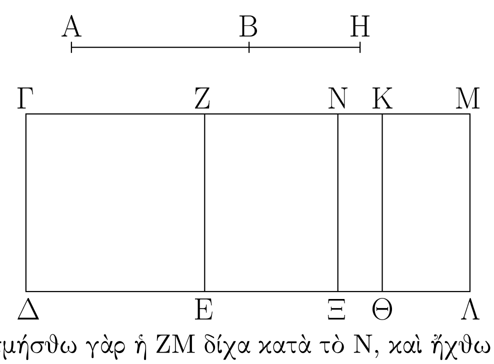
ποιοῦν τὴν ΓΖ· λέγω, ὅτι ἡ ΓΖ ἀποτομή ἐστι δευτέρα.

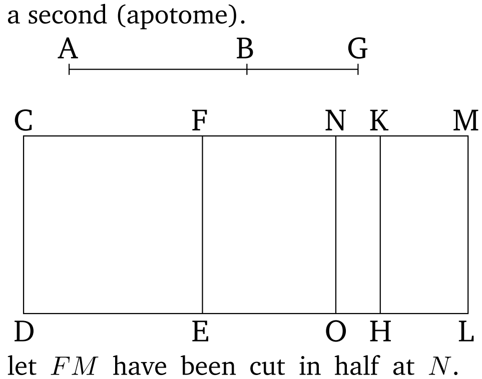
Ἔστω γὰρ τῇ ΑΒ προσαρμόζουσα ἡ ΒΗ· αἱ ἄρα ΑΗ, ΗΒ μέσαι εἰσὶ δυνάμει μόνον σύμμετροι ῥητὸν περιέχουσαι. καὶ τῷ μὲν ἀπὸ τῆς ΑΗ ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω τὸ ΓΘ πλάτος ποιοῦν τὴν ΓΚ, τῷ δὲ ἀπὸ τῆς ΗΒ ἴσον τὸ ΚΛ πλάτος ποιοῦν τὴν ΚΜ· ὅλον ἄρα τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ· μέσον ἄρα καὶ τὸ ΓΛ. καὶ παρὰ ῥητὴν τὴν ΓΔ παράκειται πλάτος ποιοῦν τὴν ΓΜ· ῥητὴ ἄρα ἐστὶν ἡ ΓΜ καὶ ἀσύμμετρος τῇ ΓΔ μήκει. καὶ ἐπεὶ τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ, ὧν τὸ ἀπὸ τῆς ΑΒ ἴσον ἐστὶ τῷ ΓΕ, λοιπὸν ἄρα τὸ δὶς ὑπὸ τῶν ΑΗ, ΗΒ ἴσον ἐστὶ τῷ ΖΛ. ῥητὸν δέ [ἐστι] τὸ δὶς ὑπὸ τῶν ΑΗ, ΗΒ· ῥητὸν ἄρα τὸ ΖΛ. καὶ παρὰ ῥητὴν τὴν ΖΕ παράκειται πλάτος ποιοῦν τὴν ΖΜ· ῥητὴ ἄρα ἐστὶ καὶ ἡ ΖΜ καὶ σύμμετρος τῇ ΓΔ μήκει. ἐπεὶ οὖν τὰ μὲν ἀπὸ τῶν ΑΗ, ΗΒ, τουτέστι τὸ ΓΛ, μέσον ἐστίν, τὸ δὲ δὶς ὑπὸ τῶν ΑΗ, ΗΒ, τουτέστι τὸ ΖΛ, ῥητόν ἀσύμμετρον ἄρα ἐστὶ τὸ ΓΛ τῷ ΖΛ. ὡς δὲ τὸ ΓΛ πρὸς τὸ ΖΛ, οὕτως ἐστὶν ἡ ΓΜ πρὸς τὴν ΖΜ· ἀσύμμετρος ἄρα ἡ ΓΜ τῇ ΖΜ μήκει. καί εἰσιν ἀμφότεραι ῥηταί· αἱ ἄρα ΓΜ, ΜΖ ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἡ ΓΖ ἄρα ἀποτομή ἐστιν. λέγω δή, ὅτι καὶ δευτέρα.




Τετμήσθω γὰρ ἡ ΖΜ δίχα κατὰ τὸ Ν, καὶ ἤχθω διὰ τοῦ Ν τῇ ΓΔ παράλληλος ἡ ΝΞ· ἑκάτερον ἄρα τῶν ΖΞ, ΝΛ ἴσον ἐστὶ τῷ ὑπὸ τῶν ΑΗ, ΗΒ. καὶ ἐπεὶ τῶν ἀπὸ τῶν ΑΗ, ΗΒ τετραγώνων μέσον ἀνάλογόν ἐστι τὸ ὑπὸ τῶν ΑΗ, ΗΒ, καί ἐστιν ἴσον τὸ μὲν ἀπὸ τῆς ΑΗ τῷ ΓΘ, τὸ δὲ ὑπὸ τῶν ΑΗ, ΗΒ τῷ ΝΛ, τὸ δὲ ἀπὸ τῆς ΒΗ τῷ ΚΛ, καὶ τῶν ΓΘ, ΚΛ ἄρα μέσον ἀνάλογόν ἐστι τὸ ΝΛ· ἔστιν ἄρα ὡς τὸ ΓΘ πρὸς τὸ ΝΛ, οὕτως τὸ ΝΛ πρὸς τὸ ΚΛ. ἀλλ᾽ ὡς μὲν τὸ ΓΘ πρὸς

and $CD$ a rational (straight-line). And let $CE$, equal to the (square) on $AB$, have been applied to $CD$, producing $CF$ as breadth. I say that $CF$ is a second apotome.

For let $BG$ be an attachment to $AB$. Thus, $AG$ and $GB$ are medial (straight-lines which are) commensurable in square only, containing a rational (area) [Prop. 10.74]. And let $CH$, equal to the (square) on $AG$, have been applied to $CD$, producing $CK$ as breadth, and $KL$, equal to the (square) on $GB$, producing $KM$ as breadth. Thus, the whole of $CL$ is equal to the (sum of the squares) on $AG$ and $GB$. Thus, $CL$ (is) also a medial (area) [Props. 10.15, 10.23 corr.]. And it is applied to the rational (straight-line) $CD$, producing $CM$ as breadth. $CM$ is thus rational, and incommensurable in length with $CD$ [Prop. 10.22]. And since $CL$ is equal to the (sum of the squares) on $AG$ and $GB$, of which the (square) on $AB$ is equal to $CE$, the remainder, twice the (rectangle contained) by $AG$ and $GB$, is thus equal to $FL$ [Prop. 2.7]. And twice the (rectangle contained) by $AG$ and $GB$ [is] rational. Thus, $FL$ (is) rational. And it is applied to the rational (straight-line) $FE$, producing $FM$ as breadth. $FM$ is thus also rational, and commensurable in length with $CD$ [Prop. 10.20]. Therefore, since the (sum of the squares) on $AG$ and $GB$—that is to say, $CL$—is medial, and twice the (rectangle contained) by $AG$ and $GB$—that is to say, $FL$—(is) rational, $CL$ is thus incommensurable with $FL$. And as $CL$ (is) to $FL$, so $CM$ is to $FM$ [Prop. 6.1]. Thus, $CM$ (is) incommensurable in length with $FM$ [Prop. 10.11]. And they are both rational (straight-lines). Thus, $CM$ and $MF$ are rational (straight-lines which are) commensurable in square only. $CF$ is thus an apotome [Prop. 10.73]. So, I say that (it is) also a second (apotome).




For let $FM$ have been cut in half at $N$. And let $NO$ have been drawn through (point) $N$, parallel to $CD$. Thus, $FO$ and $NL$ are each equal to the (rectangle contained) by $AG$ and $GB$. And since the (rectangle contained) by $AG$ and $GB$ is the mean proportional to the squares on $AG$ and $GB$ [Prop. 10.21 lem.], and the (square) on $AG$ is equal to $CH$, and the (rectangle contained) by $AG$ and $GB$ to $NL$, and the (square) on τὸ ΝΛ, οὕτως ἐστὶν ἡ ΓΚ πρὸς τὴν ΝΜ, ὡς δὲ τὸ ΝΛ πρὸς τὸ ΚΛ, οὕτως ἐστὶν ἡ ΝΜ πρὸς τὴν ΜΚ· ὡς ἄρα ἡ ΓΚ πρὸς τὴν ΝΜ, οὕτως ἐστὶν ἡ ΝΜ πρὸς τὴν ΚΜ· τὸ ἄρα ὑπὸ τῶν ΓΚ, ΚΜ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΝΜ, τουτέστι τῷ τετάρτῳ μέρει τοῦ ἀπὸ τῆς ΖΜ [καὶ ἐπεὶ σύμμετρόν ἐστι τὸ ἀπὸ τῆς ΑΗ τῷ ἀπὸ τῆς ΒΗ, σύμμετρόν ἐστι καὶ τὸ ΓΘ τῷ ΚΛ, τουτέστιν ἡ ΓΚ τῇ ΚΜ]. ἐπεὶ οὖν δύο εὐθεῖαι ἄνισοί εἰσιν αἱ ΓΜ, ΜΖ, καὶ τῷ τετάρτῳ μέρει τοῦ ἀπὸ τῆς ΜΖ ἴσον παρὰ τὴν μείζονα τὴν ΓΜ παραβέβληται ἐλλεῖπον εἴδει τετραγώνῳ τὸ ὑπὸ τῶν ΓΚ, ΚΜ καὶ εἰς σύμμετρα αὐτὴν διαιρεῖ, ἡ ἄρα ΓΜ τῆς ΜΖ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ μήκει. καί ἐστιν ἡ προσαρμόζουσα ἡ ΖΜ σύμμετρος μήκει τῇ ἐκκειμένῃ ῥητῇ τῇ ΓΔ· ἡ ἄρα ΓΖ ἀποτομή ἐστι δευτέρα.

