

C A S U S I.

CAP. IX.

224. Habeat ergo membrum supremum unicum Factorem realem  $u$ , quod evenit si  $\zeta\zeta$  sit minor quam  $4\alpha\gamma$ : atque, posito  $t$  infinito, erit  $au + d = 0$ , quæ est æquatio pro Asymtota recta. Præbeat hæc æquatio valorem  $u = c$ ; eritque,

$$at(u - c) + t(\zeta c + \epsilon c + \eta) + \gamma c^3 + \zeta c + \theta c + i = 0,$$

quæ est æquatio pro natura Asymtotæ. Hinc, prout  $\zeta c + \epsilon c + \eta$  vel non fuerit  $= 0$ , vel sit  $= 0$ , duplex Asymtotæ indoles prodit; nempe vel  $u - c = \frac{A}{t}$ , vel  $u - c = \frac{A}{t^2}$ ; unde duæ primæ Linearum tertii ordinis species formantur, quæ ita se habebunt.

1.

PRIMA Species unicam habet Asymtotam rectam speciei  $u = \frac{A}{t}$ .

2.

SECUNDA Species unicam habet Asymtotam rectam speciei  $u = \frac{A}{t^2}$ .

C A S U S 2.

225. Sint membri supremi tres Factores simplices reales & inter se inæquales; quod evenit si in æquatione

$$atu + \zeta tu + \gamma u^3 + dt + etu + \zeta u + \eta t + \theta u + i = 0,$$

fuerit  $\zeta\zeta$  major quam  $4\alpha\gamma$ . Hoc igitur casu de unoquoque Factore eadem sunt tenenda; quæ modo de unico Factore sunt exposita. Unusquisque scilicet suppeditat binos ramos hyperbolicos vel speciei  $u = \frac{A}{t}$ , vel speciei  $u = \frac{A}{t^2}$ , unde



primum fit  $u = \frac{\alpha\epsilon + \zeta\gamma}{\alpha\delta - \zeta\gamma} = c$ , qui valor si loco  $u$  in CAP. IX.  
 secundo membro continente  $t$  substituitur, ostendet ex hoc  
 Factore  $u$  seu  $\alpha y - \zeta x$  Asymtotam oriri formæ  $u = \frac{A}{t}$   
 nisi fuerit

$$\frac{\alpha\eta + \zeta\theta}{\zeta} + \frac{(\alpha\epsilon + \zeta\gamma)(\gamma\epsilon + \delta\zeta)}{(\alpha\delta - \zeta\gamma)^2} = 0.$$

Simili modo Factor  $\gamma y - \delta x$  Asymtotam præbebit formæ  
 $u = \frac{A}{t}$  nisi fuerit

$$\frac{\gamma\eta + \delta\theta}{\delta} + \frac{(\alpha\epsilon + \zeta\gamma)(\gamma\epsilon + \delta\zeta)}{(\alpha\delta - \zeta\gamma)^2} = 0.$$

227. Hinc patet fieri utique posse ut neque  $\eta$  neque utraque formula modo inventa evanescat, ex quo species tertia utique erit possibilis. Quod ad speciem quartam attinet, ponatur  $\eta = 0$ , quo una Asymtota formæ  $u = \frac{A}{tt}$  prodeat; tum autem ambæ reliquæ expressiones in unam coalescunt, ideoque binæ reliquæ Asymtotæ erunt formæ  $u = \frac{A}{t}$ , nisi fuerit  $\theta + \frac{(\alpha\epsilon + \zeta\gamma)(\gamma\epsilon + \delta\zeta)}{(\alpha\delta - \zeta\gamma)^2} = 0$ ; unde & species quarta est possibilis. At, si præter  $\eta = 0$ , una ex binis reliquis expressionibus reduatur  $= 0$ , simul altera evanescit; quam ob rem fieri non potest, ut duæ Asymtotæ fiant formæ  $u = \frac{A}{tt}$ , quin simul tertia eandem formam induat; ex quo species quinta est impossibilis. Sexta autem ob hoc ipsum erit possibilis, quia oritur, si  $\eta = 0$ , &  $\theta = \frac{-(\alpha\epsilon + \zeta\gamma)(\gamma\epsilon + \delta\zeta)}{(\alpha\delta - \zeta\gamma)^2}$ . Hi ergo duo casus quinque tantum præbuerunt species Linearum tertii ordinis, quod ea, quam quintam posuimus, prætermitti debet, &

5.

QUINTA Species tres habet Asymtotas speciei  $u = \frac{A}{tt}$ .

CASUS

228. Habeat membrum supremum duos Factores  $u$  æquales; quod evenit, si in æquatione casus præcedentis primus terminus  $\alpha t t u$  evanescat. Æquatio ergo generalis ad hunc casum pertinens erit hujusmodi,

$$\alpha t u u - \zeta u^3 + \gamma t t + \delta t u + \epsilon u u + \zeta t + \eta u + \theta = 0,$$

habet ergo membrum supremum duos Factores  $u$  æquales, ac tertium  $\alpha u - \zeta u$  reliquis inæqualem. Iste tertius Factor producet Asymptotam vel formæ  $u = \frac{A}{t}$ , vel formæ  $u =$

$\frac{A}{t t}$ , prout fuerit hæc expressio

$$(\alpha \delta + 2 \zeta \gamma)(\alpha^2 \epsilon + \alpha \zeta \delta + \zeta \zeta \gamma) - \alpha^3 (\alpha \eta + \zeta \zeta)$$

vel non  $= 0$ , vel  $= 0$ .

