LIB. II.

$$\begin{aligned} o = t^5u + t^4uu &- 2t^3u^3 - 2ttu^4 + tu^5 + u^6 \\ - 2t^4 & \qquad\qquad + 2u^4 \\ + 4 & \end{aligned}$$

ex hac æquatione, facto $t = \infty$, invenitur $u = o$; ideoque reliqui termini, præter hos duos $t^5u - 2t^4$, evanefcunt; unde pro Afymtota curvilinea erit $u = \frac{2}{t}$. Ob hunc ergo Factorem Curva quæfita duos habebit ramos bB, cC in infinitum excurrentes.

216. Sumantur nunc Factores æquales gemini x^2; eritque, ob $xx = \frac{xy(yy+xx)-1}{y^3(y-x)}$. Axe ergo fumto recta AD ad priorem XY normali, fiet $y = t$ & $x = u$, pro quo ifta æquatio refultat

$$\begin{aligned} o &= t^4u^2 - t^3u^3 \\ &- t^3u - tu^3 \\ &+ 1 \end{aligned}$$

quæ, facto t infinito, abit in $t^4u^2 - t^3u + 1 = o$, unde duæ nafcuntur æquationes $u = \frac{1}{t}$ & $u = \frac{1}{t^3}$. Quare hic Factor quatuor præbet ramos in infinitum excurrentes; primo nempe duos dD, eE ex æquatione $u = \frac{1}{t}$; & duos ad eafdem partes fitos δD & εE ex æquatione $u = \frac{1}{t^3}$.

217. Tres Factores æquales y^3 referuntur ad ipfum Axem XY, fietque $t = x$ & $y = u$, unde nafcitur æquatio hæc,

$$o = -t^3u^3 + ttu^4 - t^3u - tu^3 + 1:$$

quæ, pofito t infinito, dat $t^3u^3 + t^3u = o$, feu $u(uu+1) = o$; unde, ob $uu+1 = o$ æquationem impoffibilem, unica obtinetur Afymtota recta $u = o$, conveniens cum ipfo Axe XY, cujus indoles exprimetur hac æquatione $t^3u = 1$ feu $u = \frac{1}{t^3}$; ac propterea ifte Factor triplex duos tantum præbet ramos yY & xX in infinitum excurrentes. Omnino ergo Curva

Curva quæſita octo ramos in infinitum extenſos habebit, qui quomodo in ſpatio finito inter ſe conjungantur hujus non eſt loci explicare. CAP. VIII.

218. Ex hoc ergo & præcedente Capite ramorum in infinitum extenſorum varietas luculenter perſpicitur. Primum enim hi rami Curvarum vel ad Lineam quampiam rectam tanquam Aſymtotam convergunt, uti fit in Hyperbola, vel Aſymtotam rectam non habent, uti Parabola. Priori caſu rami Curvarum vocantur *hyperbolici*, poſteriori *parabolici*. Utriuſque claſſis innumerabiles dantur ſpecies; ramorum enim hyperbolicorum ſpecies his exprimuntur æquationibus, inter Coordinatas t & u, quarum illa t ſtatuitur infinita.

$$u = \frac{A}{t};\ u = \frac{A}{tt};\ u = \frac{A}{t^3};\ u = \frac{A}{t^4},\ \&c.$$
$$u^2 = \frac{A}{t};\ u^2 = \frac{A}{tt};\ u^2 = \frac{A}{t^3};\ u^2 = \frac{A}{t^4},\ \&c.$$
$$u^3 = \frac{A}{t};\ u^3 = \frac{A}{tt};\ u^3 = \frac{A}{t^3};\ u^3 = \frac{A}{t^4},\ \&c.$$
&c.

Ramorum vero parabolicorum ſpecies indicantur ſequentibus æquationibus.

$$u^2 = At;\ u^3 = At;\ u^4 = At;\ u^5 = At,\ \&c.$$
$$u^3 = At^2;\ u^4 = At^2;\ u^5 = At^2;\ u^6 = At^2,\ \&c.$$
$$u^4 = At^3;\ u^5 = At^3;\ u^6 = At^3;\ u^7 = At^3,\ \&c.$$
&c.

Quælibet autem æquatio harum expoſitarum, ad minimum, duos exhibet ramos in infinitum excurrentes, ſi exponentium ipſarum t & u non uterque fuerit numerus par; ſin autem uterque exponens fuerit numerus par, tum vel nullum ramum infinitum prabet, vel quatuor: illud ſcilicet evenit, ſi æquatio ſit impoſſibilis, hoc vero ſi ſit realis.

LIB. II.

CAPUT IX.

De Linearum tertii ordinis ſubdiviſione in ſpecies.

219. NAtura atque numerus ramorum in infinitum extenſorum merito eſſentiale diſcrimen in Lineis curvis conſtituere cenſetur, atque ex hoc fonte commodiſſime deſumitur ratio ſubdiviſionis Linearum cujuſque ordinis in ſuas ſpecies diverſas. Hinc enim quoque oritur eadem Linearum ſecundi ordinis diviſio in ſuas ſpecies, quam ipſa rei natura ſupra ſuppeditaverat.

Sit enim propoſita æquatio generalis pro Lineis ſecundi ordinis

$$\alpha yy + \beta yx + \gamma xx + \delta y + \varepsilon x + \zeta = 0,$$

cujus ſupremum membrum $\alpha yy + \beta yx + \gamma xx$, potiſſimum ſpectetur, utrum habeat Factores ſimplices reales an ſecus. Quod ſi enim careat Factoribus, naſcitur prima ſpecies, *Ellipſis* dicta, ſin autem Factores ſint reales, videndum eſt utrum ſint inæquales, an æquales; illo caſu oritur *Hyperbola*, hoc vero *Parabola*.

