5. (a) Eigenvalues 2, 2; eigenvectors $t(1, 0)$, $t \neq 0$. If $C = \begin{bmatrix} a & b \\ -b & 0 \end{bmatrix}$, $b \neq 0$, then

$$C^{-1}AC = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

(b) Eigenvalues 3, 3; eigenvectors $t(1, 1)$, $t \neq 0$. If $C = \begin{bmatrix} a & b \\ a+b & b \end{bmatrix}$, $b \neq 0$, then

$$C^{-1}AC = \begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix}$$

6. Eigenvalues 1, 1, 1; eigenvectors $t(1, -1, -1)$, $t \neq 0$

Chapter 5

5.5 Exercises (page 118)

3. (b) T^n is Hermitian if n is even, skew-Hermitian if n is odd
7. (a) Symmetric (b) Neither (c) Symmetric (d) Symmetric
9. (d) $Q(x + ty) = Q(x) + t\bar{t}Q(y) + \bar{t}(T(x), y) + t(T(y), x)$

5.11 Exercises (page 124)

1. (a) Symmetric and Hermitian
   (b) None of the four types
   (c) Skew-symmetric
   (d) Skew-symmetric and skew-Hermitian

4. (b) $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

5. Eigenvalues $\lambda_1 = 0$, $\lambda_2 = 25$; orthonormal eigenvectors $u_1 = \frac{1}{5}(4, -3)$, $u_2 = \frac{1}{5}(3, 4)$.

$$C = \frac{1}{5}\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}$$

6. Eigenvalues $\lambda_1 = 2i$, $\lambda_2 = -2i$; orthonormal eigenvectors

$$u_1 = \frac{1}{\sqrt{2}}(1, -i), \quad u_2 = \frac{1}{\sqrt{2}}(1, i). \qquad C = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix}$$

7. Eigenvalues $\lambda_1 = 1$, $\lambda_2 = 3$, $\lambda_3 = -4$; orthonormal eigenvectors

$$u_1 = \frac{1}{\sqrt{10}}(1, 0, 3), \quad u_2 = \frac{1}{\sqrt{14}}(3, 2, -1), \quad u_3 = \frac{1}{\sqrt{35}}(3, -5, -1).$$

$$C = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{14}} & \frac{3}{\sqrt{35}} \\ 0 & \frac{2}{\sqrt{14}} & \frac{-5}{\sqrt{35}} \\ \frac{3}{\sqrt{10}} & \frac{-1}{\sqrt{14}} & \frac{-1}{\sqrt{35}} \end{bmatrix}$$

8. Eigenvalues $\lambda_1 = 1$, $\lambda_2 = 6$, $\lambda_3 = -4$; orthonormal eigenvectors

$$u_1 = \tfrac{1}{5}(0, 4, -3), \quad u_2 = \frac{1}{\sqrt{50}}(5, 3, 4), \quad u_3 = \frac{1}{\sqrt{50}}(5, -3, -4).$$

$$C = \frac{1}{\sqrt{50}}\begin{bmatrix} 0 & 5 & 5 \\ -3\sqrt{2} & 4 & -4 \end{bmatrix}$$

9. (a), (b), (c) are unitary; (b), (c) are orthogonal

11. (a) Eigenvalues $\lambda_1 = ia$, $\lambda_2 = -ia$; orthonormal eigenvectors

$$u_1 = \frac{1}{\sqrt{2}}(1, i), \quad u_2 = \frac{1}{\sqrt{2}}(1, -i). \qquad \text{(b)} \quad C = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$$

5.15 Exercises (page 134)

1. (a) $A = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ (b) $\lambda_1 = 0$, $\lambda_2 = 5$ (c) $u_1 = \frac{1}{\sqrt{5}}(1, -2)$, $u_2 = \frac{1}{\sqrt{5}}(2, 1)$

(d) $C = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

2. (a) $A = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$ (b) $\lambda_1 = \frac{1}{2}$, $\lambda_2 = -\frac{1}{2}$

(c) $u_1 = \frac{1}{\sqrt{2}}(1, 1)$, $u_2 = \frac{1}{\sqrt{2}}(1, -1)$ (d) $C = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

