expelled from the Savilian Chair of Geometry of Oxford in 1649, Wallis was appointed in his place. At last his dormant mathematical ability had a chance to develop, and from then on he was active in mathematics almost continually until the end of his life.

Isaac Newton (Figure 9.4) was born on Christmas Day, 1642, at Woolsthorpe, Lincolnshire. His family background and early life did not augur well for future greatness. Newton's father, also named Isaac, was fairly well off but illiterate, and he died three months before Newton was born. His mother, Hannah Ayscough, remarried when Newton was three, only to abandon him on the insistence of his stepfather. The boy was left in the care of the Ayscough family, a circumstance that helped his education (Hannah's brother William had studied at Cambridge, and eventually he directed Newton there) but did not compensate emotionally for the absence of his father and mother. Newton became intensely neurotic, secretive, and suspicious in later life; he never married and tended to make enemies rather than friends.

The young Newton was more interested in building intricate machines, such as model windmills, than academic studies, though once he set his mind to it he became top of his school. In 1661 he entered Trinity College, Cambridge, as a sizar. Sizars had to earn their keep as servants to the wealthier students, and it was indicative of his mother's meanness that he had to become one, for she could afford to support him but chose not to. Newton's early studies were in Aristotle, the standard curriculum of the time. The first thinker to make an impression on him was Descartes, whose works were then creating a stir in Cambridge. By 1664, in a series of notes he called *Quaestiones quaedam philosophicae*, Newton was absorbed with questions of mechanics, optics, and the physiology of vision. He was also struck by Descartes' geometry, preferring it to Euclid, which in his first encounter "he despised ... as a trifling book" (according to later reminiscences of de Moivre).

The years 1664 to 1666 were the most important in Newton's mathematical development and perhaps the most creative period in the life of any mathematician. In 1664 he devoured the mathematics of Descartes, Viète, and Wallis and began his own investigations. Late in 1664 he conceived the idea of curvature, from which much of differential geometry was to grow (see Chapter 17). The university was closed in 1665, which was the disastrous plague year in much of England. Newton returned to Woolsthorpe, where his mathematical reflections became an all-consuming passion. Fifty years later, Newton recalled the time as follows:

> In the beginning of the year 1665 I found the Method of approximating series & the Rule for reducing any dignity of any Binomial into such a series. The same year in May I found the method of Tangents of Gregory & Slusius, & in November had the direct method of fluxions & the next year in January had the Theory of Colours & in May following I had entrance into y$^{e}$ inverse method of fluxions. And in the same year I began to think of gravity extending to y$^{e}$ orb of the Moon & ... from Keplers rule of the periodical times of the Planets ... I deduced that the forces w$^{ch}$ keep the Planets in the Orbs must [be] reciprocally as the squares of their distance from the centers. ... All this was in the two plague years of 1665–1666. For in those days I was in the prime of my age for invention & minded Mathematicks & Philosophy more then [*sic*] at any time since.
>
> [Whiteside (1966), p. 32]

In addition to the achievements mentioned, Newton's discoveries in this period included the series for $\log(1+x)$ and, at least in preliminary form, the classification of cubic curves.

As we have seen, Newton's first attempts to publish his results were unsuccessful; nevertheless, there were some who read them and recognized his genius. In 1669 the Lucasian Professor of Mathematics at Trinity, Isaac Barrow, resigned to devote himself to theology, and Newton was appointed to the chair on Barrow's recommendation. Newton held the position until 1696, when he made the puzzling decision to accept the position of master of the Mint in London. The outstanding achievement of his Lucasian professorship was the classic *Principia* [Newton (1687)], or, to give it its full title, *Philosophiae naturalis principia mathematica* (Mathematical Principles of Natural Philosophy).

The *Principia*, which developed the theory of gravitation based on Newton's inverse square law of 1665, owes its existence to a visit by Edmund Halley to Cambridge in 1684. The hypothesis of the inverse square law was in the air at this stage—Wren, Hooke, and Halley himself had thought of it—but a mathematical derivation of its consequences was lacking. Halley asked Newton what curve a planet would describe under this law and was delighted to learn that Newton had calculated it to be an ellipse. When asked to supply his demonstration, Newton had some trouble reconstructing it, eventually sending Halley a nine-page paper, *De motu corporum in gyrum* (On the Motion of Bodies in an Orbit), three months later. *De motu* was the *Principia* in embryonic form.

Figure 9.4: Isaac Newton

Realizing the importance of Newton's results, Halley communicated them to the Royal Society and urged Newton to expand them for publication. His prodding came at just the right time. The excitement over Newton's early discoveries had died down, and for the preceding six or seven years he had been wasting his time on alchemical experiments. With his interest in mathematics rekindled, Newton devoted the next 18 months almost exclusively to *Principia* "so intent, so serious upon his studies, y$^{t}$ he eat very sparingly, nay, ofttimes he has forget to eat at all," as a Cambridge contemporary noticed [see Westfall (1980), p. 406]. When Book I was delivered to the Royal Society in April 1686, they were still reluctant to publish, and it took heroic efforts from Halley to bring them round. He not only risked his own money on the venture, but had to coax Newton to go through with it, as Newton flew into tantrums when Hooke raised his own claims of priority. Finally in 1687 the *Principia* was published and Newton's fame was secure, at least in Britain.

