10.9 Exercises (page 331)

1. $\frac{23}{8}$

2. $2a^3$

3. $a = (3c/2)^{\frac{1}{2}}$

4. 0

5. $8\pi(\sin\theta - \cos\theta)$

6. $\pi a^3/4$

7. $-\sqrt{2}$

8. $256a^3/15$

9. $2\pi^2 a^3(1 + 2\pi^2)$

10. $[(2 + t_0^2)^{3/2} - 2\sqrt{2}]/3$

12. moment of inertia $= 4a^4$

13. $2\pi/3$

14. $\dfrac{600 - 36\sqrt{2} - 49\log(9 - 4\sqrt{2})}{64[6\sqrt{2} + \log(3 + 2\sqrt{2})]}$

15. $x = \dfrac{6ab^2}{3a^2 + 4\pi^2 b^2}$; $\bar{y} = -\dfrac{6\pi ab^2}{3a^2 + 4\pi^2 b^2}$

16. $I_x = (a^2 + b^2)^{\frac{1}{2}}[\pi a^4 + (4\pi^3 - \pi/2)a^2 b^2 + 32\pi^5 b^4/5]$
    $I_y = (a^2 + b^2)^{\frac{1}{2}}[\pi a^4 + (4\pi^3 + \pi/2)a^2 b^2 + 32\pi^5 b^4/5]$

10.13 Exercises (page 336)

1. All except (f) are connected

6. (a) Not conservative
   (b) $(2e^{2\pi} - 5e^{\pi} - 5\pi - 3)/10$

7. (b) 3

8. $\frac{8}{5}$

10. $4b^2 - 8\pi b + 4$; minimum occurs when $b = \pi$

10.18 Exercises (page 345)

1. $\varphi(x, y) = \frac{1}{2}(x^2 + y^2) + c$

2. $\varphi(x, y) = x^3 y + c$

3. $\varphi(x, y) = x^2 e^y + xy - y^2 + C$

4. $\varphi(x, y) = x\sin y + y\cos x + (x^2 + y^2)/2 + C$

5. $\varphi(x, y) = x\sin(xy) + C$

6. $\varphi(x, y, z) = (x^2 + y^2 + z^2)/2 + C$

7. $\varphi(x, y, z) = x^2/2 - y^2/2 + xz - yz + c$

8. f is not a gradient

9. f is not a gradient

10. f is not a gradient

11. $\varphi(x, y, z) = y^2\sin x + xz^3 - 4y + 2z + C$

12. $\varphi(x, y, z) = x + 2x^2 y - x^3 z^2 + 2y - z^3 + c$

13. (b) $\varphi(x, y) = \dfrac{ar^{n+1}}{n + 1} + C$ if $n \neq -1$; $\varphi(x, y) = a\log r + C$ if $n = -1$

15. $\varphi(x) = \dfrac{r^{p+2}}{p + 2} + C$ if $p \neq -2$; $\varphi(x) = \log r + C$ if $p = -2$

16. $\varphi(x) = g(r) + C$

10.20 Exercises (page 349)

1. $x^2/2 + 2xy + y^2/2 = C$

2. $x^2 y = c$

3. $x^3/3 - xy$ -$y/2 + (\sin 2y)/4 = C$
4. $\cos 2x \sin 3y = C$
5. $x^3y + 4x^2y^2 - 12e^y + 12ye^y = C$
6. $\int Q(x)e^{\int P(x)dx} \, dx - ye^{\int P(x)dx} = C$
8. (a) $x + y = C'$
   (b) $y^3/x^3 - 3 \log |x| = C$
9. (a) $6(xy)^{\frac{1}{3}} - (y/x)^{\frac{3}{2}} = C$; $(x^5y)^{-\frac{1}{3}}$ is an integrating factor
   (b) $x + e^{-x} \sin y = C$; $e^{-x} \cos y$ is an integrating factor
10. $x^3y^4 + x^4y^5 = c$, $10x^3y^4 + x^5y^5 = C$,   respectively; x^2y^3 is a common integrating factor

