the middle of the 20th century, when Nicolas Bourbaki issued a collection of books that purported to cover the elements of all the mathematics we study now.

None of the theorems contained in the 13 books can with certainty be ascribed to Euclid himself. It is believed that the Pythagoreans, including Archytas, were responsible for much of what appears in Books I, II, VI, VII, VIII, IX and XI and that Hippocrates was behind Books III and IV. For Books V and XII we are to thank Eudoxus, and Books X and XIII are said to be based on the work of Theaetetus.

However, the logical organization of the *Elements* is undoubtedly Euclid's contribution. Its success can be measured by the fact that, after more than 2,000 years, it was still used as a textbook in British schools. Moreover, throughout the ages, its structure was often imitated. Thomas Aquinas used a similar axiomatic presentation in his *Summa*, Newton's *Principia* is written in the style of the *Elements* and Spinoza's *Ethics* follows its logical arrangement. Undoubtedly the *Elements* has been the most influential scientific textbook in history.

Euclid's grandiose plan was to deduce all of mathematics from a small number of initial definitions and assumptions. The assumptions are subdivided into *axioms*, dealing with mathematics in general, and *postulates*, dealing with geometry in particular.

His treatment illustrated the ideal described by Aristotle at the beginning of his *Posterior Analytics*: sure knowledge is obtained by the rigorous deduction of the consequences of basic truths. To Euclid, these basic truths were either definitions or basic assumptions, largely assertions of unique existence. Let us take a closer look at his definitions, axioms and postulates.

The *Elements* begins with a list of 23 definitions, of which we will mention the first four:

1. A *point* is that which has no parts.

2. A *line* is length without width.

3. The extremities of a line are points.

4. A *straight line* is a line which lies evenly with the points on itself.

These statements are not definitions in the modern sense, though they make it clear that a point has no extension, that a line is not necessarily straight and that it is of finite length. Today we prefer to regard points and straight lines as undefined primitive concepts and leave the definition of curved lines to more advanced mathematics. The obscurity of Definition 4 may be due to the translation.

Euclid's axioms are intended to apply to all of mathematics, not just to geometry. A typical axiom asserts: 'If equals are added to equals, their sums are equal.' One cannot quarrel with this statement, though today we might derive it from axioms of equality and the view of addition as an operation.

Euclid lists five postulates, which we shall now state and comment upon.

I. To draw a straight line from any point to any other point.

Presumably this means that there exists a unique straight line joining two distinct given points. Thus, a 'straight line' cannot be interpreted as referring to a great circle on a sphere, as there are many great circles joining two antipodes, e.g., the meridians passing through the two poles on the globe. The way to get around this objection is to *identify* antipodal points; one then obtains *elliptic geometry*, which also satisfies Postulate I.

II. To produce a finite straight line continuously in a straight line.

Here 'continuously' is usually interpreted to imply 'indefinitely', thus ruling out not only spherical, but also elliptic geometry.

III. To describe a circle with any center and any distance [as radius].

Like Postulate I, this is a construction, or unique existence statement, the word 'circle' having previously been defined, in Definition 15, as 'a plane figure contained by one line [i.e. curve] such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another.' It would appear that by a circle Euclid means not just its circumference but also its interior.

IV. That all right angles are equal to one another.

The status of this assertion as a postulate is rather dubious, and it has been argued, already in antiquity, that it should be listed as an axiom instead.

V. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles.

This is the most famous of Euclid's postulates and it is to his credit that he recognized its significance. It will be discussed at length in Chapter 17.

It is on these definitions and assumptions that Euclid plans to erect his impressive edifice of logical deductions. Here is how he begins:

Proposition 1.
On a finite straight line to construct an equilateral triangle.

In his proof he considers a segment AB and constructs circles with centers A and B and radius AB. He then considers the point C in which the two circles intersect and goes on to show that $\triangle ABC$ is equilateral.