

LIB. II. area Sectoris $ACD = \frac{1}{2} s$, a qua auferatur Triangulum

$$ACD = \frac{1}{2} AC \cdot DE = \frac{1}{2} \cdot \sin.s, \text{ remanebitque Segmentum}$$

$$AD = \frac{1}{2} s - \frac{1}{2} \cdot \sin.s, \text{ quod æquale esse debet semissi semicirculi } ADB, \text{ at area semicirculi est } = \frac{1}{2} \pi: \text{ unde}$$

$$\text{erit } s - \sin.s = \frac{1}{2} \pi = 90^\circ, \text{ ideoque } s - 90^\circ = \sin.s. \text{ Ponatur } s - 90^\circ = u; \text{ erit } \sin.s = \cos.u, \text{ & hanc ob rem } u = \cos.u.$$

Per Problema ergo primum erit $u = 42^\circ, 20', 47'', 14'''$; hincque $s = \text{angulo } ACD = 132^\circ, 20', 47'', 14'''$, & angulus $BCD = 47^\circ, 39', 12'', 46'''$. Ipsa vero Corda AD erit =

1, 8295422. Q. E. F.

535. Sic igitur in Circulo Segmentum absinditur cujus area sit totius Circuli pars quarta, Segmentum autem semissi Circuli æquale est ipse semicirculus ejusque Corda Diameter. Simili modo Segmentum inveniri potest, quod sit triens totius Circuli, quod sequenti Problemate investigemus.

P R O B L E M A V.

TAB. XXIX. Ex puncto Peripherie A educere duas Cordas AB, AC, quibus area Circuli in tres partes æquales dividatur.
Fig. 115.

S O L U T I O.

Posito Circuli Radio = 1, & hemiperipheria = π , sit Arcus AB vel $AC = s$; eritque area Segmenti AEB vel $AFC = \frac{1}{2} s - \frac{1}{2} \cdot \sin.s$: at area Circuli est = π ; unde, cum Segmenti AEB area debeat esse triens Circuli, fiet $\frac{1}{2} s - \frac{1}{2} \cdot \sin.s = \frac{\pi}{3} = 60^\circ$; seu, $s - \sin.s = 120^\circ$, ideoque $s - 120^\circ = \sin.s$. Sit $s - 120^\circ = u$, erit $u = \sin.(u + 120) = \sin.(60 - u)$. Arcus

Arcus ergo u quare debet, qui sit æqualis sinui anguli 60° — u . CAP.
Erit ergo u minor quam 60° ; ad quem Arcum inveniendum fa- XXII.
ciamus sequentes positiones

$60 - u = 20^\circ$	$60 - u = 30^\circ$	$60 - u = 40^\circ$
$l.u = 1,3010300$	$1,4771213$	$1,6020600$
subtrahe $1,7581226$	$1,7581226$	$1,7581226$
$l.u = 9,5429074$	$9,7189987$	$9,8439374$
$l.fin.(60 - u) = 9,8080575$	$9,6989700$	$9,5340517$
$+ 2651601$	200287	3098857

Patet ergo angulum u aliquanto esse minorem quam 30° , &, calculo subducto, major esse debet quam 29° sit ergo $u = 29^\circ$.

$$\begin{aligned}
 60 - u &= 31^\circ \\
 l.u &= 1,4623980 \\
 \text{subtrahe } &1,7581226 \\
 l.u &= 9,7042754 \\
 l.fin.(60 - u) &= 9,7118393 \\
 + &75639 \\
 \hline
 &200287 \\
 275926 : 75639 &= 1^\circ : 16' : 26''.
 \end{aligned}$$

Foret ergo angulus $u = 29^\circ, 16', 26''$, ad quem accuratius inveniendum, faciamus has hypotheses uno tantum minuto differentes

$u = 29^\circ, 16'$ seu	$u = 29^\circ, 17'$ seu
$u = 1756'$	$u = 1757'$
$60 - u = 30^\circ 44'$	$60 - u = 30^\circ 43'$
$l.u = 3,2445245$	$3,2447718$
subtrahe $3,5362739$	$3,5362739$
$l.u = 9,7082506$	$9,7084979$
$l.fin.(60 - u) = 9,7084575$	$9,7082450$
$+ 2069$	2529
	$4598 : 2069 = 1' : 27'' : 0'''.$

