per quartam, secundam per tertiam et subtrahendo, $(\alpha\delta - \beta\gamma)(a^\lambda b^\mu - b^\lambda a^\mu)^2 = (a^\lambda b^\mu - b^\lambda a^\mu)(c^\lambda d^\mu - d^\lambda c^\mu) = k(a^\lambda b^\mu - b^\lambda a^\mu)^2$, vnde necessario $\alpha\delta - \beta\gamma = k$. *Q. E. S.*

235. Si forma $AXX + 2BXY + CYY \ldots F$ transit in productum e duabus formis $axx + 2bxy + cyy \ldots f$, et $a'x'x' + 2b'x'y' + c'y'y' \ldots f'$ per substitutionem talem $X = pxx' + p'xy' + p''yx' + p'''yy'$, $Y = qxx' + q'xy' + q''yx' + q'''yy'$ (quod breuitatis causa in sequentibus semper ita exprimemus: Si F transit in ff' per substitutionem $p, p', p'', p'''; q, q', q'', q'''$ *)), dicemus simpliciter, formam F *transformabilem* esse in ff'; si insuper haec transformatio ita est comparata, vt sex numeri $pq' - qp'$, $pq'' - qp''$, $pq''' - qp'''$, $p'q'' - q'p''$, $p'q''' - q'p'''$, $p''q''' - q''p'''$ diuisorem communem non habeant: formam F e formis f, f' *compositam* vocabimus.

Inchoabimus hanc disquisitionem a suppositione generalissima, formam F in ff' transire per substitutionem $p, p', p'', p'''; q, q', q'', q'''$ et quae inde sequantur euoluemus. Manifesto huic suppositioni ex asse aequiualebunt sequentes nouem aequationes (i. e. simulac hae aequationes locum habent, F per substitutionem dictam transibit in ff', et vice versa):

*) In hac igitur designatione ad ordinem tum coefficientium p, p' etc. tum formarum f, f' probe respicere oportet. Facile autem perspicietur, si ordo formarum f, f' conuertatur vt prior fiat posterior, coefficientes p', q' cum his p'', q'' commutandos esse, reliquos suo quemlibet loco manere.

$$App + 2Bpq + Cqq = aa' \ldots\ldots\ldots\ldots [1]$$
$$Ap'p' + 2Bp'q' + Cq'q' = ac' \ldots\ldots\ldots [2]$$
$$Ap''p'' + 2Bp''q'' + Cq''q'' = ca' \ldots\ldots\ldots [3]$$
$$Ap'''p''' + 2Bp'''q''' + Cq'''q''' = cc' \ldots\ldots [4]$$
$$App' + B(pq' + qp') + Cqq' = ab' \ldots\ldots [5]$$
$$App'' + B(pq'' + qp'') + Cqq'' = ba' \ldots [6]$$
$$Ap'p''' + B(p'q''' + q'p''') + Cq'q''' = bc' \; [7]$$
$$Ap''p''' + B(p''q''' + q''p''') + Cq''q''' = cb' \; [8]$$
$$A(pp''' + p'p'') + B(pq''' + qp''' + p'q'' + q'p'') + C(qq''' + q'q'') = 2bb' \ldots [9]$$

Sint determinantes formarum F, f, f' resp. D, d, d'; diuisores communes maximi numerorum A, $2B$, C; a, $2b$, c; a', $2b'$, c' resp. M, m, m' (quos omnes positiue acceptos supponimus). Porro determinentur sex numeri integri $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{A}'$, $\mathfrak{B}'$, $\mathfrak{C}'$ ita vt sit $\mathfrak{A}a + 2\mathfrak{B}b + \mathfrak{C}c = m$, $\mathfrak{A}'a' + 2\mathfrak{B}'b' + \mathfrak{C}'c' = m'$. Denique designentur numeri $pq' - qp'$, $pq'' - qp''$, $pq''' - qp'''$, $p'q'' - q'p''$, $p'q''' - q'p'''$, $p''q''' - q''p'''$ resp. per P, Q, R, S, T, U, sitque ipsorum diuisor communis maximus positiue acceptus $= k$. — Iam ponendo

$$App''' + B(pq''' + qp''') + Cqq''' = bb' + \Delta \; [10]$$

fit ex aequ. 9

$$Ap'p'' + B(p'q'' + q'p'') + Cq'q'' = bb' - \Delta \; [11]$$

Ex his vndecim aequationibus 1 ... 11, sequentes nouas euoluimus *):

*) Origo harum aequationum haec est: 12 ex 5. 5 — 1. 2; 13 ex 5. 9 — 1. 7 — 2. 6; 14 ex 10. 11 — 6. 7; 15 ex 5. 8 + 5. 8 + 10. 10 + 11. 11 — 1. 4 — 2. 3 —

$$DPP = d'aa \ldots\ldots [12]$$
$$DP(R-S) = 2d'ab \ldots\ldots [13]$$
$$DPU = d'ac - (\Delta\Delta - dd') \ldots\ldots [14]$$
$$D(R-S)^2 = 4d'bb + 2(\Delta\Delta - dd') \quad [15]$$
$$D(R-S)U = 2d'bc \ldots\ldots [16]$$
$$DUU = d'cc \ldots\ldots [17]$$
$$DQQ = da'a' \ldots\ldots [18]$$
$$DQ(R+S) = 2da'b' \ldots\ldots [19]$$
$$DQT = da'c' - (\Delta\Delta - dd') \ldots\ldots [20]$$
$$D(R+S)^2 = 4db'b' + 2(\Delta\Delta - dd') \quad [21]$$
$$D(R+S)T = 2db'c' \ldots\ldots [22]$$
$$DTT = dc'c' \ldots\ldots [23]$$

Hinc rursus deducuntur hae duae:

$$0 = 2d'aa(\Delta\Delta - dd')$$
$$0 = (\Delta\Delta - dd')^2 - 2d'ac(\Delta\Delta - dd')$$

scilicet prior ex 12. 15 — 13. 13, posterior ex 14. 14 — 12. 17; vnde facile perspicitur, necessario esse $\Delta\Delta - dd' = 0$, siue sit $a = 0$, siue non sit $= 0$ *). Supponemus itaque, in aequatt. 14, 15, 20, 21 ad dextram deleri $\Delta\Delta - dd'$.

Iam statuendo

$$\mathfrak{A}P + \mathfrak{B}(R-S) + \mathfrak{C}U = mn'$$
$$\mathfrak{A}'Q + \mathfrak{B}'(R+S) + \mathfrak{C}'T = m'n$$

6. 7 — 6. 7; 16 ex 8. 9 — 3. 7 — 4. 6; 17 ex 8. 8 — 3. 4. Deductio sex reliquarum eodem modo adornatur, si modo aequationes 2, 5, 7 cum aequationibus 3, 6, 8 resp. commutantur, et reliquae 1, 4, 9, 10, 11 eodem loco deinceps retinentur, puta 18 ex 6. 6 — 1. 3 etc.

*) Haec deriuatio aequationis $\Delta\Delta = dd'$ ad institutum praesens sufficit; alioquin analysin elegantiorem sed hic nimis prolixam tradere possemus, directe deducendo ex aequationibus 1 ... 11 hanc $0 = (\Delta\Delta - dd')^2$.