

- computation, 454
- Condillac, 274
- Condorcet, 312
- conformal mapping, 256
 - and mapmaking, 297
- conformal model, 348
 - as part of \mathbb{C} , 354
 - disk, 348
 - half-plane, 348
 - distance, 349
 - hemisphere, 348
 - in half-space, 350
- congruence, 67
 - and groups of motions, 371
 - modulo n , 67, 196, 426
- conic sections, 28
 - attributed to Menaechmus, 28
 - instrument for drawing, 29
 - projective view, 131
 - second degree equations, 106
- conjugates, 266
 - of quaternions, 393
- Connelly, 314
- constructible
 - number, 27
 - points, 89
 - polygons, 26
- construction
 - of equations, 111
 - ruler and compass, 25
 - of double circle arc, 218
 - of double lemniscate arc, 218
- continued fraction
 - and Pell's equation, 46, 76
 - definition, 46
 - for π , 154
 - periodic, 47
- continuity, 266
 - and axiom of choice, 464
 - and velocity, 233
- continuous
 - magnitude, 54
 - Dedekind definition, 54
 - process, 2
- continuum hypothesis, 458, 463
 - consistency, 458
 - independence, 458
- coordinates, 5, 12
 - in Hipparchus, 104
 - in Oresme, 105
- Copernicus, 234
- costruzione legittima, 122
- Cotes, 263
 - and complex logarithms, 294
 - and complex numbers, 263
 - Harmonia mensurarum*, 294
 - theorem on n -gon, 263
- countability, 456
- countable additivity, 461
- counting board, 84
- covering, 282
 - of orientable surface, 444
 - of projective plane, 443
 - of pseudosphere, 443
 - of torus, 443
 - projection map, 285
 - sheets of, 282
 - and integration, 302
 - universal, 443
- Cramer
 - and Bézout's theorem, 114
 - and permutations, 366
- Cramer's rule, 85
- cross-ratio
 - in Desargues, 126
 - Möbius invariance proof, 127
- cryptography
 - and Fermat's little theorem, 196
 - in Cardano, 101
 - in Viète, 103
 - Wallis, 162
- cube, 21
 - duplication of *see* duplication of the cube 25
 - symmetry group, 369
- cubic curves

- and Fermat's last theorem, 199
 - as tori, 286
 - five types, 111
 - Newton classification, 110
 - of genus 0, 205
 - parameterization, 207, 213
 - projective classification, 307
 - projective view, 131
- cubic equations, 91
 - and circular functions, 93
 - and complex functions, 293
 - and complex numbers, 94, 257
 - and trisection, 93
 - have real roots, 257
 - in Cardano, 92
 - in Viète, 93
 - solution, 92
- curl, 247
- curvature, 146
 - and Euler characteristic, 440
 - center of, 320
 - constant
 - surface of, 324, 344
 - due to Newton, 163
 - Gaussian, 323
 - and solid angle, 439
 - integral of, 329
 - geodesic, 326
 - intrinsic, 323
 - Kaestner definition, 321
 - negative
 - and noneuclidean geometry, 325
 - surface of, 324
 - Newton formula, 320
 - of plane curves, 319
 - of polyhedron, 438
 - of surfaces, 322
 - principal, 323
 - radius of, 320
 - Riemann, 291
- curve
 - algebraic, 34, 107, 271
 - behavior at infinity, 131
 - complex, 276
 - cubic, 110
 - degree, 107
 - equidistant, 347
 - in conformal model, 349
 - geometric, 107
 - mechanical, 107, 236
 - on projective plane, 136
 - projective, 129
 - projective completion of, 136
 - transcendental, 107, 241, 315
 - and differential geometry, 315
- curves
 - and elimination, 85
- cusp, 108
 - of cissoid, 31, 108
 - of semicubical parabola, 111
- cycloid, 145, 236, 237
 - arc length, 318
 - as brachistochrone, 239
 - as tautochrone, 238
 - is own involute, 321
- d'Alembert
 - and complex functions, 295
 - and conjugate solutions, 266
 - and Lagrange, 275
 - and Laplace, 275
 - and the *Encyclopédie*, 274
 - fundamental theorem of algebra, 266
 - lemma, 268
 - life story, 272
 - on algebra in geometry, 110
 - wave equation, 242
- de la Hire, 143
- de Moivre
 - and generating functions, 181
 - formula, 95
 - formula for Fibonacci numbers, 181
 - inversion formula, 157
 - solution by radicals, 95