Chapter 11

11.9 Exercises (page 362)

1. $\frac{1}{3}$

2. 1

3. $2\sqrt{3} - \frac{38}{3}$

4. $\pi^2/4$

5. 2

6. 2π

7. 6

8. $t^{-3}(e^{t^2} - e^t) + t^{-2} - t^{-1}$

10. $\frac{1}{6}$

11. $\frac{1}{5}(\frac{21}{8} - \sqrt{2})$

12. $\pi/2$

13. $(\log 2)/6$

11.15 Exercises (page 371)

1. $-3\pi/2$

2. $\frac{3}{2} + \cos 1 + \sin 1 - \cos 2 - 2 \sin 2$

3. $e - e^{-1}$

4. $\frac{7}{3} \log 2$

5. $\pi^2 - \frac{40}{9}$

6. 6

7. $\frac{80}{3}$

8. (a) $\frac{8}{3}$ (b) 2   (c) 320π

9. $\int_0^1 \left[\int_x^1 f(x, y) \, dy \right] dx$

10. $\int_0^4 \left[\int_{x/2}^{\sqrt{x}} f(x, y) \, dy \right] dx$

11. $\int_1^2 \left[\int_1^{y^2} f(x, y) \, dx \right] dy$

12. $\int_0^1 \left[\int_{2-y}^{1+\sqrt{1-y^2}} f(x, y) \, dx \right] dy$

13. $\int_{-1}^0 \left[\int_{-\sqrt{4y+4}}^{\sqrt{4y+4}} f(x, y) \, dx \right] dy + \int_0^8 \left[\int_{-\sqrt{4y+4}}^{2-y} f(x, y) \, dx \right] dy$

14. $\int_0^1 \left[\int_{e^y}^e f(x, y) \, dx \right] dy$

15. $\int_{-1}^0 \left[\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) \, dx \right] dy + \int_0^1 \left[\int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x, y) \, dx \right] dy$

16. $\int_0^1 \left[\int_{y^{1/2}}^{y^{1/3}} f(x, y) \, dx \right] dy$

17. $\int_{-1}^0 \left[\int_{-2 \arcsin y}^{\pi} f(x, y) \, dx \right] dy + \int_0^1 \left[\int_{\arcsin y}^{\pi - \arcsin y} f(x, y) \, dx \right] dy$

18. $\int_{-2}^0 \left[\int_{2x+4}^{4-x^2} f(x, y) \, dy \right] dx$

19. $\int_0^1\left[\int_x^{2-x}(x^2+y^2)\,dy\right]dx=\frac{4}{3}$

20. $y=0$, $y=x\tan c$, $x^2+y^2=a^2$, $x^2+y^2=b^2$

21. (a) $\int_1^8\left[\int_{y^{1/3}}^y f(x,y)\,dx\right]dy$

 (b) $4e^8+2e/3$

22. $m=2$; $n=1$

11.18 Exercises (page 377)

1. $\bar{x}=-\frac{1}{2}$, $\bar{y}=\frac{8}{5}$
2. $\bar{x}=1$, $\bar{y}=0$
3. $\bar{x}=\frac{18}{13}$, $\bar{y}=\frac{50}{39}$
4. $\bar{x}=\pi/2$, $\bar{y}=\pi/8$

5. $\bar{x}=(\sqrt{2}+1)\left(\frac{\pi\sqrt{2}}{4}-1\right)=\frac{\pi}{2}+\frac{\pi\sqrt{2}}{4}-1-\sqrt{2}$, $\bar{y}=\frac{\sqrt{2}+1}{4}$

6. $\bar{x}=\dfrac{2a^2\log a-a^2+1}{4(a\log a-a+1)}$, $\bar{y}=\dfrac{a(\log a)^2}{2(a\log a-a+1)}-1$

7. $\bar{x}=\bar{y}=\frac{1}{5}$
8. $\bar{x}=\bar{y}=256/(315\pi)$
9. $\frac{26}{3}-\frac{15}{2}\log 3$
10. $\bar{x}=\frac{2}{3}\|\overrightarrow{AB}\|$, $\bar{y}=\frac{2}{3}\|\overrightarrow{AD}\|$; assuming the x- and y-axes are chosen along sides AB and AD, respectively