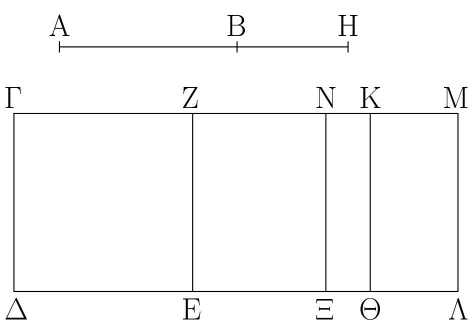
Τὸ ἄρα ἀπὸ μέσης ἀποτομῆς πρώτης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν δευτέραν· ὅπερ ἔδει δεῖξαι.

$BG$ to $KL$, $NL$ is thus also the mean proportional to $CH$ and $KL$. Thus, as $CH$ is to $NL$, so $NL$ (is) to $KL$ [Prop. 5.11]. But, as $CH$ (is) to $NL$, so $CK$ is to $NM$, and as $NL$ (is) to $KL$, so $NM$ is to $MK$ [Prop. 6.1]. Thus, as $CK$ (is) to $NM$, so $NM$ is to $KM$ [Prop. 5.11]. The (rectangle contained) by $CK$ and $KM$ is thus equal to the (square) on $NM$ [Prop. 6.17]—that is to say, to the fourth part of the (square) on $FM$ [and since the (square) on $AG$ is commensurable with the (square) on $BG$, $CH$ is also commensurable with $KL$—that is to say, $CK$ with $KM$]. Therefore, since $CM$ and $MF$ are two unequal straight-lines, and the (rectangle contained) by $CK$ and $KM$, equal to the fourth part of the (square) on $MF$, has been applied to the greater $CM$, falling short by a square figure, and divides it into commensurable (parts), the square on $CM$ is thus greater than (the square on) $MF$ by the (square) on (some straight-line) commensurable in length with ($CM$) [Prop. 10.17]. The attachment $FM$ is also commensurable in length with the (previously) laid down rational (straight-line) $CD$. $CF$ is thus a second apotome [Def. 10.16].

Thus, the (square) on a first apotome of a medial (straight-line), applied to a rational (straight-line), produces a second apotome as breadth. (Which is) the very thing it was required to show.

## ϟθʹ.

Τὸ ἀπὸ μέσης ἀποτομῆς δευτέρας παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν τρίτην.

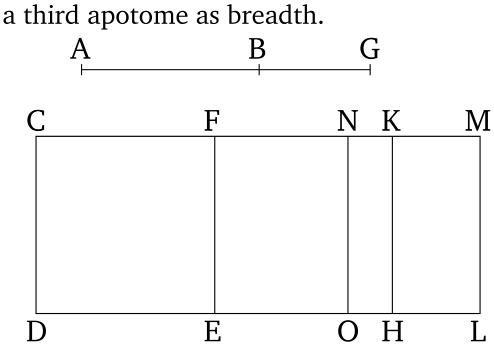



Ἔστω μέσης ἀποτομὴ δευτέρα ἡ ΑΒ, ῥητὴ δὲ ἡ ΓΔ, καὶ τῷ ἀπὸ τῆς ΑΒ ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω τὸ ΓΕ πλάτος ποιοῦν τὴν ΓΖ· λέγω, ὅτι ἡ ΓΖ ἀποτομή ἐστι τρίτη.

Ἔστω γὰρ τῇ ΑΒ προσαρμόζουσα ἡ ΒΗ· αἱ ἄρα ΑΗ, ΗΒ μέσαι εἰσὶ δυνάμει μόνον σύμμετροι μέσον περιέχουσαι. καὶ τῷ μὲν ἀπὸ τῆς ΑΗ ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω τὸ ΓΘ πλάτος ποιοῦν τὴν ΓΚ, τῷ δὲ ἀπὸ τῆς ΒΗ ἴσον παρὰ τὴν ΚΘ παραβεβλήσθω τὸ ΚΛ πλάτος ποιοῦν τὴν ΚΜ· ὅλον ἄρα τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ [καί ἐστι μέσα τὰ ἀπὸ τῶν ΑΗ, ΗΒ]· μέσον ἄρα καὶ τὸ ΓΛ. καὶ παρὰ ῥητὴν τὴν

## Proposition 99

The (square) on a second apotome of a medial (straight-line), applied to a rational (straight-line), produces a third apotome as breadth.




Let $AB$ be the second apotome of a medial (straight-line), and $CD$ a rational (straight-line). And let $CE$, equal to the (square) on $AB$, have been applied to $CD$, producing $CF$ as breadth. I say that $CF$ is a third apotome.

For let $BG$ be an attachment to $AB$. Thus, $AG$ and $GB$ are medial (straight-lines which are) commensurable in square only, containing a medial (area) [Prop. 10.75]. And let $CH$, equal to the (square) on $AG$, have been applied to $CD$, producing $CK$ as breadth. And let $KL$, ΓΔ παραβέβληται πλάτος ποιοῦν τὴν ΓΜ· ῥητὴ ἄρα ἐστὶν ἡ ΓΜ καὶ ἀσύμμετρος τῇ ΓΔ μήκει. καὶ ἐπεὶ ὅλον τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ, ὧν τὸ ΓΕ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΒ, λοιπὸν ἄρα τὸ ΛΖ ἴσον ἐστὶ τῷ δὶς ὑπὸ τῶν ΑΗ, ΗΒ. τετμήσθω οὖν ἡ ΖΜ δίχα κατὰ τὸ Ν σημεῖον, καὶ τῇ ΓΔ παράλληλος ἤχθω ἡ ΝΞ· ἑκάτερον ἄρα τῶν ΖΞ, ΝΛ ἴσον ἐστὶ τῷ ὑπὸ τῶν ΑΗ, ΗΒ. μέσον δὲ τὸ ὑπὸ τῶν ΑΗ, ΗΒ· μέσον ἄρα ἐστὶ καὶ τὸ ΖΛ. καὶ παρὰ ῥητὴν τὴν ΕΖ παράκειται πλάτος ποιοῦν τὴν ΖΜ· ῥητὴ ἄρα καὶ ἡ ΖΜ καὶ ἀσύμμετρος τῇ ΓΔ μήκει. καὶ ἐπεὶ αἱ ΑΗ, ΗΒ δυνάμει μόνον εἰσὶ σύμμετροι, ἀσύμμετρος ἄρα [ἐστὶ] μήκει ἡ ΑΗ τῇ ΗΒ· ἀσύμμετρον ἄρα ἐστὶ καὶ τὸ ἀπὸ τῆς ΑΗ τῷ ὑπὸ τῶν ΑΗ, ΗΒ. ἀλλὰ τῷ μὲν ἀπὸ τῆς ΑΗ σύμμετρά ἐστι τὰ ἀπὸ τῶν ΑΗ, ΗΒ, τῷ δὲ ὑπὸ τῶν ΑΗ, ΗΒ τὸ δὶς ὑπὸ τῶν ΑΗ, ΗΒ· ἀσύμμετρα ἄρα ἐστὶ τὰ ἀπὸ τῶν ΑΗ, ΗΒ τῷ δὶς ὑπὸ τῶν ΑΗ, ΗΒ. ἀλλὰ τοῖς μὲν ἀπὸ τῶν ΑΗ, ΗΒ ἴσον ἐστὶ τὸ ΓΛ, τῷ δὲ δὶς ὑπὸ τῶν ΑΗ, ΗΒ ἴσον ἐστὶ τὸ ΖΛ· ἀσύμμετρον ἄρα ἐστὶ τὸ ΓΛ τῷ ΖΛ. ὡς δὲ τὸ ΓΛ πρὸς τὸ ΖΛ, οὕτως ἐστὶν ἡ ΓΜ πρὸς τὴν ΖΜ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΓΜ τῇ ΖΜ μήκει. καί εἰσιν ἀμφότεραι ῥηταί· αἱ ἄρα ΓΜ, ΜΖ ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΓΖ. λέγω δή, ὅτι καὶ τρίτη.

Ἐπεὶ γὰρ σύμμετρόν ἐστι τὸ ἀπὸ τῆς ΑΗ τῷ ἀπὸ τῆς ΗΒ, σύμμετρον ἄρα καὶ τὸ ΓΘ τῷ ΚΛ· ὥστε καὶ ἡ ΓΚ τῇ ΚΜ. καὶ ἐπεὶ τῶν ἀπὸ τῶν ΑΗ, ΗΒ μέσον ἀνάλογόν ἐστι τὸ ὑπὸ τῶν ΑΗ, ΗΒ, καί ἐστι τῷ μὲν ἀπὸ τῆς ΑΗ ἴσον τὸ ΓΘ, τῷ δὲ ἀπὸ τῆς ΗΒ ἴσον τὸ ΚΛ, τῷ δὲ ὑπὸ τῶν ΑΗ, ΗΒ ἴσον τὸ ΝΛ, καὶ τῶν ΓΘ, ΚΛ ἄρα μέσον ἀνάλογόν ἐστι τὸ ΝΛ· ἔστιν ἄρα ὡς τὸ ΓΘ πρὸς τὸ ΝΛ, οὕτως τὸ ΝΛ πρὸς τὸ ΚΛ. ἀλλ᾿ ὡς μὲν τὸ ΓΘ πρὸς τὸ ΝΛ, οὕτως ἐστὶν ἡ ΓΚ πρὸς τὴν ΝΜ, ὡς δὲ τὸ ΝΛ πρὸς τὸ ΚΛ, οὕτως ἐστὶν ἡ ΝΜ πρὸς τὴν ΚΜ· ὡς ἄρα ἡ ΓΚ πρὸς τὴν ΜΝ, οὕτως ἐστὶν ἡ ΜΝ πρὸς τὴν ΚΜ· τὸ ἄρα ὑπὸ τῶν ΓΚ, ΚΜ ἴσον ἐστὶ τῷ [ἀπὸ τῆς ΜΝ, τουτέστι τῷ] τετάρτῳ μέρει τοῦ ἀπὸ τῆς ΖΜ. ἐπεὶ οὖν δύο εὐθεῖαι ἄνισοί εἰσιν αἱ ΓΜ, ΜΖ, καὶ τῷ τετάρτῳ μέρει τοῦ ἀπὸ τῆς ΖΜ ἴσον παρὰ τὴν ΓΜ παραβέβληται ἐλλεῖπον εἴδει τετραγώνῳ καὶ εἰς σύμμετρα αὐτὴν διαιρεῖ, ἡ ΓΜ ἄρα τῆς ΜΖ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ. καὶ οὐδετέρα τῶν ΓΜ, ΜΖ σύμμετρός ἐστι μήκει τῇ ἐκκειμένῃ ῥητῇ τῇ ΓΔ· ἡ ἄρα ΓΖ ἀποτομή ἐστι τρίτη.

