116. Si ergo in Sectione conica binæ Diametri conjugatæ habeantur, GI, EF & gi, ef, erit primo CAP. V.

$$Cg : Ce = CG. \sin. ECe : CG. \sin. GCg.$$

Ergo

$$\sin. GCg : \sin. ECe = CE. Ce : CG. Cg.$$

& si chordæ Ee & Gg ducantur, fiet hinc Triangulum $CGg =$ Triangulo CEe. Deinde erit $Cg : Ce = CG. \sin. GCe : CE. \sin. gCE$, seu $Ce. CG. \sin. GCe = CE. Cg. \sin. gCE$: unde, si ducantur chordæ Ge & gE, erunt Triangula GCe & gCE inter se æqualia, seu e regione erit Triangulum $ICf =$ Triangulo iCF. Ultima vero æquatio $ab. \sin. (q + \pi - p) = fg. \sin. q$ dabit $Cg. Ce. \sin. gCe = CG. CE. \sin. GCE$. Quod si ergo ducantur chordæ EG & eg, vel e regione FI & fi erunt pariter Triangula ICF & iCf æqualia: unde sequitur omnia parallelogramma, quæ circa binas Diametros conjugatas describuntur, inter se esse æqualia.

117. Habentur ergo tria triangulorum paria inter se æqualia, nempe,

I. Triangulum FCf æquale Triangulo ICi.
II. Triangulum fCI æquale Triangulo FCi.
III. Triangulum FCI æquale Triangulo fCi.

Unde sequitur fore trapezia $FfCI$ & $iICf$ inter se æqualia; a quibus si auferatur idem triangulum fCI, erit Triangulum $FIf =$ Triangulo Ifi: quæ cum super eadem basi fI sint constituta, necesse est ut sit chorda Fi chordæ fI parallela. Porro itaque erit Triangulum $FIi =$ Triangulo ifF, ad quæ si addantur triangula æqualia FCI & fCi, erunt quoque hæc trapezia inter se æqualia $FCIi = iCfF$.

118. Hinc etiam deducitur methodus ad quodvis Lineæ secundi ordinis punctum M tangentem MT ducendi. Sumta enim Diametro GI pro Axe, cui conjugatæ semissis sit EC, ex puncto M ipsi CE parallela ad Axem ducatur MP, quæ erit semiordinata, ac $PN = PM$. Ducta CM, quæ erit Semidiameter, quæratur ejus Semidiameter conjugata CK, cui tangens MT quæsita erit parallela. Sit angulus $GCE = q$; TAB. VII. Fig. 27.

LIB. II. $GCM = p$ & $ECK = \varpi$; erit, uti vidimus, $\frac{EC^2}{GC^2} = \frac{\sin.p.\sin.(q+\pi)}{\sin.\pi.\sin.(q-p)}$ & $MC = CG\sqrt{\frac{\sin.q.\sin.(q+\pi)}{\sin.(q-p).\sin.(q+\pi-p)}}$. At in Triangulo CMP est $MC^2 = CP^2 + MP^2 + 2PM \times CP.\cos.q.$ & $MP : MC = \sin.p : \sin.q.$ & $MP : CP = \sin.p : \sin.(q-p)$. Deinde in Triangulo CMT, ob angulos datos, erit $CM : CT : MT = \sin.(q+\pi) : \sin.(q+\pi-p) : \sin.p$. Hinc, angulis eliminatis, erit $MC = CG\sqrt{\frac{MC}{CP}.\frac{CM}{CT}}$ seu $CG^2 = CP.CT$. Hinc erit $CP : CG = CG : CT$, unde positio tangentis expedite invenitur. Erit autem ex hac proportione *dividendo* $CP : PG = CG : TG$; & ob $CG = CI$ *componendo* $CP : IP = CG : TI$.

119. Cum sit $\frac{CE^2}{CG^2} = \frac{\sin.p.\sin.(q+\varpi)}{\sin.\varpi.\sin.(q-p)}$ & $\frac{CK^2}{CM^2} = \frac{\sin.p.\sin.(q-p)}{\sin.\pi.\sin.(q+\pi)}$, itemque $\frac{CM^2}{CG^2} = \frac{\sin.q.\sin.(q+\pi)}{\sin.(q-p).\sin(q+\pi-p)}$ & $\frac{CK^2}{CE^2} = \frac{\sin.q.\sin.(q-p)}{\sin.(q+\varpi).\sin.(q+\pi-p)}$, erit $\frac{CE^2+CG^2}{CG^2} = \frac{\sin.p.\sin.(q+\varpi)+\sin.\varpi.\sin.(q-p)}{\sin.\pi.\sin.(q-p)}$, & $\frac{CK^2+CM^2}{CM^2} = \frac{\sin.p.\sin.(q-p)+\sin.\pi.\sin.(q+\varpi)}{\sin.\varpi.\sin.(q+\pi)}$. At est $\sin.A.\sin.B = \frac{1}{2}\cos.(A-B) - \frac{1}{2}\cos.(A+B)$ & vicissim $\frac{1}{2}\cos.A - \frac{1}{2}\cos.B = \sin.\frac{A+B}{2}.\sin.\frac{B-A}{2}$. Unde erit $\sin.p.\sin.(q+\pi) + \sin.\varpi.\sin.(q-p) = \frac{1}{2}\cos.(q+\varpi-p) - \frac{1}{2}\cos.(q+\pi+p) + \frac{1}{2}\cos.(q-\pi-p) - \frac{1}{2}\cos.(q+\pi-p) = \frac{1}{2}\cos.(q-\varpi-p) - \frac{1}{2}\cos.(q+\pi+p) = \sin.q.\sin.(p+\pi)$. Atque $\sin.p.\sin.(q-p) + \sin.\pi.\sin.(q+\varpi) = \frac{1}{2}\cos.(q-2p) - \frac{1}{2}\cos.q + \frac{1}{2}\cos.q -$

