

Καὶ ἐπεὶ εἰς παραλλήλους τὰς ΑΔ, ΕΓ εὐθεῖα ἐνέπεσεν ἡ ΑΓ, ἡ ἄρα ὑπὸ ΑΓΕ γωνία ἵση ἐστὶ τῇ ὑπὸ ΓΑΔ. ἀλλ᾽ ἡ ὑπὸ ΓΑΔ τῇ ὑπὸ ΒΑΔ ὑπόκειται ἵση· καὶ ἡ ὑπὸ ΒΑΔ ἄρα τῇ ὑπὸ ΑΓΕ ἐστιν ἵση. πάλιν, ἐπεὶ εἰς παραλλήλους τὰς ΑΔ, ΕΓ εὐθεῖα ἐνέπεσεν ἡ ΒΑΕ, ἡ ἐκτὸς γωνία ἡ ὑπὸ ΒΑΔ ἵση ἐστὶ τῇ ἐντὸς τῇ ὑπὸ ΑΕΓ. ἐδείχθη δὲ καὶ ἡ ὑπὸ ΑΓΕ τῇ ὑπὸ ΒΑΔ ἵση· καὶ ἡ ὑπὸ ΑΓΕ ἄρα γωνία τῇ ὑπὸ ΑΕΓ ἐστιν ἵση· ὥστε καὶ πλευρὰ ἡ ΑΕ πλευρῷ τῇ ΑΓ ἐστιν ἵση. καὶ ἐπεὶ τριγώνου τοῦ ΒΓΕ παρὰ μίαν τῶν πλευρῶν τὴν ΕΓ ἥκται ἡ ΑΔ, ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΒΑ πρὸς τὴν ΑΕ. ἵση δὲ ἡ ΑΕ τῇ ΑΓ· ὡς ἄρα ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΒΑ πρὸς τὴν ΑΓ.

Ἄλλὰ δὴ ἔστω ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΒΑ πρὸς τὴν ΑΓ, καὶ ἐπεζεύχθω ἡ ΑΔ· λέγω, ὅτι δίχα τέτμηται ἡ ὑπὸ ΒΑΓ γωνία ὑπὸ τῆς ΑΔ εὐθείας.

Τῶν γὰρ αὐτῶν κατασκευασθέντων, ἐπεὶ ἐστιν ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΒΑ πρὸς τὴν ΑΓ, ἀλλὰ καὶ ὡς ἡ ΒΔ πρὸς τὴν ΔΓ, οὕτως ἐστὶν ἡ ΒΑ πρὸς τὴν ΑΕ· τριγώνου γὰρ τοῦ ΒΓΕ παρὰ μίαν τὴν ΕΓ ἥκται ἡ ΑΔ· καὶ ὡς ἄρα ἡ ΒΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΒΑ πρὸς τὴν ΑΕ. ἵση ἄρα ἡ ΑΓ τῇ ΑΕ· ὥστε καὶ γωνία ἡ ὑπὸ ΑΕΓ τῇ ὑπὸ ΑΓΕ ἐστιν ἵση. ἀλλ᾽ ἡ μὲν ὑπὸ ΑΕΓ τῇ ἐκτὸς τῇ ὑπὸ ΒΑΔ [ἐστιν] ἵση, ἡ δὲ ὑπὸ ΑΓΕ τῇ ἐναλλάξ τῇ ὑπὸ ΓΑΔ ἐστιν ἵση· καὶ ἡ ὑπὸ ΒΑΔ ἄρα τῇ ὑπὸ ΓΑΔ ἐστιν ἵση. ἡ ἄρα ὑπὸ ΒΑΓ γωνία δίχα τέτμηται ὑπὸ τῆς ΑΔ εὐθείας.

Ἐὰν ἄρα τριγώνου ἡ γωνία δίχα τμηθῇ, ἡ δὲ τέμνουσα τὴν γωνίαν εὐθεῖα τέμνη καὶ τὴν βάσιν, τὰ τῆς βάσεως τμήματα τὸν αὐτὸν ἔξει λόγον ταῖς λοιπαῖς τοῦ τριγώνου πλευραῖς καὶ ἐὰν τὰ τῆς βάσεως τμήματα τὸν αὐτὸν ἔχῃ λόγον ταῖς λοιπαῖς τοῦ τριγώνου πλευραῖς, ἡ ἀπὸ τῆς κορυφῆς ἐπὶ τὴν τομὴν ἐπιζευγνυμένη εὐθεῖα δίχα τέμνει τὴν τοῦ τριγώνου γωνίαν· ὅπερ ἔδει δεῖξαι.

(CE) at (point) E.[†]

And since the straight-line AC falls across the parallel (straight-lines) AD and EC , angle ACE is thus equal to CAD [Prop. 1.29]. But, (angle) CAD is assumed (to be) equal to BAD . Thus, (angle) BAD is also equal to ACE . Again, since the straight-line BAE falls across the parallel (straight-lines) AD and EC , the external angle BAD is equal to the internal (angle) AEC [Prop. 1.29]. And (angle) ACE was also shown (to be) equal to BAD . Thus, angle ACE is also equal to AEC . And, hence, side AE is equal to side AC [Prop. 1.6]. And since AD has been drawn parallel to one of the sides EC of triangle BCE , thus, proportionally, as BD is to DC , so BA (is) to AE [Prop. 6.2]. And AE (is) equal to AC . Thus, as BD (is) to DC , so BA (is) to AC .

And so, let BD be to DC , as BA (is) to AC . And let AD have been joined. I say that angle BAC has been cut in half by the straight-line AD .

For, by the same construction, since as BD is to DC , so BA (is) to AC , then also as BD (is) to DC , so BA is to AE . For AD has been drawn parallel to one (of the sides) EC of triangle BCE [Prop. 6.2]. Thus, also, as BA (is) to AC , so BA (is) to AE [Prop. 5.11]. Thus, AC (is) equal to AE [Prop. 5.9]. And, hence, angle AEC is equal to ACE [Prop. 1.5]. But, AEC [is] equal to the external (angle) BAD , and ACE is equal to the alternate (angle) CAD [Prop. 1.29]. Thus, (angle) BAD is also equal to CAD . Thus, angle BAC has been cut in half by the straight-line AD .

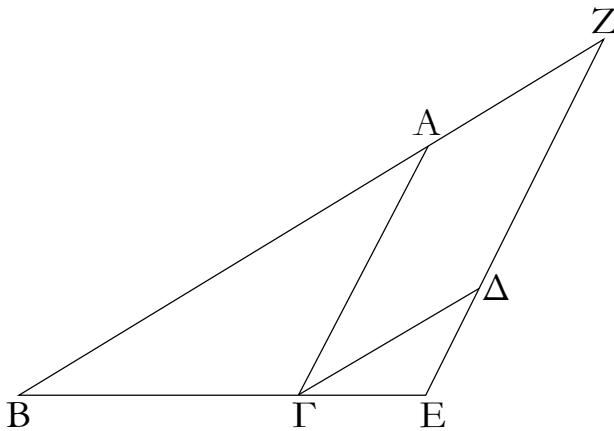
Thus, if an angle of a triangle is cut in half, and the straight-line cutting the angle also cuts the base, then the segments of the base will have the same ratio as the remaining sides of the triangle. And if the segments of the base have the same ratio as the remaining sides of the triangle, then the straight-line joining the vertex to the cutting (point) will cut the angle of the triangle in half. (Which is) the very thing it was required to show.

[†] The fact that the two straight-lines meet follows because the sum of ACE and CAE is less than two right-angles, as can easily be demonstrated.

See Post. 5.

δ' .

Τῶν ἴσογωνίων τριγώνων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας καὶ ὁμόλογοι αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι.



Ἐστω ἴσογωνια τρίγωνα τὰ ABC , ΔGE ἵσην ἔχοντα τὴν μὲν ὑπὸ ABC γωνίαν τῇ ὑπὸ ΔGE , τὴν δὲ ὑπὸ BAG τῇ ὑπὸ ΓDE καὶ ἔτι τὴν ὑπὸ ΔABG τῇ ὑπὸ ΓED λέγω, ὅτι τῶν ABG , ΔGE τριγώνων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας καὶ ὁμόλογοι αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι.

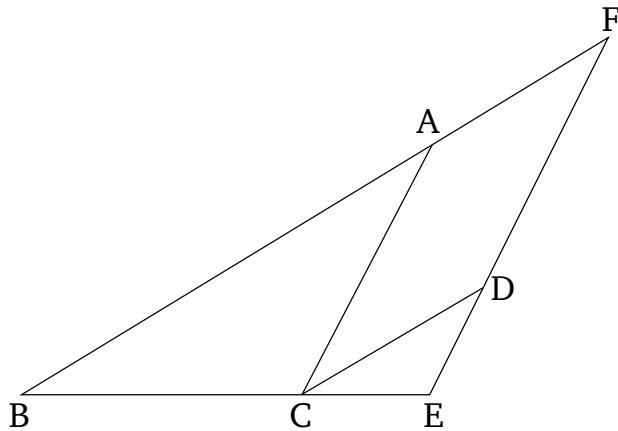
Κείσθω γάρ ἐπ' εὐθείας ἡ BG τῇ GE . καὶ ἐπεὶ αἱ ὑπὸ ABG , AGB γωνίαι δύο ὄρθῳ ἐλάττονές εἰσιν, ἵση δὲ ἡ ὑπὸ AGB τῇ ὑπὸ ΔEG , αἱ ἄρα ὑπὸ ABG , ΔEG δύο ὄρθῳ ἐλάττονές εἰσιν· αἱ BA , ED ἄρα ἐκβαλλόμεναι συμπεσοῦνται. ἐκβεβλήσθωσαν καὶ συμπιπτέωσαν κατὰ τὸ Z .

Καὶ ἐπεὶ ἵση ἐστὶν ἡ ὑπὸ ΔGE γωνία τῇ ὑπὸ ABG , παράλληλος ἐστιν ἡ BZ τῇ ΓD . πάλιν, ἐπεὶ ἵση ἐστὶν ἡ ὑπὸ ΔGB τῇ ΔEG , παράλληλος ἐστιν ἡ AG τῇ ZE . παραλλήλογραμμον ἄρα ἐστὶ τὸ $ZAGD$. ἵση ἄρα ἡ μὲν ZA τῇ ΔG , ἡ δὲ AG τῇ ZD . καὶ ἐπεὶ τριγώνου τοῦ ZBE παρὰ μίαν τὴν ZE ἥκται ἡ AG , ἐστιν ἄρα ὡς ἡ BA πρὸς τὴν AZ , οὕτως ἡ BG πρὸς τὴν GE . ἵση δὲ ἡ AZ τῇ ΓD . ὡς ἄρα ἡ BA πρὸς τὴν ΓD , οὕτως ἡ BG πρὸς τὴν GE , καὶ ἐναλλάξ ὡς ἡ AB πρὸς τὴν BG , οὕτως ἡ ΔG πρὸς τὴν GE . πάλιν, ἐπεὶ παράλληλος ἐστιν ἡ ΓD τῇ BZ , ἐστιν ἄρα ὡς ἡ BG πρὸς τὴν GE , οὕτως ἡ ZD πρὸς τὴν ΔE . ἵση δὲ ἡ ZD τῇ AG . ὡς ἄρα ἡ BG πρὸς τὴν GE , οὕτως ἡ AB πρὸς τὴν BG , οὕτως ἡ ΔG πρὸς τὴν ΔE , καὶ ἐναλλάξ ὡς ἡ BG πρὸς τὴν ΓA , οὕτως ἡ GE πρὸς τὴν $E\Delta$. ἐπεὶ οὖν ἐδείχθη ὡς μὲν ἡ AB πρὸς τὴν BG , οὕτως ἡ ΔG πρὸς τὴν GE , ὡς δὲ ἡ BG πρὸς τὴν ΓA , οὕτως ἡ GE πρὸς τὴν $E\Delta$, διὸ ἵσου ἄρα ὡς ἡ BA πρὸς τὴν AG , οὕτως ἡ ΓD πρὸς τὴν ΔE .

Τῶν ἄρα ἴσογωνίων τριγώνων ἀνάλογόν εἰσιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας καὶ ὁμόλογοι αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι· ὅπερ ἔδει δεῖξαι.

Proposition 4

In equiangular triangles the sides about the equal angles are proportional, and those (sides) subtending equal angles correspond.



Let ABC and DCE be equiangular triangles, having angle ABC equal to DCE , and (angle) BAC to CDE , and, further, (angle) ACB to CED . I say that in triangles ABC and DCE the sides about the equal angles are proportional, and those (sides) subtending equal angles correspond.

Let BC be placed straight-on to CE . And since angles ABC and ACB are less than two right-angles [Prop 1.17], and ACB (is) equal to DEC , thus ABC and DEC are less than two right-angles. Thus, BA and ED , being produced, will meet [C.N. 5]. Let them have been produced, and let them meet at (point) F .

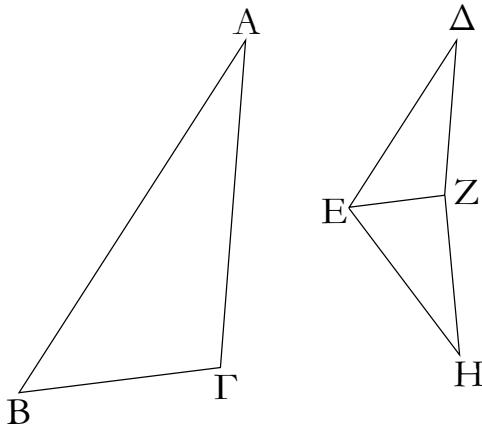
And since angle DCE is equal to ABC , BF is parallel to CD [Prop. 1.28]. Again, since (angle) ACB is equal to DEC , AC is parallel to FE [Prop. 1.28]. Thus, $FACD$ is a parallelogram. Thus, FA is equal to DC , and AC to FD [Prop. 1.34]. And since AC has been drawn parallel to one (of the sides) FE of triangle FBE , thus as BA is to AF , so BC (is) to CE [Prop. 6.2]. And AF (is) equal to CD . Thus, as BA (is) to CD , so BC (is) to CE , and, alternately, as AB (is) to BC , so DC (is) to CE [Prop. 5.16]. Again, since CD is parallel to BF , thus as BC (is) to CE , so FD (is) to DE [Prop. 6.2]. And FD (is) equal to AC . Thus, as BC is to CE , so AC (is) to DE , and, alternately, as BC (is) to CA , so CE (is) to ED [Prop. 6.2]. Therefore, since it was shown that as AB (is) to BC , so DC (is) to CE , and as BC (is) to CA , so CE (is) to ED , thus, via equality, as BA (is) to AC , so CD (is) to DE [Prop. 5.22].

Thus, in equiangular triangles the sides about the equal angles are proportional, and those (sides) subtend-

ing equal angles correspond. (Which is) the very thing it was required to show.

ε' .

Ἐάν δύο τρίγωνα τὰς πλευρὰς ἀνάλογον ἔχῃ, ισογώνια ἔσται τὰ τρίγωνα καὶ ἵσας ἔξει τὰς γωνίας, ὑφ' ἀς αἱ ὁμόλογοι πλευραὶ ὑποτείνουσιν.



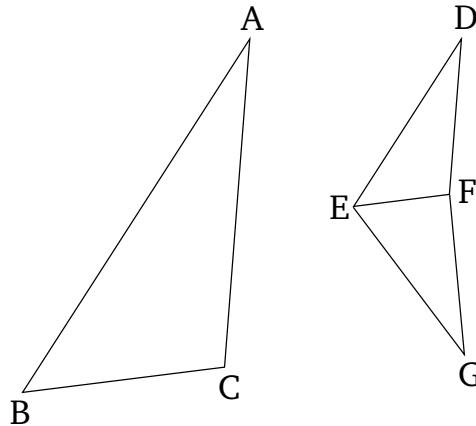
Ἔστω δύο τρίγωνα τὰ ABC , ΔEZ τὰς πλευρὰς ἀνάλογον ἔχοντα, ὡς μὲν τὴν AB πρὸς τὴν BG , οὕτως τὴν ΔE πρὸς τὴν EZ , ὡς δὲ τὴν BG πρὸς τὴν GA , οὕτως τὴν EZ πρὸς τὴν ZD , καὶ ἔτι ὡς τὴν BA πρὸς τὴν AG , οὕτως τὴν ED πρὸς τὴν ΔZ . λέγω, ὅτι ισογώνιόν ἐστι τὸ ABC τρίγωνον τῷ ΔEZ τριγώνῳ καὶ ἵσας ἔξουσι τὰς γωνίας, ὑφ' ἀς αἱ ὁμόλογοι πλευραὶ ὑποτείνουσιν, τὴν μὲν ὑπὸ ABC τῇ ὑπὸ ΔEZ , τὴν δὲ ὑπὸ BGA τῇ ὑπὸ EZH καὶ ἔτι τὴν ὑπὸ BAF τῇ ὑπὸ $E\Delta Z$.

Συνεστάτω γάρ πρὸς τῇ EZ εὐθείᾳ καὶ τοῖς πρὸς αὐτῇ σημείοις τοῖς E , Z τῇ μὲν ὑπὸ ABC γωνίᾳ ἵση ἡ ὑπὸ ZEH , τῇ δὲ ὑπὸ AGB ἵση ἡ ὑπὸ EZH . λοιπὴ ἄρα ἡ πρὸς τῷ A λοιπῇ τῇ πρὸς τῷ H ἔστιν ἵση.

