = -\dfrac{1}{2}\dfrac{\partial f}{\partial u} + \dfrac{1}{2}\dfrac{\partial f}{\partial v}$

3. (a) $\dfrac{\partial F}{\partial s} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial s} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial s}$; $\dfrac{\partial F}{\partial t} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial t}$

(c) $\dfrac{\partial^2 F}{\partial s\,\partial t} = \dfrac{\partial^2 f}{\partial x^2}\dfrac{\partial X}{\partial s}\dfrac{\partial X}{\partial t} + \dfrac{\partial^2 f}{\partial x\,\partial y}\left(\dfrac{\partial X}{\partial s}\dfrac{\partial Y}{\partial t} + \dfrac{\partial X}{\partial t}\dfrac{\partial Y}{\partial s}\right) + \dfrac{\partial^2 f}{\partial y^2}\dfrac{\partial Y}{\partial s}\dfrac{\partial Y}{\partial t} + \dfrac{\partial f}{\partial x}\dfrac{\partial^2 X}{\partial s\,\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial^2 Y}{\partial s\,\partial t}$

4. (a) $\dfrac{\partial F}{\partial s} = \dfrac{\partial f}{\partial x} + t\dfrac{\partial f}{\partial y}$; $\dfrac{\partial F}{\partial t} = \dfrac{\partial f}{\partial x} + s\dfrac{\partial f}{\partial y}$; $\dfrac{\partial^2 F}{\partial s^2} = \dfrac{\partial^2 f}{\partial x^2} + 2t\dfrac{\partial^2 f}{\partial x\,\partial y} + t^2\dfrac{\partial^2 f}{\partial y^2}$;

$\dfrac{\partial^2 F}{\partial t^2} = \dfrac{\partial^2 f}{\partial x^2} + 2s\dfrac{\partial^2 f}{\partial x\,\partial y} + s^2\dfrac{\partial^2 f}{\partial y^2}$; $\dfrac{\partial^2 F}{\partial s\,\partial t} = \dfrac{\partial^2 f}{\partial x^2} + (s + t)\dfrac{\partial^2 f}{\partial x\,\partial y} + st\dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial f}{\partial y}$

(b) $\dfrac{\partial F}{\partial s} = t\dfrac{\partial f}{\partial x} + \dfrac{1}{t}\dfrac{\partial f}{\partial y}$; $\dfrac{\partial F}{\partial t} = s\dfrac{\partial f}{\partial x} - \dfrac{s}{t^2}\dfrac{\partial f}{\partial y}$; $\dfrac{\partial^2 F}{\partial s^2} = t^2\dfrac{\partial^2 f}{\partial x^2} + 2\dfrac{\partial^2 f}{\partial x\,\partial y} + \dfrac{1}{t^2}\dfrac{\partial^2 f}{\partial y^2}$;

$\dfrac{\partial^2 F}{\partial t^2} = s^2\dfrac{\partial^2 f}{\partial x^2} - 2\dfrac{s^2}{t^2}\dfrac{\partial^2 f}{\partial x\,\partial y} + \dfrac{s^2}{t^4}\dfrac{\partial^2 f}{\partial y^2} + \dfrac{2s}{t^3}\dfrac{\partial f}{\partial y}$; $\dfrac{\partial^2 F}{\partial s\,\partial t} = st\dfrac{\partial^2 f}{\partial x^2} - \dfrac{s}{t^3}\dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial f}{\partial x} - \dfrac{1}{t^2}\dfrac{\partial f}{\partial y}$

(c) $\dfrac{\partial F}{\partial s} = \dfrac{1}{2}\dfrac{\partial f}{\partial x} + \dfrac{1}{2}\dfrac{\partial f}{\partial y}$; $\dfrac{\partial F}{\partial t} = -\dfrac{1}{2}\dfrac{\partial f}{\partial x} + \dfrac{1}{2}\dfrac{\partial f}{\partial y}$; $\dfrac{\partial^2 F}{\partial s\,\partial t} = -\dfrac{1}{4}\dfrac{\partial^2 f}{\partial x^2} + \dfrac{1}{4}\dfrac{\partial^2 f}{\partial y^2}$;

