quidem positiuus. Quia vero $t^\circ = m$, $t' = T$, $u^\circ = o$, $u' = U$, adeoque integri: omnes t'', t''' etc. u'', u''' etc. etiam integri erunt. Porro perspicuum est, quia $TT > mm$, omnes t°, t', t'', t''' etc. positiuos et continuo in infinitum crescentes esse, nec non omnes u°, u', u'', u''' etc.

III. Supponamus, dari adhuc alios valores positiuos ipsorum t, u qui in progressione t°, t', t'' etc. u°, u', u'' etc. non contenti sint, puta $\mathfrak{T}$, $\mathfrak{U}$. Manifestum est, quum progressio u°, u' etc. a o in infinitum crescat, $\mathfrak{U}$ necessario inter duos terminos proximos, u^n et u^{n+1} situm fore, ita vt sit $\mathfrak{U} > u^n$ et $\mathfrak{U} < u^{n+1}$. Vt absurditatem huius suppositionis demonstremus, obseruamus

1° Aequationi $tt - Duu = mm$ satisfactum iri etiam ponendo $t = \frac{1}{m}(\mathfrak{T}t^n - D\mathfrak{U}u^n)$, $u = \frac{1}{m}(\mathfrak{U}t^n - \mathfrak{T}u^n)$. Hoc quidem nullo negotio per substitutionem confirmatur: quod vero hi valores quos ponemus breuitatis gratia $=\tau$, υ, semper sunt numeri *integri* ita ostendimus. Si (M, N, P) est forma determinantis D, atque m diuisor communis numerorum M, $2N$, P: erit tum $\mathfrak{T} + N\mathfrak{U}$ tum $t^n + Nu^n$ per m diuisibilis adeoque etiam $\mathfrak{U}(t^n + Nu^n) - u^n(\mathfrak{T} + N\mathfrak{U})$ siue $\mathfrak{U}t^n - \mathfrak{T}u^n$. Quare υ erit integer et proin etiam τ, quia $\tau\tau = D\upsilon\upsilon + mm$.

2° Patet υ non posse esse $= o$; hinc enim sequeretur $\mathfrak{U}\mathfrak{U}t^n t^n = \mathfrak{T}\mathfrak{T}u^n u^n$ siue $\mathfrak{U}\mathfrak{U}(Du^n u^n + mm) = u^n u^n(D\mathfrak{U}\mathfrak{U} + mm)$ siue $\mathfrak{U}\mathfrak{U} =$

$u^n u^n$, contra hyp. ex qua $\mathfrak{U} > u^n$. Quum igitur praeter valorem o, minimus valor ipsius u sit U, erit v certe non minor quam U.

3° Facile ex valoribus ipsorum t^n, t^{n+1}, u^n, u^{n+1} confirmari potest esse $m U = u^{n+1} t^n - t^{n+1} u^n$. Quare $\mathfrak{U} t^n - \mathfrak{T} u^n$ certe non erit minor quam $u^{n+1} t^n - t^{n+1} u^n$.

4° Iam ex aequatione $\mathfrak{T}\mathfrak{T} - D\mathfrak{U}\mathfrak{U} = mm$ habetur $\frac{\mathfrak{T}}{\mathfrak{U}} = \sqrt{}(D + \frac{mm}{\mathfrak{U}\mathfrak{U}})$ et similiter $\frac{t^{n+1}}{u^{n+1}} = \sqrt{}(D + \frac{mm}{u^{n+1} u^{n+1}})$, vnde facile deducitur esse $\frac{\mathfrak{T}}{\mathfrak{U}} > \frac{t^{n+1}}{u^{n+1}}$. Hinc vero et ex conclusione in 3° sequitur $(\mathfrak{U} t^n - \mathfrak{T} u^n)(t^n + u^n \frac{\mathfrak{T}}{\mathfrak{U}}) > (u^{n+1} t^n - t^{n+1} u^n)(t^n + u^n \frac{t^{n+1}}{u^{n+1}})$, siue, euolutione facta, et loco ipsorum $\mathfrak{T}\mathfrak{T}$, $t^n t^n$, $t^{n+1} t^{n+1}$ substitutis valoribus suis $D\mathfrak{U}\mathfrak{U} + mm$, $D u^n u^n + mm$, $D u^{n+1} u^{n+1} + mm$,

$$\frac{1}{\mathfrak{U}}(\mathfrak{U}\mathfrak{U} - u^n u^n) > \frac{1}{u^{n+1}}(u^{n+1} u^{n+1} - u^n u^n),$$

vnde, quoniam vtraque quantitas manifesto positiua, fit transponendo $\mathfrak{U} + \frac{u^n u^n}{u^{n+1}} > u^{n+1} + \frac{u^n u^n}{\mathfrak{U}}$, *Q. E. A.*, quia quantitatis prioris pars prima *minor* est quam pars prima quantitatis secundae, nec non illius secunda minor quam secunda huius. Quamobrem suppositio consistere nequit et progressiones t°, t', t'', etc. u°, u', u'', etc. omnes valores positiuos ipsorum t, u exhibebunt.

Ex. Pro $D = 61$, $m = 2$ valores minimos positiuos ipsorum t, u inuenimus 1523, 195: quare *omnes* valores positiui exhibebuntur per has formulas $t = (\frac{1523}{2} + \frac{195}{2}\sqrt{61})^e + (\frac{1523}{2} - \frac{195}{2}\sqrt{61})^e$, $u = \frac{1}{\sqrt{61}}((\frac{1523}{2} + \frac{195}{2}\sqrt{61})^e - (\frac{1523}{2} - \frac{195}{2}\sqrt{61})^e)$. Inuenitur autem $t^\circ = 2$, $t' = 1523$, $t'' = 1523\,t' - t^\circ = 2319527$, $t''' = 1523\,t'' - t' = 3532618098$ etc.; $u^\circ = 0$, $u' = 195$, $u'' = 1523\,u' - u^\circ = 296985$, $u''' = 1523\,u'' - u' = 452307960$ etc.

201. Circa problema in artt. praecc. tractatum sequentes obseruationes adhuc adiicimus.

1) Quum aequationem $tt - Duu = mm$ pro omnibus casibus soluere docuerimus, vbi m est diuisor communis maximus trium numerorum M, $2N$, P, talium vt $NN - MP = D$: operae pretium est omnes numeros qui tales diuisores esse possunt siue omnes valores ipsius m pro valore dato ipsius D assignare. Ponatur $D = nnD'$, ita vt D' a factoribus quadraticis omnino sit liber, quod obtinetur si pro nn assumitur maximum quadratum ipsum D metiens: sin vero D