Ἔστιν ἵση τὸ ABC τρίγωνον τῷ EHZ [τριγώνῳ]. τῶν ἄρα ABC , EHZ τριγώνων ἀνάλογον εἰσὶν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας καὶ ὁμόλογοι αἱ ὑπὸ τὰς ἵσας γωνίας ὑποτείνουσαι· ἔστιν ἄρα ὡς ἡ AB πρὸς τὴν BG , [οὕτως] ἡ HE πρὸς τὴν EZ . ἀλλ' ὡς ἡ AB πρὸς τὴν BG , οὕτως ὑπόκειται ἡ ΔE πρὸς τὴν EZ . ὡς ἄρα ἡ ΔE πρὸς τὴν EZ , οὕτως ἡ HE πρὸς τὴν EZ . ἐκατέρᾳ ἄρα τῶν ΔE , HE πρὸς τὴν EZ τὸν αὐτὸν ἔχει λόγον· ἵση ἄρα ἔστιν ἡ ΔE τῇ HE . διὰ τὰ αὐτὰ δὴ καὶ ἡ ΔZ τῇ HZ ἔστιν ἵση. ἐπεὶ οὖν ἵση ἔστιν ἡ ΔE τῇ EH , κοινὴ δὲ ἡ EZ , δύο δὴ αἱ ΔE , EZ δυσὶ ταῖς HE , EZ ἵσαι εἰσὶν· καὶ βάσις ἡ ΔZ βάσει τῇ HZ [ἔστιν] ἵση· γωνία ἄρα ἡ ὑπὸ ΔEZ γωνίᾳ τῇ ὑπὸ HEZ ἔστιν ἵση, καὶ τὸ ΔEZ τρίγωνον τῷ HEZ τριγώνῳ ἵσον, καὶ αἱ λοιπαὶ γωνίαι ταῖς λοιπαῖς γωνίαις ἵσαι, ὑφ' ἀς αἱ ἵσαι πλευραὶ ὑποτείνουσιν. ἵση ἄρα ἔστι καὶ ἡ μὲν ὑπὸ ΔEZ γωνία τῇ ὑπὸ HZE , ἡ δὲ ὑπὸ $E\Delta Z$ τῇ ὑπὸ EHZ . καὶ ἐπεὶ ἡ μὲν ὑπὸ $ZE\Delta$ τῇ ὑπὸ HEZ ἔστιν ἵση, ἀλλ' ἡ ὑπὸ HEZ τῇ ὑπὸ ABC , καὶ ἡ ὑπὸ

Proposition 5

If two triangles have proportional sides then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal.



Let ABC and DEF be two triangles having proportional sides, (so that) as AB (is) to BC , so DE (is) to EF , and as BC (is) to CA , so EF (is) to FD , and, further, as BA (is) to AC , so ED (is) to DF . I say that triangle ABC is equiangular to triangle DEF , and (that the triangles) will have the angles which corresponding sides subtend equal. (That is), (angle) ABC (equal) to DEF , BCA to EFD , and, further, BAC to EDF .

For let (angle) FEG , equal to angle ABC , and (angle) EFG , equal to ACB , have been constructed on the straight-line EF at the points E and F on it (respectively) [Prop. 1.23]. Thus, the remaining (angle) at A is equal to the remaining (angle) at G [Prop. 1.32].

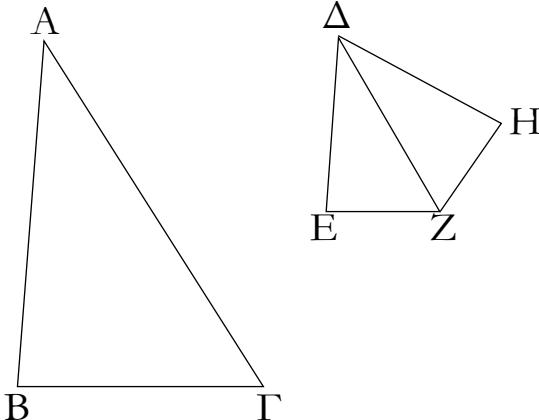
Thus, triangle ABC is equiangular to [triangle] EGF . Thus, for triangles ABC and EGF , the sides about the equal angles are proportional, and (those) sides subtending equal angles correspond [Prop. 6.4]. Thus, as AB is to BC , [so] GE (is) to EF . But, as AB (is) to BC , so, it was assumed, (is) DE to EF . Thus, as DE (is) to EF , so GE (is) to EF [Prop. 5.11]. Thus, DE and GE each have the same ratio to EF . Thus, DE is equal to GE [Prop. 5.9]. So, for the same (reasons), DF is also equal to GF . Therefore, since DE is equal to EG , and EF (is) common, the two (sides) DE , EF are equal to the two (sides) GE , EF (respectively). And base DF [is] equal to base FG . Thus, angle DEF is equal to angle GEF [Prop. 1.8], and triangle DEF (is) equal to triangle GEF , and the remaining angles (are) equal to the remaining angles which the equal sides subtend [Prop. 1.4]. Thus, angle DFE is also equal to GFE , and

ΑΒΓ ἄρα γωνία τῇ ὑπὸ ΔEZ ἐστιν ἵση. διὰ τὰ αὐτὰ δὴ καὶ ἡ ὑπὸ ΑΓΒ τῇ ὑπὸ ΔZE ἐστιν ἵση, καὶ ἔτι ἡ πρὸς τῷ Α τῇ πρὸς τῷ Δ· ἴσογώνιον ἄρα ἐστὶ τὸ ΑΒΓ τρίγωνον τῷ ΔEZ τριγώνῳ.

Ἐὰν ἄρα δύο τρίγωνα τὰς πλευράς ἀνάλογον ἔχῃ, ἴσογώνια ἔσται τὰ τρίγωνα καὶ ἵσας ἔξει τὰς γωνίας, ὥφετος ἂν ὅμοιοι πλευραὶ ὑποτείνουσιν· ὅπερ ἔδει δεῖξαι.

5'.

Ἐὰν δύο τρίγωνα μίαν γωνίαν μιᾷ γωνίᾳ ἵσην ἔχῃ, περὶ δὲ τὰς ἵσας γωνίας τὰς πλευράς ἀνάλογον, ἴσογώνια ἔσται τὰ τρίγωνα καὶ ἵσας ἔξει τὰς γωνίας, ὥφετος ὅμοιοι πλευραὶ ὑποτείνουσιν.



Ἐστω δύο τρίγωνα τὰ ΑΒΓ, ΔEZ μίαν γωνίαν τὴν ὑπὸ ΒΑΓ μιᾷ γωνίᾳ τῇ ὑπὸ ΕΔΖ ἵσην ἔχοντα, περὶ δὲ τὰς ἵσας γωνίας τὰς πλευράς ἀνάλογον, ὡς τὴν ΒΑ πρὸς τὴν ΑΓ, οὕτως τὴν ΕΔ πρὸς τὴν ΔΖ· λέγω, ὅτι ἴσογώνιόν ἐστι τὸ ΑΒΓ τρίγωνον τῷ ΔEZ τριγώνῳ καὶ ἵσην ἔξει τὴν ὑπὸ ΑΒΓ γωνίαν τῇ ὑπὸ ΔEZ, τὴν δὲ ὑπὸ ΑΓΒ τῇ ὑπὸ ΔΖE.

Συνεστάτω γάρ πρὸς τῇ ΔΖ εὐθείᾳ καὶ τοῖς πρὸς αὐτῇ σημείοις τοῖς Δ, Ζ όποτέρᾳ μὲν τῶν ὑπὸ ΒΑΓ, ΕΔΖ ἵση ὡς ΖΔΗ, τῇ δὲ ὑπὸ ΑΓΒ ἵση ἡ ὑπὸ ΔΖΗ· λοιπὴ ἄρα πρὸς τῷ Β γωνία λοιπὴ τῇ πρὸς τῷ Η ἵση ἐστίν.

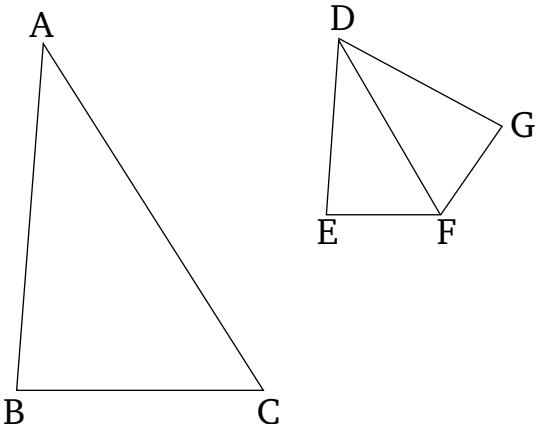
Ἴσογώνιον ἄρα ἐστὶ τὸ ΑΒΓ τρίγωνον τῷ ΔΖΗ τριγώνῳ. ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΒΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΗΔ πρὸς τὴν ΔΖ. ὑπόκειται δὲ καὶ ὡς ἡ ΒΑ πρὸς τὴν ΑΓ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΖ· καὶ ὡς ἄρα ἡ ΕΔ πρὸς τὴν ΔΖ, οὕτως ἡ ΗΔ πρὸς τὴν ΔΖ. Ἱση ἄρα ἡ ΕΔ τῇ ΔΗ· καὶ κοινὴ ἡ ΔΖ· δύο δὴ αἱ ΕΔ, ΔΖ δυσὶ ταῖς ΗΔ, ΔΖ ἵσας εἰσὶν· καὶ γωνία ἡ ὑπὸ ΕΔΖ γωνίᾳ τῇ ὑπὸ ΗΔΖ [ἐστιν] Ἱση· βάσις ἄρα ἡ ΕΖ βάσει τῇ ΗΖ ἐστιν Ἱση, καὶ τὸ ΔEZ τρίγωνον τῷ ΗΔΖ τριγώνῳ ἵσον ἐστίν, καὶ αἱ λοιπαὶ γωνίαι ταῖς λοιπαῖς γωνίαις ἵσας ἐσονται, ὥφετος ὅμοιοι πλευραὶ ὑποτείνουσιν. Ἱση ἄρα ἐστὶν ἡ μὲν ὑπὸ ΔΖΗ τῇ ὑπὸ ΔΖΕ, ἡ δὲ ὑπὸ ΔΗΖ

(angle) EDF to EGF. And since (angle) FED is equal to GEF, and (angle) GEF to ABC, angle ABC is thus also equal to DEF. So, for the same (reasons), (angle) ACB is also equal to DFE, and, further, the (angle) at A to the (angle) at D. Thus, triangle ABC is equiangular to triangle DEF.

Thus, if two triangles have proportional sides then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal. (Which is) the very thing it was required to show.

Proposition 6

If two triangles have one angle equal to one angle, and the sides about the equal angles proportional, then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal.



Let ABC and DEF be two triangles having one angle, BAC , equal to one angle, EDF (respectively), and the sides about the equal angles proportional, (so that) as BA (is) to AC , so ED (is) to DF . I say that triangle ABC is equiangular to triangle DEF, and will have angle ABC equal to DEF , and (angle) ACB to DFE .

For let (angle) FDG , equal to each of BAC and EDF , and (angle) DFG , equal to ACB , have been constructed on the straight-line AF at the points D and F on it (respectively) [Prop. 1.23]. Thus, the remaining angle at B is equal to the remaining angle at G [Prop. 1.32].

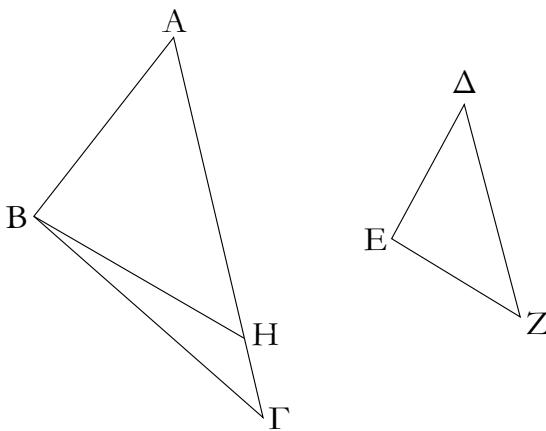
Thus, triangle ABC is equiangular to triangle DFG. Thus, proportionally, as BA (is) to AC , so GD (is) to DF [Prop. 6.4]. And it was also assumed that as BA (is) to AC , so ED (is) to DF . And, thus, as ED (is) to DF , so GD (is) to DF [Prop. 5.11]. Thus, ED (is) equal to DG [Prop. 5.9]. And DF (is) common. So, the two (sides) ED , DF are equal to the two (sides) GD , DF (respectively). And angle EDF [is] equal to angle GDF . Thus, base EF is equal to base GF , and triangle DEF is equal to triangle GDF, and the remaining angles

τῇ ὑπὸ ΔEZ. ἀλλ’ ἡ ὑπὸ ΔZH τῇ ὑπὸ ΑΓΒ ἐστιν ἵση· καὶ ἡ ὑπὸ ΑΓΒ ἄρα τῇ ὑπὸ ΔZE ἐστιν ἵση. ὑπόκειται δὲ καὶ ἡ ὑπὸ ΒΑΓ τῇ ὑπὸ EΔZ ἵση· καὶ λοιπὴ ἄρα ἡ πρὸς τῷ B λοιπῇ τῇ πρὸς τῷ E ἴση ἐστιν· ἰσογώνιον ἄρα ἐστὶ τὸ ABC τρίγωνον τῷ ΔEZ τριγώνῳ.

Ἐὰν ἄρα δύο τρίγωνα μίαν γωνίαν μιᾷ γωνίᾳ ἵσην ἔχῃ, περὶ δὲ τὰς ἵσας γωνίας τὰς πλευράς ἀνάλογον, ἰσογώνια ἐσται τὰ τρίγωνα καὶ ἵσας ἔξει τὰς γωνίας, ὑφ’ ἀς αἱ ὁμόλογοι πλευραὶ ὑποτείνουσιν· ὅπερ ἔδει δεῖξαι.

ζ'.

Ἐὰν δύο τρίγωνα μίαν γωνίαν μιᾷ γωνίᾳ ἵσην ἔχῃ, περὶ δὲ ἄλλας γωνίας τὰς πλευράς ἀνάλογον, τῶν δὲ λοιπῶν ἑκατέρων ἅμα ἥτοι ἐλάσσονα ἢ μὴ ἐλάσσονα ὅρθής, ἰσογώνια ἐσται τὰ τρίγωνα καὶ ἵσας ἔξει τὰς γωνίας, περὶ ἀς ἀνάλογον εἰσιν αἱ πλευραί.



Ἐστω δύο τρίγωνα τὰ ABC, ΔEZ μίαν γωνίαν μιᾷ γωνίᾳ ἵσην ἔχοντα τὴν ὑπὸ ΒΑΓ τῇ ὑπὸ EΔZ, περὶ δὲ ἄλλας γωνίας τὰς ὑπὸ ABC, ΔEZ τὰς πλευράς ἀνάλογον, ὡς τὴν AB πρὸς τὴν BG, οὕτως τὴν ΔE πρὸς τὴν EZ, τῶν δὲ λοιπῶν τῶν πρὸς τοῖς Γ, Z πρότερον ἑκατέρων ἅμα ἐλάσσονα ὅρθῆς· λέγω, ὅτι ἰσογώνιόν ἐστι τὸ ABC τρίγωνον τῷ ΔEZ τριγώνῳ, καὶ ἵση ἔσται ἡ ὑπὸ ABC γωνία τῇ ὑπὸ ΔEZ, καὶ λοιπὴ δηλονότι ἡ πρὸς τῷ Γ λοιπὴ τῇ πρὸς τῷ Z ἵση.

Εἰ γάρ ἄνισός ἐστιν ἡ ὑπὸ ABC γωνία τῇ ὑπὸ ΔEZ, μία αὐτῶν μείζων ἐστίν. ἔστω μείζων ἡ ὑπὸ ABC. καὶ συνεστάτω πρὸς τῇ AB εὐθείᾳ καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ B τῇ ὑπὸ ΔEZ γωνίᾳ ἵση ἡ ὑπὸ ABC.

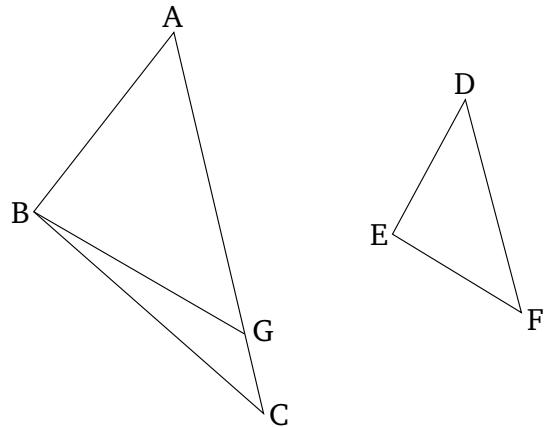
Καὶ ἐπεὶ ἵση ἐστιν ἡ μὲν A γωνία τῇ Δ, ἡ δὲ ὑπὸ ABH τῇ ὑπὸ ΔEZ, λοιπὴ ἄρα ἡ ὑπὸ AHB λοιπῇ τῇ ὑπὸ ΔZE ἐστιν ἵση. ἰσογώνιον ἄρα ἐστὶ τὸ ABH τρίγωνον τῷ ΔEZ

will be equal to the remaining angles which the equal sides subtend [Prop. 1.4]. Thus, (angle) DFG is equal to DFE, and (angle) DGF to DEF. But, (angle) DFG is equal to ACB. Thus, (angle) ACB is also equal to DFE. And (angle) BAC was also assumed (to be) equal to EDF. Thus, the remaining (angle) at B is equal to the remaining (angle) at E [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle DEF.

Thus, if two triangles have one angle equal to one angle, and the sides about the equal angles proportional, then the triangles will be equiangular, and will have the angles which corresponding sides subtend equal. (Which is) the very thing it was required to show.

Proposition 7

If two triangles have one angle equal to one angle, and the sides about other angles proportional, and the remaining angles either both less than, or both not less than, right-angles, then the triangles will be equiangular, and will have the angles about which the sides are proportional equal.