11. $I_x=\dfrac{5\pi}{12}$, $I_y=\dfrac{2\pi^3}{3}-\pi$

12. $I_x=\frac{1}{12}b^3(a-c)$, $I_y=\frac{1}{12}b(a^3-c^3)$
13. $I_x=I_y=(1-5\pi/16)r^4$
14. $I_x=I_y=\frac{9}{8}$
15. $I_x=\frac{1}{64}[(4a-1)e^{4a}-1]$, $I_y=\frac{1}{32}[(a^3-3a^2+6a-6)e^{2a}+6]$
16. $I_x=\frac{72}{105}$, $I_y=\frac{148}{45}$
19. $\frac{1}{3}h[\sqrt{2}+\log(1+\sqrt{2})]$
20. $h^2+\frac{1}{2}r^2$
21. (a) $(\frac{13}{6},1)$
 (b) $(\frac{7}{3},\frac{9}{2})$
 (c) $(\frac{11}{4},\frac{11}{4})$
 (d) $(\frac{19}{8},\frac{13}{8})$
22. $h=2\sqrt{3}$
23. $h>r\sqrt{2}$

11.22 Exercises (page 385)

1. (a) -4
 (b) 4
 (c) 8
 (d) 4π
 (e) $3\pi/2$

2. 0

3. $n = 3$

4. $-\pi$

9. $g(x, y) = \pm[P^2(x, y) + Q^2(x, y)]^{1/2}$

★**11.25** Exercises (page 391)

1. (b) 0

2. 0, 2π, -2π

3. As many as three

4. As many as seven

5. (a) - 3

6. 2π

11.28 Exercises (page 399)

1. $\int_0^{2\pi} \left[\int_0^a f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta$

2. $\int_{-\pi/2}^{\pi/2} \left[\int_0^{2\cos\theta} f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta$

3. $\int_0^{2\pi} \left[\int_a^b f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta$

4. $\int_0^{\pi/2} \left[\int_0^{g(\theta)} f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta$,   where $g(\theta) = 1/(\cos\theta + \sin\theta)$

5. $\int_0^{\pi/4} \left[\int_0^{\tan\theta\sec\theta} f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta + \int_{\pi/4}^{3\pi/4} \left[\int_0^{\csc\theta} f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta$

$+ \int_{3\pi/4}^{\pi} \left[\int_0^{\tan\theta\sec\theta} f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta$

6. $\frac{3}{4}\pi a^4$

7. $\frac{1}{6}a^3[\sqrt{2} + \log(1 + \sqrt{2})]$

8. $\sqrt{2} - 1$

9. $\pi a^4/8$

10. $\int_0^{\pi/4} \left[\int_0^{\sec\theta} f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta + \int_{\pi/4}^{\pi/2} \left[\int_0^{\csc\theta} f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta$

11. $\int_{\pi/4}^{\pi/3} \left[\int_0^{2\sec\theta} f(r)r\,dr \right] d\theta$

12. $\int_0^{\pi/2} \left[\int_{g(\theta)}^{1} f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta$,   where $g(0) = 1/(\cos\theta + \sin\theta)$

13. $\int_0^{\pi/4} \left[\int_{\tan\theta\sec\theta}^{\sec\theta} f(r\cos\theta, r\sin\theta)r\,dr \right] d\theta$

14. $\pi^4/3$

15. (a) $u = 7x - y,\; v = -5x + y$

    (b) 60

17. (a) $1 + 2u$

    (c) $\frac{14}{3}$

    (d) $2 + \dfrac{2}{\sqrt{3}}\left(\arctan\dfrac{1}{\sqrt{3}} - \arctan\dfrac{5}{\sqrt{3}} \right)$

18. (a) $4(u^2 + v^2)$

    (c) 0

19. $\dfrac{\pi}{1-p}\,[(p^2 + r^2)^{1-p} - p^{2(1-p)}]$ if $p \neq 1$; $\pi \log (1 + r^2)$ if $p = 1$.

