9. (a) One needs $n-1$ multiplications; in each case the partial product 3^j has at most $O(n)$ digits and 3 has 2 digits, so there are $O(n)$ bit operations; thus, the total is $O(n^2)$. (b) Here the partial product has $O(n\,log\,n)$ digits, so each multiplication takes $O(n\,log^2 n)$ bit operations; the total is $O(n^2 log^2 n)$.
10. $O(n^2 log^2 N)$.
11. (a) $O(n\,log^2 n)$; (b) $O(log^2 n)$.
12. $O(rsn(log^2 m + log\,n))$.
13. (a) The product of $O(n/log\,n)$ numbers each with $O(log\,n)$ digits has $O(n/log\,n)\cdot O(log\,n) = O(n)$ digits. (b) $O(n\,log\,n)$; (c) $O(n^2)$.
14. (a) $O(\sqrt{n}\,log^2 n)$; (b) $O(\sqrt{n}\,log\,n)$.
15. $O(m\,log\,n)$.
16. Suppose that n has $k+1$ bits. As a first approximation to $m = [\sqrt{n}]$ take a 1 followed by $[k/2]$ zeros. Find the digits of m from left to right after the 1 by each time trying to change the zero to 1, and if the square of the resulting m is larger than n, putting it back to 0.

§ **I.2.**

1. (b) A simple counterexample: let $b = -a$.
2. 16 divisors: 1, 3, 5, 7, 9, 15, 21, 27, 35, 45, 63, 105, 135, 189, 315, 945.
3. (a) When $a|n$ write $n = ab$ and let $a \longleftrightarrow b$. (b) Given $n = ab$ with $a \geq b$, set $s = (a+b)/2$ and $t = (a-b)/2$. Conversely, given $n = s^2 - t^2$, set $a = s+t$, $b = s-t$ to get the reverse correspondence. (c) $473^2 - 472^2$; $159^2 - 156^2$; $97^2 - 92^2$; $71^2 - 64^2$; $57^2 - 48^2$; $39^2 - 24^2$; $33^2 - 12^2$; $31^2 - 4^2$.
4. (b) $100! = 2^{97}\cdot 3^{48}\cdot 5^{24}\cdot 7^{16}\cdot 11^9\cdot 13^7\cdot 17^5\cdot 19^5\cdot 23^4\cdot 29^3\cdot 31^3\cdot 37^2\cdot 41^2\cdot 43^2\cdot 47^2\cdot 53\cdot 59\cdot 61\cdot 67\cdot 71\cdot 73\cdot 79\cdot 83\cdot 89\cdot 97$. (c) The formula is $(n - S_p(n))/(p-1)$. To prove this, write $n = d_{k-1}p^{k-1} + \cdots + d_1 p + d_0$, and note that for each j: $[n/p^j] = d_{k-1}p^{k-1-j} + \cdots + d_{j+1}p + d_j$. Then use the formula in part (a).
6. (a) $1 = 11\cdot 19 - 8\cdot 26$; (b) $17 = 1\cdot 187 - 5\cdot 34$; (c) $1 = 205\cdot 160 - 39\cdot 841$; (d) $13 = 65\cdot 2171 - 54\cdot 2613$.
7. For example, here's a comparison between the two ways in the case of part (d):

$$\begin{aligned} 2613 &= 2171 + 442 \\ 2171 &= 4\cdot 442 + 403 \\ 442 &= 403 + 39 \\ 403 &= 10\cdot 39 + 13 \\ 39 &= 3\cdot 13. \end{aligned} \qquad \begin{aligned} 2613 &= 2171 + 442 \\ 2171 &= 5\cdot 442 - 39 \\ 442 &= 11\cdot 39 + 13 \\ 39 &= 3\cdot 13. \end{aligned}$$