Let ABC and DEF be two triangles having one angle, BAC , equal to one angle, EDF (respectively), and the sides about (some) other angles, ABC and DEF (respectively), proportional, (so that) as AB (is) to BC , so DE (is) to EF , and the remaining (angles) at C and F , first of all, both less than right-angles. I say that triangle ABC is equiangular to triangle DEF, and (that) angle ABC will be equal to DEF , and (that) the remaining (angle) at C (will be) manifestly equal to the remaining (angle) at F .

For if angle ABC is not equal to (angle) DEF then one of them is greater. Let ABC be greater. And let (angle) ABG , equal to (angle) DEF , have been constructed on the straight-line AB at the point B on it [Prop. 1.23].

And since angle A is equal to (angle) D , and (angle) ABG to DEF , the remaining (angle) AGB is thus equal

τριγώνωφ. ἔστιν ἄρα ὡς ἡ AB πρὸς τὴν BH , οὕτως ἡ ΔE πρὸς τὴν EZ . ὡς δὲ ἡ ΔE πρὸς τὴν EZ , [οὕτως] ὑπόκειται ἡ AB πρὸς τὴν BG . ἡ AB ἄρα πρὸς ἐκατέραν τῶν BG , BH τὸν αὐτὸν ἔχει λόγον: Ἰση ἄρα ἡ BG τῇ BH . ὥστε καὶ γωνία ἡ πρὸς τῷ Γ γωνίᾳ τῇ ὑπὸ BHG ἔστιν Ἰση. ἐλάττων δὲ ὁρθῆς ὑπόκειται ἡ πρὸς τῷ Γ : ἐλάττων ἄρα ἔστιν ὁρθῆς καὶ ὑπὸ BHG . ὥστε ἡ ἐφεξῆς αὐτῇ γωνία ἡ ὑπὸ AHB μείζων ἔστιν ὁρθῆς. καὶ ἐδείχθη Ἰση οὖσα τῇ πρὸς τῷ Z καὶ ἡ πρὸς τῷ Z ἄρα μείζων ἔστιν ὁρθῆς. ὑπόκειται δὲ ἐλάσσοναν ὁρθῆς· ὅπερ ἔστιν ἄτοπον. οὐκ ἄρα ἀνισός ἔστιν ἡ ὑπὸ ABG γωνία τῇ ὑπὸ ΔEZ : Ἰση ἄρα. ἔστι δὲ καὶ ἡ πρὸς τῷ A Ἰση τῇ πρὸς τῷ Δ : καὶ λοιπὴ ἄρα ἡ πρὸς τῷ Γ λοιπὴ τῇ πρὸς τῷ Z Ἰση ἔστιν. Ισογώνιον ἄρα ἔστι τὸ ABG τρίγωνον τῷ ΔEZ τριγώνῳ.

Ἄλλὰ δὴ πάλιν ὑποκείσθω ἐκατέρα τῶν πρὸς τοῖς Γ , Z μὴ ἐλάσσοναν ὁρθῆς· λέγω πάλιν, ὅτι καὶ οὕτως ἔστιν Ισογώνιον τὸ ABG τρίγωνον τῷ ΔEZ τριγώνῳ.

Τῶν γὰρ αὐτῶν κατασκευασθέντων ὄμοιώς δεῖξομεν, ὅτι Ἰση ἔστιν ἡ BG τῇ BH : ὥστε καὶ γωνία ἡ πρὸς τῷ Γ τῇ ὑπὸ BHG Ἰση ἔστιν. οὐκ ἐλάττων δὲ ὁρθῆς ἡ πρὸς τῷ Γ : οὐκ ἐλάττων ἄρα ὁρθῆς οὐδὲ ἡ ὑπὸ BHG . τριγώνου δὴ τοῦ BHG αἱ δύο γωνίαι δύο ὁρθῶν οὐκ εἰσιν ἐλάττονες· ὅπερ ἔστιν ἀδύνατον. οὐκ ἄρα πάλιν ἀνισός ἔστιν ἡ ὑπὸ ABG γωνία τῇ ὑπὸ ΔEZ : Ἰση ἄρα. ἔστι δὲ καὶ ἡ πρὸς τῷ A τῇ πρὸς τῷ Δ Ἰση· λοιπὴ ἄρα ἡ πρὸς τῷ Γ λοιπὴ τῇ πρὸς τῷ Z Ἰση ἔστιν. Ισογώνιον ἄρα ἔστι τὸ ABG τρίγωνον τῷ ΔEZ τριγώνῳ.

Ἐὰν ἄρα δύο τρίγωνα μίαν γωνίαν μιᾷ γωνίᾳ Ἰσην ἔχῃ, περὶ δὲ ἄλλας γωνίας τὰς πλευρὰς ἀνάλογον, τῶν δὲ λοιπῶν ἐκατέραν ἀμα ἐλάττονα ἢ μὴ ἐλάττονα ὁρθῆς, Ισογώνια ἔσται τὰ τρίγωνα καὶ Ἰσας ἔξει τὰς γωνίας, περὶ δὲς ἀνάλογον εἰσιν αἱ πλευραί· ὅπερ ἔδει δεῖξαι.

to the remaining (angle) DFE [Prop. 1.32]. Thus, triangle ABG is equiangular to triangle DEF . Thus, as AB is to BG , so DE (is) to EF [Prop. 6.4]. And as DE (is) to EF , [so] it was assumed (is) AB to BC . Thus, AB has the same ratio to each of BC and BG [Prop. 5.11]. Thus, BC (is) equal to BG [Prop. 5.9]. And, hence, the angle at C is equal to angle BGC [Prop. 1.5]. And the angle at C was assumed (to be) less than a right-angle. Thus, (angle) BGC is also less than a right-angle. Hence, the adjacent angle to it, AGB , is greater than a right-angle [Prop. 1.13]. And (AGB) was shown to be equal to the (angle) at F . Thus, the (angle) at F is also greater than a right-angle. But it was assumed (to be) less than a right-angle. The very thing is absurd. Thus, angle ABC is not unequal to (angle) DEF . Thus, (it is) equal. And the (angle) at A is also equal to the (angle) at D . And thus the remaining (angle) at C is equal to the remaining (angle) at F [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle DEF .

But, again, let each of the (angles) at C and F be assumed (to be) not less than a right-angle. I say, again, that triangle ABC is equiangular to triangle DEF in this case also.

For, with the same construction, we can similarly show that BC is equal to BG . Hence, also, the angle at C is equal to (angle) BGC . And the (angle) at C (is) not less than a right-angle. Thus, BGC (is) not less than a right-angle either. So, in triangle BGC the (sum of) two angles is not less than two right-angles. The very thing is impossible [Prop. 1.17]. Thus, again, angle ABC is not unequal to DEF . Thus, (it is) equal. And the (angle) at A is also equal to the (angle) at D . Thus, the remaining (angle) at C is equal to the remaining (angle) at F [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle DEF .

Thus, if two triangles have one angle equal to one angle, and the sides about other angles proportional, and the remaining angles both less than, or both not less than, right-angles, then the triangles will be equiangular, and will have the angles about which the sides (are) proportional equal. (Which is) the very thing it was required to show.

η'.

Ἐὰν ἐν ὁρθογωνίῳ τριγώνῳ ἀπό τῆς ὁρθῆς γωνίας ἐπὶ τὴν βάσιν κάθετος ἀχθῆ, τὰ πρὸς τῇ καθέτῳ τρίγωνα ὄμοιά ἔστι τῷ τε ὅλῳ καὶ ἀλλήλοις.

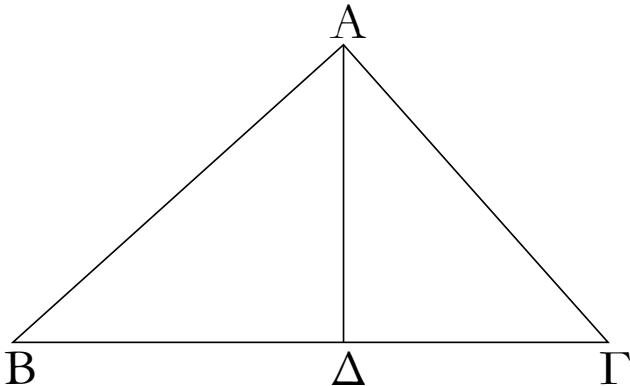
Ἐστω τρίγωνον ὁρθογωνίον τὸ ABG ὁρθὴν ἔχον τὴν ὑπὸ BAG γωνίαν, καὶ ἡχθῶ ἀπὸ τοῦ A ἐπὶ τὴν BG κάθετος ἡ $A\Delta$: λέγω, ὅτι ὄμοιόν ἔστιν ἐκάτερον τῶν $AB\Delta$, $A\Delta G$

Proposition 8

If, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the triangles around the perpendicular are similar to the whole (triangle), and to one another.

Let ABC be a right-angled triangle having the angle BAC a right-angle, and let AD have been drawn from

τριγώνων ὅλω τῷ ΑΒΓ καὶ ἔτι ἀλλήλοις.



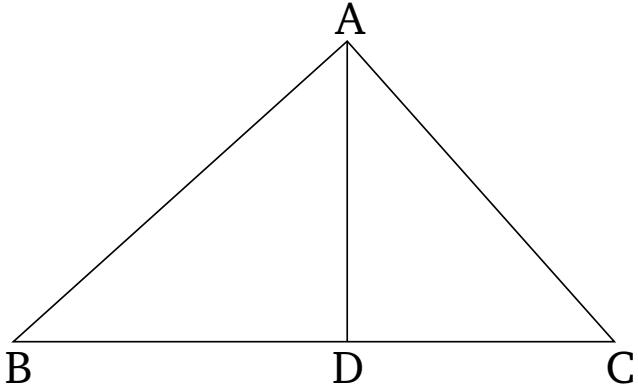
Ἐπεὶ γὰρ ἵση ἐστὶν ἡ ὑπὸ ΒΑΓ τῇ ὑπὸ ΑΔΒ· ὥρθῃ γὰρ ἐκατέρᾳ· καὶ κοινὴ τῶν δύο τριγώνων τοῦ τε ΑΒΓ καὶ τοῦ ΑΒΔ ἡ πρὸς τῷ Β, λοιπὴ ἄρα ἡ ὑπὸ ΑΓΒ λοιπὴ τῇ ὑπὸ ΒΑΔ ἐστὶν ἵση· ἵσογώνιον ἄρα ἐστὶ τὸ ΑΒΓ τρίγωνον τῷ ΑΒΔ τριγώνῳ. ἔστιν ἄρα ὡς ἡ ΒΓ ὑποτείνουσα τὴν ὥρθὴν τοῦ ΑΒΓ τριγώνου πρὸς τὴν ΒΑ ὑποτείνουσαν τὴν ὥρθὴν τοῦ ΑΒΔ τριγώνου, οὕτως αὐτὴ ἡ ΑΒ ὑποτείνουσα τὴν πρὸς τῷ Γ γωνίαν τοῦ ΑΒΓ τριγώνου πρὸς τὴν ΒΔ ὑποτείνουσαν τὴν ἵσην τὴν ὑπὸ ΒΑΔ τοῦ ΑΒΔ τριγώνου, καὶ ἔτι ἡ ΑΓ πρὸς τὴν ΑΔ ὑποτείνουσαν τὴν πρὸς τῷ Β γωνίαν κοινὴν τῶν δύο τριγώνων. τὸ ΑΒΓ ἄρα τρίγωνον τῷ ΑΒΔ τριγώνῳ ἵσογώνιόν τε ἐστὶ καὶ τὰς περὶ τὰς ἵσας γωνίας πλευράς ἀνάλογον ἔχει. ὅμοιον ἄμα [ἔστι] τὸ ΑΒΓ τριγώνον τῷ ΑΒΔ τριγώνῳ. ὅμοιώς δὴ δεῖξομεν, ὅτι καὶ τῷ ΑΔΓ τριγώνῳ ὅμοιόν ἐστι τὸ ΑΒΓ τριγώνον ἐκάτερον ἄρα τῶν ΑΒΔ, ΑΔΓ [τριγώνων] ὅμοιόν ἐστιν ὅλω τῷ ΑΒΓ.

Λέγω δή, ὅτι καὶ ἀλλήλοις ἐστὶν ὅμοια τὰ ΑΒΔ, ΑΔΓ τρίγωνα.

Ἐπεὶ γὰρ ὥρθῃ ἡ ὑπὸ ΒΔΑ ὥρθῃ τῇ ὑπὸ ΑΔΓ ἐστὶν ἵση, ἀλλὰ μὴν καὶ ἡ ὑπὸ ΒΑΔ τῇ πρὸς τῷ Γ ἐδείχθη ἵση, καὶ λοιπὴ ἄρα ἡ πρὸς τῷ Β λοιπὴ τῇ ὑπὸ ΔΑΓ ἐστὶν ἵση· ἵσογώνιον ἄρα ἐστὶ τὸ ΑΒΔ τριγώνον τῷ ΑΔΓ τριγώνῳ. ἔστιν ἄρα ὡς ἡ ΒΔ τοῦ ΑΒΔ τριγώνου ὑποτείνουσα τὴν ὑπὸ ΒΑΔ πρὸς τὴν ΔΑ τοῦ ΑΔΓ τριγώνου ὑποτείνουσαν τὴν πρὸς τῷ Γ ἵσην τῇ ὑπὸ ΒΑΔ, οὕτως αὐτὴ ἡ ΑΔ τοῦ ΑΒΔ τριγώνου ὑποτείνουσα τὴν πρὸς τῷ Β γωνίαν πρὸς τὴν ΔΓ ὑποτείνουσαν τὴν ὑπὸ ΔΑΓ τοῦ ΑΔΓ τριγώνου ἵσην τῇ πρὸς τῷ Β, καὶ ἔτι ἡ ΒΑ πρὸς τὴν ΑΓ ὑποτείνουσαι τὰς ὥρθάς· ὅμοιον ἄρα ἐστὶ τὸ ΑΒΔ τριγώνον τῷ ΑΔΓ τριγώνῳ.

Ἐὰν ἄρα ἐν ὥρθογωνίῳ τριγώνῳ ἀπὸ τῆς ὥρθης γωνίας ἐπὶ τὴν βάσιν κάθετος ἀχθῆ, τὰ πρὸς τῇ καθέτῳ τρίγωνα ὅμοιά ἐστι τῷ τε ὅλῳ καὶ ἀλλήλοις [ὅπερ ἔδει δεῖξαι].

A, perpendicular to BC [Prop. 1.12]. I say that triangles ABD and ADC are each similar to the whole (triangle) ABC and, further, to one another.



For since (angle) BAC is equal to ADB —for each (are) right-angles—and the (angle) at B (is) common to the two triangles ABC and ABD , the remaining (angle) ACB is thus equal to the remaining (angle) BAD [Prop. 1.32]. Thus, triangle ABC is equiangular to triangle ABD . Thus, as BC , subtending the right-angle in triangle ABC , is to BA , subtending the right-angle in triangle ABD , so the same AB , subtending the angle at C in triangle ABC , (is) to BD , subtending the equal (angle) BAD in triangle ABD , and, further, (so is) AC to AD , (both) subtending the angle at B common to the two triangles [Prop. 6.4]. Thus, triangle ABC is equiangular to triangle ABD , and has the sides about the equal angles proportional. Thus, triangle ABC [is] similar to triangle ABD [Def. 6.1]. So, similarly, we can show that triangle ABC is also similar to triangle ADC . Thus, [triangles] ABD and ADC are each similar to the whole (triangle) ABC .

So I say that triangles ABD and ADC are also similar to one another.

For since the right-angle BDA is equal to the right-angle ADC , and, indeed, (angle) BAD was also shown (to be) equal to the (angle) at C , thus the remaining (angle) at B is also equal to the remaining (angle) DAC [Prop. 1.32]. Thus, triangle ABD is equiangular to triangle ADC . Thus, as BD , subtending (angle) BAD in triangle ABD , is to DA , subtending the (angle) at C in triangle ADC , (which is) equal to (angle) BAD , so (is) the same AD , subtending the angle at B in triangle ABD , to DC , subtending (angle) DAC in triangle ADC , (which is) equal to the (angle) at B , and, further, (so is) BA to AC , (each) subtending right-angles [Prop. 6.4]. Thus, triangle ABD is similar to triangle ADC [Def. 6.1].

Thus, if, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base

then the triangles around the perpendicular are similar to the whole (triangle), and to one another. [(Which is) the very thing it was required to show.]

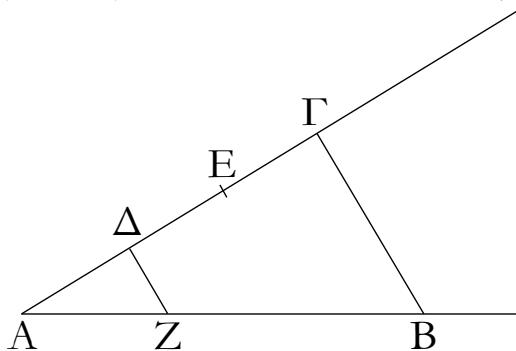
Πόρισμα.

Ἐκ δὴ τούτου φανερόν, ὅτι ἔὰν ἐν ὁρθογωνίῳ τριγώνῳ ἀπὸ τῆς ὁρθῆς γωνίας ἐπὶ τὴν βάσις κάθετος ἀχθῆ, ἡ ἀχθεῖσα τῶν τῆς βάσεως τμημάτων μέση ἀνάλογόν ἐστιν· ὅπερ ἔδει δεῖξαι.

[†] In other words, the perpendicular is the geometric mean of the pieces.

θ'.

Τῆς δοθείσης εὐθείας τὸ προσταχθὲν μέρος ἀφελεῖν.



