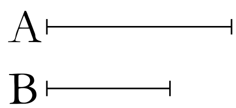
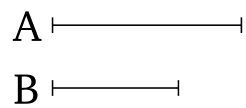


Εἰ γὰρ ἔχει τὸ Α πρὸς τὸ Β λόγον, ὃν ἀριθμὸς πρὸς ἀριθμόν, σύμμετρον ἔσται τὸ Α τῷ Β. οὐκ ἔστι δέ· οὐκ ἄρα τὸ Α πρὸς τὸ Β λόγον ἔχει, ὃν ἀριθμὸς πρὸς ἀριθμόν.

Τὰ ἄρα ἀσύμμετρα μεγέθη πρὸς ἄλληλα λόγον οὐκ ἔχει, καὶ τὰ ἑξῆς.

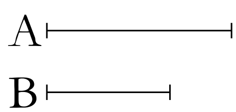


For if A has to B the ratio which (some) number (has) to (some) number then A will be commensurable with B [Prop. 10.6]. But it is not. Thus, A does not have to B the ratio which (some) number (has) to (some) number.

Thus, incommensurable numbers do not have to one another, and so on ....

η´.

Ἐὰν δύο μεγέθη πρὸς ἄλληλα λόγον μὴ ἔχῃ, ὃν ἀριθμὸς πρὸς ἀριθμόν, ἀσύμμετρα ἔσται τὰ μεγέθη.



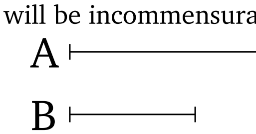
Δύο γὰρ μεγέθη τὰ Α, Β πρὸς ἄλληλα λόγον μὴ ἐχέτω, ὃν ἀριθμὸς πρὸς ἀριθμόν· λέγω, ὅτι ἀσύμμετρά ἐστι τὰ Α, Β μεγέθη.

Εἰ γὰρ ἔσται σύμμετρα, τὸ Α πρὸς τὸ Β λόγον ἕξει, ὃν ἀριθμὸς πρὸς ἀριθμόν. οὐκ ἔχει δέ. ἀσύμμετρα ἄρα ἐστὶ τὰ Α, Β μεγέθη.

Ἐὰν ἄρα δύο μεγέθη πρὸς ἄλληλα, καὶ τὰ ἑξῆς.

Proposition 8

If two magnitudes do not have to one another the ratio which (some) number (has) to (some) number then the magnitudes will be incommensurable.



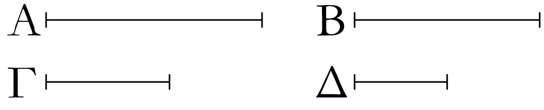
For let the two magnitudes A and B not have to one another the ratio which (some) number (has) to (some) number. I say that the magnitudes A and B are incommensurable.

For if they are commensurable, A will have to B the ratio which (some) number (has) to (some) number [Prop. 10.5]. But it does not have (such a ratio). Thus, the magnitudes A and B are incommensurable.

Thus, if two magnitudes ... to one another, and so on ....

θ´.

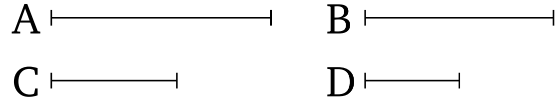
Τὰ ἀπὸ τῶν μήκει συμμέτρων εὐθειῶν τετράγωνα πρὸς ἄλληλα λόγον ἔχει, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν· καὶ τὰ τετράγωνα τὰ πρὸς ἄλληλα λόγον ἔχοντα, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν, καὶ τὰς πλευρὰς ἕξει μήκει συμμέτρους. τὰ δὲ ἀπὸ τῶν μήκει ἀσυμμέτρων εὐθειῶν τετράγωνα πρὸς ἄλληλα λόγον οὐκ ἔχει, ὅνπερ τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν· καὶ τὰ τετράγωνα τὰ πρὸς ἄλληλα λόγον μὴ ἔχοντα, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν, οὐδὲ τὰς πλευρὰς ἕξει μήκει συμμέτρους.



Ἔστωσαν γὰρ αἱ Α, Β μήκει σύμμετροι· λέγω, ὅτι τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Β τετράγωνον λόγον ἔχει, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν.

Proposition 9

Squares on straight-lines (which are) commensurable in length have to one another the ratio which (some) square number (has) to (some) square number. And squares having to one another the ratio which (some) square number (has) to (some) square number will also have sides (which are) commensurable in length. But squares on straight-lines (which are) incommensurable in length do not have to one another the ratio which (some) square number (has) to (some) square number. And squares not having to one another the ratio which (some) square number (has) to (some) square number will not have sides (which are) commensurable in length either.



For let A and B be (straight-lines which are) commensurable in length. I say that the square on A has to the square on B the ratio which (some) square number (has) to (some) square number.

Ἐπεὶ γὰρ σύμμετρός ἐστιν ἡ Α τῇ Β μήκει, ἡ Α ἄρα πρὸς τὴν Β λόγον ἔχει, ὃν ἀριθμὸς πρὸς ἀριθμόν. ἐχέτω, ὃν ὁ Γ πρὸς τὸν Δ. ἐπεὶ οὖν ἐστιν ὡς ἡ Α πρὸς τὴν Β, οὕτως ὁ Γ πρὸς τὸν Δ, ἀλλὰ τοῦ μὲν τῆς Α πρὸς τὴν Β λόγου διπλασίων ἐστὶν ὁ τοῦ ἀπὸ τῆς Α τετραγώνου πρὸς τὸ ἀπὸ τῆς Β τετράγωνον· τὰ γὰρ ὅμοια σχήματα ἐν διπλασίονι λόγῳ ἐστὶ τῶν ὁμολόγων πλευρῶν· τοῦ δὲ τοῦ Γ [ἀριθμοῦ] πρὸς τὸν Δ [ἀριθμὸν] λόγου διπλασίων ἐστὶν ὁ τοῦ ἀπὸ τοῦ Γ τετραγώνου πρὸς τὸν ἀπὸ τοῦ Δ τετράγωνον· δύο γὰρ τετραγώνων ἀριθμῶν εἷς μέσος ἀνάλογόν ἐστιν ἀριθμός, καί ὁ τετράγωνος πρὸς τὸν τετράγωνον [ἀριθμὸν] διπλασίονα λόγον ἔχει, ἤπερ ἡ πλευρὰ πρὸς τὴν πλευράν· ἔστιν ἄρα καὶ ὡς τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Β τετράγωνον, οὕτως ὁ ἀπὸ τοῦ Γ τετράγωνος [ἀριθμὸς] πρὸς τὸν ἀπὸ τοῦ Δ [ἀριθμοῦ] τετράγωνον [ἀριθμόν].

Ἀλλὰ δὴ ἔστω ὡς τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Β, οὕτως ὁ ἀπὸ τοῦ Γ τετράγωνος πρὸς τὸν ἀπὸ τοῦ Δ [τετράγωνον]· λέγω, ὅτι σύμμετρός ἐστιν ἡ Α τῇ Β μήκει.

Ἐπεὶ γάρ ἐστιν ὡς τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Β [τετράγωνον], οὕτως ὁ ἀπὸ τοῦ Γ τετράγωνος πρὸς τὸν ἀπὸ τοῦ Δ [τετράγωνον], ἀλλ᾽ ὁ μὲν τοῦ ἀπὸ τῆς Α τετραγώνου πρὸς τὸ ἀπὸ τῆς Β [τετράγωνον] λόγος διπλασίων ἐστὶ τοῦ τῆς Α πρὸς τὴν Β λόγου, ὁ δὲ τοῦ ἀπὸ τοῦ Γ [ἀριθμοῦ] τετραγώνου [ἀριθμοῦ] πρὸς τὸν ἀπὸ τοῦ Δ [ἀριθμοῦ] τετράγωνον [ἀριθμὸν] λόγος διπλασίων ἐστὶ τοῦ τοῦ Γ [ἀριθμοῦ] πρὸς τὸν Δ [ἀριθμὸν] λόγου, ἔστιν ἄρα καὶ ὡς ἡ Α πρὸς τὴν Β, οὕτως ὁ Γ [ἀριθμὸς] πρὸς τὸν Δ [ἀριθμόν]. ἡ Α ἄρα πρὸς τὴν Β λόγον ἔχει, ὃν ἀριθμὸς ὁ Γ πρὸς ἀριθμὸν τὸν Δ· σύμμετρος ἄρα ἐστὶν ἡ Α τῇ Β μήκει.

Ἀλλὰ δὴ ἀσύμμετρος ἔστω ἡ Α τῇ Β μήκει· λέγω, ὅτι τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Β [τετράγωνον] λόγον οὐκ ἔχει, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν.

Εἰ γὰρ ἔχει τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Β [τετράγωνον] λόγον, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν, σύμμετρος ἔσται ἡ Α τῇ Β. οὐκ ἔστι δέ· οὐκ ἄρα τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Β [τετράγωνον] λόγον ἔχει, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν.

Πάλιν δὴ τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Β [τετράγωνον] λόγον μὴ ἐχέτω, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν· λέγω, ὅτι ἀσύμμετρός ἐστιν ἡ Α τῇ Β μήκει.

Εἰ γάρ ἐστι σύμμετρος ἡ Α τῇ Β, ἕξει τὸ ἀπὸ τῆς Α πρὸς τὸ ἀπὸ τῆς Β λόγον, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν. οὐκ ἔχει δέ· οὐκ ἄρα σύμμετρός ἐστιν ἡ Α τῇ Β μήκει.

Τὰ ἄρα ἀπὸ τῶν μήκει συμμέτρων, καὶ τὰ ἑξῆς.

For since A is commensurable in length with B, A thus has to B the ratio which (some) number (has) to (some) number [Prop. 10.5]. Let it have (that) which C (has) to D. Therefore, since as A is to B, so C (is) to D. But the (ratio) of the square on A to the square on B is the square of the ratio of A to B. For similar figures are in the squared ratio of (their) corresponding sides [Prop. 6.20 corr.]. And the (ratio) of the square on C to the square on D is the square of the ratio of the [number] C to the [number] D. For there exits one number in mean proportion to two square numbers, and (one) square (number) has to the (other) square [number] a squared ratio with respect to (that) the side (of the former has) to the side (of the latter) [Prop. 8.11]. And, thus, as the square on A is to the square on B, so the square [number] on the (number) C (is) to the square [number] on the [number] D.†

And so let the square on A be to the (square) on B as the square (number) on C (is) to the [square] (number) on D. I say that A is commensurable in length with B.

For since as the square on A is to the [square] on B, so the square (number) on C (is) to the [square] (number) on D. But, the ratio of the square on A to the (square) on B is the square of the (ratio) of A to B [Prop. 6.20 corr.]. And the (ratio) of the square [number] on the [number] C to the square [number] on the [number] D is the square of the ratio of the [number] C to the [number] D [Prop. 8.11]. Thus, as A is to B, so the [number] C also (is) to the [number] D. A, thus, has to B the ratio which the number C has to the number D. Thus, A is commensurable in length with B [Prop. 10.6].‡

And so let A be incommensurable in length with B. I say that the square on A does not have to the [square] on B the ratio which (some) square number (has) to (some) square number.

For if the square on A has to the [square] on B the ratio which (some) square number (has) to (some) square number then A will be commensurable (in length) with B. But it is not. Thus, the square on A does not have to the [square] on the B the ratio which (some) square number (has) to (some) square number.

So, again, let the square on A not have to the [square] on B the ratio which (some) square number (has) to (some) square number. I say that A is incommensurable in length with B.

For if A is commensurable (in length) with B then the (square) on A will have to the (square) on B the ratio which (some) square number (has) to (some) square number. But it does not have (such a ratio). Thus, A is not commensurable in length with B.

Thus, (squares) on (straight-lines which are) commensurable in length, and so on ....

Πόρισμα.

Καὶ φανερὸν ἐκ τῶν δεδειγμένων ἔσται, ὅτι αἱ μήκει σύμμετροι πάντως καὶ δυνάμει, αἱ δὲ δυνάμει οὐ πάντως καὶ μήκει.

Corollary

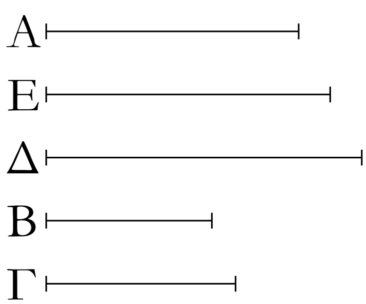
And it will be clear, from (what) has been demonstrated, that (straight-lines) commensurable in length (are) always also (commensurable) in square, but (straight-lines commensurable) in square (are) not always also (commensurable) in length.

† There is an unstated assumption here that if $\alpha : \beta :: \gamma : \delta$ then $\alpha^2 : \beta^2 :: \gamma^2 : \delta^2$.

‡ There is an unstated assumption here that if $\alpha^2 : \beta^2 :: \gamma^2 : \delta^2$ then $\alpha : \beta :: \gamma : \delta$.

ιʹ.

Τῇ προτεθείσῃ εὐθείᾳ προσευρεῖν δύο εὐθείας ἀσυμμέτρους, τὴν μὲν μήκει μόνον, τὴν δὲ καὶ δυνάμει.



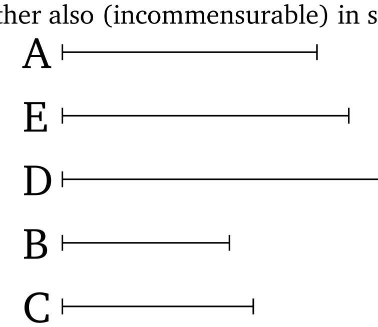
Ἔστω ἡ προτεθεῖσα εὐθεῖα ἡ Α· δεῖ δὴ τῇ Α προσευρεῖν δύο εὐθείας ἀσυμμέτρους, τὴν μὲν μήκει μόνον, τὴν δὲ καὶ δυνάμει.

