

LIB. II. area Sectoris  $ACD = \frac{1}{2} s$ , a qua auferatur Triangulum

$ACD = \frac{1}{2} AC \cdot DE = \frac{1}{2} \cdot \sin s$ , remanebitque Segmen-

tum  $AD = \frac{1}{2} s - \frac{1}{2} \cdot \sin s$ , quod æquale esse debet se-

missi semicirculi  $ADB$ , at area semicirculi est  $= \frac{1}{2} \pi$ ; unde

erit  $s - \sin s = \frac{1}{2} \pi = 90^\circ$ , ideoque  $s - 90^\circ = \sin s$ . Po-

natur  $s - 90^\circ = u$ ; erit  $\sin s = \cos u$ , & hanc ob rem  $u =$

$\cos u$ . Per Problema ergo primum erit  $u = 42^\circ, 20', 47'', 14'''$ ;

hincque  $s =$  angulo  $ACD = 132^\circ, 20', 47'', 14'''$ , & angulus

$BCD = 47^\circ, 39', 12'', 46'''$ . Ipsa vero Corda  $AD$  erit  $=$

1, 8295422. Q. E. F.

535. Sic igitur in Circulo Segmentum abscinditur cujus area sit totius Circuli pars quarta, Segmentum autem semissi Circuli æquale est ipse semicirculus ejusque Corda Diameter. Simili modo Segmentum inveniri potest, quod sit triens totius Circuli, quod sequenti Problemate investigemus.

## PROBLEMA V.

TAB. XXIX. Ex puncto Peripheria A educere duas Cordas AB, AC, quibus area Circuli in tres partes æquales dividatur.  
Fig. 115.

## SOLUTIO.

Posito Circuli Radius  $= 1$ , & hemiperipheria  $= \pi$ , sit Arcus  $AB$  vel  $AC = s$ ; eritque area Segmenti  $AEB$  vel  $AFC =$

$\frac{1}{2} s - \frac{1}{2} \sin s$ ; at area Circuli est  $= \pi$ ; unde, cum Segmenti

$AEB$  area debeat esse triens Circuli, fiet  $\frac{1}{2} s - \frac{1}{2} \sin s =$

$\frac{\pi}{3} = 60^\circ$ ; seu,  $s - \sin s = 120^\circ$ , ideoque  $s - 120^\circ =$

$\sin s$ . Sit  $s - 120^\circ = u$ , erit  $u = \sin(u + 120^\circ) = \sin(60^\circ - u)$ .

Arcus

Arcus ergo  $u$  quæri debet, qui sit æqualis sinui anguli  $60^\circ$  —  $u$ . CAP. XXII.  
Erit ergo  $u$  minor quam  $60^\circ$ ; ad quem Arcum inveniendum faciamus sequentes positiones

$u = 20^\circ$	$u = 30^\circ$	$u = 40^\circ$
$60 - u = 40^\circ$	$60 - u = 30^\circ$	$60 - u = 20^\circ$
$l. u = 1,3010300$	$1,4771213$	$1,6020600$
subtrahe $1,7581226$	$1,7581226$	$1,7581226$
$l. u = 9,5429074$	$9,7189987$	$9,8439374$
$l. \sin.(60 - u) = 9,8080575$	$9,6989700$	$9,5340517$
$+ 2651601$	$- 200287$	$- 3098857$

Patet ergo angulum  $u$  aliquanto esse minorem quam  $30^\circ$ , &, calculo subducto, major esse debet quam  $29^\circ$  sit ergo  $u = 29^\circ$

$60 - u = 31^\circ$
$l. u = 1,4623980$
subtrahe $1,7581226$
$l. u = 9,7042754$
$l. \sin.(60 - u) = 9,7118393$
$+ 75639$
$- 200287$
$275926 : 75639 = 1^\circ : 16', 26''.$

Foret ergo angulus  $u = 29^\circ, 16', 26''$ , ad quem accuratius inveniendum, faciamus has hypotheses uno tantum minuto differentes

$u = 29^\circ, 16'$	$u = 29^\circ, 17'$
icu	feu
$u = 1756'$	$u = 1757'$
$60 - u = 30^\circ, 44'$	$60 - u = 30^\circ, 43'$
$l. u = 3,2445245$	$3,2447718$
subtrahe $3,5362739$	$3,5362739$
$l. u = 9,7082506$	$9,7084979$
$l. \sin.(60 - u) = 9,7084575$	$9,7082450$
$+ 2069$	$- 2529$
$2529$	

$$4598 : 2069 = 1' : 27'', 0'''.$$

Erit ergo vere  $u = 29^\circ, 16', 27'', 0'''$ ,

hincque

LIB. II.

