

eclipsed by Newton's for many years. It is only in modern quantum mechanics that we acknowledge that light may be viewed as waves as well as particles.

Often a promising young mathematician is denied a job in mathematics because all the places are filled by older professors who have not done a thing since the day they were given tenure. These old professors might learn a lesson from Isaac Barrow. In 1669, he resigned his professorship of geometry at Cambridge, so that the young Newton might take his place. Thereafter, Barrow devoted himself to divinity.

Gregory used the method of Wallis to obtain 'Gregory's series':

$$x = \tan x - \frac{1}{3} \tan^3 x + \frac{1}{5} \tan^5 x - \dots$$

Putting  $x = \pi/4$ , this gives

$$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \dots,$$

an interesting but slowly converging series for  $\pi$ . (This tan series had also been obtained by Indian mathematicians, and is found in a book called the *Tantrasangraha-vyakhya* (c. 1530).)

Newton never knew his father, who had died before his birth. Newton was supposed to learn farming, but spent his time doing experiments and building mechanical models. Finally, his uncle relented and sent him to Cambridge, where he read Euclid, Descartes, Kepler and Wallis, and attended the lectures of Isaac Barrow.

By the time he got his B.A., Newton had discovered derivatives, which he called 'fluxions', and had established the Binomial Theorem for integer and fractional exponents (without, however, giving a rigorous proof).

Newton used the Binomial Theorem to expand certain functions  $f(x)$  into power series. When he wanted to find the area under the curve  $y = f(x)$  he could then apply Wallis's method, replacing  $x^n$  by  $x^{n+1}/(n+1)$ . At this time, there was little knowledge about the conditions under which one can treat an infinite sum in the same way as a finite sum. Indeed, there is no evidence that Newton worried about the convergence of the series in his generalized Binomial Theorem.

During the plague of 1665–66, Newton withdrew to the family farm and thought about gravitation. Back in Cambridge, he first helped Barrow with some lecture notes, and then took over his professorship in 1669. Around this time he discovered the decomposition of white light by a prism. From 1673 to 1683 he lectured on algebra and the theory of equations.

Newton's greatest contribution to knowledge was his theory of universal gravitation, which once and for all provided a rational explanation for the

apparently erratic motions of the heavenly bodies. The fundamental idea was simple: the same force which causes an apple to fall must act on the moon and the planets. (Note that Newton had to reject the Aristotelian idea that an apple falls simply because it is the nature of ‘earthy’ things to go downwards.)

Similar ideas had previously occurred to Hooke, Huygens, Halley and Wren. These thinkers realized that Kepler’s laws implied that any ‘force of gravity’ would have to obey an inverse square law, such as that given by Newton. What Newton did was to show that, conversely, an inverse square law implies Kepler’s laws.

The *law of gravitation* asserts that, given two bodies with masses  $M$  and  $m$ , at a distance  $x$  apart, the force between them is given by

$$F = kMmx^{-2},$$

where  $k$  is a universal constant. If  $M$  is large compared with  $m$ , we may think of this force as being exerted by  $M$  on  $m$ . Since Newton had defined *force* to mean rate of change of momentum, in this case  $F = -m\ddot{x}$ , it follows that the acceleration  $\ddot{x} = -kMx^{-2}$  of the smaller body does not depend on its mass  $m$ . This result is still valid today, even if Newton’s law has to be slightly modified to conform to the general theory of relativity. If  $M$  is the mass of the earth, assumed to be concentrated at its center, and  $m$  is the mass of the apple on or near the surface,  $\ddot{x}$  is practically constant, confirming Galileo’s original observation.

At first Newton worked out the planetary motions from the assumption that the sun and the planets were points. However, he was not happy about this assumption, and so he did not publish his results immediately. It was only in 1685, about twenty years later, that he was able to prove that the gravitational force due to a solid sphere is the same as if the entire mass were concentrated in the center. He assumed that the density of matter at a point inside the sphere depends only on its distance from the center, this presumably being the case with all the planets in our solar system.

Having overcome this last difficulty, Newton finally published his epoch-making *Principia* in 1686. To avoid all controversy about his methods, he replaced his original arguments involving the infinitesimal calculus by classical geometrical arguments in the style of Euclid, which his contemporaries were able to understand. Unfortunately, for this very reason, today the *Principia* is difficult to read.

The second volume of the *Principia* dealt with hydrostatics and hydrodynamics. It showed that Descartes’s theory of vortices did not work. Newton’s theories were soon accepted everywhere. Even in France, Voltaire advocated Newton against Descartes (in 1733).

It was only in 1692 that Newton published two letters on ‘fluxions’, as he called derivatives. He wrote  $\dot{x}$ ,  $\ddot{x}$  for the first and second derivatives with respect to a parameter  $t$  (for time). He wrote  $o$  for  $dt$  and  $\dot{x}o$  for  $dx$ .