Ἐκκείσθωσαν γὰρ δύο αριθμοὶ οἱ Β, Γ πρὸς ἀλλήλους λόγον μὴ ἔχοντες, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν, τουτέστι μὴ ὅμοιοι ἐπίπεδοι, καὶ γεγονέτω ὡς ὁ Β πρὸς τὸν Γ, οὕτως τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Δ τετράγωνον· ἐμάθομεν γάρ· σύμμετρον ἄρα τὸ ἀπὸ τῆς Α τῷ ἀπὸ τῆς Δ. καὶ ἐπεὶ ὁ Β πρὸς τὸν Γ λόγον οὐκ ἔχει, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν, οὐδ᾽ ἄρα τὸ ἀπὸ τῆς Α πρὸς τὸ ἀπὸ τῆς Δ λόγον ἔχει, ὃν τετράγωνος ἀριθμὸς πρὸς τετράγωνον ἀριθμόν· ἀσύμμετρος ἄρα ἐστὶν ἡ Α τῇ Δ μήκει. εἰλήφθω τῶν Α, Δ μέση ἀνάλογον ἡ Ε· ἔστιν ἄρα ὡς ἡ Α πρὸς τὴν Δ, οὕτως τὸ ἀπὸ τῆς Α τετράγωνον πρὸς τὸ ἀπὸ τῆς Ε. ἀσύμμετρος δέ ἐστιν ἡ Α τῇ Δ μήκει· ἀσύμμετρον ἄρα ἐστὶ καὶ τὸ ἀπὸ τῆς Α τετράγωνον τῷ ἀπὸ τῆς Ε τετραγώνῳ· ἀσύμμετρος ἄρα ἐστὶν ἡ Α τῇ Ε δυνάμει.

Τῇ ἄρα προτεθείσῃ εὐθείᾳ τῇ Α προσεύρηνται δύο εὐθεῖαι ἀσύμμετροι αἱ Δ, Ε, μήκει μὲν μόνον ἡ Δ, δυνάμει δὲ καὶ μήκει δηλαδὴ ἡ Ε [ὅπερ ἔδει δεῖξαι].

Proposition 10†

To find two straight-lines incommensurable with a given straight-line, the one (incommensurable) in length only, the other also (incommensurable) in square.



Let A be the given straight-line. So it is required to find two straight-lines incommensurable with A, the one (incommensurable) in length only, the other also (incommensurable) in square.

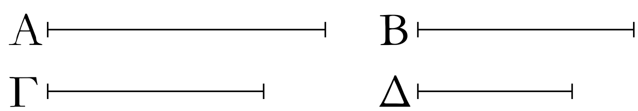
For let two numbers, B and C, not having to one another the ratio which (some) square number (has) to (some) square number—that is to say, not (being) similar plane (numbers)—have been taken. And let it be contrived that as B (is) to C, so the square on A (is) to the square on D. For we learned (how to do this) [Prop. 10.6 corr.]. Thus, the (square) on A (is) commensurable with the (square) on D [Prop. 10.6]. And since B does not have to C the ratio which (some) square number (has) to (some) square number, the (square) on A thus does not have to the (square) on D the ratio which (some) square number (has) to (some) square number either. Thus, A is incommensurable in length with D [Prop. 10.9]. Let the (straight-line) E (which is) in mean proportion to A and D have been taken [Prop. 6.13]. Thus, as A is to D, so the square on A (is) to the (square) on E [Def. 5.9]. And A is incommensurable in length with D. Thus, the square on A is also incommensurble with the square on E [Prop. 10.11]. Thus, A is incommensurable in square with E.

Thus, two straight-lines, D and E, (which are) incommensurable with the given straight-line A, have been found, the one, D, (incommensurable) in length only, the other, E, (incommensurable) in square, and, clearly, also in length. [(Which is) the very thing it was required to show.]

† This whole proposition is regarded by Heiberg as an interpolation into the original text.

ιαʹ.

Ἐὰν τέσσαρα μεγέθη ἀνάλογον ᾖ, τὸ δὲ πρῶτον τῷ δευτέρῳ σύμμετρον ᾖ, καὶ τὸ τρίτον τῷ τετάρτῳ σύμμετρον ἔσται· κἂν τὸ πρῶτον τῷ δευτέρῳ ἀσύμμετρον ᾖ, καὶ τὸ τρίτον τῷ τετάρτῳ ἀσύμμετρον ἔσται.



Ἔστωσαν τέσσαρα μεγέθη ἀνάλογον τὰ Α, Β, Γ, Δ, ὡς τὸ Α πρὸς τὸ Β, οὕτως τὸ Γ πρὸς τὸ Δ, τὸ Α δὲ τῷ Β σύμμετρον ἔστω· λέγω, ὅτι καὶ τὸ Γ τῷ Δ σύμμετρον ἔσται.

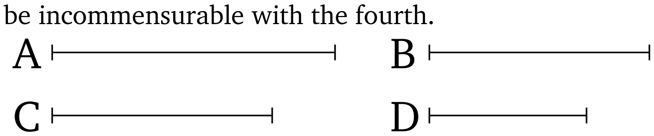
Ἐπεὶ γὰρ σύμμετρόν ἐστι τὸ Α τῷ Β, τὸ Α ἄρα πρὸς τὸ Β λόγον ἔχει, ὃν ἀριθμὸς πρὸς ἀριθμόν. καί ἐστιν ὡς τὸ Α πρὸς τὸ Β, οὕτως τὸ Γ πρὸς τὸ Δ· καὶ τὸ Γ ἄρα πρὸς τὸ Δ λόγον ἔχει, ὃν ἀριθμὸς πρὸς ἀριθμόν· σύμμετρον ἄρα ἐστὶ τὸ Γ τῷ Δ.

Ἀλλὰ δὴ τὸ Α τῷ Β ἀσύμμετρον ἔστω· λέγω, ὅτι καὶ τὸ Γ τῷ Δ ἀσύμμετρον ἔσται. ἐπεὶ γὰρ ἀσύμμετρόν ἐστι τὸ Α τῷ Β, τὸ Α ἄρα πρὸς τὸ Β λόγον οὐκ ἔχει, ὃν ἀριθμὸς πρὸς ἀριθμόν. καί ἐστιν ὡς τὸ Α πρὸς τὸ Β, οὕτως τὸ Γ πρὸς τὸ Δ· οὐδὲ τὸ Γ ἄρα πρὸς τὸ Δ λόγον ἔχει, ὃν ἀριθμὸς πρὸς ἀριθμόν· ἀσύμμετρον ἄρα ἐστὶ τὸ Γ τῷ Δ.

Ἐὰν ἄρα τέσσαρα μεγέθη, καὶ τὰ ἑξῆς.

Proposition 11

If four magnitudes are proportional, and the first is commensurable with the second, then the third will also be commensurable with the fourth. And if the first is incommensurable with the second, then the third will also be incommensurable with the fourth.



Let A, B, C, D be four proportional magnitudes, (such that) as A (is) to B, so C (is) to D. And let A be commensurable with B. I say that C will also be commensurable with D.

For since A is commensurable with B, A thus has to B the ratio which (some) number (has) to (some) number [Prop. 10.5]. And as A is to B, so C (is) to D. Thus, C also has to D the ratio which (some) number (has) to (some) number. Thus, C is commensurable with D [Prop. 10.6].

And so let A be incommensurable with B. I say that C will also be incommensurable with D. For since A is incommensurable with B, A thus does not have to B the ratio which (some) number (has) to (some) number [Prop. 10.7]. And as A is to B, so C (is) to D. Thus, C does not have to D the ratio which (some) number (has) to (some) number either. Thus, C is incommensurable with D [Prop. 10.8].

Thus, if four magnitudes, and so on ....

ιβʹ.

Τὰ τῷ αὐτῷ μεγέθει σύμμετρα καὶ ἀλλήλοις ἐστὶ σύμμετρα.

Ἑκάτερον γὰρ τῶν Α, Β τῷ Γ ἔστω σύμμετρον. λέγω, ὅτι καὶ τὸ Α τῷ Β ἐστι σύμμετρον.

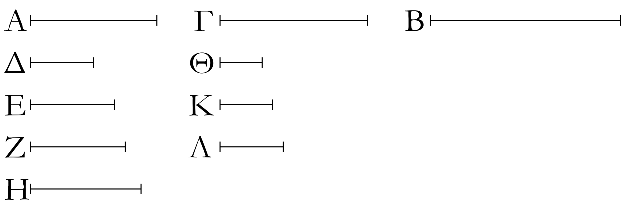
Ἐπεὶ γὰρ σύμμετρόν ἐστι τὸ Α τῷ Γ, τὸ Α ἄρα πρὸς τὸ Γ λόγον ἔχει, ὃν ἀριθμὸς πρὸς ἀριθμόν. ἐχέτω, ὃν ὁ Δ πρὸς τὸν Ε. πάλιν, ἐπεὶ σύμμετρόν ἐστι τὸ Γ τῷ Β, τὸ Γ ἄρα πρὸς τὸ Β λόγον ἔχει, ὃν ἀριθμὸς πρὸς ἀριθμόν. ἐχέτω, ὃν ὁ Ζ πρὸς τὸν Η. καὶ λόγων δοθέντων ὁποσωνοῦν τοῦ τε, ὃν ἔχει ὁ Δ πρὸς τὸν Ε, καὶ ὁ Ζ πρὸς τὸν Η εἰλήφθωσαν ἀριθμοὶ ἑξῆς ἐν τοῖς δοθεῖσι λόγοις οἱ Θ, Κ, Λ· ὥστε εἶναι

Proposition 12

(Magnitudes) commensurable with the same magnitude are also commensurable with one another.

For let A and B each be commensurable with C. I say that A is also commensurable with B.

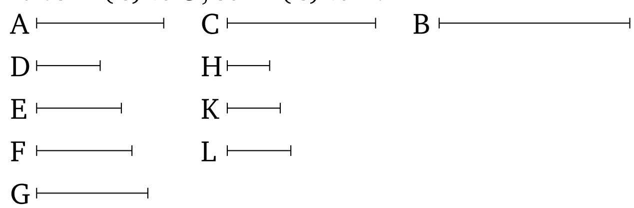
For since A is commensurable with C, A thus has to C the ratio which (some) number (has) to (some) number [Prop. 10.5]. Let it have (the ratio) which D (has) to E. Again, since C is commensurable with B, C thus has to B the ratio which (some) number (has) to (some) number [Prop. 10.5]. Let it have (the ratio) which F (has) to G. And for any multitude whatsoever ὡς μὲν τὸν Δ πρὸς τὸν Ε, οὕτως τὸν Θ πρὸς τὸν Κ, ὡς δὲ τὸν Ζ πρὸς τὸν Η, οὕτως τὸν Κ πρὸς τὸν Λ.



Ἐπεὶ οὖν ἐστιν ὡς τὸ Α πρὸς τὸ Γ, οὕτως ὁ Δ πρὸς τὸν Ε, ἀλλ᾽ ὡς ὁ Δ πρὸς τὸν Ε, οὕτως ὁ Θ πρὸς τὸν Κ, ἔστιν ἄρα καὶ ὡς τὸ Α πρὸς τὸ Γ, οὕτως ὁ Θ πρὸς τὸν Κ. πάλιν, ἐπεί ἐστιν ὡς τὸ Γ πρὸς τὸ Β, οὕτως ὁ Ζ πρὸς τὸν Η, ἀλλ᾽ ὡς ὁ Ζ πρὸς τὸν Η, [οὕτως] ὁ Κ πρὸς τὸν Λ, καὶ ὡς ἄρα τὸ Γ πρὸς τὸ Β, οὕτως ὁ Κ πρὸς τὸν Λ. ἔστι δὲ καὶ ὡς τὸ Α πρὸς τὸ Γ, οὕτως ὁ Θ πρὸς τὸν Κ· δι᾽ ἴσου ἄρα ἐστὶν ὡς τὸ Α πρὸς τὸ Β, οὕτως ὁ Θ πρὸς τὸν Λ. τὸ Α ἄρα πρὸς τὸ Β λόγον ἔχει, ὃν ἀριθμὸς ὁ Θ πρὸς ἀριθμὸν τὸν Λ· σύμμετρον ἄρα ἐστὶ τὸ Α τῷ Β.

Τὰ ἄρα τῷ αὐτῷ μεγέθει σύμμετρα καὶ ἀλλήλοις ἐστὶ σύμμετρα· ὅπερ ἔδει δεῖξαι.

of given ratios—(namely,) those which D has to E, and F to G—let the numbers H, K, L (which are) continuously (proportional) in the(se) given ratios have been taken [Prop. 8.4]. Hence, as D is to E, so H (is) to K, and as F (is) to G, so K (is) to L.

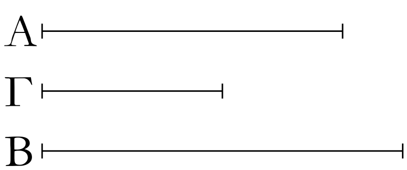


Therefore, since as A is to C, so D (is) to E, but as D (is) to E, so H (is) to K, thus also as A is to C, so H (is) to K [Prop. 5.11]. Again, since as C is to B, so F (is) to G, but as F (is) to G, [so] K (is) to L, thus also as C (is) to B, so K (is) to L [Prop. 5.11]. And also as A is to C, so H (is) to K. Thus, via equality, as A is to B, so H (is) to L [Prop. 5.22]. Thus, A has to B the ratio which the number H (has) to the number L. Thus, A is commensurable with B [Prop. 10.6].

Thus, (magnitudes) commensurable with the same magnitude are also commensurable with one another. (Which is) the very thing it was required to show.

ιγʹ.

Ἐὰν ᾖ δύο μεγέθη σύμμετρα, τὸ δὲ ἕτερον αὐτῶν μεγέθει τινὶ ἀσύμμετρον ᾖ, καὶ τὸ λοιπὸν τῷ αὐτῷ ἀσύμμετρον ἔσται.



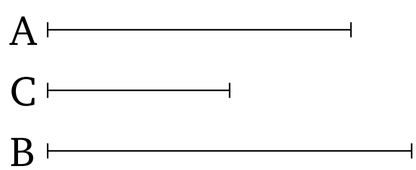
Ἔστω δύο μεγέθη σύμμετρα τὰ Α, Β, τὸ δὲ ἕτερον αὐτῶν τὸ Α ἄλλῳ τινὶ τῷ Γ ἀσύμμετρον ἔστω· λέγω, ὅτι καὶ τὸ λοιπὸν τὸ Β τῷ Γ ἀσύμμετρόν ἐστιν.