$\frac{1}{2}$

$\frac{1}{2}\cos.(q+2\varpi) = \frac{1}{2}\cos.(q-2p) - \frac{1}{2}\cos.(q+2\varpi) =$ CAP. V.
$\sin.(q+\varpi-p).\sin.(p+\varpi)$.

Hinc ergo erit

$$\frac{CE^2+CG^2}{CG^2} = \frac{\sin.q.\sin.(p+\pi)}{\sin.\pi.\sin.(q-p)},$$

&

$$\frac{CK^2+CM^2}{CM^2} = \frac{\sin.(q+\varpi-p).\sin.(p+\pi)}{\sin.\varpi.\sin.(q+\pi)},$$

unde conficitur

$$\frac{CE^2+CG^2}{CK^2+CM^2} = \frac{CG^2}{CM^2}\cdot\frac{\sin.q.\sin.(q+\pi)}{\sin(q-p).\sin.(q+\varpi-p)} = \frac{CG^2}{CM^2}\cdot\frac{CM^2}{CG^2}.$$

Quare erit $CE^2+CG^2 = CK^2+CM^2$, ideoque in eadem Linea ſecundi ordinis ſumma quadratorum binarum Diametrorum conjugatarum ſemper eſt conſtans.

120. Cum igitur dentur duæ Semidiametri conjugatæ CG & CE, pro Semidiametro CM ad lubitum aſſumta ſtatim reperitur ejus ſemidiameter conjugata CK ſumendo $CK = \sqrt{(CE^2+CG^2-CM^2)}$. Ex ſuperioribus ergo Sectionum conicarum proprietatibus erit $TG.TI : TM^2 = CG.CI : CK^2 = CG^2 : CK^2 = CG^2 : CE^2+CG^2-CM^2$; ideoque $TM^2 = CG\sqrt{\frac{CE^2+CG^2-CM^2}{TG.TI}}$. Simili modo, ſi producta Ordinata MN, ducatur tangens NT, ambæ tangentes MT & NT Axi TI in eodem puncto T occurrent. Erit enim pro utraque $CP : CG = CG : CT$. At vero ducta recta CN erit $TN = CG\sqrt{\frac{CE^2+CG^2-CM^2}{TG.TI}}$, adeoque $TM^2 : TN^2 = CE^2+CG^2-CM^2 : CE^2+CG^2-CN^2$. Erit vero, ob biſectam MN in P, $\sin.CTM : \sin.CTN = TN : TM = \sqrt{(CE^2+CG^2-CN^2)} : \sqrt{(CE^2+CG^2-CM^2)}$.

121. Ducantur in terminis Diametri A & B tangentes AK, TAB.
BL, ac producatur tangens quæcunque MT' donec utramque VII.
tangentem ſecet in punctis K & L. Sit ECF Diameter con- Fig. 28.
jugata, cui tum Applicatæ MP tum tangentes AK & BL,

 erunt

LIB. II. erunt parallelæ. Cum jam ſit, ex natura tangentis, $CP : CA = CA : CT$ ob $CB = CA$ erit $CP : AP = CA : AT$, & $CP : BP = CA : BT$ ergo $CP : CA = CA : CT = AP : AT = BP : BT$, hincque $AT : BT = AP : BP$. At eſt $AT : BT = AK : BL$, ergo $AK : BL = AP : BP$. Deinde eſt $AT = \frac{CA \cdot AP}{CP}$; $BT = \frac{CA \cdot BP}{CP}$; & $PT = \frac{CA \cdot AP}{CP} + AP = \frac{AP \cdot BP}{CP}$ ergo $AT : PT = CA : BP = AK : PM$; ſimilique modo erit $BT : PT = CA : AP = BL : PM$; unde fit $AK = \frac{CA \cdot PM}{BP}$, $BL = \frac{CA \cdot PM}{AP}$ & $AK \cdot BL = \frac{CA^2 \cdot PM^2}{AP \cdot BP}$. At eſt $AP \cdot BP : PM^2 = AC^2 : CE^2$, unde conſequitur iſta egregia proprietas $AK \cdot BL = CE^2$, ex qua porro fit $AK = CE\sqrt{\frac{AP}{BP}}$ & $BL = CE\sqrt{\frac{BP}{AP}}$, & $AP : BP = AK^2 : CE^2 = CE^2 : BL^2 = KM : ML$, atque $AK : BL = KM : LM$.

