bows, Archimedes designed devices which dropped huge stones on the Roman ships. He even constructed a crane which could lift a ship from the water, and drop it back in, stern-first. When Syracuse finally fell (212 BC), the Roman general Marcellus gave orders to bring Archimedes to him unharmed. These were not obeyed. Archimedes, it seems, was slain by an unknown soldier. There are various accounts of this story; the best known is that by Plutarch in his biography of Marcellus. Who today would know of Marcellus were it not for Archimedes?

Archimedes wrote on many subjects: the circle, the parabola, the spiral, the sphere, the cylinder, arithmetic, mechanics, statics and hydrostatics. One of his more interesting books, the *Method*, was rediscovered only in 1906.

By considering a regular 96-gon inscribed in a circle, Archimedes showed that $\pi < 3\frac{1}{7}$. By considering a regular 96-gon circumscribing a circle, he showed that $3\frac{10}{71} < \pi$. He was aware that one could calculate π to any desired accuracy by letting the number of sides of the regular polygon tend to infinity.

He proved that the area of a circle is πr^2 and that the volume of a sphere is $\frac{4}{3}\pi r^3$. He knew how to calculate the area bounded by a parabola and a chord, the area of a sector of a spiral, the volume of an ellipsoid of revolution, the volume of a segment of a sphere, the centroid of a hemisphere and, perhaps most remarkably, the volume common to two equal right circular cylinders intersecting at right angles. All these are calculus problems and, indeed, Archimedes was using what we would now call the technique of 'integration'.

As an example of Archimedes's mathematics, let us see how he proved that the area of a circle is πr^2. He started with the following assumptions and theorems:

1. Circles and circle segments have areas.

2. The area of a set of pairwise disjoint triangles and circle segments equals the sum of the areas of those triangles and circle segments; thus if we dissect a circle into triangles and circle segments, the area of the circle is the sum of the areas of the triangles and circle segments into which it has been dissected; also the area of the circle is greater than the sum of the areas of any proper subset of those triangles and circle segments.

3. Given any circle, there is a straight line segment which is longer than the perimeter of any polygon inscribed in the circle, and shorter than the perimeter of any polygon circumscribing the circle; this is the 'circumference' of the circle. (The first person to give the name π to the circumference of the circle with unit diameter was not Archimedes, but William Jones, in 1706.)

4. Given any areas e and f, there is a natural number m such that $me > f$; (this assumption is found at the beginning of Archimedes's 'On the Sphere and the Cylinder', so it is often called the 'Axiom of Archimedes'; however, it is also found at the beginning of Book V of Euclid's *Elements* and, even earlier, at 266b in Aristotle's *Physics*).

5. A regular 2^n-gon inscribed in a circle takes up more than $1 - 1/2^{n-1}$ of its area; a regular 2^n-gon circumscribed about a circle has an area less than $1 + 1/2^{n-2}$ of that of the circle.

6. The area of a circle is proportional to its diameter squared (see Euclid's *Elements* XII 2).

Using these assumptions and theorems, Archimedes derived the formula for the area of a circle by first obtaining two contradictions:

(A) Suppose the circle has area greater than that of a right triangle T whose legs equal the radius and circumference of the circle.

By (4) and (5) we can find a natural number n such that

$$\text{circle area} - \text{inscribed regular } 2^n\text{-gon area} < \text{circle area} - \text{area of } T$$

and hence

$$\text{area of } T < 2^n\text{-gon area.}$$

Let AB be a side of the inscribed regular 2^n-gon, and ON a perpendicular from the center O of the circle to AB (with N being the midpoint of AB). Then ON is less than the radius of the circle. Using (3), we have

$$\begin{aligned} 2^n\text{-gon area} &= 2^n(\tfrac{1}{2}AB \cdot ON) \\ &= \tfrac{1}{2}(2^n AB)ON \\ &< \tfrac{1}{2}\text{circumference} \times \text{radius} \\ &= \text{area of } T. \end{aligned}$$

Contradiction. Thus (A) must be rejected.

(B) Suppose the circle has area less than T.

By (4) and (5) there is a natural number n such that the area of $T >$ circumscribed regular 2^n-gon area. However, if AB is a side of the circumscribing regular 2^n-gon, then, by (3),

$$\begin{aligned} 2^n\text{-gon area} &= 2^n(\tfrac{1}{2}AB \times \text{circle radius}) \\ &> \tfrac{1}{2}\text{circumference} \times \text{radius} \\ &= \text{area of } T. \end{aligned}$$

Contradiction. Thus (B) must be rejected.