8

Projective Geometry

8.1 Perspective

Perspective may be simply described as the realistic representation of spatial scenes on a plane. This of course has been a concern of painters since ancient times, and some Roman artists seem to have achieved correct perspective by the first century BCE; an impressive example is shown in Wright (1983), p. 38. However, this may have been a stroke of individual genius rather than the success of a theory, because the vast majority of ancient paintings show incorrect perspective. If indeed there was a classical theory of perspective, it was well and truly lost during the Dark Ages. Medieval artists made some charming attempts at perspective but always got it wrong, and errors persisted well into the fifteenth century. [Errors still survive in twentieth-century mathematics texts. Figure 8.1 shows a fifteenth-century artistic example from Wright (1983), p. 41, alongside a twentieth-century mathematical example from the exposé of Grünbaum (1985).]

The discovery of a method for correct perspective is usually attributed to the Florentine painter-architect Brunelleschi (1377–1446), around 1420. The first published method appears in the treatise *On Painting* by Alberti (1436). The latter method, which became known as *Alberti's veil*, used a piece of transparent cloth, stretched on a frame, and set in front of the scene to be painted. Then, viewing the scene with one eye, in a fixed position, one could trace the scene directly onto the veil. Figure 8.2 shows this method, with a peephole to maintain a fixed eye position, as depicted by Dürer (1525).

Figure 8.1: Errors in perspective

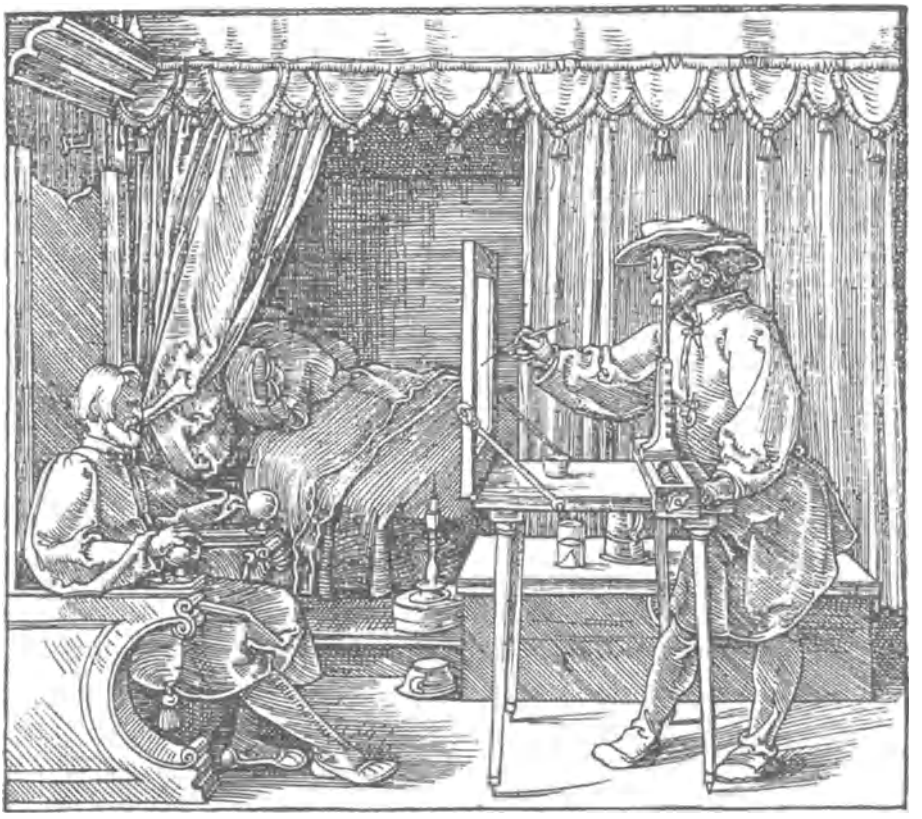

Figure 8.2: Dürer's depiction of Alberti's veil

Alberti's veil was fine for painting actual scenes, but to paint an imaginary scene in perspective some theory was required. The basic principles Renaissance artists used were the following:

(i) A straight line in perspective remains straight.

(ii) Parallel lines either remain parallel or converge to a single point (their *vanishing point*).

These principles suffice to solve a problem artists frequently encountered: the perspective depiction of a square-tiled floor. Alberti (1436) solved the special case of this problem in which one set of floor lines is horizontal, that is, parallel to the horizon. His method, which became known as the *costruzione legittima*, is indicated in simplified form in Figure 8.3. The nonhorizontal floor lines are determined by spacing them equally along the base line (imagined to touch the floor) and letting them converge to a vanishing point on the horizon. The horizontal floor lines are then determined by choosing one of them arbitrarily, thus determining one tile in the floor, and then producing the diagonal of this tile to the horizon. The intersections of this diagonal with the nonhorizontal lines are the points through which the horizontal lines pass. This is certainly true on the actual floor (Figure 8.4), hence it remains true in the perspective view.

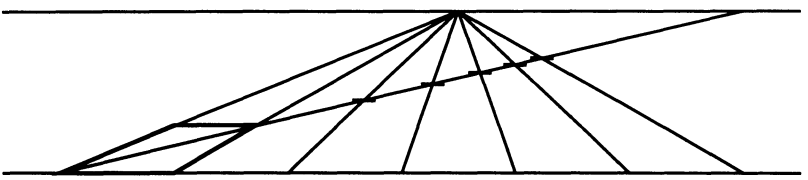

Figure 8.3: The *costruzione legittima*

EXERCISES

In almost all paintings of tiled floors, one set of lines is parallel to the horizon. However, the principles (i) and (ii) suffice to generate a perspective view of a tiled floor given an arbitrarily situated tile, and they show that no measurement is needed to achieve equal spacing along the base line in the *costruzione legittima*.

8.1.1 Use the lines shown in Figure 8.5 to determine all lines in a pavement generated by the given tile.