$\dfrac{\partial^2 F}{\partial s^2} = \dfrac{1}{4}\dfrac{\partial^2 f}{\partial x^2} + \dfrac{1}{2}\dfrac{\partial^2 f}{\partial x\,\partial y} + \dfrac{1}{4}\dfrac{\partial^2 f}{\partial y^2}$; $\dfrac{\partial^2 F}{\partial t^2} = \dfrac{1}{4}\dfrac{\partial^2 f}{\partial x^2} - \dfrac{1}{2}\dfrac{\partial^2 f}{\partial x\,\partial y} + \dfrac{1}{4}\dfrac{\partial^2 f}{\partial y^2}$

5. $\dfrac{\partial^2\varphi}{\partial r^2} = \cos^2\theta\dfrac{\partial^2 f}{\partial x^2} + \cos\theta\sin\theta\left(\dfrac{\partial^2 f}{\partial x\,\partial y} + \dfrac{\partial^2 f}{\partial y\,\partial x}\right) + \sin^2\theta\dfrac{\partial^2 f}{\partial y^2}$;

$\dfrac{\partial^2\varphi}{\partial r\,\partial\theta} = -r\cos\theta\sin\theta\dfrac{\partial^2 f}{\partial x^2} + r\cos^2\theta\dfrac{\partial^2 f}{\partial x\,\partial y} - r\sin^2\theta\dfrac{\partial^2 f}{\partial y\,\partial x} + r\cos\theta\sin\theta\dfrac{\partial^2 f}{\partial y^2}$

$-\sin\theta\dfrac{\partial f}{\partial x} + \cos\theta\dfrac{\partial f}{\partial y}$;

6. $\dfrac{\partial F}{\partial r} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial r} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial r} + \dfrac{\partial f}{\partial z}\dfrac{\partial Z}{\partial r}$; $\dfrac{\partial F}{\partial s} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial s} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial s} + \dfrac{\partial f}{\partial z}\dfrac{\partial Z}{\partial s}$;

$\dfrac{\partial F}{\partial t} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial t} + \dfrac{\partial f}{\partial z}\dfrac{\partial Z}{\partial t}$

7. (a) $\dfrac{\partial F}{\partial r} = \dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + 2\dfrac{\partial f}{\partial z}$; $\dfrac{\partial F}{\partial s} = \dfrac{\partial f}{\partial x} - 2\dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z}$; $\dfrac{\partial F}{\partial t} = \dfrac{\partial f}{\partial x} + 3\dfrac{\partial f}{\partial y} - \dfrac{\partial f}{\partial z}$

(b) $\dfrac{\partial F}{\partial r} = 2r\left(\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z}\right)$; $\dfrac{\partial F}{\partial s} = 2s\left(\dfrac{\partial f}{\partial x} - \dfrac{\partial f}{\partial y} - \dfrac{\partial f}{\partial z}\right)$; $\dfrac{\partial F}{\partial t} = 2t\left(\dfrac{\partial f}{\partial x} - \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z}\right)$

8. $\dfrac{\partial F}{\partial s} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial s} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial s} + \dfrac{\partial f}{\partial z}\dfrac{\partial Z}{\partial s}$; $\dfrac{\partial F}{\partial t} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial t} + \dfrac{\partial f}{\partial z}\dfrac{\partial Z}{\partial t}$

9. (a) $\dfrac{\partial F}{\partial s} = 2s\dfrac{\partial f}{\partial x} + 2s\dfrac{\partial f}{\partial y} + 2t\dfrac{\partial f}{\partial z}$; $\dfrac{\partial F}{\partial t} = 2t\dfrac{\partial f}{\partial x} - 2t\dfrac{\partial f}{\partial y} + 2s\dfrac{\partial f}{\partial z}$

