etc. Designentur $\frac{b+b'}{a}$, $\frac{b'+b''}{a'}$, $\frac{b''+b'''}{a''}$ etc. respectiue per h', h'', h''' etc. Sint indeterminatae formarum F, F', F'' etc. x, y; x', y'; x'', y'' etc. Ponatur F transmutari

in F' positis $x = \alpha'x' + \beta'y'$, $y = \gamma'x + \delta'y'$
F'' . . . $x = \alpha''x'' + \beta''y''$, $y = \gamma''x'' + \delta''y''$
F''' . . . $x = \alpha'''x''' + \beta'''y'''$, $y = \gamma'''x''' + \delta'''y'''$
etc.

Tum quia F transit in F' positis $x = -y'$, $y = x' + h'y'$; F' in F'' positis $x' = -y''$, $y' = x'' + h''y''$; F'' in F''' positis $x'' = -y'''$, $y'' = x''' + h'''y'''$ etc. (art. 160), facile eruetur sequens algorithmus (art. 159):

$\alpha' = 0$	$\beta' = -1$	$\gamma' = 1$	$\delta' = h'$
$\alpha'' = \beta'$	$\beta'' = h''\beta' - \alpha'$	$\gamma'' = \delta'$	$\delta'' = h''\delta' - \gamma'$
$\alpha''' = \beta''$	$\beta''' = h'''\beta'' - \alpha''$	$\gamma''' = \delta''$	$\delta''' = h'''\delta'' - \gamma''$
$\alpha^{\text{IV}} = \beta'''$	$\beta^{\text{IV}} = h^{\text{IV}}\beta''' - \alpha'''$	$\gamma^{\text{IV}} = \delta'''$	$\delta^{\text{IV}} = h^{\text{IV}}\delta''' - \gamma'''$

etc.

siue

$\alpha' = 0$	$\beta' = -1$	$\gamma' = 1$	$\delta' = h'$
$\alpha'' = \beta'$	$\beta'' = h''\beta'$	$\gamma'' = \delta'$	$\delta'' = h''\delta' - 1$
$\alpha''' = \beta''$	$\beta''' = h'''\beta'' - \beta'$	$\gamma''' = \delta''$	$\delta''' = h'''\delta'' - \delta'$
$\alpha^{\text{IV}} = \beta'''$	$\beta^{\text{IV}} = h^{\text{IV}}\beta''' - \beta''$	$\gamma^{\text{IV}} = \delta'''$	$\delta^{\text{IV}} = h^{\text{IV}}\delta''' - \delta''$

etc.

Omnes has transformationes esse proprias tum ex ipsarum formatione tum ex art. 159 nullo negotio deduci potest.

Algorithmus hic perquam simplex et ad calculum expeditus, algorithmo in art. 27 exposito est analogus, ad quem etiam reduci potest *). Ceterum solutio haec ad formas determinantis negatiui non est restricta, sed ad omnes casus patet, si modo nullus numerorum a', a'', a''' etc. $= 0$.

178. Problema. *Propositis duabus formis F, f, eiusdem determinantis negatiui, proprie aequiualentibus: inuenire transformationem aliquam propriam alterius in alterum.*

Sol. Supponamus formam F esse (A, B, A') et per methodum art. 171 inuentam esse progressionem formarum (A', B', A''), (A'', B'', A''') etc. vsque ad (A^m, B^m, A^{m+1}) quae sit reducta: similiterqun f esse (a, b, a') et per eandem methodum inuentam seriem (a', b', a''), (a'', b'', a''') vsque ad (a_n, b_n, a^{n+1}), quae sit reducta. Tum duo casus locum habere possunt.

I. Si formae (A^m, B^m, A^{m+1}), (a^n, b^n, a^{n+1}) sunt aut identicae, aut oppositae simulque ancipites. Tum formae (A^{m-1}, B^{m-1}, A^m), $(a^n, -b^{n-1}, a^{n-1})$ erunt contiguae (designante A^{m-1} terminum progressionis $A, A', A'' \ldots A^m$ penultimum, similiaque $B^{m-1}, a^{n-1}, b^{n-1}$). Nam

*) Erit scilicet in signis art. 27, $\mathfrak{G}^n = \pm [-h'', h''' - h^{IV} \ldots \pm h^n]$, vbi signa ambigue posita, esse debent $- -$; $- +$; $+ -$; $+ +$; prout n formae $4k + 0$; 1; 2; 3 — et $\mathfrak{d}^n = \pm [h' - h'', h''' \ldots \ldots \pm h^n]$, vbi signa ambigua esse debent $+ -$; $+ +$; $- -$; $- +$ prout n formae $4k + 0$; 1; 2; 3. Sed hoc, quod quiuis facile ipse confirmare poterit, fusius exsequi, nobis breuitas non permittit.

$A^m = a^n$, $B^{m-1} \equiv - B^m$ (mod. A^m), $b^{n-1} \equiv - b^n$ (mod. a^n siue A^m), vnde $B^{m-1} - b^{n-1} \equiv b^n - B^m$ adeoque $\equiv 0$, si formae (A^m, B^m, A^{m+1}), (a^n, b^n, a^{n+1}) sunt identicae et $\equiv 2b^n$ adeoque $\equiv 0$, si sunt oppositae et ancipites. Quare in progressione formarum (A, B, A'), (A', B', A'') .... (A^{m-1}, B^{m-1}, A^m), $(a^n, - b^{n-1}, a^{n-1})$, $(a^{n-1}, - b^{n-2}, a^{n-2})$...$(a', - b, a)$, (a, b, a') quaeuis forma praecedenti contigua erit, adeoque per art. praec. transformatio propria primae F in vltimam f inueniri poterit.

II. Si formae (A^m, B^m, A^{m+1}), (a^n, b^n, a^{n+1}) non identicae, sed oppositae simulque $A^m = A^{m+1} = a^n = a^{n+1}$. Tum progressio formarum (A, B, A'), (A', B', A'')....(A^m, B^m, A^{m+1}), $(a^n, - b^{n-1}, a^{n-1})$, $(a^{n-1}, - b^{n-2}, a^{n-2})$.... $(a', - b, a)$, (a, b, a') eadem proprietate erit praedita. Nam $A^{m+1} = a^n$, et $B^m - b^{n-1} = - (b^n + b^{n-1})$ per a^n diuisibilis. Vnde per art. praec. inuenietur transformatio propria formae primae F in vltimam f.

Ex. Ita pro formis (23, 38, 63), (15, 20, 27) habetur progressio (23, 38, 63), (63, 25, 10), (10, 5, 3), (3, 1, 2), (2, — 7, 27) (27, — 20, 15), (15, 20, 27), quare $h' = 1$, $h'' = 3$, $h''' = 2$, $h^{iv} = -3$, $h^{v} = -1$, $h^{vi} = 0$. Hinc deducitur transformatio formae $23xx + 76xy + 63yy$ in $15tt + 40tu + 27uu$ haec: $x = - 13t - 18u$, $y = 8t + 11u$.