primo in secundum erit $y$, productum e tertio in quartum $y'$.

*Ex.* II. Si, omnibus reliquis manentibus, $\varphi$ sinum indicare supponitur, ita vt $Z = x^{16} - \frac{17}{4}x^{14} + \frac{119}{16}x^{12} - \frac{221}{32}x^{10} + \frac{935}{256}x^{8} - \frac{561}{512}x^{6} + \frac{357}{2048}x^{4} - \frac{51}{4096}xx + \frac{17}{65536}$ in duos factores 8 dimensionum $y$, $y'$ resoluere oporteat, erit $y$ productum e quatuor factoribus duplicibus $xx - (\varphi\omega)^2$, $xx - (\varphi 9\omega)^2$, $xx - (\varphi 13\omega)^2$, $xx - (\varphi 15\omega)^2$. Iam quum sit $\varphi k\omega = -\frac{1}{2}i[k] + \frac{1}{2}i[n - k]$, erit $(\varphi k\omega)^2 = -\frac{1}{4}[2k] + \frac{1}{2}[n] - \frac{1}{4}[2n - 2k] = \frac{1}{2} - \frac{1}{4}[2k] - \frac{1}{4}[2n - 2k]$; hinc deducitur summa quadratorum radicum $\varphi\omega$, $9\omega$, $\varphi 13\omega$, $\varphi 15\omega$ haec $2 - \frac{1}{4}(8, 1)$, earundem biquadratorum summa $= \frac{3}{2} - \frac{3}{16}(8, 1)$, summa potestatum sextarum $= \frac{5}{4} - \frac{9}{64}(8, 1) - \frac{1}{64}(8, 3)$, summa octauarum $\frac{35}{32} - \frac{25}{256}(8, 1) - \frac{1}{32}(8, 3)$. Hinc fit $y = x^8 - (2 - \frac{1}{4}(8, 1))x^6 + (\frac{3}{2} - \frac{5}{16}(8, 1) + \frac{1}{8}(8, 3))x^4 - (\frac{1}{2} - \frac{9}{64}(8, 1) + \frac{5}{64}(8, 3))xx + \frac{1}{16} - \frac{5}{256}(8, 1) + \frac{3}{256}(8, 3)$; $y'$ deriuatur ex $y$ commutando $(8, 1)$, $(8, 3)$, ita vt per substitutionem valorum horum aggregatorum habeatur

$$y = x^8 - (\tfrac{17}{8} - \tfrac{1}{8}\sqrt{17})x^6 + (\tfrac{51}{32} - \tfrac{7}{32}\sqrt{17})x^4 - (\tfrac{17}{32} - \tfrac{7}{64}\sqrt{17})xx + \tfrac{17}{256} - \tfrac{1}{64}\sqrt{17}$$

$$y' = x^8 - (\tfrac{17}{8} + \tfrac{1}{8}\sqrt{17})x^6 + (\tfrac{51}{32} + \tfrac{7}{32}\sqrt{17})x^4 - (\tfrac{17}{32} + \tfrac{7}{64}\sqrt{17})xx + \tfrac{17}{256} + \tfrac{1}{64}\sqrt{17}$$

Perinde $Z$ in quatuor factores resolui potest, quorum coëfficientes per aggregata quatuor terminorum exprimi possunt, et quidem productum e duobus erit $y$, productum e duobus reliquis $y'$.

365. Reduximus itaque, per disquisitiones praecedentes, sectionem circuli in $n$ partes, si $n$ est numerus primus, ad solutionem tot aequationum, in quot factores resoluere licet numerum $n - 1$, quarum aequationem gradus per magnitudinem factorum determinantur. Quoties itaque $n - 1$ est potestas numeri 2, quod euenit pro valoribus ipsius $n$ his 3, 5, 17, 257, 65537 etc., sectio circuli ad solas aequationes quadraticas reducetur, functionesque trigonometricae angulorum $\frac{P}{n}$, $\frac{2P}{n}$ etc. per radices quadraticas plus minusue complicatas (pro magnitudine ipsius $n$) exhiberi poterunt; quocirca in his casibus sectio circuli in $n$ partes, siue descriptio polygoni regularis $n$ laterum manifesto per constructiones geometricas absolui poterit. Ita e. g. pro $n = 17$ ex artt. 354, 361 facile pro cosinu anguli $\frac{1}{17}P$ expressio haec deriuatur:

$$-\tfrac{1}{16} + \tfrac{1}{16}\sqrt{17} + \tfrac{1}{16}\sqrt{(34 - 2\sqrt{17})} - \tfrac{1}{8}\sqrt{(17 + 3\sqrt{17} - \sqrt{(34 - 2\sqrt{17})} - 2\sqrt{(34 + 2\sqrt{17})})}$$

cosinus multiplorum illius anguli formam similem, sinus autem vno signo radicali plus habent. Magnopere sane est mirandum, quod, quum iam Euclidis temporibus circuli diuisibilitas geometrica in tres et quinque partes nota fuerit, nihil his inuentis interuallo 2000 annorum adiectum sit, omnesque geometrae tamquam certum pronunciauerint, praeter illas sectiones easque quae sponte inde demanant, puta sectiones in 15, $3.2^{\mu}$, $5.2^{\mu}$, $15.2^{\mu}$ nec non in $2^{\mu}$ partes, nullas alias per constructiones geometricas absolui posse. — Ceterum facile probatur, si numerus primus $n$ sit $= 2^m + 1$, etiam exponentem $m$ alios factores primos quam numerum 2 implicare non posse, adeoque vel $= 1$ vel $= 2$ vel altiori potestati numeri 2 aequalem esse debere; si enim $m$ per vllum numerum imparem $\zeta$ (vnitate maiorem) diuisibilis esset atque $m = \zeta\eta$, foret $2^m + 1$ diuisibilis per $2^{\eta} + 1$, adeoque necessario compositus. Omnes itaque valores ipsius $n$, pro quibus ad meras aequationes quadraticas deferimur, sub forma $2^{2^{\nu}} + 1$ continentur; ita quinque numeri 3, 5, 17, 257, 65537 prodeunt statuendo $\nu = 0, 1, 2, 3, 4$ siue $m = 1, 2, 4, 8, 16$. Neutiquam vero pro *omnibus* numeris sub illa forma contentis sectio circuli geometrice perficitur, sed pro iis tantum qui sunt numeri primi. Fermatius quidem inductione deceptus affirmauerat, omnes numeros sub illa forma contentos necessario primos esse; at ill. Euler hanc regulam iam pro $\nu = 5$ siue $m = 32$ erroneam esse, numero $2^{32} + 1 = 4294967297$ factorem 641 inuoluente, primus animaduertit.

Quoties autem $n - 1$ alios factores primos praeter 2 implicat, semper ad aequationes altiores deferimur; puta ad vnam pluresue cubicas quando 3 semel aut pluries inter factores primos ipsius $n - 1$ reperitur; ad aequationes quinti gradus, quando $n - 1$ diuisibilis est per 5 etc., OMNIQVE RIGORE DEMONSTRARE POSSVMVS, HAS AEQVATIONES