LIB. II. & $\sqrt{(aa-pp)(pp-bb)} = -pq.cos.s$, unde fit $tang.2ACM = \frac{-2qq.cos.s}{pp+qq.cos.2s}$, quia $cos.s$ eſt negativus. Tandem eſt $CK^2 = MT.Mt$; ex ſuperioribus vero eruitur $MV = q\sqrt{\frac{AP}{BP}}$ & $AV = b\sqrt{\frac{AP}{BP}}$; unde erit $AV : MV = b : q = CE : CK$. Ergo rectæ, ſi ducantur, AM & EK, inter ſe erunt parallelæ.

146. Quia eſt $pq.sin.s = ab$, erit pq major quam ab; &, cum ſit $pp + qq = aa + bb$, quantitates p & q magis ad rationem æqualitatis accedunt, quam a & b, unde inter omnes Diametros conjugatas, illæ quæ ſunt orthogonales maxime a ſe invicem diſcrepant. Dabuntur ergo duæ Diametri conjugatæ inter ſe æquales, ad quas inveniendas ſit $q = p$, eritque $2pp = aa + bb$, & $p = q = \sqrt{\frac{aa+bb}{2}}$, & $sin.s = \frac{2ab}{aa+bb}$, atque $cos.s = \frac{-aa+bb}{aa-bb}$; unde fit $sin.\frac{1}{2}s = \sqrt{\frac{aa}{aa-bb}}$, $cos.\frac{1}{2}s = \sqrt{\frac{bb}{aa-bb}}$, ergo $tang.\frac{1}{2}s = \frac{a}{b} = tang.CEB$, & $MCK = 2CEB = AEB$. Porro $CP = \frac{a}{\sqrt{2}}$, $CM = \frac{b}{\sqrt{2}}$, quare Semidiametri conjugatæ inter ſe æquales CM, CK erunt parallelæ Cordis AE & BE.

147. Si Abſciſſæ a Vertice A computentur, ponaturque $AP = x$, $PM = y$, cum nunc ſit $a - x$ quod ante erat x, habebitur iſta æquatio $yy = \frac{bb}{aa}(2ax - xx) = \frac{2bb}{a}x - \frac{bb}{aa}xx$, ubi patet eſſe $\frac{2bb}{a}$ Parametrum ſeu latus rectum Ellipſis. Ponatur Semilatus rectum, ſeu Applicata in Foco $= c$, & diſtantia Foci a Vertice $AF = d$, erit $\frac{bb}{a} = c$ & $a - \sqrt{(aa-bb)} = d = a - \sqrt{(aa-ac)}$, unde fit $2ad - dd = ac$ & $a = \frac{dd}{2d-c}$. Hinc erit $yy = 2cx - \frac{c(2d-c)xx}{dd}$, quæ eſt

æquatio

æquatio pro Ellipsi inter Coordinatas orthogonales x & y, Abscissis x in Axe principali AB a Vertice A computatis, quæ obtinetur ex datis distantia Foci a Vertice $AF = d$ & Semilatere recto $= c$; ubi notandum est semper esse debere $2d$ majorem quam c, quia est $AC = a = \frac{dd}{2d - c}$, & $CD = b = d\sqrt{\frac{c}{2d - c}}$. Cap. VI.

148. Quod si ego fuerit $2d = c$ erit $yy = 2cx$, quam æquationem supra vidimus esse pro Parabola: æquatio enim superior $yy = \alpha + \beta x$ ad hanc formam reducitur, initio Abscissarum intervallo $= \frac{\alpha}{\beta}$ mutato. Sit igitur MAN Parabola, cujus natura inter Abscissam $AP = x$, & Applicatam $PM = y$ hac æquatione exprimatur $yy = 2cx$. Erit ergo distantia Foci a Vertice $AF = d = \frac{1}{2}c$, & Semiparameter $FH = c$, atque ubique $PM^2 = 2FH \cdot AP$: unde, posita Abscissa AP infinita, simul Applicatæ PM & PN in infinitum excrescunt; ideoque Curva ad utramque Axis AP partem in infinitum extenditur. Posita autem Abscissa x negativa Applicata fit imaginaria, hincque Axi ultra A versus T nulla Curvæ portio respondet. Tab. VIII. Fig. 32.

