

- claw ($K_{1,3}$) 12, 15, 18, 37, 87, 199, 279, 285–6, 333, 341–3, 348
 claw-free 49, 117–8, 147, 173, 217, 281–3, 285, 297, 341–2
 Clebsch graph 466
 CLIQUE 502
 clique 4, 9, 62, 123, 137, 153, 173, 192–217, 224–31, 263, 275, 280–3, 286, 288, 291, 319–48, 381, 384–7, 394–400, 413–4, 420, 422, 426, 439–41, 447–8, 453, 465, 470, 496, 502
 clique cover(ing) 226, 319–21, 326, 339, 342, 344, 422
 clique cover(ing) number 226, 319
 clique decomposition 397
 clique identification 344
 clique number 192, 199, 231, 319, 335, 339, 439–41, 447
 clique tree 327–8, 345
 clique-vertex incidence matrix 328–9, 346
 closed ear 164–5, 172
 closed neighborhood 116, 341, 468
 closed set (in the plane) 233–41, 245, 254, 267–8, 389–90, 397, 452, 468
 closed set (matroids) 360, 362, 367–8, 371–2, 375
 closed trail 20, 26–7, 30–1, 34, 57–60, 172–3, 290–1, 295, 313, 506
 closed walk 20, 24, 32, 48, 63, 65, 99, 237, 239, 455
 closure (Hamiltonian) 289–90, 298, 419, 449
 closure function (matroids) 360
 closure operator 360
 co-critical vertex pair 339
 cograph 202, 344
 cobase 360–2
 cocircuit 360, 362, 375
 cocycle (matroids) 362
 cograph 202
 color 4, 191–2, 204, 275, 380
 color class 191–3, 200, 203–4, 207, 217, 275, 331, 339
 color sum 204
 color-critical 192, 199, 206, 210, 215, 218, 344
 2-COLORABILITY 495, 505
 3-COLORABILITY 500, 504–5
 k -COLORABILITY 495, 501, 505
 k -colorable 191–2, 202, 204, 211, 309, 363, 408
 k -coloring 191–4, 198, 200, 205, 207, 210–1, 216–7, 219–24, 229, 309, 380–3, 386, 393–4, 449
 column matroid 351–2, 375
 combinatorial design 11, 465, 470
 common system of distinct representatives (CSDR) 119, 171–2, 353, 368–9
 comparability graph 228, 231, 329–31
 compatible pair 232
 complement (graph) 4, 10–2, 15, 38, 49, 52, 71, 77, 80, 115, 121, 201, 207, 215–6, 226–7, 245, 255, 283, 297, 312, 320, 322, 334–5, 340–1, 344, 360–2, 366, 375, 379, 393, 400, 422, 456, 461, 465–70
 complement (set) 474
 complement reducible 344
 complete bipartite graph 9–10, 14, 33, 41, 104, 409, 413, 416
 complete graph 9–11, 16, 26, 32, 50, 62, 79, 83–4, 87, 104, 108, 149, 193–4, 197–204, 207, 214, 217–8, 221, 224, 230–1, 263, 290, 293, 298, 329, 336, 344, 381, 386, 398–9, 419, 426, 459, 470, 487
 complete loopless digraph 393
 complete multipartite graph 207, 215
 complete subgraph 26, 280–1, 381, 386, 397 (see *clique*)
 completely labeled cell 388–9, 391, 395
 complexity 125, 269, 286, 425, 494, 496, 499
 component 22–32
 composition (of functions) 9, 18, 485–7
 composition (of graphs) 284, 332–4, 393
 conclusion 477
 conditional statement 248, 477–9, 481
 conditional probability 443, 448
 configuration (reducible) 258–61, 265, 270
 conflict graph 252, 254, 256
 congruence (modulo n) 52, 64, 88, 94, 194, 204, 217, 269, 272, 274, 303, 309, 464, 490–1
 conjunction 477
 connected dominating set 117, 122–3
 connected graph 5, 21
 2-connected graph 150, 155, 158, 161–4, 173–5, 198, 204, 213, 240, 243–4, 247–8, 250, 252–4, 287–8, 293, 295–6, 298, 312–4, 317, 348, 417–9
 3-connected graph 150, 158–9, 166, 174–5, 213, 218, 237, 247–52, 256, 292, 295, 301–4, 316, 376, 505
 k -connected 149, 151, 158–62, 164, 169–70, 174–5, 283, 298, 440, 450–1
 connected to 21–2, 31
 connection relation 21–2, 29, 34, 59, 63
 CONNECTIVITY 149, 152, 164, 439, 495
 connectivity 149–53, 158–9, 163–9, 174, 182, 211, 215, 248, 274, 292, 301–2, 304, 313–4, 406, 439–41, 463
 connector 391–2
 consecutive 1s property 328–9, 346–7
 conservation constraints 176–7, 184, 186–8, 307
 consistent rounding 186, 190
 construction procedure 30, 324
 contains 6, 21, 24, 471
 contraction (edge) 84–5, 143–5, 213, 218, 221–3,

- 239, 241, 249–51, 256,
269, 305, 317, 324,
contraction (in matroids)
363–6, 375–7
contrapositive 38, 77, 110–1,
159, 200, 249, 290, 324,
478, 491
converse (of conditional) 477
convex combination 395
convex embedding 248–50,
255
convex function 443
convex polygon 247–8, 256
copy 10
cost **95–7, 100, 103, 126–30,**
185, 494, 496–500, 505
counting arguments 34–7,
47–50, 68, 79–85, 92, 108,
111, 138, 219, 223–4,
229, 241, 263, 272, 279,
322, 335, 385, 420, 427,
436, 458, 463, 473, 485–9
Coupon Collector 451
cover/covering (see *edge cover, vertex cover, etc.*)
COVERING CIRCUIT 506
covering set 127–8
critical 94, 122, 147, 192,
196, 198–9, 201, 203,
206, 210–3, 215, 217–8,
334–6, 339–44, 348, 506
 α -critical 122, 506
 k -critical 192, 196, 198–9,
203, 210–3, 215, 217–8
critical edge 122, 340, 342–3
critically connected
cross edges (in Petersen graph) 276–7
crossing 234
crossing number **262–4, 269**
cryptomorphism 360
CSDR (see *Common Syst. of Distinct Representatives*)
cube Q_3 3, 35–6, 49, 51, 76,
105, 119, 150, 236, 243,
255, 271, 295–6, 379, 390,
397, 401–3, 422, 468
cubic graph 304–11
curve 1–2, 48, 54, **233–9,**
241, 245–6, 254, 268
cut (see edge cut, source/sink cut, vertex cut)
 x, y -cut 166–8, 172
cut-edge 23, 43–4, 48–9, 52,
68–70, 75, 77, 104, 139,
147, 155, 158, 165, 173–
5, 237–8, 300–1, 304,
307–8, 313
cut-vertex 23, 29, 31–2, 77,
146, 155–6, 158, 160,
162, 198, 212, 240, 243,
247, 284, 420, 506
cycle 5–6, **9–20, 23–37, 43,**
49, 55–60, 63–71, 75–9,
84–7, 96, 103–5, 108–10,
118–9, 122, 140, 147, 155,
159–65, 170–5, 192–200,
203–4, 213, 216–7, 224–35,
238–45, 250–9, 270–7,
284–306, **310–8, 323,**
326–30, 341–4, 349–65,
373–76, 379, 391, 394–5,
408–24, 429, **436–7, 440–**
1, 452–5, 460, 467–8,,
492–4, 497–9, 502, 505
 n -cycle 9, 12, 35, 49, 92, 94,
306, 417–8, 460, 468
4-cycle 14, 23, 25, 34, 48–
9, 70, 94, 193, 221, 223,
228, 270, 305, 329, 345,
394–5, 408, 460, 467, 505
5-cycle 11–4, 18, 50, 92, 108,
114, 119, 142, 192–3, 199,
205–6, 210, 215, 234,
252, 270, 276–7, 312, 318,
323, 336, 344–5, 348,
384, 394, 422, 460, 470
6-cycle 10, 37, 49, 216, 234,
318, 487
cycle double cover (CDC)
312–4, 317–8
cycle matroid 313, **350–5,**
358, 360, 362–5, 373–6,
406
cycle space 313, 452, 467
cycle-power 337–43

