8. (b)

$g.c.d.(101000110101,\ 100001111011)$
$= g.c.d.(110111010,\ 100001111011)$
$= g.c.d.(11011101,\ 100001111011) = g.c.d.(11011101,\ 11110011110)$
$= g.c.d.(11011101,\ 1111001111) = g.c.d.(11011101,\ 1011110010)$
$= g.c.d.(11011101,\ 101111001) = g.c.d.(11011101,\ 10011100)$
$= g.c.d.(11011101,\ 100111) = g.c.d.(10110110,\ 100111)$
$= g.c.d.(1011011,\ 100111) = g.c.d.(110100,\ 100111)$
$= g.c.d.(1101,\ 100111) = g.c.d.(1101,\ 11010)$
$= g.c.d.(1101,\ 1101) = 1101.$

(c) Consider the product $ab$, and show that every two steps must decrease the product of the two numbers whose g.c.d. you're taking at least by a factor of 2. Thus, there are $O(\log a)$ steps. Each step is at most a subtraction, so takes $O(\log a)$ bit operations. (Notice that no division or multiplication is involved.) (d) It doesn't give a way of expressing the g.c.d. as an integer combination of the original two numbers. However, it can be modified so as to do this: see "Extending the Binary GCD Algorithm" by G. H. Norton in *Algebraic Algorithms and Error Correcting Codes*, Springer-Verlag, 1986, 363–372.

9. $O(\log a \log b + \log^3 b)$.

10. (a) The remainders decrease at the slowest rate when all of the quotients are 1. (b) Write $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = BAB^{-1}$, where $A = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha' \end{pmatrix}$ is the diagonal matrix made up from the eigenvalues and $B$ is a matrix whose columns are eigenvectors, e.g., $B = \begin{pmatrix} \alpha & \alpha' \\ 1 & 1 \end{pmatrix}$. (c) Since $\sqrt{5}\,a \geq \sqrt{5}\,f_{k+2} = \alpha^{k+2} - \alpha'^{k+2} > \alpha^{k+2} - 1$, it follows that $k < \big(log(1 + \sqrt{5}\,a)/log\,\alpha\big) - 2$; we can also get the simpler estimate $k < \log a/\log \alpha$. The latter estimate is equal to $1.44042\cdots log_2 a$, while the estimate in the proof of Proposition I.2.1 is $2\,log_2 a$.

11. (b) In the sum of $(\log r_i)(1 + \log q_{i+1})$, use the inequalities $r_i \leq b$ and $\prod q_{i+1} \leq a$. Conclude that the sum is bounded by $O\big((\log b)(\log a + \log a)\big)$.

12. (a) $x^4 + x^2 + 1 = (x^2)(x^2 + 1) + 1$; $1 = 1(x^4 + x^2 + 1) - x^2(x^2 + 1)$. (b) $x^4 - 4x^3 + 6x^2 - 4x + 1 = (x - 3)(x^3 - x^2 + x - 1) + (2x^2 - 2)$, $x^3 - x^2 + x - 1 = (\frac{1}{2}x - \frac{1}{2})(2x^2 - 2) + (2x - 2)$, $2x^2 - 2 = (x+1)(2x-2)$, so the g.c.d. is $x - 1$; $x - 1 = (-\frac{1}{4}x + \frac{1}{4})f + (\frac{1}{4}x^2 - x + \frac{5}{4})g$.

13. $g.c.d.(f, f') = x^2 - x - 1$, and the multiple roots are the golden ratio and its conjugate $(1 \pm \sqrt{5})/2$.

14. (a) $5+6i = 2i(3-2i)+1$; $1 = 1(5+6i) - 2i(3-2i)$. (b) $8-19i = 2(7-11i)+(-6+3i)$, $7-11i = (-2+i)(-6+3i)+(-2+i)$, $-6+3i = 3(-2+i)$, so $-2+i$ is the g.c.d.; $-2+i = (-3+2i)(7-11i) + (2-i)(8-19i)$.

15. (a) $12^2 + 25^2$; (b) $54^2 + 31^2$; (c) $116^2 + 159^2$.