229. Quod ad duos Factores æquales attinet, primum casus occurrit, si  $\gamma$  non fuerit  $= 0$ ; tum enim, facto  $t = \infty$ , fiet  $\alpha u u + \gamma t = 0$ , quæ est æquatio pro Asymptota parabolica speciei  $u u = A t$ . Hinc istæ duæ nascentur species novæ Linearum tertii ordinis, nempe.

6.

SEXTA Species habet unam Asymptotam speciei  $u = \frac{A}{t}$  & unam Asymptotam speciei  $u u = A t$ .

7.

SEPTIMA Species habet unam Asymptotam speciei  $u = \frac{A}{t t}$  & unam parabolicam speciei  $u u = A t$ .

230. Sit jam  $\gamma = 0$ ; atque Factor tertius  $\alpha t - \zeta u$  dabit Asymptotam formæ  $u = \frac{A}{t t}$ , si fuerit

$$\delta(\alpha \epsilon + \zeta \delta) = \alpha(\alpha \eta + \zeta \zeta)$$

sin autem hæc æqualitas non habeat locum, Asymptota erit formæ  $u = \frac{A}{t}$ . Habebimus ergo hanc æquationem

+  $\alpha t u u$

$$\begin{array}{rcl} + \alpha t u u & - & \zeta u^3 \\ + \delta t u & + & \varepsilon u u = 0 \\ + \zeta t & + & \eta u \\ + \theta & & \end{array}$$

Hic, facto  $t = \infty$ , fiet  $\alpha u u + \delta u + \zeta = 0$ .

Sit primum  $\delta \delta$  minor quam  $4 \alpha \zeta$ , atque hinc nulla orietur Afymtota; quare ex hoc casu duæ oriuntur Species.

8.

OCTAVA Species habet unicam Afymtotam speciei

$$u = \frac{A}{t}.$$

9.

NONA Species habet unicam Afymtotam speciei

$$u = \frac{A}{t}.$$

231. Sint æquationis  $\alpha u u + \delta u + \zeta = 0$ , ambæ radices reales & inæquales, nempe  $\delta \delta$  major quam  $4 \alpha \zeta$ ; atque hinc duæ prodibunt Afymtotæ rectæ inter se parallelæ, utraque formæ  $u = \frac{A}{t}$ , qui casus denuo duas suppeditat Species.

10.

DECIMA Species habet unam Afymtotam speciei  $u = \frac{A}{t}$  & duas inter se parallelas speciei  $u = \frac{A}{t}$ .

11.

UNDECIMA Species habet unam Afymtotam speciei  $u = \frac{A}{t}$  & duas inter se parallelas speciei  $u = \frac{A}{t}$ .

232. Sint æquationis  $\alpha u u + \delta u + \zeta = 0$ , ambæ radices inter se æquales, seu  $\delta \delta = 4 \alpha \zeta$ , seu  $\alpha u u + \delta u + \zeta = \alpha (u - c)^2$ , fietque  $\alpha t (u - c)^2 = \zeta c^3 - \varepsilon c c - \eta c - \theta$ , unde oritur Afymtota recta una speciei  $u u = \frac{A}{t}$ . Hinc ergo duæ nascuntur Species novæ.

12.

DUODECIMA Species habet unam Afymtotam speciei

Euleri *Introduc.* in *Anal. infin. Tom. II.*

Q  $u =$

LIB. II.  $u = \frac{A}{t}$  & unam speciei  $uu = \frac{A}{t}$ .

13.

DECIMATERTIA Species habet unam Asymptotam speciei  $u = \frac{A}{tt}$  & unam speciei  $uu = \frac{A}{t}$ .

## CASUS IV.

233. Quid si membri supremi omnes tres Factores fuerint æquales, æquatio habebit hujusmodi formam,

$$\alpha u^3 + \zeta tt + \gamma tu + \delta uu + \epsilon t + \zeta u + \eta = 0,$$

Hic primum spectandus est terminus  $\zeta tt$ , qui si non desit, Curva habebit Asymptotam parabolicam speciei  $u^3 = At$ , sicque una oritur Species.

14.

DECIMAQUARTA Species habet unicam Asymptotam parabolicam speciei  $u^3 = At$ .

234. Desit jam terminus  $\zeta tt$ , eritque

$$\alpha u^3 + \gamma tu + \delta uu + \epsilon t + \zeta u + \eta = 0;$$

unde, posito  $t$  infinito, fiet  $\alpha u^3 + \gamma tu + \epsilon t = 0$ , nisi sint  $\gamma$  &  $\epsilon = 0$ . Non igitur sit  $\gamma = 0$ , atque in hac æquatione duæ continentur æquationes  $\alpha uu + \gamma t = 0$ , &  $\gamma u + \epsilon = 0$ ; prior est pro Asymptota parabolica speciei  $uu = At$ ; posterior vero, si ponatur  $\frac{\epsilon}{\gamma} = c$ , dabit æquationem hanc

$$\gamma t(u - c) + \alpha c^3 + \delta cc + \zeta c + \eta = 0,$$

eritque ergo pro Asymptota hyperbolica speciei  $u = \frac{A}{t}$ , unde.

15.

DECIMAQUINTA Species unam habet Asymptotam parabolicam speciei  $uu = At$ , & unam rectam speciei  $u = \frac{A}{t}$