deadheading 130
de Bruijn cycle 60, 94
de Bruijn graph 61, 63
decision problem 494–5
decomposition 11–2, 18, 25,
30–1, 34, 56, 64, 76, 87–8,
94, 140, 147, 155, 163–5,
172–5, 248, 252, 261, 271,
276, 280–1, 284, 286, 302,
314, 324, 371, 397–8,
413–5, 460
F-decomposition 397, 413–4
decomposition procedure 324
deficiency 121, 146
defined on 483–4
 k -degenerate graph 269
degree (of vertex) 6
degree sequence 44–6, 59, 62,
76, 94, 141, 195, 290–1,
297–8, 345, 418, 438
degree set
degree-sum formula 35, 40,
43–4, 51, 58, 214, 238,
242, 365, 385
deletion ($G - e$, $G - v$) 23
deletion (matroids) 362–6
deletion method 428–9,
449–50
demand 130, 184, 187
density bound 390–1, 396
density of graph 435–6
dependence (linear) 400, 457
dependence (matroids) 352,
359, 373
dependent edge 232
dependent sets 313, 349–50
depth-first search (DFS)
156–7, 402, 404
descendant 100
determinant 85–7, 92, 452–
4, 462, 469
diagonal Ramsey number
385, 394, 450
DIAMETER 495
diameter 71–2, 75–9, 99,
105, 114, 122, 147, 153,
160, 209, 216, 244, 379,
396, 424, 432, 458, 464
difference (of sets) 473
digraph 53
Dijkstra's Algorithm 97–100,
105
dilation 390
Dilworth's Theorem 413, 424
 k -dimensional cube Q_k 35–6,
48–9, 71, 76, 105, 108,
119, 150, 174, 193, 282,
296, 329–30, 379, 390
 k -dimensional simplex 395
Dinitz Conjecture 410
Dirac's Theorem 218, 417–8,
441
direct method (of proof) 478

- direct sum (matroids) 369–70, 406
 directed graph 53, 66, 90, 189, 377, 406, 422, 506
DIRECTED HAMILTONIAN CYCLE 503
DIRECTED HAMILTONIAN PATH 500, 502–3
 Directed Matrix Tree Theorem 89
 discharging 261, 304
 disconnected graph 6, 12, 15, 21–2, 25, 31, 38–9, 50, 52, 63, 71, 78, 85, 149–50, 156, 165, 173, 241, 247, 249, 333, 347, 431–2, 437, 470
 disconnecting set 152, 155, 159, 168
 discrepancy 402–3
 discrete system 54
 disjoint (sets) 473
 disjoint union ($G + H$) 39, 48, 104, 137–8, 155, 193, 199, 271, 306, 313, 359, 371, 399, 419, 465, 470
 disjointness graph 13–4, 17–8, 276
 disjunction 477
 distance 5, 46, 57–8, 70–3, 78, 95, 97–9, 105, 130, 137, 190, 192, 198, 201, 209, 217, 225, 235, 246, 265, 271, 294, 302, 345, 379, 390–2, 400–3, 419, 421, 449, 452, 468
 distance-preserving 400–1
 distinct 489
 DNA chains 328
 dodecahedron 243, 245, 286, 295
 domain 437, 483–5
 dominating set 116–8, 122–3, 428–9, 506
 domination number (see *dominating set*)
 dot product 86, 306, 317, 338, 400
 double cover 312–4, 317–8
 double jump 437
 double torus 267–8
 double triangle 281–2, 285–6
 double-star 78
 double-torus 266
 doubly stochastic matrix 120
 drawing 2, 9–12, 30, 54, 233–5, 242, 262–6, 272, 449, 504
 dual augmentation property 362
 dual edge 236, 238, 244–5, 360, 363–4
 dual graph **236–9**, 241–5, 300, 309, 314–5, 317, 360, 376
 dual matroid 349, **360–5**, 375–7
 dual problem (optimization) 113–4, 118, 125–6, 135, 166, 172, 179, 188, **323**
Duality Theorem 323
 ear 163–5, 172–3, 175, 248
 eccentricity 71–2, 78, 99, 105
 edge 2, 53
 edge cover 114–5, 122
 edge cut 152–5, 159–60, 164–5, 181, 190, 211, 238, 283, 301, 303–7, 312, 317, 452, 467
 edge-choosability 409
 edge-chromatic number 275, 283
 edge-coloring 274–9, 282–5, 296, 299–305, 310–1, 381, 409
 k -edge-colorable 275, 296, 411
3-EDGE-COLORABILITY 505
 k -edge-coloring 275, 284–5, 296, 381
 $\Delta(G)$ -EDGE-COLORING
 2-edge-connected 164–5, 172–3, 243, 300–2, 305, 312–4, 317, 424
 k -edge-connected 152, 158, 160, 164–6, 174–5, 283
 edge-connectivity 152–3, **165–9**, 274, 301–2, 406
 edge-transitive 18
 Edmonds' Blossom Algorithm 144
 Edmonds' Branching Theorem 405–6, 422
 eigenvalue 401, **453–70**
 eigenvector 453, **455–70**
 element (of set) 471
 embedding **234–56**, 266–72, 283, 302, 313, 376, 400–1, 453
 empty set 472
 encoding 26, 101, 389, 397–8, 400–3, 494
 endpoint 2, 20, 53
 entropy 103
 equality of sets 472
 equality relation 490
 equality subgraph 126–9
 equitable edge-coloring 285
 equivalence class 9, 22, 33, 63, 173, 313, 490–1
 equivalence relation 8–9, 22, 63, 173–4, 490
 erasure 43, 53
Erdős–Faber–Lovász Conjecture 202
Erdős–Gallai condition 141, 148, 185
Erdős–Szekeres Theorem 203, 379, 382
Euler's Formula 233, 241–2, 245, 255, 268, 272, 316, 375
EULERIAN CIRCUIT 495, 499
Eulerian circuit **27–34**, 42, 60–1, 64, 77, 89–91, 99, 140, 273, 285, 298–9, 498
Eulerian digraph 60, 64, 90–1, 130
Eulerian graph 27–31, 34, 60, 77, 244, 295, 298, 308, 495
Eulerian trail 27, 60, 64
 even cycle 109–10, 138, 174, 204, 217, 276, 318
 even digraph 318
 even graph 27–31, 33–4, 48, 50, 308, 311–3, 414
 even numbers 472–3
 even pair 348
 even triangle 281
 even vertex 26, 36, 100, 140
 even walk 24
 event (probability) 425–7, 443–50
 evolution 193, 436–8
 excess (matrix) 126–30, 141, 176–7, 179–80
 existential quantifier 475–6