Εἰ γάρ ἐστι σύμμετρον τὸ Β τῷ Γ, ἀλλὰ καὶ τὸ Α τῷ Β σύμμετρόν ἐστιν, καὶ τὸ Α ἄρα τῷ Γ σύμμετρόν ἐστιν. ἀλλὰ καὶ ἀσύμμετρον· ὅπερ ἀδύνατον. οὐκ ἄρα σύμμετρόν ἐστι τὸ Β τῷ Γ· ἀσύμμετρον ἄρα.

Ἐὰν ἄρα ᾖ δύο μεγέθη σύμμετρα, καὶ τὰ ἑξῆς.

Proposition 13

If two magnitudes are commensurable, and one of them is incommensurable with some magnitude, then the remaining (magnitude) will also be incommensurable with it.



Let A and B be two commensurable magnitudes, and let one of them, A, be incommensurable with some other (magnitude), C. I say that the remaining (magnitude), B, is also incommensurable with C.

For if B is commensurable with C, but A is also commensurable with B, A is thus also commensurable with C [Prop. 10.12]. But, (it is) also incommensurable (with C). The very thing (is) impossible. Thus, B is not commensurable with C. Thus, (it is) incommensurable.

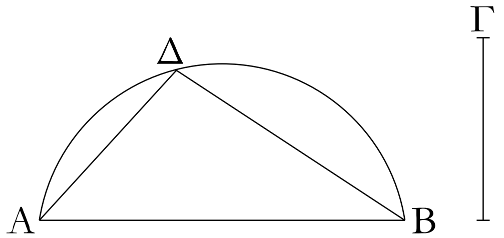
Thus, if two magnitudes are commensurable, and so on ....

Λῆμμα.

Δύο δοθεισῶν εὐθειῶν ἀνίσων εὑρεῖν, τίνι μεῖζον δύναται ἡ μείζων τῆς ἐλάσσονος.

Lemma

For two given unequal straight-lines, to find by (the square on) which (straight-line) the square on the greater



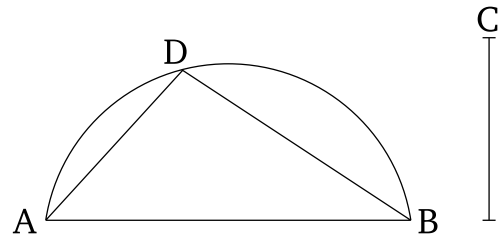
Ἔστωσαν αἱ δοθεῖσαι δύο ἄνισοι εὐθεῖαι αἱ ΑΒ, Γ, ὧν μείζων ἔστω ἡ ΑΒ· δεῖ δὴ εὑρεῖν, τίνι μεῖζον δύναται ἡ ΑΒ τῆς Γ.

Γεγράφθω ἐπὶ τῆς ΑΒ ἡμικύκλιον τὸ ΑΔΒ, καὶ εἰς αὐτὸ ἐνηρμόσθω τῇ Γ ἴση ἡ ΑΔ, καὶ ἐπεζεύχθω ἡ ΔΒ. φανερὸν δή, ὅτι ὀρθή ἐστιν ἡ ὑπὸ ΑΔΒ γωνία, καὶ ὅτι ἡ ΑΒ τῆς ΑΔ, τουτέστι τῆς Γ, μεῖζον δύναται τῇ ΔΒ.

Ὁμοίως δὲ καὶ δύο δοθεισῶν εὐθειῶν ἡ δυναμένη αὐτὰς εὑρίσκεται οὕτως.

Ἔστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι αἱ ΑΔ, ΔΒ, καὶ δέον ἔστω εὑρεῖν τὴν δυναμένην αὐτάς. κείσθωσαν γάρ, ὥστε ὀρθὴν γωνίαν περιέχειν τὴν ὑπὸ ΑΔ, ΔΒ, καὶ ἐπεζεύχθω ἡ ΑΒ· φανερὸν πάλιν, ὅτι ἡ τὰς ΑΔ, ΔΒ δυναμένη ἐστὶν ἡ ΑΒ· ὅπερ ἔδει δεῖξαι.

(straight-line is) larger than (the square on) the lesser.†



Let AB and C be the two given unequal straight-lines, and let AB be the greater of them. So it is required to find by (the square on) which (straight-line) the square on AB (is) greater than (the square on) C.

Let the semi-circle ADB have been described on AB. And let AD, equal to C, have been inserted into it [Prop. 4.1]. And let DB have been joined. So (it is) clear that the angle ADB is a right-angle [Prop. 3.31], and that the square on AB (is) greater than (the square on) AD—that is to say, (the square on) C—by (the square on) DB [Prop. 1.47].

And, similarly, the square-root of (the sum of the squares on) two given straight-lines is also found likeso.

Let AD and DB be the two given straight-lines. And let it be necessary to find the square-root of (the sum of the squares on) them. For let them have been laid down such as to encompass a right-angle—(namely), that (angle encompassed) by AD and DB. And let AB have been joined. (It is) again clear that AB is the square-root of (the sum of the squares on) AD and DB [Prop. 1.47]. (Which is) the very thing it was required to show.

† That is, if α and β are the lengths of two given straight-lines, with α being greater than β, to find a straight-line of length γ such that $\alpha^2 = \beta^2 + \gamma^2$. Similarly, we can also find γ such that $\gamma^2 = \alpha^2 + \beta^2$.

ιδʹ.

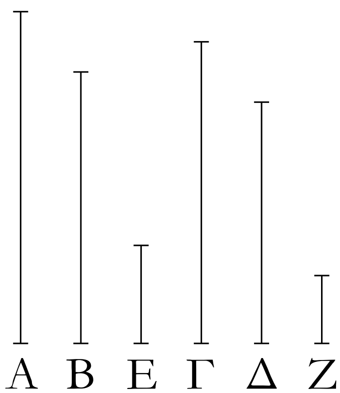
Ἐὰν τέσσαρες εὐθεῖαι ἀνάλογον ὦσιν, δύνηται δὲ ἡ πρώτη τῆς δευτέρας μεῖζον τῷ ἀπὸ συμμέτρου ἑαυτῇ [μήκει], καὶ ἡ τρίτη τῆς τετάρτης μεῖζον δυνήσεται τῷ ἀπὸ συμμέτρου ἑαυτῇ [μήκει]. καὶ ἐὰν ἡ πρώτη τῆς δευτέρας μεῖζον δύνηται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ [μήκει], καὶ ἡ τρίτη τῆς τετάρτης μεῖζον δυνήσεται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ [μήκει].

Ἔστωσαν τέσσαρες εὐθεῖαι ἀνάλογον αἱ Α, Β, Γ, Δ, ὡς ἡ Α πρὸς τὴν Β, οὕτως ἡ Γ πρὸς τὴν Δ, καὶ ἡ Α μὲν τῆς Β μεῖζον δυνάσθω τῷ ἀπὸ τῆς Ε, ἡ δὲ Γ τῆς Δ μεῖζον δυνάσθω τῷ ἀπὸ τῆς Ζ· λέγω, ὅτι, εἴτε σύμμετρός ἐστιν ἡ Α τῇ Ε, σύμμετρός ἐστι καὶ ἡ Γ τῇ Ζ, εἴτε ἀσύμμετρός ἐστιν ἡ Α τῇ Ε, ἀσύμμετρός ἐστι καὶ ὁ Γ τῇ Ζ.

Proposition 14

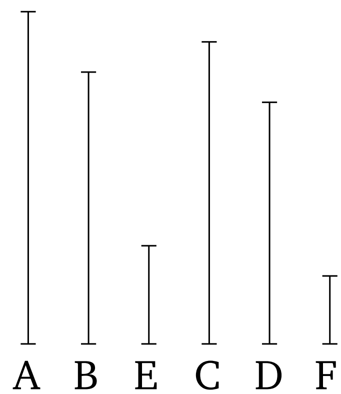
If four straight-lines are proportional, and the square on the first is greater than (the square on) the second by the (square) on (some straight-line) commensurable [in length] with the first, then the square on the third will also be greater than (the square on) the fourth by the (square) on (some straight-line) commensurable [in length] with the third. And if the square on the first is greater than (the square on) the second by the (square) on (some straight-line) incommensurable [in length] with the first, then the square on the third will also be greater than (the square on) the fourth by the (square) on (some straight-line) incommensurable [in length] with the third.

Let A, B, C, D be four proportional straight-lines, (such that) as A (is) to B, so C (is) to D. And let the square on A be greater than (the square on) B by the (square) on E, and let the square on C be greater than (the square on) D by the (square) on F. I say that A is either commensurable (in length) with E, and C is also commensurable with F, or A is incommensurable (in length) with E, and C is also incommensurable with F.



Ἐπεὶ γάρ ἐστιν ὡς ἡ Α πρὸς τὴν Β, οὕτως ἡ Γ πρὸς τὴν Δ, ἔστιν ἄρα καὶ ὡς τὸ ἀπὸ τῆς Α πρὸς τὸ ἀπὸ τῆς Β, οὕτως τὸ ἀπὸ τῆς Γ πρὸς τὸ ἀπὸ τῆς Δ. ἀλλὰ τῷ μὲν ἀπὸ τῆς Α ἴσα ἐστὶ τὰ ἀπὸ τῶν Ε, Β, τῷ δὲ ἀπὸ τῆς Γ ἴσα ἐστὶ τὰ ἀπὸ τῶν Δ, Ζ. ἔστιν ἄρα ὡς τὰ ἀπὸ τῶν Ε, Β πρὸς τὸ ἀπὸ τῆς Β, οὕτως τὰ ἀπὸ τῶν Δ, Ζ πρὸς τὸ ἀπὸ τῆς Δ· διελόντι ἄρα ἐστὶν ὡς τὸ ἀπὸ τῆς Ε πρὸς τὸ ἀπὸ τῆς Β, οὕτως τὸ ἀπὸ τῆς Ζ πρὸς τὸ ἀπὸ τῆς Δ· ἔστιν ἄρα καὶ ὡς ἡ Ε πρὸς τὴν Β, οὕτως ἡ Ζ πρὸς τὴν Δ· ἀνάπαλιν ἄρα ἐστὶν ὡς ἡ Β πρὸς τὴν Ε, οὕτως ἡ Δ πρὸς τὴν Ζ. ἔστι δὲ καὶ ὡς ἡ Α πρὸς τὴν Β, οὕτως ἡ Γ πρὸς τὴν Δ· δι᾿ ἴσου ἄρα ἐστὶν ὡς ἡ Α πρὸς τὴν Ε, οὕτως ἡ Γ πρὸς τὴν Ζ. εἴτε οὖν σύμμετρός ἐστιν ἡ Α τῇ Ε, συμμετρός ἐστι καὶ ἡ Γ τῇ Ζ, εἴτε ἀσύμμετρός ἐστιν ἡ Α τῇ Ε, ἀσύμμετρός ἐστι καὶ ἡ Γ τῇ Ζ.

Ἐὰν ἄρα, καὶ τὰ ἑξῆς.



For since as A is to B, so C (is) to D, thus as the (square) on A is to the (square) on B, so the (square) on C (is) to the (square) on D [Prop. 6.22]. But the (sum of the squares) on E and B is equal to the (square) on A, and the (sum of the squares) on D and F is equal to the (square) on C. Thus, as the (sum of the squares) on E and B is to the (square) on B, so the (sum of the squares) on D and F (is) to the (square) on D. Thus, via separation, as the (square) on E is to the (square) on B, so the (square) on F (is) to the (square) on D [Prop. 5.17]. Thus, also, as E is to B, so F (is) to D [Prop. 6.22]. Thus, inversely, as B is to E, so D (is) to F [Prop. 5.7 corr.]. But, as A is to B, so C also (is) to D. Thus, via equality, as A is to E, so C (is) to F [Prop. 5.22]. Therefore, A is either commensurable (in length) with E, and C is also commensurable with F, or A is incommensurable (in length) with E, and C is also incommensurable with F [Prop. 10.11].

Thus, if, and so on ....

ιεʹ.

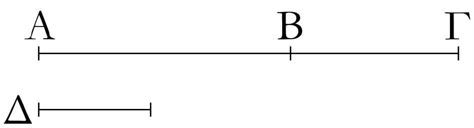
Ἐὰν δύο μεγέθη σύμμετρα συντεθῇ, καὶ τὸ ὅλον ἑκατέρῳ αὐτῶν σύμμετρον ἔσται· κἂν τὸ ὅλον ἑνὶ αὐτῶν σύμμετρον ᾖ, καὶ τὰ ἐξ ἀρχῆς μεγέθη σύμμετρα ἔσται.

Συγκείσθω γὰρ δύο μεγέθη σύμμετρα τὰ ΑΒ, ΒΓ· λέγω, ὅτι καὶ ὅλον τὸ ΑΓ ἑκατέρῳ τῶν ΑΒ, ΒΓ ἐστι σύμμετρον.

Proposition 15

If two commensurable magnitudes are added together then the whole will also be commensurable with each of them. And if the whole is commensurable with one of them then the original magnitudes will also be commensurable (with one another).

For let the two commensurable magnitudes AB and BC be laid down together. I say that the whole AC is also commensurable with each of AB and BC.

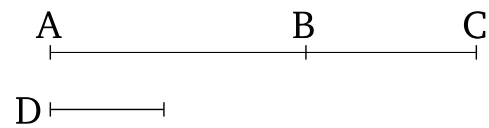


Ἐπεὶ γὰρ σύμμετρά ἐστι τὰ ΑΒ, ΒΓ, μετρήσει τι αὐτὰ μέγεθος. μετρείτω, καὶ ἔστω τὸ Δ. ἐπεὶ οὖν τὸ Δ τὰ ΑΒ, ΒΓ μετρεῖ, καὶ ὅλον τὸ ΑΓ μετρήσει. μετρεῖ δὲ καὶ τὰ ΑΒ, ΒΓ. τὸ Δ ἄρα τὰ ΑΒ, ΒΓ, ΑΓ μετρεῖ· σύμμετρον ἄρα ἐστὶ τὸ ΑΓ ἑκατέρῳ τῶν ΑΒ, ΒΓ.

Ἀλλὰ δὴ τὸ ΑΓ ἔστω σύμμετρον τῷ ΑΒ· λέγω δή, ὅτι καὶ τὰ ΑΒ, ΒΓ σύμμετρά ἐστιν.

