

CASUS I.

CAP. IX.

224. Habet ergo membrum supremum unicum Factorem realem μ , quod evenit si $\epsilon\epsilon$ sit minor quam $\alpha\gamma$: atque, posito t infinito, erit $\mu + \delta = 0$, quæ est æquatio pro Asymtota recta. Præbeat hæc æquatio valorem $\mu = c$; eritque,

$$\alpha tt(\mu - c) + t(\epsilon cc + \epsilon c + \eta) + \gamma c^3 + \zeta cc + \theta c + \iota = 0,$$

quæ est æquatio pro natura Asymtotæ. Hinc, prout $\epsilon cc + \epsilon c + \eta$ vel non fuerit $= 0$, vel sit $= 0$, duplex Asymtotæ indoles prodit; nempe vel $\mu - c = \frac{A}{t}$, vel $\mu - c = \frac{A}{tt}$; unde duæ primæ Linearum tertii ordinis species formantur, quæ ita se habebunt.

I.

PRIMA Species unicam habet Asymtotam rectam speciei $\mu = \frac{A}{t}$.

2.

SECUNDA Species unicam habet Asymtotam rectam speciei $\mu = \frac{A}{tt}$.

CASUS 2.

225. Sint membra supremi tres Factores simplices reales & inter se inæquales; quod evenit si in æquatione

$$\alpha tt\mu + \epsilon t\mu + \gamma \mu^3 + \delta tt + \epsilon t\mu + \zeta \mu + \eta t + \theta \mu + \iota = 0,$$

fuerit $\epsilon\epsilon$ major quam $\alpha\gamma$. Hoc igitur casu de unoquoque Factore eadem sunt tenenda; quæ modo de unico Factore sunt exposita. Unusquisque scilicet suppeditat binos ramos hyperbolicos vel speciei $\mu = \frac{A}{t}$, vel speciei $\mu = \frac{A}{tt}$, unde

LIB. II. in hoc casu quatuor diverse species Linearum tertii ordinis continentur, tribus Asymtotis rectis ad se invicem utcunque inclinatis præditæ, quæ species sunt.

3.

TERTIA Species tres habet Asymtotas speciei $u = \frac{A}{t}$.

4.

QUARTA Species duas habet Asymtotas speciei $u = \frac{A}{t^2}$ & unam speciei $u = \frac{A}{tt}$.

Quinta Species unam habet Asymtotam speciei $u = \frac{A}{t}$ & duas speciei $u = \frac{A}{tt}$. *

Sexta Species tres habet Asymtotas speciei $u = \frac{A}{tt}$.

226. Videamus autem an hæ omnes species sint possibiles; quem in finem sumamus hanc æquationem latissime patentem,

$$\gamma(\alpha y - \epsilon x)(\gamma y - dx) + exy + \zeta yy + \eta x + \theta y + \dots = 0,$$

cujus supremum membrum tres habet Factores reales; quamquam enim terminus xx est omissus, tamen æquatio non minus late patet. Ex præcedentibus autem intelligitur, Factorem y præbere Asymtotam formæ $u = \frac{A}{t}$, si non fuerit $\eta = 0$. Quare videamus cujusmodi Asymtotam præbeat Factor $\alpha y - \epsilon x$. Ad hoc ponamus $y = au + \epsilon t$, & $x = at - \epsilon u$; sitque, brevitas ergo, $\alpha^2 + \epsilon^2 = 1$, quod semper assumete licet; atque æquatio transformabitur in hanc formam.

$$\begin{aligned} & \epsilon(\epsilon y - \alpha u)tu + (2\alpha\epsilon y - (\alpha a - \epsilon\epsilon)\delta)tu + \alpha(\alpha y + \epsilon d)u^3 \\ & + \epsilon(\alpha e + \epsilon\zeta)tu + (2\alpha\epsilon\zeta + (\alpha a - \epsilon\epsilon)\epsilon)tu + \alpha(\alpha\zeta - \epsilon e)u^2 = 0 \\ & + (\alpha\eta + \epsilon\theta)t \quad \quad \quad + (\alpha\theta - \epsilon\eta)u \end{aligned}$$

Hic Factor $\alpha y - \epsilon x$ transit in u ; ex quo, posito t infinito, primum

* Vide infra pag. 119.