- expander 453, 463, 469
 Expansion Lemma 162, 170, 175
 expansion operation 43–4, 52–3, 175
 expansive property 358–60
 expectation (of random variable) 427–34, 440, 443–6, 449, 452
 extremal problem 38–9, 41, 116, 209, 396, 413
 extremality method 28–9, 32, 34, 40–1, 63, 68, 137, 249, 289, 294, 299
 face 235–50, 253–6, 267–72, 295, 300–3, 307–9, 312–5, 353, 360, 401, 412, 424
 face length 238–9, 241
 face-coloring 300–1, 307, 309
 factor 136
 1-factor 136–41, 145–8, 159, 276, 283–4, 308, 310, 318
 2-factor 136, 140, 147, 276–7, 285, 288, 315
 f -factor 140–1, 148
 k -factor 140, 146
 1-factorable 276, 284
 1-factorization 276, 279, 284–5, 310
 1-factorization Conjecture 279, 284–5
 1-factorization 276, 279, 284–5, 310
 factor-critical 147
 factorial 107, 220, 294, 386, 428, 434–5, 486–9
 fan 170–1, 213
 Fáry's Theorem 247, 251, 255
 fat triangle 275
 feasible flow 176–80, 184–8
 feasible solution 323, 497
 finite automaton 54
 finite graph 3
 finite set 473
 finite state machine 54, 57
 Five Color Theorem 257–8
 flat 266–8, 360
 floor function 39, 483, 491
 flow (in network) 176–89, 495
 flow (in graph) 307–18
 flow number 309
 k -flow 307–12, 317–8
 k -flowable 309
 flower 142, 306, 317
 forbidden substructure 323, 365
 Ford–Fulkerson Labeling Algorithm 179–82, 186–9, 438–9
 Ford–Fulkerson Theorem 180–5
 forest 67, 75–80, 96–7, 104, 160, 206, 214, 217, 219, 244, 297, 327, 345, 351, 353–4, 362–3, 372, 413, 424, 434, 436, 468
 Four Color Theorem 213, 259–61, 268–70, 300–4, 311, 314, 411, 469
 H -fragment 252–4, 256
 fraternal orientation 345
 H -free 41, 348
 P_4 -free 52, 202, 344, 347
 free matroid 357
 Friendship Theorem 453, 465–7
 Fulkerson's Conjecture 318
 function 483
 functional digraph 55, 64
 fundamental set of circuits 374
 Gale–Ryser Theorem 185, 190
 Gale–Shapley Algorithm 131–2, 135–6
 Gallai's Theorem 376
 Gallai–Roy–Vitaver Theorem 196
 Gallai–Milgram Theorem 413
 gambler 444–5
 games 48, 51, 57, 73–4, 106, 119–20, 183–4, 274, 286, 366, 445
 gammoid 377
 gas–water–electricity 233
 generalized coloring 199
 generalized cover 146
 generalized partition matroid 370
 generalized Petersen graph 316
 GENUS 266, 495
 genus 266–7, 272, 283
 geometric dual 365, 376
 Gewirtz graph 466
 Ghouila–Hour's Theorem 420
 girth 13–4, 17, 37, 49, 79, 105, 119, 147, 206, 216–7, 219, 232, 245, 255, 297, 304–6, 312–4, 365, 396, 412, 429
 good algorithm 124–5, 142, 196, 219, 274, 276, 279, 292, 493–4, 504
 good characterization 495
 gossip problem 406–8, 422–3
 graceful labeling 87–8, 92–4
 Graceful Tree Conjecture 87, 94
 graph 2
 graph transformation 64, 138, 141, 285, 422
 graphic matroid 350, 357, 375–6
 graphic sequence 44–5, 48, 148, 185
 greedy algorithm 96, 116, 195, 349, 354–7, 366, 373–4, 429, 441–2, 496–7
 greedy coloring 194–202, 227, 276, 324, 331–2, 344, 442, 459
 greedy decomposition 397–8
 greedy ear decomposition 173
 grid ($P_m \square P_n$) 193, 316, 390, 396
 grid (positions) 73, 251, 265, 370, 410–1, 425, 446, 460, 490
 Grinberg graph 302, 316
 Grinberg's condition 303, 315–6
 group 18, 309, 449, 452–3, growth rate 265, 431, 483
 Grötzsch graph 205–6, 215, 218, 294
 Grötzsch's Theorem 270
 Gyárfás–Sumner Conjecture 206, 215
 Hadwiger's Conjecture 213
 Hajós' Conjecture 213, 414, 442
 Hajós' construction 217

- Hall's Condition **110–3**, 121, 146–7, 368, 377, 463
 Hall's Theorem **110–3**, 120–1, 146–7, 171, 175, 219, 376
 Hamiltonian closure 298, 419, 449
HAMILTONIAN CYCLE 494–500, 503, 505–6
 Hamiltonian cycle/graph **286–99**, 302–4, 314–7, 395–6, 416–21, 437, 440–1, 449, 493–4, 497–9, 502–3, 506
HAMILTONIAN PATH 495, 500, 502–3, 505–6
 Hamiltonian path 292, 295–7, 299, 303, 316–7, 428, 497, 502
 Hamiltonian-connected 297–8
 handle 266–8, 313
 handshake party problem 481
 Harary graph 150, 153
 Harper's bound 390–1
 Havel–Hakimi Theorem 45, 52, 59
 head **53–61**, 86, 90, 94, 164–5, 168, 178, 307–8, 357–8, 406, 484, 503
 head partition matroid 357
 Helly property 80, 346
 hereditary family of graphs 226–8, 275, 325, 332, 334, 341, 344, 353, 357, 371
 hereditary system 349–55, 357–63, 366, 369–71, 373–4, 377
 heuristic algorithm 496
 homogeneous set 380–1
 Huffman's Algorithm 101–3
 Huffman code 103, 106
 Hungarian Algorithm 127–9, 132, 134–5
 hunter/farmer problem 121
 hypercube 35–6, 49, 71, 108, 122, 150, 174, 193, 350
 hypergraph 449
 hyperplane 360–2, 375, 395
 hypobase 360–1, 375
 hypothesis 477
 icosahedron 214, 243, 315
 ideal (of sets) 349
 idempotence property 359
 if (in definitions) 473
 image 8, 55, 147, 234, 377, 401, **483–6**
 imperfect graph 232, 320–3, 333–6, 343–4, 347
 in-neighborhood 58
 in-tree 89–91
 incidence matrix 6, 17, 56, 86, 323, 328–9, 337, 346, 375, 469
 incidence relation 234, 322, 489–90
 incidence vector 338, 452
 incident 6
 inclusion-exclusion principle 223, 230
 incorporation property 359–61, 367, 374
 increasing path 406–7, 423
 increasing trail 393
 indegree **58–65**, 89, 130, 190, 331, 404, 410, 503
 independence number **113–4**, 192, 194, 199, 319, 441
 independent dominating set 117–8, 122–3
 independent events 426
 independent set (in graph) 4–5, 9–10, 15, 23–5, 29, 32, 36–7, 75, 113–8, 121–2, 192–4, 199, 203, 205, 208, 211, 215–6, 218–21, 226, 230, 273, 293, 319, 333, 384–5, 393, 395, 410–1, 413, 428–9, 449, 493–6, 502, 506
 independent set (in matroid) **349–77**
INDEPENDENT SET 494–6, 500, 502
k-INDEPENDENT SET 495
 independent vectors 353, 400
 index of summation 485
 indicator variable **427–34**, 448, 452
 indirect proof 151, 478
 induced by 23
 induced circuit property 354–5, 360, 374
 induced subgraph 23, 32–4, 37, 41–2, 50, 64, 75, 175, 204, 211, 219, 225–6, 231, 281, 285–6, 315, 319–21, 324, 330–4, 340–1, 343, 345, 410, 434, 450, 454, 458–9, 470
 induction 19–21, 24–34, 40–7, 479–83
 induction hypothesis **19**, 479–82
 induction parameter **19**, 42, 480
 induction step **19**, 479–82
 induction trap 42–4, 68, 481–2
 infinite graph 3
 infinite set 473
 integer 471, 474
 integer lattice 393, 474
 integer linear program 323
 integrality condition 465, 470
Integrality Theorem 181, 183
Interlacing Theorem 458
 internal vertex 20–1, 69, 72, 151, 161, 163, 166–7, 173, 177, 270, 412, 415
 internally disjoint paths 158, 161–2, **166–75**, 182–3, 212, 218, 274, 417
 intersection graph 324, 327–8, 341, 344–5, 451
 intersection number 397
 intersection of matroids 366
 intersection of sets 473
 intersection representation 324, 345, 397
t-interval 451
 interval graph 195–6, 204, 224, 226–7, 231, 328, 330, 346–7
 interval number 451
 interval representation 195–6, 226, 328–9, 346
 intractable 495
 inverse 18, 53, 267, 390, 484
 inversion 33
 involution 470
 isolated vertices 22, 31–2, 51, 60, 77, 80, 90, 96, 114–8, 121–2, 138, 155, 210, 223, 230, 376–7,