Ἐπεὶ γὰρ σύμμετρά ἐστι τὰ ΑΓ, ΑΒ, μετρήσει τι αὐτὰ μέγεθος. μετρείτω, καὶ ἔστω τὸ Δ. ἐπεὶ οὖν τὸ Δ τὰ ΓΑ, ΑΒ μετρεῖ, καὶ λοιπὸν ἄρα τὸ ΒΓ μετρήσει. μετρεῖ δὲ καὶ τὸ ΑΒ· τὸ Δ ἄρα τὰ ΑΒ, ΒΓ μετρήσει· σύμμετρα ἄρα ἐστὶ τὰ ΑΒ, ΒΓ.

Ἐὰν ἄρα δύο μεγέθη, καὶ τὰ ἑξῆς.



For since AB and BC are commensurable, some magnitude will measure them. Let it (so) measure (them), and let it be D. Therefore, since D measures (both) AB and BC, it will also measure the whole AC. And it also measures AB and BC. Thus, D measures AB, BC, and AC. Thus, AC is commensurable with each of AB and BC [Def. 10.1].

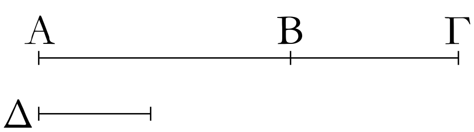
And so let AC be commensurable with AB. I say that AB and BC are also commensurable.

For since AC and AB are commensurable, some magnitude will measure them. Let it (so) measure (them), and let it be D. Therefore, since D measures (both) CA and AB, it will thus also measure the remainder BC. And it also measures AB. Thus, D will measure (both) AB and BC. Thus, AB and BC are commensurable [Def. 10.1].

Thus, if two magnitudes, and so on ....

ις´.

Ἐὰν δύο μεγέθη ἀσύμμετρα συντεθῇ, καὶ τὸ ὅλον ἑκατέρῳ αὐτῶν ἀσύμμετρον ἔσται· κἂν τὸ ὅλον ἑνὶ αὐτῶν ἀσύμμετρον ᾖ, καὶ τὰ ἐξ ἀρχῆς μεγέθη ἀσύμμετρα ἔσται.



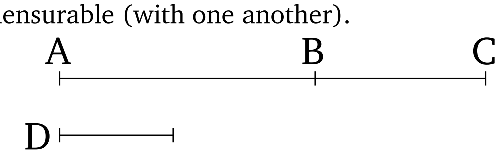
Συγκείσθω γὰρ δύο μεγέθη ἀσύμμετρα τὰ ΑΒ, ΒΓ· λέγω, ὅτι καὶ ὅλον τὸ ΑΓ ἑκατέρῳ τῶν ΑΒ, ΒΓ ἀσύμμετρόν ἐστιν.

Εἰ γὰρ μή ἐστιν ἀσύμμετρα τὰ ΓΑ, ΑΒ, μετρήσει τι [αὐτὰ] μέγεθος. μετρείτω, εἰ δυνατόν, καὶ ἔστω τὸ Δ. ἐπεὶ οὖν τὸ Δ τὰ ΓΑ, ΑΒ μετρεῖ, καὶ λοιπὸν ἄρα τὸ ΒΓ μετρήσει. μετρεῖ δὲ καὶ τὸ ΑΒ· τὸ Δ ἄρα τὰ ΑΒ, ΒΓ μετρεῖ. σύμμετρα ἄρα ἐστὶ τὰ ΑΒ, ΒΓ· ὑπέκειντο δὲ καὶ ἀσύμμετρα· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα τὰ ΓΑ, ΑΒ μετρήσει τι μέγεθος· ἀσύμμετρα ἄρα ἐστὶ τὰ ΓΑ, ΑΒ. ὁμοίως δὴ δείξομεν, ὅτι καὶ τὰ ΑΓ, ΓΒ ἀσύμμετρά ἐστιν. τὸ ΑΓ ἄρα ἑκατέρῳ τῶν ΑΒ, ΒΓ ἀσύμμετρόν ἐστιν.

Ἀλλὰ δὴ τὸ ΑΓ ἑνὶ τῶν ΑΒ, ΒΓ ἀσύμμετρον ἔστω. ἔστω δὴ πρότερον τῷ ΑΒ· λέγω, ὅτι καὶ τὰ ΑΒ, ΒΓ ἀσύμμετρά ἐστιν. εἰ γὰρ ἔσται σύμμετρα, μετρήσει τι αὐτὰ μέγεθος. μετρείτω, καὶ ἔστω τὸ Δ. ἐπεὶ οὖν τὸ Δ τὰ ΑΒ, ΒΓ μετρεῖ, καὶ ὅλον ἄρα τὸ ΑΓ μετρήσει. μετρεῖ δὲ καὶ τὸ ΑΒ· τὸ Δ ἄρα τὰ ΓΑ, ΑΒ μετρεῖ. σύμμετρα ἄρα ἐστὶ τὰ

Proposition 16

If two incommensurable magnitudes are added together then the whole will also be incommensurable with each of them. And if the whole is incommensurable with one of them then the original magnitudes will also be incommensurable (with one another).



For let the two incommensurable magnitudes AB and BC be laid down together. I say that that the whole AC is also incommensurable with each of AB and BC.

For if CA and AB are not incommensurable then some magnitude will measure [them]. If possible, let it (so) measure (them), and let it be D. Therefore, since D measures (both) CA and AB, it will thus also measure the remainder BC. And it also measures AB. Thus, D measures (both) AB and BC. Thus, AB and BC are commensurable [Def. 10.1]. But they were also assumed (to be) incommensurable. The very thing is impossible. Thus, some magnitude cannot measure (both) CA and AB. Thus, CA and AB are incommensurable [Def. 10.1]. So, similarly, we can show that AC and CB are also incommensurable. Thus, AC is incommensurable with each of AB and BC.

And so let AC be incommensurable with one of AB and BC. So let it, first of all, be incommensurable with

ΓΑ, ΑΒ· ὑπέκειτο δὲ καὶ ἀσύμμετρα· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα τὰ ΑΒ, ΒΓ μετρήσει τι μέγεθος· ἀσύμμετρα ἄρα ἐστὶ τὰ ΑΒ, ΒΓ.

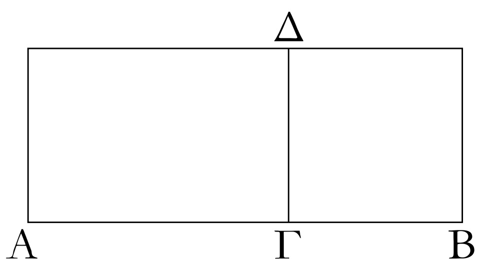
Ἐὰν ἄρα δύο μεγέθη, καὶ τὰ ἑξῆς.

AB. I say that AB and BC are also incommensurable. For if they are commensurable then some magnitude will measure them. Let it (so) measure (them), and let it be D. Therefore, since D measures (both) AB and BC, it will thus also measure the whole AC. And it also measures AB. Thus, D measures (both) CA and AB. Thus, CA and AB are commensurable [Def. 10.1]. But they were also assumed (to be) incommensurable. The very thing is impossible. Thus, some magnitude cannot measure (both) AB and BC. Thus, AB and BC are incommensurable [Def. 10.1].

Thus, if two... magnitudes, and so on ....

Λῆμμα.

Ἐὰν παρά τινα εὐθεῖαν παραβληθῇ παραλληλόγραμμον ἐλλεῖπον εἴδει τετραγώνῳ, τὸ παραβληθὲν ἴσον ἐστὶ τῷ ὑπὸ τῶν ἐκ τῆς παραβολῆς γενομένων τμημάτων τῆς εὐθείας.



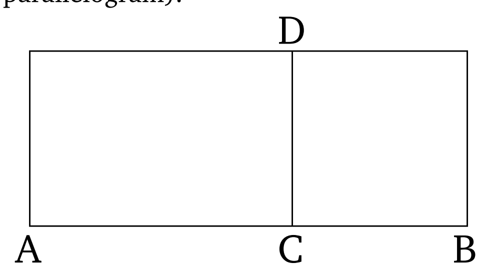
Παρὰ γὰρ εὐθεῖαν τὴν ΑΒ παραβεβλήσθω παραλληλόγραμμον τὸ ΑΔ ἐλλεῖπον εἴδει τετραγώνῳ τῷ ΔΒ· λέγω, ὅτι ἴσον ἐστὶ τὸ ΑΔ τῷ ὑπὸ τῶν ΑΓ, ΓΒ.

Καί ἐστιν αὐτόθεν φανερόν· ἐπεὶ γὰρ τετράγωνόν ἐστι τὸ ΔΒ, ἴση ἐστὶν ἡ ΔΓ τῇ ΓΒ, καί ἐστι τὸ ΑΔ τὸ ὑπὸ τῶν ΑΓ, ΓΔ, τουτέστι τὸ ὑπὸ τῶν ΑΓ, ΓΒ.

Ἐὰν ἄρα παρά τινα εὐθεῖαν, καὶ τὰ ἑξῆς.

Lemma

If a parallelogram,† falling short by a square figure, is applied to some straight-line then the applied (parallelogram) is equal (in area) to the (rectangle contained) by the pieces of the straight-line created via the application (of the parallelogram).



For let the parallelogram AD, falling short by the square figure DB, have been applied to the straight-line AB. I say that AD is equal to the (rectangle contained) by AC and CB.

And it is immediately obvious. For since DB is a square, DC is equal to CB. And AD is the (rectangle contained) by AC and CD—that is to say, by AC and CB.

Thus, if ... to some straight-line, and so on ....

† Note that this lemma only applies to rectangular parallelograms.

ιζʹ.

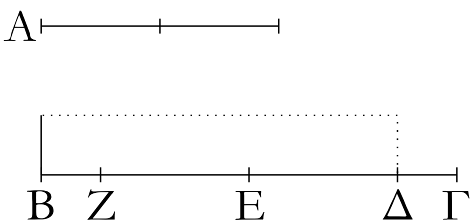
Ἐὰν ὦσι δύο εὐθεῖαι ἄνισοι, τῷ δὲ τετράτῳ μέρει τοῦ ἀπὸ τῆς ἐλάσσονος ἴσον παρὰ τὴν μείζονα παραβληθῇ ἐλλεῖπον εἴδει τετραγώνῳ καὶ εἰς σύμμετρα αὐτὴν διαιρῇ μήκει, ἡ μείζων τῆς ἐλάσσονος μεῖζον δυνήσεται τῷ ἀπὸ συμμέτου ἑαυτῇ [μήκει]. καὶ ἐὰν ἡ μείζων τῆς ἐλάσσονος μεῖζον δύνηται τῷ ἀπὸ συμμέτρου ἑαυτῇ [μήκει], τῷ δὲ τετράρτῳ τοῦ ἀπὸ τῆς ἐλάσσονος ἴσον παρὰ τὴν μείζονα παραβληθῇ ἐλλεῖπον εἴδει τετραγώνῳ, εἰς σύμμετρα αὐτὴν διαιρεῖ μήκει.

Ἔστωσαν δύο εὐθεῖαι ἄνισοι αἱ Α, ΒΓ, ὧν μείζων ἡ

Proposition 17†

If there are two unequal straight-lines, and a (rectangle) equal to the fourth part of the (square) on the lesser, falling short by a square figure, is applied to the greater, and divides it into (parts which are) commensurable in length, then the square on the greater will be larger than (the square on) the lesser by the (square) on (some straight-line) commensurable [in length] with the greater. And if the square on the greater is larger than (the square on) the lesser by the (square) on (some straight-line) commensurable [in length] with the

ΒΓ, τῷ δὲ τετράρτῳ μέρει τοῦ ἀπὸ ἐλάσσονος τῆς Α, τουτέστι τῷ ἀπὸ τῆς ἡμισείας τῆς Α, ἴσον παρὰ τὴν ΒΓ παραβεβλήσθω ἐλλεῖπον εἴδει τετραγώνῳ, καὶ ἔστω τὸ ὑπὸ τῶν ΒΔ, ΔΓ, σύμμετρος δὲ ἔστω ἡ ΒΔ τῇ ΔΓ μήκει· λέγω, ὅτι ἡ ΒΓ τῆς Α μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ.



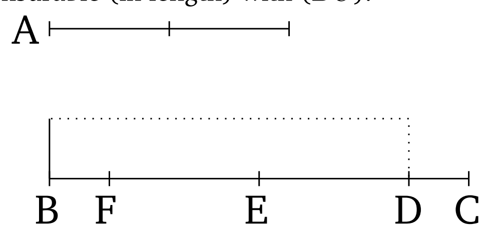
Τετμήσθω γὰρ ἡ ΒΓ δίχα κατὰ τὸ Ε σημεῖον, καὶ κείσθω τῇ ΔΕ ἴση ἡ ΕΖ. λοιπὴ ἄρα ἡ ΔΓ ἴση ἐστὶ τῇ ΒΖ. καὶ ἐπεὶ εὐθεῖα ἡ ΒΓ τέτμηται εἰς μὲν ἴσα κατὰ τὸ Ε, εἰς δὲ ἄνισα κατὰ τὸ Δ, τὸ ἄρα ὑπὸ ΒΔ, ΔΓ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΕΔ τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τῆς ΕΓ τετραγώνῳ· καὶ τὰ τετραπλάσια· τὸ ἄρα τετράκις ὑπὸ τῶν ΒΔ, ΔΓ μετὰ τοῦ τετραπλασίου τοῦ ἀπὸ τῆς ΔΕ ἴσον ἐστὶ τῷ τετράκις ἀπὸ τῆς ΕΓ τετραγώνῳ. ἀλλὰ τῷ μὲν τετραπλασίῳ τοῦ ὑπὸ τῶν ΒΔ, ΔΓ ἴσον ἐστὶ τὸ ἀπὸ τῆς Α τετράγωνον, τῷ δὲ τετραπλασίῳ τοῦ ἀπὸ τῆς ΔΕ ἴσον ἐστὶ τὸ ἀπὸ τῆς ΔΖ τετράγωνον· διπλασίων γάρ ἐστιν ἡ ΔΖ τῆς ΔΕ. τῷ δὲ τετραπλασίῳ τοῦ ἀπὸ τῆς ΕΓ ἴσον ἐστὶ τὸ ἀπὸ τῆς ΒΓ τετράγωνον· διπλασίων γάρ ἐστι πάλιν ἡ ΒΓ τῆς ΓΕ. τὰ ἄρα ἀπὸ τῶν Α, ΔΖ τετράγωνα ἴσα ἐστὶ τῷ ἀπὸ τῆς ΒΓ τετράγωνῳ· ὥστε τὸ ἀπὸ τῆς ΒΓ τοῦ ἀπὸ τῆς Α μεῖζόν ἐστι τῷ ἀπὸ τῆς ΔΖ· ἡ ΒΓ ἄρα τῆς Α μεῖζον δύναται τῇ ΔΖ. δεικτέον, ὅτι καὶ σύμμετρός ἐστιν ἡ ΒΓ τῇ ΔΖ. ἐπεὶ γὰρ σύμμετρός ἐστιν ἡ ΒΔ τῇ ΔΓ μήκει, σύμμετρος ἄρα ἐστὶ καὶ ἡ ΒΓ τῇ ΓΔ μήκει. ἀλλὰ ἡ ΓΔ ταῖς ΓΔ, ΒΖ ἐστι σύμμετρος μήκει· ἴση γάρ ἐστιν ἡ ΓΔ τῇ ΒΖ. καὶ ἡ ΒΓ ἄρα σύμμετρός ἐστι ταῖς ΒΖ, ΓΔ μήκει· ὥστε καὶ λοιπῇ τῇ ΖΔ σύμμετρός ἐστιν ἡ ΒΓ μήκει· ἡ ΒΓ ἄρα τῆς Α μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ.