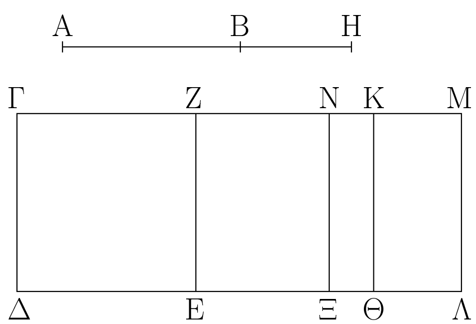
Τὸ ἄρα ἀπὸ μέσης ἀποτομῆς δευτέρας παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν τρίτην· ὅπερ ἔδει δεῖξαι.

equal to the (square) on $BG$, have been applied to $KH$, producing $KM$ as breadth. Thus, the whole of $CL$ is equal to the (sum of the squares) on $AG$ and $GB$ [and the (sum of the squares) on $AG$ and $GB$ is medial]. $CL$ (is) thus also medial [Props. 10.15, 10.23 corr.]. And it has been applied to the rational (straight-line) $CD$, producing $CM$ as breadth. Thus, $CM$ is rational, and incommensurable in length with $CD$ [Prop. 10.22]. And since the whole of $CL$ is equal to the (sum of the squares) on $AG$ and $GB$, of which $CE$ is equal to the (square) on $AB$, the remainder $LF$ is thus equal to twice the (rectangle contained) by $AG$ and $GB$ [Prop. 2.7]. Therefore, let $FM$ have been cut in half at point $N$. And let $NO$ have been drawn parallel to $CD$. Thus, $FO$ and $NL$ are each equal to the (rectangle contained) by $AG$ and $GB$. And the (rectangle contained) by $AG$ and $GB$ (is) medial. Thus, $FL$ is also medial. And it is applied to the rational (straight-line) $EF$, producing $FM$ as breadth. $FM$ is thus rational, and incommensurable in length with $CD$ [Prop. 10.22]. And since $AG$ and $GB$ are commensurable in square only, $AG$ [is] thus incommensurable in length with $GB$. Thus, the (square) on $AG$ is also incommensurable with the (rectangle contained) by $AG$ and $GB$ [Props. 6.1, 10.11]. But, the (sum of the squares) on $AG$ and $GB$ is commensurable with the (square) on $AG$, and twice the (rectangle contained) by $AG$ and $GB$ with the (rectangle contained) by $AG$ and $GB$. The (sum of the squares) on $AG$ and $GB$ is thus incommensurable with twice the (rectangle contained) by $AG$ and $GB$ [Prop. 10.13]. But, $CL$ is equal to the (sum of the squares) on $AG$ and $GB$, and $FL$ is equal to twice the (rectangle contained) by $AG$ and $GB$. Thus, $CL$ is incommensurable with $FL$. And as $CL$ (is) to $FL$, so $CM$ is to $FM$ [Prop. 6.1]. $CM$ is thus incommensurable in length with $FM$ [Prop. 10.11]. And they are both rational (straight-lines). Thus, $CM$ and $MF$ are rational (straight-lines which are) commensurable in square only. $CF$ is thus an apotome [Prop. 10.73]. So, I say that (it is) also a third (apotome).

For since the (square) on $AG$ is commensurable with the (square) on $GB$, $CH$ (is) thus also commensurable with $KL$. Hence, $CK$ (is) also (commensurable in length) with $KM$ [Props. 6.1, 10.11]. And since the (rectangle contained) by $AG$ and $GB$ is the mean proportional to the (squares) on $AG$ and $GB$ [Prop. 10.21 lem.], and $CH$ is equal to the (square) on $AG$, and $KL$ equal to the (square) on $GB$, and $NL$ equal to the (rectangle contained) by $AG$ and $GB$, $NL$ is thus also the mean proportional to $CH$ and $KL$. Thus, as $CH$ is to $NL$, so $NL$ (is) to $KL$. But, as $CH$ (is) to $NL$, so $CK$ is to $NM$, and as $NL$ (is) to $KL$, so $NM$ (is) to $KM$ [Prop. 6.1].

## ρʹ.

Τὸ ἀπὸ ἐλάσσονος παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν τετάρτην.




Ἔστω ἐλάσσων ἡ ΑΒ, ῥητὴ δὲ ἡ ΓΔ, καὶ τῷ ἀπὸ τῆς ΑΒ ἴσον παρὰ ῥητὴν τὴν ΓΔ παραβεβλήσθω τὸ ΓΕ πλάτος ποιοῦν τὴν ΓΖ· λέγω, ὅτι ἡ ΓΖ ἀποτομή ἐστι τετάρτη.

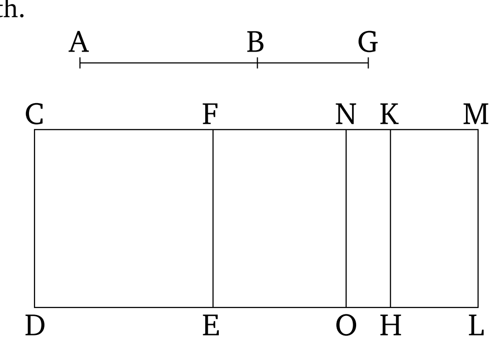
Ἔστω γὰρ τῇ ΑΒ προσαρμόζουσα ἡ ΒΗ· αἱ ἄρα ΑΗ, ΗΒ δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὸ μὲν συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΗ, ΗΒ τετραγώνων ῥητόν, τὸ δὲ δὶς ὑπὸ τῶν ΑΗ, ΗΒ μέσον. καὶ τῷ μὲν ἀπὸ τῆς ΑΗ ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω τὸ ΓΘ πλάτος ποιοῦν τὴν ΓΚ, τῷ δὲ ἀπὸ τῆς ΒΗ ἴσον τὸ ΚΛ πλάτος ποιοῦν τὴν ΚΜ· ὅλον ἄρα τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ. καί ἐστι τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΗ, ΗΒ ῥητόν· ῥητὸν ἄρα ἐστὶ καὶ τὸ ΓΛ. καὶ παρὰ ῥητὴν τὴν ΓΔ παράκειται πλάτος ποιοῦν τὴν ΓΜ· ῥητὴ ἄρα καὶ ἡ ΓΜ καὶ σύμμετρος τῇ ΓΔ μήκει. καὶ ἐπεὶ ὅλον τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ, ὧν τὸ ΓΕ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΒ, λοιπὸν ἄρα τὸ ΖΛ ἴσον ἐστὶ τῷ δὶς ὑπὸ τῶν ΑΗ, ΗΒ. τετμήσθω οὖν ἡ ΖΜ δίχα κατὰ τὸ Ν σημεῖον, καὶ ἤχθω δια τοῦ Ν ὁποτέρᾳ τῶν ΓΔ,

Thus, as $CK$ (is) to $MN$, so $MN$ is to $KM$ [Prop. 5.11]. Thus, the (rectangle contained) by $CK$ and $KM$ is equal to the [(square) on $MN$—that is to say, to the] fourth part of the (square) on $FM$ [Prop. 6.17]. Therefore, since $CM$ and $MF$ are two unequal straight-lines, and (some area), equal to the fourth part of the (square) on $FM$, has been applied to $CM$, falling short by a square figure, and divides it into commensurable (parts), the square on $CM$ is thus greater than (the square on) $MF$ by the (square) on (some straight-line) commensurable (in length) with ($CM$) [Prop. 10.17]. And neither of $CM$ and $MF$ is commensurable in length with the (previously) laid down rational (straight-line) $CD$. $CF$ is thus a third apotome [Def. 10.13].

Thus, the (square) on a second apotome of a medial (straight-line), applied to a rational (straight-line), produces a third apotome as breadth. (Which is) the very thing it was required to show.

## Proposition 100

The (square) on a minor (straight-line), applied to a rational (straight-line), produces a fourth apotome as breadth.




Let $AB$ be a minor (straight-line), and $CD$ a rational (straight-line). And let $CE$, equal to the (square) on $AB$, have been applied to the rational (straight-line) $CD$, producing $CF$ as breadth. I say that $CF$ is a fourth apotome.

For let $BG$ be an attachment to $AB$. Thus, $AG$ and $GB$ are incommensurable in square, making the sum of the squares on $AG$ and $GB$ rational, and twice the (rectangle contained) by $AG$ and $GB$ medial [Prop. 10.76]. And let $CH$, equal to the (square) on $AG$, have been applied to $CD$, producing $CK$ as breadth, and $KL$, equal to the (square) on $BG$, producing $KM$ as breadth. Thus, the whole of $CL$ is equal to the (sum of the squares) on $AG$ and $GB$. And the sum of the (squares) on $AG$ and $GB$ is rational. $CL$ is thus also rational. And it is applied to the rational (straight-line) $CD$, producing $CM$ as breadth. Thus, $CM$ (is) also rational, and commensurable in length with $CD$ [Prop. 10.20]. And since the

ΜΛ παράλληλος ἡ ΝΞ· ἑκάτερον ἄρα τῶν ΖΞ, ΝΛ ἴσον ἐστὶ τῷ ὑπὸ τῶν ΑΗ, ΗΒ. καὶ ἐπεὶ τὸ δὶς ὑπὸ τῶν ΑΗ, ΗΒ μέσον ἐστὶ καί ἐστιν ἴσον τῷ ΖΛ, καὶ τὸ ΖΛ ἄρα μέσον ἐστίν. καὶ παρὰ ῥητὴν τὴν ΖΕ παράκειται πλάτος ποιοῦν τὴν ΖΜ· ῥητὴ ἄρα ἐστὶν ἡ ΖΜ καὶ ἀσύμμετρος τῇ ΓΔ μήκει. καὶ ἐπεὶ τὸ μὲν συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΗ, ΗΒ ῥητόν ἐστιν, τὸ δὲ δὶς ὑπὸ τῶν ΑΗ, ΗΒ μέσον, ἀσύμμετρα [ἄρα] ἐστὶ τὰ ἀπὸ τῶν ΑΗ, ΗΒ τῷ δὶς ὑπὸ τῶν ΑΗ, ΗΒ. ἴσον δέ [ἐστι] τὸ ΓΛ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ, τῷ δὲ δὶς ὑπὸ τῶν ΑΗ, ΗΒ ἴσον τὸ ΖΛ· ἀσύμμετρον ἄρα [ἐστὶ] τὸ ΓΛ τῷ ΖΛ. ὡς δὲ τὸ ΓΛ πρὸς τὸ ΖΛ, οὕτως ἐστὶν ἡ ΓΜ πρὸς τὴν ΜΖ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΓΜ τῇ ΜΖ μήκει. καί εἰσιν ἀμφότεραι ῥηταί· αἱ ἄρα ΓΜ, ΜΖ ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΓΖ. λέγω [δή], ὅτι καὶ τετάρτη.