3. (a) $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (b) $\lambda_1 = \sqrt{2}$, $\lambda_2 = -\sqrt{2}$

(c) $u_1 = t(1 + \sqrt{2}, 1)$, $u_2 = t(-1, 1 + \sqrt{2})$, where $t = 1/\sqrt{4 + 2\sqrt{2}}$

(d) $C = t\begin{bmatrix} 1 + \sqrt{2} & -1 \\ 1 & 1 + \sqrt{2} \end{bmatrix}$, where $t = 1/\sqrt{4 + 2\sqrt{2}}$

4. (a) $A = \begin{bmatrix} 34 & -12 \\ -12 & 41 \end{bmatrix}$ (b) $\lambda_1 = 50$, $\lambda_2 = 25$

(c) $u_1 = \frac{1}{5}(3, -4)$, $u_2 = \frac{1}{5}(4, 3)$ (d) $C = \frac{1}{5}\begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$

5. (a) $A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (b) $\lambda_1 = 0$, $\lambda_2 = \frac{3}{2}$, $\lambda_3 = -\frac{1}{2}$

(c) $u_1 = \dfrac{1}{\sqrt{3}}(1, -1, -1)$, $u_2 = \dfrac{1}{\sqrt{6}}(2, 1, 1)$, $u_3 = \dfrac{1}{\sqrt{2}}(0, 1, -1)$

(c) $C = \dfrac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & 2 & 0 \\ -\sqrt{2} & 1 & \sqrt{3} \\ -\sqrt{2} & 1 & -\sqrt{3} \end{bmatrix}$

6. (a) $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$ (b) $\lambda_1 = 1$, $\lambda_2 = 3$, $\lambda_3 = -2$

(c) $u_1 = (0, 1, 0)$, $u_2 = \dfrac{1}{\sqrt{5}}(2, 0, 1)$, $u_3 = \dfrac{1}{\sqrt{5}}(1, 0, -2)$

(d) $C = \dfrac{1}{\sqrt{5}}\begin{bmatrix} 0 & 2 & 1 \\ \sqrt{5} & 0 & 0 \\ 0 & 1 & -2 \end{bmatrix}$

7. (a) $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ (b) $\lambda_1 = \lambda_2 = -1$, $\lambda_3 = 8$

(c) $u_1 = \dfrac{1}{\sqrt{2}}(1, 0, -1)$, $u_2 = \dfrac{1}{3\sqrt{2}}(-1, 4, -1)$, $u_3 = \dfrac{1}{3}(2, 1, 2)$

(d) $C = \dfrac{1}{3\sqrt{2}}\begin{bmatrix} 3 & -1 & 2\sqrt{2} \\ 0 & 4 & \sqrt{2} \\ -3 & -1 & 2\sqrt{2} \end{bmatrix}$

8. Ellipse; center at $(0, 0)$
9. Hyperbola; center at $(-\frac{5}{2}, -\frac{5}{2})$
10. Parabola; vertex at $(\frac{5}{16}, -\frac{15}{16})$
11. Ellipse; center at $(0, 0)$
12. Ellipse; center at $(6, -4)$
13. Parabola; vertex at $(\frac{2}{25}, \frac{11}{25})$
14. Ellipse; center at $(0, 0)$
15. Parabola; vertex at $(\frac{3}{4}, \frac{3}{4})$
16. Ellipse; center at $(-1, \frac{1}{2})$
17. Hyperbola; center at $(0, 0)$
18. Hyperbola; center at $(-1, 2)$
19. -14

5.20 Exercises (page 141)

8. $a = \pm\frac{1}{3}\sqrt{3}$

13. (a), (b), and (e)

Chapter 6

6.3 Exercises (page 144)

1. $y = e^{3x} - e^{2x}$
2. $y = \frac{2}{3}x^2 + \frac{1}{3}x^5$
3. $y = 4\cos x - 2\cos^2 x$
4. Four times the initial amount
5. $f(x) = Cx^n$, or $f(x) = Cx^{1/n}$
6. (b) $y = e^{4x} - e^{-x^3/3}$
7. $y = c_1 e^{2x} + c_2 e^{-2x}$
8. $y = c_1 \cos 2x + c_2 \sin 2x$
9. $y = e^{x}(c_1 \cos 2x + c_2 \sin 2x)$
10. $y = e^{-x}(c_1 + c_2 x)$