Ἀλλὰ δὴ ἡ ΒΓ τῆς Α μεῖζον δυνάσθω τῷ ἀπὸ συμμέτρου ἑαυτῇ, τῷ δὲ τετράτρῳ τοῦ ἀπὸ τῆς Α ἴσον παρὰ τὴν ΒΓ παραβεβλήσθω ἐλλεῖπον εἴδει τετραγώνῳ, καὶ ἔστω τὸ ὑπὸ τῶν ΒΔ, ΔΓ. δεικτέον, ὅτι σύμμετρός ἐστιν ἡ ΒΔ τῇ ΔΓ μήκει.

Τῶν γὰρ αὐτῶν κατασκευασθέντων ὁμοίως δείξομεν, ὅτι ἡ ΒΓ τῆς Α μεῖζον δύναται τῷ ἀπὸ τῆς ΖΔ. δύναται δὲ ἡ

greater, and a (rectangle) equal to the fourth (part) of the (square) on the lesser, falling short by a square figure, is applied to the greater, then it divides it into (parts which are) commensurable in length.

Let A and BC be two unequal straight-lines, of which (let) BC (be) the greater. And let a (rectangle) equal to the fourth part of the (square) on the lesser, A—that is, (equal) to the (square) on half of A—falling short by a square figure, have been applied to BC. And let it be the (rectangle contained) by BD and DC [see previous lemma]. And let BD be commensurable in length with DC. I say that that the square on BC is greater than the (square on) A by (the square on some straight-line) commensurable (in length) with (BC).



For let BC have been cut in half at the point E [Prop. 1.10]. And let EF be made equal to DE [Prop. 1.3]. Thus, the remainder DC is equal to BF. And since the straight-line BC has been cut into equal (pieces) at E, and into unequal (pieces) at D, the rectangle contained by BD and DC, plus the square on ED, is thus equal to the square on EC [Prop. 2.5]. (The same) also (for) the quadruples. Thus, four times the (rectangle contained) by BD and DC, plus the quadruple of the (square) on DE, is equal to four times the square on EC. But, the square on A is equal to the quadruple of the (rectangle contained) by BD and DC, and the square on DF is equal to the quadruple of the (square) on DE. For DF is double DE. And the square on BC is equal to the quadruple of the (square) on EC. For, again, BC is double CE. Thus, the (sum of the) squares on A and DF is equal to the square on BC. Hence, the (square) on BC is greater than the (square) on A by the (square) on DF. Thus, BC is greater in square than A by DF. It must also be shown that BC is commensurable (in length) with DF. For since BD is commensurable in length with DC, BC is thus also commensurable in length with CD [Prop. 10.15]. But, CD is commensurable in length with CD plus BF. For CD is equal to BF [Prop. 10.6]. Thus, BC is also commensurable in length with BF plus CD [Prop. 10.12]. Hence, BC is also commensurable in length with the remainder FD [Prop. 10.15]. Thus, the square on BC is greater than (the square on) A by the (square) on (some straight-line) commensurable (in length) with (BC).

ΒΓ τῆς Α μεῖζον τῷ ἀπὸ συμμέτρου ἑαυτῇ. σύμμετρος ἄρα ἐστὶν ἡ ΒΓ τῇ ΖΔ μήκει· ὥστε καὶ λοιπῇ συναμφοτέρῳ τῇ ΒΖ, ΔΓ σύμμετρός ἐστιν ἡ ΒΓ μήκει. ἀλλὰ συναμφότερος ἡ ΒΖ, ΔΓ σύμμετρός ἐστι τῇ ΔΓ [μήκει]. ὥστε καὶ ἡ ΒΓ τῇ ΓΔ σύμμετρός ἐστι μήκει· καὶ διελόντι ἄρα ἡ ΒΔ τῇ ΔΓ ἐστι σύμμετρος μήκει.

Ἐὰν ἄρα ὦσι δύο εὐθεῖαι ἄνισοι, καὶ τὰ ἑξῆς.

And so let the square on BC be greater than the (square on) A by the (square) on (some straight-line) commensurable (in length) with (BC). And let a (rectangle) equal to the fourth (part) of the (square) on A, falling short by a square figure, have been applied to BC. And let it be the (rectangle contained) by BD and DC. It must be shown that BD is commensurable in length with DC.

For, similarly, by the same construction, we can show that the square on BC is greater than the (square on) A by the (square) on FD. And the square on BC is greater than the (square on) A by the (square) on (some straight-line) commensurable (in length) with (BC). Thus, BC is commensurable in length with FD. Hence, BC is also commensurable in length with the remaining sum of BF and DC [Prop. 10.15]. But, the sum of BF and DC is commensurable [in length] with DC [Prop. 10.6]. Hence, BC is also commensurable in length with CD [Prop. 10.12]. Thus, via separation, BD is also commensurable in length with DC [Prop. 10.15].

Thus, if there are two unequal straight-lines, and so on ....

† This proposition states that if $\alpha\,x - x^2 = \beta^2/4$ (where $\alpha = BC$, $x = DC$, and $\beta = A$) then α and $\sqrt{\alpha^2 - \beta^2}$ are commensurable when $\alpha - x$ are x are commensurable, and *vice versa*.

ιηʹ.

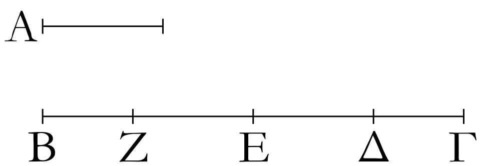
Ἐὰν ὦσι δύο εὐθεῖαι ἄνισοι, τῷ δὲ τετάρτῳ μέρει τοῦ ἀπὸ τῆς ἐλάσσονος ἴσον παρὰ τὴν μείζονα παραβληθῇ ἐλλεῖπον εἴδει τετραγώνῳ, καὶ εἰς ἀσύμμετρα αὐτὴν διαιρῇ [μήκει], ἡ μείζων τῆς ἐλάσσονος μεῖζον δυνήσεται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. καὶ ἐὰν ἡ μείζων τῆς ἐλάσσονος μεῖζον δύνηται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ, τῷ δὲ τετάρτῳ τοῦ ἀπὸ τῆς ἐλάσσονος ἴσον παρὰ τὴν μείζονα παραβληθῇ ἐλλεῖπον εἴδει τετραγώνῳ, εἰς ἀσύμμετρα αὐτὴν διαιρεῖ [μήκει].

Ἔστωσαν δύο εὐθεῖαι ἄνισοι αἱ Α, ΒΓ, ὧν μείζων ἡ ΒΓ, τῷ δὲ τετάρτῳ [μέρει] τοῦ ἀπὸ τῆς ἐλάσσονος τῆς Α ἴσον παρὰ τὴν ΒΓ παραβεβλήσθω ἐλλεῖπον εἴδει τετραγώνῳ, καὶ ἔστω τὸ ὑπὸ τῶν ΒΔΓ, ἀσύμμετρος δὲ ἔστω ἡ ΒΔ τῇ ΔΓ μήκει· λέγω, ὅτι ἡ ΒΓ τῆς Α μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ.

Proposition 18†

If there are two unequal straight-lines, and a (rectangle) equal to the fourth part of the (square) on the lesser, falling short by a square figure, is applied to the greater, and divides it into (parts which are) incommensurable [in length], then the square on the greater will be larger than the (square on the) lesser by the (square) on (some straight-line) incommensurable (in length) with the greater. And if the square on the greater is larger than the (square on the) lesser by the (square) on (some straight-line) incommensurable (in length) with the greater, and a (rectangle) equal to the fourth (part) of the (square) on the lesser, falling short by a square figure, is applied to the greater, then it divides it into (parts which are) incommensurable [in length].

Let A and BC be two unequal straight-lines, of which (let) BC (be) the greater. And let a (rectangle) equal to the fourth [part] of the (square) on the lesser, A, falling short by a square figure, have been applied to BC. And let it be the (rectangle contained) by BDC. And let BD be incommensurable in length with DC. I say that that the square on BC is greater than the (square on) A by the (square) on (some straight-line) incommensurable (in length) with (BC).

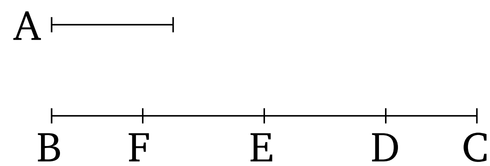


Τῶν γὰρ αὐτῶν κατασκευασθέντων τῷ πρότερον ὁμοίως δείξομεν, ὅτι ἡ ΒΓ τῆς Α μεῖζον δύναται τῷ ἀπὸ τῆς ΖΔ. δεικτέον [οὖν], ὅτι ἀσύμμετρός ἐστιν ἡ ΒΓ τῇ ΔΖ μήκει. ἐπεὶ γὰρ ἀσύμμετρός ἐστιν ἡ ΒΔ τῇ ΔΓ μήκει, ἀσύμμετρος ἄρα ἐστὶ καὶ ἡ ΒΓ τῇ ΓΔ μήκει. ἀλλὰ ἡ ΔΓ σύμμετρός ἐστι συναμφοτέραις ταῖς ΒΖ, ΔΓ· καὶ ἡ ΒΓ ἄρα ἀσύμμετρός ἐστι συναμφοτέραις ταῖς ΒΖ, ΔΓ. ὥστε καὶ λοιπῇ τῇ ΖΔ ἀσύμμετρός ἔστιν ἡ ΒΓ μήκει. καὶ ἡ ΒΓ τῆς Α μεῖζον δύναται τῷ ἀπὸ τῆς ΖΔ· ἡ ΒΓ ἄρα τῆς Α μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ.

Δυνάσθω δὴ πάλιν ἡ ΒΓ τῆς Α μεῖζον τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ, τῷ δὲ τετάρτῳ τοῦ ἀπὸ τῆς Α ἴσον παρὰ τὴν ΒΓ παραβεβλήσθω ἐλλεῖπον εἴδει τετραγώνῳ, καὶ ἔστω τὸ ὑπὸ τῶν ΒΔ, ΔΓ. δεικτέον, ὅτι ἀσύμμετρός ἐστιν ἡ ΒΔ τῇ ΔΓ μήκει.

Τῶν γὰρ αὐτῶν κατασκευασθέντων ὁμοίως δείξομεν, ὅτι ἡ ΒΓ τῆς Α μεῖζον δύναται τῷ ἀπὸ τῆς ΖΔ. ἀλλὰ ἡ ΒΓ τῆς Α μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. ἀσύμμετρος ἄρα ἐστὶν ἡ ΒΓ τῇ ΖΔ μήκει· ὥστε καὶ λοιπῇ συναμφοτέρῳ τῇ ΒΖ, ΔΓ ἀσύμμετρός ἐστιν ἡ ΒΓ. ἀλλὰ συναμφότερος ἡ ΒΖ, ΔΓ τῇ ΔΓ σύμμετρός ἐστι μήκει· καὶ ἡ ΒΓ ἄρα τῇ ΔΓ ἀσύμμετρός ἐστι μήκει· ὥστε καὶ διελόντι ἡ ΒΔ τῇ ΔΓ ἀσύμμετρός ἐστι μήκει.

Ἐὰν ἄρα ὦσι δύο εὐθεῖαι, καὶ τὰ ἑξῆς.



For, similarly, by the same construction as before, we can show that the square on BC is greater than the (square on) A by the (square) on FD. [Therefore] it must be shown that BC is incommensurable in length with DF. For since BD is incommensurable in length with DC, BC is thus also incommensurable in length with CD [Prop. 10.16]. But, DC is commensurable (in length) with the sum of BF and DC [Prop. 10.6]. And, thus, BC is incommensurable (in length) with the sum of BF and DC [Prop. 10.13]. Hence, BC is also incommensurable in length with the remainder FD [Prop. 10.16]. And the square on BC is greater than the (square on) A by the (square) on FD. Thus, the square on BC is greater than the (square on) A by the (square) on (some straight-line) incommensurable (in length) with (BC).

So, again, let the square on BC be greater than the (square on) A by the (square) on (some straight-line) incommensurable (in length) with (BC). And let a (rectangle) equal to the fourth [part] of the (square) on A, falling short by a square figure, have been applied to BC. And let it be the (rectangle contained) by BD and DC. It must be shown that BD is incommensurable in length with DC.

For, similarly, by the same construction, we can show that the square on BC is greater than the (square) on A by the (square) on FD. But, the square on BC is greater than the (square) on A by the (square) on (some straight-line) incommensurable (in length) with (BC). Thus, BC is incommensurable in length with FD. Hence, BC is also incommensurable (in length) with the remaining sum of BF and DC [Prop. 10.16]. But, the sum of BF and DC is commensurable in length with DC [Prop. 10.6]. Thus, BC is also incommensurable in length with DC [Prop. 10.13]. Hence, via separation, BD is also incommensurable in length with DC [Prop. 10.16].

