teristic and minimal polynomials, respectively. What all this tells us is the following. If we consider the relation of similarity on the set of 3×3 matrices over F, the equivalence classes are in one-one correspondence with ordered pairs (f, p) of monic polynomials over F which satisfy (a) and (b).

A.6. The Axiom of Choice

Loosely speaking, the Axiom of Choice is a rule (or principle) of thinking which says that, given a family of non-empty sets, we can choose one element out of each set. To be more precise, suppose that we have an index set A and for each α in A we have an associated set S_α, which is non-empty. To 'choose' one member of each S_α means to give a rule f which associates with each α some element $f(\alpha)$ in the set S_α. The axiom of choice says that this is possible, i.e., given the family of sets $\{S_\alpha\}$, there exists a function f from A into

$$\bigcup_\alpha S_\alpha$$

such that $f(\alpha)$ is in S_α for each α. This principle is accepted by most mathematicians, although many situations arise in which it is far from clear how any explicit function f can be found.

The Axiom of Choice has some startling consequences. Most of them have little or no bearing on the subject matter of this book; however, one consequence is worth mentioning: Every vector space has a basis. For example, the field of real numbers has a basis, as a vector space over the field of rational numbers. In other words, there is a subset S of R which is linearly independent over the field of rationals and has the property that each real number is a rational linear combination of some finite number of elements of S. We shall not stop to derive this vector space result from the Axiom of Choice. For a proof, we refer the reader to the book by Kelley in the bibliography.

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Index

A

B

C

G

H

I

J

K

L