neraliter pro $k$ per $e$ diuisibili fit $= 1$; cuiuis alii valori ipsius $k$ radix ab 1 diuersa respondet.

II. Quum sit $(\cos\frac{kP}{e} + i\sin\frac{kP}{e})^\lambda = \cos\frac{\lambda kP}{e} + i\sin\frac{\lambda kP}{e}$, patet, si $R$ sit radix talis quae respondeat valori ipsius $k$ ad $e$ primo, in progressione $R$, $RR$, $R^3$ etc. terminum $e^{\text{tum}}$ quidem esse $= 1$, omnes antecedentes vero ab 1 diuersos. Hinc statim sequitur, omnes $e$ quantitates $1, R, RR, R^3 \ldots R^{e-1}$ inaequales esse, et quum manifesto omnes aequationi $x^e - 1 = 0$ satisfaciant, exhibebunt omnes radices huius aequationis.

III. Denique in eadem suppositione aggregatum $1 + R^\lambda + R^{2\lambda} \ldots + R^{\lambda(e-1)}$ fit $= 0$, pro quouis valore integro ipsius $\lambda$ per $e$ non diuisibili; etenim est $= \frac{1 - R^{\lambda e}}{1 - R^\lambda}$, cuius fractionis numerator fit $= 0$, denominator vero non $= 0$. Quando vero $\lambda$ per $e$ diuisibilis est, illud aggregatum manifesto fit $= e$.

360. Sit, vt semper in praecc., $n$ numerus primus, $g$ radix primitiua pro modulo $n$, atque $n - 1$ productum e tribus integris positiuis; breuitatis caussa disquisitionem ita statim instituemus, vt etiam ad casus vbi $\alpha$ aut $\gamma = 1$ pateat; quando $\gamma = 1$, pro aggregatis $(\gamma, 1)$, $(\gamma, g)$ etc. radices $[1]$, $[g]$ etc. accipere oportebit. Supponamus itaque, ex omnibus $\alpha$ aggregatis $\beta\gamma$ terminorum cognitis $(\beta\gamma, 1)$, $(\beta\gamma, g)$,

$(\beta\gamma, gg)\ldots(\beta\gamma, g^{\alpha-1})$ deducenda esse aggregata $\gamma$ terminorum, quod negotium supra ad aequationem affectam $\beta^{\text{ti}}$ gradus reduximus, nunc vero per puram aeque altam absoluere docebimus. Ad abbreuiandum pro aggregatis $(\gamma, 1)$, $(\gamma, g^{\alpha})$, $(\gamma, g^{2\alpha})\ldots(\gamma, g^{\alpha\beta-\alpha})$, quae sub $(\beta\gamma, 1)$ contenta sunt, scribemus $a, b, c\ldots m$ resp.; pro his $(\gamma, g)$, $(\gamma, g^{\alpha+1})\ldots(\gamma, g^{\alpha\beta-\alpha+1})$ sub $(\beta\gamma, g)$ contentis resp. $a', b'\ldots m'$; pro his $(\gamma, gg)$, $(\gamma, g^{\alpha+2})\ldots(\gamma, g^{\alpha\beta-\alpha+2})$ resp. $a''$, $b''\ldots m''$ etc. vsque ad ea quae sub $(\beta\gamma, g^{\alpha-1})$ continentur.

I. Iam designet $R$ indefinite radicem aequationis $x^{\beta}-1=0$, supponamusque ex euolutione potestatis $\beta^{\text{tae}}$ functionis $t=a+Rb+RRc\ldots+R^{\beta-1}m$ oriri per praecepta art. 345.

$$\begin{aligned}N&+Aa+Bb+Cc\ldots+Mm\\&+A'a'+B'b'+C'c'\ldots+M'm'\\&+A''a''+B''b''+C''c''\ldots+M''m''\\&+\text{etc.}=T\end{aligned}$$

vbi omnes coëfficientes $N$, $A$, $B$, $A'$ etc. erunt functiones rationales integrae ipsius $R$. Supponantur etiam potestates $\beta^{\text{tae}}$ duarum aliarum functionum $u=R^{\beta}a+Rb+RRc\ldots+R^{\beta-1}m$, $u'=b+Rc+R''d\ldots+R^{\beta-2}m+R^{\beta-1}a$ resp. euolui in $U$ et $U'$, perspicieturque facile ex art. 350., quum $u'$ oriatur ex $t$ commutando aggregata $a, b, c\ldots m$ resp. cum $b, c, d\ldots a$, fore $U'=$

$$\begin{aligned}N &+ Ab + Bc + Cd \ldots + Ma\\ &+ A'b' + B'c' + C'd' \ldots + M'a'\\ &+ A''b'' + B''c'' + C''d'' \ldots + M''a''\\ &+ \text{etc.}\end{aligned}$$

Porro patet quum sit $u = Ru'$, fore $U = R^{\varepsilon}U'$, quare propter $R^{\varepsilon} = 1$ coëfficientes correspondentes in $U$ et $U'$ aequales erunt; denique quum $t$ et $u$ in eo tantum differant, quod $a$ in $t$ per vnitatem, in $u$ per $R^{\varepsilon}$ multiplicatur, facile intelligetur, omnes coëfficientes correspondentes (i. e. qui eadem aggregata multiplicant) in $T$ et $U$ aequales esse, et proin etiam omnes coëfficientes correspondentes in $T$ et $U'$. Hinc tandem colligitur $A = B = C$ etc. $= M$; $A' = B' = C'$ etc., $A'' = B'' = C''$ etc. etc., vnde $T$ reducitur ad formam talem $N + A(\varepsilon\gamma, 1) + A'(\varepsilon\gamma, g) + A''(\varepsilon\gamma, gg)$ etc., vbi singuli coëfficientes $N$, $A$, $A'$ etc. sub formam talem reducere licet $pR^{\varepsilon-1} + p'R^{\varepsilon-2} + p''R^{\varepsilon-3}$ + etc. ita vt $p$, $p'$, $p''$ etc. sint numeri integri dati.

II. Si pro $R$ accipitur radix determinata aequationis $x^{\varepsilon} - 1 = 0$ (cuius solutionem iam haberi supponimus), et quidem talis cuius nulla inferior potestas quam $\varepsilon^{\text{ta}}$ vnitati aequalis est, etiam $T$ quantitas determinata erit, ex qua $t$ per aequationem puram $t^{\varepsilon} - T = 0$ deriuare licet. At quum haec aequatio $\varepsilon$ radices habeat, quae erunt $t$, $Rt$, $RRt \ldots R^{\varepsilon-1}t$, dubium videri potest, quamnam radicem adoptare oporteat. Hoc vero prorsus rbitrarium esse, ita facile apparebit. Meminisse oportet, postquam omnia aggregata