Thus, if there are two . . . straight-lines, and so on . . . .

† This proposition states that if $\alpha\,x - x^2 = \beta^2/4$ (where $\alpha = BC$, $x = DC$, and $\beta = A$) then α and $\sqrt{\alpha^2 - \beta^2}$ are incommensurable when $\alpha - x$ are x are incommensurable, and *vice versa*.

ιθʹ.

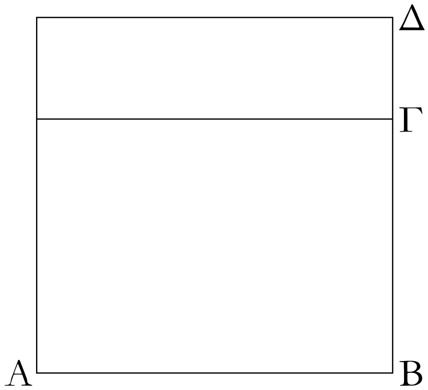
Τὸ ὑπὸ ῥητῶν μήκει συμμέτρων εὐθειῶν περιεχόμενον ὀρθογώνιον ῥητόν ἐστιν.

Ὑπὸ γὰρ ῥητῶν μήκει συμμέτρων εὐθειῶν τῶν ΑΒ, ΒΓ

Proposition 19

The rectangle contained by rational straight-lines (which are) commensurable in length is rational.

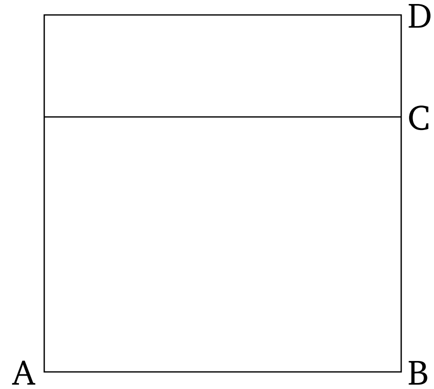
For let the rectangle AC have been enclosed by the ὀρθογώνιον περιεχέσθω τὸ ΑΓ· λέγω, ὅτι ῥητόν ἐστι τὸ ΑΓ.



Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΑΒ τετράγωνον τὸ ΑΔ· ῥητὸν ἄρα ἐστὶ τὸ ΑΔ. καὶ ἐπεὶ σύμμετρός ἐστιν ἡ ΑΒ τῇ ΒΓ μήκει, ἴση δέ ἐστιν ἡ ΑΒ τῇ ΒΔ, σύμμετρος ἄρα ἐστὶν ἡ ΒΔ τῇ ΒΓ μήκει. καί ἐστιν ὡς ἡ ΒΔ πρὸς τὴν ΒΓ, οὕτως τὸ ΔΑ πρὸς τὸ ΑΓ. σύμμετρον ἄρα ἐστὶ τὸ ΔΑ τῷ ΑΓ. ῥητὸν δὲ τὸ ΔΑ· ῥητὸν ἄρα ἐστὶ καὶ τὸ ΑΓ.

Τὸ ἄρα ὑπὸ ῥητῶν μήκει συμμέτρων, καὶ τὰ ἑξῆς.

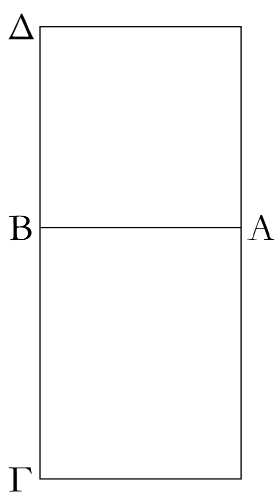
rational straight-lines AB and BC (which are) commensurable in length. I say that AC is rational.



For let the square AD have been described on AB. AD is thus rational [Def. 10.4]. And since AB is commensurable in length with BC, and AB is equal to BD, BD is thus commensurable in length with BC. And as BD is to BC, so DA (is) to AC [Prop. 6.1]. Thus, DA is commensurable with AC [Prop. 10.11]. And DA (is) rational. Thus, AC is also rational [Def. 10.4]. Thus, the . . . by rational straight-lines . . . commensurable, and so on . . . .

κ΄.

Ἐὰν ῥητὸν παρὰ ῥητὴν παραβληθῇ, πλάτος ποιεῖ ῥητὴν καὶ σύμμετρον τῇ, παρ᾽ ἣν παράκειται, μήκει.

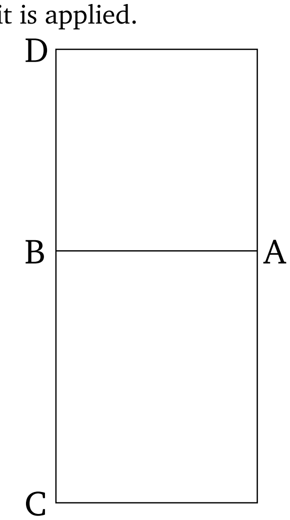


Ῥητὸν γὰρ τὸ ΑΓ παρὰ ῥητὴν τὴν ΑΒ παραβεβλήσθω πλάτος ποιοῦν τὴν ΒΓ· λέγω, ὅτι ῥητή ἐστιν ἡ ΒΓ καὶ σύμμετρος τῇ ΒΑ μήκει.

Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΑΒ τετράγωνον τὸ ΑΔ· ῥητὸν ἄρα ἐστὶ τὸ ΑΔ. ῥητὸν δὲ καὶ τὸ ΑΓ· σύμμετρον ἄρα

Proposition 20

If a rational (area) is applied to a rational (straight-line) then it produces as breadth a (straight-line which is) rational, and commensurable in length with the (straight-line) to which it is applied.



For let the rational (area) AC have been applied to the rational (straight-line) AB, producing the (straight-line) BC as breadth. I say that BC is rational, and commensurable in length with BA.

For let the square AD have been described on AB.

ἐστὶ τὸ ΔΑ τῷ ΑΓ. καί ἐστιν ὡς τὸ ΔΑ πρὸς τὸ ΑΓ, οὕτως ἡ ΔΒ πρὸς τὴν ΒΓ. σύμμετρος ἄρα ἐστὶ καὶ ἡ ΔΒ τῇ ΒΓ· ἴση δὲ ἡ ΔΒ τῇ ΒΑ· σύμμετρος ἄρα καὶ ἡ ΑΒ τῇ ΒΓ. ῥητὴ δέ ἐστιν ἡ ΑΒ· ῥητὴ ἄρα ἐστὶ καὶ ἡ ΒΓ καὶ σύμμετρος τῇ ΑΒ μήκει.

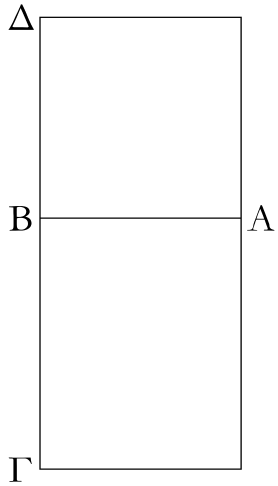
Ἐὰν ἄρα ῥητὸν παρὰ ῥητὴν παραβληθῇ, καὶ τὰ ἑξῆς.

AD is thus rational [Def. 10.4]. And AC (is) also rational. DA is thus commensurable with AC. And as DA is to AC, so DB (is) to BC [Prop. 6.1]. Thus, DB is also commensurable (in length) with BC [Prop. 10.11]. And DB (is) equal to BA. Thus, AB (is) also commensurable (in length) with BC. And AB is rational. Thus, BC is also rational, and commensurable in length with AB [Def. 10.3].

Thus, if a rational (area) is applied to a rational (straight-line), and so on ....

καʹ.

Τὸ ὑπὸ ῥητῶν δυνάμει μόνον συμμέτρων εὐθειῶν περιεχόμενον ὀρθογώνιον ἄλογόν ἐστιν, καὶ ἡ δυναμένη αὐτὸ ἄλογός ἐστιν, καλείσθω δὲ μέση.

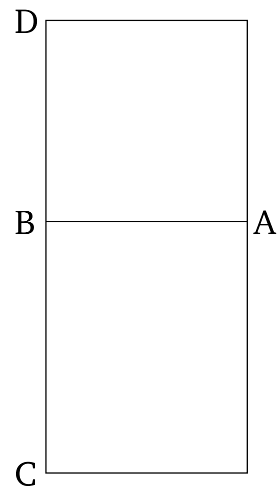


Ὑπὸ γὰρ ῥητῶν δυνάμει μόνον συμμέτρων εὐθειῶν τῶν ΑΒ, ΒΓ ὀρθογώνιον περιεχέσθω τὸ ΑΓ· λέγω, ὅτι ἄλογόν ἐστι τὸ ΑΓ, καὶ ἡ δυναμένη αὐτὸ ἄλογός ἐστιν, καλείσθω δὲ μέση.

Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΑΒ τετράγωνον τὸ ΑΔ· ῥητὸν ἄρα ἐστὶ τὸ ΑΔ. καὶ ἐπεὶ ἀσύμμετρός ἐστιν ἡ ΑΒ τῇ ΒΓ μήκει· δυνάμει γὰρ μόνον ὑπόκεινται σύμμετροι· ἴση δὲ ἡ ΑΒ τῇ ΒΔ, ἀσύμμετρος ἄρα ἐστὶ καὶ ἡ ΔΒ τῇ ΒΓ μήκει. καί ἐστιν ὡς ἡ ΔΒ πρὸς τὴν ΒΓ, οὕτως τὸ ΑΔ πρὸς τὸ ΑΓ· ἀσύμμετρον ἄρα [ἐστὶ] τὸ ΔΑ τῷ ΑΓ. ῥητὸν δὲ τὸ ΔΑ· ἄλογον ἄρα ἐστὶ τὸ ΑΓ· ὥστε καὶ ἡ δυναμένη τὸ ΑΓ [τουτέστιν ἡ ἴσον αὐτῷ τετράγωνον δυναμένη] ἄλογός ἐστιν, καλείσθω δε μέση· ὅπερ ἔδει δεῖξαι.

Proposition 21

The rectangle contained by rational straight-lines (which are) commensurable in square only is irrational, and its square-root is irrational—let it be called medial.†



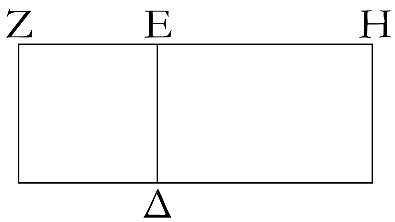
For let the rectangle AC be contained by the rational straight-lines AB and BC (which are) commensurable in square only. I say that AC is irrational, and its square-root is irrational—let it be called medial.

For let the square AD have been described on AB. AD is thus rational [Def. 10.4]. And since AB is incommensurable in length with BC. For they were assumed to be commensurable in square only. And AB (is) equal to BD. DB is thus also incommensurable in length with BC. And as DB is to BC, so AD (is) to AC [Prop. 6.1]. Thus, DA [is] incommensurable with AC [Prop. 10.11]. And DA (is) rational. Thus, AC is irrational [Def. 10.4]. Hence, its square-root [that is to say, the square-root of the square equal to it] is also irrational [Def. 10.4]—let it be called medial. (Which is) the very thing it was required to show.

† Thus, a medial straight-line has a length expressible as $k^{1/4}$.

Λῆμμα.

Ἐὰν ὦσι δύο εὐθεῖαι, ἔστιν ὡς ἡ πρώτη πρὸς τὴν δευτέραν, οὕτως τὸ ἀπὸ τῆς πρώτης πρὸς τὸ ὑπὸ τῶν δύο εὐθειῶν.

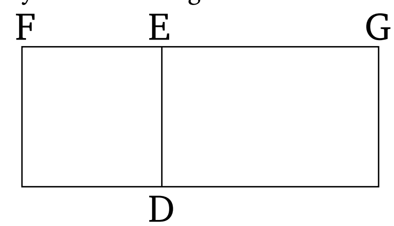


Ἔστωσαν δύο εὐθεῖαι αἱ ΖΕ, ΕΗ. λέγω, ὅτι ἐστὶν ὡς ἡ ΖΕ πρὸς τὴν ΕΗ, οὕτως τὸ ἀπὸ τῆς ΖΕ πρὸς τὸ ὑπὸ τῶν ΖΕ, ΕΗ.

Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΖΕ τετράγωνον τὸ ΔΖ, καὶ συμπεπληρώσθω τὸ ΗΔ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΖΕ πρὸς τὴν ΕΗ, οὕτως τὸ ΖΔ πρὸς τὸ ΔΗ, καί ἐστι τὸ μὲν ΖΔ τὸ ἀπὸ τῆς ΖΕ, τὸ δὲ ΔΗ τὸ ὑπὸ τῶν ΔΕ, ΕΗ, τουτέστι τὸ ὑπὸ τῶν ΖΕ, ΕΗ, ἔστιν ἄρα ὡς ἡ ΖΕ πρὸς τὴν ΕΗ, οὕτως τὸ ἀπὸ τῆς ΖΕ πρὸς τὸ ὑπὸ τῶν ΖΕ, ΕΗ. ὁμοίως δὲ καὶ ὡς τὸ ὑπὸ τῶν ΗΕ, ΕΖ πρὸς τὸ ἀπὸ τῆς ΕΖ, τουτέστιν ὡς τὸ ΗΔ πρὸς τὸ ΖΔ, οὕτως ἡ ΗΕ πρὸς τὴν ΕΖ· ὅπερ ἔδει δεῖξαι.

Lemma

If there are two straight-lines then as the first is to the second, so the (square) on the first (is) to the (rectangle contained) by the two straight-lines.

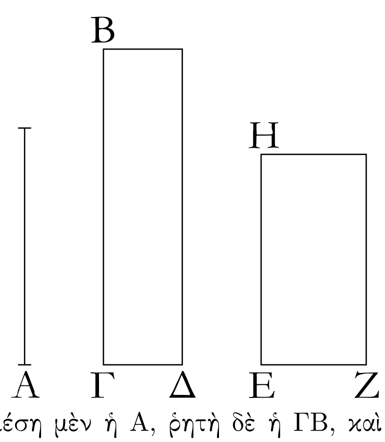


Let FE and EG be two straight-lines. I say that as FE is to EG, so the (square) on FE (is) to the (rectangle contained) by FE and EG.