De Morgan, 385

Dedekind

- and irrationals, 12
- and Peano axioms, 426
- and Riemann surfaces, 426
- cut, 54, 267, 426, 455
 - for irrational, 55
 - for rational, 55
- defined algebraic integers, 409
- defined ideals, 412
- definition of $\sqrt{2}$, 54
- definition of continuity, 54
- definition of field, 386
- friend of Riemann, 424
- life story, 424
- number fields, 422
- product of ideals, 417
- proved two-square theorem, 419
- rigor, 53
- student of Gauss, 337, 424
- student of Riemann, 291
- supplemented Dirichlet, 425

Degen, 226

- eight-square identity, 395

degree

- of curve, 107
- of field, 422

Dehn

- and hyperbolic geometry, 449
- combinatorial group theory, 376
- solved Hilbert's third problem, 58

Desargues, 124

- and cross-ratio, 126
- and projective lines, 136
- Brouillon projet*, 126, 142
- life story, 142
- projective geometry, 125
- theorem, 126
 - and algebra, 397, 428
 - and foundations, 126
 - planar case, 128
 - statement, 126

used epicyclic curves, 143

Descartes

- and analytic geometry, 105
- coordinate method, 19
- factor theorem, 97, 266
- folium, 108
- Géométrie*, 105
- in the stove, 118
- integration formula, 149
- life story, 116
- notation for powers, 97
- polyhedron formula, 432
 - and Gauss–Bonnet, 438

descriptive geometry, 125

determinant, 139

diagonal argument, 464

- and computability, 465
- and Gödel's theorem, 471
- and rate of growth, 465
- for real numbers, 465
- for sets, 464

Diderot, 274

differentiability, 234

differential equations

- and catenary, 237
- and elastica, 239
- and mechanics, 235
- for geodesics, 326
- partial, 242

differential geometry, 315

- and calculus, 315
- and curvature, 163, 319
- and hyperbolic geometry, 344

differentiation, 146

Diocles, 31

Diophantine

- equations, 6
 - cubic, 48
 - linear, 43
 - no algorithm, 6, 37
 - quadratic, 43
 - rational solutions, 6
- problems, 6

- Diophantus, 3
 - Arithmetic*
 - Bachet edition, 50
 - in Bombelli's *Algebra*, 50
 - and complex numbers, 382
 - and Diophantine problems, 6
 - and Pythagorean triples, 7
 - and sums of squares, 390
 - chord and tangent methods, 83
 - chord method, 48
 - on folium, 109
 - composition, 74
 - identity, 72
 - life story, 49
 - method, 6, 79
 - and elliptic functions, 207
 - and Fermat, 6
 - and Newton, 6
 - geometric interpretation, 48
 - solution of $y^3 = x^2 + 2$, 405
 - tangent method, 48, 151
 - and Viète, 49
 - two-square identity, 383, 418
- Dirichlet
 - and algebraic integers, 409
 - and Fermat's last theorem, 199
 - class number formula, 425
 - function, 459, 462
 - principle, 290
 - and Riemann mapping theorem, 298
 - justified by Hilbert, 298
 - replaced Gauss, 424
 - taught Riemann, 290
 - theorem on primes, 244
 - Vorlesungen*, 425
- discrete process, 2
- discriminant, 416
 - invariance of, 420, 427
- distance, 116
 - and coordinates, 12
 - and Pythagoras' theorem, 13
 - definition of, 14, 116
- divergence, 248
 - of harmonic series, 171
- divisibility
 - and Pythagorean triples, 5
 - in Euclid, 5
- division of stakes, 195
- division property, 407
- dodecahedron, 24
 - dual to icosahedron, 24
 - symmetry group, 369
- Donaldson, 432
- double periodicity, 222
 - and complex integration, 302
 - and Riemann, 223, 290, 303
 - of Weierstrass \wp -function, 304
- double point, 108, 205
- double root, 206
- doubling the arc
 - of circle, 218
 - of lemniscate, 217, 218
- du Bois-Reymond, 465
- duplication of the cube, 25
 - by cissoid, 32
 - by intersecting conics, 29
 - by Menaechmus, 29
- Dürer, 120
- Dyck
 - concept of group, 367
 - groups and tessellations, 371
- e
 - transcendence, 26
- Einstein, 402
- Eisenstein, 290
 - and algebraic integers, 409
 - series, 303
 - student of Gauss, 337
- elastica, 215, 236, 239
 - pictures, 240
- elimination, 83, 114
 - and linear algebra, 139
 - and polynomial equations, 84
 - Gaussian, 84
- ellipse, 28