122. In quocunque ergo Curvæ puncto M ducatur tangens occurrens tangentibus parallelis AK, BL in K & L, erit ſemper Semidiameter CE tangentibus AK & BL parallela media proportionalis inter AK & BL, ſeu erit $CE^2 = AK \times BL$. Quod ſi ergo in alio quocunque Curvæ puncto m ſimili modo ducatur tangens kml, erit quoque $CE^2 = Ak \cdot Bl$, ideoque $AK : Ak = Bl : BL$; hincque erit quoque $AK : Kk = Bl : Ll$. Secent tangentes KL & kl ſe mutuo in o, eritque $AK : Bl = Ak : BL = Kk : Ll = ko : lo = Ko : Lo$. Atque hæ ſunt præcipuæ Sectionum conicarum proprietates, ex quibus NEWTONUS plurima inſignia problemata reſolvit in *Principiis*.

123. Cum ſit $AK : Bl = Ko : Lo$, ſi tangens LB producatur in I ut ſit $BI = AK$, erit I punctum, ubi tangens ex altera parte ipſi KL parallela hanc tangentem LB eſſet ſectura, quemadmodum K in tangente LK eſt punctum, ubi ea

a

a tangente AK ipsi BL parallela secatur. Transibit ergo recta IK per Centrum C, ibique bifariam secabitur. Quodsi igitur duæ quæcunque tangentes BL, ML, modo præscripto in I & K producantur, eæque a tertia tangente lmo in punctis l & o secentur, erit $BI : Bl = Ko : Lo$, &, *componendo*, $IB : Il = Ko : KL$, ubicunque ergo tertia tangens lmo ducatur erit perpetuo $IB . KL = Il . Ko$. Ducta ergo quarta tangente quacunque $\lambda\mu\omega$ binas primum assumtas IL & KL secante in λ & ω, erit pariter $IB . KL = I\lambda . K\omega$, ideoque $Il . Ko = I\lambda . K\omega$ seu $Il : I\lambda = K\omega : Ko$. Ductis ergo rectis $l\omega$, λo, in qua ratione hæ secabuntur, recta per sectionum puncta transiens in eadem ratione secabit rectam IK. Quare si rectæ $l\omega$ & λo bisecentur, recta per bisectionis puncta transiens, bisecabit quoque rectam IK ideoque per Centrum Sectionis conicæ C transibit. Cap. V.

124. Quod recta nmH, quæ rectas $l\omega$, λo in data ratione secat, in eadem ratione secare debeat rectam KI, si quidem fuerit $Il : I\lambda = K\omega : Ko$, seu $I\lambda : \lambda l = Ko : o\omega$, hoc modo ex Geometria ostendetur. Secet recta mn utramque $l\omega$ & λo in ratione $m : n$, seu sit $\lambda m : mo = ln : n\omega = m : n$, & ea producta trajiciat tangentes IL, & KL in Q & R; eritque $\sin. Q : \sin. R = \frac{ln}{Ql} : \frac{n\omega}{R\omega} = \frac{\lambda m}{Q\lambda} : \frac{mo}{Ro} = \frac{m}{Ql} : \frac{n}{R\omega}$, ergo $Ql : R\omega = Q\lambda : Ro$, & *dividendo* $l\lambda : o\omega = Q\lambda : Ro = Ql : R\omega$. Cum vero sit $l\lambda : o\omega = I\lambda : Ko$, erit quoque $QI : RK = l\lambda : o\omega$, & $\sin. Q : \sin. R = \frac{m}{l\lambda} : \frac{n}{o\omega}$. At est quoque $\sin. Q : \sin. R = \frac{HI}{QI} : \frac{HK}{KR} = \frac{HI}{l\lambda} : \frac{HK}{o\omega}$, unde fit $HI : HK = m : n = \lambda m : mo = ln : n\omega$. Tab. VIII. Fig. 30.

125. Datis duabus Semidiametris conjugatis $CG . CE$, quæ angulum obliquum $GCE = q$ inter se comprehendant, semper reperiri poterunt duæ aliæ Semidiametri conjugatæ CM & CK quæ angulum MCK rectum constituant. Sit angulus $GCM = p$; & posito $ECK = \pi$, erit $q + \pi - p = 90°$ ideoque Tab. VII. Fig. 27.