

tituatur, & Chorda $A\epsilon$ jungatur: ita ut nunc rectæ MQ & RMS , C.A.P. V.
 ductæ, ut ante, per M lâteribus AB & BD parallelæ, latera
 quadrilateri $ABDc$ secant in punctis p , Q , R & s ; similis
 proprietas locum habebit. Cum enim sit $MP \cdot MQ : BQ \times$
 $DQ = cG \cdot cH : BH \cdot DH$ seu $MP \cdot MQ : MR \cdot MS =$
 $cG \cdot cH : BH \cdot DH$, ob rectam RS ipsi BD parallelam &
 æqualem. Triangula vero similia APP , AGc & DSS , cHD ,
 præbent has proportiones $Pp : AP = Gc : AG$; seu, ob $AP :$
 $AG = BQ : BH$, hanc $Pp : BQ = Gc : BH$: altera similitudo
 dat hanc $DS(MQ) : Ss = cH : DH$, quibus conjunctis fit

$$MQ \cdot Pp : MR \cdot Ss = cG \cdot cH : BH \cdot DH, \text{ ob } BQ = MR.$$

Hæc proportio cum superiori collata præbet

$$MP \cdot MQ : MR \cdot MS = Pp \cdot MQ : MR \cdot Ss,$$

unde addendo antecedentes & consequentes fit

$$MP \cdot MQ : MR \cdot MS = Mp \cdot MQ : MR \cdot Ms,$$

ubicunque ergo sumantur puncta c & M in Curva, erit semper
 ratio $Mp \cdot MQ$ ad $MR \cdot Ms$ eadem, dummodo rectæ MQ
 & Rs per M ducantur Chordis AB & BD parallelæ. Ex su-
 periore vero proportione sequitur fore $MP : MS = Mp : Ms$.
 Cum igitur, variato puncto c , tantum puncta p & s mutentur,
 erit Mp ad Ms in data ratione, utcunque punctum c varietur,
 dum punctum M fixum servatur.

99. Quod si quatuor quæcunque puncta A , B , C , D in TAB. VI.
 Linea secundi ordinis fuerint data, eaque jungantur rectis, Fig. 23.
 ut habeatur trapezium inscriptum $ABDC$, proprietas Sectio-
 num conicarum latissime patens ex præcedenti deducitur. Sci-
 licet, si ex Curvæ puncto M ad singula trapezii latera
 sub datis angulis ducantur rectæ MP , MQ , MR & MS ,
 erunt semper rectangula binarum harum linearum ad opposita la-
 tera ductarum inter se in data ratione, nempe erit $MP \cdot MQ$
 ad

L I B . II. ad $MR.MS$ in data ratione eadem, ubicunque punctum M in Curva capiatur, dummodo anguli ad $P, Q, R, & S$ iidem serventur. Ad hoc ostendendum ducantur per M duas rectas $Mq & rs$, illa lateri AB hæc lateri BD parallela, ac notentur intersectionum cum lateribus trapezii puncta $p, q, r, & s$: eritque per prius inventum Mp, Mq ad Mr, Ms in data ratione. Propter omnes autem angulos datos datae erunt ratios $MP: Mp, MQ: Mq, MR: Mr, & MS: Ms$, ex quibus sequitur fore MP, MQ ad MR, MS in data quoque ratione.

T A B . VI. 100. Quoniam supra vidimus, si Ordinatae parallelae MN ,
Fig. 24. m_n producantur, quoad tangentis cuipiam CPp occurrand in $P & p$, fore $PM.PN: CP^2 = pm. pn: Cp^2$. Quare, si puncta $L & l$ notentur, ut sit PL media proportionalis inter $PM & PN$, pariterque pl media proportionalis inter $pm & pn$, erit $PL^2: CP^2 = pl^2: Cp^2$; ideoque erit $PL: CP = pl: Cp$, unde patet omnia puncta L, l in Linea recta per punctum contactus C transeunte esse sita. Quare, si una Applicata PMN ita secetur in L ut sit $PL^2 = PM.PN$, recta CLD per puncta $C & L$ ducta omnes reliquas Applicatas pmn ita quoque secabit in l ut sit pl media proportionalis inter $pm & pn$. Vel, si duas Applicatae $PN & pn$ ita in punctis $L & l$ secentur, ut sit $PL^2 = PM.PN & pl^2 = pm.pn$ recta per $L & l$ producta per punctum contactus C transibit, atque omnes reliquas Applicatas illis parallelas in eadem ratione secabit.