- 398–9, 408, 414, 422–3, 433–4, 437, 451, 455
isometric embedding 400–1
isomorphic to 7
isomorphism **7–17**, 38, 49, 56, 75, 78, 81, 92, 94, 207, 234, 243, 276, 364, 430, 438–9, 441, 453, 485, 490
isomorphism class 9, 12–3, 81, 207, 234
isomorphism relation 8–9
join $G \vee H$ 138, 146, 150, 155, 193, 199, 210, 215–6, 236, 264, 271, 291, 298, 310, 334, 360, 380, 387, 436–7
joined 22
Jordan Curve Theorem 235, 238, 241, 258, 301
junk 328, 340
- K_3 10–2, 26, 138, 155, 220–1, 240, 286, 344, 384, 386–7, 395, 467
 K_4 11–2, 25, 31, 43–4, 53, 175, 209, 212–3, 215, 218, 236, 240, 250, 256, 272, 302, 314, 349, 352, 357, 374, 401
 K_5 9–12, 140, 214, 234, 242–3, 246–7, 250–2, 256, 258, 263, 267, 269, 283, 363, 365
 $K_{3,3}$ 10, 43, 150, 159, 233–4, 242–3, 246–7, 250–2, 256, 267, 269, 272, 363, 365, 422
Kempe chain 258–60
kernel 57–8, 64, 410–1
kernel-perfect 410–1
king **62–3**, 65–6, 190, 450
kite 12, 23, 50, 84–6, 92, 223–4, 279–81, 349, 397
knight's tour 295
Kotzig's Theorem 284
Krausz decomposition 286
Kruskal's Algorithm 95–7, 104, 498
Kuratowski subgraph 247–52, 255
Kuratowski's Theorem 246–8, 251–2, 255–6, 269, 364
König's Other Theorem 376
König–Egerváry Theorem **113–5**, 121, 123, 146, 168, 174, 189, 210–1, 227, 368, 373, 413, 424
Königsberg Bridge Problem 1–2, 19–20, 26
Lagrangian multiplier 456
Laplacian matrix 463, 469
Laplacian eigenvalue 469
Las Vergnas' condition 298, 418
lattice 349, 360, 393, 474
leaf 67–73, 76, 80–3, 86, 89, 93, 101–3, 106, 115, 156, 174, 198, 214, 219–20, 324–5, 331, 468
leaf block 156, 198
left child 101
left subtree 101
length (of object in graph) 20, 237
length (of encoding of graph) 397, 398, 401
lexicographic product 393
lg 97, 202, 400, 422–3, 434, 449, 451
line digraph 168
line graph 168, 227, 273–5, **279–86**, 295, 320, 409, 422, 493, 505–6
linear matroid 351, 353, 357, 360
linear programming 179, 376
linearity of expectation 427–8, 432
list 474
list k -colorable 408
list chromatic index 409
list chromatic number 408–12
list coloring 199, 408–12, 423
list edge-coloring 409–11
literal (in logical formula) 499–501, 506
S-lobe 211–3, 218, 247–8
local density 390, 396
local search 497
logical formula 499–500
longest cycle 173, 292, 294, 298, 416
longest path 34, 71, 147, 196–7, 228, 294, 416–9
loop (in graph) 2, 6, 8, 20, 24, 27, 34–5, 44, 69, 76–7, 84–5, 107, 149, 192, 223, 236–9, 241, 267–8, 275, 284, 286, 294, 300, 305
loop (in digraph) 54, 58–9, 61, 64, 299
loop (in matroid) 351, 366–7, 370, 372–3, 375
loopless digraph 56, 62, 64, 66, 89, 302, 393, 413, 420
loopless graph 6, 17, 34–5, 40, 44, 49–52, 75, 85–6, 155, 192, 203, 265, 275, 285, 302
Mader's Theorem 175
magnifier graph 463–4, 469
map (in the plane) 1, 5–8, 191, 219, 233–8, 258–60
Markov chain 55
Markov's Inequality 432–3, 444, 448, 452
Marriage Theorem 111
martingale 443–7, 452
martingale tail inequality 443–7, 452
MATCHING 495, 506
matching 100, **107–48**, 150, 166, 168, 175, 179–80, 211, 216, 227, 273–6, 283, 295–6, 308, 310, 318, 349, 352, 357, 366, 368–9, **373–7**, 395, 399, 408, 411, 419, 424, 436, 449, 451–2, 463, 468–9, 493, 499, 505–6
mates 107, 131, 337–40
matrix 490
matrix rounding 186, 190
Matrix Tree Theorem 85–6, 89, 92–4, 453, 462–3, 469
0,1-matrix 120, 322, 328, 454
matroid 74, 313, **349–78**, 406, 506
matroid basis graph 376
Matroid Covering Theorem 372
Matroid Intersection Theorem **366–71**, 376–7, 413

