Figure 11.5: Pierre de Fermat

cuit, and was buried there. However, in 1675 his remains were transferred to the Fermat family vault in the Church of the Augustines in Toulouse.

Fermat's apparent refusal to put mathematics ahead of his profession makes the depth and range of his mathematical achievement all the more perplexing. We may never know enough about Fermat to understand his mathematical thought, but the attempts that have been made so far raise hopes that more can be done. Mahoney (1973) gives a survey of all of Fermat's mathematics but fails to do justice to the number theory. Weil (1984) has a brilliant analysis of Fermat's number theory, but other facets of Fermat's mathematics have yet to be analyzed with comparable insight.

12

Elliptic Functions

12.1 Elliptic and Circular Functions

The story of elliptic functions is one of the most curious in the history of mathematics, beginning with a complicated analytic idea—integrals of the form $\int R(t, \sqrt{p(t)})\, dt$, where R is a rational function and p is a polynomial of degree 3 or 4—and reaching a climax with a simple geometric idea—the torus surface. Perhaps the best way to understand it is to compare it with a fictitious history of circular functions which begins with the integral $\int dt/\sqrt{1-t^2}$ and ends with the discovery of the circle. Unlikely as this fiction is, it was paralleled by the actual development of elliptic functions between the 1650s and the 1850s.

The late recognition of the geometric nature of elliptic functions was due to late recognition of the existence and geometric nature of complex numbers. In fact, the later history of elliptic functions unfolds alongside the development of complex numbers, which is the subject of Chapters 14 to 16. In the present chapter we are concerned mainly with the history up to 1800, before complex numbers entered in a really essential way. However, there are some subplots of the main story that do not require complex numbers for their understanding and nicely show the parallel with the fictitious history of circular functions. It is convenient to relate one of these now, because it illustrates the parallel in a simplified way and also ties up a loose end from Chapter 11—the parameterization of cubic curves.