Ἐπεὶ γὰρ αἱ ΑΗ, ΗΒ δυνάμει εἰσὶν ἀσύμμετροι, ἀσύμμετρον ἄρα καὶ τὸ ἀπὸ τῆς ΑΗ τῷ ἀπὸ τῆς ΗΒ. καί ἐστι τῷ μὲν ἀπὸ τῆς ΑΗ ἴσον τὸ ΓΘ, τῷ δὲ ἀπὸ τῆς ΗΒ ἴσον τὸ ΚΛ· ἀσύμμετρον ἄρα ἐστὶ τὸ ΓΘ τῷ ΚΛ. ὡς δὲ τὸ ΓΘ πρὸς τὸ ΚΛ, οὕτως ἐστὶν ἡ ΓΚ πρὸς τὴν ΚΜ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΓΚ τῇ ΚΜ μήκει. καὶ ἐπεὶ τῶν ἀπὸ τῶν ΑΗ, ΗΒ μέσον ἀνάλογόν ἐστι τὸ ὑπὸ τῶν ΑΗ, ΗΒ, καί ἐστιν ἴσον τὸ μὲν ἀπὸ τῆς ΑΗ τῷ ΓΘ, τὸ δὲ ἀπὸ τῆς ΗΒ τῷ ΚΛ, τὸ δὲ ὑπὸ τῶν ΑΗ, ΗΒ τῷ ΝΛ, τῶν ἄρα ΓΘ, ΚΛ μέσον ἀνάλογόν ἐστι τὸ ΝΛ· ἔστιν ἄρα ὡς τὸ ΓΘ πρὸς τὸ ΝΛ, οὕτως τὸ ΝΛ πρὸς τὸ ΚΛ. ἀλλ᾽ ὡς μὲν τὸ ΓΘ πρὸς τὸ ΝΛ, οὕτως ἐστίν ἡ ΓΚ πρὸς τὴν ΝΜ, ὡς δὲ τὸ ΝΛ πρὸς τὸ ΚΛ, οὕτως ἐστὶν ἡ ΝΜ πρὸς τὴν ΚΜ· ὡς ἄρα ἡ ΓΚ πρὸς τὴν ΜΝ, οὕτως ἐστὶν ἡ ΜΝ πρὸς τὴν ΚΜ· τὸ ἄρα ὑπὸ τῶν ΓΚ, ΚΜ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΜΝ, τουτέστι τῷ τετάρτῳ μέρει τοῦ ἀπὸ τῆς ΖΜ. ἐπεὶ οὖν δύο εὐθεῖαι ἄνισοί εἰσιν αἱ ΓΜ, ΜΖ, καὶ τῷ τετράρτῳ μέρει τοῦ ἀπὸ τῆς ΜΖ ἴσον παρὰ τὴν ΓΜ παραβέβληται ἐλλεῖπον εἴδει τετραγώνῳ τὸ ὑπὸ τῶν ΓΚ, ΚΜ καὶ εἰς ἀσύμμετρα αὐτὴν διαιρεῖ, ἡ ἄρα ΓΜ τῆς ΜΖ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. καί ἐστιν ὅλη ἡ ΓΜ σύμμετρος μήκει τῇ ἐκκειμένῃ ῥητῇ τῇ ΓΔ· ἡ ἄρα ΓΖ ἀποτομή ἐστι τετάρτη.

Τὸ ἄρα ἀπὸ ἐλάσσονος καὶ τὰ ἑξῆς.

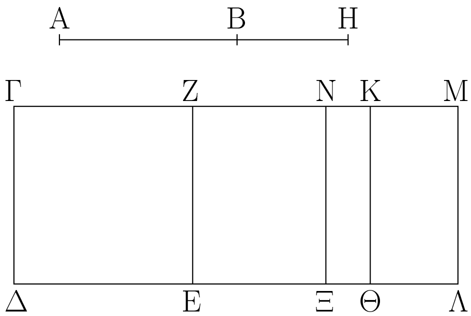
whole of $CL$ is equal to the (sum of the squares) on $AG$ and $GB$, of which $CE$ is equal to the (square) on $AB$, the remainder $FL$ is thus equal to twice the (rectangle contained) by $AG$ and $GB$ [Prop. 2.7]. Therefore, let $FM$ have been cut in half at point $N$. And let $NO$ have been drawn through $N$, parallel to either of $CD$ or $ML$. Thus, $FO$ and $NL$ are each equal to the (rectangle contained) by $AG$ and $GB$. And since twice the (rectangle contained) by $AG$ and $GB$ is medial, and is equal to $FL$, $FL$ is thus also medial. And it is applied to the rational (straight-line) $FE$, producing $FM$ as breadth. Thus, $FM$ is rational, and incommensurable in length with $CD$ [Prop. 10.22]. And since the sum of the (squares) on $AG$ and $GB$ is rational, and twice the (rectangle contained) by $AG$ and $GB$ medial, the (sum of the squares) on $AG$ and $GB$ is [thus] incommensurable with twice the (rectangle contained) by $AG$ and $GB$. And $CL$ (is) equal to the (sum of the squares) on $AG$ and $GB$, and $FL$ equal to twice the (rectangle contained) by $AG$ and $GB$. $CL$ [is] thus incommensurable with $FL$. And as $CL$ (is) to $FL$, so $CM$ is to $MF$ [Prop. 6.1]. $CM$ is thus incommensurable in length with $MF$ [Prop. 10.11]. And both are rational (straight-lines). Thus, $CM$ and $MF$ are rational (straight-lines which are) commensurable in square only. $CF$ is thus an apotome [Prop. 10.73]. [So], I say that (it is) also a fourth (apotome).

For since $AG$ and $GB$ are incommensurable in square, the (square) on $AG$ (is) thus also incommensurable with the (square) on $GB$. And $CH$ is equal to the (square) on $AG$, and $KL$ equal to the (square) on $GB$. Thus, $CH$ is incommensurable with $KL$. And as $CH$ (is) to $KL$, so $CK$ is to $KM$ [Prop. 6.1]. $CK$ is thus incommensurable in length with $KM$ [Prop. 10.11]. And since the (rectangle contained) by $AG$ and $GB$ is the mean proportional to the (squares) on $AG$ and $GB$ [Prop. 10.21 lem.], and the (square) on $AG$ is equal to $CH$, and the (square) on $GB$ to $KL$, and the (rectangle contained) by $AG$ and $GB$ to $NL$, $NL$ is thus the mean proportional to $CH$ and $KL$. Thus, as $CH$ is to $NL$, so $NL$ (is) to $KL$. But, as $CH$ (is) to $NL$, so $CK$ is to $NM$, and as $NL$ (is) to $KL$, so $NM$ is to $KM$ [Prop. 6.1]. Thus, as $CK$ (is) to $MN$, so $MN$ is to $KM$ [Prop. 5.11]. The (rectangle contained) by $CK$ and $KM$ is thus equal to the (square) on $MN$—that is to say, to the fourth part of the (square) on $FM$ [Prop. 6.17]. Therefore, since $CM$ and $MF$ are two unequal straight-lines, and the (rectangle contained) by $CK$ and $KM$, equal to the fourth part of the (square) on $MF$, has been applied to $CM$, falling short by a square figure, and divides it into incommensurable (parts), the square on $CM$ is thus greater than (the square on) $MF$ by the (square) on (some straight-line) incommensurable (in length) with ($CM$) [Prop. 10.18]. And the whole of $CM$ is commensurable in length with the (previously) laid down rational (straight-line) $CD$. Thus, $CF$ is a fourth apotome [Def. 10.14].

Thus, the (square) on a minor, and so on ...

## ρα´.

Τὸ ἀπὸ τῆς μετὰ ῥητοῦ μέσον τὸ ὅλον ποιούσης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν πέμπτην.



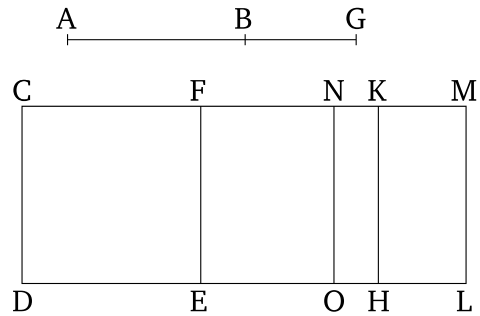

Ἔστω ἡ μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσα ἡ ΑΒ, ῥητὴ δὲ ἡ ΓΔ, καὶ τῷ ἀπὸ τῆς ΑΒ ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω τὸ ΓΕ πλάτος ποιοῦν τὴν ΓΖ· λέγω, ὅτι ἡ ΓΖ ἀποτομή ἐστι πέμπτη.

