

- any $z \notin L$ the point $(\wp(z), \wp'(z))$ lies on the elliptic curve E ;
- 3. two complex numbers z_1 and z_2 give the same point $(\wp(z), \wp'(z))$ on E if and only if $z_1 - z_2 \in L$;
- 4. the map that associates any $z \notin L$ to the corresponding point $(\wp(z), \wp'(z))$ on E and associates any $z \in L$ to the point at infinity $O \in E$ gives a 1-to-1 correspondence between E and the quotient of the complex plane by the subgroup L (denoted \mathbb{C}/L);
- 5. this 1-to-1 correspondence is an isomorphism of abelian groups. In other words, if z_1 corresponds to the point $P \in E$ and z_2 corresponds to $Q \in E$, then the complex number $z_1 + z_2$ corresponds to the point $P + Q$.

Thus, we can think of the abelian group E as equivalent to the complex plane modulo a suitable lattice. To visualize the latter group, note that every equivalence class $z + L$ has one and only one representative in the “fundamental parallelogram” consisting of complex numbers of the form $a\omega_1 + b\omega_2$, $0 \leq a, b < 1$ (for example, if L is the Gaussian integers, the fundamental parallelogram is the unit square). Since opposite points on the parallel sides of the boundary of the parallelogram differ by a lattice point, they are equal in \mathbb{C}/L . That is, we think of them as “glued together.” If we visualize this — folding over one side of the parallelogram to meet the opposite side (obtaining a segment of a cylinder) and then folding over again and gluing the opposite circles — we see that we obtain a “torus” (donut), pictured below.