149. Cum æquatio pro Ellipsi abeat in Parabolam, facto $2d = c$, manifestum est Parabolam nil aliud esse præter Ellipsin, cujus Semiaxis $a = \frac{dd}{2d - c}$ fit infinitus; quam ob rem proprietates omnes, quas pro Ellipsi invenimus, ad Parabolam transferentur, posito Axe a infinito. Primum autem, cum sit $AF = \frac{1}{2}c$, erit $FP = x - \frac{1}{2}c$, hinc ducta ex Foco F ad Curvæ punctum M recta FM erit, $FM^2 = xx - cx + \frac{1}{4}cc + yy = xx + cx + \frac{1}{4}cc$, ideoque $FM = x +$

 $\frac{1}{2}c =$

 $\frac{1}{2}c = AP + AF$, quæ est præcipua proprietas Foci in Parabola.

150. Quoniam Parabola nascitur ex Ellipsi, Axe majore in infinitum aucto; consideremus Parabolam, tanquam esset Ellipsis, sitque ejus Semiaxis $AC = a$, existente a quantitate infinita, ita ut Centrum C infinite distet a Vertice A. Ad M ducatur tangens Curvæ MT Axi occurrens in T; quia erat $CP : CA = CA : CT$, erit $CT = \frac{aa}{a-x}$, ob $CP = a - x$; hincque $AT = \frac{ax}{a-x}$. At, cum sit a quantitas infinita, Abscissa x præ ea evanescet, eritque $a - x = a$, ideoque $AT = x = AP$: quod idem hoc modo ostendi potest, cum sit $AT = \frac{ax}{a-x}$, erit $AT = x + \frac{xx}{a-x}$, at quia fractionis $\frac{xx}{a-x}$ denominator est infinitus, numeratore existente finito, valor fractionis erit evanescens, ideoque $AT = AP = x$.

151. Quod si ergo ex puncto M ad Centrum Parabolæ C infinite distans ducatur Linea MC, quæ erit Axi AC parallela, ea quoque erit Diameter Curvæ omnes Chordas tangenti MT parallelas bisecans. Scilicet, si ducatur Chorda seu Ordinata mn tangenti MT parallela, ea a Diametro Mp bisecabitur in p. Omnis ergo recta Axi AP parallela ducta in Parabola erit Diameter obliquangula. Ad hujusmodi Diametrorum naturam eruendam sit $Mp = t$, $pm = u$, ducatur ex m ad Axem normalis msr; erit, ob $PT = 2x$, & $MT = \surd(4xx + 2cx)$, $\surd(4xx + 2cx) : 2x : \surd 2cx = pm : ps : ms$, unde obtinetur $ps = \frac{2xu}{\surd(4xx + 2cx)} = u\surd\frac{2x}{2x+c}$, & $ms = u\surd\frac{c}{2x+c}$; hinc erit $Ar = x + t + u\surd\frac{2x}{2x+c}$, & $mr = \surd 2cx + u\surd\frac{c}{2x+c}$. Quia vero est $mr^2 = 2c \cdot Ar$,

erit

erit $2cx+2cu\sqrt{\frac{2x}{2x+c}}+\frac{cuu}{2x+c}=2cx+2ct+2cu\sqrt{\frac{2x}{2x+c}}$, hincque $uu=2t(2x+c)=4FM.t$, seu $pm^2=4FM.Mp$. At anguli obliquitatis mps erit Sinus $=\sqrt{\frac{c}{2x+c}}=\sqrt{\frac{AF}{FM}}$, Cosinus $=\sqrt{\frac{2x}{2x+c}}=\sqrt{\frac{AP}{FM}}$, ideoque $\mathit{fin.}\ 2mps=\frac{2\sqrt{2cx}}{2x+c}=\frac{y}{FM}=\mathit{fin.}\ MFp$, ergo erit angulus $mps=MTP=\frac{1}{2}MFr$.