For let the square DF have been described on FE. And let GD have been completed. Therefore, since as FE is to EG, so FD (is) to DG [Prop. 6.1], and FD is the (square) on FE, and DG the (rectangle contained) by DE and EG—that is to say, the (rectangle contained) by FE and EG—thus as FE is to EG, so the (square) on FE (is) to the (rectangle contained) by FE and EG. And also, similarly, as the (rectangle contained) by GE and EF is to the (square on) EF—that is to say, as GD (is) to FD—so GE (is) to EF. (Which is) the very thing it was required to show.

κβ´.

Τὸ ἀπὸ μέσης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ῥητὴν καὶ ἀσύμμετρον τῇ, παρ᾽ ἣν παράκειται, μήκει.

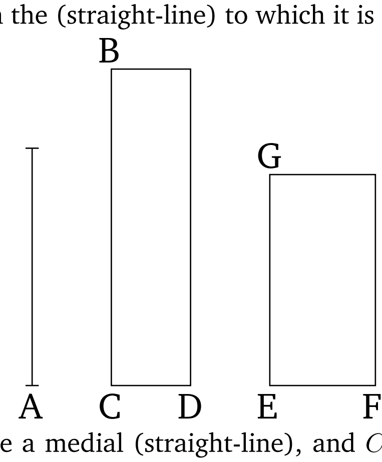


Ἔστω μέση μὲν ἡ Α, ῥητὴ δὲ ἡ ΓΒ, καὶ τῷ ἀπὸ τῆς Α ἴσον παρὰ τὴν ΒΓ παραβεβλήσθω χωρίον ὀρθογώνιον τὸ ΒΔ πλάτος ποιοῦν τὴν ΓΔ· λέγω, ὅτι ῥητή ἐστιν ἡ ΓΔ καὶ ἀσύμμετρος τῇ ΓΒ μήκει.

Ἐπεὶ γὰρ μέση ἐστὶν ἡ Α, δύναται χωρίον περιεχόμενον ὑπὸ ῥητῶν δυνάμει μόνον συμμέτρων. δυνάσθω τὸ ΗΖ.

Proposition 22

The square on a medial (straight-line), being applied to a rational (straight-line), produces as breadth a (straight-line which is) rational, and incommensurable in length with the (straight-line) to which it is applied.



Let A be a medial (straight-line), and CB a rational (straight-line), and let the rectangular area BD, equal to the (square) on A, have been applied to BC, producing CD as breadth. I say that CD is rational, and incommensurable in length with CB.

For since A is medial, the square on it is equal to a δύναται δὲ καὶ τὸ ΒΔ· ἴσον ἄρα ἐστὶ τὸ ΒΔ τῷ ΗΖ. ἔστι δὲ αὐτῷ καὶ ἰσογώνιον· τῶν δὲ ἴσων τε καὶ ἰσογωνίων παραλληλογράμμων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας· ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΒΓ πρὸς τὴν ΕΗ, οὕτως ἡ ΕΖ πρὸς τὴν ΓΔ. ἔστιν ἄρα καὶ ὡς τὸ ἀπὸ τῆς ΒΓ πρὸς τὸ ἀπὸ τῆς ΕΗ, οὕτως τὸ ἀπὸ τῆς ΕΖ πρὸς τὸ ἀπὸ τῆς ΓΔ. σύμμετρον δέ ἐστι τὸ ἀπὸ τῆς ΓΒ τῷ ἀπὸ τῆς ΕΗ· ῥητὴ γάρ ἐστιν ἑκατέρα αὐτῶν· σύμμετρον ἄρα ἐστὶ καὶ τὸ ἀπὸ τῆς ΕΖ τῷ ἀπὸ τῆς ΓΔ. ῥητὸν δέ ἐστι τὸ ἀπὸ τῆς ΕΖ· ῥητὸν ἄρα ἐστὶ καὶ τὸ ἀπὸ τῆς ΓΔ· ῥητὴ ἄρα ἐστὶν ἡ ΓΔ. καὶ ἐπεὶ ἀσύμμετρός ἐστιν ἡ ΕΖ τῇ ΕΗ μήκει· δυνάμει γὰρ μόνον εἰσὶ σύμμετροι· ὡς δὲ ἡ ΕΖ πρὸς τὴν ΕΗ, οὕτως τὸ ἀπὸ τῆς ΕΖ πρὸς τὸ ὑπὸ τῶν ΖΕ, ΕΗ, ἀσύμμετρον ἄρα [ἐστὶ] τὸ ἀπὸ τῆς ΕΖ τῷ ὑπὸ τῶν ΖΕ, ΕΗ. ἀλλὰ τῷ μὲν ἀπὸ τῆς ΕΖ σύμμετρόν ἐστι τὸ ἀπὸ τῆς ΓΔ· ῥηταὶ γάρ εἰσι δυνάμει· τῷ δὲ ὑπὸ τῶν ΖΕ, ΕΗ σύμμετρόν ἐστι τὸ ὑπὸ τῶν ΔΓ, ΓΒ· ἴσα γάρ ἐστι τῷ ἀπὸ τῆς Α· ἀσύμμετρον ἄρα ἐστὶ καὶ τὸ ἀπὸ τῆς ΓΔ τῷ ὑπὸ τῶν ΔΓ, ΓΒ. ὡς δὲ τὸ ἀπὸ τῆς ΓΔ πρὸς τὸ ὑπὸ τῶν ΔΓ, ΓΒ, οὕτως ἐστὶν ἡ ΔΓ πρὸς τὴν ΓΒ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΔΓ τῇ ΓΒ μήκει. ῥητὴ ἄρα ἐστὶν ἡ ΓΔ καὶ ἀσύμμετρος τῇ ΓΒ μήκει· ὅπερ ἔδει δεῖξαι.

(rectangular) area contained by rational (straight-lines which are) commensurable in square only [Prop. 10.21]. Let the square on (A) be equal to GF. And the square on (A) is also equal to BD. Thus, BD is equal to GF. And (BD) is also equiangular with (GF). And for equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional [Prop. 6.14]. Thus, proportionally, as BC is to EG, so EF (is) to CD. And, also, as the (square) on BC is to the (square) on EG, so the (square) on EF (is) to the (square) on CD [Prop. 6.22]. And the (square) on CB is commensurable with the (square) on EG. For they are each rational. Thus, the (square) on EF is also commensurable with the (square) on CD [Prop. 10.11]. And the (square) on EF is rational. Thus, the (square) on CD is also rational [Def. 10.4]. Thus, CD is rational. And since EF is incommensurable in length with EG. For they are commensurable in square only. And as EF (is) to EG, so the (square) on EF (is) to the (rectangle contained) by FE and EG [see previous lemma]. The (square) on EF [is] thus incommensurable with the (rectangle contained) by FE and EG [Prop. 10.11]. But, the (square) on CD is commensurable with the (square) on EF. For they are rational in square. And the (rectangle contained) by DC and CB is commensurable with the (rectangle contained) by FE and EG. For they are (both) equal to the (square) on A. Thus, the (square) on CD is also incommensurable with the (rectangle contained) by DC and CB [Prop. 10.13]. And as the (square) on CD (is) to the (rectangle contained) by DC and CB, so DC is to CB [see previous lemma]. Thus, DC is incommensurable in length with CB [Prop. 10.11]. Thus, CD is rational, and incommensurable in length with CB. (Which is) the very thing it was required to show.

† Literally, "rational".

κγ΄.

Ἡ τῇ μέσῃ σύμμετρος μέση ἐστίν.

Ἔστω μέση ἡ Α, καὶ τῇ Α σύμμετρος ἔστω ἡ Β· λέγω, ὅτι καὶ ἡ Β μέση ἐστίν.

Ἐκκείσθω γὰρ ῥητὴ ἡ ΓΔ, καὶ τῷ μὲν ἀπὸ τῆς Α ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω χωρίον ὀρθογώνιον τὸ ΓΕ πλάτος ποιοῦν τὴν ΕΔ· ῥητὴ ἄρα ἐστὶν ἡ ΕΔ καὶ ἀσύμμετρος τῇ ΓΔ μήκει. τῷ δὲ ἀπὸ τῆς Β ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω χωρίον ὀρθογώνιον τὸ ΓΖ πλάτος ποιοῦν τὴν ΔΖ. ἐπεὶ οὖν σύμμετρός ἐστιν ἡ Α τῇ Β, σύμμετρόν ἐστι καὶ τὸ ἀπὸ τῆς Α τῷ ἀπὸ τῆς Β. ἀλλὰ τῷ μὲν ἀπὸ τῆς Α ἴσον ἐστὶ τὸ ΕΓ, τῷ δὲ ἀπὸ τῆς Β ἴσον ἐστὶ τὸ ΓΖ· σύμμετρον ἄρα ἐστὶ τὸ ΕΓ τῷ ΓΖ. καί ἐστιν ὡς τὸ ΕΓ πρὸς τὸ ΓΖ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΖ·

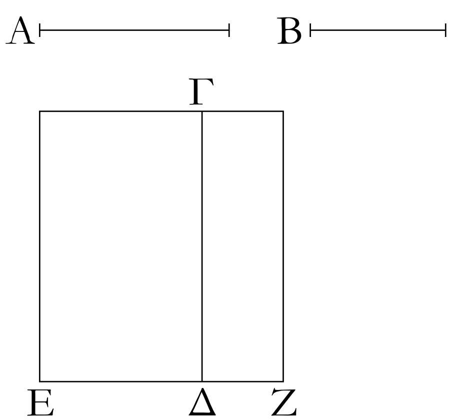
Proposition 23

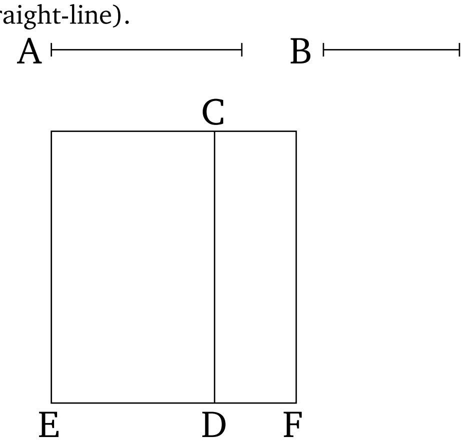
A (straight-line) commensurable with a medial (straight-line) is medial.

Let A be a medial (straight-line), and let B be commensurable with A. I say that B is also a medial (staight-line).

Let the rational (straight-line) CD be set out, and let the rectangular area CE, equal to the (square) on A, have been applied to CD, producing ED as width. ED is thus rational, and incommensurable in length with CD [Prop. 10.22]. And let the rectangular area CF, equal to the (square) on B, have been applied to CD, producing DF as width. Therefore, since A is commensurable with B, the (square) on A is also commensurable with σύμμετρος ἄρα ἐστὶν ἡ ΕΔ τῇ ΔΖ μήκει. ῥητὴ δέ ἐστιν ἡ ΕΔ καὶ ἀσύμμετρος τῇ ΔΓ μήκει· ῥητὴ ἄρα ἐστὶ καὶ ἡ ΔΖ καὶ ἀσύμμετρος τῇ ΔΓ μήκει· αἱ ΓΔ, ΔΖ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι. ἡ δὲ τὸ ὑπὸ ῥητῶν δυνάμει μόνον συμμέτρων δυναμένη μέση ἐστίν. ἡ ἄρα τὸ ὑπὸ τῶν ΓΔ, ΔΖ δυναμένη μέση ἐστίν· καὶ δύναται τὸ ὑπὸ τῶν ΓΔ, ΔΖ ἡ Β· μέση ἄρα ἐστὶν ἡ Β.

the (square) on B. But, EC is equal to the (square) on A, and CF is equal to the (square) on B. Thus, EC is commensurable with CF. And as EC is to CF, so ED (is) to DF [Prop. 6.1]. Thus, ED is commensurable in length with DF [Prop. 10.11]. And ED is rational, and incommensurable in length with CD. DF is thus also rational [Def. 10.3], and incommensurable in length with DC [Prop. 10.13]. Thus, CD and DF are rational, and commensurable in square only. And the square-root of a (rectangle contained) by rational (straight-lines which are) commensurable in square only is medial [Prop. 10.21]. Thus, the square-root of the (rectangle contained) by CD and DF is medial. And the square on B is equal to the (rectangle contained) by CD and DF. Thus, B is a medial (straight-line).





Πόρισμα.

Ἐκ δὴ τούτου φανερόν, ὅτι τὸ τῷ μέσῳ χωρίῳ σύμμετρον μέσον ἐστίν.

Corollary

And (it is) clear, from this, that an (area) commensurable with a medial area† is medial.

† A medial area is equal to the square on some medial straight-line. Hence, a medial area is expressible as $k^{1/2}$.

κδʹ.

Τὸ ὑπὸ μέσων μήκει συμμέτρων εὐθειῶν περιεχόμενον ὀρθογώνιον μέσον ἐστίν.

Ὑπὸ γὰρ μέσων μήκει συμμέτρων εὐθειῶν τῶν ΑΒ, ΒΓ περιεχέσθω ὀρθογώνιον τὸ ΑΓ· λέγω, ὅτι τὸ ΑΓ μέσον ἐστίν.

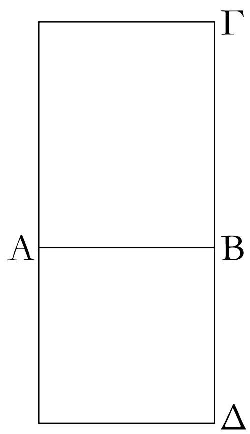
Ἀναγεγράφθω γὰρ ἀπὸ τῆς ΑΒ τετράγωνον τὸ ΑΔ· μέσον ἄρα ἐστὶ τὸ ΑΔ. καὶ ἐπεὶ σύμμετρός ἐστιν ἡ ΑΒ τῇ ΒΓ μήκει, ἴση δὲ ἡ ΑΒ τῇ ΒΔ, σύμμετρος ἄρα ἐστὶ καὶ ἡ ΔΒ τῇ ΒΓ μήκει· ὥστε καὶ τὸ ΔΑ τῷ ΑΓ σύμμετρόν ἐστιν. μέσον δὲ τὸ ΔΑ· μέσον ἄρα καὶ τὸ ΑΓ· ὅπερ ἔδει δεῖξαι.