    $I(p, r)$ tends to a finite limit when $p > 1$

11.34 Exercises (page 413)

1. $\frac{1}{364}$
2. $\log \sqrt{2} - \frac{5}{16}$
3. $\frac{1}{48}$
4. $\frac{4}{5}\pi abc$
5. $\pi/6$

6. $\displaystyle\int_0^1 \left\{ \int_0^x \left[\int_0^{1-x} f(x, y, z)\, dy \right] dz + \int_x^1 \left[\int_{z-x}^{1-x} f(x, y, z)\, dy \right] dz \right\} dx$

7. $\displaystyle\int_0^1 \left\{ \int_{-z}^z \left[\int_{-\sqrt{z^2-x^2}}^{\sqrt{z^2-x^2}} f(x, y, z)\, dy \right] dx \right\} dz$

8. $\displaystyle\int_0^1 \left\{ \int_0^{x^2} \left[\int_0^1 f(x, y, z)\, dy \right] dz + \int_{x^2}^{1+x^2} \int_{\sqrt{z-x^2}}^1 \left[f(x, y, z)\, dy \right] dz \right\} dx$

10. $16\pi/3$
11. $\frac{1}{6}$
12. $\frac{1}{60}\pi a^2 h(3a^2 + 2h^2)$
13. $\frac{4}{3}\pi a^3$
14. $\frac{4}{3}\pi(b^3 - a^3)$
15. $\frac{4}{3}\pi R^3 (a^2 + b^2 + c^2)^{-\frac{1}{2}}$
18. $\frac{2}{3}\pi(5\sqrt{5} - 4)$
19. $\frac{32}{9}$
20. $\frac{4}{5}\pi(b^5 - a^5)$
22. On the axis at distance $\frac{2}{5}h$ from the base
23. On the axis at distance $\frac{1}{5}h$ from the base

24. On the axis of symmetry at distance $\dfrac{3}{8} \cdot \dfrac{b^4 - a^4}{b^3 - a^3}$ from the "cutting plane" of the hemispheres

25. $\bar{x} = \bar{y} = \bar{z} = \frac{7}{12}h$ (assuming the specified corner is at the origin)
26. $\frac{3}{20}M(a^2 + 4h^2)$
27. $\frac{2}{5}MR^2$
28. $\frac{3}{5}Ma^2$
29. $2\frac{1}{4}$

Chapter 12

12.4 Exercises (page 424)

1. $(a_2 b_3 - a_3 b_2)(x - x_0) + (a_3 b_1 - a_1 b_3)(y - y_0) + (a_1 b_2 - a_2 b_1)(z - z_0) = 0$;

    $\dfrac{\partial r}{\partial u} \times \dfrac{\partial r}{\partial v} = (a_2 b_3 - a_3 b_2)\mathbf{i} + (a_3 b_1 - a_1 b_3)\mathbf{j} + (a_1 b_2 - a_2 b_1)\mathbf{k}$

2. $x^2/a^2 + y^2/b^2 = z$; $\dfrac{\partial r}{\partial u} \times \dfrac{\partial r}{\partial v} = -2bu^2 \cos v\, \mathbf{i} - 2au^2 \sin v\, \mathbf{j} + ubu\, \mathbf{k}$

3. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$; $\dfrac{\partial r}{\partial u} \times \dfrac{\partial r}{\partial v} = abc \, \sin u \left(\dfrac{\sin u \cos v}{a} i + \dfrac{\sin u \sin v}{b} j + \dfrac{\cos u}{c} k \right)$

4. $z = f(\sqrt{x^2 + y^2})$; $\dfrac{\partial r}{\partial u} \times \dfrac{\partial r}{\partial v} = -uf'(u) \cos vi - uf'(u) \sin vj + uk$

5. $\dfrac{y^2}{a^2} + \dfrac{z^2}{b^2} = 1$; $\dfrac{\partial r}{\partial u} \times \dfrac{\partial r}{\partial v} = b \sin vj + a \cos vk$

6. $(\sqrt{x^2 + y^2} - a)^2 + z^2 = b^2$;

$\dfrac{\partial r}{\partial u} \times \dfrac{\partial r}{\partial v} = b(a + b \cos u)(\cos u \sin vi + \cos u \cos vj + \sin uk)$

7. $|abc| \cosh v \left[\left(\dfrac{\sin^2 u}{a^2} + \dfrac{\cos^2 u}{b^2} \right) \cosh^2 v + \dfrac{\sinh^2 v}{c^2} \right]^{1/2}$

8. $\sqrt{128v^2 + 4}$
9. $|u - v| \sqrt{36u^2v^2 + 9(u + v)^2 + 4}$
10. $\sqrt{u^4 + u^2}$