Erit ergo vere $u = 29^\circ, 16', 27'', 0'''$,
hincque

Euleri *Introduct. in Anal. infin. Tom. II.* R r Arcus

L I B . II.Arcus $s = AEB = 149^\circ, 16', 27'', 0''' = AFC;$
unde resultatArcus $BC = 61^\circ, 27', 6'', 0''',$
ipsa veroChorda $AB = AC = 19285340.$ Q. E. F.

536. His Problematis, quibus Arcus quispiam queritur dato Sinu vel Cosinui æqualis, adjungamus sequens, quo quidem idem negotium proponitur, attamen major difficultas occurrit.

P R O B L E M A V I .

T A B .
XXIX.
Fig. 116. *In semicirculo AEB Arcum AE abscindere, ita ut, ducto ejus Sinu ED, Arcus AE sit æqualis summe rectarum AD + DE.*

S O L U T I O .

Quoniam statim patet hunc Arcum quadrante esse majorem, queramus ejus Complementum BE , & vocemus Arcum $BE = s$, ita ut sit Arcus $AE = 180^\circ - s$, atque $\angle AC = 1$, $CD = \cos s$, $DE = \sin s$, erit $180^\circ - s = 1 + \cos s + \sin s$. At, est $\sin s = 2 \sin \frac{1}{2} s \cdot \cos \frac{1}{2} s$, & $1 + \cos s = 2 \cos \frac{1}{2} s \cdot \cos \frac{1}{2} s$; unde fit $180^\circ - s = 2 \cos \frac{1}{2} s (\sin \frac{1}{2} s + \cos \frac{1}{2} s)$. At, est $\cos(45^\circ - \frac{1}{2} s) = \sqrt{2} \cdot \cos \frac{1}{2} s + \frac{1}{\sqrt{2}} \cdot \sin \frac{1}{2} s$: ergo $\sin \frac{1}{2} s + \cos \frac{1}{2} s = \sqrt{2} \cdot \cos(45^\circ - \frac{1}{2} s)$: unde erit $180^\circ - s = 2\sqrt{2} \cdot \cos \frac{1}{2} s \cdot \cos(45^\circ - \frac{1}{2} s)$. Hac facta reductione, faciamus sequentes positiones

$$\frac{1}{2} s =$$

$\frac{1}{2} s = 20^\circ$	$\frac{1}{2} s = 21^\circ$	C A P. X X I I .
$45^\circ - \frac{1}{2} s = 25^\circ$	$45^\circ - \frac{1}{2} s = 24^\circ$	
$180 - s = 140^\circ$	$180 - s = 138^\circ$	
$l.(180 - s) = 2, 1461280$	$2, 1398791$	
subtrahe $1, 7581226$	$1, 7581226$	
$l.(180 - s) = 0, 3880054$	$0, 3817565$	
$l.co\sqrt{\cdot} \frac{1}{2} s = 9, 9729898$	$9, 9701517$	
$l.co\sqrt{\cdot} (45^\circ - \frac{1}{2} s) = 9, 9572757$	$9, 9607302$	
$l.2\sqrt{2} = 0, 4515450$	$0, 4515450$	
Error $+ 61989$	$0, 3824269$	
	6704	
$68693 : 61989 = 1^\circ : 54'$		

Hinc continetur $\frac{1}{2} s$ intra limites $20^\circ, 54'$, & $20^\circ, 55'$,
 ideoque sequentes hypotheses fiant