11. $k = n^2\pi^2$; $f_k(x) = C \sin n\pi x$ $(n = 1, 2, 3, \ldots)$
13. (a) $y'' - y = 0$
 (b) $y'' - 4y' + 4y = 0$
 (c) $y'' + y' + \frac{5}{4}y = 0$
 (d) $y'' + 4y = 0$
 (e) $y'' - y = 0$
14. $y = \frac{1}{3}\sqrt{6},\ y'' = -12y = -4\sqrt{6}$

6.9 Exercises (page 154)

1. $y = c_1 + c_2e^{-x} + c_3e^{3x}$
2. $y = c_1 + c_2e^{x} + c_3e^{-x}$
3. $y = c_1 + (c_2 + c_3x)e^{-2x}$
4. $y = (c_1 + c_2x + c_3x^2)e^{x}$
5. $y = (c_1 + c_2x + c_3x^2 + c_4x^3)e^{-x}$
6. $y = c_1e^{2x} + c_2e^{-2x} + c_3 \cos 2x + c_4 \sin 2x$
7. $y = e^{\sqrt{2}x}(c_1 \cos \sqrt{2}x + c_2 \sin \sqrt{2}x) + e^{-\sqrt{2}x}(c_3 \cos\sqrt{2}x + c_4 \sin \sqrt{2}x)$
8. $y = c_1e^{x} + e^{-x/2}(c_2 \cos \frac{1}{2}\sqrt{3}x + c_3 \sin \frac{1}{2}\sqrt{3}x)$
9. $y = e^{-x}[(c_1 + c_2x) \cos x + (c_3 + c_4x) \sin x]$
10. $y = (c_1 + c_2x) \cos x + (c_3 + c_4x) \sin x$
11. $y = c_1 + c_2x + (c_3 + c_4x) \cos \sqrt{2}x + (c_5 + c_6x) \sin \sqrt{2}x$
12. $y = c_1 + c_2x + (c_3 + c_4x) \cos 2x + (c_5 + c_6x) \sin 2x$

13. $f(x) = \dfrac{1}{2m^2}(e^{mx} - \cos mx - \sin mx)$

15. (a) $y^{(4)} - 5y'' + 4y = 0$
 (b) $y''' + 6y'' + 12y' + 8y = 0$
 (c) $y^{(4)} - 2y''' + y'' = 0$
 (d) $y^{(4)} - 2y''' + y'' = 0$
 (e) $y^{(5)} - 2y^{(4)} + y''' = 0$
 (f) $y^{(4)} + 8y''' + 33y'' + 68y' + 52y = 0$
 (g) $y^{(4)} - 2y'' + y = 0$
 (h) $y^{(6)} + 4y'' = 0$

6.15 Exercises (page 166)

1. $y_1 = -2x - x^2 - \frac{1}{3}x^3$
2. $y_1 = \frac{1}{4}xe^{2x}$
3. $y_1 = (x - \frac{4}{3})e^{x}$
4. $y_1 = \frac{1}{3} \sin x$
5. $y_1 = \frac{1}{2}x^2e^{x} + e^{2x}$
6. $y_1 = \frac{1}{2}xe^{x}$
7. $y_1 = x \cosh x$
8. $y_1 = \frac{1}{24}x^4e^{-x}$
9. $50y_1 = (11 - 5x)e^{x} \sin 2x + (2 - 10x)e^{x} \cos 2x$
10. $y_1 = -(\frac{5}{8}x + \frac{3}{8}x^2 + \frac{1}{12}x^3)e^{-x}$