Ἔστω γὰρ τῇ ΑΒ προσαρμόζουσα ἡ ΒΗ· αἱ ἄρα ΑΗ, ΗΒ εὐθεῖαι δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὸ μὲν συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων μέσον, τὸ δὲ δὶς ὑπ᾽ αὐτῶν ῥητόν, καὶ τῷ μὲν ἀπὸ τῆς ΑΗ ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω τὸ ΓΘ, τῷ δὲ ἀπὸ τῆς ΗΒ ἴσον τὸ ΚΛ· ὅλον ἄρα τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ. τὸ δὲ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΗ, ΗΒ ἅμα μέσον ἐστίν· μέσον ἄρα ἐστὶ τὸ ΓΛ. καὶ παρὰ ῥητὴν τὴν ΓΔ παράκειται πλάτος ποιοῦν τὴν ΓΜ· ῥητὴ ἄρα ἐστὶν ἡ ΓΜ καὶ ἀσύμμετρος τῇ ΓΔ. καὶ ἐπεὶ ὅλον τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ, ὧν τὸ ΓΕ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΑΒ, λοιπὸν ἄρα τὸ ΖΛ ἴσον ἐστὶ τῷ δὶς ὑπὸ τῶν ΑΗ, ΗΒ. τετμήσθω οὖν ἡ ΖΜ δίχα κατὰ τὸ Ν, καὶ ἤχθω διὰ τοῦ Ν ὁποτέρᾳ τῶν ΓΔ, ΜΛ παράλληλος ἡ ΝΞ· ἑκάτερον ἄρα τῶν ΖΞ, ΝΛ ἴσον ἐστὶ τῷ ὑπὸ τῶν ΑΗ, ΗΒ, καὶ ἐπεὶ τὸ δὶς ὑπὸ τῶν ΑΗ, ΗΒ ῥητόν ἐστι καί [ἐστιν] ἴσον τῷ ΖΛ, ῥητὸν ἄρα ἐστὶ τὸ ΖΛ. καὶ παρὰ ῥητὴν τὴν ΕΖ παράκειται πλάτος ποιοῦν τὴν ΖΜ· ῥητὴ ἄρα ἐστὶν ἡ ΖΜ καὶ σύμμετρος τῇ ΓΔ μήκει. καὶ ἐπεὶ τὸ μὲν ΓΛ μέσον ἐστίν, τὸ δὲ ΖΛ ῥητόν, ἀσύμμετρον ἄρα ἐστὶ τὸ ΓΛ τῷ ΖΛ. ὡς δὲ τὸ ΓΛ πρὸς τὸ ΖΛ, οὕτως ἡ ΓΜ πρὸς τὴν ΜΖ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΓΜ τῇ ΜΖ μήκει. καί εἰσιν ἀμφότεραι ῥηταί· αἱ ἄρα ΓΜ, ΜΖ ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΓΖ. λὲγω δή, ὅτι καὶ πέμπτη.

Ὁμοίως γὰρ δείξομεν, ὅτι τὸ ὑπὸ τῶν ΓΚΜ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΝΜ, τουτέστι τῷ τετάρτῳ μέρει τοῦ ἀπὸ τῆς

## Proposition 101

The (square) on that (straight-line) which with a rational (area) makes a medial whole, applied to a rational (straight-line), produces a fifth apotome as breadth.




Let $AB$ be that (straight-line) which with a rational (area) makes a medial whole, and $CD$ a rational (straight-line). And let $CE$, equal to the (square) on $AB$, have been applied to $CD$, producing $CF$ as breadth. I say that $CF$ is a fifth apotome.

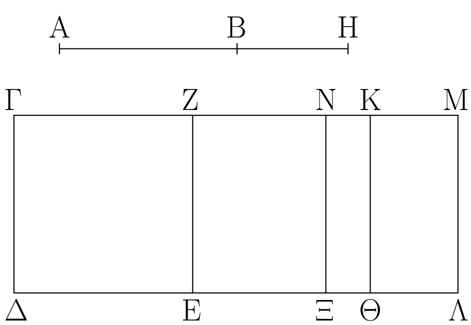
Let $BG$ be an attachment to $AB$. Thus, the straight-lines $AG$ and $GB$ are incommensurable in square, making the sum of the squares on them medial, and twice the (rectangle contained) by them rational [Prop. 10.77]. And let $CH$, equal to the (square) on $AG$, have been applied to $CD$, and $KL$, equal to the (square) on $GB$. The whole of $CL$ is thus equal to the (sum of the squares) on $AG$ and $GB$. And the sum of the (squares) on $AG$ and $GB$ together is medial. Thus, $CL$ is medial. And it has been applied to the rational (straight-line) $CD$, producing $CM$ as breadth. $CM$ is thus rational, and incommensurable (in length) with $CD$ [Prop. 10.22]. And since the whole of $CL$ is equal to the (sum of the squares) on $AG$ and $GB$, of which $CE$ is equal to the (square) on $AB$, the remainder $FL$ is thus equal to twice the (rectangle contained) by $AG$ and $GB$ [Prop. 2.7]. Therefore, let $FM$ have been cut in half at $N$. And let $NO$ have been drawn through $N$, parallel to either of $CD$ or $ML$. Thus, $FO$ and $NL$ are each equal to the (rectangle contained) by $AG$ and $GB$. And since twice the (rectangle contained) by $AG$ and $GB$ is rational, and [is] equal to $FL$, $FL$ is thus rational. And it is applied to the rational (straight-line) $EF$, producing $FM$ as breadth. Thus, $FM$ is rational, and commensurable in length with $CD$ [Prop. 10.20]. And since $CL$ is medial, and $FL$ rational, ΖΜ. καὶ ἐπεὶ ἀσύμμετρόν ἐστι τὸ ἀπὸ τῆς ΑΗ τῷ ἀπὸ τῆς ΗΒ, ἴσον δὲ τὸ μὲν ἀπὸ τῆς ΑΗ τῷ ΓΘ, τὸ δὲ ἀπὸ τῆς ΗΒ τῷ ΚΛ, ἀσύμμετρον ἄρα τὸ ΓΘ τῷ ΚΛ. ὡς δὲ τὸ ΓΘ πρὸς τὸ ΚΛ, οὕτως ἡ ΓΚ πρὸς τὴν ΚΜ· ἀσύμμετρος ἄρα ἡ ΓΚ τῇ ΚΜ μήκει. ἐπεὶ οὖν δύο εὐθεῖαι ἄνισοί εἰσιν αἱ ΓΜ, ΜΖ, καὶ τῷ τετάρτῳ μέρει τοῦ ἀπὸ τῆς ΖΜ ἴσον παρὰ τὴν ΓΜ παραβέβληται ἐλλεῖπον εἴδει τετραγώνῳ καὶ εἰς ἀσύμμετρα αὐτὴν διαιρεῖ, ἡ ἄρα ΓΜ τῆς ΜΖ μεῖζον δύναται τῷ ἀπὸ ἀσύμμέτρου ἑαυτῇ. καί ἐστιν ἡ προσαρμόζουσα ἡ ΖΜ σύμμετρος τῇ ἐκκειμένῃ ῥητῇ τῇ ΓΔ· ἡ ἄρα ΓΖ ἀποτομή ἐστι πέμπτη· ὅπερ ἔδει δεῖξαι.

$CL$ is thus incommensurable with $FL$. And as $CL$ (is) to $FL$, so $CM$ (is) to $MF$ [Prop. 6.1]. $CM$ is thus incommensurable in length with $MF$ [Prop. 10.11]. And both are rational. Thus, $CM$ and $MF$ are rational (straight-lines which are) commensurable in square only. $CF$ is thus an apotome [Prop. 10.73]. So, I say that (it is) also a fifth (apotome).

For, similarly (to the previous propositions), we can show that the (rectangle contained) by $CKM$ is equal to the (square) on $NM$—that is to say, to the fourth part of the (square) on $FM$. And since the (square) on $AG$ is incommensurable with the (square) on $GB$, and the (square) on $AG$ (is) equal to $CH$, and the (square) on $GB$ to $KL$, $CH$ (is) thus incommensurable with $KL$. And as $CH$ (is) to $KL$, so $CK$ (is) to $KM$ [Prop. 6.1]. Thus, $CK$ (is) incommensurable in length with $KM$ [Prop. 10.11]. Therefore, since $CM$ and $MF$ are two unequal straight-lines, and (some area), equal to the fourth part of the (square) on $FM$, has been applied to $CM$, falling short by a square figure, and divides it into incommensurable (parts), the square on $CM$ is thus greater than (the square on) $MF$ by the (square) on (some straight-line) incommensurable (in length) with ($CM$) [Prop. 10.18]. And the attachment $FM$ is commensurable with the (previously) laid down rational (straight-line) $CD$. Thus, $CF$ is a fifth apotome [Def. 10.15]. (Which is) the very thing it was required to show.

## ρβ΄.

Τὸ ἀπὸ τῆς μετὰ μέσου μέσον τὸ ὅλον ποιούσης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν ἕκτην.

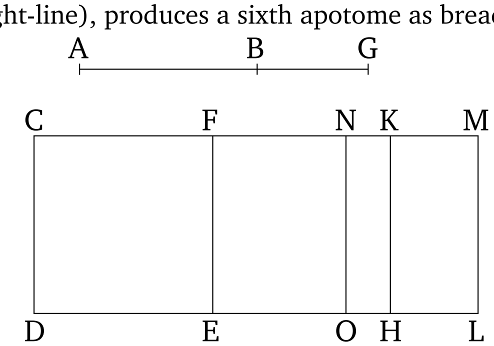



Ἔστω ἡ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσα ἡ ΑΒ, ῥητὴ δὲ ἡ ΓΔ, καὶ τῷ ἀπὸ τῆς ΑΒ ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω τὸ ΓΕ πλάτος ποιοῦν τὴν ΓΖ· λέγω, ὅτι ἡ ΓΖ ἀποτομή ἐστιν ἕκτη.

Ἔστω γὰρ τῇ ΑΒ προσαρμόζουσα ἡ ΒΗ· αἱ ἄρα ΑΗ, ΗΒ δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τό τε συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων μέσον καὶ τὸ δὶς ὑπὸ τῶν ΑΗ, ΗΒ μέσον καὶ ἀσύμμετρον τὰ ἀπὸ τῶν ΑΗ, ΗΒ τῷ

## Proposition 102

The (square) on that (straight-line) which with a medial (area) makes a medial whole, applied to a rational (straight-line), produces a sixth apotome as breadth.




Let $AB$ be that (straight-line) which with a medial (area) makes a medial whole, and $CD$ a rational (straight-line). And let $CE$, equal to the (square) on $AB$, have been applied to $CD$, producing $CF$ as breadth. I say that $CF$ is a sixth apotome.