primum sit $\alpha = \frac{\alpha\epsilon + \zeta\delta}{\alpha\delta - \zeta\gamma} = c$, qui valor si loco α in CAP. IX.
secundo membro continente t substituatur, ostendet ex hoc
Factore α seu $\alpha\gamma - \zeta x$ Alymtotam oriri formæ $\alpha = \frac{A}{t}$
nisi fuerit

$$\frac{\alpha\eta + \zeta\theta}{\zeta} + \frac{(\alpha\epsilon + \zeta\delta)(\gamma\epsilon + \delta\zeta)}{(\alpha\delta - \zeta\gamma)^2} = 0.$$

Simili modo Factor $\gamma\gamma - \delta x$ Asymtotam præbebit formæ
 $\alpha = \frac{A}{t}$ nisi fuerit

$$\frac{\gamma\eta + \delta\theta}{\delta} + \frac{(\alpha\epsilon + \zeta\delta)(\gamma\epsilon + \delta\zeta)}{(\alpha\delta - \zeta\gamma)^2} = 0.$$

227. Hinc patet fieri utique posse ut neque η neque utraque formula modo inventa evanescat, ex quo species tertia utique erit possibilis. Quod ad speciem quartam attinet, ponatur $\eta = 0$, quo una Asymtota formæ $\alpha = \frac{A}{tt}$ prodeat; tum autem ambæ reliquæ expressiones in unam coalescunt, ideoque binæ reliquæ Asymtotæ erunt formæ $\alpha = \frac{A}{t}$, nisi fuerit $\theta + \frac{(\alpha\epsilon + \zeta\delta)(\gamma\epsilon + \delta\zeta)}{(\alpha\delta - \zeta\gamma)^2} = 0$; unde & species quarta est possibilis. At, si præter $\eta = 0$, una ex binis reliquis expressionibus reduciatur $= 0$, simul alteria evanescit; quam ob rem fieri non potest, ut duæ Asymtotæ fiant formæ $\alpha = \frac{A}{tt}$, quin simul tertia eandem formam induat; ex quo species quinta est impossibilis. Sexta autem ob hoc ipsum erit possibilis, quia oritur, si $\eta = 0$, & $\theta = -\frac{(\alpha\epsilon + \zeta\delta)(\gamma\epsilon + \delta\zeta)}{(\alpha\delta - \zeta\gamma)^2}$. Hi ergo duo casus quinque tantum præbuerunt species Linearum tertii ordinis, quod ea, quam quintam posuimus, prætermitti debet, &

5.

QUINTA Species tres habet Asymtotas speciei $\alpha = \frac{A}{tt}$.

CASUS

CASUS 3.

228. Habeat membrum supremum duos Factores α æquales; quod evenit, si in æquatione casus præcedentis primus terminus $\alpha t u$ evanescat. Æquatio ergo generalis ad hunc casum pertinens erit hujusmodi,

$$\alpha t u u - \epsilon u^3 + \gamma t t + \delta t u + \epsilon u u + \zeta t + \eta u + \theta = 0,$$

habet ergo membrum supremum duos Factores α æquales, ac tertium αu — ϵu reliquis inæqualem. Iste tertius Factor producet Asymtotam vel formæ $u = \frac{A}{t}$, vel formæ $u = \frac{A}{t t}$, prout fuerit hæc expressio

$$(\alpha \delta + 2\epsilon \gamma)(\alpha^2 \epsilon + \alpha \epsilon \delta + \epsilon \epsilon \gamma) - \alpha^3 (\alpha \gamma + \epsilon \zeta)$$

vel non $= 0$, vel $= 0$.

229. Quod ad duos Factores æquales attinet, primum casus occurrit, si γ non fuerit $= 0$; tum enim, facto $t = \infty$, fit $\alpha u u + \gamma t = 0$, quæ est æquatio pro Asymtota parabolica speciei $u u = At$. Hinc istæ duæ nascentur species novæ Linearum tertii ordinis, nempe.

6.

SEXTA Species habet unam Asymtotam speciei $u = \frac{A}{t}$
& unam Asymtotam speciei $u u = At$.

7.

SEPTIMA Species habet unam Asymtotam speciei $u = \frac{A}{t t}$
& unam parabolicam speciei $u u = At$.