12.6 Exercises (page 429)

2. $\dfrac{\pi a^2 \sqrt{3}}{3}$
3. $(2\pi - 4a^2$
4: 4
5. (a) A circular paraboloid
   (b) $-2u^2 \cos vi - 2u^2 \sin vj + uk$
   (c) $n = 6$
6. $\sqrt{2}\,\pi a^2/4$
7. $2\pi\sqrt{6}$
8. $2\pi a^2(3\sqrt{3} - 1)/3$
9. $4\pi^2 ab$
11. (a) A unit circle in the xy-plane; a unit semicircle in the xz-plane, with $z \leq 0$; a unit semicircle in the plane $x = y$ with $z \leq 0$
    (b) The hemisphere $x^2 + y^2 + z^2 = 1, z \leq 0$
    (c) The sphere $x^2 + y^2 + z^2 = 1$ except for the North Pole; the line joining the North Pole and (x, y, z) intersects the xy-plane at (u, v, 0)

12.10 Exercises (page 436)

1. $4\pi/3$
3. $\bar{x} = \bar{y} = \bar{z} = a/2$
4. $\tfrac{1}{2}$
7. 0
8. $\pi\sqrt{2}$
9. On the axis of the cone, at a distance $\tfrac{1}{4}a(1 - \cos \alpha)/[1 - \cos (\alpha/2)]$ from the center of the sphere
10. $\pi a^3 h + \tfrac{2}{3}\pi a h^3$
11. $3\pi a^3 h + \tfrac{2}{3}\pi a h^3$
12. $2\pi/3$
13. $-\pi/3$

12.13 Exercises (page 442)

1. 0
2. $-\pi$

3. - 4
4. $\frac{4}{3}$

12.15 Exercises (page 447)

1. (a) div $F(x, y, z) = 2x + 2y + 2z$; curl $F(x, y, z) = 0$
   (b) div $F(x, y, z) = 0$; curl $F(x, y, z) = 2i + 4j + 6k$
   (c) div $F(x, y, z) = -x \sin y$; curl $F(x, y, z) = i + j$
   (d) div $F(x, y, z) = ye^{xy} - x \sin(xy) - 2xz \sin(xz^2)$;
       curl $F(x, y, z) = z^2 \sin(xz^2)j - [xe^{xy} + y \sin(xy)]k$
   (e) div $F(x, y, z) = 2x \sin y + 2y \sin(xz) - xy \sin z \cos(\cos z)$;
       curl $F(x, y, z) = [x \sin(\cos z) - xy^2 \cos(xz)]i - y \sin(\cos z)j + [y^2z \cos(xz) - x^2 \cos y]k$

2. 0
4. $n = -3$
5. No such vector field
10. One such field is $\mu(x, y, z) = (xyz)^{-2}$
11. div $(V \times r) = 0$; curl $(V \times r) = (c + 1)V$
13. $16(a + b)$

★**12.17** Exercises (page 452)

1. $(3x - 2z)j - xk$ is one such field
2. $(x^2/2 - xy - yz + z^2/2)j + (x^2/2 - xz)k$ is one such field
3. $(x^2y/2 + z^2/2)j + \nabla f(x, y)$ for some f independent of z

5. $G(x, y, z)\dfrac{yz}{r(x^2 + y^2)}i - \dfrac{xz}{r(x^2 + y^2)}j$ satisfies curl $G = r^{-3}r$ at all points not on the z-axis

6. $f(r) = Cr^{-3}$
9. $F(x, y, z) = -\frac{1}{3}(z^3i + x^3j + y^3k)$, $G(x, y, z) = \frac{1}{3}\nabla(x^3y + y^3z + z^3x)$
10. (c) $3\pi/2$

12.21 Exercises (page 462)

1. 3
2. (a) 14477
   (b) -16π
   (c) 128π
15. 8π

3. (a) $3|V|$
   (b) $9|V|\bar{z}$
   (c) $|V|\bar{x}$
   (d) $4I_z$

Chapter 13

13.4 Exercises (page 472)

2. $A_1 \cup A_2 \cup A_3 = (A_1 \cap A_2' \cap A_3') \cup (A_2 \cap A_3') \cup A_3$; $\bigcup\limits_{k=1}^{n} A_k = \bigcup\limits_{k=1}^{n-1} (A_k \cap \bigcap\limits_{j=k+1}^{n} A_j') \cup A_n$

3.