For let $BG$ be an attachment to $AB$. Thus, $AG$ and $GB$ are incommensurable in square, making the sum of the squares on them medial, and twice the (rectangle δὶς ὑπὸ τῶν ΑΗ, ΗΒ. παραβεβλήσθω οὖν παρὰ τὴν ΓΔ τῷ μὲν ἀπὸ τῆς ΑΗ ἴσον τὸ ΓΘ πλάτος ποιοῦν τὴν ΓΚ, τῷ δὲ ἀπὸ τῆς ΒΗ τὸ ΚΛ· ὅλον ἄρα τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ· μέσον ἄρα [ἐστὶ] καὶ τὸ ΓΛ. καὶ παρὰ ῥητὴν τὴν ΓΔ παράκειται πλάτος ποιοῦν τὴν ΓΜ· ῥητὴ ἄρα ἐστὶν ἡ ΓΜ καὶ ἀσύμμετρος τῇ ΓΔ μήκει. ἐπεὶ οὖν τὸ ΓΛ ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΑΗ, ΗΒ, ὧν τὸ ΓΕ ἴσον τῷ ἀπὸ τῆς ΑΒ, λοιπὸν ἄρα τὸ ΖΛ ἴσον ἐστὶ τῷ δὶς ὑπὸ τῶν ΑΗ, ΗΒ. καί ἐστι τὸ δὶς ὑπὸ τῶν ΑΗ, ΗΒ μέσον· καὶ τὸ ΖΛ ἄρα μέσον ἐστίν. καὶ παρὰ ῥητὴν τὴν ΖΕ παράκειται πλάτος ποιοῦν τὴν ΖΜ· ῥητὴ ἄρα ἐστὶν ἡ ΖΜ καὶ ἀσύμμετρος τῇ ΓΔ μήκει. καὶ ἐπεὶ τὰ ἀπὸ τῶν ΑΗ, ΗΒ ἀσύμμετρά ἐστι τῷ δὶς ὑπὸ τῶν ΑΗ, ΗΒ, καί ἐστι τοῖς μὲν ἀπὸ τῶν ΑΗ, ΗΒ ἴσον τὸ ΓΛ, τῷ δὲ δὶς ὑπὸ τῶν ΑΗ, ΗΒ ἴσον τὸ ΖΛ, ἀσύμμετρος ἄρα [ἐστὶ] τὸ ΓΛ τῷ ΖΛ. ὡς δὲ τὸ ΓΛ πρὸς τὸ ΖΛ, οὕτως ἐστὶν ἡ ΓΜ πρὸς τὴν ΜΖ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΓΜ τῇ ΜΖ μήκει. καί εἰσιν ἀμφότεραι ῥηταί. αἱ ΓΜ, ΜΖ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΓΖ. λέγω δή, ὅτι καὶ ἕκτη.

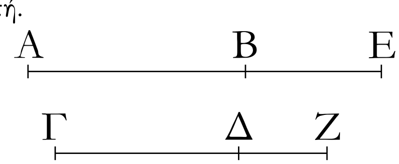
Ἐπεὶ γὰρ τὸ ΖΛ ἴσον ἐστὶ τῷ δὶς ὑπὸ τῶν ΑΗ, ΗΒ, τετμήσθω δίχα ἡ ΖΜ κατὰ τὸ Ν, καὶ ἤχθω διὰ τοῦ Ν τῇ ΓΔ παράλληλος ἡ ΝΞ· ἑκάτερον ἄρα τῶν ΖΞ, ΝΛ ἴσον ἐστὶ τῷ ὑπὸ τῶν ΑΗ, ΗΒ. καὶ ἐπεὶ αἱ ΑΗ, ΗΒ δυνάμει εἰσὶν ἀσύμμετροι, ἀσύμμετρον ἄρα ἐστὶ τὸ ἀπὸ τῆς ΑΗ τῷ ἀπὸ τῆς ΗΒ. ἀλλὰ τῷ μὲν ἀπὸ τῆς ΑΗ ἴσον ἐστὶ τὸ ΓΘ, τῷ δὲ ἀπὸ τῆς ΗΒ ἴσον ἐστὶ τὸ ΚΛ· ἀσύμμετρον ἄρα ἐστὶ τὸ ΓΘ τῷ ΚΛ. ὡς δὲ τὸ ΓΘ πρὸς τὸ ΚΛ, οὕτως ἐστὶν ἡ ΓΚ πρὸς τὴν ΚΜ· ἀσύμμετρος ἄρα ἐστὶν ἡ ΓΚ τῇ ΚΜ. καὶ ἐπεὶ τῶν ἀπὸ τῶν ΑΗ, ΗΒ μέσον ἀνάλογόν ἐστι τὸ ὑπὸ τῶν ΑΗ, ΗΒ, καὶ ἐστι τῷ μὲν ἀπὸ τῆς ΑΗ ἴσον τὸ ΓΘ, τῷ δὲ ἀπὸ τῆς ΗΒ ἴσον τὸ ΚΛ, τῷ δὲ ὑπὸ τῶν ΑΗ, ΗΒ ἴσον τὸ ΝΛ, καὶ τῶν ἄρα ΓΘ, ΚΛ μέσον ἀνάλογόν ἐστι τὸ ΝΛ· ἔστιν ἄρα ὡς τὸ ΓΘ πρὸς τὸ ΝΛ, οὕτως τὸ ΝΛ πρὸς τὸ ΚΛ. καὶ διὰ τὰ αὐτὰ ἡ ΓΜ τῆς ΜΖ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. καὶ οὐδετέρα αὐτῶν σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ τῇ ΓΔ· ἡ ΓΖ ἄρα ἀποτομή ἐστιν ἕκτη· ὅπερ ἔδει δεῖξαι.

contained) by $AG$ and $GB$ medial, and the (sum of the squares) on $AG$ and $GB$ incommensurable with twice the (rectangle contained) by $AG$ and $GB$ [Prop. 10.78]. Therefore, let $CH$, equal to the (square) on $AG$, have been applied to $CD$, producing $CK$ as breadth, and $KL$, equal to the (square) on $BG$. Thus, the whole of $CL$ is equal to the (sum of the squares) on $AG$ and $GB$. $CL$ [is] thus also medial. And it is applied to the rational (straight-line) $CD$, producing $CM$ as breadth. Thus, $CM$ is rational, and incommensurable in length with $CD$ [Prop. 10.22]. Therefore, since $CL$ is equal to the (sum of the squares) on $AG$ and $GB$, of which $CE$ (is) equal to the (square) on $AB$, the remainder $FL$ is thus equal to twice the (rectangle contained) by $AG$ and $GB$ [Prop. 2.7]. And twice the (rectangle contained) by $AG$ and $GB$ (is) medial. Thus, $FL$ is also medial. And it is applied to the rational (straight-line) $FE$, producing $FM$ as breadth. $FM$ is thus rational, and incommensurable in length with $CD$ [Prop. 10.22]. And since the (sum of the squares) on $AG$ and $GB$ is incommensurable with twice the (rectangle contained) by $AG$ and $GB$, and $CL$ equal to the (sum of the squares) on $AG$ and $GB$, and $FL$ equal to twice the (rectangle contained) by $AG$ and $GB$, $CL$ [is] thus incommensurable with $FL$. And as $CL$ (is) to $FL$, so $CM$ is to $MF$ [Prop. 6.1]. Thus, $CM$ is incommensurable in length with $MF$ [Prop. 10.11]. And they are both rational. Thus, $CM$ and $MF$ are rational (straight-lines which are) commensurable in square only. $CF$ is thus an apotome [Prop. 10.73]. So, I say that (it is) also a sixth (apotome).

For since $FL$ is equal to twice the (rectangle contained) by $AG$ and $GB$, let $FM$ have been cut in half at $N$, and let $NO$ have been drawn through $N$, parallel to $CD$. Thus, $FO$ and $NL$ are each equal to the (rectangle contained) by $AG$ and $GB$. And since $AG$ and $GB$ are incommensurable in square, the (square) on $AG$ is thus incommensurable with the (square) on $GB$. But, $CH$ is equal to the (square) on $AG$, and $KL$ is equal to the (square) on $GB$. Thus, $CH$ is incommensurable with $KL$. And as $CH$ (is) to $KL$, so $CK$ is to $KM$ [Prop. 6.1]. Thus, $CK$ is incommensurable (in length) with $KM$ [Prop. 10.11]. And since the (rectangle contained) by $AG$ and $GB$ is the mean proportional to the (squares) on $AG$ and $GB$ [Prop. 10.21 lem.], and $CH$ is equal to the (square) on $AG$, and $KL$ equal to the (square) on $GB$, and $NL$ equal to the (rectangle contained) by $AG$ and $GB$, $NL$ is thus also the mean proportional to $CH$ and $KL$. Thus, as $CH$ is to $NL$, so $NL$ (is) to $KL$. And for the same (reasons as the preceding propositions), the square on $CM$ is greater than (the square on) $MF$ by the (square) on (some straight-line)

## ργ΄.

Ἡ τῇ ἀποτομῇ μήκει σύμμετρος ἀποτομή ἐστι καὶ τῇ τάξει ἡ αὐτή.




Ἔστω ἀποτομὴ ἡ AB, καὶ τῇ AB μήκει σύμμετρος ἔστω ἡ ΓΔ· λέγω, ὅτι καὶ ἡ ΓΔ ἀποτομή ἐστι καὶ τῇ τάξει ἡ αὐτὴ τῇ AB.

Ἐπεὶ γὰρ ἀποτομή ἐστιν ἡ AB, ἔστω αὐτῇ προσαρμόζουσα ἡ BE· αἱ AE, EB ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι. καὶ τῷ τῆς AB πρὸς τὴν ΓΔ λόγῳ ὁ αὐτὸς γεγονέτω ὁ τῆς BE πρὸς τὴν ΔZ· καὶ ὡς ἓν ἄρα πρὸς ἕν, πάντα [ἐστὶ] πρὸς πάντα· ἔστιν ἄρα καὶ ὡς ὅλη ἡ AE πρὸς ὅλην τὴν ΓZ, οὕτως ἡ AB πρὸς τὴν ΓΔ. σύμμετρος δὲ ἡ AB τῇ ΓΔ μήκει· σύμμετρος ἄρα καὶ ἡ AE μὲν τῇ ΓZ, ἡ δὲ BE τῇ ΔZ. καὶ αἱ AE, EB ῥηταί εἰσι δυνάμει μόνον σύμμετροι· καὶ αἱ ΓZ, ZΔ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι [ἀποτομὴ ἄρα ἐστὶν ἡ ΓΔ. λέγω δή, ὅτι καὶ τῇ τάξει ἡ αὐτὴ τῇ AB].