T A B . VI. 101. His Linearum secundi ordinis proprietatibus, quæ ex forma æquationis immediate consequuntur, expositis; progrediamur ad alias magis reconditas investigandas. Sit igitur proposita æquatio pro his Lineis secundi ordinis generalis

$$yy + \frac{(\epsilon x + \gamma)}{\zeta}y + \frac{\delta xx + \epsilon x + \alpha}{\zeta} = 0,$$

ex qua cum cuivis Abscissæ $AP=x$, duplex Applicata y
nempe

nempe PM & PN respondeat, positio Diametri omnes Or- CAP. V.
dinatas MN bifariam secantis definiri potest. Sit enim IG —
ista Diameter, quæ Ordinatam MN secabit in puncto medio
 L , quod ergo punctum est in Diametro. Ponatur $PL = z$;
&, cum sit $z = \frac{1}{2} PM + \frac{1}{2} PN$, erit $z = \frac{\varepsilon x - \gamma}{2\zeta}$,
seu $z\zeta - \varepsilon x + \gamma = 0$, quæ est æquatio positionem Diametri
 IG præbens.

102. Hinc porro longitudo Diametri IG definiri poterit,
quæ dat loca bina in Curva, ubi puncta M & N coincidunt,
seu ubi sit $PM = PN$. Ex æquatione vero dantur $PM +$
 $PN = \frac{\varepsilon x - \gamma}{2\zeta}$ & $PM \cdot PN = \frac{\delta xx + \varepsilon x + \alpha}{2\zeta}$, unde fit
 $(PM - PN)^2 = (PM + PN)^2 - 4PM \cdot PN =$
 $(\varepsilon\varepsilon - 4\delta\zeta)xx + 2(\varepsilon\gamma - 2\varepsilon\zeta)x + (\gamma\gamma - 4\alpha\zeta) = 0$, seu
 $xx - \frac{2(2\varepsilon\zeta - \varepsilon\gamma)}{\varepsilon\varepsilon - 4\delta\zeta}x + \frac{\gamma\gamma - 4\alpha\zeta}{\varepsilon\varepsilon - 4\delta\zeta} = 0$, cuius æquatio-
nis propterea radices sunt AK & AH ita ut sit $AK + AH =$
 $\frac{4\varepsilon\zeta - 2\varepsilon\gamma}{\varepsilon\varepsilon - 4\delta\zeta}$ & $AK \cdot AH = \frac{\gamma\gamma - 4\alpha\zeta}{\varepsilon\varepsilon - 4\delta\zeta}$: hinc fit $(AH -$
 $AK)^2 = KH^2 = \frac{4(2\varepsilon\zeta - \varepsilon\gamma)^2 - 4(\varepsilon\varepsilon - 4\delta\zeta)(\gamma\gamma - 4\alpha\zeta)}{(\varepsilon\varepsilon - 4\delta\zeta)^2}$.

At est $IG^2 = \frac{\varepsilon\varepsilon + 4\zeta\zeta}{4\zeta\zeta} KH^2$, si quidem Applicatae ad A-
xem normales statuantur.

103. Sint ista Applicatae, quas hic sumus contemplati, nor-
males ad Axem AH ; nunc vero hinc quæramus æquationem
pro Applicatis obliquangulis. Ducatur ergo ex quovis Curvæ
puncto M ad Axem Applicata obliquangula Mp faciens cum
Axe angulum MpH , cuius Sinus sit $= \mu$ & Cosinus $= v$.

Sit nova Abscissa $Ap = t$, Applicata $pM = u$, erit $\frac{y}{u} = \mu$
& $\frac{Pp}{u} = v$, unde erit $y = \mu u$ & $x = t + v u$, qui valores

L I B . II . in æquatione inter x & y , quæ erat $o = \alpha + \epsilon x + \gamma y + \delta xx + \epsilon xy + \zeta yy$, substituti præbent

$$\begin{aligned} o = & \alpha + \epsilon t + \nu \epsilon u + \delta tt + 2 \nu \delta tu + \nu \nu \delta uu \\ & + \mu \gamma uu \quad + \mu \epsilon tu \quad + \mu \nu \epsilon uu \\ & + \mu \mu \zeta uu \end{aligned}$$

seu

$$uu + \frac{((\mu \epsilon + 2 \nu \delta) t + \mu \gamma + \nu \epsilon) u + \delta tt + \epsilon t + \alpha}{\mu \mu \zeta + \mu \nu \epsilon + \nu \nu \delta} = o.$$