In the early 1690s Newton worked on revising the *Principia* and bringing some of his earlier investigations into order. As we have seen, the final form of his classification of cubic curves dates from this period. In 1693 he had a nervous breakdown, and this may have influenced him to leave Cambridge for the Mint in 1696. He did not completely abandon science, becoming president of the Royal Society in 1703, but his mathematical activity was mainly confined to the priority dispute with Leibniz over the invention of the calculus. Newton died in 1727 and was buried in Westminster Abbey. Westfall (1980) is an excellent recent biography.

Gottfried Wilhelm Leibniz (Figure 9.5) was born in Leipzig in 1646 and died in Hannover in 1716. His father, Friedrich, was professor of moral philosophy at Leipzig and his mother, Katherina Schmuck, also came from an academic family. From the age of six Leibniz was given free access to his father's library, and he became a voracious reader. At 15 he entered the University of Leipzig and received a doctorate in law from Altdorf in 1666 (Leipzig refused him a doctorate because of his youth). During 1663, on a summer visit to the University of Jena, he learned a little of Euclid, but otherwise his studies were in law and philosophy, the subjects that were to be the basis of his subsequent career. The lack of early practice in mathematics left its mark on Leibniz's later mathematical style, in which good ideas are sometimes insufficiently developed through lack of technical skill. Often he seemed to lack not only the technique but also the patience to develop the ideas conceived by his wide-ranging imagination.

It now appears that Leibniz was a pioneer in combinatorics, mathematical logic, and topology, but his ideas in these fields were too fragmentary to be of use to his contemporaries.

An interest in logic led Leibniz to his first mathematical venture, the essay *Dissertatio de arte combinatoria* [Leibniz (1666)]. His aim was "a general method in which all truths of reason would be reduced to a kind of calculation." Leibniz foresaw that permutations and combinations would be involved, but he did not make enough progress to interest seventeenth-century mathematicians in the project. The dream of a universal logical calculus was revived in the nineteenth century but finally shattered by the results of Gödel (1931) (see Chapter 23). Nevertheless, Leibniz benefited greatly from his work on combinatorics; it led him to his ideas in calculus.

Figure 9.5: Gottfried Wilhelm Leibniz

Following his doctorate in law, Leibniz had commenced a legal career in the service of the elector of Mainz. In 1672 his duties took him to Paris, where he met Huygens and for the first time gained a firm grasp of mathematics. The years 1672 to 1676 were crucial in Leibniz' mathematical life and have been covered in detail by Hofmann (1974). Beginning with

"Pascal's triangle," which he had used in his *Dissertatio* [Leibniz (1666)], Leibniz became interested in the differences between successive terms of series. Using differences, he developed a method of interpolation for functions which, as we shall see in Section 10.2, was also the independent discovery of Newton and Gregory. Leibniz showed his discovery to Huygens, who encouraged him to use differences in the summation of infinite series by posing the problem of evaluating $\sum_{n=1}^{\infty} 1/n(n+1)$. Leibniz succeeded (after some time) and went on to use the same method successfully in other cases. This was his introduction to the infinite processes of the calculus and also, perhaps, the origin of his preference for "closed-form" solutions. In 1673 he advanced to a higher level with the discovery of

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

and

$$\frac{1}{2}\log 2 = \frac{1}{2.4} + \frac{1}{6.8} + \frac{1}{10.12} + \cdots$$

by term-by-term integration. By 1676 he had virtually completed his formulation of the calculus, including the fundamental theorem, the $dx$ notation, and the integral sign.

The first period of Leibniz's mathematical activity came to an end in 1676. He had failed to obtain an academic position in Paris or London and, seeking a better salary, he moved to Hannover to enter the service of the Duke of Brunswick-Lüneburg. His main duties were to act as adviser, librarian, and consultant on certain engineering works. When the duke died in 1679 his successor commissioned Leibniz to compile a genealogy of the House of Brunswick in order to bolster the family's dynastic claims. Leibniz threw himself into this project with a zeal that is hard to admire, given the purpose of the genealogy, though it did enable him to travel, visit libraries, and meet scholars throughout Europe. He helped found the journal *Acta Eruditorum* in 1682 and used it to publish his discoveries in calculus, as well as those of his brilliant successors, Jakob and Johann Bernoulli. This led to the rapid spread of Leibniz's notation and methods throughout the Continent.

With the succession of a new Duke of Brunswick in 1698, Leibniz fell somewhat from favor, though he retained his job and, with the support of other family members, founded the Berlin Academy in 1700 and became its first president. His final years were embittered by the priority dispute over the calculus and his employer's neglect. He was still doggedly trying to complete the history of the House of Brunswick when he died in 1716. His secretary was the only person to attend his funeral, and the history was not published until 1843.