Arcus  $s = AEB = 149^\circ, 16', 27'', 0''' = AFC$ ;  
unde resultat

Arcus  $BC = 61^\circ, 27', 6'', 0'''$ ,  
ipfa vero

Chorda  $AB = AC = 19285340$ . Q. E. F.

536. His Problematis, quibus Arcus quispian queritur dato Sinui vel Cofinui æqualis, adjungamus sequens, quo quidem idem negotium proponitur, attamen major difficultas occurrit.

## PROBLEMA VI.

TAB.

XXIX.

Fig. 116.

In semicirculo AEB Arcum AE abscindere, ita ut, ducto ejus Sinu ED, Arcus AE sit æqualis summe rectorum AD + DE.

## SOLUTIO.

Quoniam statim patet hunc Arcum quadrante esse majorem, quæramus ejus Complementum  $BE$ , & vocemus Arcum  $BE = s$ , ita ut sit Arcus  $AE = 180^\circ - s$ , atque ob  $AC = 1$ ,  $CD = \cos s$ ,  $DE = \sin s$ , erit  $180^\circ - s = 1 + \cos s + \sin s$ . At, est  $\sin s = 2 \sin \frac{1}{2} s \cos \frac{1}{2} s$ , &  $1 + \cos s = 2 \cos \frac{1}{2} s \cos \frac{1}{2} s$ ; unde fit  $180^\circ - s = 2 \cos \frac{1}{2} s (\sin \frac{1}{2} s + \cos \frac{1}{2} s)$ . At, est  $\cos (45^\circ - \frac{1}{2} s) = \frac{1}{\sqrt{2}} \cos \frac{1}{2} s + \frac{1}{\sqrt{2}} \sin \frac{1}{2} s$ : ergo  $\sin \frac{1}{2} s + \cos \frac{1}{2} s = \sqrt{2} \cos (45^\circ - \frac{1}{2} s)$ : unde erit  $180^\circ - s = 2\sqrt{2} \cos \frac{1}{2} s \cos (45^\circ - \frac{1}{2} s)$ . Hac facta reductione, faciamus sequentes positiones

$$\frac{1}{2} s =$$

$\frac{1}{2} s = 20^\circ$	$\frac{1}{2} s = 21^\circ$
$45^\circ - \frac{1}{2} s = 25^\circ$	$45^\circ - \frac{1}{2} s = 24^\circ$
$180 - s = 140^\circ$	$180 - s = 138^\circ$
$l.(180 - s) = 2, 1461280$	$2, 1398791$
subtrahe $1, 7581226$	$1, 7581226$
$l.(180 - s) = 0, 3880054$	$0, 3817565$
$l.\cos. \frac{1}{2} s = 9, 9729858$	$9, 9701517$
$l.\cos. (45^\circ - \frac{1}{2} s) = 9, 9572757$	$9, 9607302$
$12 \sqrt{2} = 0, 4515450$	$0, 4515450$
$0, 3818065$	$0, 3824269$
Error $+ 61989$	$- 6704$
$6704$	
$68693 : 61989 = 1^\circ : 54'$	

Hinc continetur  $\frac{1}{2} s$  intra limites  $20^\circ, 54',$  &  $20^\circ, 55',$   
ideoque sequentes hypothescs fiant

$\frac{1}{2} s = 20^\circ, 54'$	$\frac{1}{2} s = 20^\circ, 55'$
$45^\circ - \frac{1}{2} s = 24^\circ, 6'$	$45^\circ - \frac{1}{2} s = 24^\circ, 5'$
$s = 41^\circ, 48'$	$s = 41^\circ, 50'$
$180 - s = 138^\circ, 12'$	$180 - s = 138^\circ, 10'$
feu	feu
$180 - s = 8292'$	$180 - s = 8290'$
$l.(180 - s) = 3, 9186593$	$3, 9185545$
subtrahe $3, 5362739$	$3, 5362739$
$0, 3823854$	$0, 3822806$
$l.\cos. \frac{1}{2} s = 9, 9704419$	$9, 9703937$
$l.\cos. (45^\circ - \frac{1}{2} s) = 9, 9603919$	$9, 9604484$
$12 \sqrt{2} = 0, 4515450$	$0, 4515450$
$0, 3823788$	$0, 3823871$
Error $+ 66$	$- 1065$
$1065$	
$1131 : 66 = 1' : 3'', 30'''$	