	(i)	(ii)	(iii)	(iv)	(v)
(a)	$A' \cap B'$	$A \cap B'$	$A \cup B$	$(A \cap B') \cup (A' \cap B)$	$A' \cup B'$
(b)	500	200	500	300	800

6. $\mathscr{B}_2 = \{\varnothing, A_1, A_2, A_1 \cup A_2, A_1', A_2', A_1' \cap A_2', S\}$

7. $\mathscr{B}_3 = \{\varnothing, A_1, A_2, A_3, A_1 \cup A_2, A_2 \cup A_3, A_1 \cup A_3, A_1 \cup A_2 \cup A_3, A_1', A_2', A_3', A_1' \cap A_2',$
$A_1' \cap A_3', A_1' \cap A_3', A_1' \cap A_2' \cap A_3', S\}$ (if $n > 3$)

13.7 Exercises (page 477)

1. $A \subseteq B'$
2. $x \in A' \cap B' \cap C'$
3. $x \in A \cap B' \cap C'$
4. $x \in A \cup B \cup C$
5. $x \in (A \cap B' \cap C') \cup (A' \cap B \cap C') \cup (A' \cap B' \cap C)$
6. $x \in (A' \cap B') \cup (B' \cap C') \cup (A' \cap C')$
7. $x \in (A \cap B) \cup (A \cap C) \cup (B \cap C)$
8. $x \in (A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C)$
9. $x \in (A \cap B \cap C)'$
10. $x \in A \cap C \cap B'$
11. $x \in A \cap B \cap C$
12. $x \in A \cup B \cup C$
15. (a) $1 - a$ (d) $1 - c$
    (b) $1 - b$ (e) $1 - a + c$
    (c) $a + b - c$ (f) $a - c$

13.9 Exercises (page 479)

2. (a) $\frac{5}{14}$ 5. $\frac{9}{47}$
   (b) $\frac{45}{91}$ 6. $\frac{8}{47}$
   (c) $\frac{10}{91}$ 7. $\frac{4}{47}$
   (d) $\frac{36}{91}$ 8. (a) $A/(A + B)$
3. (a) $\frac{23}{36}$ (b) $B/(A + B)$
   (b) $\frac{1}{6}$ (c) $(C + 1)/(C + D + 1)$
   (c) $\frac{1}{3}$ (d) $C/(C + D + 1)$
   (d) $\frac{5}{36}$ 9. (a) $\frac{1}{9}$
   (e) $\frac{13}{36}$ (b) $\frac{4}{9}$
   (f) $\frac{2}{3}$ (c) $\frac{5}{9}$
4. $\frac{1}{4}$ (d) $\frac{8}{9}$
10. $P_0 = 1 - P(A) - P(B) + P(A \cap B)$, $P_1 = P(A) + P(B) - 2P(A \cap B)$, $P_2 = P(A \cap B)$
12. (a) 5 to 9
   (b) 45 to 46
   (c) 10 to 81
   (d) 36 to 55

13.11 Exercises (page 485)

1. $\{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$
2. 1326
3: 54
4. $\{H, T\} \times \{H, T\} \times \{1, 2, 3, 4, 5, 6\}$;   24 outcomes
5. $52!/(13!)^4$
6. 36
   (a) 18
   (b) 12
   (c) 24

7. (a) 13 . 12 11 . 72 = 123552 (not including triplets or quadruplets)
   (b) 5148
   (c) 36 (not including $10JQKA$)
   (d) 4

8. (a) $4\binom{13}{5}\Big/\binom{52}{5}$ (b) $36\Big/\binom{52}{5}$ (c) $4\Big/\binom{52}{5}$

9. (a) $\dfrac{2 \cdot 98!}{(49!)^2}$ (b) $\dfrac{98!}{48! \cdot 50!}$

10. $\binom{98}{48}\Big/\binom{100}{50}$

11. 16
12. n^k

13.14 Exercises (page 490)

2. (a) $P(A) = \tfrac{3}{10}$; $P(B \mid A) = \tfrac{6}{11}$; $P(A \cap B) = \tfrac{9}{55}$

4. $\dfrac{\binom{98}{48}}{\binom{100}{50} - \binom{98}{50}}$

5. $\tfrac{4}{11}$

6. (a) $1 - \dfrac{26! \cdot 34!}{21! \cdot 39!} = 1 - \dfrac{\binom{34}{13}}{\binom{39}{13}} = 1 - \dfrac{\binom{62}{5}}{\binom{39}{5}}$

