

metically or geometrically. Omar Khayyam (1100 A.D.) had developed a method for drawing a line segment whose length was a positive real root of a given cubic polynomial. In 1225, Leonardo of Pisa gave an arithmetical solution to $x^3 + 2x^2 + 10x = 20$. Because he used arithmetic rather than geometry, Leonardo was able to obtain an approximation to the positive root which was accurate to nine decimal places.

The first person to develop anything like a complete method for solving cubic equations — one that could in principle handle negative and imaginary roots as well as positive real roots — was alleged to have been Scipione Ferro of Bologna (1465–1526). He could solve any equation of the form $x^3 + bx = c$, giving the answers to any degree of accuracy. Ferro kept his method a secret — so that he would have an advantage over other mathematicians in mathematical contests — but, just before he died, he passed it on to one Antonio Fiore.

In 1530 Zuanne da Coi sent the following problems to Niccolo Tartaglia (1500–1557):

$$\begin{aligned}x^3 + 3x^2 &= 5, \\x^3 + 6x^2 + 8x &= 1000.\end{aligned}$$

Tartaglia announced that he could solve these equations and was promptly challenged by Fiore to a contest. Each contestant had to deposit a certain amount of money with a notary and propose a number of problems for his rival to solve. Whoever solved more of the problems within 30 days was to get all the money.

Tartaglia, suspecting that Fiore would pose equations of the form $x^3 + bx = c$, quickly worked out a general method for solving such equations. Indeed, Fiore's problems were of this nature, and Tartaglia was able to solve them all. He himself posed equations of the form $x^3 + ax^2 = c$, which he could solve, but which were too difficult for Fiore.

Tartaglia was born in Brescia, Italy. This town was captured by the French in 1512 and most of the inhabitants were massacred. Tartaglia's jaws were split by a soldier's sword and it was thus that he acquired his name, which means 'the stammerer'. He lectured at Verona and Venice, becoming famous through his victory over Fiore. He published a book on ballistics in 1537, in which he correctly stated that a projectile achieves its maximum range when fired at an angle of 45° . However, he gave no proof of this. In 1560 Tartaglia wrote a book on number theory, which contained some amusing puzzles, for example:

Three couples wish to cross a river in a boat that holds only two people. How can this be done if no woman is to be left with a man unless her husband is present?

Three people wish to share the oil in a 24 ounce jar. They have empty measuring jars of capacity 5, 11 and 13 ounces. How can they divide the oil?

The later part of Tartaglia's life was embittered by a quarrel with Girolamo Cardano (1501–1576), another Italian, whose autobiography has been republished by Dover. Cardano was a famous physician in Milan and once travelled to Scotland to cure an Archbishop of asthma. He applied his mathematics to mechanics, gambling and astrology. Indeed, he might be called the discoverer of probability theory. Cardano's oldest son was executed for having poisoned his wife, and Cardano himself was imprisoned, in 1570, for heresy, for having published a horoscope of Jesus Christ. (This was heretical because it suggested that God was subject to the stars.) Cardano was freed only after he recanted. There is a story that he foretold the day of his own death, using astrology, and then felt compelled to commit suicide to make his prediction come true.

Cardano had persuaded Tartaglia to tell him the secret method for solving cubic equations. This Tartaglia did only on condition that Cardano would never reveal it. However, some years later Cardano learned of the prior work by Ferro, and he decided to publish the secret method in his *Ars Magna* (1545). Cardano gave due credit to Tartaglia for the method, but Tartaglia was upset that Cardano had broken his promise to keep it secret. Henceforth, Tartaglia would have no special advantage in mathematical contests!

Annoyed, Tartaglia challenged Cardano to a competition. The latter did not show up, being represented instead by his student, Ferrari (1522–1565). It seems that Ferrari did better than Tartaglia, and Tartaglia lost both prestige and income.

Cardano's *Ars Magna* was the best book on algebra so far. It still used geometry to prove the algebraic identity

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b),$$

and it still shied away from negative numbers, listing the following equations separately:

$$x^3 + px = q, \quad x^3 = px + q, \quad x^3 + px + q = 0, \quad x^3 + q = px.$$

Nonetheless, the *Ars Magna* contained a full explanation of the cubic equation, including a treatment of imaginary numbers.

Ferri came from Bologna in Italy. He extended the work of Tartaglia and Cardano, solving the general fourth degree equation. His solution appears in the *Ars Magna*. Ferrari became rich in the service of the Cardinal Fernando Gonzalo, but ill health forced him to retire to Bologna, in 1565, to teach mathematics. According to W.W. Rouse Ball in *A Short Account*