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Alexandria from 300 BC to 200 BC

The school of mathematics established by Euclid in Alexandria produced some first rate mathematicians in the third century BC. Among them were the following:

- Aristarchus of Samos, 310 – 250 BC,
- Archimedes of Syracuse, 287 – 212 BC,
- Apollonius of Perga, 260 – 190 BC,
- Eratosthenes of Cyrene, 275 – 195 BC.

Aristarchus came from Samos, the same Greek island Pythagoras came from. He gave an interesting application of mathematics to astronomy. Let SEM be the triangle whose vertices are the sun (S), the earth (E) and the moon (M) (Figure 18.1). Aristarchus noted that when the moon is at its first quarter, the angle SME is a right angle. This is why we see exactly half of the part of the moon's surface that faces the earth. When the moon is in its first quarter, one can see the sun and the moon together in the sky, at the same time. Thus Aristarchus was able to measure the angle SEM. He found it to be 29/30 of a right angle. (A more accurate value is 0.9981 of a right angle.) Constructing a right triangle with an acute angle of 29/30 of a right angle — there is a ruler and compass construction for this — Aristarchus found that the ratio of its short side to its hypotenuse is about 1/19. He concluded that the distance from the earth to the sun is about 19 times greater than the distance from the earth to the moon. Had his

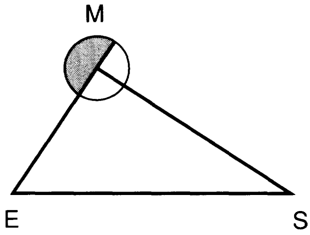

FIGURE 18.1. Relative distances of the sun and moon

measurement of the angle SEM been correct, he would have found that SE is about 400 times SM.

His calculation would have been easier had he used trigonometry, which was only developed a century later. If $\angle SEM = 87°$, then $\angle ESM = 3° = \pi/60$ radians, hence $EM/ES = \sin(\pi/60) \approx \pi/60 \approx 1/19$, as the sine of a small angle is approximately equal to that angle when expressed in radian measure.

Since the apparent sizes of the sun and the moon are approximately equal, as is seen during a solar eclipse, their actual diameters are in the same ratio as their distance from the earth.

By looking at the shadow cast by the earth upon the moon during a lunar eclipse one may also compare the size of the moon with that of the earth. (Since the sun is far away, the size of the earth is approximately the same as that of its shadow.) Aristarchus found

$$\frac{\text{diameter of the earth}}{\text{diameter of the moon}} \approx 7.$$

The actual figure is about 4. According to Plutarch, Aristarchus also proposed the hypothesis that 'the earth moves in an oblique circle about the sun at the same time as it turns around its axis'. It seems that Copernicus suppressed his acquaintance with the work of Aristarchus!

Although Archimedes is assumed to have studied in Alexandria, his productive life was spent in Syracuse. We shall leave him to Chapter 19.

Apollonius (260–190 BC) came from Perga in the south of what is now Turkey. He wrote a treatise on conics which contained 400 propositions. These were arranged in eight books, four of which survived in the original Greek, and three of which survived in Arabic translation. We do not read this treatise anymore, because we feel we can do the same things more easily using analytic geometry.

According to the modern definition of a *conic section*, it is the set of all points P in the plane such that P's distance from a fixed point, called the *focus*, bears a constant ratio to its distance from a fixed line, called