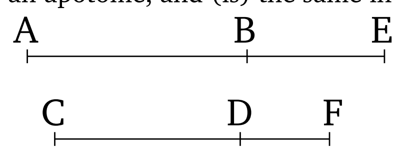
Ἐπεὶ οὖν ἐστιν ὡς ἡ AE πρὸς τὴν ΓZ, οὕτως ἡ BE πρὸς τὴν ΔZ, ἐναλλὰξ ἄρα ἐστὶν ὡς ἡ AE πρὸς τὴν EB, οὕτως ἡ ΓZ πρὸς τὴν ZΔ. ἤτοι δὴ ἡ AE τῆς EB μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ ἢ τῷ ἀπὸ ἀσυμμέτρου. εἰ μὲν οὖν ἡ AE τῆς EB μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ, καὶ ἡ ΓZ τῆς ZΔ μεῖζον δυνήσεται τῷ ἀπὸ συμμέτρου ἑαυτῇ. καὶ εἰ μὲν σύμμετρός ἐστιν ἡ AE τῇ ἐκκειμένῃ ῥητῇ μήκει, καὶ ἡ ΓZ, εἰ δὲ ἡ BE, καὶ ἡ ΔZ, εἰ δὲ οὐδετέρα τῶν AE, EB, καὶ οὐδετέρα τῶν ΓZ, ZΔ. εἰ δὲ ἡ AE [τῆς EB] μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ, καὶ ἡ ΓZ τῆς ZΔ μεῖζον δυνήσεται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. καὶ εἰ μὲν σύμμετρός ἐστιν ἡ AE τῇ ἐκκειμένῃ ῥητῇ μήκει, καὶ ἡ ΓZ, εἰ δὲ ἡ BE, καὶ ἡ ΔZ, εἰ δὲ οὐδετέρα τῶν AE, EB, οὐδετέρα τῶν ΓZ, ZΔ.

Ἀποτομὴ ἄρα ἐστὶν ἡ ΓΔ καὶ τῇ τάξει ἡ αὐτὴ τῇ AB· ὅπερ ἔδει δεῖξαι.

incommensurable (in length) with ($CM$) [Prop. 10.18]. And neither of them is commensurable with the (previously) laid down rational (straight-line) $CD$. Thus, $CF$ is a sixth apotome [Def. 10.16]. (Which is) the very thing it was required to show.

## Proposition 103

A (straight-line) commensurable in length with an apotome is an apotome, and (is) the same in order.




Let $AB$ be an apotome, and let $CD$ be commensurable in length with $AB$. I say that $CD$ is also an apotome, and (is) the same in order as $AB$.

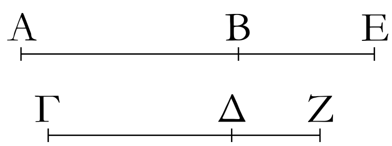
For since $AB$ is an apotome, let $BE$ be an attachment to it. Thus, $AE$ and $EB$ are rational (straight-lines which are) commensurable in square only [Prop. 10.73]. And let it have been contrived that the (ratio) of $BE$ to $DF$ is the same as the ratio of $AB$ to $CD$ [Prop. 6.12]. Thus, also, as one is to one, (so) all [are] to all [Prop. 5.12]. And thus as the whole $AE$ is to the whole $CF$, so $AB$ (is) to $CD$. And $AB$ (is) commensurable in length with $CD$. $AE$ (is) thus also commensurable (in length) with $CF$, and $BE$ with $DF$ [Prop. 10.11]. And $AE$ and $BE$ are rational (straight-lines which are) commensurable in square only. Thus, $CF$ and $FD$ are also rational (straight-lines which are) commensurable in square only [Prop. 10.13]. [$CD$ is thus an apotome. So, I say that (it is) also the same in order as $AB$.]

Therefore, since as $AE$ is to $CF$, so $BE$ (is) to $DF$, thus, alternately, as $AE$ is to $EB$, so $CF$ (is) to $FD$ [Prop. 5.16]. So, the square on $AE$ is greater than (the square on) $EB$ either by the (square) on (some straight-line) commensurable, or by the (square) on (some straight-line) incommensurable, (in length) with ($AE$). Therefore, if the (square) on $AE$ is greater than (the square on) $EB$ by the (square) on (some straight-line) commensurable (in length) with ($AE$) then the square on $CF$ will also be greater than (the square on) $FD$ by the (square) on (some straight-line) commensurable (in length) with ($CF$) [Prop. 10.14]. And if $AE$ is commensurable in length with a (previously) laid down rational (straight-line) then so (is) $CF$ [Prop. 10.12], and if $BE$ (is commensurable), so (is) $DF$, and if neither of $AE$ or $EB$ (are commensurable), neither (are) either of $CF$ or $FD$ [Prop. 10.13]. And if the (square) on $AE$ is greater [than (the square on) $EB$] by the (square) on (some straight-line) incommensurable (in length) with ($AE$) then the (square) on $CF$ will also be greater than (the square on) $FD$ by the (square) on (some straight-line) incommensurable (in length) with ($CF$) [Prop. 10.14]. And if $AE$ is commensurable in length with a (previously) laid down rational (straight-line), so (is) $CF$ [Prop. 10.12], and if $BE$ (is commensurable), so (is) $DF$, and if neither of $AE$ or $EB$ (are commensurable), neither (are) either of $CF$ or $FD$ [Prop. 10.13].

Thus, $CD$ is an apotome, and (is) the same in order as $AB$ [Defs. 10.11—10.16]. (Which is) the very thing it was required to show.

## ρδ′.

Ἡ τῇ μέσης ἀποτομῇ σύμμετρος μέσης ἀποτομή ἐστι καὶ τῇ τάξει ἡ αὐτή.




Ἔστω μέσης ἀποτομὴ ἡ ΑΒ, καὶ τῇ ΑΒ μήκει σύμμετρος ἔστω ἡ ΓΔ· λέγω, ὅτι καὶ ἡ ΓΔ μέσης ἀποτομή ἐστι καὶ τῇ τάξει ἡ αὐτὴ τῇ ΑΒ.

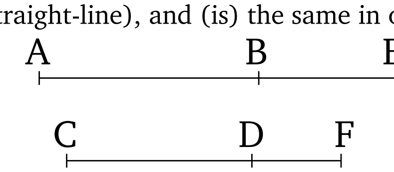
Ἐπεὶ γὰρ μέσης ἀποτομή ἐστιν ἡ ΑΒ, ἔστω αὐτῇ προσαρμόζουσα ἡ ΕΒ. αἱ ΑΕ, ΕΒ ἄρα μέσαι εἰσὶ δυνάμει μόνον σύμμετροι. καὶ γεγονέτω ὡς ἡ ΑΒ πρὸς τὴν ΓΔ, οὕτως ἡ ΒΕ πρὸς τὴν ΔΖ· σύμμετρος ἄρα [ἐστὶ] καὶ ἡ ΑΕ τῇ ΓΖ, ἡ δὲ ΒΕ τῇ ΔΖ. αἱ δὲ ΑΕ, ΕΒ μέσαι εἰσὶ δυνάμει μόνον σύμμετροι· καὶ αἱ ΓΖ, ΖΔ ἄρα μέσαι εἰσὶ δυνάμει μόνον σύμμετροι· μέσης ἄρα ἀποτομή ἐστιν ἡ ΓΔ. λέγω δή, ὅτι καὶ τῇ τάξει ἐστὶν ἡ αὐτὴ τῇ ΑΒ.

Ἐπεὶ [γάρ] ἐστιν ὡς ἡ ΑΕ πρὸς τὴν ΕΒ, οὕτως ἡ ΓΖ πρὸς τὴν ΖΔ [ἀλλ᾽ ὡς μὲν ἡ ΑΕ πρὸς τὴν ΕΒ, οὕτως τὸ ἀπὸ τῆς ΑΕ πρὸς τὸ ὑπὸ τῶν ΑΕ, ΕΒ, ὡς δὲ ἡ ΓΖ πρὸς τὴν ΖΔ, οὕτως τὸ ἀπὸ τῆς ΓΖ πρὸς τὸ ὑπὸ τῶν ΓΖ, ΖΔ], ἔστιν ἄρα καὶ ὡς τὸ ἀπὸ τῆς ΑΕ πρὸς τὸ ὑπὸ τῶν ΑΕ, ΕΒ, οὕτως τὸ ἀπὸ τῆς ΓΖ πρὸς τὸ ὑπὸ τῶν ΓΖ, ΖΔ [καὶ ἐναλλὰξ ὡς τὸ ἀπὸ τῆς ΑΕ πρὸς τὸ ἀπὸ τῆς ΓΖ, οὕτως τὸ ὑπὸ τῶν ΑΕ, ΕΒ πρὸς τὸ ὑπὸ τῶν ΓΖ, ΖΔ]. σύμμετρον δὲ τὸ ἀπὸ τῆς ΑΕ τῷ ἀπὸ τῆς ΓΖ· σύμμετρον ἄρα ἐστὶ καὶ τὸ ὑπὸ τῶν ΑΕ, ΕΒ τῷ ὑπὸ τῶν ΓΖ, ΖΔ. εἴτε οὖν ῥητόν ἐστι τὸ ὑπὸ τῶν ΑΕ, ΕΒ, ῥητὸν ἔσται καὶ τὸ ὑπὸ τῶν ΓΖ, ΖΔ, εἴτε μέσον [ἐστὶ] τὸ ὑπὸ τῶν ΑΕ, ΕΒ, μέσον [ἐστὶ] καὶ τὸ ὑπὸ τῶν ΓΖ, ΖΔ.

Μέσης ἄρα ἀποτομή ἐστιν ἡ ΓΔ καὶ τῇ τάξει ἡ αὐτὴ τῇ ΑΒ· ὅπερ ἔδει δεῖξαι.