(b) $\dfrac{\partial F}{\partial s} = \dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + t\dfrac{\partial f}{\partial z}$; $\dfrac{\partial F}{\partial t} = \dfrac{\partial f}{\partial x} - \dfrac{\partial f}{\partial y} + s\dfrac{\partial f}{\partial z}$

10. $\dfrac{\partial F}{\partial r} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial r} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial r}$; $\dfrac{\partial F}{\partial s} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial s} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial s}$; $\dfrac{\partial F}{\partial t} = \dfrac{\partial f}{\partial x}\dfrac{\partial X}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial Y}{\partial t}$

11. (a) $\dfrac{\partial F}{\partial r} = \dfrac{\partial f}{\partial x}$; $\dfrac{\partial F}{\partial s} = \dfrac{\partial f}{\partial x}$; $\dfrac{\partial F}{\partial t} = \dfrac{\partial f}{\partial y}$

(b) $\dfrac{\partial F}{\partial r} = \dfrac{\partial f}{\partial x} + 2r\dfrac{\partial f}{\partial y}$; $\dfrac{\partial F}{\partial s} = \dfrac{\partial f}{\partial x} + 2s\dfrac{\partial f}{\partial y}$; $\dfrac{\partial F}{\partial t} = \dfrac{\partial f}{\partial x} + 2t\dfrac{\partial f}{\partial y}$

(c) $\dfrac{\partial F}{\partial r} = \dfrac{1}{s}\dfrac{\partial f}{\partial x}$; $\dfrac{\partial F}{\partial s} = \dfrac{-r}{s^2}\dfrac{\partial f}{\partial x} + \dfrac{1}{t}\dfrac{\partial f}{\partial y}$; $\dfrac{\partial F}{\partial t} = \dfrac{-s}{t^2}\dfrac{\partial f}{\partial y}$

13. (a) $f(x, y, z) = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$, plus any constant vector
(b) $f(x, y, z) = P(x)\boldsymbol{i} + Q(y)\boldsymbol{j} + R(z)\boldsymbol{k}$, where P, Q, R are any three functions satisfying $P' = p$, $Q' = q$, $R' = r$

14. (a) $Df(x, y) = \begin{bmatrix} e^{x+2y} & 2e^{x+2y} \\ 2\cos(y + 2x) & \cos(y + 2x) \end{bmatrix}$; $Dg(u, v, w) = \begin{bmatrix} 1 & 4v & 9w^2 \\ -2u & 2 & 0 \end{bmatrix}$

(b) $h(u, v, w) = e^{u+2v^2+3w^3+4v-2u^2}\boldsymbol{i} + \sin(2v - u^2 + 2u + 4v^2 + 6w^3)\boldsymbol{j}$

(c) $Dh(1, -1, 1) = \begin{bmatrix} -3 & 0 & 9 \\ 0 & -6\cos 9 & 18\cos 9 \end{bmatrix}$

15. (a) $Df(x, y, z) = \begin{bmatrix} 2x & 1 & \\ 2 & 1 & 2z \end{bmatrix}$; $Dg(u, v, w) = \begin{bmatrix} v^2w^2 & 2uvw^2 & 2uv^2w \\ 0 & w^2\cos v & 2w\sin v \\ 2ue^v & u^2e^v & 0 \end{bmatrix}$

(b) $h(u, v, w) = (u^2v^4w^4 + w^2\sin v + u^2e^v)\boldsymbol{i} + (2uv^2w^2 + w^2\sin v + u^4e^{2v})\boldsymbol{j}$

(c) $Dh(u, 0, w) = \begin{bmatrix} 2u & w^2 + u^2 & 0 \\ 4u^3 & w^2 + 2u^4 & 0 \end{bmatrix}$