- arc length, 215
- as planetary orbit, 29
 - versus Cassini oval, 33
- focus of, 30
- not an elliptic curve, 215
- string construction, 30
- elliptic
 - curves, 215, 305
 - addition of points, 307
 - and Fermat's last theorem, 305
 - isomorphic to \mathbb{C}/Λ , 307
 - parameterized by \wp, \wp' , 307
 - functions, 105, 160, 200, 207, 213, 215
 - addition theorem, 209
 - and complex numbers, 222
 - and elastica, 239
 - and the torus, 286
 - birth day, 220
 - by inverting integrals, 222
 - double periodicity, 222, 302
 - series expansions, 303
 - integrals, 215
 - addition theorem, 220
 - not elementary, 215
- elliptic modular functions *see* modular functions 97
- empty set, 457
- epicycles, 33
 - in astronomy, 234
 - used by Desargues, 143
- equation
 - cubic, 91
 - solution, 92
 - linear, 84
 - Pell's, 43, 83, 404
 - polynomial, 83
 - quadratic
 - Brahmagupta formula, 42
 - in Babylon, 82
 - in Euclid, 82
 - quartic, 96
 - quintic, 96
 - van Roomen, 103
- equivalence relation, 19
 - defined by group, 373
- Euclid, 3
 - Elements*, 4, 191
 - Book V, 53
 - common notions, 18, 373
 - postulates, 18
 - Tartaglia's translation, 100
 - life story, 35
 - perfect number theorem, 40
 - and geometric series, 63
 - proofs of Pythagoras' theorem, 9
 - Pythagorean triples
 - formula, 4
 - theory of divisibility, 5
 - theory of irrationals, 90
 - view of quadratic equations, 82
- Euclidean
 - algorithm, 41
 - as "pulverizer", 46
 - criterion for irrationality, 66
 - for Gaussian integers, 408
 - for polynomials, 210
 - in Asia, 66
 - geometry, 115
 - on horosphere, 344
 - on torus, 445
 - plane, 115
 - rigid motions, 352
 - tessellations, 352
- Eudoxus, 35
 - definition of equality, 53
 - method of exhaustion, 55
 - theory of proportions, 53
- Euler
 - addition theorems, 209, 220
 - Algebra*, 191
 - and Bézout's theorem, 114
 - and Chinese remainder theorem, 70
 - and chord-tangent construction,

- 207
- and complex logarithms, 294
- and complex numbers, 261
- and conformal mapping, 297
- and Fermat's last theorem, 199
- characteristic, 432
 - and curvature, 440
 - and genus, 436
 - Poincaré generalization, 433
- constant, 173
- continued fraction formula, 155
- cotangent series, 303
- formula for e^{ix} , 295
- four-square identity, 389
- geodesic differential equation, 326
- life story, 188
- pentagonal number theorem, 39, 191
- perfect number theorem, 40
- pictures of elastica, 240
- polyhedron formula, 432
 - Legendre proof, 433
- product formula, 184
- proof of Fermat's little theorem, 197
- proved two-square theorem, 419
- rigid surface conjecture, 314
- student of Johann Bernoulli, 188
- summed $\sum 1/n^2$, 178
- theorem on $y^3 = x^2 + 2$, 405
- values of $\zeta(s)$, 185
- zeta function formula, 184
- exhaustion *see* method of exhaustion 55
- exponential function, 157
 - addition formula, 295
 - complex, 295
 - periodicity, 295
- extreme value theorem, 267, 269
- factor theorem, 97, 178
- Fagnano, 207
 - addition theorem, 209
 - duplication formula, 218
 - studied by Euler, 220
 - lemniscate division, 224
- Faltings, 199
- Fano plane, 398
- Fermat
 - and analytic geometry, 105
 - and Diophantus, 48, 50
 - and Diophantus' method, 6
 - and rational right triangles, 200
 - example of Pell's equation, 76
 - infinite descent, 200
 - integration formula, 149
 - last theorem, 198
 - and cyclotomic integers, 421
 - and elliptic curves, 305
 - and Faltings, 199
 - attempt by Lamé, 421
 - attempt by Lindemann, 26
 - for $n = 4$, 199, 200
 - proof by Wiles, 200, 421
 - special cases, 199
 - life story, 211
 - little theorem, 196
 - proof using inverses, 363
 - Observations on Diophantus*, 198
 - tangent method, 150
 - applied to folium, 151
 - theorem on $y^3 = x^2 + 2$, 405
 - theorems on sums of squares, 419
 - two-square theorem, 419
- Ferrari, 91
 - dispute with Tartaglia, 99
 - poisoned, 101
 - solution of quartic, 96
- Fibonacci
 - and cubic irrationals, 90
 - Book of Squares*, 383
 - sequence, 181, 404
- field, 365, 385, 422
 - definition, 386
 - of finite degree, 422
 - of rational numbers, 422