12. $y_1 = \dfrac{x^m e^{\alpha x}}{P^{(m)}(\alpha)}$

15. (b) $2D$ (c) $3D^2$ (d) nD^{n-1}

16. $y = Ae^{x} + Be^{-x} + \dfrac{1}{2} e^{x} \displaystyle\int \frac{e^{-x}}{x}\, dx - \frac{1}{2} e^{-x} \int \frac{e^{x}}{x}\, dx$

17. $y = (A + \frac{1}{2}x) \sin 2x + (B + \frac{1}{4} \log |\cos 2x|) \cos 2x$
18. $y = Ae^{x} + Be^{-x} + \frac{1}{2} \sec x$

19. $y = (A + Bx)e^{x} + e^{e^x} - xe^{x} \displaystyle\int e^{e^x}\, dx + e^{x} \int xe^{e^x}\, dx$

20. $y = -\frac{1}{8}\log|x| + \frac{1}{3}e^x\int\frac{e^{-x}}{x}\,dx - \frac{1}{4}e^{2x}\int\frac{e^{-2x}}{x}\,dx$
$+\frac{1}{24}e^{4x}\int\frac{e^{-4x}}{x}\,dx + Ae^x + Be^{2x} + Ce^{4x}$

6.16 Miscellaneous exercises on linear differential equations (page 167)

1. $u(x) = 6(e^{4x} - e^{-x})/5$; $v(x) = e^x - e^{-5x}$
2. $u(x) = \frac{1}{2}e^{2x-\pi}\sin 5x$; $v(x) = \frac{5}{6}e^{-2x-\pi}\sin 3x$
3. $u(x) = e^{-x^2}$; $Q(x) = 4x^2 + 2$
5. $y = (A + Bx^3)e^x + (x^2 - 2x + 2)e^{2x}$
6. $y = Ae^{4x}\int e^{-4x-x^3/3}\,dx + Be^{4x}$
7. $y = Ax^{1/2} + Bx^{-1/2}$
8. $y = Ae^x + Bx^2e^{-x} - x$
9. $y = A(x^2 - 2) + B/x$
10. $y = x^{-2}[A + B(x-1)^3 + \frac{1}{9}x^3 + \frac{2}{3}x^2 - \frac{7}{6}x + \frac{1}{2} - (x-1)^3\log|x-1|]$
11. $a = 1, -1$; $y = [Ae^{g(x)} + Be^{-g(x)}]/x$

6.21 Exercises (page 177)

2. $f(x) = u_1(x)$ $(\alpha = 1)$
3. (a) $A = (a - b)/2$, $B = (a + b)/2$

   (b) $\frac{d}{dt}\left[(t^2-1)\frac{dy}{dt}\right] - \alpha(\alpha+1)y = 0$, where $\alpha = 1$ or -2, and $x = (t+1)/2$

4. $u_1(x) = 1 + \sum_{m=1}^{\infty}(-1)^m 2^m\frac{\alpha(\alpha-2)\,(\alpha-2m+2)}{(2m)!}x^{2m}$ for all x;

   $u_2(x) = x + \sum_{m=1}^{\infty}(-1)^m 2^m\frac{(\alpha-1)(\alpha-3)\ldots(\alpha-2m+1)}{(2m+1)!}x^{2m+1}$ for all x

5. $u_1(x) = 1 + \sum_{m=1}^{\infty}\frac{(-1)^m}{(3m+2)(3m-1)\ldots 8\ 5}x^{3m}$ for all x;

   $u_2(x) = x^{-2}\left(1 + \sum_{n=1}^{\infty}\frac{(-1)^n}{3^n n!}x^{3n}\right)$ for all $x \neq 0$

6. $y = x^2\left(\frac{1}{6} + \sum_{n=1}^{\infty}\frac{(\alpha-2)(\alpha-3)\cdots(\alpha-n-1)}{n!(n+3)!}x^n\right)$ for all x

11. (b) $f(x) = \frac{1}{5}P_0(x) + \frac{4}{7}P_2(x) + \frac{8}{35}P_4(x)$

15. (b) $\frac{2n}{4n^2-1}$

6.24 **Exercises (page 188)**

5. $J_{-3/2}(x) = -\left(\frac{2}{\pi x}\right)^{1/2}\left(\frac{\cos x}{x} + \sin x\right)$