Proposition 24

A rectangle contained by medial straight-lines (which are) commensurable in length is medial.

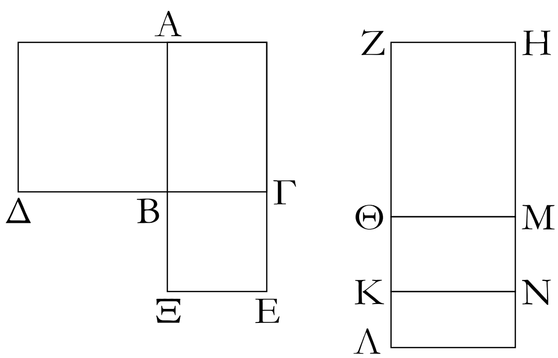
For let the rectangle AC be contained by the medial straight-lines AB and BC (which are) commensurable in length. I say that AC is medial.

For let the square AD have been described on AB. AD is thus medial [see previous footnote]. And since AB is commensurable in length with BC, and AB (is) equal to BD, DB is thus also commensurable in length with BC. Hence, DA is also commensurable with AC [Props. 6.1, 10.11]. And DA (is) medial. Thus, AC (is) also medial [Prop. 10.23 corr.]. (Which is) the very thing it was required to show.



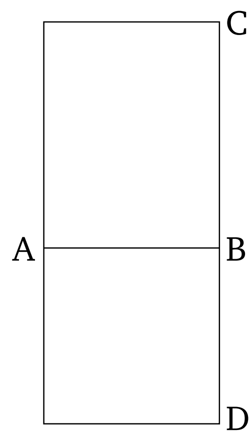
κε΄.

Τὸ ὑπὸ μέσων δυνάμει μόνον συμμέτρων εὐθειῶν περιεχόμενον ὀρθογώνιον ἤτοι ῥητὸν ἢ μέσον ἐστίν.



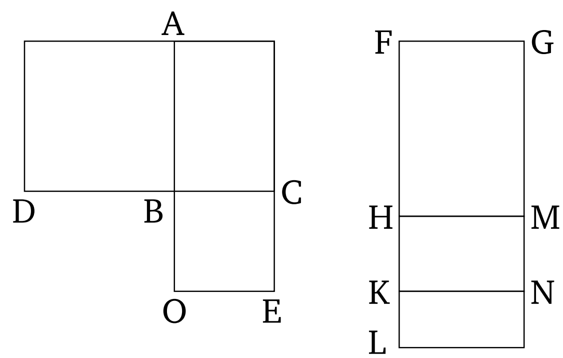
Ὑπὸ γὰρ μέσων δυνάμει μόνον συμμέτρων εὐθειῶν τῶν ΑΒ, ΒΓ ὀρθογώνιον περιεχέσθω τὸ ΑΓ· λέγω, ὅτι τὸ ΑΓ ἤτοι ῥητὸν ἢ μέσον ἐστίν.

Ἀναγεγράφθω γὰρ ἀπὸ τῶν ΑΒ, ΒΓ τετράγωνα τὰ ΑΔ, ΒΕ· μέσον ἄρα ἐστὶν ἑκάτερον τῶν ΑΔ, ΒΕ. καὶ ἐκκείσθω ῥητὴ ἡ ΖΗ, καὶ τῷ μὲν ΑΔ ἴσον παρὰ τὴν ΖΗ παραβεβλήσθω ὀρθογώνιον παραλληλόγραμμον τὸ ΗΘ πλάτος ποιοῦν τὴν ΖΘ, τῷ δὲ ΑΓ ἴσον παρὰ τὴν ΘΜ παραβεβλήσθω ὀρθογώνιον παραλληλόγραμμον τὸ ΜΚ πλάτος ποιοῦν τὴν ΘΚ, καὶ ἔτι τῷ ΒΕ ἴσον ὁμοίως παρὰ τὴν ΚΝ παραβεβλήσθω τὸ ΝΛ πλάτος ποιοῦν τὴν ΚΛ· ἐπ᾿ εὐθείας ἄρα εἰσὶν αἱ ΖΘ, ΘΚ, ΚΛ. ἐπεὶ οὖν μέσον ἐστὶν ἑκάτερον τῶν ΑΔ, ΒΕ, καί ἐστιν ἴσον τὸ μὲν ΑΔ τῷ ΗΘ, τὸ δὲ ΒΕ τῷ ΝΛ, μέσον ἄρα καὶ ἑκάτερον τῶν ΗΘ, ΝΛ. καὶ παρὰ ῥητὴν τὴν ΖΗ παράκειται· ῥητὴ ἄρα ἐστὶν ἑκατέρα τῶν ΖΘ, ΚΛ καὶ ἀσύμμετρος τῇ ΖΗ μήκει. καὶ ἐπεὶ σύμμετρόν ἐστι τὸ ΑΔ τῷ ΒΕ, σύμμετρον ἄρα ἐστὶ καὶ τὸ ΗΘ τῷ ΝΛ. καί ἐστιν ὡς τὸ ΗΘ πρὸς τὸ ΝΛ, οὕτως ἡ ΖΘ πρὸς τὴν ΚΛ· σύμμετρος ἄρα ἐστὶν ἡ ΖΘ τῇ ΚΛ μήκει. αἱ ΖΘ, ΚΛ ἄρα ῥηταί εἰσι μήκει σύμμετροι· ῥητὸν ἄρα ἐστὶ τὸ ὑπὸ τῶν ΖΘ, ΚΛ. καὶ



Proposition 25

The rectangle contained by medial straight-lines (which are) commensurable in square only is either rational or medial.



For let the rectangle AC be contained by the medial straight-lines AB and BC (which are) commensurable in square only. I say that AC is either rational or medial.

For let the squares AD and BE have been described on (the straight-lines) AB and BC (respectively). AD and BE are thus each medial. And let the rational (straight-line) FG be laid out. And let the rectangular parallelogram GH, equal to AD, have been applied to FG, producing FH as breadth. And let the rectangular parallelogram MK, equal to AC, have been applied to HM, producing HK as breadth. And, finally, let NL, equal to BE, have similarly been applied to KN, producing KL as breadth. Thus, FH, HK, and KL are in a straight-line. Therefore, since AD and BE are each medial, and AD is equal to GH, and BE to NL, GH and NL (are) thus each also medial. And they are applied to the rational (straight-line) FG. FH and KL are thus each rational, and incommensurable in length with FG [Prop. 10.22]. And since AD is commensurable with BE, GH is thus also commensurable with NL. And as ἐπεὶ ἴση ἐστὶν ἡ μὲν ΔΒ τῇ ΒΑ, ἡ δὲ ΞΒ τῇ ΒΓ, ἔστιν ἄρα ὡς ἡ ΔΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΞ. ἀλλ᾽ ὡς μὲν ἡ ΔΒ πρὸς τὴν ΒΓ, οὕτως τὸ ΔΑ πρὸς τὸ ΑΓ· ὡς δὲ ἡ ΑΒ πρὸς τὴν ΒΞ, οὕτως τὸ ΑΓ πρὸς τὸ ΓΞ· ἔστιν ἄρα ὡς τὸ ΔΑ πρὸς τὸ ΑΓ, οὕτως τὸ ΑΓ πρὸς τὸ ΓΞ. ἴσον δέ ἐστι τὸ μὲν ΑΔ τῷ ΗΘ, τὸ δὲ ΑΓ τῷ ΜΚ, τὸ δὲ ΓΞ τῷ ΝΛ· ἔστιν ἄρα ὡς τὸ ΗΘ πρὸς τὸ ΜΚ, οὕτως τὸ ΜΚ πρὸς τὸ ΝΛ· ἔστιν ἄρα καὶ ὡς ἡ ΖΘ πρὸς τὴν ΘΚ, οὕτως ἡ ΘΚ πρὸς τὴν ΚΛ· τὸ ἄρα ὑπὸ τῶν ΖΘ, ΚΛ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΘΚ. ῥητὸν δὲ τὸ ὑπὸ τῶν ΖΘ, ΚΛ· ῥητὸν ἄρα ἐστὶ καὶ τὸ ἀπὸ τῆς ΘΚ· ῥητὴ ἄρα ἐστὶν ἡ ΘΚ. καὶ εἰ μὲν σύμμετρός ἐστι τῇ ΖΗ μήκει, ῥητόν ἐστι τὸ ΘΝ· εἰ δὲ ἀσύμμετρός ἐστι τῇ ΖΗ μήκει, αἱ ΚΘ, ΘΜ ῥηταί εἰσι δυνάμει μόνον σύμμετροι· μέσον ἄρα τὸ ΘΝ. τὸ ΘΝ ἄρα ἤτοι ῥητὸν ἢ μέσον ἐστίν. ἴσον δὲ τὸ ΘΝ τῷ ΑΓ· τὸ ΑΓ ἄρα ἤτοι ῥητὸν ἢ μέσον ἐστίν.

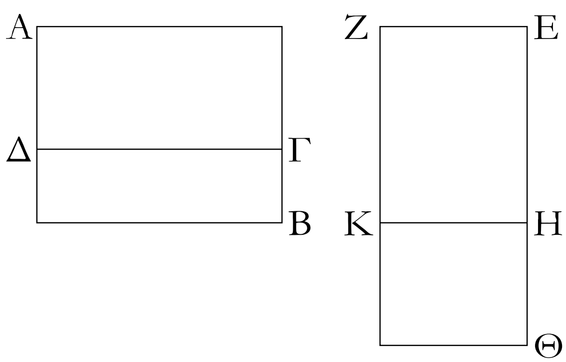
Τὸ ἄρα ὑπὸ μέσων δυνάμει μόνον συμμέτρων, καὶ τὰ ἑξῆς.

GH is to NL, so FH (is) to KL [Prop. 6.1]. Thus, FH is commensurable in length with KL [Prop. 10.11]. Thus, FH and KL are rational (straight-lines which are) commensurable in length. Thus, the (rectangle contained) by FH and KL is rational [Prop. 10.19]. And since DB is equal to BA, and OB to BC, thus as DB is to BC, so AB (is) to BO. But, as DB (is) to BC, so DA (is) to AC [Props. 6.1]. And as AB (is) to BO, so AC (is) to CO [Prop. 6.1]. Thus, as DA is to AC, so AC (is) to CO. And AD is equal to GH, and AC to MK, and CO to NL. Thus, as GH is to MK, so MK (is) to NL. Thus, also, as FH is to HK, so HK (is) to KL [Props. 6.1, 5.11]. Thus, the (rectangle contained) by FH and KL is equal to the (square) on HK [Prop. 6.17]. And the (rectangle contained) by FH and KL (is) rational. Thus, the (square) on HK is also rational. Thus, HK is rational. And if it is commensurable in length with FG then HN is rational [Prop. 10.19]. And if it is incommensurable in length with FG then KH and HM are rational (straight-lines which are) commensurable in square only: thus, HN is medial [Prop. 10.21]. Thus, HN is either rational or medial. And HN (is) equal to AC. Thus, AC is either rational or medial.

Thus, the . . . by medial straight-lines (which are) commensurable in square only, and so on . . . .

κϛ΄.

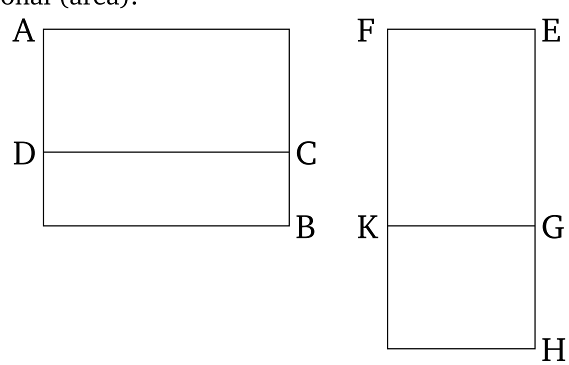
Μέσον μέσου οὐχ ὑπερέχει ῥητῷ.



Εἰ γὰρ δυνατόν, μέσον τὸ ΑΒ μέσου τοῦ ΑΓ ὑπερεχέτω ῥητῷ τῷ ΔΒ, καὶ ἐκκείσθω ῥητὴ ἡ ΕΖ, καὶ τῷ ΑΒ ἴσον παρὰ τὴν ΕΖ παραβεβλήσθω παραλληλόγραμμον ὀρθογώνιον τὸ ΖΘ πλάτος ποιοῦν τὴν ΕΘ, τῷ δὲ ΑΓ ἴσον ἀφῃρήσθω τὸ ΖΗ· λοιπὸν ἄρα τὸ ΒΔ λοιπῷ τῷ ΚΘ ἐστιν ἴσον. ῥητὸν δέ ἐστι τὸ ΔΒ· ῥητὸν ἄρα ἐστὶ καὶ τὸ ΚΘ. ἐπεὶ οὖν μέσον ἐστὶν ἑκάτερον τῶν ΑΒ, ΑΓ, καί ἐστι τὸ μὲν ΑΒ τῷ ΖΘ ἴσον, τὸ δὲ ΑΓ τῷ ΖΗ, μέσον ἄρα καὶ ἑκάτερον τῶν ΖΘ, ΖΗ. καὶ παρὰ ῥητὴν τὴν ΕΖ παράκειται· ῥητὴ ἄρα ἐστὶν ἑκατέρα τῶν ΘΕ, ΕΗ καὶ ἀσύμμετρος τῇ ΕΖ μήκει. καὶ ἐπεὶ ῥητόν ἐστι

Proposition 26

A medial (area) does not exceed a medial (area) by a rational (area).†



For, if possible, let the medial (area) AB exceed the medial (area) AC by the rational (area) DB. And let the rational (straight-line) EF be laid down. And let the rectangular parallelogram FH, equal to AB, have been applied to to EF, producing EH as breadth. And let FG, equal to AC, have been cut off (from FH). Thus, the remainder BD is equal to the remainder KH. And DB is rational. Thus, KH is also rational. Therefore, since AB and AC are each medial, and AB is equal to FH, and AC to FG, FH and FG are thus each also medial.