

presented some original insights. However, on some points our opinions diverge; so, in a spirit of compromise, we have agreed to excise some of our more extreme views. Some of these divergent opinions have been expressed in Anglin [1994] and Lambek [1994].

Mathematics

One of the challenges one faces in offering a course on the history and philosophy of mathematics is to persuade one's colleagues that the course contains some genuine mathematics. For this reason, we have included some mathematical topics, usually not treated in standard courses, for example, the renaissance method for solving cubic equations and an elementary proof of the impossibility of trisecting arbitrary angles by ruler and compass constructions. We have taken the liberty of presenting many mathematical ideas in modern garb, with the hindsight inspired by more recent developments, since a presentation faithful to the original sources, while catering to the serious scholar, would bore most students to tears.

In Part I we deal essentially with the history of mathematics up to about 1800. This is because thereafter mathematics tends to become more specialized and too advanced for the students we have in mind. However, we make occasional excursions into more modern mathematics, partly to relieve the tedium associated with a strictly chronological development and partly to present modern answers to some ancient questions, whenever this can be done without overly taxing the students' ability.

In Part II we deal with some selected topics from the nineteenth and twentieth centuries. In that period, mathematics became rather specialized and made spectacular progress in different directions, but we confine attention to questions in the foundations and philosophy of mathematics.

The more universal aspects of mathematics are sketched briefly in the last five sections. We introduce the language of category theory, which attempts a kind of unification of different branches of mathematics, albeit at a very basic and abstract level.

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