9. (a) $y = x^{1/2}[c_1 J_{1/3}(\frac{2}{3}x^{3/2}) + c_2 J_{-1/3}(\frac{2}{3}x^{3/2})]$
(b) $y = x^{1/2}[c_1 J_{1/4}(\frac{1}{2}x^2) + c_2 J_{-1/4}(\frac{1}{2}x^2)]$
(c) $y = x^{1/2}[c_1 J_\alpha(2\alpha x^{1+m/2}) + c_2 J_{-\alpha}(2\alpha x^{1+m/2})]$, where $\alpha = 1/(m+2)$, provided that $1/(m+2)$ is not an integer; otherwise replace the appropriate J by K
(d) $y = x^{1/2}[c_1 J_\alpha(\frac{1}{2}x^2) + c_2 J_{-\alpha}(\frac{1}{2}x^2)]$, where $\alpha = \sqrt{2}/8$
10. $y = g_\alpha$ satisfies $x^2y'' + (1 - 2c)xy' + (a^2b^2x^{2b} + c^2 - \alpha^2b^2)y = 0$
(a) $y = x^{-5/2}[c_1 J_5(2x^{1/2}) + c_2 K_5(2x^{1/2})]$
(b) $y = x^{-5/2}[c_1 J_{5/2}(x) + c_2 J_{-5/2}(x)]$
(c) $y = x^{-5/2}[c_1 J_1(\frac{2}{5}x^{5/2}) + c_2 K_1(\frac{2}{5}x^{5/2})]$
(d) $y = x[c_1 J_0(2x^{1/2}) + c_2 K_0(2x^{1/2})]$
11. $a = 2$, $c = 0$
12. $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} x^n$; $y = J_0(2x^{1/2})$ if $x > 0$
13. $b = (p_0 - a_0)/a_0$, $c = q_0/a_0$
14. $y = x^{1/2}$
15. $t = 1$: $y = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n!}{(2n)!} (2x)^n$

$t = \frac{1}{2}$: $y = x^{1/2} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{x}{2}\right)^n = x^{1/2}e^{-x/2}$
16. $u_0(x) = \cos x$; $u_1(x) = \frac{1}{2} - \frac{1}{6}\cos x - \frac{1}{3}\cos 2x$

Chapter 7

7.4 Exercises (page 195)

3. (b) $(P^k)' = \sum_{m=0}^{k-1} P^m P' P^{k-1-m}$

7.12 Exercises (page 205)

1. (a) $A^{-1} = 2I - A$, $A^n = nA - (n - 1)Z$

(b) $e^{tA} = e^t(1 - t)Z + te^tA = e^t\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}$

2. (a) $A^{-1} = \frac{3}{2}I - \frac{1}{2}A$, $A^n = (2^n - 1)A - (2^n - 2)Z$

(b) $e^{tA} = (2e^t - e^{2t})I + (e^{2t} - e^t)A = \begin{bmatrix} e^t & 0 \\ e^{2t} - e^t & e^{2t} \end{bmatrix}$

3. (a) $A^{-1} = A$, $A^n = \frac{1 + (-1)^n}{2} I + \frac{1 - (-1)^n}{2} A$

(b) $e^{tA} = (\cosh t)Z + (\sinh t)A = \begin{bmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{bmatrix}$

4. (a) $A^{-1} = A, \quad A^n = \dfrac{1+(-1)^n}{2} I + \dfrac{1-(-1)^n}{2} A$

(b) $e^{tA} = (\cosh t)I + (\sinh t)A = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{t} \end{bmatrix}$

5. (b) $e^{tA} = e^{at}\begin{bmatrix} \cos bt & \sin bt \\ -\sin bt & \cos bt \end{bmatrix}$

7. $e^{A(t)} = Z + (e-1)A(t)\,; \quad (e^{A(t)})' = (e-1)A'(t) = \begin{bmatrix} 0 & e-1 \\ 0 & 0 \end{bmatrix};$

$e^{A(t)}A'(t) = \begin{bmatrix} 0 & e \\ 0 & 0 \end{bmatrix}; \quad A'(t)e^{A(t)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