230. Sit jam $\gamma = 0$; atque Factor tertius $\alpha t - \epsilon u$ dabit Asymtotam formæ $u = \frac{A}{t t}$, si fuerit

$$\delta(\alpha \epsilon + \epsilon \delta) = \alpha(\alpha \gamma + \epsilon \zeta)$$

sin autem hæc æqualitas non habeat locum, Asymtota erit formæ $u = \frac{A}{t}$. Habebimus ergo hanc æquationem

+ $\alpha t u u$

$$\begin{array}{rcl} + \alpha t u u & - & 6 u^3 \\ + \delta t u & + & \epsilon u u = 0 \\ + \zeta t & + & \eta u \\ + \theta & & \end{array}$$

Hic, facta $t = \infty$, fiet $\alpha u u + \delta u + \zeta = 0$.

Sit primum $\delta\delta$ minor quam $4\alpha\zeta$, atque hinc nulla orietur Asymtota; quare ex hoc casu duæ oriuntur species.

8.

OCTAVA Species habet unicam Asymtotam speciei

$$u = \frac{A}{t}.$$

9.

NONA Species habet unicam Asymtotam speciei

$$u = \frac{A}{tt}.$$

231. Sint æquationis $\alpha u u + \delta u + \zeta = 0$, ambæ radices reales & inæquales, nempe $\delta\delta$ major quam $4\alpha\zeta$; atque hinc duæ prodibunt Asymtotæ rectæ inter se parallelæ, utraque formæ $u = \frac{A}{t}$, qui casus denuo duas suppeditat Species.

10.

DECIMA Species habet unam Asymtotam speciei $u = \frac{A}{t}$, & duas inter se parallelas speciei $u = \frac{A}{t}$.

11.

UNDECIMA Species habet unam Asymtotam speciei $u = \frac{A}{t}$, & duas inter se parallelas speciei $u = \frac{A}{t}$.

232. Sint æquationis $\alpha u u + \delta u + \zeta = 0$, ambæ radices inter se æquales, seu $\delta\delta = 4\alpha\zeta$, seu $\alpha u u + \delta u + \zeta = \alpha(u - c)^2$, fietque $\alpha t(u - c)^2 = 6c^3 - \epsilon cc - \eta c - \theta$, unde oritur Asymtota recta una speciei $u u = \frac{A}{t}$. Hinc ergo duæ nascuntur Species novæ.

12.

DUODECIMA Species habet unam Asymtotam speciei Euleri *Introduct. in Anal. infin. Tom. II.*

Q

 $u =$

L I B. II. $u = \frac{A}{t}$ & unam speciei $uu = \frac{A}{t}$.

13.

DECIMATERTIA Species habet unam Asymtotam speciei $u = \frac{A}{tt}$ & unam speciei $uu = \frac{A}{t}$.

C A S U S I V.

233. Quod si membra supremi omnes tres Factores fuerint æquales, æquatio habebit hujusmodi formam,

$$\alpha u^3 + \epsilon tt + \gamma tu + \delta uu + \epsilon t + \zeta u + \eta = 0,$$

Hic primum spectandus est terminus ϵtt , qui si non desit, Curva habebit Asymtotam parabolicam speciei $u^3 = Att$, sive una oritur Species.

14.

DECIMAQUARTA Species habet unicam Asymtotam parabolicam speciei $u^3 = Att$.

234. Desit jam terminus ϵtt , eritque

$$\alpha u^3 + \gamma tu + \delta uu + \epsilon t + \zeta u + \eta = 0;$$

unde, posito t infinito, fiet $\alpha u^3 + \gamma tu + \epsilon t = 0$, nisi sint γ & $\epsilon = 0$. Non igitur sit $\gamma = 0$, atque in hac æquatione duæ continentur æquationes $\alpha uu + \gamma t = 0$, & $\gamma u + \epsilon = 0$; prior est pro Asymtota parabolica speciei $uu = At$; posterior vero, si ponatur $\frac{-\epsilon}{\gamma} = c$, dabit æquationem hanc

$$\gamma t(u - c) + \alpha c^3 + \delta cc + \zeta c + \eta = 0,$$

eritque ergo pro Asymtota hyperbolica speciei $u = \frac{A}{t}$, unde.

15.

DECIMAQUINTA Species unam habet Asymtotam parabolicam speciei $uu = At$, & unam rectam speciei $u = \frac{A}{t}$

$$\frac{A}{t}$$