   (b) $1 - \dfrac{\binom{34}{13} + 5\binom{34}{12}}{\binom{39}{13}}$ (c) $\dfrac{3\binom{26}{5}}{\binom{39}{5}\binom{39}{13}}$

9. $\tfrac{1}{2}$
15. (a) P(A) = P(B) = P(C) = $\tfrac{1}{2}$; $P(A \cap B) = P(A \cap C) = P(B \cap C) = \tfrac{1}{4}$;
    $P(A \cap B \cap C) = 0$

13.18 Exercises (page 499)

1. (a) $P(H, H) = p_1 p_2$; $P(H, T) = p_1(1 - p_2)$; $P(T, H) = (1 - p_1)p_2$;
   $P(T, T) = (1 - p_1)(1 - p_2)$
   (b) Yes
   (c) No
   (d) H_1 and H_2, H_1 and T_2, H_2 and T_1, T_1 and T_2
2. (a) $\tfrac{319}{512}$
   (b) $\tfrac{11}{1024}$
   (c) 6

3. $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\dfrac{5'}{-6^{10}}$ $\dfrac{390625}{2519424}$ —

4. (a) $\frac{5}{16}$
   (b) $\frac{1}{2}$
   (c) $\frac{3}{16}$

5. (a) $(5!)^2/10! = \frac{1}{252}$
   (b) $\frac{1}{2}$

6. (a) $36p^{10} - 80p^9 + 45p^8$
   (b) $\frac{7}{128}$

7. It is advantageous to bet even money

15. (a) $f(p) = (1 - p)^2 + p^3$
    (b) $(\sqrt{31} - 4)/3$

8. $\begin{pmatrix} n \\ k \end{pmatrix} \dfrac{w^k b^{n-k}}{(w + b)^n}$

9. $\begin{pmatrix} 8 \\ 3 \end{pmatrix} \dfrac{17^5}{18^8} = \dfrac{9938999}{1377495072}$

10. $\frac{193}{512}$

11. $1 - (19/20)^{10} = 0.4013$

12. $\frac{193}{512}$

14. $59 \leq n \leq 65$

13.20 Exercises (page 504)

1. (a) $f(k) = 2k$
   (b) $f(k) = 3$"
   (c) $f(k) = p_k$, where p_k is the kth prime ≥ 2
   (d) one such function is $f(k) = (g(k), h(k))$, where

$$g(k) = \frac{m^2(k) + 3m(k)}{2} - k + 2, \quad h(k) = k - \frac{m^2(k) + m(k)}{2},$$

and

$$m(k) = \left[\frac{\sqrt{8k - 7} - 1}{2} \right]$$

where $[x]$ denotes the greatest integer $\leq x$

(e) $f(k) = 2^{g(k)} 3^{h(k)}$, where $g(k)$ and $h(k)$ are as defined in part (d)

13.22 Exercises (page 507)

1. $n = 0$: max $= 1$,   min $= \frac{1}{2}$
   $n = 1$: max $= \frac{1}{4}$,   min $= 0$
   $n = 2$: max $= \frac{1}{8}$,   min $= 0$
   $n = 3$: max $= \frac{1}{16}$,   min $= 0$

3. (a) $1 - qp^3 - pq^3$
   (b) $\frac{7}{8}$
   (c) $\frac{3}{5}$

4. (a) $3pq/(pq + 2)$
   (b) $\frac{1}{3}$
   (c) $2 \log 2 - 1$

13.23 Miscellaneous exercises on probability (page 507)

1. $\frac{1}{12}$

2. (a) $\frac{2}{19}$
   (b) $\frac{1}{10}$

3. (a) $\frac{1}{4}$
   (b) I--

4. (b) $\frac{15}{13}$
   (c) $\frac{13}{165}$

5. $\frac{64}{73}$

7. 0.65

8. (a) $\frac{5}{9}$
   (b) 6
   (c) $\frac{2}{9}$
   (d) No

9. $p^3 + 6p \left(\dfrac{1 - p}{2} \right)^3$

10. $np(1 - p)^{n-1} + np^{n-1}(1 - p)$

11. $\dfrac{n}{2^{n-1}} \left(1 - \dfrac{n}{2^{n-1}} \right)^{m-1}$