- 3-MATROID INTERSECTION 506
 Matroid Packing Theorem 372
 Matroid Partition Problem 378
 Matroid Union Theorem 366, 369–72, 377–8
 Max-flow Min-cut Theorem 180–5
 maximal 29
 maximal clique 31, 231, 281, 323, 327, 329–31, 345
 maximal forest 351
 maximal matching 108, 118, 122
 maximal outerplanar graph 243, 256
 maximal path 27–9, 31, 34, 60, 163, 204, 293, 298
 maximal planar graph 242–3, 245, 271
 maximal trail 29, 31, 33, 64
 maximum 29
 maximum clique 215, 322, 333, 336–40, 342–3, 347–8, 439
 maximum degree 29, 34, 41, 47–8, 52, 67, 75–9, 114, 200, 202, 204, 251, 256, 284–5, 291, 345, 390, 439, 463, 476
 maximum density 435
 maximum flow 1, **176–83**, 186–8
MAXIMUM MATCHING 495
 maximum matching 100, **108–29**, 132–4, 139, 141–2, 145, 147, 349, 493
 maximum stable set 321, 323, 336–40, 343–4, 348
 maximum independent set 29, 31, 114–5, 121–2, 203, 356, 496
 maximum weighted matching 125–30
 median 78
 member (of set) 471
Menger's Theorem **167–75**, 181–3, 189, 227, 274, 368, 377, 404–6, 422, 495
 method of contradiction 478
 Meyniel graph 330–1, 347–8
 Meyniel's Theorem 420, 424
 Min-cost Flow Problem 185
 min-max relation 113, 138, 274, 320, 323, 366, 368, 371, 495
 minimal imperfect graph 320, 322, 333–4, 347
 minimal nonplanar graph 247–8
 minimal vertex separator 231, 345
 minimally k -connected 175
 minimally k -edge-connected 175
 minimax 104
 minimum cut 167, 179, 182–3, 189
 minimum degree 34, 49, 51, 70, 79, 116–7, 122, 152–3, 159, 202, 213–4, 218, 243, 245, 256, 261, 285, 288–9, 293, 296–8, 343, 386, 428–9, 440, 457, 496
 minimum polynomial 457–8, 461
MINIMUM SPANNING CYCLE 494
 minimum spanning tree (MST) 95, 104, 349, 496, 498
MINIMUM VERTEX COVER 496
 minimum weighted cover 126–9
 minor (of graph) 251, 256, 269
 minor (of matroid) 363, 365, 375, 462
 Minty's Theorem 203
 Model A 430–5, 438, 446–7
 Model B 430, 433–4, 436
 modular 3-orientation 317
 modulus 490
 monochromatic 386–7, 393–5, 449–50
 monotone property 433
 monotone subsequence 203, 390
 monotone tournament 393
 mountain range 48
 multigraph xiv
 multinomial coefficient 489
 multiple edges 2, 6, 44, 52, 54, 59, 76, 84–5, 111, 168, 182, 185, 192, 213, 221–3, 236–9, 273–6, 279, 286, 298, 317, 351, 405–6
 multiplicity (of edge) 166, 265, 275, 279, 395, 453, 455, 460–1, 466, 468–70
Mycielski's construction 205–6, 215, 258
Nash-Williams' Orientation Theorem 175
 natural number 471
 nearest-insertion algorithm 497, 505
 nearest-neighbor algorithm 496–7
 necessary condition 24
 necklace 173
 negation 477
 neighborhood 34, 45, 58, 116, 121, 162, 215–6, 224, 325, 341, 346, 348, 368, 390, 438, 463, 468
 neighbor 2
 net demand 184
 network 1, 149, 161, 165, **176–90**, 463, 495
 network flow 176, 182, 184; 186, 189, 495
 node 55, 101, **176–88**, 388–9, 391, 449
 NODUP scheme 423
 NOHO property 407–8, 423
 nondeterministic 494–5
 nonplanar graph 243, 247–8, 252, 269, 365
 nonseparating 251
 nontrivial graph 22
 Nordhaus–Gaddum Theorem nowhere-zero flow 307–8
 NP 369, 390, 439–41, 495–7, 499–506
 NP-complete 369, 440, 495, 497, 499–506
 NP-hard 390, 439, 495, 499–500, 502, 506
 null graph 3, 435
 $O(f)$ 94, 106, 124, 228, 387–8, 437, 494
 o-triangulated 330, 347
 obstruction 269, 278, 331–2
 obstruction-free ordering 331–2
 odd antihole 340, 343

- odd component 136–9, 147
 odd cycle 24–8, 32, 41, 49,
 57–8, 63–7, 112, 122,
 138, 142, 174, 192, 195–
 7, 199–200, 203–4, 239,
 276, 285, 320, 330, 334,
339–44, 347, 357, 410,
 455, 472, 475–8
 odd degree 15, 30, 34–5,
 43, 47, 77, 100, 385, 389,
 414, 498–9
 odd hole 340–1, 343
 odd number 473
 odd triangle 281
 odd vertex 30, 36, 100
 odd walk 24–5, 57–8
 one-to-one correspondence 7,
 37, 50, 81, 473, 484
One-Way Street Problem
 130, 165, 422
 open set 235
 optimal coloring 192, 194,
 197, 199, 202, 207, 215,
 217, 227, 324, 326, 331,
 336, 340, 344, 348
 optimization problem 39,
 113–4, 179, 322, 396
 order (of a graph) 35
 order (of recurrence) 483
 order-preserving property
 358–60
 ordered graph 406, 423
 ordered pair 8, 21, 53–4,
 56, 61, 93, 190, 294, 299,
 309, 393, 420, 440, 474,
 489–90
Ore's condition 297
Ore's Theorem 417–8, 420,
 424
 orientable cycle double cover
 318
 orientation 62–5, 86, 89, 94,
 147, 165–6, 174–5, 196–
 7, 203, **228–32**, 244, 293,
 307–8, 311, 317, 329–32,
 345, 376, 379, 393, 410–
 3, 424, 449, 484
 oriented graph 62
 orthonormal 456–7, 461
 out-neighborhood 58
 out-tree 89–91, 98
 outdegree **58–66**, 89, 130,
 190, 299, 410, 449, 503
 outer face 235, 240, 412
 outerplanar graph 239–40,
 243, 256, 269–71
 outerplane graph 239–40,
 244–5
 Overfull Conjecture 278–9,
 285
 overfull subgraph 278–9,
 284–5