- theory, 404
- Fior, 91
- Fischer, 429
- flow
 - incompressible, 247
 - and divergence, 248
 - irrotational, 247
 - and curl, 247
- focus, 30
 - in astronomy, 31
- folium
 - asymptote, 108, 109
 - double point, 108
 - drawn by Huygens, 108
 - has genus 0, 205
 - of Descartes, 108
 - parameterization, 109
 - tangent of, 151
- foundations
 - arithmetic and set-theoretic, 54
 - geometric, 54
 - of geometry, 126, 428
- four-square theorem, 39
- Fourier series, 241
 - and integrals, 459
 - and theory of heat, 244
- Frege, 470
- Freudenthal, 396
- Frey, 199
- Frobenius, 396
- Fuchs, 452
- function
 - algebraic, 159, 174
 - choice, 462
 - computable, 466
 - Dirichlet, 459, 462
 - elementary, 215
 - elliptic, 160, 213
 - hyperbolic, 74, 342
 - many-valued, 180
 - modular, 97, 223
 - rational, 174
 - symmetric, 364
 - theta, 39, 223
 - transcendental, 159, 241
 - zeta, 184
- fundamental group, 448
 - as group of motions, 449
 - defined by Poincaré, 449
 - generators and relations, 449
 - higher-dimensional, 453
- fundamental polygon, 436
 - and universal covering, 445
 - for genus 2, 446
 - for torus, 445
- fundamental theorem
 - of algebra, 266
 - and Bézout's theorem, 271, 278
 - and intersections, 277
 - d'Alembert proof, 266
 - Gauss proofs, 266, 269
 - motivated by integration, 265
 - real version, 266
 - of arithmetic, 42
 - of calculus, 159
 - and Gregory, 187
 - generalized, 300
 - in Leibniz formalism, 159
- Fürtwangler, 477
- Galileo, 232
 - and catenary, 237
 - and projectile, 232
 - observed Neptune, 382
 - principle of inertia, 233
- Galois
 - and the quintic, 96, 97, 366, 378
 - field concept, 422
 - introduced group concept, 365
 - life story, 377
 - studied Legendre's *Geometry*, 378
 - theory, 365
 - and construction problems, 26
 - and regular polyhedra, 21
 - in Dedekind, 425
 - theory of ambiguity, 381