Ἐστω ἡ δοθείσα εὐθεία ἡ AB . δεῖ δὴ τῆς AB τὸ προσταχθὲν μέρος ἀφελεῖν.

Ἐπιτετάχθω δὴ τὸ τρίτον. [καὶ] διήθυνται τις ἀπὸ τοῦ A εὐθεία ἡ AC γωνίαν περιέχουσα μετὰ τῆς AB τυχοῦσαν· καὶ εἰλήφθω τυχὸν σημεῖον ἐπὶ τῆς AC τὸ D , καὶ κείσθωσαν τῇ AD ἵσαι αἱ $\Delta E, EG$. καὶ ἐπεζεύχθω ἡ BG , καὶ διὰ τοῦ Δ παράλληλος αὐτῇ ἥχθω ἡ ΔZ .

Ἐπεὶ οὖν τριγώνου τοῦ ABC παρὰ μίαν τῶν πλευρῶν τὴν BG ἤκται ἡ ΔZ , ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΓD πρὸς τὴν ΔA , οὕτως ἡ BZ πρὸς τὴν ZA . διπλὴ δὲ ἡ ΓD τῆς ΔA . διπλὴ ἄρα καὶ ἡ BZ τῆς $Z A$. τριπλὴ ἄρα ἡ BA τῆς $Z A$.

Τῆς ἄρα δοθείσης εὐθείας τῆς AB τὸ ἐπιταχθὲν τρίτον μέρος ἀφήρηται τὸ AZ . ὅπερ ἔδει ποιῆσαι.

ι'.

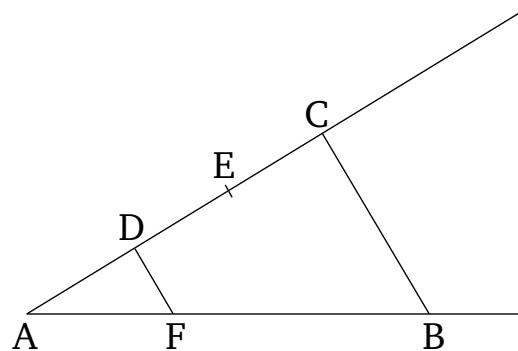
Τὴν δοθεῖσαν εὐθεῖαν ἄτμητον τῇ δοθείσῃ τετμημένη ὁμοίως τεμεῖν.

Corollary

So (it is) clear, from this, that if, in a right-angled triangle, a (straight-line) is drawn from the right-angle perpendicular to the base then the (straight-line so) drawn is in mean proportion to the pieces of the base.[†] (Which is) the very thing it was required to show.

Proposition 9

To cut off a prescribed part from a given straight-line.



Let AB be the given straight-line. So it is required to cut off a prescribed part from AB .

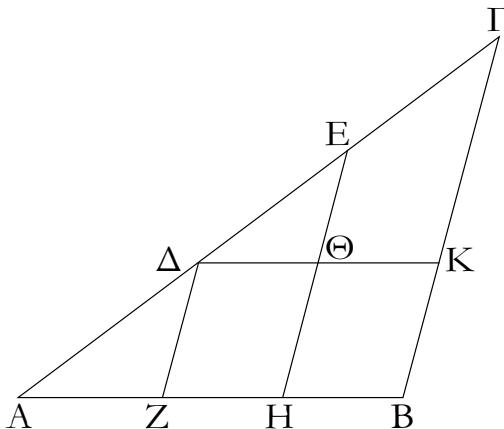
So let a third (part) have been prescribed. [And] let some straight-line AC have been drawn from (point) A , encompassing a random angle with AB . And let a random point D have been taken on AC . And let DE and EC be made equal to AD [Prop. 1.3]. And let BC have been joined. And let DF have been drawn through D parallel to it [Prop. 1.31].

Therefore, since FD has been drawn parallel to one of the sides, BC , of triangle ABC , then, proportionally, as CD is to DA , so BF (is) to FA [Prop. 6.2]. And CD (is) double DA . Thus, BF (is) also double FA . Thus, BA (is) triple AF .

Thus, the prescribed third part, AF , has been cut off from the given straight-line, AB . (Which is) the very thing it was required to do.

Proposition 10

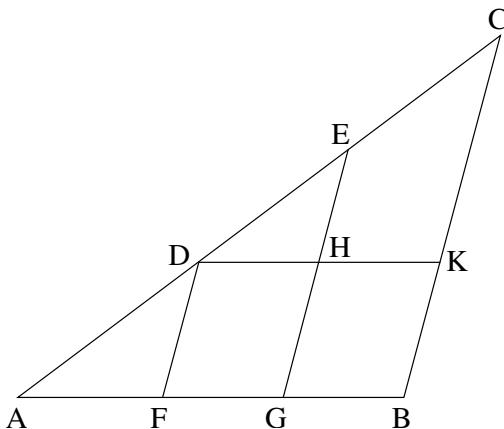
To cut a given uncut straight-line similarly to a given cut (straight-line).



Ἐστω ἡ μὲν δοθεῖσα εὐθεῖα ἄτμητος ἡ AB , ἡ δὲ τετμημένη ἡ AG κατὰ τὰ Δ , E σημεῖα, καὶ κείσινωσαν ὥστε γωνίαν τυχοῦσαν περιέχειν, καὶ ἐπεζεύχθω ἡ GB , καὶ διὰ τῶν Δ , E τῇ BG παράλληλοι ἤχθωσαν αἱ ΔZ , EH , διὰ δὲ τοῦ Δ τῇ AB παράλληλος ἤχθω ἡ $\Delta \Theta K$.

Παραλληλόγραμμον ἄρα ἔστιν ἐκάτερον τῶν $Z\Theta$, ΘB . ἵση ἄρα ἡ μὲν $\Delta \Theta$ τῇ ZH , ἡ δὲ ΘK τῇ HB . καὶ ἐπει τριγώνου τοῦ $\Delta K\Gamma$ παρὰ μίαν τῶν πλευρῶν τὴν $K\Gamma$ εὐθεῖα ἤκται ἡ ΘE , ἀνάλογον ἄρα ἔστιν ὡς ἡ ΓE πρὸς τὴν $E\Delta$, οὕτως ἡ $K\Theta$ πρὸς τὴν $\Theta\Delta$. ἵση δὲ ἡ μὲν $K\Theta$ τῇ BH , ἡ δὲ $\Theta\Delta$ τῇ HZ . ἔστιν ἄρα ὡς ἡ ΓE πρὸς τὴν $E\Delta$, οὕτως ἡ BH πρὸς τὴν HZ . πάλιν, ἐπει τριγώνου τοῦ AHE παρὰ μίαν τῶν πλευρῶν τὴν HE ἤκται ἡ $Z\Delta$, ἀνάλογον ἄρα ἔστιν ὡς ἡ $E\Delta$ πρὸς τὴν ΔA , οὕτως ἡ HZ πρὸς τὴν ZA . ἐδείχθη δὲ καὶ ὡς ἡ ΓE πρὸς τὴν $E\Delta$, οὕτως ἡ BH πρὸς τὴν HZ . ἔστιν ἄρα ὡς μὲν ἡ ΓE πρὸς τὴν $E\Delta$, οὕτως ἡ BH πρὸς τὴν HZ , ὡς δὲ ἡ $E\Delta$ πρὸς τὴν ΔA , οὕτως ἡ HZ πρὸς τὴν ZA .

Ἡ ἄρα δοθεῖσα εὐθεῖα ἄτμητος ἡ AB τῇ δοθείσῃ εὐθεῖᾳ τετμημένῃ τῇ AG ὁμοίως τέτμηται· ὅπερ ἔδει ποιῆσαι·



Let AB be the given uncut straight-line, and AC a (straight-line) cut at points D and E , and let (AC) be laid down so as to encompass a random angle (with AB). And let CB have been joined. And let DF and EG have been drawn through (points) D and E (respectively), parallel to BC , and let DHK have been drawn through (point) D , parallel to AB [Prop. 1.31].

Thus, FH and HB are each parallelograms. Thus, DH (is) equal to FG , and HK to GB [Prop. 1.34]. And since the straight-line HE has been drawn parallel to one of the sides, KC , of triangle DKC , thus, proportionally, as CE is to ED , so KH (is) to HD [Prop. 6.2]. And KH (is) equal to BG , and HD to GF . Thus, as CE is to ED , so BG (is) to GF . Again, since FD has been drawn parallel to one of the sides, GE , of triangle AGE , thus, proportionally, as ED is to DA , so GF (is) to FA [Prop. 6.2]. And it was also shown that as CE (is) to ED , so BG (is) to GF . Thus, as CE is to ED , so BG (is) to GF , and as ED (is) to DA , so GF (is) to FA .

Thus, the given uncut straight-line, AB , has been cut similarly to the given cut straight-line, AC . (Which is) the very thing it was required to do.

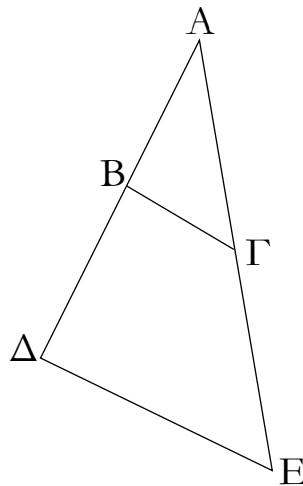
ια'.

Proposition 11

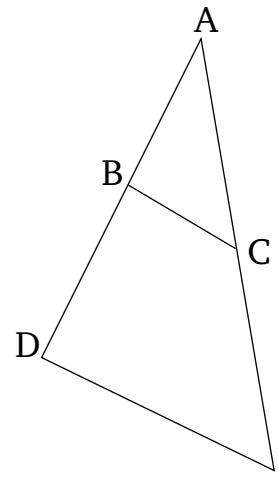
To find a third (straight-line) proportional to two given straight-lines.

Let BA and AC be the [two] given [straight-lines], and let them be laid down encompassing a random angle. So it is required to find a third (straight-line) proportional to BA and AC . For let $(BA$ and $AC)$ have been produced to points D and E (respectively), and let BD be made equal to AC [Prop. 1.3]. And let BC have been joined. And let DE have been drawn through (point) D parallel to it [Prop. 1.31].

Therefore, since BC has been drawn parallel to one of the sides DE of triangle ADE , proportionally, as AB is to BD , so AC (is) to CE [Prop. 6.2]. And BD (is) equal



Δύο ἄρα δοθεισῶν εὐθειῶν τῶν AB , AC τρίτη ἀνάλογον αὐταῖς προσεύρηται ἡ GE . ὅπερ ἔδει ποιῆσαι.

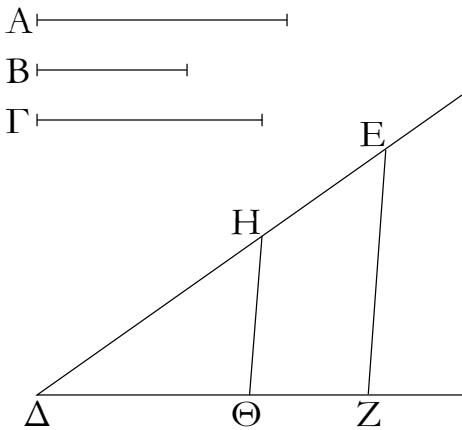


to AC . Thus, as AB is to AC , so AC (is) to CE .

Thus, a third (straight-line), CE , has been found (which is) proportional to the two given straight-lines, AB and AC . (Which is) the very thing it was required to do.

β'.

Τριῶν δοθεισῶν εὐθειῶν τετάρτην ἀνάλογον προσευρεῖν.



Ἐστωσαν αἱ δοθεῖσαι τρεῖς εὐθεῖαι αἱ A , B , Γ . δεῖ δὴ τῶν A , B , Γ τετάρτην ἀνάλογον προσευρεῖν.

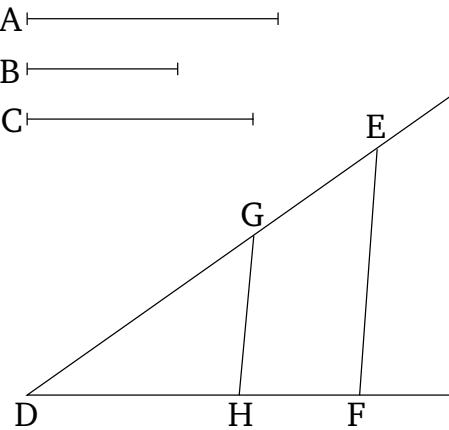
Ἐκκείσθωσαν δύο εὐθεῖαι αἱ ΔE , ΔZ γωνίαν περιέχουσαι [τυχοῦσαν] τὴν ὑπὸ $E\Delta Z$ · καὶ κείσθω τῇ μὲν A ἵση ἡ ΔH , τῇ δὲ B ἵση ἡ HE , καὶ ἔτι τῇ Γ ἵση ἡ $\Delta \Theta$ · καὶ ἐπιζευχθείσῃς τῇς $H\Theta$ παράλληλος αὐτῇ ἔχθω διὰ τοῦ E ἡ EZ .

Ἐπεὶ οὖν τριγώνου τοῦ ΔEZ παρὰ μίαν τὴν EZ ἤκται ἡ $H\Theta$, ἔστιν ἄρα ὡς ἡ ΔH πρὸς τὴν HE , οὕτως ἡ $\Delta \Theta$ πρὸς τὴν ΘZ . Ἱση δὲ ἡ μὲν ΔH τῇ A , ἡ δὲ HE τῇ B , ἡ δὲ $\Delta \Theta$ τῇ Γ . ἔστιν ἄρα ὡς ἡ A πρὸς τὴν B , οὕτως ἡ Γ πρὸς τὴν ΘZ .

Τριῶν ἄρα δοθεισῶν εὐθειῶν τῶν A , B , Γ τετάρτη ἀνάλογον προσεύρηται ἡ ΘZ . ὅπερ ἔδει ποιῆσαι.

Proposition 12

To find a fourth (straight-line) proportional to three given straight-lines.



Let A , B , and C be the three given straight-lines. So it is required to find a fourth (straight-line) proportional to A , B , and C .

Let the two straight-lines DE and DF be set out encompassing the [random] angle EDF . And let DG be made equal to A , and GE to B , and, further, DH to C [Prop. 1.3]. And GH being joined, let EF have been drawn through (point) E parallel to it [Prop. 1.31].

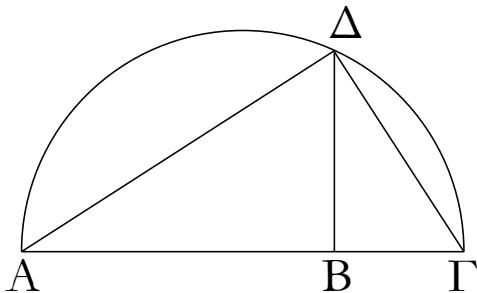
Therefore, since GH has been drawn parallel to one of the sides EF of triangle DEF , thus as DG is to GE , so DH (is) to HF [Prop. 6.2]. And DG (is) equal to A , and GE to B , and DH to C . Thus, as A is to B , so C (is)

to HF .

Thus, a fourth (straight-line), HF , has been found (which is) proportional to the three given straight-lines, A , B , and C . (Which is) the very thing it was required to do.

ιγ'.

Δύο δοθεισῶν εὐθειῶν μέσην ἀνάλογον προσευρεῖν.

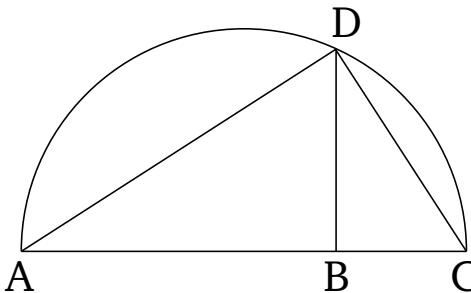


Ἐστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι αἱ AB , BC . δεῖ δὴ τῶν AB , BC μέσην ἀνάλογον προσευρεῖν.

Κείσθωσαν ἐπὶ εὐθείας, καὶ γεγράφω ἐπὶ τῆς AC ἡμικύκλιον τὸ $A\Delta\Gamma$, καὶ ἡχθω ἀπὸ τοῦ B σημείου τῇ AC εὐθείᾳ πρὸς ὁρθὰς ἡ BA , καὶ ἐπεζεύχθωσαν αἱ $A\Delta$, $\Delta\Gamma$.

Ἐπει ἐν ἡμικύκλῳ γωνίᾳ ἔστιν ἡ ὑπὸ $A\Delta\Gamma$, ὁρθή ἔστιν. καὶ ἐπει ἐν ὁρθογωνίῳ τριγώνῳ τῷ $A\Delta\Gamma$ ἀπὸ τῆς ὁρθῆς γωνίας ἐπὶ τὴν βάσιν κάθετος ἡ ΔB , ἡ ΔB ἥρα τῶν τῆς βάσεως τμημάτων τῶν AB , BC μέση ἀνάλογόν ἔστιν.

Δύο ἥρα δοθεισῶν εὐθειῶν τῶν AB , BC μέση ἀνάλογον προσεύρηται ἡ ΔB . ὅπερ ἔδει ποιῆσαι.



Let AB and BC be the two given straight-lines. So it is required to find the (straight-line) in mean proportion to AB and BC .

Let (AB and BC) be laid down straight-on (with respect to one another), and let the semi-circle ADC have been drawn on AC [Prop. 1.10]. And let BD have been drawn from (point) B , at right-angles to AC [Prop. 1.11]. And let AD and DC have been joined.

And since ADC is an angle in a semi-circle, it is a right-angle [Prop. 3.31]. And since, in the right-angled triangle ADC , the (straight-line) DB has been drawn from the right-angle perpendicular to the base, DB is thus the mean proportional to the pieces of the base, AB and BC [Prop. 6.8 corr.].

