

$q^6 - 6q^4 + 9qq - 2, (2, 7) = q^7 - 7q^5$   
 $14q^3 - 7q, (2, 8) = q^8 - 8q^6 + 20q^4 -$   
 $16qq + 2, (2, 9) = q^9 - 9q^7 + 27q^5 -$   
 $30q^3 + 9q.$  Commodius quam per praecepta  
 art. 346 hae aequationes in casu praesenti per  
 reflexiones sequentes euolui possunt. Supponen-  
 do [1] =  $\cos \frac{kP}{19} + i \sin \frac{kP}{19}$ , fit [18] =  
 $\cos \frac{18kP}{19} + i \sin \frac{18kP}{19} = \cos \frac{kP}{19} - i \sin \frac{kP}{19},$   
 adeoque  $(2, 1) = 2\cos \frac{kP}{19};$  nec non ge-  
 neraliter  $[\lambda] = \cos \frac{\lambda kP}{19} + i \sin \frac{\lambda kP}{19},$  adeoque  
 $(2, \lambda) = [\lambda] + [18\lambda] = [\lambda] + [-\lambda] =$   
 $2\cos \frac{\lambda kP}{19}.$  Quare si  $\frac{1}{2}q = \cos \omega,$  erit  $(2, 2) = 2\cos 2\omega, (2, 3) = 2\cos 3\omega$  etc., vnde per ae-  
 quationes notas pro cosinibus angulorum multi-  
 plicium eadem formulae vt supra deriuantur. —  
 Iam ex his formulis valores numerici sequentes  
 eliciuntur:

$(2, 2) = -0,1651586909$	$(2, 6) = 0,4909709743$
$(2, 3) = 1,5782810188$	$(2, 7) = -1,7589475024$
$(2, 4) = -1,9727226068$	$(2, 8) = 1,8916344834$
$(2, 5) = 1,0938963162$	$(2, 9) = -0,8033908493$

Valores ipsorum  $(2, 7), (2, 8)$  etiam ex ae-  
 quatione (B), cuius duae reliquae radices sunt,  
 elici possunt, dubiumque, *vtra* harum radicum  
 fiat  $(2, 7)$  et *vtra*  $(2, 8)$ , vel per calculum  
 approximatum secundum formulas praecc., vel  
 per tabulas sinuum tolletur, quae obiter tantum  
 consultae ostendunt, fieri  $(2, 1) = 2\cos \omega$  po-

nendo  $\omega = \frac{7}{19}P$ , vnde fieri oportet  $(2, 7) =$   
 $2\cos \frac{49}{19}P = 2\cos \frac{8}{19}P$ , et  $(2, 8) = 2\cos \frac{56}{19}P$   
 $= 2\cos \frac{1}{19}P$ . — Similiter aggregata  $(2, 2)$ ,  $(2,$   
 $3)$ ,  $(2, 5)$  etiam per aequationem  $x^3 - (6,$   
 $2)xx + ((6, 1) + (6, 2))x - 2 - (6, 4)$   
 $= 0$ , cuius radices sunt, inuenire licet, incertitudineque, quaenam radices illis aggregatis resp.  
aequales statuendae sint, prorsus eodem modo  
remouebitur, vt ante; et perinde etiam ag-  
gregata  $(2, 4)$ ,  $(2, 6)$ ,  $(2, 9)$  per aequationem  
 $x^3 - (6, 4)xx + ((6, 2) + (6, 4))x - 2$   
 $- (6, 1) = 0$  elici poterunt.

Denique [1] et [18] sunt radices aequatio-  
nis  $xx - (2, 1)x + 1 = 0$ , quarum altera  
fit  $= \frac{1}{2}(2, 1) + i\sqrt{(1 - \frac{1}{4}(2, 1)^2)} = \frac{1}{2}(2,$   
 $1) + i\sqrt{(\frac{1}{2} - \frac{1}{4}(2, 2))}$ , altera  $= \frac{1}{2}(2, 1) -$   
 $i\sqrt{(\frac{1}{2} - \frac{1}{4}(2, 2))}$ , hinc valores numerici  $=$   
 $- 0,6772815716 \pm 0,7357239107i$ . Sedecim  
radices reliquae vel ex euolutione potestatum  
vtriusuis harum radicum, vel e solutione octo  
aliarum similium aequationum deduci possunt,  
vbi in methodo posteriori vel per tabulas sinuum  
vel per artificium in ex. sq. explicandum decidi  
debet, pro vtra radice parti imaginariae  
signum positium et pro vtra negatiuum praefi-  
gendum sit. Hoc modo inuenti sunt valores  
sequentes, vbi signum superius radici priori,  
inferius posteriori respondere supponitur.

$$\begin{aligned}
 [1] \text{ et } [18] &= -0,6772815716 \pm 0,7357239107i \\
 [2] \text{ et } [17] &= -0,0825793455 \pm 0,9965844930i \\
 [3] \text{ et } [16] &= 0,7891405094 \pm 0,6142127127i \\
 [4] \text{ et } [15] &= -0,9863613034 \pm 0,1645945903i \\
 [5] \text{ et } [14] &= 0,5469481581 \pm 0,8371664783i \\
 [6] \text{ et } [13] &= 0,2454854871 \pm 0,9694002659i \\
 [7] \text{ et } [12] &= -0,8794737512 \pm 0,4759473930i \\
 [8] \text{ et } [11] &= 0,9458172417 \pm 0,3246994692i \\
 [9] \text{ et } [10] &= -0,4016954247 \pm 0,9157733267i
 \end{aligned}$$

354. *Exemplum secundum pro n = 17.*  
 Hic habetur  $n - 1 = 2 \cdot 2 \cdot 2 \cdot 2$ , quamobrem calculus radicum  $\Omega$  ad quatuor aequationes quadráticas reducendus erit. Pro radice primitiva hic accipiemus numerum 3, cuius potestates residua minima sequentia secundum modulum 17 suppeditant:

$$\begin{array}{ccccccccc}
 0. & 1. & 2. & 3. & 4. & 5. & 6. & 7. & 8. & 9. & 10. & 11. & 12. & 13. & 14. & 15. \\
 1. & 3. & 9. & 10. & 1 & 3. & 5. & 15. & 11. & 16. & 14. & 8. & 7. & 4. & 12. & 2. & 6
 \end{array}$$

Hinc emergunt distributiones sequentes complexus  $\Omega$  in periodos duas octonorum, quatuor quaternionorum, octo binorum terminorum:

$$\Omega = (16, 1) \left\{ \begin{array}{l} (4, 1) ((2, 1) \dots [1], [16] \\ (4, 13) ((2, 13) \dots [4], [13]) \\ (4, 9) ((2, 9) \dots [8], [9]) \\ (4, 15) ((2, 15) \dots [2], [15]) \\ (4, 5) ((2, 3) \dots [3], [14]) \\ (4, 5) ((2, 5) \dots [5], [12]) \\ (4, 10) ((2, 10) \dots [7], [10]) \\ (4, 10) ((2, 11) \dots [6], [11]) \end{array} \right.$$

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