- theory of fields, 365, 386
- gamma function, 185
- Gauss
 - and algebraic integers, 410
 - and Chinese remainder theorem, 70
 - and circle division, 26
 - and complex integration, 299
 - and conformal mapping, 298
 - and elliptic functions, 222
 - and lemniscate division, 224
 - and modular functions, 97, 223
 - and quadratic forms, 420, 425
 - and the agM, 222
 - and unique prime factorization, 420
 - area of hyperbolic circle, 342
 - arithmetic-geometric mean, 188
 - construction of 17-gon, 26
 - curvature, 323
 - Disquisitiones arithmeticae*, 335
 - formula for sphere motion, 353, 391
 - fundamental theorem of algebra, 266
 - geodesic curvature, 327
 - geodesy, 323, 336
 - life story, 334
 - proved two-square theorem, 419
 - sphere, 281
 - taught Dedekind, 337
 - taught Eisenstein, 337
 - theorema egregium*, 323
 - triangle tessellation, 355, 448
- Gauss–Bonnet theorem, 327, 342
- polyhedral form, 438
- Gaussian
 - integer, 406
 - divisibility criterion, 408
 - division property, 407
 - Euclidean algorithm, 408
 - prime, 406
 - factorization, 407
- generating function, 181
 - for combinations, 195
 - of Fibonacci sequence, 181
- generators and relations, 371, 373
 - and topology, 376
 - read off tessellation, 374
- genus, 199
 - and Euler characteristic, 436
 - and rational functions, 205
 - as number of holes, 286
 - implicit in Abel, 226
 - topological meaning, 281
- geodesic, 326
 - curvature, 326
 - differential equation, 326
 - mapped to straight line, 344
 - on cone, 327
 - on cylinder, 327
 - on pseudosphere, 327
 - on sphere, 326
- geometric series, 157
 - and area of parabola, 61
 - and bodily substance, 251
 - and volume of tetrahedron, 60
 - in Euclid, 63
- geometric-harmonic mean, 187
- geometry
 - algebraic, 31
 - analytic, 12, 104
 - complex interpretation, 352
 - descriptive, 125
 - differential, 315
 - foundations of, 126, 428
 - hyperbolic, 342
 - noneuclidean, 18, 105, 256, 325, 338
 - of surfaces, 344
 - projective, 114, 125
 - spherical, 341
- Gibbs, 403
- Gödel, 167
 - and axiom of choice, 478
 - and continuum hypothesis, 458

- and relativity theory, 478
 - incompleteness theorem, 429, 469
 - life story, 475
 - “miracle” of computability, 465
 - second theorem, 472
 - in Hilbert and Bernays, 429, 473
- golden ratio, 27
- golden rectangle, 21
 - constructibility, 89
- Gordan, 427
- Goursat, 301
- Grandi, 109
- Graves
 - John, 385
 - discovered octonions, 393
 - read literature on squares, 389
 - Robert, 391
- great circle
 - and projective line, 136
- Green, 300
- Green’s theorem, 300
 - implies Cauchy’s theorem, 300
- Gregory, 174
 - and interpolation, 176
 - and Taylor’s theorem, 175
 - and transcendence, 187
 - geometric-harmonic mean, 187
 - life story, 186
 - Vera quadratura*, 187
- group
 - associativity, 367
 - cancellation, 367
 - concept of Galois, 365
 - defining properties, 361
 - fundamental, 448
 - identity, 361
 - inverse, 361
 - isomorphism, 368
 - isomorphism problem, 450
 - of motions, 371
 - of permutations, 365
 - of rigid motions, 352
 - of transformations, 371
 - on a cubic curve, 367
 - polyhedral, 368
 - and theory of equations, 369
 - presentation, 371
 - S_n , 364
 - symmetry, 368
 - word problem, 468
- group theory, 309, 361
 - and theory of equations, 365
 - combinatorial, 373
- Hadamard, 263
- Hahn, 477
- Halcke, 81
- Halley, 164, 235
- halting problem, 468
- Hamilton
 - defined complex numbers, 384
 - discovered quaternions, 388
 - dynamics, 402
 - life story, 399
 - optics, 401
 - predicted conical refraction, 402
 - presented icosahedral group, 371
 - sought product of triples, 387
- handle, 437
- harmonic series, 171, 172
- harmony
 - and integer ratios, 10, 15
 - and Pythagoras, 10
 - of the spheres, 16
- Harnack, 460
- Harriot
 - and interpolation, 177
 - and logarithmic spiral, 316
 - and stereographic projection, 297
 - life story, 331
 - theorem on spherical area, 329, 341, 441
- Hausdorff, 463
- Heath, 200
- Heaviside, 403
- Hermite