152. Quia eſt $MF=AP+AF$, ob $AP=AT$, erit $FM=FT$; ideoque triangulum MFT iſoſceles, & angulus $MFr=2MTA$, ut modo invenimus. Cum deinde ſit $MT=2\sqrt{x(x+\frac{1}{2}c)}$, erit $MT=2\sqrt{AP.FM}$, hinc ex Foco F in tangentem demiſſo perpendiculo erit $MS=TS=\sqrt{AP.FM}=\sqrt{AT.TF}$, unde erit $AT:TS=TS:TF$. Ex qua analogia perſpicitur punctum S fore in recta AS ad Axem in Vertice A normali. Erit vero $AS=\frac{1}{2}PM$, & $AS:TS=AF:FS$, ergo $FS=\sqrt{AF.FM}$ & FS erit media proportionalis inter AF & FM. Præterea vero erit $AS:MS=AS:TS=FS:FM=\sqrt{AF}:\sqrt{FM}$. Quod, ſi ducatur ad tangentem in M normalis MW Axem ſecans in W, erit $PT:PM=PM:PW$, ſeu $2x:\sqrt{2cx}=\sqrt{2cx}:PW$; unde fit $PW=c$, ubique igitur intervallum PW, quod in Axe inter Applicatam PM & normalem WM intercipitur, conſtantem habet magnitudinem atque æquale eſt ſemiſſi Lateris recti, ſeu Applicatæ FH. Erit autem $FW=FT=FM$ & $MW=2\sqrt{AF.FM}$.

153. Pervenimus jam ad Hyperbolam, cujus natura exprimitur hac æquatione $yy=\alpha+\beta x+\gamma xx$, Abſciſſis ſuper Diametro orthogonali ſumtis. Quod ſi autem initium Abſciſſarum transferatur intervallo $\frac{\beta}{2\gamma}$, orietur ejuſmodi æquatio

LIB. II. $yy = \alpha + \gamma xx$, in qua Abſciſſæ a Centro computantur. Debet autem γ eſſe quantitas affirmativa; quod vero ad α attinet, perinde eſt ſive ea ſit quantitas affirmativa ſive negativa, permutatis enim Coordinatis x & y, affirmatio quantitatis α in negationem mutatur & viciſſim. Quam ob rem ſit α quantitas negativa, & $yy = \gamma xx - \alpha$, atque apparet Applicatam y bis evaneſcere: ſcilicet, ſi fuerit $x = +\sqrt{\frac{\alpha}{\gamma}}$ & $x = -\sqrt{\frac{\alpha}{\gamma}}$. Denotante ergo C Centro, ſint A & B loca, ubi Axis a Curva trajicitur; ac, poſito Semiaxe $CA = CB = a$, erit $a = \sqrt{\frac{\alpha}{\gamma}}$, & $\alpha = \gamma aa$, unde fit $yy = \gamma xx - \gamma aa$. Quamdiu ergo eſt x^2 minor quam a^2, Applicata erit imaginaria, unde toti Axi AB nulla Curvæ portio reſpondet. Sumto vero xx majore quam aa, Applicatæ continuo creſcunt, atque tandem in infinitum abeunt, habebit ergo Hyperbola quatuor ramos AI, Ai, BK, Bk in infinitum excurrentes & inter ſe ſimiles atque æquales, quæ eſt proprietas principalis Hyperbolarum.

TAB. IX. Fig. 33.

154. Quia, poſito $x = 0$, fit $yy = -\gamma aa$, Hyperbola non inſtar Ellipſis habebit Axem conjugatum, quod in Centro C Applicata eſt imaginaria. Erit ergo ipſe Axis conjugatus imaginarius, quem, ut aliquam ſimilitudinem Ellipſis ſervemus, ponamus $= b\sqrt{-1}$, ita ut ſit $\gamma aa = bb$, & $\gamma = \frac{bb}{aa}$. Vocata ergo Abſciſſa $CP = x$, & Applicata $PM = y$, erit $yy = \frac{bb}{aa}(xx - aa)$, ideoque æquatio pro Ellipſi ante tractata $yy = \frac{bb}{aa}(aa - xx)$ tranſmutatur in æquationem pro Hyperbola ponendo $-bb$ loco bb. Ob hanc ergo affinitatem proprietates Ellipſis ante inventæ facile ad Hyperbolam transferuntur. Ac primo quidem, cum pro Ellipſi diſtantia Focorum a Centro eſſet $= \sqrt{(aa - bb)}$, pro Hyperbola erit