Thus, DB has been found (which is) in mean proportion to the two given straight-lines, AB and BC . (Which is) the very thing it was required to do.

[†] In other words, to find the geometric mean of two given straight-lines.

ιδ'.

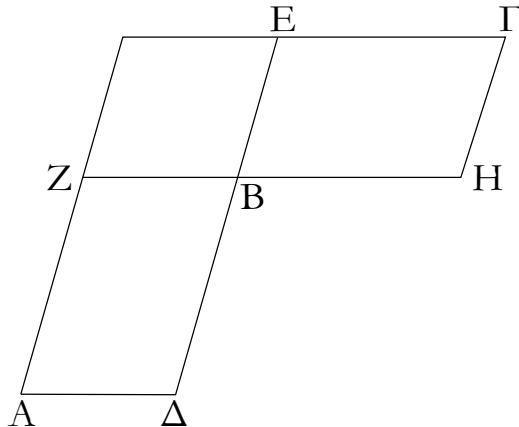
Τῶν ἵσων τε καὶ ἵσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας· καὶ δῶν ἵσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας, ἵσα ἔστιν ἔκεινα.

Ἐστω ἵσα τε καὶ ἵσογώνια παραλληλόγραμμα τὰ AB , BC ἵσας ἔχοντα τὰς πρὸς τῷ B γωνίας, καὶ κείσθωσαν ἐπὶ εὐθείας αἱ ΔB , BE . ἐπὶ εὐθείας ἥρα εἰσὶ καὶ αἱ ZB , BH . λέγω, ὅτι τῶν AB , BC ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας, τουτέστιν, ὅτι ἔστιν ὡς ἡ ΔB πρὸς τὴν BE , οὕτως ἡ $H B$ πρὸς τὴν BZ .

Proposition 14

In equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional. And those equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.

Let AB and BC be equal and equiangular parallelograms having the angles at B equal. And let DB and BE be laid down straight-on (with respect to one another). Thus, FB and BG are also straight-on (with respect to one another) [Prop. 1.14]. I say that the sides of AB and

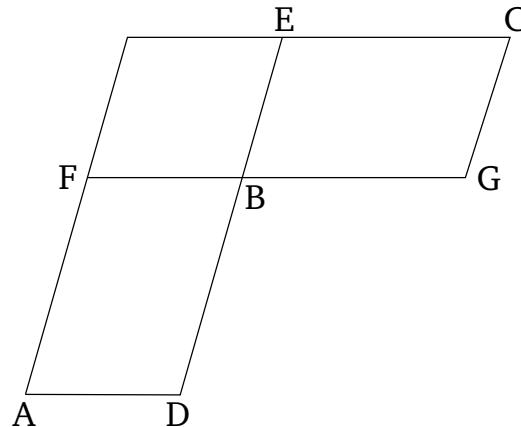


Συμπεπληρώσθω γάρ τὸ ΖΕ παραλληλόγραφμον. ἐπεὶ οὖν ἵστι τὸ ΑΒ παραλληλόγραφμον τῷ ΒΓ παραλληλογράμμῳ, ἀλλο δέ τι τὸ ΖΕ, ἔστιν ἄρα ως τὸ ΑΒ πρὸς τὸ ΖΕ, οὕτως τὸ ΒΓ πρὸς τὸ ΖΕ. ἀλλ' ως μὲν τὸ ΑΒ πρὸς τὸ ΖΕ, οὕτως ἡ ΔΒ πρὸς τὴν ΒΕ, ως δέ τὸ ΒΓ πρὸς τὸ ΖΕ, οὕτως ἡ ΗΒ πρὸς τὴν ΒΖ· καὶ ως ἄρα ἡ ΔΒ πρὸς τὴν ΒΕ, οὕτως ἡ ΗΒ πρὸς τὴν ΒΖ. τῶν ἄρα ΑΒ, ΒΓ παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας.

Αλλὰ δὴ ἔστω ὡς ἡ ΔΒ πρὸς τὴν ΒΕ, οὕτως ἡ ΗΒ πρὸς τὴν ΒΖ· λέγω, ὅτι ἵσον ἔστι τὸ ΑΒ παραλληλόγραμμον τῷ ΒΓ παραλληλογράμμῳ.

Ἐπεὶ γάρ ἐστιν ὡς ή ΔΒ πρὸς τὴν BE, οὔτως ή HB πρὸς τὴν BZ, ἀλλ᾽ ὡς μὲν ή ΔΒ πρὸς τὴν BE, οὔτως τὸ AB παραλληλόγραμμον πρὸς τὸ ZE παραλληλόγραμμον, ὡς δὲ ή HB πρὸς τὴν BZ, οὔτως τὸ BG παραλληλόγραμμον πρὸς τὸ ZE παραλληλόγραμμον, καὶ ὡς ἄρα τὸ AB πρὸς τὸ ZE, οὔτως τὸ BG πρὸς τὸ ZE· ἵσον ἄρα ἐστὶ τὸ AB παραλληλόγραμμον τῷ BG παραλληλογράμμῳ.

Τῶν ἄρα Ἰσων τε καὶ Ἰσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς Ἰσας γωνίας· καὶ ὅν Ἰσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς Ἰσας γωνίας, ἵσα ἐστὶν ἔκεινα· ὅπερ ἔδει δεῖξαι.



For let the parallelogram FE have been completed. Therefore, since parallelogram AB is equal to parallelogram BC , and FE (is) some other (parallelogram), thus as (parallelogram) AB is to FE , so (parallelogram) BC (is) to FE [Prop. 5.7]. But, as (parallelogram) AB (is) to FE , so DB (is) to BE , and as (parallelogram) BC (is) to FE , so GB (is) to BF [Prop. 6.1]. Thus, also, as DB (is) to BE , so GB (is) to BF . Thus, in parallelograms AB and BC the sides about the equal angles are reciprocally proportional.

And so, let DB be to BE , as GB (is) to BF . I say that parallelogram AB is equal to parallelogram BC .

For since as DB is to BE , so GB (is) to BF , but as DB (is) to BE , so parallelogram AB (is) to parallelogram FE , and as GB (is) to BF , so parallelogram BC (is) to parallelogram FE [Prop. 6.1], thus, also, as (parallelogram) AB (is) to FE , so (parallelogram) BC (is) to FE [Prop. 5.11]. Thus, parallelogram AB is equal to parallelogram BC [Prop. 5.9].

Thus, in equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional. And those equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal. (Which is) the very thing it was required to show.

Proposition 15

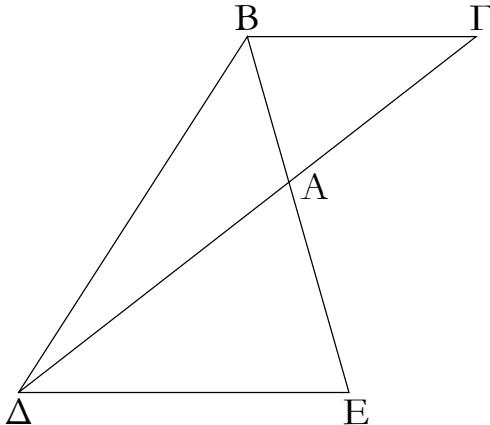
Τῶν Ἰσων καὶ μίαν μιᾷ Ἰσην ἔχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς Ἰσας γωνίας· καὶ διὰ μίαν μιᾷ Ἰσην ἔχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς Ἰσας γωνίας, ἵσα ἐστὶν ἐκεῖνα.

Ἐστω ἡσα τριγώνα τὰ ΑΒΓ, ΑΔΕ μίαν μιᾷ ἵσην ἔχοντα γωνίαν τὴν ὑπὸ ΒΑΓ τῇ ὑπὸ ΔΑΕ· λέγω, ὅτι τῶν ΑΒΓ, ΑΔΕ τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἡσας γωνίας, τουτέστιν, ὅτι ἐστὶν ὡς ἡ ΓΑ πρὸς τὴν ΑΔ, οὕτως

In equal triangles also having one angle equal to one (angle) the sides about the equal angles are reciprocally proportional. And those triangles having one angle equal to one angle for which the sides about the equal angles (are) reciprocally proportional are equal.

Let ABC and ADE be equal triangles having one angle equal to one (angle), (namely) BAC (equal) to DAE . I say that, in triangles ABC and ADE , the sides about the

ἡ EA πρὸς τὴν AB.



Κείσθω γάρ ὥστε ἐπ’ εὐθείας εἶναι τὴν ΓΑ τῇ ΑΔ· ἐπ’ εὐθείας ἄρα ἔστι καὶ ἡ EA τῇ AB. καὶ ἐπεζεύχθω ἡ BΔ.

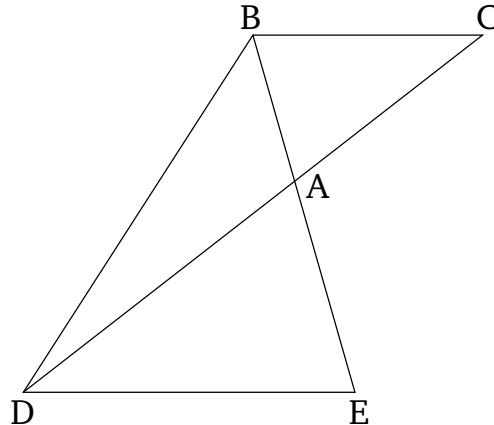
Ἐπεὶ οὖν ἵστη τὸ ΑΒΓ τρίγωνον τῷ ΑΔΕ τριγώνῳ, ἀλλο δέ τι τὸ ΒΑΔ, ἔστιν ἄρα ὡς τὸ ΓΑΒ τρίγωνον πρὸς τὸ ΒΑΔ τρίγωνον, οὕτως τὸ ΕΑΔ τρίγωνον πρὸς τὸ ΒΑΔ τρίγωνον. ἀλλ’ ὡς μὲν τὸ ΓΑΒ πρὸς τὸ ΒΑΔ, οὕτως ἡ ΓΑ πρὸς τὴν ΑΔ, ὡς δὲ τὸ ΕΑΔ πρὸς τὸ ΒΑΔ, οὕτως ἡ EA πρὸς τὴν AB. καὶ ὡς ἄρα ἡ ΓΑ πρὸς τὴν ΑΔ, οὕτως ἡ EA πρὸς τὴν AB. τῶν ΑΒΓ, ΑΔΕ ἄρα τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας.

Ἄλλὰ δὴ ἀντιπεπονθέτωσαν αἱ πλευραὶ τῶν ΑΒΓ, ΑΔΕ τριγώνων, καὶ ἔστω ὡς ἡ ΓΑ πρὸς τὴν ΑΔ, οὕτως ἡ EA πρὸς τὴν AB· λέγω, ὅτι ἵστη τὸ ΑΒΓ τρίγωνον τῷ ΑΔΕ τριγώνῳ.

Ἐπιζευχθείσης γάρ πάλιν τῆς ΒΔ, ἐπεὶ ἔστιν ὡς ἡ ΓΑ πρὸς τὴν ΑΔ, οὕτως ἡ EA πρὸς τὴν AB, ἀλλ’ ὡς μὲν ἡ ΓΑ πρὸς τὴν ΑΔ, οὕτως τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΒΑΔ τρίγωνον, ὡς δὲ ἡ EA πρὸς τὴν AB, οὕτως τὸ ΕΑΔ τρίγωνον πρὸς τὸ ΒΑΔ τρίγωνον, ὡς ἄρα τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΒΑΔ τρίγωνον, οὕτως τὸ ΕΑΔ τρίγωνον πρὸς τὸ ΒΑΔ τρίγωνον. ἐκάτερον ἄρα τῶν ΑΒΓ, ΕΑΔ πρὸς τὸ ΒΑΔ τὸν αὐτὸν ἔχει λόγον. ἵσων ἄρα ἔστι τὸ ΑΒΓ [τρίγωνον] τῷ ΕΑΔ τριγώνῳ.

Τῶν ἄρα ἵσων καὶ μίαν μιᾷ ἵσην ἔχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας· καὶ ὡς μίαν μιᾳ ἵσην ἔχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας, ἔχεινα ἵσα ἔστιν· ὅπερ ἔδει δεῖξαι.

equal angles are reciprocally proportional, that is to say, that as CA is to AD, so EA (is) to AB.



For let CA be laid down so as to be straight-on (with respect) to AD. Thus, EA is also straight-on (with respect) to AB [Prop. 1.14]. And let BD have been joined.

Therefore, since triangle ABC is equal to triangle ADE, and BAD (is) some other (triangle), thus as triangle CAB is to triangle BAD, so triangle EAD (is) to triangle BAD [Prop. 5.7]. But, as (triangle) CAB (is) to BAD, so CA (is) to AD, and as (triangle) EAD (is) to BAD, so EA (is) to AB [Prop. 6.1]. And thus, as CA (is) to AD, so EA (is) to AB. Thus, in triangles ABC and ADE the sides about the equal angles (are) reciprocally proportional.

And so, let the sides of triangles ABC and ADE be reciprocally proportional, and (thus) let CA be to AD, as EA (is) to AB. I say that triangle ABC is equal to triangle ADE.

For, BD again being joined, since as CA is to AD, so EA (is) to AB, but as CA (is) to AD, so triangle ABC (is) to triangle BAD, and as EA (is) to AB, so triangle EAD (is) to triangle BAD [Prop. 6.1], thus as triangle ABC (is) to triangle BAD, so triangle EAD (is) to triangle BAD. Thus, (triangles) ABC and EAD each have the same ratio to BAD. Thus, [triangle] ABC is equal to triangle EAD [Prop. 5.9].

Thus, in equal triangles also having one angle equal to one (angle) the sides about the equal angles (are) reciprocally proportional. And those triangles having one angle equal to one angle for which the sides about the equal angles (are) reciprocally proportional are equal. (Which is) the very thing it was required to show.

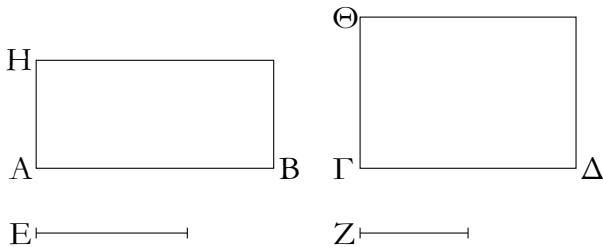
17'.

Ἐὰν τέσσαρες εὐθεῖαι ἀνάλογοι ὥσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὁρθογώνιον ἵστη ἔστι τῷ ὑπὸ τῶν μέσων περιεχομένῳ ὁρθογώνῳ· καὶ τὸ ὑπὸ τῶν ἄκρων

Proposition 16

If four straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the rectangle contained by the middle (two). And if the rect-

περιεχόμενον ὁρθογώνιον ἵσον ἡ τῷ ὑπὸ τῶν μέσων περιεχομένῳ ὁρθογώνιῳ, αἱ τέσσαρες εὐθεῖαι ἀνάλογον ἔσονται.



Ἐστασαν τέσσαρες εὐθεῖαι ἀνάλογον αἱ AB, ΓΔ, E, Z, ως ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ E πρὸς τὴν Z· λέγω, ὅτι τὸ ὑπὸ τῶν AB, Z περιεχόμενον ὁρθογώνιον ἵσον ἔστι τῷ ὑπὸ τῶν ΓΔ, E περιεχομένῳ ὁρθογώνιῳ.

Ὑχθωσαν [γάρ] ἀπὸ τῶν A, Γ σημείων ταῖς AB, ΓΔ εὐθεῖαις πρὸς ὁρθὰς αἱ AH, ΓΘ, καὶ κείσθω τῇ μὲν Z ἵση ἡ AH, τῇ δὲ E ἵση ἡ ΓΘ. καὶ συμπεπληρώσθω τὰ BH, ΔΘ παραλληλόγραμμα.

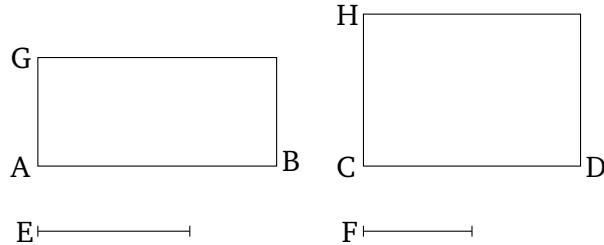
Καὶ ἐπεὶ ἔστιν ως ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ E πρὸς τὴν Z, ἵση δὲ ἡ μὲν E τῇ ΓΘ, ἡ δὲ Z τῇ AH, ἔστιν ἄρα ως ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ ΓΘ πρὸς τὴν AH. τῶν BH, ΔΘ ἄρα παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας. ὃν δὲ ἵσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνάς, ἵσα ἔστιν ἔκεινα· ἵσον ἄρα ἔστι τὸ BH παραλληλόγραμμον τῷ ΔΘ παραλληλογράμμῳ. καὶ ἔστι τὸ μὲν BH τὸ ὑπὸ τῶν AB, Z· ἵση γάρ ἡ AH τῇ Z· τὸ δὲ ΔΘ τὸ ὑπὸ τῶν ΓΔ, E· ἵση γάρ ἡ E τῇ ΓΘ· τὸ ἄρα ὑπὸ τῶν AB, Z περιεχόμενον ὁρθογώνιον ἵσον ἔστι τῷ ὑπὸ τῶν ΓΔ, E περιεχομένῳ ὁρθογώνιῳ.

Ἄλλὰ δὴ τὸ ὑπὸ τῶν AB, Z περιεχόμενον ὁρθογώνιον ἵσον ἔστω τῷ ὑπὸ τῶν ΓΔ, E περιεχομένῳ ὁρθογώνιῳ. λέγω, ὅτι αἱ τέσσαρες εὐθεῖαι ἀνάλογον ἔσονται, ως ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ E πρὸς τὴν Z.