8.24 Miscellaneous exercises (page 281)

1. One example: $f(x, y) = 3x$ when $x = y$, $f(x, y) = 0$ otherwise
2. $D_1 f(0, 0) = 0$; $D_2 f(0, 0) = -1$; $D_{2,1} f(0, 0) = 0$; $D_{1,2} f(0, 0)$ does not exist
3. (a) If $a = (a_1, a_2)$, then $f'(O; a) = a_2^3/a_1^2$ if $a_1 \neq 0$, and $f'(O; a) = 0$ if $a_1 = 0$
   (b) Not continuous at the origin
4. $\dfrac{\partial f}{\partial x} = \dfrac{1}{2} e^{-xy} x^{-1/2} y^{1/2}$; $\dfrac{\partial f}{\partial y} = \dfrac{1}{2} e^{-xy} x^{1/2} y^{-1/2}$
5. $$F'''(t) = \frac{\partial^3 f}{\partial x^3}[X'(t)]^3 + 3\frac{\partial^3 f}{\partial x^2 \partial y}[X'(t)]^2 Y'(t) + 3\frac{\partial^3 f}{\partial x \partial y^2} X'(t)[Y'(t)]^2$$
   $$+ \frac{\partial^3 f}{\partial y^3}[Y'(t)]^3 + 3\frac{\partial^2 f}{\partial x^2} X'(t)X''(t) + 3\frac{\partial^2 f}{\partial x \partial y}[X''(t)Y'(t) + X'(t)Y''(t)]$$
   $$+ 3\frac{\partial^2 f}{\partial y^2} Y'(t)Y''(t) + \frac{\partial f}{\partial x} X'''(t) + \frac{\partial f}{\partial y} Y'''(t),$$
   assuming the mixed partial derivatives are independent of the order of differentiation
6. 8
7. (a) $\dfrac{\partial g}{\partial u} = \dfrac{\partial f}{\partial x} v + \dfrac{\partial f}{\partial y}$; $\dfrac{\partial g}{\partial v} = \dfrac{\partial f}{\partial x} u - \dfrac{\partial f}{\partial y} v$; $\dfrac{\partial^2 g}{\partial u \partial v} = uv\dfrac{\partial^2 f}{\partial x^2} + (u^2 - v^2)\dfrac{\partial^2 f}{\partial x \partial y}$
   $- uv\dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial f}{\partial x}$ (b) $a = \frac{1}{2}$, $b = -\frac{1}{2}$

10. (a) $\varphi'(t) = A'(t)\int_c^{B(t)} f[A(t), y]\,dy + B'(t)\int_a^{A(t)} f[x, B(t)]\,dx$
    (b) $\varphi'(t) = 2te^{t^2}(2e^{t^2} - e^a - e^c)$
13. A sphere with center at the origin and radius $\sqrt{2}$
14. $f(x) = x^2$

Chapter 9

9.3 Exercises (page 286)

1. $f(x, y) = \sin(x - \frac{4}{3}y)$
2. $f(x, y) = e^{x+5y/2} - 1$
3. (a) $u(x, y) = x^2 y^2 e^{xy}$
   (b) $v(x, y) = 2 + \log\left|\dfrac{x}{y}\right|$

5. $A = B = C = 1$, $D = -3$; $f(x, y) = \varphi_1(3x + y) + \varphi_2(x - y)$
6. $G(x, y) = x - y$

9.8 Exercises (page 302)

1. $\partial X/\partial v = (1 + xu)/(x - y)$; $\partial Y/\partial u = (1 - yv)/(x - y)$; $\partial Y/\partial v = (1 + yu)/(y - x)$
2. $\partial X/\partial y = -(1 + xu)/(1 + u)$; $\partial V/\partial u = (1 - yv)/(1 + yu)$; $\partial V/\partial y = (1 - x)/(1 + u)$
3. $\dfrac{\partial X}{\partial v} = \dfrac{\partial(F, G)}{\partial(y, v)}\Big/\dfrac{\partial(F, G)}{\partial(x, y)}$, $\dfrac{\partial Y}{\partial u} = \dfrac{\partial(F, G)}{\partial(u, x)}\Big/\dfrac{\partial(F, G)}{\partial(x, y)}$; $\dfrac{\partial Y}{\partial v} = \dfrac{\partial(F, G)}{\partial(v, x)}\Big/\dfrac{\partial(F, G)}{\partial(x, y)}$
4. $T = \pm\dfrac{1}{\sqrt{151}}(24i - 4\sqrt{7}\,j + 3\sqrt{7}\,k)$