8. (a) $A^n = 0$ if $n \geq 3$

(b) $e^{tA} = I + tA + \dfrac{1}{2}t^2A^2 = \begin{bmatrix} 1 & t & t+\frac{1}{2}t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$

9. (a) $A^n = A$ if $n \geq 1$

(b) $e^{tA} = I + (e^t - 1)A = \begin{bmatrix} 1 & e^t-1 & e^t-1 \\ 0 & e^t & e^t-1 \\ 0 & 0 & 1 \end{bmatrix}$

10. (a) $A^3 = 4A^2 - 5A + 2I; \quad A^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 1 & 0 \\ 0 & n & 1 \end{bmatrix}$

(b) $e^{tA} = \begin{bmatrix} e^{2t} & 0 & 0 \\ 0 & e^t & 0 \\ 0 & te^t & e^t \end{bmatrix}$

11. $e^{tA} = Z + tA + \frac{1}{2}t^2A^2$

13. $e^Ae^B = \begin{bmatrix} e^2 & -(e-1)^2 \\ 0 & 1 \end{bmatrix}; \quad e^Be^A = \begin{bmatrix} e^2 & (e-1)^2 \\ 0 & 1 \end{bmatrix}; \quad e^{A+B} = \begin{bmatrix} e^2 & 0 \\ 0 & 1 \end{bmatrix}$

7.15 Exercises @age 211)

1. $e^{tA} = \frac{1}{2}(3e^t - e^{3t})I + \frac{1}{2}(e^{3t} - e^t)A$

2. $e^{tA} = (\cosh \sqrt{5}\,t)I + \dfrac{1}{\sqrt{5}}(\sinh \sqrt{5}\,t)A$

3. $e^{tA} = \frac{1}{2}e^t\{(t^2 - 2t + 2)Z + (-2t^2 + 2t)A + t^2A^2\}$

4. $e^{tA} = (3e^{-t} - 3e^{-2t} + e^{-3t})I + (\frac{5}{2}e^{-t} - 4e^{-2t} + \frac{3}{2}e^{-3t})A + (\frac{1}{2}e^{-t} - e^{-2t} + \frac{1}{2}e^{-3t})A^2$

5. $e^{tA} = (4e^t - 3e^{2t} + 2te^{2t})I + (4e^{2t} - 3te^{2t} - 4e^t)A + (e^t - e^{2t} + te^{2t})A^2$

6. $e^{tA} = (4e^t - 6e^{2t} + 4e^{3t} - e^{4t})I + (-\frac{13}{3}e^t + \frac{19}{2}e^{2t} - 7e^{3t} + \frac{11}{6}e^{4t})A$
$\quad + (\frac{3}{2}e^t - 4e^{2t} + \frac{7}{2}e^{3t} - e^{4t})A^2 + (-\frac{1}{6}e^t + \frac{1}{2}e^{2t} - \frac{1}{2}e^{3t} + \frac{1}{6}e^{4t})A^3$

7. (b) $e^{tA} = \frac{1}{6}e^{\lambda t}\{(6 - 6\lambda t + 3\lambda^2t^2 - \lambda^3t^3)I + (6t - 6\lambda t^2 + 3\lambda^2t^3)A + (3t^2 - 3\lambda t^3)A^2 + t^3A^3\}$

8. $y_1 = c_1 \cosh \sqrt{5}\, t + \dfrac{c_1 + 2c_2}{\sqrt{5}} \sinh \sqrt{5}\, t$, $y_2 = c_2 \cosh \sqrt{5}\, t + \dfrac{2c_1 - c_2}{\sqrt{5}} \sinh \sqrt{5}\, t$