Τῶν γάρ αὐτῶν κατασκευασθέντων, ἐπεὶ τὸ ὑπὸ τῶν AB, Z ἵσον ἔστι τῷ ὑπὸ τῶν ΓΔ, E, καὶ ἔστι τὸ μὲν ὑπὸ τῶν AB, Z τὸ BH· ἵση γάρ ἔστιν ἡ AH τῇ Z· τὸ δὲ ὑπὸ τῶν ΓΔ, E τὸ ΔΘ· ἵση γάρ ἡ ΓΘ τῇ E· τὸ ἄρα BH ἵσον ἔστι τῷ ΔΘ. καὶ ἔστιν ἵσογωνία. τῶν δὲ ἵσων καὶ ἵσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἵσας γωνίας. ἔστιν ἄρα ως ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ ΓΘ πρὸς τὴν AH. ἵση δὲ ἡ μὲν ΓΘ τῇ E, ἡ δὲ AH τῇ Z· ἔστιν ἄρα ως ἡ AB πρὸς τὴν ΓΔ, οὕτως ἡ E πρὸς τὴν Z.

Ἐδίν ἄρα τέσσαρες εὐθεῖαι ἀνάλογον ὥσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὁρθογώνιον ἵσον ἔστι τῷ ὑπὸ τῶν μέσων περιεχομένῳ ὁρθογώνιῳ· καὶ τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὁρθογώνιον ἵσον ἡ τῷ ὑπὸ τῶν μέσων περιεχομένῳ ὁρθογώνιῳ, αἱ τέσσαρες εὐθεῖαι ἀνάλογον ἔσονται· ὅπερ ἔδει δεῖξαι.

angle contained by the (two) outermost is equal to the rectangle contained by the middle (two) then the four straight-lines will be proportional.



Let AB, CD, E, and F be four proportional straight-lines, (such that) as AB (is) to CD, so E (is) to F. I say that the rectangle contained by AB and F is equal to the rectangle contained by CD and E.

[For] let AG and CH have been drawn from points A and C at right-angles to the straight-lines AB and CD (respectively) [Prop. 1.11]. And let AG be made equal to F, and CH to E [Prop. 1.3]. And let the parallelograms BG and DH have been completed.

And since as AB is to CD, so E (is) to F, and E (is) equal CH, and F to AG, thus as AB is to CD, so CH (is) to AG. Thus, in the parallelograms BG and DH the sides about the equal angles are reciprocally proportional. And those equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal [Prop. 6.14]. Thus, parallelogram BG is equal to parallelogram DH. And BG is the (rectangle contained) by AB and F. For AG (is) equal to F. And DH (is) the (rectangle contained) by CD and E. For E (is) equal to CH. Thus, the rectangle contained by AB and F is equal to the rectangle contained by CD and E.

And so, let the rectangle contained by AB and F be equal to the rectangle contained by CD and E. I say that the four straight-lines will be proportional, (so that) as AB (is) to CD, so E (is) to F.

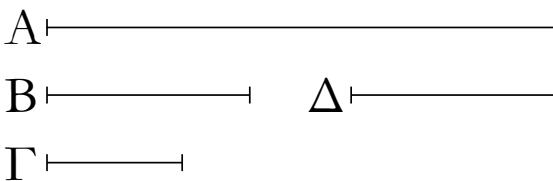
For, with the same construction, since the (rectangle contained) by AB and F is equal to the (rectangle contained) by CD and E. And BG is the (rectangle contained) by AB and F. For AG is equal to F. And DH (is) the (rectangle contained) by CD and E. For CH (is) equal to E. BG is thus equal to DH. And they are equiangular. And in equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional [Prop. 6.14]. Thus, as AB is to CD, so CH (is) to AG. And CH (is) equal to E, and AG to F. Thus, as AB is to CD, so E (is) to F.

Thus, if four straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the rectangle contained by the middle (two). And if the rectangle contained by the (two) outermost is equal to

the rectangle contained by the middle (two) then the four straight-lines will be proportional. (Which is) the very thing it was required to show.

ιζ'.

Ἐὰν τρεῖς εὐθεῖαι ἀνάλογον ὕσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὁρθογώνιον ἵσον ἔστι τῷ ἀπὸ τῆς μέσης τετραγώνῳ· καὶ τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὁρθογώνιον ἵσον ἥτις τῷ ἀπὸ τῆς μέσης τετραγώνῳ, αἱ τρεῖς εὐθεῖαι ἀνάλογον ἔσονται.



Ἐστωσαν τρεῖς εὐθεῖαι ἀνάλογον αἱ Α, Β, Γ, ὡς ἥτις πρὸς τὴν Β, οὕτως ἥτις Β πρὸς τὴν Γ· λέγω, ὅτι τὸ ὑπὸ τῶν Α, Γ περιεχόμενον ὁρθογώνιον ἵσον ἔστι τῷ ἀπὸ τῆς Β τετραγώνῳ.

Κείσθω τῇ Β ἵση ἥ Δ.

Καὶ ἐπεὶ ἔστων ὡς ἥτις Α πρὸς τὴν Β, οὕτως ἥτις Β πρὸς τὴν Γ, ἵση δὲ ἥτις Β πρὸς τὴν Δ, ἔστιν ἄρα ὡς ἥτις Α πρὸς τὴν Β, ἥτις Δ πρὸς τὴν Γ· ἐὰν δὲ τέσσαρες εὐθεῖαι ἀνάλογον ὕσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον [ὁρθογώνιον] ἵσον ἔστι τῷ ὑπὸ τῶν μέσων περιεχόμενῳ ὁρθογώνῳ· τὸ ἄρα ὑπὸ τῶν Α, Γ ἵσον ἔστι τῷ ὑπὸ τῶν Β, Δ· ἀλλὰ τὸ ὑπὸ τῶν Β, Δ τὸ ἀπὸ τῆς Β ἔστιν· ἵση γάρ ἥτις Β πρὸς τὴν Δ· τὸ ἄρα ὑπὸ τῶν Α, Γ περιεχόμενον ὁρθογώνιον ἵσον ἔστι τῷ ἀπὸ τῆς Β τετραγώνῳ.

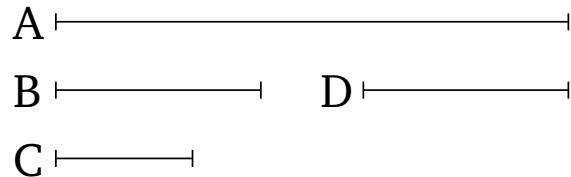
Ἀλλὰ δὴ τὸ ὑπὸ τῶν Α, Γ ἵσον ἔστω τῷ ἀπὸ τῆς Β· λέγω, ὅτι ἔστων ὡς ἥτις Α πρὸς τὴν Β, οὕτως ἥτις Β πρὸς τὴν Γ.

Τῶν γὰρ αὐτῶν κατασκευασθέντων, ἐπεὶ τὸ ὑπὸ τῶν Α, Γ ἵσον ἔστι τῷ ἀπὸ τῆς Β, ἀλλὰ τὸ ἀπὸ τῆς Β τὸ ὑπὸ τῶν Β, Δ ἔστιν· ἵση γάρ ἥτις Β πρὸς τὴν Δ· τὸ ἄρα ὑπὸ τῶν Α, Γ ἵσον ἔστι τῷ ὑπὸ τῶν Β, Δ· ἐὰν δὲ τὸ ὑπὸ τῶν ἄκρων ἵσον ἥτις τῷ ὑπὸ τῶν μέσων, αἱ τέσσαρες εὐθεῖαι ἀνάλογον εἰσιν. ἔστιν ἄρα ὡς ἥτις Α πρὸς τὴν Β, οὕτως ἥτις Δ πρὸς τὴν Γ· ἵση δὲ ἥτις Β πρὸς τὴν Δ· ὡς ἄρα ἥτις Α πρὸς τὴν Β, οὕτως ἥτις Β πρὸς τὴν Γ.

Ἐὰν ἄρα τρεῖς εὐθεῖαι ἀνάλογον ὕσιν, τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὁρθογώνιον ἵσον ἔστι τῷ ἀπὸ τῆς μέσης τετραγώνῳ· καὶ τὸ ὑπὸ τῶν ἄκρων περιεχόμενον ὁρθογώνιον ἵσον ἥτις τῷ ἀπὸ τῆς μέσης τετραγώνῳ, αἱ τρεῖς εὐθεῖαι ἀνάλογον ἔσονται· ὅπερ ἔδει δεῖξαι.

Proposition 17

If three straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the square on the middle (one). And if the rectangle contained by the (two) outermost is equal to the square on the middle (one) then the three straight-lines will be proportional.



Let A , B and C be three proportional straight-lines, (such that) as A (is) to B , so B (is) to C . I say that the rectangle contained by A and C is equal to the square on B .

Let D be made equal to B [Prop. 1.3].

And since as A is to B , so B is to C , and B (is) equal to D , thus as A is to B , (so) D (is) to C . And if four straight-lines are proportional then the [rectangle] contained by the (two) outermost is equal to the rectangle contained by the middle (two) [Prop. 6.16]. Thus, the (rectangle contained) by A and C is equal to the (rectangle contained) by B and D . But, the (rectangle contained) by B and D is the (square) on B . For B (is) equal to D . Thus, the rectangle contained by A and C is equal to the square on B .

And so, let the (rectangle contained) by A and C be equal to the (square) on B . I say that as A is to B , so B (is) to C .

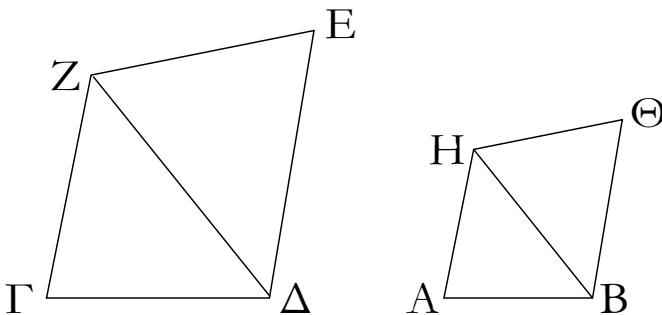
For, with the same construction, since the (rectangle contained) by A and C is equal to the (square) on B . But, the (square) on B is the (rectangle contained) by B and D . For B (is) equal to D . The (rectangle contained) by A and C is thus equal to the (rectangle contained) by B and D . And if the (rectangle contained) by the (two) outermost is equal to the (rectangle contained) by the middle (two) then the four straight-lines are proportional [Prop. 6.16]. Thus, as A is to B , so D (is) to C . And B (is) equal to D . Thus, as A (is) to B , so B (is) to C .

Thus, if three straight-lines are proportional then the rectangle contained by the (two) outermost is equal to the square on the middle (one). And if the rectangle contained by the (two) outermost is equal to the square on the middle (one) then the three straight-lines will be proportional. (Which is) the very thing it was required to

show.

ιη'.

Ἄπο τῆς δοιθείσης εὐθείας τῷ δοιθέντι εὐθυγράμμῳ ὅμοιόν τε καὶ ὁμοίως κείμενον εὐθύγραμμον ἀναγράψαι.

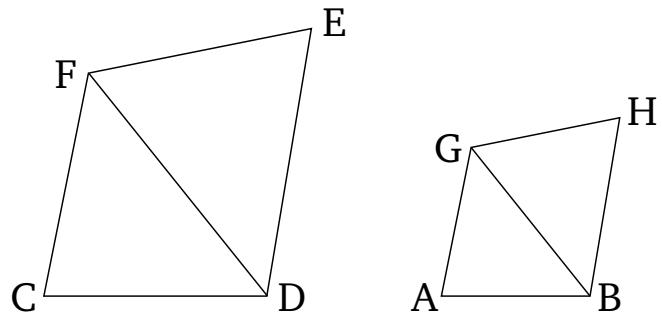


Ἐστω ἡ μὲν δοιθείσα εὐθεῖα ἡ AB, τὸ δὲ δοιθέν εὐθυγράμμῳ τὸ ΓΕ· δεῖ δὴ ἀπὸ τῆς AB εὐθείας τῷ ΓΕ εὐθυγράμμῳ ὅμοιόν τε καὶ ὁμοίως κείμενον εὐθύγραμμον ἀναγράψαι.

Ἐπεζεύχθω ἡ ΔΖ, καὶ συνεστάτω πρὸς τῇ AB εὐθείᾳ καὶ τοῖς πρὸς αὐτῇ σημείοις τοῖς A, B τῇ μὲν πρὸς τῷ Γ γωνίᾳ ἵση ἡ ὑπὸ HAB, τῇ δὲ ὑπὸ ΓΔΖ ἵση ἡ ὑπὸ ABH. λοιπὴ ἄρα ἡ ὑπὸ ΓΖΔ τῇ ὑπὸ AHB ἐστιν ἵση· ἴσογώνιον ἄρα ἐστὶ τὸ ZΓΔ τρίγωνον τῷ HAB τριγώνῳ. ἀνάλογον ἄρα ἐστὶν ὡς ἡ ZΔ πρὸς τὴν HB, οὕτως ἡ ZΓ πρὸς τὴν HA, καὶ ἡ ΓΔ πρὸς τὴν AB. πάλιν συνεστάτω πρὸς τῇ BH εὐθείᾳ καὶ τοῖς πρὸς αὐτῇ σημείοις τοῖς B, H τῇ μὲν ὑπὸ ΔΖΕ γωνίᾳ ἵση ἡ ὑπὸ BHΘ, τῇ δὲ ὑπὸ ZΔΕ ἵση ἡ ὑπὸ HBΘ. λοιπὴ ἄρα ἡ πρὸς τῷ E λοιπῇ τῇ πρὸς τῷ Θ ἐστιν ἵση· ἴσογώνιον ἄρα ἐστὶ τὸ ZΔΕ τρίγωνον τῷ HΘB τριγώνῳ. ἀνάλογον ἄρα ἐστὶν ὡς ἡ ZΔ πρὸς τὴν HB, οὕτως ἡ ZΕ πρὸς τὴν HΘ καὶ ἡ EΔ πρὸς τὴν ΘB. ἐδείχθη δὲ καὶ ὡς ἡ ZΔ πρὸς τὴν HB, οὕτως ἡ ZΓ πρὸς τὴν HA καὶ ἡ ΓΔ πρὸς τὴν AB· καὶ ὡς ἄρα ἡ ZΓ πρὸς τὴν AH, οὕτως ἡ τε ΓΔ πρὸς τὴν AB καὶ ἡ ZΕ πρὸς τὴν HΘ καὶ ἔτι ἡ EA πρὸς τὴν ΘB. καὶ ἐπειὶ ἵση ἐστὶν ἡ μὲν ὑπὸ ΓΖΔ γωνία τῇ ὑπὸ AHB, ἡ δὲ ὑπὸ ΔΖΕ τῇ ὑπὸ BHΘ, ὅλη ἄρα ἡ ὑπὸ ΓΔΕ τῇ ὑπὸ ABΘ ἐστιν ἵση. ἔστι δὲ καὶ ἡ μὲν πρὸς τῷ Γ πρὸς τῷ A ἵση, ἡ δὲ πρὸς τῷ E τῇ πρὸς τῷ Θ. ἴσογώνιον ἄρα ἐστὶ τὸ AΘ τῷ ΓΕ· καὶ τὰς περὶ τὰς ἵσας γωνίας αὐτῶν πλευρὰς ἀνάλογον ἔχει· ὅμοιον ἄρα ἐστὶ τὸ AΘ εὐθύγραμμον τῷ ΓΕ εὐθυγράμμῳ.

Ἀπὸ τῆς δοιθείσης ἄρα εὐθείας τῆς AB τῷ δοιθέντι εὐθυγράμμῳ τῷ ΓΕ ὅμοιόν τε καὶ ὁμοίως κείμενον εὐθύγραμμον ἀναγέγραπται τὸ AΘ· ὅπερ ἔδει ποιῆσαι.

To describe a rectilinear figure similar, and similarly laid down, to a given rectilinear figure on a given straight-line.



Let AB be the given straight-line, and CE the given rectilinear figure. So it is required to describe a rectilinear figure similar, and similarly laid down, to the rectilinear figure CE on the straight-line AB .

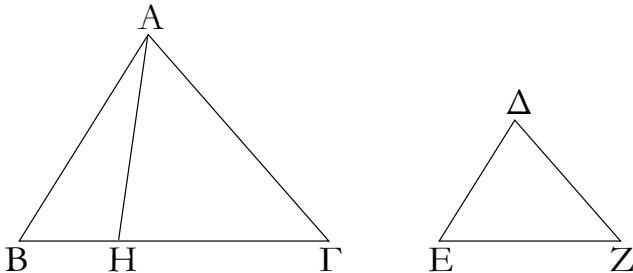
Let DF have been joined, and let GAB , equal to the angle at C , and ABG , equal to (angle) CDF , have been constructed on the straight-line AB at the points A and B on it (respectively) [Prop. 1.23]. Thus, the remaining (angle) CFD is equal to AGB [Prop. 1.32]. Thus, triangle FCD is equiangular to triangle GAB . Thus, proportionally, as FD is to GB , so FC (is) to GA , and CD to AB [Prop. 6.4]. Again, let BGH , equal to angle DFA , and GBH equal to (angle) FDE , have been constructed on the straight-line BG at the points G and B on it (respectively) [Prop. 1.23]. Thus, the remaining (angle) at E is equal to the remaining (angle) at H [Prop. 1.32]. Thus, triangle FDE is equiangular to triangle GHB . Thus, proportionally, as FD is to GB , so FE (is) to GH , and ED to HB [Prop. 6.4]. And it was also shown (that) as FD (is) to GB , so FC (is) to GA , and CD to AB . Thus, also, as FC (is) to AG , so CD (is) to AB , and FE to GH , and, further, ED to HB . And since angle CFD is equal to AGB , and DFE to BGH , thus the whole (angle) CFE is equal to the whole (angle) AGH . So, for the same (reasons), (angle) CDE is also equal to ABH . And the (angle) at C is also equal to the (angle) at A , and the (angle) at E to the (angle) at H . Thus, (figure) AH is equiangular to CE . And (the two figures) have the sides about their equal angles proportional. Thus, the rectilinear figure AH is similar to the rectilinear figure CE [Def. 6.1].