220. Caſu ergo, quo membri ſupremi Factores ſunt reales & inæquales, Curva duas habebit Aſymtotas rectas; ad quarum naturam inveſtigandam ſit $\alpha yy + \beta yx + \gamma xx = (ay - bx)(cy - dx)$, ita ut ſit

$$(ay - bx)(cy - dx) + \delta y + \varepsilon x + \zeta = 0.$$

Conſideretur primum Factor $ay - bx$, qui in infinito dat $\frac{y}{x} = \frac{b}{a}$, fiet itaque

$$ay - bx + \frac{\delta b + \varepsilon a}{bc - ad} + \frac{\zeta}{cy - dx} = 0,$$

unde

unde æquatio $ay - bx + \frac{\delta b + \varepsilon a}{bc - ad} = 0$, definit positionem unius Asymtotæ rectæ; similique modo æquatio hæc $cy - dx + \frac{\delta d + \varepsilon c}{ad - bc} = 0$, ostendet Asymtotam alteram. CAP. IX.

221. Ad naturam cujusque Asymtotæ scrutandam, æquationem ad alium Axem transferamus ponendo $y = \frac{au + bt}{\sqrt{(aa + bb)}}$ & $x = \frac{at - bu}{\sqrt{(aa + bb)}}$, sitque $\sqrt{(aa + bb)} = g$, erit $u((ac + bd)u + (bc - ad)t) + \frac{(\delta a - \varepsilon b)u + (\delta b + \varepsilon a)t}{g} + \zeta = 0$, ideoque

$$g(bc - ad)tu + g(ac + bd)uu +$$
$$(\delta b + \varepsilon a)t + (\delta a - \varepsilon b)u + \zeta g = 0.$$

Hinc, posito in reliquis membris $u = -\frac{\delta b - \varepsilon a}{g(bc - ad)}$, erit $(g(bc - ad)u + \delta b + \varepsilon a)t + \frac{(ac + bd)(\delta b + \varepsilon a)^2}{g(bc - ad)^2}$ $\frac{(\delta a - \varepsilon b)(\delta b + \varepsilon a)}{g(bc - ad)} + \zeta g = 0$, seu $g(bc - ad)u + \delta b + \varepsilon a + \frac{g(\delta b + \varepsilon c)(\delta b + \varepsilon a)}{(bc - ad)^2 t} + \frac{\zeta g}{t} = 0$: erit ergo Asymtota hyperbolica generis $u = \frac{A}{t}$. Simili vero modo Asymtota altera ex Factore $cy - dx$ oriunda definietur, unde Curva habebit duo ramorum in infinitum extensorum paria, utrumque æquatione $u = \frac{A}{t}$ expressum.

222. Sint jam ambo Factores æquales, seu $\alpha yy + \beta xy + \gamma xx = (ay - bx)^2$; atque, facta eadem ad alium Axem translatione, qua fit $y = \frac{ay + bt}{g}$, & $x = \frac{at - bu}{g}$, erit $gguu + \frac{(\delta a - \varepsilon b)u}{g} + \frac{(\delta b + \varepsilon a)t}{g} + \zeta = 0$; &, facto t infinito, erit $uu + \frac{(\delta b + \varepsilon a)t}{g^3} = 0$, quæ æquatio ostendit

 duos

duos ramos parabolicos ſpeciei $uu = At$, quippe Curva ipſa erit Parabola, ipſaque ſua Aſymtota. Sin autem eſſet $\delta b + \varepsilon a = 0$, tum æquatio foret $ggzuu + \frac{\delta gu}{a} + \zeta = 0$, pro duabus rectis inter ſe parallelis, qui eſt caſus, quo æquatio ſecundi ordinis tota in duos Factores ſimplices eſt reſolubilis.

Sic igitur ſpecies Linearum ſecundi ordinis inveniſſemus, etiam ſi nondum erutæ fuiſſent.

223. Eodem igitur modo aggrediamur Lineas tertii ordinis, quarum æquatio generalis eſt

$$\alpha y^3 + \beta y^2 x + \gamma yxx + \delta x^3 + \varepsilon yy + \zeta yx + \eta xx + \theta y + \iota x + \varkappa = 0.$$

Supremum igitur membrum $\alpha y^3 + \beta y^2 x + \gamma y x^2 + \delta x^3$, quia eſt imparium dimenſionum, vel unum habet Factorem ſimplicem realem, vel omnes tres Factores ſimplices erunt reales. Sequentes igitur caſus ſunt evolvendi.

I.

Si unicus extet Factor ſimplex realis.

II.

Si omnes tres ſint reales, & inter ſe inæquales.

III.

Si duo Factores fuerint æquales.

IV.

Si omnes tres Factores fuerint æquales.

Quoniam vero in quovis caſu ad unicum Factorem calculum accommodaſſe ſufficit; ſit iſte Factor, ſive ſolus adſit ſive cum aliis ſui æqualibus inæqualibuſve, $ay - bx$; atque ad hunc poſitio Axis ita immutetur, ut hactenus fecimus; quo facto, oriatur hæc æquatio, qua vice ſuperioris utamur cum æque late pateat

$$\alpha ttu + \beta tuu + \gamma u^3 + \delta tt + \varepsilon tu + \zeta uu + \eta t + \theta u + \iota = 0,$$

ubi membrum ſupremum $\alpha ttu + \beta tuu + \gamma u^3$, unum certe habet Factorem u.

CASUS I.