104. Hic ergo iterum quævis Applicata duplicem habebit valorem, nempe pM & pn : quare Ordinatarum Mn Diameter ilg pari modo ut ante definietur. Scilicet, bisecta Ordinata Mn in l erit l , punctum in Diametro. Ponatur ergo $pl = v$, erit $v = \frac{pM + pn}{2} = \frac{(\mu \epsilon + 2 \nu \delta) t - \mu \gamma - \nu \epsilon}{2(\mu \mu \zeta + \mu \nu \epsilon + \nu \nu \delta)}$. Demittatur ex l in Axem AH perpendiculum lg , ac ponatur $Aq = p$, $ql = q$, erit $\mu = \frac{q}{v}$ & $v = \frac{pq}{q} = \frac{p}{\mu} t$, unde fit $v = \frac{q}{\mu}$, & $t = p - rv = p - \frac{vq}{\mu}$. Substituantur hi valores in æquatione inter t & v ante inventa, & prodibit $\frac{q}{\mu} = \frac{-\mu \epsilon p - 2 \nu \delta p + \nu \epsilon q + 2 \nu \nu \delta q}{2\mu \mu \zeta + 2\mu \nu \epsilon + 2\nu \nu \delta}$

seu

$$(2\mu \mu \zeta + \mu \nu \epsilon)q + (\mu \mu \epsilon + 2\mu \nu \delta)p + \mu \mu \gamma + \mu \nu \epsilon = o,$$

$$(2\mu \zeta + \nu \epsilon)q + (\mu \epsilon + 2\nu \delta)p + \gamma \mu + \nu \epsilon = o,$$

qua æquatione positio Diametri ig definitur.

105. Prior Diameter IG , cuius positio per hanc æquationem determinabatur $2\zeta z + \epsilon x + \gamma = o$, producta cum Axe concurrat in O , eritque $AO = \frac{-\gamma}{\epsilon}$; hinc fit $PO = \frac{-\gamma}{\epsilon} - x$, & anguli LOP tangens erit $= \frac{2}{PO} = \frac{-\epsilon z}{\epsilon x + \gamma}$

$= \frac{\epsilon}{2\zeta}$, & tangens anguli MLG , sub quo Diameter IG CAP. V.
 Ordinatas MN bisecat erit $= \frac{2\zeta}{\epsilon}$. Altera vero Diameter ig
 producta Axi occurrat in o , eritque $Ao = \frac{\mu\gamma - \nu\epsilon}{\mu\epsilon + 2\nu\delta}$, &
 anguli Aol tangens erit $= \frac{\mu\epsilon + 2\nu\delta}{2\mu\zeta + \nu\epsilon}$. Cum jam arguli AOL
 tangens sit $= \frac{\epsilon}{2\zeta}$, ambæ Diametri se mutuo intersecabunt in
 puncto quodam C , facientque angulum $OCo = AOL - AOL$,
 cuius propterea tangens est $= \frac{4\nu\delta\zeta - \gamma\epsilon\epsilon}{4\mu\zeta + 2\nu\delta\epsilon + 2\nu\zeta + \mu\epsilon\epsilon}$. An-
 gulus autem, sub quo hæc altera Diameter suas Ordinatas bi-
 secat, est $Mlo = 180^\circ - lpo - AOL$: hujus propterea tan-
 gens est $= \frac{2\mu\mu\zeta + 2\mu\nu\epsilon + 2\nu\nu\delta}{\mu\mu\epsilon + 2\mu\nu\delta - 2\mu\nu\zeta - \nu\nu\epsilon}$.

106. Inquiramus autem in punctum C , ubi hæc duæ Dia-
 metri se mutuo intersecant: ex quo ad Axem perpendicularum
 CD demittatur, ac vocetur $AD = g$, $CD = h$; eritque
 primo, quod C in Diametro IG extat, $2\zeta h + \epsilon g + \gamma = 0$.
 Deinde, quia C quoque in Diametro ig reperitur, erit

$$(2\mu\zeta + \nu\epsilon)h + (\mu\epsilon + 2\nu\delta)g + \mu\gamma + \nu\epsilon = 0.$$

Subtrahatur hinc prior æquatio per μ multiplicata, ac remanebit

$$\nu\epsilon h + 2\nu\delta g + \nu\epsilon = 0, \text{ seu } \epsilon h + 2\delta g + \epsilon = 0.$$

Ex his fit $h = \frac{-\epsilon\epsilon - \gamma}{2\zeta} = \frac{2\delta g - \epsilon}{\epsilon}$, ideoque
 $(\epsilon\epsilon - 4\delta\zeta)g = 2\zeta\zeta - \gamma\epsilon$, & $g = \frac{2\zeta\zeta - \gamma\epsilon}{\epsilon\epsilon - 4\delta\zeta}$ & $h =$
 $\frac{2\gamma\delta - \epsilon\epsilon}{\epsilon\epsilon - 4\delta\zeta}$. In quibus determinationibus cum non insint
 quantitates μ & ν , a quibus obliquitas Applicatarum ρMn
 pendet, manifestum est punctum C idem manere, utcunque
 obliquitas varietur.