Thus, the rectilinear figure AH , similar, and similarly laid down, to the given rectilinear figure CE has been constructed on the given straight-line AB . (Which is) the

very thing it was required to do.

iθ'.

Τὰ ὄμοια τρίγωνα πρὸς ἄλληλα ἐν διπλασίονι λόγῳ ἔστι τῶν ὁμολόγων πλευρῶν.



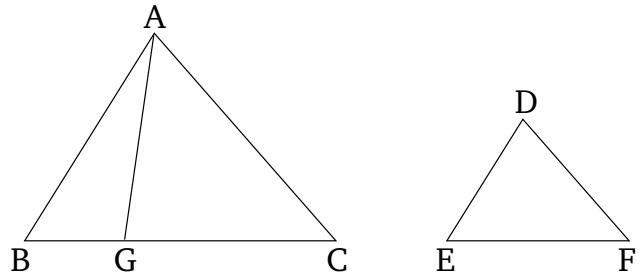
Ἐστω ὄμοια τρίγωνα τὰ ABG , ΔEZ ἵσην ἔχοντα τὴν πρὸς τῷ B γωνίαν τῇ πρὸς τῷ E , ὡς δὲ τὴν AB πρὸς τὴν BG , οὕτως τὴν ΔE πρὸς τὴν EZ , ὡστε ὁμόλογον εἶναι τὴν BG τῇ EZ · λέγω, ὅτι τὸ ABG τρίγωνον πρὸς τὸ ΔEZ τρίγωνον διπλασίονα λόγον ἔχει ἥπερ ἢ BG πρὸς τὴν EZ .

Εἰλήφθω γάρ τῶν BG , EZ τρίτη ἀνάλογον ἢ BH , ὡστε εἶναι ὡς τὴν BG πρὸς τὴν EZ , οὕτως τὴν EZ πρὸς τὴν BH · καὶ ἐπεζεύχθω ἢ AH .

Ἐπεὶ οὖν ἔστιν ὡς ἢ AB πρὸς τὴν BG , οὕτως ἢ ΔE πρὸς τὴν EZ , ἐναλλάξ ἄρα ἔστιν ὡς ἢ AB πρὸς τὴν ΔE , οὕτως ἢ BG πρὸς τὴν EZ . ἀλλ᾽ ὡς ἢ BG πρὸς EZ , οὕτως ἔστιν ἢ EZ πρὸς BH . καὶ ὡς ἄρα ἢ AB πρὸς ΔE , οὕτως ἢ EZ πρὸς BH · τῶν ABH , ΔEZ ἄρα τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνάις. ὃν δὲ μίαν μιᾷ ἵσην ἔχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνάις, ἵσα ἔστιν ἔκεινα. ἵσον ἄρα ἔστι τὸ ABH τρίγωνον τῷ ΔEZ τριγώνῳ. καὶ ἐπεὶ ἔστιν ὡς ἢ BG πρὸς τὴν EZ , οὕτως ἢ EZ πρὸς τὴν BH , ἐὰν δὲ τρεῖς εὐθεῖαι ἀνάλογον ὕσιν, ἢ πρώτη πρὸς τὴν τρίτην διπλασίονα λόγον ἔχει ἥπερ πρὸς τὴν δευτέραν, ἢ BG ἄρα πρὸς τὴν BH διπλασίονα λόγον ἔχει ἥπερ ἢ ΓB πρὸς τὴν EZ . ὡς δὲ ἢ ΓB πρὸς τὴν BH , οὕτως τὸ ABG τρίγωνον πρὸς τὸ ABH τρίγωνον· καὶ τὸ ABG ἄρα τριγώνον πρὸς τὸ ABH διπλασίονα λόγον ἔχει ἥπερ ἢ BG πρὸς τὴν EZ . ἵσον δὲ τὸ ABH τρίγωνον τῷ ΔEZ τριγώνῳ. καὶ τὸ ABG ἄρα τριγώνον πρὸς τὸ ΔEZ τριγώνον διπλασίονα λόγον ἔχει ἥπερ ἢ BG πρὸς τὴν EZ .

Τὰ ἄρα ὄμοια τρίγωνα πρὸς ἄλληλα ἐν διπλασίονι λόγῳ ἔστι τῶν ὁμολόγων πλευρῶν. [ὅπερ ἔδει δεῖξαι.]

Similar triangles are to one another in the squared[†] ratio of (their) corresponding sides.



Let ABC and DEF be similar triangles having the angle at B equal to the (angle) at E , and AB to BC , as DE (is) to EF , such that BC corresponds to EF . I say that triangle ABC has a squared ratio to triangle DEF with respect to (that side) BC (has) to EF .

For let a third (straight-line), BG , have been taken (which is) proportional to BC and EF , so that as BC (is) to EF , so EF (is) to BG [Prop. 6.11]. And let AG have been joined.

Therefore, since as AB is to BC , so DE (is) to EF , thus, alternately, as AB is to DE , so BC (is) to EF [Prop. 5.16]. But, as BC (is) to EF , so EF is to BG . And, thus, as AB (is) to DE , so EF (is) to BG . Thus, for triangles ABG and DEF , the sides about the equal angles are reciprocally proportional. And those triangles having one (angle) equal to one (angle) for which the sides about the equal angles are reciprocally proportional are equal [Prop. 6.15]. Thus, triangle ABG is equal to triangle DEF . And since as BC (is) to EF , so EF (is) to BG , and if three straight-lines are proportional then the first has a squared ratio to the third with respect to the second [Def. 5.9], BC thus has a squared ratio to BG with respect to (that) CB (has) to EF . And as CB (is) to BG , so triangle ABC (is) to triangle ABG [Prop. 6.1]. Thus, triangle ABC also has a squared ratio to (triangle) ABG with respect to (that side) BC (has) to EF . And triangle ABG (is) equal to triangle DEF . Thus, triangle ABC also has a squared ratio to triangle DEF with respect to (that side) BC (has) to EF .

Thus, similar triangles are to one another in the squared ratio of (their) corresponding sides. [(Which is) the very thing it was required to show].

Πόρισμα.

Ἐκ δὴ τούτου φανερόν, ὅτι, ἐὰν τρεῖς εὐθεῖαι ἀνάλογον ὕσιν, ἔστιν ὡς ἢ πρώτη πρὸς τὴν τρίτην, οὕτως τὸ ἀπὸ

Corollary

So it is clear, from this, that if three straight-lines are proportional, then as the first is to the third, so the figure

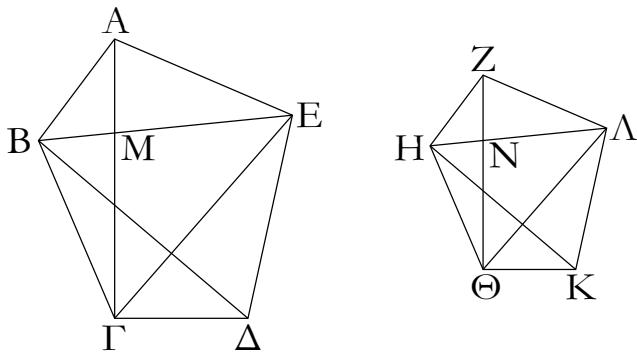
τῆς πρώτης εἶδος πρὸς τὸ ἀπὸ τῆς δευτέρας τὸ ὄμοιον καὶ ὄμοιῶς ἀναγραφόμενον. ὅπερ ἔδει δεῖξαι.

(described) on the first (is) to the similar, and similarly described, (figure) on the second. (Which is) the very thing it was required to show.

[†] Literally, “double”.

x' .

Τὰ ὄμοια πολύγωνα εἰς τε ὄμοια τρίγωνα διαιρεῖται καὶ εἰς ἵσα τὸ πλῆθος καὶ ὄμοιογα τοῖς ὅλοις, καὶ τὸ πολύγωνον πρὸς τὸ πολύγωνον διπλασίονα λόγον ἔχει ἡπερ ἡ ὄμοιογος πλευρὰ πρὸς τὴν ὄμοιογον πλευράν.



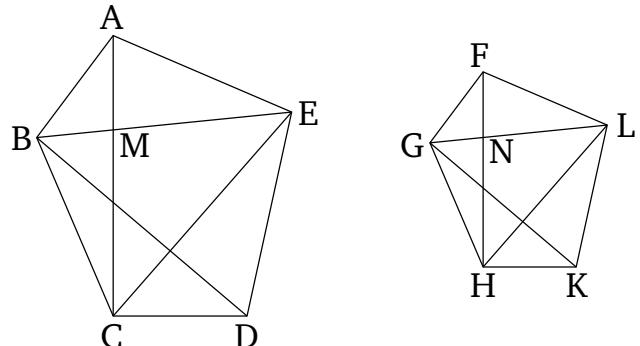
Ἐστω ὄμοια πολύγωνα τὰ ABCDE, ZHΘKL, ὄμοιογος δὲ ἔστω ἡ AB τῇ ZH· λέγω, ὅτι τὰ ABCDE, ZHΘKL πολύγωνα εἰς τε ὄμοια τρίγωνα διαιρεῖται καὶ εἰς ἵσα τὸ πλῆθος καὶ ὄμοιογα τοῖς ὅλοις, καὶ τὸ ABCDE πολύγωνον πρὸς τὸ ZHΘKL πολύγωνον διπλασίονα λόγον ἔχει ἡπερ ἡ AB πρὸς τὴν ZH.

Ἐπεζεύχθωσαν αἱ BE, EG, HL, ΛΘ.

Καὶ ἐπεὶ ὄμοιόν ἐστι τὸ ABCDE πολύγωνον τῷ ZHΘKL πολυγώνῳ, ἵση ἔστιν ἡ ὑπὸ BAE γωνία τῇ ὑπὸ HZΛ. καὶ ἐστιν ὡς ἡ BA πρὸς AE, οὕτως ἡ HZ πρὸς ΖΛ. ἐπεὶ οὖν δύο τρίγωνά ἔστι τὰ ABE, ZHL μίαν γωνίαν μιᾷ γωνίᾳ ἵσην ἔχοντα, περὶ δὲ τὰς ἵσας γωνίας τὰς πλευρὰς ἀνάλογον, ἴσογώνιον ἄρα ἔστι τὸ ABE τρίγωνον τῷ ZHL τριγώνῳ· ὥστε καὶ ὄμοιον· ἵση ἄρα ἔστιν ἡ ὑπὸ ABE γωνία τῇ ὑπὸ ZHL. ἔστι δὲ καὶ ὅλη ἡ ὑπὸ ABΓ ὅλη τῇ ὑπὸ ZHΘ ἵση διὰ τὴν ὄμοιότητα τῶν πολυγώνων· λοιπὴ ἄρα ἡ ὑπὸ EBΓ γωνία τῇ ὑπὸ ΛΗΘ ἕστιν ἵση. καὶ ἐπεὶ διὰ τὴν ὄμοιότητα τῶν ABE, ZHL τριγώνων ἔστιν ὡς ἡ EB πρὸς BA, οὕτως ἡ ΛΗ πρὸς HZ, ἀλλὰ μὴν καὶ διὰ τὴν ὄμοιότητα τῶν πολυγώνων ἔστιν ὡς ἡ AB πρὸς BΓ, οὕτως ἡ ZH πρὸς ΗΘ, διὸ ἵσου ἄρα ἔστιν ὡς ἡ EB πρὸς BΓ, οὕτως ἡ ΛΗ πρὸς ΗΘ, καὶ περὶ τὰς ἵσας γωνάις τὰς ὑπὸ EBΓ, ΛΗΘ αἱ πλευρὰὶ ἀνάλογόν εἰσιν· ἴσογώνιον ἄρα ἔστι τὸ EBΓ τρίγωνον τῷ ΛΗΘ τριγώνῳ· ὥστε καὶ ὄμοιόν ἔστι τὸ EBΓ τρίγωνον τῷ ΛΗΘ τριγώνῳ. διὰ τὰ αὐτὰ δὴ καὶ τὸ ΕΓΔ τρίγωνον ὄμοιόν ἔστι τῷ ΛΘΚ τριγώνῳ. τὰ ἄρα ὄμοια πολύγωνα τὰ ABCDE, ZHΘKL εἰς τε ὄμοια τρίγωνα διήρηται καὶ εἰς ἵσα

Proposition 20

Similar polygons can be divided into equal numbers of similar triangles corresponding (in proportion) to the wholes, and one polygon has to the (other) polygon a squared ratio with respect to (that) a corresponding side (has) to a corresponding side.



Let ABCDE and FGHKL be similar polygons, and let AB correspond to FG. I say that polygons ABCDE and FGHKL can be divided into equal numbers of similar triangles corresponding (in proportion) to the wholes, and (that) polygon ABCDE has a squared ratio to polygon FGHKL with respect to that AB (has) to FG.

Let BE, EC, GL, and LH have been joined.

And since polygon ABCDE is similar to polygon FGHKL, angle BAE is equal to angle GFL, and as BA is to AE, so GF (is) to FL [Def. 6.1]. Therefore, since ABE and FGL are two triangles having one angle equal to one angle and the sides about the equal angles proportional, triangle ABE is thus equiangular to triangle FGL [Prop. 6.6]. Hence, (they are) also similar [Prop. 6.4, Def. 6.1]. Thus, angle ABE is equal to (angle) FGL. And the whole (angle) ABC is equal to the whole (angle) FGH, on account of the similarity of the polygons. Thus, the remaining angle EBC is equal to LGH. And since, on account of the similarity of triangles ABE and FGL, as EB is to BA, so LG (is) to GF, but also, on account of the similarity of the polygons, as AB is to BC, so FG (is) to GH, thus, via equality, as EB is to BC, so LG (is) to GH [Prop. 5.22], and the sides about the equal angles, EBC and LGH, are proportional. Thus, triangle EBC is equiangular to triangle LGH [Prop. 6.6]. Hence, triangle EBC is also similar to triangle LGH [Prop. 6.4, Def. 6.1]. So, for the same (reasons), triangle ECD is also similar

τὸ πλῆθος.

Λέγω, ὅτι καὶ ὁμόλογα τοῖς ὅλοις, τουτέστιν ὥστε ἀνάλογον εἶναι τὰ τρίγωνα, καὶ ἡγούμενα μὲν εἶναι τὰ ABE, EBG, EΓΔ, ἐπόμενα δὲ αὐτῶν τὰ ZΗΔ, ΛΗΘ, ΛΘΚ, καὶ ὅτι τὸ ΑΒΓΔΕ πολύγωνον πρὸς τὸ ZΗΘΚΛ πολύγωνον διπλασίονα λόγον ἔχει ἡπερ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν, τουτέστιν ἡ AB πρὸς τὴν ZH.