- and algebraic integers, 409
 - followed Galois' hint, 366
 - preserved works of Galois, 378
 - solution of quintic, 97
 - transcendence of e , 26
- Heron, 30, 80
- Heuraet, 318
- Higman, 469
- Hilbert
 - algebra of projective planes, 397
 - arithmetic and geometry, 12
 - basis theorem, 428
 - foundations of geometry, 115, 126
 - justified Dirichlet principle, 298
 - life story, 426
 - problems, 58
 - first, 458
 - second, 428
 - third, 58
 - program, 428
 - rectified flaws in Euclid, 18
 - theorem on constant curvature, 324
 - Zahlbericht*, 428
- Hipparchus, 104
- Hobbes
 - denounced Wallis' *Conics*, 115
 - in love with geometry, 17
 - on *Arithmetica infinitorum*, 152
 - on Torricelli's result, 149
- Holbein, 124
- Hölder, 258
- Holmboe, 225
- homeomorphism, 431
 - problem, 450, 469
- homogeneous
 - coordinates, 134
 - extend cartesian plane, 134
 - polynomial, 135
- homomorphism, 423
- homotopic paths, 448
- Hooke, 164, 250
 - and catenary, 237
- horocycle, 347
 - in conformal model, 349
- horosphere, 344
 - in half-space model, 350
 - is Euclidean, 344
- Hudde, 151
- Hurewicz, 453
- Hurwitz, 396, 427
- Huygens
 - and catenary, 237
 - and pseudosphere, 324
 - description of tractrix, 318
 - drew folium, 108
 - found tautochrone, 238
 - on discoveries in geometry, 146
 - pendulum clocks, 238
- hydrodynamics, 245
 - and complex functions, 295
- hydrostatics, 231
- hyperbola, 28
 - arc length, 215
 - area of segment, 63
 - points at infinity, 131
 - quadrature of, 156
- hyperbolic
 - circle, 347
 - in conformal model, 349
 - function, 342
 - geometry, 342
 - and differential geometry, 344
 - complex interpretation, 355
 - conformal models, 348
 - named by Klein, 342
 - projective model, 344
 - plane, 342
 - as covering, 446
 - rigid motions, 347
 - tessellations, 354
 - space, 350
 - rigid motions, 356
 - tessellation, 448
 - trigonometry, 342

- hypercomplex numbers, 382
 - algebraic properties, 396
- hypergeometric, 154
 - differential equation, 355
- icosahedron
 - constructibility, 89
 - Pacioli construction, 21
 - symmetry group, 369
 - tessellation, 353
- ideal numbers, 411
- ideals, 411
 - as kernels, 423
 - classes of, 416
 - containment and division, 413
 - defined by Dedekind, 412
 - factorization of, 416
 - gcd of, 416
 - in \mathbb{Z} , 413
 - in $\mathbb{Z}[\sqrt{-5}]$, 414
 - in $\mathbb{Z}[i]$, 414
 - maximal, 416
 - principal, 413
 - product of, 417
 - shape of, 414
 - sum of, 413, 415
- identity, 361
- incommensurable *see* irrational 11
- indivisibles, 149
 - in *Arithmetica infinitorum*, 152
- induction
 - and infinite descent, 203
 - characterizes natural numbers, 426
 - in Levi ben Gershon, 193
 - in Pascal, 195
- inertia
 - and Galileo, 233
 - and Newton, 233
- infinite
 - completed, 51
 - and limits, 52
 - and set theory, 54
 - descent, 200
 - in Greek mathematics, 51
 - potential, 51
 - processes
 - for finding volume, 58
 - rejected by Greeks, 51
 - product, 153, 179
 - reasoning about, 51
 - sequence, 52
 - set of points, 52
- infinite series, 146, 170
 - for algebraic functions, 174
 - for circular functions, 156, 172
 - for log, 157
 - for π , 172
 - in Greek mathematics, 170
 - inversion, 157
 - by de Moivre, 157
 - Newton's calculus of, 147
- infinitesimals, 53, 159
 - of Robinson, 148
- infinity
 - behavior of curves at, 131
 - inflection at, 132
 - line at, 126, 136
 - point at, 125
- infinity *see* infinite 51
- inflection, 108, 132
- inradius, 22
- integer
 - algebraic, 409
 - cyclotomic, 420
 - Gaussian, 406
 - quadratic, 420
 - rational, 405, 410
- integral
 - arcsine, 216
 - elliptic, 215
 - Lebesgue, 460
 - lemniscatic, 216
 - Riemann, 459
- integration, 146
 - and arc length, 318
 - and partial fractions, 265