P
 P_3 11, 32, 48, 64, 163, 173,
 199, 223, 333, 386, 395,
 417–8, 465
 P_4 11–2, 15–8, 23, 33–4, 50,
 52, 108, 138, 147, 163,
 202, 221, 344, 347, 417
 P_5 12, 23, 231, 404
 p-critical 334–6, 339–44, 348
 $P = NP?$ 441, 495, 497
 pair-disjoint 447–8
 pairwise 489
 parallel elements (in ma-
 troid) 351–2, 373, 375
 parallel computation (see
 nondeterministic)
 parent 100–1, 147, 157, 220,
 402
 parity 17, 27, 36, 46, 137–9,
 142, 148, 236, 239, 271–
 2, 301–2, 310, 312, 317,
 347, 388, 473, 490
 parity graph 330
 parity lemma 148
 parity subgraph 312, 317
 partial transversal 127–8,
 353
 partite set 4
 k -partite 5, 51, 192, 207, 296
 partition 473
 partition matroid 357–8,
 366, 368, 370–1, 373
 partitionable graph **335–42**,
 347–8
 Pascal's Formula 73, 488
 path **5–34**, 38, 43, 47, **55–**
 64, 67–81, 84, **88–106**,
 109–12, 119, 123–4, **129–**
 34, 142–5, 147, **151–83**
 u, v -path 20
 X, Y -path 166, 170, 175
 paw 12, 31, 236, 279–81
 pendant vertex 67
 perfect graph 204, 224, 226–
 8, 319–24, 328, 330–7,
 341, 343–4, 347, 413
 α -perfect 319–22
 β -perfect 335
 γ -perfect 319–22
 perfect elimination ordering
 224
Perfect Graph Theorem
 (PGT) 226–7, **320–2**,
 334–5, 344, 413
 perfect matching **107–8**,
 111–4, 118–22, 125–9,
 131, **134–6**, 139, 141,
 146, 148, 211, 274–6,
 283, 295, 318, 374, 424,
 451–2, 469
 perfect order 331–2
 perfectly orderable graph
 331, 347
 performance ratio 202, 496,
 498–9, 505
 permutation 8, 14, 18, 32–3,
 55, 64, 107, 120, 332, 390,
 448–9, 453–4, 470, 486
 permutation matrix 120, 470
Petersen graph **13–8**, 37, 41,
 50, 71, 79, 87, 108, 119,
 122, 139, 159, 175, 192,
 197, 203, 230, 245, 251,
 255, 269, 276, 279, 283–
 4, 288, 292, 295–7, **304–**
 18, 470, 487
PGT (see *Perfect Graph The-
 orem*)
 pigeonhole principle 151,
 171, 230, 261, **378–93**,
 491–2
 pigeonhole property 427
 planar graph 5, **234–62**,
 266, 269–72, 274, 301–
 4, 307, 309, 312, 315–6,
 341, 349, **358–65**, 376,
 411–2, 423–4, 469, 504–6
PLANAR 3-COLORABILITY
 planar embedding 235–6,
 241–8, 251–4, 271, 376
 planar map 238
PLANARITY 252, 495
 plane graph **235–45**, 254–6,
 270, 300–3, 307–9, 312,
 314, 360, 363–5, 375, 412
 planted tree 101
 Platonic solid 242
 Poisson distribution 434
 polygon 242, 247–8, 255–6,
 270–1, 452

- polygonal curve 48, 234–5,
 245
 polyhedron 242–3
 polynomial-time algorithm
 124–5, 253, 269, 282,
 377, 438–9, 493–500,
 504–5
 positional game 120
 positive k -flow 307, 318
 k th power (of graph) 296
 predecessor 54, 58, 62, 98,
 294, 417
 predecessor set 58
 prefix-free code 101–3, 106
 pretzel 266
 Prim's Algorithm 97, 104
 prime snark 305
 principal submatrix 454, 458
 probabilistic analysis 425
 probabilistic method 117,
 206, 385, 410, 425–52
 probability model 425–7,
 430
 probability space 426–7,
 430, 436, 443, 445
 product dimension 397–9,
 422
 product representation
 398–9
 proper 191, 275, 300, 388,
 472
 proper coloring 191–2, 196–
 201, 204–5, 217–20, 223,
 227, 300, 321, 408, 410,
 412, 423–4, 449, 483
 proper edge-coloring 275–7,
 285, 381, 409
 proper face-coloring 300
 proper interval graph 347
 proper labeling 388–9, 391,
 395
 proper subgraph 192, 212,
 247, 472
 proper subset 120, 152, 155,
 356, 464, 472
 Proposal Algorithm 131,
 134–5, 411
 Prüfer code 81–3, 92, 345
 quadratic growth 483
 quota 380–5
 radius 71–2, 75, 78, 265
 Ramsey multiplicity 395
 Ramsey number 206, 380–8,
 394, 426, 428, 437, 450
 Ramsey number (for graphs)
 386
 Ramsey's Theorem 378,
 380–6, 388, 393
 random graph 196, 425–6,
 430, 432, 436–40, 445–7,
 450, 463
 random variable 427–33,
 442–3, 445–7, 452, 469
 rank function (of matroid)
 349–50, 354–61, 364,
 366–77, 406
 rank (of matrix) 453–5
 rational numbers 471–2
 real numbers 120, 129, 203,
 471–2
 reciprocity 229
 Reconstruction Conjecture
 38
 recurrence 84–5, 94, 106,
 221–3, 228–30, 232, 272,
 468, 483
 reduces to 499, 506
 reducible configuration 258–
 61, 265, 270
 reduction from 499–500,
 505–6
 reflexive property (relations)
 490
 region 1, 5, 191, 233, 235,
 238–9, 245, 247, 255,
 258, 268, 316, 391, 474,
 479–80
 regular graph 34–44, 48–53,
 79, 87, 92, 116, 122, 136,
 139–40, 147, 153, 175,
 190, 198, 201, 204, 242,
 276, 283–5, 295–305, 308,
 314, 317, 385, 387, 434,
 453, 460–6, 470, 505–6
 3-regular 37, 40, 43–4, 49,
 52–3, 92, 122, 136, 139,
 146–7, 153, 158–9, 173,
 175, 243–5, 271–2, 276,
 292, 295–6, 300–5, 308,
 314–8, 385, 424, 505–6
 k -regular 34–6, 49, 51, 79,
 111, 116, 136, 140, 146–
 7, 151, 159, 198, 276,
 284–5, 296, 411, 428,
 460–2, 464, 466, 469
 regular embedding 272
 relation 8, 489
 remainder classes 491
 Replacement Lemma 334
 representable matroid 351
 restriction matroid 363–4,
 370, 375–7
 restriction martingale 445–7
 reverse edge 66
 right child 101
 right subtree 101
 ring (in planar configura-
 tion) 258–60, 270
 Ringel's Conjecture 87
 road network 5–6, 99, 112,
 165
 Robbins' Theorem 166
 root 67, 89–90, 94, 100–1,
 106, 147, 157, 198, 220,
 229, 345–6, 402, 406,
 433, 449, 453, 455, 466
 rootable family 345
 rooted plane tree 101, 106
 rooted tree 100, 106, 404
 rule of product 486
 rule of sum 485
 running time 97, 124–5,
 132, 425, 430, 494
 2-SAT 500, 505
 3-SAT 500–1, 505–6
 SATISFIABILITY 499–500,
 506
 satisfiable formula 499–500,
 506
 saturated vertex 107, 110,
 118, 124, 133, 139, 142–
 4, 352, 377
 Schur's Theorem 393
 score sequence 62
 SDR (see *System of Distinct
 Representatives*)
 Second Moment Method
 433–4, 437, 440–3, 450
 selection (subset) 486
 self-complementary 11–2,
 17, 32, 245, 271, 320
 Semi-strong Perfect Graph
 Theorem 344
 separating set 149–50, 153,
 158, 162, 164, 169, 183,
 200, 218, 231, 251
 separator 166, 231, 345
 sequence 483
 set 471