5. $2i + j + \sqrt{3}\,k$, or any nonzero scalar multiple thereof
6. $\partial x/\partial u = 0$, $\partial x/\partial v = \pi/12$
8. $n = 2$
9. $\partial f/\partial x = -1/(2y + 2z + 1)$; $\partial f/\partial y = -2(y + z)/(2y + 2z + 1)$;
$\partial^2 f/(\partial x\,\partial y) = 2/(2y + 2z + 1)^3$
10. $\partial^2 z/(\partial x\,\partial y) = [\sin(x + y)\cos^2(y + z) + \sin(y+z)\cos^2(x + y)]/\cos^3(y+z)$
11. $$\frac{\partial f}{\partial x} = -\frac{D_1F + 2xD_2F}{D_1F + 2zD_2F}\,;\quad \frac{\partial f}{\partial y} = -\frac{D_1F + 2yD_2F}{D_1F + 2zD_2F}$$
12. $D_1F = f'[x + g(y)]$; $D_2F = f'[x + g(y)]g'(y)$; $D_{1,1}F = f''[x + g(y)]$;
$D_{1,2}F = f''[x + g(y)]g'(y)$; $D_{2,2}F = f''[x + g(y)]\,[g'(y)]^2 + f'[x + g(y)]g''(y)$

9.13 Exercises (page 313)

1. Absolute minimum at (0, 1)
2. Saddle point at (0, 1)
3. Saddle point at (0, 0)
4. Absolute minimum at each point of the line $y = x + 1$
5. Saddle point at (1, 1)
6. Absolute minimum at (1, 0)
7. Saddle point at (0, 0)
8. Saddle points at (0, 6) and at (x, 0), all x; relative minima at (0, y), $0 < y < 6$; relative maxima at (2, 3) and at (0, y) for $y < 0$ and $y > 6$
9. Saddle point at (0, 0); relative minimum at (1, 1)
10. Saddle points at $(n\pi + \pi/2, 0)$, where n is any integer
11. Absolute minimum at (0, 0); saddle point at $(-\frac{1}{4}, -\frac{1}{2})$
12. Absolute minimum at $(-\frac{1}{26}, -\frac{3}{26})$; absolute maximum at (1, 3)
13. Absolute maximum at $(\pi/3, \pi/3)$; absolute minimum at $(2\pi/3, 2\pi/3)$; relative maximum at (π, π); relative minimum at (0, 0); saddle points at $(0, \pi)$ and $(\pi, 0)$
14. Saddle point at (1, 1)
15. Absolute maximum at each point of the circle $x^2 + y^2 = 1$; absolute minimum at (0, 0)
17. (c) Relative maximum at (2, 2); no relative minima; saddle points at (0, 3), (3, 0), and (3, 3)
18. Relative maximum $\frac{1}{8}$ at $(\frac{1}{2}, \frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$; relative minimum $-\frac{1}{8}$ at $(\frac{1}{2}, -\frac{1}{2})$ and $(-\frac{1}{2}, \frac{1}{2})$; saddle points at (0, 0), $(\pm 1, 0)$, and $(0, \pm 1)$; absolute maximum 1 at (1, -1) and (-1, 1); absolute minimum -1 at (1, 1) and $(-1, -1)$
19. (a) $a = 1$, $b = -\frac{1}{6}$
(b) $a = 6\log 2 - 3\pi/2$, $b = \pi - 3\log 2$
21. Let $x^* = \frac{1}{n}\sum_{i=1}^{n} x_i$, $y^* = \frac{1}{n}\sum_{i=1}^{n} y_i$, $u_i = x_i - x^*$. Then $a = \left(\sum_{i=1}^{n} y_i u_i\right)\Big/\left(\sum_{i=1}^{n} u_i^2\right)$,
and $b = y^* - ax^*$
22. Let $x^* = \frac{1}{n}\sum_{i=1}^{n} x_i$, $y^* = \frac{1}{n}\sum_{i=1}^{n} y_i$ $z^* = \frac{1}{n}\sum_{i=1}^{n} z_i$, $u_i = x_i - x^*$, $v_i = y_i - y^*$, and let