Ἐπεζεύχθωσαν γάρ οἱ ΑΓ, ΖΘ. καὶ ἐπεὶ διὰ τὴν ὁμοιότητα τῶν πολυγώνων ἵση ἐστὶν ἡ ὑπὸ AΒΓ γωνία τῇ ὑπὸ ZΗΘ, καὶ ἐστιν ὡς ἡ AB πρὸς BG, οὕτως ἡ ZH πρὸς HΘ, ἵσογώνιον ἐστι τὸ AΒΓ τρίγωνον τῷ ZΗΘ τριγώνῳ· ἵση ἄρα ἐστὶν ἡ μὲν ὑπὸ BAG γωνία τῇ ὑπὸ HZΘ, ἡ δὲ ὑπὸ BΓΑ τῇ ὑπὸ HΘΖ. καὶ ἐπεὶ ἵση ἐστὶν ἡ ὑπὸ BAM γωνία τῇ ὑπὸ HZN, ἐστι δὲ καὶ ἡ ὑπὸ ABM τῇ ὑπὸ ZHN ἵση, καὶ λοιπὴ ἄρα ἡ ὑπὸ AMB λοιπῇ τῇ ὑπὸ ZNH ἵση ἐστὶν ἵσογώνιον ἄρα ἐστὶ τὸ ABM τρίγωνον τῷ ZHN τριγώνῳ. ὁμοίως δὴ δεῖξομεν, ὅτι καὶ τὸ BMG τρίγωνον ἵσογώνιον ἐστι τῷ HNΘ τριγώνῳ. ἀνάλογον ἄρα ἐστὶν, ὡς μὲν ἡ AM πρὸς MB, οὕτως ἡ ZN πρὸς NH, ὡς δὲ ἡ BM πρὸς MT, οὕτως ἡ HN πρὸς NΘ· ὥστε καὶ δι’ ἵσου, ὡς ἡ AM πρὸς MG, οὕτως ἡ ZN πρὸς NΘ. ἀλλ’ ὡς ἡ AM πρὸς MG, οὕτως τὸ ABM [τρίγωνον] πρὸς τὸ MBG, καὶ τὸ AME πρὸς τὸ EΜG· πρὸς ἄλληλα γάρ εἰσιν ὡς οἱ βάσεις. καὶ ὡς ἄρα ἐν τῶν ἡγουμένων πρὸς ἐν τῶν ἐπόμενων, οὕτως ἀπαντα τὰ ἡγούμενα πρὸς ἀπαντα τὰ ἐπόμενα· ὡς ἄρα τὸ AMB τρίγωνον πρὸς τὸ BMG, οὕτως τὸ ABE πρὸς τὸ ΓΒΕ. ἀλλ’ ὡς τὸ AMB πρὸς τὸ BMG, οὕτως ἡ AM πρὸς MG· καὶ ὡς ἄρα ἡ AM πρὸς MG, οὕτως τὸ ABE τρίγωνον πρὸς τὸ EΒΓ τρίγωνον. διὰ τὰ αὐτὰ δὴ καὶ ὡς ἡ ZN πρὸς NΘ, οὕτως τὸ ZΗΔ τρίγωνον πρὸς τὸ HΛΘ τριγώνον. καὶ ἐστιν ὡς ἡ AM πρὸς MG, οὕτως ἡ ZN πρὸς NΘ· καὶ ὡς ἄρα τὸ ABE τρίγωνον πρὸς τὸ BEΓ τρίγωνον, οὕτως τὸ ZΗΔ τρίγωνον πρὸς τὸ HΛΘ τριγώνον, καὶ ἐναλλάξ ὡς τὸ ABE τρίγωνον πρὸς τὸ ZΗΔ τρίγωνον, οὕτως τὸ BEΓ τρίγωνον πρὸς τὸ HΛΘ τριγώνον. ὁμοίως δὴ δεῖξομεν ἐπιζευχθεισῶν τῶν BD, HK, ὅτι καὶ ὡς τὸ BEΓ τρίγωνον πρὸς τὸ ΛΗΘ τρίγωνον, οὕτως τὸ EΓΔ τρίγωνον πρὸς τὸ ΛΘΚ τριγώνον. καὶ ἐπεὶ ἐστιν ὡς τὸ ABE τρίγωνον πρὸς τὸ ZΗΔ τριγώνον, οὕτως τὸ EΒΓ πρὸς τὸ ΛΗΘ, καὶ ἐτι τὸ EΓΔ πρὸς τὸ ΛΘΚ, καὶ ὡς ἄρα ἐν τῶν ἡγουμένων πρὸς ἐν τῶν ἐπόμενων, οὕτως ἀπαντα τὰ ἡγούμενα πρὸς ἀπαντα τὰ ἐπόμενα· ἐστιν ἄρα ὡς τὸ ABE τρίγωνον πρὸς τὸ ZΗΔ τρίγωνον, οὕτως τὸ ΑΒΓΔΕ πολύγωνον πρὸς τὸ ZΗΘΚΛ πολύγωνον. ἀλλὰ τὸ ABE τρίγωνον πρὸς τὸ ZΗΔ τριγώνον διπλασίονα λόγον ἔχει ἡπερ ἡ AB ὁμόλογος πλευρὰ πρὸς τὴν ZH ὁμόλογον πλευράν. καὶ τὸ ΑΒΓΔΕ ἄρα πολύγωνον πρὸς τὸ ZΗΘΚΛ πολύγωνον διπλασίονα λόγον ἔχει ἡπερ ἡ AB ὁμόλογος πλευρὰ πρὸς τὴν ZH ὁμόλογον πλευράν.

Τὰ ἄρα ὅμοια πολύγωνα εἰς τε ὅμοια τρίγωνα διαιρεῖται καὶ εἰς ἵσα τὸ πλῆθος καὶ ὁμόλογα τοῖς ὅλοις, καὶ τὸ

to triangle LHK . Thus, the similar polygons $ABCDE$ and $FGHKL$ have been divided into equal numbers of similar triangles.

I also say that (the triangles) correspond (in proportion) to the wholes. That is to say, the triangles are proportional: ABE , EBC , and ECD are the leading (magnitudes), and their (associated) following (magnitudes are) FGL , LGH , and LHK (respectively). (I) also (say) that polygon $ABCDE$ has a squared ratio to polygon $FGHKL$ with respect to (that) a corresponding side (has) to a corresponding side—that is to say, (side) AB to FG .

For let AC and FH have been joined. And since angle ABC is equal to FGH , and as AB is to BC , so FG (is) to GH , on account of the similarity of the polygons, triangle ABC is equiangular to triangle FGH [Prop. 6.6]. Thus, angle BAC is equal to GFH , and (angle) BCA to GHF . And since angle BAM is equal to GFN , and (angle) ABM is also equal to FGN (see earlier), the remaining (angle) AMB is thus also equal to the remaining (angle) FNG [Prop. 1.32]. Thus, triangle ABM is equiangular to triangle FGN . So, similarly, we can show that triangle BMC is also equiangular to triangle GNH . Thus, proportionally, as AM is to MB , so FN (is) to NG , and as BM (is) to MC , so GN (is) to NH [Prop. 6.4]. Hence, also, via equality, as AM (is) to MC , so FN (is) to NH [Prop. 5.22]. But, as AM (is) to MC , so [triangle] ABM is to MBC , and AME to EMC . For they are to one another as their bases [Prop. 6.1]. And as one of the leading (magnitudes) is to one of the following (magnitudes), so (the sum of) all the leading (magnitudes) is to (the sum of) all the following (magnitudes) [Prop. 5.12]. Thus, as triangle AMB (is) to MBC , so (triangle) ABE (is) to CBE . But, as (triangle) AMB (is) to MBC , so AM (is) to MC . Thus, also, as AM (is) to MC , so triangle ABE (is) to triangle EBC . And so, for the same (reasons), as FN (is) to NH , so triangle FGL (is) to triangle GLH . And as AM is to MC , so FN (is) to NH . Thus, also, as triangle ABE (is) to triangle BEC , so triangle FGL (is) to triangle GLH , and, alternately, as triangle ABE (is) to triangle FGL , so triangle BEC (is) to triangle GLH [Prop. 5.16]. So, similarly, we can also show, by joining BD and GK , that as triangle BEC (is) to triangle LGH , so triangle ECD (is) to triangle LHK . And since as triangle ABE is to triangle FGL , so (triangle) EBC (is) to LGH , and, further, (triangle) ECD to LHK , and also as one of the leading (magnitudes is) to one of the following, so (the sum of) all the leading (magnitudes is) to (the sum of) all the following [Prop. 5.12], thus as triangle ABE is to triangle FGL , so polygon $ABCDE$ (is) to polygon $FGHKL$. But, triangle ABE has a squared ratio

πολύγωνον πρὸς τὸ πολύγωνον διπλασίονα λόγον ἔχει ἥπερ ἡ ὁμόλογος πλευρὰ πρὸς τὴν ὁμόλογον πλευράν [ὅπερ ἔδει δεῖξαι].

to triangle FGL with respect to (that) the corresponding side AB (has) to the corresponding side FG . For, similar triangles are in the squared ratio of corresponding sides [Prop. 6.14]. Thus, polygon $ABCDE$ also has a squared ratio to polygon $FGHKL$ with respect to (that) the corresponding side AB (has) to the corresponding side FG .

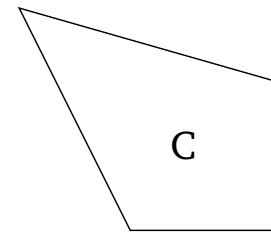
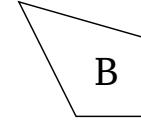
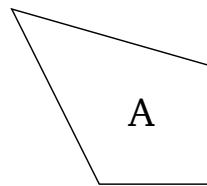
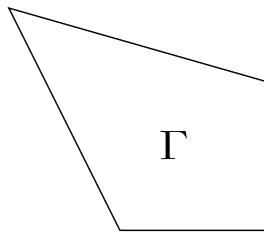
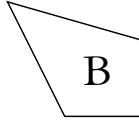
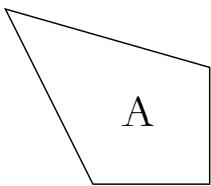
Thus, similar polygons can be divided into equal numbers of similar triangles corresponding (in proportion) to the wholes, and one polygon has to the (other) polygon a squared ratio with respect to (that) a corresponding side (has) to a corresponding side. [(Which is) the very thing it was required to show].

Πόρισμα.

Ωσαύτως δὲ καὶ ἐπὶ τῶν [ὅμοιών] τετραπλεύρων δειχθῆσται, ὅτι ἐν διπλασίονι λόγῳ εἰσὶ τῶν ὁμολόγων πλευρῶν. ἐδείχθη δὲ καὶ ἐπὶ τῶν τριγώνων ὡστε καὶ καθόλου τὰ ὄμοια εὐθύγραμμα σχήματα πρὸς ἄλληλα ἐν διπλασίονι λόγῳ εἰσὶ τῶν ὁμολόγων πλευρῶν. ὅπερ ἔδει δεῖξαι.

$\chi\alpha'$.

Τὰ τῷ αὐτῷ εὐθύγραμμῳ ὄμοια καὶ ἄλλήλοις ἔστιν ὄμοια.



Ἐστω γὰρ ἔκάτερον τῶν A , B εὐθύγραμμῶν τῷ Γ ὄμοιον· λέγω, ὅτι καὶ τὸ A τῷ B ἔστιν ὄμοιον.

Ἐπεὶ γὰρ ὄμοιον ἔστι τὸ A τῷ Γ , ἴσογώνιόν τέ ἔστιν αὐτῷ καὶ τὰς περὶ τὰς ἵσας γωνίας πλευράς ἀνάλογον ἔχει. πάλιν, ἐπεὶ ὄμοιον ἔστι τὸ B τῷ Γ , ἴσογώνιόν τέ ἔστιν αὐτῷ καὶ τὰς περὶ τὰς ἵσας γωνίας πλευράς ἀνάλογον ἔχει. ἔκάτερον ἄρα τῶν A , B τῷ Γ ἴσογώνιόν τέ ἔστι καὶ τὰς περὶ τὰς ἵσας γωνίας πλευράς ἀνάλογον ἔχει [ὡστε καὶ τὸ A τῷ B ἴσογώνιόν τέ ἔστι καὶ τὰς περὶ τὰς ἵσας γωνίας

And, in the same manner, it can also be shown for [similar] quadrilaterals that they are in the squared ratio of (their) corresponding sides. And it was also shown for triangles. Hence, in general, similar rectilinear figures are also to one another in the squared ratio of (their) corresponding sides. (Which is) the very thing it was required to show.

Proposition 21

(Rectilinear figures) similar to the same rectilinear figure are also similar to one another.

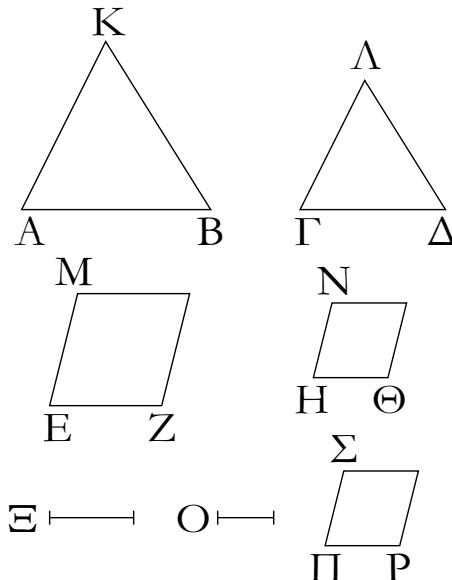
Let each of the rectilinear figures A and B be similar to (the rectilinear figure) C . I say that A is also similar to B .

For since A is similar to C , (A) is equiangular to (C), and has the sides about the equal angles proportional [Def. 6.1]. Again, since B is similar to C , (B) is equiangular to (C), and has the sides about the equal angles proportional [Def. 6.1]. Thus, A and B are each equiangular to C , and have the sides about the equal angles

πλευράς ἀνάλογον ἔχει]. ὅμοιον ἄρα ἐστὶ τὸ Α τῷ Β· ὅπερ
ἔδει δεῖξαι.

$\chi\beta'$.

Ἐὰν τέσσαρες εὐθεῖαι ἀνάλογον ὁσιν, καὶ τὰ ἀπὸ αὐτῶν
εὐθύγραμμα ὅμοιά τε καὶ ὁμοίως ἀναγεγραμμένα ἀνάλογον
ἐσται· καὶ τὰ ἀπὸ αὐτῶν εὐθύγραμμα ὅμοιά τε καὶ ὁμοίως
ἀναγεγραμμένα ἀνάλογον ἥ, καὶ αὐτὰi αἱ εὐθεῖαι ἀνάλογον
ἐσονται.



Ἐστωσαν τέσσαρες εὐθεῖαι ἀνάλογον αἱ ΑΒ, ΓΔ, ΕΖ,
ΗΘ, ὡς ἡ ΑΒ πρὸς τὴν ΓΔ, οὕτως ἡ ΕΖ πρὸς τὴν ΗΘ, καὶ
ἀναγεγράφωσαν ἀπὸ μὲν τῶν ΑΒ, ΓΔ ὅμοιά τε καὶ ὁμοίως
κείμενα εὐθύγραμμα τὰ ΚΑΒ, ΛΓΔ, ἀπὸ δὲ τῶν ΕΖ, ΗΘ
ὅμοιά τε καὶ ὁμοίως κείμενα εὐθύγραμμα τὰ ΜΖ, ΝΘ· λέγω,
ὅτι ἐστὶν ὡς τὸ ΚΑΒ πρὸς τὸ ΛΓΔ, οὕτως τὸ ΜΖ πρὸς τὸ
ΝΘ.

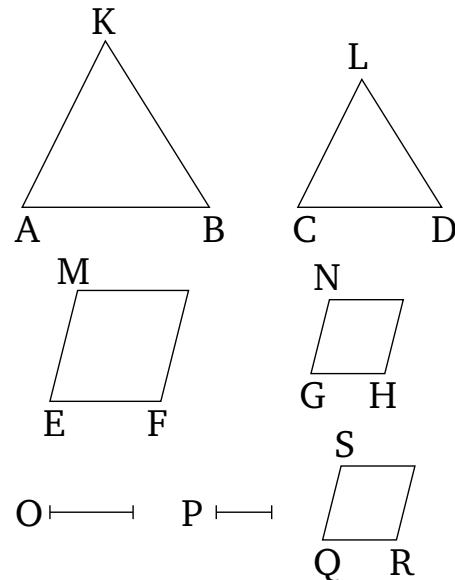
Εἰλήφω γάρ τῶν μὲν ΑΒ, ΓΔ τρίτη ἀνάλογον ἡ Ξ, τῶν
δὲ ΕΖ, ΗΘ τρίτη ἀνάλογον ἡ Ο. καὶ ἐπεὶ ἐστὶν ὡς μὲν ἡ ΑΒ
πρὸς τὴν ΓΔ, οὕτως ἡ ΕΖ πρὸς τὴν ΗΘ, ὡς δὲ ἡ ΓΔ πρὸς
τὴν Ξ, οὕτως ἡ ΗΘ πρὸς τὴν Ο, διὸ ἵσου ἄρα ἐστὶν ὡς ἡ
ΑΒ πρὸς τὴν Ξ, οὕτως ἡ ΕΖ πρὸς τὴν Ο. ἀλλ᾽ ὡς μὲν ἡ
ΑΒ πρὸς τὴν Ξ, οὕτως [καὶ] τὸ ΚΑΒ πρὸς τὸ ΛΓΔ, ὡς δὲ
ἡ ΕΖ πρὸς τὴν Ο, οὕτως τὸ ΜΖ πρὸς τὸ ΝΘ· καὶ ὡς ἄρα
τὸ ΚΑΒ πρὸς τὸ ΛΓΔ, οὕτως τὸ ΜΖ πρὸς τὸ ΝΘ.

Ἄλλὰ δὴ ἔστω ὡς τὸ ΚΑΒ πρὸς τὸ ΛΓΔ, οὕτως τὸ ΜΖ
πρὸς τὸ ΝΘ· λέγω, ὅτι ἐστὶ καὶ ὡς ἡ ΑΒ πρὸς τὴν ΓΔ,
οὕτως ἡ ΕΖ πρὸς τὴν ΗΘ. εἰ γάρ μή ἐστιν, ὡς ἡ ΑΒ πρὸς
τὴν ΓΔ, οὕτως ἡ ΕΖ πρὸς τὴν ΗΘ, ἔστω ὡς ἡ ΑΒ πρὸς τὴν
ΓΔ, οὕτως ἡ ΕΖ πρὸς τὴν ΠΡ, καὶ ἀναγεγράψω ἀπὸ τῆς

proportional [hence, A is also equiangular to B , and has the sides about the equal angles proportional]. Thus, A is similar to B [Def. 6.1]. (Which is) the very thing it was required to show.

Proposition 22

If four straight-lines are proportional then similar, and similarly described, rectilinear figures (drawn) on them will also be proportional. And if similar, and similarly described, rectilinear figures (drawn) on them are proportional then the straight-lines themselves will also be proportional.



Let AB , CD , EF , and GH be four proportional straight-lines, (such that) as AB (is) to CD , so EF (is) to GH . And let the similar, and similarly laid out, rectilinear figures KAB and LCD have been described on AB and CD (respectively), and the similar, and similarly laid out, rectilinear figures MF and NH on EF and GH (respectively). I say that as KAB is to LCD , so MF (is) to NH .

For let a third (straight-line) O have been taken (which is) proportional to AB and CD , and a third (straight-line) P proportional to EF and GH [Prop. 6.11]. And since as AB is to CD , so EF (is) to GH , and as CD (is) to O , so GH (is) to P , thus, via equality, as AB is to O , so EF (is) to P [Prop. 5.22]. But, as AB (is) to O , so [also] KAB (is) to LCD , and as EF (is) to P , so MF (is) to NH [Prop. 5.19 corr.]. And, thus, as KAB (is) to LCD , so MF (is) to NH .

And so let KAB be to LCD , as MF (is) to NH . I say also that as AB is to CD , so EF (is) to GH . For if as AB is to CD , so EF (is) not to GH , let AB be to CD , as EF