9. $y_1 = e^t(\cos 3t - \sin 3t)$, $y_2 = e^t(\cos 3t - 3 \sin 3t)$
10. $y_1 = e^{2t} + 4te^{2t}$, $y_2 = -2e^t + e^{2t} + 4te^{2t}$, $y_3 = -2e^t + 4e^{2t}$
11. $y_1 = c_1e^{2t}$, $y_2 = c_2e^t$, $y_3 = (c_2t + c_3)e^t$
12. $y_1 = 3e^{-t} - 3e^{-2t} + e^{-3t}$, $y_2 = -3e^{-t} + 6e^{-2t} - 3e^{-3t}$, $y_3 = 3e^{-t} - 12e^{-2t} + 9e^{-3t}$
13. $y_1 = e^{5t} + 7e^{-3t}$, $y_2 = 2e^{5t} - 2e^{-3t}$, $y_3 = -e^{5t} + e^{-3t}$
14. $y_1 = -\frac{1}{2}e^t + e^{2t} + \frac{1}{2}e^{3t}$, $y_2 = e^{2t} + e^{3t}$, $y_3 = e^{3t}$, $y_4 = e^{4t}$
15. $y_1 = 2e^{2t} - 1$, $y_2 = 2e^{2t} - t - 2$, $y_3 = 2e^{2t}$, $y_4 = e^{2t}$

7.17 Exercises (page 215)

2. (c) $y_1 = (b - 1)e^x + 2(c + 1 - b)xe^x + 1$, $y_2 = ce^x + 2(c + 1 - b)xe^x$
4. $y_1 = -\frac{1}{3}e^t - \frac{1}{6}e^{4t} + \frac{1}{2}e^{2t}$, $y_2 = \frac{2}{3}e^t - \frac{1}{6}e^{4t} + \frac{1}{2}e^{2t}$

5. (a) $B_0 = B$, $B_1 = AB$, $B_2 = \dfrac{1}{2!} A^2B, \ldots,\quad B_m = \dfrac{1}{m!} A^mB$
   (b) $B = -m!\,(A^{-1})^{m+1}C$

6. (a) $Y(t) = \left(I + tA + \dfrac{1}{2} t^2A^2 + \dfrac{1}{6} t^3A^3\right)B$, where $B = -6A^{-4}C = -\dfrac{3}{128}\begin{bmatrix}1\\1\end{bmatrix}$.
   This gives the particular solution $y_1 = y_2 = -\frac{3}{128} - \frac{3}{32}t - \frac{3}{16}t^2 - \frac{1}{4}t^3$
   (b) $y_1 = y_2 = -\frac{3}{128} - \frac{3}{32}t - \frac{3}{16}t^2 - \frac{1}{4}t^3 + \frac{131}{128}e^{4t}$

7. $E = B$, $F = \dfrac{1}{\alpha}(AB + C)$

8. (a) $y_1 = -\cos 2t - \frac{1}{2} \sin 2t$, $y_2 = -\frac{1}{2} \sin 2t$
   (b) $y_1 = 2 \cosh 2t + \frac{5}{2} \sinh 2t - \cos 2t - \frac{1}{2} \sin 2t$, $y_2 = \cosh 2t + \frac{1}{2} \sinh 2t - \frac{1}{2} \sin 2t$
9. $y_1(x) = e^{2x} + e^{3x} - e^x$, $y_2(x) = -2e^{2x} - e^{3x} + 3e^x$
10. $y_1(x) = \frac{4}{25}e^x - \frac{1}{36}e^{2x} + (c_1 - \frac{119}{900})e^{-4x} + (\frac{11}{30} - c_1 - c_2)xe^{-4x}$,
    $y_2(x) = \frac{1}{25}e^x + \frac{7}{36}e^{2x} + (c_2 - \frac{211}{900})e^{-4x} + (c_1 + c_2 - \frac{11}{30})xe^{-4x}$
11. $y_1(x) = e^{-4x}(2 \cos x + \sin x) + \frac{31}{26}e^x - \frac{93}{17}$, $y_2(x) = e^{-4x}(\sin x + 3 \cos x) - \frac{2}{13}e^x + \frac{6}{17}$
12. $y_1(x) = e^{-x}(x^2 + 2x + 3) + x^2 - 3x + 3$, $y_2(x) = e^{-x}(-2x - 2) + x$,
    $y_3(x) = 2e^{-x} + x - 1$

7.20 Exercises (page 221)

4. (c) $Y(x) = e^xe^{\frac{1}{2}x^2A}B$

5. If $A(x) = \sum_{k=0}^{\infty} x^kA_k$, then $Y(x) = B + xC + \sum_{k=2}^{\infty} x^kB_k$,
   where $(k + 2)(k + 1)B_{k+2} = \sum_{r=0}^{k} A_rB_{k-r}$ for $k \geq 0$.