R r 2

Hanc

L1 B. II. Hanc ob rem erit  $\frac{1}{2} s = 20^{\circ}, 54', 3'', 30'''$ ,

inde

$$s = 41^{\circ}, 48', 7'', 0''' = BE$$

ideoque Arcus quæsitus

$$AE = 138^{\circ}, 11', 53'', 0'''.$$

Erit vero Linea

$$DE = 0,6665578, \text{ \& } AD = 1,7454535. \text{ Q. E. F.}$$

§ 37. Comparemus nunc Arcus cum suis Tangentibus; & cum in primo quadrante Tangentes sint Arcubus minores; quæramus Arcum, qui suæ Tangentis semissi sit æqualis, quo solvetur

## PROBLEMA VII.

TAB. *Abſcindere Sectorem ACD, qui ſit ſemiſſis Trianguli ACE*  
XXIX. *a Radio AC, Tangente AE & Secante CE comprehenſi.*  
*Fig. 117.*

## SOLUTIO.

Posito Arcu  $AD = s$ , erit Sector  $ACD = \frac{1}{2}s$ , Triangulum vero  $ACE = \frac{1}{2} \cdot \text{tang. } s$ : unde debet esse  $\frac{1}{2} \cdot \text{tang. } s = s$ , seu  $2s = \text{tang. } s$ . Faciamus ergo has hypotheses

$s = 60^{\circ}$	$s = 70^{\circ}$	$s = 66^{\circ}$	$s = 67^{\circ}$
$l. 2s = 2,0791812$	$2,1461280$	$2,1205739$	$2,1271048$
$1,7581226$	$1,7581226$	$1,7581226$	$1,7581226$
$l. 2s = 0,3210586$	$0,3880054$	$0,3624513$	$0,3689822$
$l. \text{tang. } s = 0,2385606$	$0,4389341$	$0,3514169$	$0,3721481$
$+ 824980$	$- 509287$	$+ 110344$	$- 31659$

Hinc ipsius  $s$  reperiuntur limites arctiores  $66^{\circ}$ ,  $46'$ , &  $66^{\circ}$ ,  $47'$ : quare fiat

$s =$

$s = 66^{\circ}, 46'$	$s = 66^{\circ}, 47'$
feu	feu
$s = 4006'$	$s = 4007'$
$2s = 8012'$	$2s = 8014'$
$l. 2s = 3, 9037409$	$3, 9038493$
$3, 5362739$	$3, 5362739$
$l. 2s = 0, 3674670$	$0, 3675754$
$l. tang. s = 0, 3672499$	$0, 3675985$
Error $+ 2171$	$- 231$
$\frac{231}{2402 : 2171 = 1' : 54'', 14'''}$	

unde erit

Arcus  $s = AD = 66^{\circ}, 46', 54'', 14'''$ ,  
hincqueTangens  $AE = 2, 3311220$ . Q. E. F.  
538. Proponatur nunc sequens.

## P R O B L E M A V I I I.

Proposito Circuli quadrante ACB invenire Arcum AE, qui  
equalis sit Chordæ suæ AE ad occursum F usque productæ.T A B.  
XXIX.  
Fig. 118.

## S O L U T I O.

Sit Arcus  $AE = s$ , erit ejus Chorda  $AE = 2 \sin. \frac{1}{2} s$ , finus versus  $AD = 1 - \cos. s = 2 \sin. \frac{1}{2} s \sin. \frac{1}{2} s$  : unde Triangula similia  $ADE, ACF$ , dabunt  $2 \sin. \frac{1}{2} s \sin. \frac{1}{2} s : 2 \sin. \frac{1}{2} s = 1 : s$ , eritque ergo  $s \sin. \frac{1}{2} s = 1$ . Fiant ergo sequentes positiones