$\frac{1}{2} s = 20^\circ, 54'$	$\frac{1}{2} s = 20^\circ, 55'$
$45^\circ - \frac{1}{2} s = 24^\circ, 6'$	$45^\circ - \frac{1}{2} s = 24^\circ, 5'$
$180 - s = 138^\circ, 12'$	$180 - s = 138^\circ, 10'$
feu	feu
$180 - s = 8292'$	$180 - s = 8290'$
$l.(180 - s) = 3, 9186593$	$3, 9185545$
subtrahe $3, 5362739$	$3, 5362739$
$0, 3823854$	$0, 3822806$
$l.co\sqrt{\cdot} \frac{1}{2} s = 9, 9704419$	$9, 9703937$
$l.co\sqrt{\cdot} (45^\circ - \frac{1}{2} s) = 9, 9603919$	$9, 9604484$
$l.2\sqrt{2} = 0, 4515450$	$0, 4515450$
Error $+ 66$	1065
$1131 : 66 = 1' : 3'', 30'''.$	

R r 2 Hanc

L I B . II. Hanc ob rem erit $\frac{1}{2}s = 20^\circ, 54', 3'', 30'''$,
inde

$$s = 41^\circ, 48', 7'', 0''' = BE$$

ideoque Arcus quæstus

$$AE = 138^\circ, 11', 53'', 0'''.$$

Erit vero Linea

$$DE = 0, 6665578, \& AD = 1, 7454535. Q. E. F.$$

§ 37. Comparemus nunc Arcus cum suis Tangentibus; &, cum in primo quadrante Tangentes sint Arcubus minores, quæramus Arcum, qui suæ Tangentis semissi sit æqualis, quo solvetur

P R O B L E M A V I I .

T A B . *Abscindere Sectorem ACD, qui sit semissis Trianguli ACE*
XXIX. *a Radio AC, Tangente AE & Secante CE comprehensi.*

Fig. 117.

S O L U T I O .

Posito Arcu $AD = s$, erit Sector $ACD = \frac{1}{2}s$, Triangulum vero $ACE = \frac{1}{2} \cdot \text{tang. } s$: unde debet esse $\frac{1}{2} \cdot \text{tang. } s = s$, seu $2s = \text{tang. } s$. Faciamus ergo has hypotheses

$s = 60^\circ$	$s = 70^\circ$	$s = 66^\circ$	$s = 67^\circ$
$l. 2s = 2,0791812$	$2,1461280$	$2,1205739$	$2,1271048$
$1,7581226$	$1,7581226$	$1,7581226$	$1,7581226$
$l. 2s = 0,3210586$	$0,3880054$	$0,3624513$	$0,3689822$
$l. \text{tang. } s = 0,2385606$	$0,4389341$	$0,3514169$	$0,3721481$
$+ 824980$	$- 509287$	$+ 110344$	$- 31659$

Hinc ipsis s reperiuntur limites arctiores $66^\circ, 46', \& 66^\circ, 47'$: quare fiat

$s =$

C A P. XXII.	
$s = 66^\circ, 46'$ feu	$s = 66^\circ, 47'$ feu
$s = 4006'$	$s = 4007'$
$2s = 8012'$	$2s = 8014'$
$l. 2s = 3, 9037409$	$3, 9038493$
$3, 5362739$	$3, 5362739$
$l. 2s = 0, 3674670$	$0, 3675754$
$l. tang. s = 0, 3672499$	$0, 3675985$
Error + 2171	— 231
	$\frac{231}{2402 : 2171 = 1' : 54'', 14''}.$

unde erit

Arcus $s = AD = 66^\circ, 46', 54'', 14''$,
hincqueTangens $AE = 2, 3311220$. Q. E. F.

538. Proponatur nunc sequens.

P R O B L E M A V I I I .

Proposito Circuli quadrante ACB invenire Arcum AE, qui
equalis sit Chordae sue AE ad occursum F usque producte.

T A B.
XXIX.

Fig. 118.

S O L U T I O .

Sit Arcus $AE = s$, erit ejus Chorda $AE = 2 \cdot \sin. \frac{1}{2} s$, si-
nus versus $AD = 1 - \cos. s = 2 \cdot \sin. \frac{1}{2} s \cdot \sin. \frac{1}{2} s$: unde
Triangula similia ADE, ACF , dabunt $2 \cdot \sin. \frac{1}{2} s \cdot \sin. \frac{1}{2} s :$
 $2 \sin. \frac{1}{2} s = 1 : s$, eritque ergo $s \cdot \sin. \frac{1}{2} s = 1$. Fiant ergo
sequentes positiones