- r*-set 380–3
 Shannon bound 103, 106, 275, 279, 285
 Shannon Switching Game 365–6
 sharp 39, 70, 117, 123, 159, 210, 216, 269, 279, 284, 298, 339, 399, 434, 450
 sharp threshold 434
 shift graph 202
SHORTEST CYCLE 495
 shortest cycle 13, 217
 shortest path 29, 34, 73, 76–7, 97–8, 100, 400
 simple curve 235
 simple digraph 54–5, 59, 61–4
 simple graph 2
 simple hereditary system (matroid) 351, 375
 simple polygon 255, 270
 simplicial construction ordering 325–6
 simplicial elimination ordering 224–7, 231, 324–7, 344
 simplicial subdivision 388–9, 391, 395
 simplicial vertex 224–7, 231, 325–7, 331, 422
 sink 1, 176–89, 373, 449, 463
 sink set 178
 sink vertex 176
 size 35, 473
 size of decomposition 414
 size of matching 114
 skew partition 347
 snark 305–7, 312, 314, 317
f-soluble 148
 source 1, 176–89, 266, 373, 413–4, 463
 source set 178, 413
 source vertex 176
 source/sink cut 178–80, 188–9
 span function (of matroid) 358–60, 375
 spanning cycle 231, 240, 252, 273–4, 276, 284, **286–98**, 303–4, 314, 317, 376, 421, 437, 493–4, 497–9, 505
 spanning path 94, 104, 200, 287, 292, 498, 502
 spanning set (of matroid) 360–1, 376–7
 spanning subgraph 67, 95, 136, 140, 160, 223, 243, 312, 343, 351, 353, 373, 399, 454, 459
 spanning tree 67–70, **73–87**, 92–8, 103–5, 123, 147, 157–8, 160, 174, 190, 198, 216, 221, 232, 244, 312, 327–8, 349, 351, 354, 360, 363, 365, 372, 377–9, 402–6, 424, 451, 462–3, 469, 483, 496, 498, 505
 spans (in matroid) 358, 365
 sparse graph 145, 437, 440
 Spectral Theorem 456–8
 spectrum 453, 455, 462, 468–70
Sperner's Lemma 378, **388–91**, 395
SPGC (see *Strong Perfect Graph Conjecture*)
 spine of caterpillar 88
 split graph 345
 split of digraph 59, 424
 squashed-cube dimension 397, 401, 403, 422, 468
 stable matching 131–2, 134–6, 411
 Stable Roommates Problem 135
 stable set 4, 319–23, 326, **330–48**, 372, 441, 447–8, 506
 standard deviation 433
 star 67, 71–2, 76, 78, 80–1, 88, 115–6, 121, 214, 275, 333–4, 344, 413, 459–60
 star-cutset 333–4, 344
 Star-Cutset Lemma 333–4, 347
r-staset 447–8
 Steinitz exchange property 358–60
 stem (of blossom) 142–3
 Stirling's approximation 440
 straight-line embedding 251, 255–6
 Street-Sweeping Problem 129
 strength of theorem 440
 strict digraph 294, 299, 420
 strict gammoid 377
 strong absorption property 355–6
 strong component 56–7, 63–4, 156, 160
 strong digraph 58, 65, 90, 165, 420
 strong duality 179–80, 323
 strong elimination property 359, 374
Strong Embedding Conjecture 313
 strong induction 19, 66, 79, 480
 strong orientation 165
Strong Perfect Graph Conjecture (SPGC) 320, 334–7, 339–44, 347–8
 strongly connected digraph 56, 60–1, 63, 65, 89, 164, 245, 420, 450
 strongly perfect graph 330–1, 347
 strongly regular graph 464–7, 470
 subconstituent 470
 subcube 36, 49, 295, 401
 subdivision 162–3, 173, 212–5, 218–9, **246–51**, 256, 269, 272, 304–5, 310–1, 314, 365, 388–9, 391, 395, 442
H-subdivision 212, 213, 218
 subgraph 6, 56
 submodular function 373
 submodularity property 354–6, 367, 370–4, 377
 subset 471
 subtree 80–3, 86–7, 93, 101, 106, 157, 324–7, 344–5, 436, 449
 subtree representation 324–5, 327
 successor 54, 57–8, 60–2, 190, 294, 345, 410–1, 421, 451
 successor set 58
 sufficient condition 24
 sum of graphs 39 (see *disjoint union*)
 sum of matroids 369–70, 406

- summand 485
 supbase (of matroid) 360–1,
 363
 superconcentrator 463
 supergraph 297
 superregular graph 470
 supply 130, 184, 187
 sweep subgraph 130
 2-switch 46–7, 53
 absorption property 351,
 357, 377
 Sylvester's Law of Inertia
 457, 459
 symmetric difference 109–
 10, 118–19, 122, 133,
 137–8, 160, 314, 348,
 352–3, 359, 374, 376,
 467, 473–4
 symmetric digraph 175, 502
 symmetric matrix 6, 456–8,
 469
 symmetric property (relations) 490
 system of distinct representatives 119, 171, 369
 Szekeres–Wilf number 231
 Szekeres–Wilf Theorem 201
 Szemerédi Regularity Lemma 388
 tail (of edge) 53–60, 86, 91,
 164–5, 168, 178, 307–8,
 357–8, 484, 503
 tail partition matroid 358
 Tait coloring 301–2, 314
 Tait's Conjecture 302, 304
 Tait's Theorem 307–9, 311,
 314
 target (of function) 483
 Tarry's Algorithm 95
 telegraph problem 423
 telephone problem 422
 tensor product 201
 ternary matroid 357
 thickness 261, 271
 threshold (in Ramsey theory)
 380–1, 387
 threshold function 425, 433–
 7, 440–1, 450–1
 Tic-Tac-Toe 120
 tolerance (of path) 177–80
 TONCAS 28–9, 44, 110, 136,
 184, 225, 246
 toroidal 266–8, 272, 341
 torus 266–9, 272, 317
 total coloring 411, 423
 Total Coloring Conjecture
 411
 total dominating set 117,
 122
 totally unimodular 469
 t -tough 288
 toughness 288, 292, 297
 tournament 62–6, 190, 200,
 293, 299, 329, 393, 413,
 428, 450–1
 trace 453–4
 traffic lights 201, 266, 328
 trail 20, 26–34, 60, 64, 77,
 90, 100, 106, 173, 295,
 313, 380, 393, 506
 u, v -trail 20, 34
 transformation 47, 59, 64,
 135, 138, 141, 168, 171,
 182–3, 186, 189, 285,
 292, 360, 422, 494, 499–
 502
 transformation from 499
 transitive digraph 228, 413,
 424
 transitive graph 14, 18
 transitive orientation 228,
 231, 331, 413
 transitive property (relations) 490
 transitivity of dependence (matroids) 359
 transportation transportation network 184–5
 Transportation Problem 130,
 185
 transposition 454
 transversal matroid 352–3,
 357, 368–9, 373, 376–7
 transversal (of matrix) 126–
 8, 135
 Traveling Salesman Problem (TSP) 452, 493–4, 496–8,
 505
 tree 67–106, 118, 122–3,
 146–7, 155–8, 174, 190,
 198, 202, 204, 214, 216,
 219–21, 224, 229, 244–5,
 296, 312, 315, 317, 323–
 4, 327–8, 344–6, 349–
 51, 354, 360, 363, 365,
 372, 377–9, 386, 390,
 393–4, 396, 402–7, 424,
 436, 449, 451, 455, 462–
 3, 467–9, 492, 498–9, 505
 k -tree 345
 triangle 12, 18, 41–2, 48–
 52, 164, 223, 230–1, 256,
 275, 279–3, 285–6, 296,
 305, 336, 349, 397, 399,
 409, 412, 422, 424, 454,
 465, 469
 triangle-free graph 41–
 2, 50–1, 159, 193, 203,
 205–6, 208–9, 216, 241–
 2, 261–3, 270, 397, 399,
 424, 469, 476
 triangle inequality 66, 78,
 498, 505
 triangle, monochromatic
 378–9, 383–9, 391, 394–5,
 449
 triangular grid 390–2
 o-triangulated graph
 triangulation 242, 245, 258–
 61, 270, 300, 307, 315,
 378, 388–9, 424
 trivial 22
 trivial edge cut 304–6
 Tucker's Algorithm 34
 k -tuple 15, 26, 32–3, 35–6,
 49, 76, 379, 400, 474, 486
 Turán graph 207–10, 216
 Turán's Theorem 209–10,
 216–7
 Tutte graph 303
 Tutte's 1-factor Theorem
 146, 283
 Tutte's Condition 136–7,
 139, 141, 146–7
 Tutte's Conjectures
 Tutte's Theorem 139, 146–8,
 250
 twin 348
 two-step method 428
 unavoidable set 258, 260–1
 underlying graph 56, 60, 66,
 89, 175, 177
 k -uniform hypergraph 449
 uniform matroid 357, 370,
 373, 376, 406
 uniformity property 354–6,
 359, 361, 374
 union of graphs 25
 union of digraphs 56
 union of matroids 369–78