$$\Delta = \begin{vmatrix} \sum u_i^2 & \sum u_i v_i \\ \sum u_i v_i & \sum v_i^2 \end{vmatrix}$$, where the sums are for $i = 1, 2, \ldots, n$. Then

$$a = \frac{1}{\Delta}\begin{vmatrix} \sum u_i z_i & \sum u_i v_i \\ \sum v_i z_i & \sum v_i^2 \end{vmatrix}, \quad b = \frac{1}{\Delta}\begin{vmatrix} \sum v_i z_i & \sum u_i v_i \\ \sum u_i z_i & \sum u_i^2 \end{vmatrix}, \quad c = z^* - ax^* - by^*$$

25. Eigenvalues 4, 16, 16; relative minimum at (1, 1, 1)

9.15 Exercises (page 318)

1. Maximum value is $\frac{1}{4}$; no minimum
2. Maximum is 2; minimum is 1
3. (a) Maximum is $\dfrac{\sqrt{a^2+b^2}}{ab}$ at $(b(a^2+b^2)^{-1/2}, a(a^2+b^2)^{-1/2})$; minimum is $-\dfrac{\sqrt{a^2+b^2}}{ab}$ at $(-b(a^2+b^2)^{-1/2}, -a(a^2+b^2)^{-1/2})$

   (b) Minimum is $a^2b^2/(a^2+b^2)$ at $\left(\dfrac{ab^2}{a^2+b^2}, \dfrac{a^2b}{a^2+b^2}\right)$; no maximum
4. Maximum is $1 + \sqrt{2}/2$ at the points $(n\pi + \pi/8, n\pi - \pi/8)$, where n is any integer; minimum is $1 - \sqrt{2}/2$ at $(n\pi + 5\pi/8, n\pi + 3\pi/8)$, where n is any integer
5. Maximum is 3 at $(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3})$; minimum is -3 at $(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3})$
6. (0, 0, 1) and (0, 0, -1)
7. 1
8. (1, 0, 0), (0, 1, 0), (−1, 0, 0), (0, −1, 0)
9. $\dfrac{a^a b^b c^c}{(a+b+c)^{a+b+c}}$ at $\left(\dfrac{a}{a+b+c}, \dfrac{b}{a+b+c}, \dfrac{c}{a+b+c}\right)$
10. $abc\sqrt{3}/2$
11. $5 \log r + 3 \log \sqrt{3}$
12. $m^2 = \dfrac{A + C - \sqrt{(A-C)^2 + 4B^2}}{2(AC - B^2)}$
13. $(4 \pm \sqrt{5})/\sqrt{2}$
14. Angle is $\pi/3$; width across the bottom is $c/3$; maximum area is $c^2/(4\sqrt{3})$

Chapter 10

10.5 Exercises (page 328)

1. $-\frac{14}{15}$
2. $-2\pi a^2$
3. $\frac{1}{35}$
4. $\frac{4}{3}$
5. 0
6. 40
7. $\frac{23}{6}$
8. $\frac{5}{2}$
9. $-\frac{369}{3x}$
10. -2π
11. 0
12. (a) $-2\sqrt{2}\pi$
    (b) $-\pi$