7.24 Exercises (page 230)

1. (a) $Y(x) = e^x$
   (b) $Y_n(x) = 2e^x - \sum_{k=0}^{n} \dfrac{x^k}{k!}$ if n is odd; $Y_n(x) = \sum_{k=0}^{n} \dfrac{x^k}{k!}$ if n is even

2. $Y,(x) = \frac{x^2}{2} + \frac{x^5}{20} + \frac{x^8}{160} + \frac{x^{11}}{4400}$

3. $Y,(x) = x + \frac{x^4}{4} + \frac{x^7}{14} + \frac{x^{10}}{160}$

4. $Y_3(x) = \frac{x^3}{3} + \frac{4x'}{63} + \frac{8x^9}{405} + \frac{184x^{11}}{51975} + \frac{4x13}{12285}$

5. (a) $Y_2(x) = 1 + x + x^2 + \frac{2x^3}{3} + \frac{x^4}{6} + \frac{2x^5}{15} + \frac{x^7}{63}$

   (b) $M = 2;\ c = \frac{1}{2}$

   (c) $Y(x) = 1 + x + x^2 + \frac{4x^3}{3} + \frac{7x^4}{6} + \frac{6x^5}{5} + \cdots$

6. (a) $Y_4(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{38x^9}{2835} + \frac{134x^{11}}{51975} + \frac{4x13}{12285} + \frac{x^{15}}{59535}$

   (d) $Y(x) = \tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \cdots$ for $|x| < \frac{\pi}{2}$

8. $Y_3(x) = 2 + x^2 + x^3 + \frac{3x^5}{20} + \frac{x^6}{10}$; $Z_3(x) = 3x^2 + \frac{3x^4}{4} + \frac{6x^5}{5} + \frac{3x^7}{28} + \frac{3x^8}{40}$

9. $Y_3(x) = 5 + x + \frac{x^4}{12} + \frac{x^6}{6} + \frac{2x^7}{63} + \frac{x^9}{72}$;

   $Z_3(x) = 1 + \frac{x^3}{3} + x^5 + \frac{2x^6}{9} + \frac{x^8}{8} + \frac{11x^9}{324} + \frac{7x^{11}}{264}$

10. (d) $Y,(x) = 0$; $\lim_{n\to\infty} Y,(x) = 0$

    (e) $Y_n(x) = \begin{cases} x^2 & \text{if } x \ge 0 \\ -x^2 & \text{if } x \le 0 \end{cases}$; $\lim_{n\to\infty} Y,(x) = \begin{cases} x^2 & \text{if } x \ge 0 \\ -x^2 & \text{if } x \le 0 \end{cases}$

    (f) $Y,(x) = \frac{2x^n}{3^n}$; $\lim_{n\to\infty} Y_n(x) = 0$

    (g) $Y_n(x) = \begin{cases} x^2 & \text{if } x \ge 0 \\ -x^2 & \text{if } x \le 0 \end{cases}$; $\lim_{n\to\infty} Y,(x) = \begin{cases} x^2 & \text{if } x \ge 0 \\ -x^2 & \text{if } x \le 0 \end{cases}$

Chapter 8

8.3 Exercises (page 245)

2. All open except (d), (e), (h), and (j)
3. All open except (d)
5. (e) One example is the collection of all 2-balls $B(O; 1/k)$, where $k = 1, 2, 3, \ldots$
6. (a) Both (b) Both (c) Closed (d) Open (e) Closed (f) Neither (g) Closed (h) Neither (i) Closed (j) Closed (k) Neither (1) Closed
8. (e) One example is the collection of all sets of the form $S_k = \{x \mid \|x\| \le 1 - 1/k\}$ for $k = 1, 2, 3, \ldots$. Their union is the open ball $B(O; 1)$
10. No