union of sets 473
 unipathic digraph 66
 unit interval graph 346
 unit-distance graph 201
 universal quantifier 475–6
 universe 223, 472–6
 unlabeled graph 9, 38
 unsaturated (vertex/edge)
 107, 109–11, 115, 123,
 129, 132, 134, 139, 142–
 5, 147, 368, 377
 unstable pair 130
 value of flow 176
 variance 433
 vector space 349, 351, 355,
 452–3, 467, 470
 vectorial matroid 351–2,
 355, 373
 Venn diagram 474
 vertex 2
 n -vertex graph 34
 vertex k -split 174
 VERTEX COVER 496, 502–
 3, 506
 vertex cover 112–8, 121,
 123–9, 146, 168, 227, 349,
 368, 413, 459, 502–3, 506

vertex cut 149–53, 164, 218,
 248, 333, 376
 vertex duplication 321–2,
 348
 vertex multiplication 320,
 322
 vertex ordering 6, 55, 194–
 202, 298, 331–2, 428, 451
 vertex separator 231, 345
 vertex set 2
 vertex split
 vertex-color-critical 218
 vertex-deleted subgraphs
 37–8
 vertex-transitive 14, 18
 Vizing's Theorem 275, 284–
 5, 399, 409
 Wagner's Theorem 269
 walk 20–2, 24–5, 31–3, 48,
 57–8, 60, 63, 65, 99, 203,
 236–9, 392, 455, 458, 461
 weak absorption property
 351, 354, 356, 374, 377
 weak elimination property
 352, 353, 359, 373–5
 weak dual 244
 weak duality 323, 367, 376

|

weak elimination property
 352–3, 355–6, 359, 373–5
 weakly chordal graph 330–1,
 334, 347
 weakly connected 56, 60
 weighted average 389, 427
 weighted cover 125–9
 weighted graph 95–8, 103–6,
 134, 190, 372, 377, 494,
 498, 506
 weighted intersection graph
 327
 weighted matching 125–6,
 145, 366
 Well Ordering Property 19,
 479
 wheel ($K_1 \vee C_{n-1}$) 174, 229
 Whitney's Theorem 166
 Wiener index 72
 winning strategy 57, 74,
 119, 366
 Woodall's Theorem 420, 424
 word form of permutation
 101, 486

zero flow 176, 180–1, 184,
 187, 307–8

| | | | |
|------------------------|--------------------------------|-------------------------------|----------------------------|
| $l(D)$ | maximum length of path | $\alpha(G)$ | independence number |
| $l(F)$ | length of a face | $\alpha'(G)$ | maximum size of matching |
| $\lg x$ | logarithm base 2 | $\beta(G)$ | vertex cover number |
| $\ln x$ | natural logarithm | $\beta'(G)$ | edge cover number |
| M | matching | $\gamma(G)$ | genus, domination number |
| $M(G)$ | incidence matrix | $\Delta(G)$ | maximum degree |
| $M(G)$ | cycle matroid of G | $\Delta^+(G), \Delta^-(G)$ | maximum out-, in-degree |
| M^* | dual hereditary system | $\delta(G)$ | minimum degree |
| $M.F$ | contraction of M to F | $\delta^+(G), \delta^-(G)$ | minimum out-, in-degree |
| $M F$ | restriction of M to F | $\partial(v)$ | demand at a vertex |
| \mathbb{N} | set of natural numbers | $\epsilon_G(u)$ | eccentricity of u in G |
| N | network | $\Theta(f)$ | growth rate |
| $N(v)N_G(v)$ | (open) neighborhood | $\theta(G)$ | clique cover number |
| $N[v]$ | closed neighborhood | $\theta'(G)$ | intersection number |
| $N^+(v), N^-(v)$ | out-, in-neighborhood | $\kappa(G)$ | (vertex) connectivity |
| $n(G)$ | order (number of vertices) | $\kappa'(G)$ | edge-connectivity |
| $\sigma(f), \sigma(f)$ | growth rate | $\kappa(x, y)$ | local connectivity |
| $\sigma(H)$ | number of odd components | $\kappa'(x, y)$ | local edge-connectivity |
| $P(A)$ | probability of an event | $\kappa(r; G)$ | local-global connectivity |
| P_n | path with n vertices | $\lambda(x, y)$ | max # disjoint paths |
| $\text{pdim } G$ | product dimension | $\lambda'(x, y)$ | max # edge-disjoint paths |
| $\text{qdim } G$ | squashed-cube dimension | $\lambda_1, \dots, \lambda_n$ | eigenvalues |
| Q_k | k -dimensional hypercube | μ_1, \dots, μ_n | eigenvalues |
| $\text{rad } G$ | radius | $\mu(e), \mu(G)$ | edge multiplicity |
| $R(k, l)$ | Ramsey number | $v(G)$ | crossing number |
| $R(G, H)$ | graph Ramsey number | \prod | product |
| \mathbb{R} | set of real numbers | $\rho(G)$ | maximum density |
| \mathbb{R}^2 | $\mathbb{R} \times \mathbb{R}$ | \sum | summation |
| r_M | rank function of matroid | σ, π, τ | permutation |
| S_γ | surface with γ handles | $\sigma(v)$ | supply at a vertex |
| $\text{Spec}(G)$ | spectrum (eigenvalues) | σ_M | span function |
| A^T | transpose of matrix | $\tau(G)$ | number of spanning trees |
| T | tree, tournament | $\Upsilon(G)$ | arboricity |
| $T_{n,i}$ | Turán graph | $\phi(G; \lambda)$ | characteristic polynomial |
| $t_r(n)$ | size of Turán graph | $\chi(G)$ | chromatic number |
| $U_{k,n}$ | uniform matroid | $\chi'(G)$ | edge-chromatic number |
| $u(e)$ | upper bound on flow | $\chi(G; k)$ | chromatic polynomial |
| $\text{val}(f)$ | value of a flow f | $\chi_l(G)$ | list chromatic number |
| $V(G)$ | vertex set | $\psi(G; \lambda)$ | minimum polynomial |
| W_n | wheel with n vertices | $\Omega(f), \omega(f)$ | growth rate |
| $w(e)$ | weight of edge | $\omega(G)$ | clique number |
| \mathbb{Z} | set of integers | | |
| \mathbb{Z}_p | integers modulo p | | |