

$q^6 - 6q^4 + 9qq - 2, (2, 7) = q^7 - 7q^5 - 14q^3 - 7q, (2, 8) = q^8 - 8q^6 + 20q^4 - 16qq + 2, (2, 9) = q^9 - 9q^7 + 27q^5 - 30q^3 + 9q$. Commodius quam per praecepta art. 346 hae aequationes in casu praesenti per reflexiones sequentes euolui possunt. Supponen-

do $[1] = \cos \frac{kP}{19} + i \sin \frac{kP}{19}$, fit $[18] =$

$\cos \frac{18kP}{19} + i \sin \frac{18kP}{19} = \cos \frac{kP}{19} - i \sin \frac{kP}{19}$,

adeoque $(2, 1) = 2\cos \frac{kP}{19}$; nec non ge-

neraliter $[\lambda] = \cos \frac{\lambda kP}{19} + i \sin \frac{\lambda kP}{19}$, adeoque

$(2, \lambda) = [\lambda] + [18\lambda] = [\lambda] + [-\lambda] =$

$2\cos \frac{\lambda kP}{19}$. Quare si $\frac{1}{2}q = \cos \omega$, erit $(2, 2)$

$= 2\cos 2\omega, (2, 3) = 2\cos 3\omega$ etc., vnde per ae-

quationes notas pro cosinibus angulorum multi-

plicium eadem formulae vt supra deriuantur. —

Iam ex his formulis valores numerici sequentes

eliciuntur:

$(2, 2) = -0,1651586909 \quad (2, 6) = 0,4909709743$

$(2, 3) = 1,5782810188 \quad (2, 7) = -1,7589475024$

$(2, 4) = -1,9727226068 \quad (2, 8) = 1,8916344834$

$(2, 5) = 1,0938963162 \quad (2, 9) = -0,8033908493$

Valores ipsorum $(2, 7), (2, 8)$ etiam ex ae-

quatione (B) , cuius duae reliquae radices sunt,

elici possunt, dubiumque, *vtra* harum radicum

fiat $(2, 7)$ et *vtra* $(2, 8)$, vel per calculum

nendo $\omega = \frac{7}{19}P$, vnde fieri oportet $(2, 7) = 2\cos \frac{49}{19}P = 2\cos \frac{8}{19}P$, et $(2, 8) = 2\cos \frac{56}{19}P = 2\cos \frac{1}{19}P$. — Similiter aggregata $(2, 2)$, $(2, 3)$, $(2, 5)$ etiam per aequationem $x^3 - (6, 2)xx + ((6, 1) + (6, 2))x - 2 - (6, 4) = 0$, cuius radices sunt, inuenire licet, incertitudoque, quatenus radices illis aggregatis *resp.* aequales statuendae sint, prorsus eodem modo remouebitur, vt ante; et perinde etiam aggregata $(2, 4)$, $(2, 6)$, $(2, 9)$ per aequationem $x^3 - (6, 4)xx + ((6, 2) + (6, 4))x - 2 - (6, 1) = 0$ elici poterunt.

Denique [1] et [18] sunt radices aequationis $xx - (2, 1)x + 1 = 0$, quarum altera fit $= \frac{1}{2}(2, 1) + i\sqrt{(1 - \frac{1}{4}(2, 1)^2)} = \frac{1}{2}(2, 1) + i\sqrt{(\frac{1}{2} - \frac{1}{4}(2, 2))}$, altera $= \frac{1}{2}(2, 1) - i\sqrt{(\frac{1}{2} - \frac{1}{4}(2, 2))}$, hinc valores numerici $= -0,6772815716 \pm 0,7357239107i$. Sedecim radices reliquae vel ex euolutione potestatum vtriusuis harum radicum, vel e solutione octo aliarum similium aequationum deduci possunt, vbi in methodo posteriori vel per tabulas sinuum vel per artificium in ex. sq. explicandum decidi debeat, pro vtra radice parti imaginariae signum positium et pro vtra negatium praefigendum sit. Hoc modo inuenti sunt valores sequentes, vbi signum superius radici priori, inferius posteriori respondere supponitur.

$$\begin{aligned}
 [1] \text{ et } [18] &= -0,6772815716 \pm 0,7357239107i \\
 [2] \text{ et } [17] &= -0,0825793455 \pm 0,9965844930i \\
 [3] \text{ et } [16] &= 0,7891405094 \pm 0,6142127127i \\
 [4] \text{ et } [15] &= -0,9863613034 \pm 0,1645945903i \\
 [5] \text{ et } [14] &= 0,5469481581 \pm 0,8371664783i \\
 [6] \text{ et } [13] &= 0,2454854871 \pm 0,9694002659i \\
 [7] \text{ et } [12] &= -0,8794737512 \pm 0,4759473930i \\
 [8] \text{ et } [11] &= 0,9458172417 \pm 0,3246994692i \\
 [9] \text{ et } [10] &= -0,4016954247 \pm 0,9157733267i
 \end{aligned}$$

354. *Exemplum secundum pro $n = 17$.*
Hic habetur $n - 1 = 2.2.2.2$, quamobrem calculus radicum Ω ad quatuor aequationes quadraticas reducendus erit. Pro radice primitiva hic accipiemus numerum 3, cuius potestates residua minima sequentia secundum modulum 17 sup-
peditant:

$$\begin{array}{cccccccccccccccc}
 0.1.2. & 3. & 4.5. & 6. & 7. & 8. & 9.10.11.12.13.14.15. \\
 1.3.9.10.13.5.15.11.16.14. & 8. & 7. & 4.12. & 2. & 6
 \end{array}$$

Hinc emergunt distributiones sequentes complexus Ω in periodos duas octonorum, quatuor quaternorum, octo binorum terminorum:

$$\Omega = (16, 1) \left\{ \begin{array}{l} (8, 1) \left\{ \begin{array}{l} (4, 1) \left(\begin{array}{l} (2, 1) \dots [1], [16] \\ (2, 13) \dots [4], [13] \end{array} \right) \\ (4, 9) \left(\begin{array}{l} (2, 9) \dots [8], [9] \\ (2, 15) \dots [2], [15] \end{array} \right) \end{array} \right. \\ (8, 3) \left\{ \begin{array}{l} (4, 3) \left(\begin{array}{l} (2, 3) \dots [3], [14] \\ (2, 5) \dots [5], [12] \end{array} \right) \\ (4, 10) \left(\begin{array}{l} (2, 10) \dots [7], [10] \\ (2, 11) \dots [6], [11] \end{array} \right) \end{array} \right. \end{array} \right.$$

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