## Proposition 104

A (straight-line) commensurable (in length) with an apotome of a medial (straight-line) is an apotome of a medial (straight-line), and (is) the same in order.




Let $AB$ be an apotome of a medial (straight-line), and let $CD$ be commensurable in length with $AB$. I say that $CD$ is also an apotome of a medial (straight-line), and (is) the same in order as $AB$.

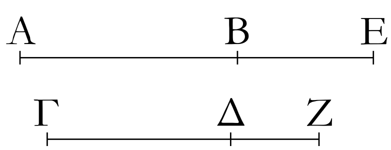
For since $AB$ is an apotome of a medial (straight-line), let $EB$ be an attachment to it. Thus, $AE$ and $EB$ are medial (straight-lines which are) commensurable in square only [Props. 10.74, 10.75]. And let it have been contrived that as $AB$ is to $CD$, so $BE$ (is) to $DF$ [Prop. 6.12]. Thus, $AE$ [is] also commensurable (in length) with $CF$, and $BE$ with $DF$ [Props. 5.12, 10.11]. And $AE$ and $EB$ are medial (straight-lines which are) commensurable in square only. $CF$ and $FD$ are thus also medial (straight-lines which are) commensurable in square only [Props. 10.23, 10.13]. Thus, $CD$ is an apotome of a medial (straight-line) [Props. 10.74, 10.75]. So, I say that it is also the same in order as $AB$.

[For] since as $AE$ is to $EB$, so $CF$ (is) to $FD$ [Props. 5.12, 5.16] [but as $AE$ (is) to $EB$, so the (square) on $AE$ (is) to the (rectangle contained) by $AE$ and $EB$, and as $CF$ (is) to $FD$, so the (square) on $CF$ (is) to the (rectangle contained) by $CF$ and $FD$], thus as the (square) on $AE$ is to the (rectangle contained) by $AE$ and $EB$, so the (square) on $CF$ also (is) to the (rectangle contained) by $CF$ and $FD$ [Prop. 10.21 lem.] [and, alternately, as the (square) on $AE$ (is) to the (square) on $CF$, so the (rectangle contained) by $AE$ and $EB$ (is) to the (rectangle contained) by $CF$ and $FD$]. And the (square) on $AE$ (is) commensurable with the (square) on $CF$. Thus, the (rectangle contained) by $AE$ and $EB$ is also commensurable with the (rectangle contained) by $CF$ and $FD$ [Props. 5.16, 10.11]. Therefore, either the (rectangle contained) by $AE$ and $EB$ is rational, and the (rectangle contained) by $CF$ and $FD$ will also be rational [Def. 10.4], or the (rectangle contained) by $AE$ and $EB$ [is] medial, and the (rectangle contained) by $CF$ and $FD$ [is] also medial [Prop. 10.23 corr.].

Therefore, $CD$ is the apotome of a medial (straight-line), and is the same in order as $AB$ [Props. 10.74, 10.75]. (Which is) the very thing it was required to show.

## ρε´.

Ἡ τῇ ἐλάσσονι σύμμετρος ἐλάσσων ἐστίν.



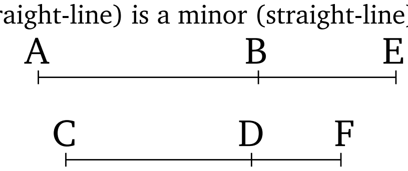

Ἔστω γὰρ ἐλάσσων ἡ ΑΒ καὶ τῇ ΑΒ σύμμετρος ἡ ΓΔ· λέγω, ὅτι καὶ ἡ ΓΔ ἐλάσσων ἐστίν.

Γεγονέτω γὰρ τὰ αὐτά· καὶ ἐπεὶ αἱ ΑΕ, ΕΒ δυνάμει εἰσὶν ἀσύμμετροι, καὶ αἱ ΓΖ, ΖΔ ἄρα δυνάμει εἰσὶν ἀσύμμετροι. ἐπεὶ οὖν ἐστιν ὡς ἡ ΑΕ πρὸς τὴν ΕΒ, οὕτως ἡ ΓΖ πρὸς τὴν ΖΔ, ἔστιν ἄρα καὶ ὡς τὸ ἀπὸ τῆς ΑΕ πρὸς τὸ ἀπὸ τῆς ΕΒ, οὕτως τὸ ἀπὸ τῆς ΓΖ πρὸς τὸ ἀπὸ τῆς ΖΔ. συνθέντι ἄρα ἐστὶν ὡς τὰ ἀπὸ τῶν ΑΕ, ΕΒ πρὸς τὸ ἀπὸ τῆς ΕΒ, οὕτως τὰ ἀπὸ τῶν ΓΖ, ΖΔ πρὸς τὸ ἀπὸ τῆς ΖΔ [καὶ ἐναλλάξ]· σύμμετρον δέ ἐστι τὸ ἀπὸ τῆς ΒΕ τῷ ἀπὸ τῆς ΔΖ· σύμμετρον ἄρα καὶ τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΕ, ΕΒ τετραγώνων τῷ συγκειμένῳ ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ τετραγώνων. ῥητὸν δέ ἐστι τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΕ, ΕΒ τετραγώνων· ῥητὸν ἄρα ἐστὶ καὶ τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ τετραγώνων. πάλιν, ἐπεί ἐστιν ὡς τὸ ἀπὸ τῆς ΑΕ πρὸς τὸ ὑπὸ τῶν ΑΕ, ΕΒ, οὕτως τὸ ἀπὸ τῆς ΓΖ πρὸς τὸ ὑπὸ τῶν ΓΖ, ΖΔ, σύμμετρον δὲ τὸ ἀπὸ τῆς ΑΕ τετράγωνον τῷ ἀπὸ τῆς ΓΖ τετραγώνῳ, σύμμετρον ἄρα ἐστὶ καὶ τὸ ὑπὸ τῶν ΑΕ, ΕΒ τῷ ὑπὸ τῶν ΓΖ, ΖΔ. μέσον δὲ τὸ ὑπὸ τῶν ΑΕ, ΕΒ· μέσον ἄρα καὶ τὸ ὑπὸ τῶν ΓΖ, ΖΔ· αἱ ΓΖ, ΖΔ ἄρα δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὸ μὲν συγκείμενον ἐκ τῶν ἀπ᾽ αὐτῶν τετραγώνων ῥητόν, τὸ δ᾽ ὑπ᾽ αὐτῶν μέσον.

Ἐλάσσων ἄρα ἐστὶν ἡ ΓΔ· ὅπερ ἔδει δεῖξαι.

## Proposition 105

A (straight-line) commensurable (in length) with a minor (straight-line) is a minor (straight-line).




For let $AB$ be a minor (straight-line), and (let) $CD$ (be) commensurable (in length) with $AB$. I say that $CD$ is also a minor (straight-line).

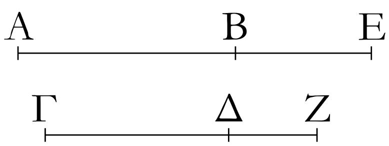
For let the same things have been contrived (as in the former proposition). And since $AE$ and $EB$ are (straight-lines which are) incommensurable in square [Prop. 10.76], $CF$ and $FD$ are thus also (straight-lines which are) incommensurable in square [Prop. 10.13]. Therefore, since as $AE$ is to $EB$, so $CF$ (is) to $FD$ [Props. 5.12, 5.16], thus also as the (square) on $AE$ is to the (square) on $EB$, so the (square) on $CF$ (is) to the (square) on $FD$ [Prop. 6.22]. Thus, via composition, as the (sum of the squares) on $AE$ and $EB$ is to the (square) on $EB$, so the (sum of the squares) on $CF$ and $FD$ (is) to the (square) on $FD$ [Prop. 5.18], [also alternately]. And the (square) on $BE$ is commensurable with the (square) on $DF$ [Prop. 10.104]. The sum of the squares on $AE$ and $EB$ (is) thus also commensurable with the sum of the squares on $CF$ and $FD$ [Prop. 5.16, 10.11]. And the sum of the (squares) on $AE$ and $EB$ is rational [Prop. 10.76]. Thus, the sum of the (squares) on $CF$ and $FD$ is also rational [Def. 10.4]. Again, since as the (square) on $AE$ is to the (rectangle contained) by $AE$ and $EB$, so the (square) on $CF$ (is) to the (rectangle contained) by $CF$ and $FD$ [Prop. 10.21 lem.], and the square on $AE$ (is) commensurable with the square on $CF$, the (rectangle contained) by $AE$ and $EB$ is thus also commensurable with the (rectangle contained) by $CF$ and $FD$. And the (rectangle contained) by $AE$ and $EB$ (is) medial [Prop. 10.76]. Thus, the (rectangle contained) by $CF$ and $FD$ (is) also medial [Prop. 10.23 corr.]. $CF$ and

$FD$ are thus (straight-lines which are) incommensurable in square, making the sum of the squares on them rational, and the (rectangle contained) by them medial.

Thus, $CD$ is a minor (straight-line) [Prop. 10.76]. (Which is) the very thing it was required to show.

## ρϛʹ.

Ἡ τῇ μετὰ ῥητοῦ μέσον τὸ ὅλον ποιούσῃ σύμμετρος μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσά ἐστιν.



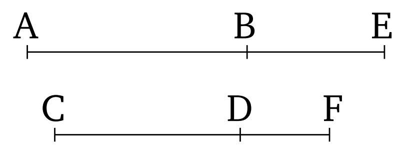

Ἔστω μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσα ἡ ΑΒ καὶ τῇ ΑΒ σύμμετρος ἡ ΓΔ· λέγω, ὅτι καὶ ἡ ΓΔ μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσά ἐστιν.

Ἔστω γὰρ τῇ ΑΒ προσαρμόζουσα ἡ ΒΕ· αἱ ΑΕ, ΕΒ ἄρα δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὸ μὲν συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΕ, ΕΒ τετραγώνων μέσον, τὸ δʼ ὑπʼ αὐτῶν ῥητόν. καὶ τὰ αὐτὰ κατεσκευάσθω. ὁμοίως δὴ δείξομεν τοῖς πρότερον, ὅτι αἱ ΓΖ, ΖΔ ἐν τῷ αὐτῷ λόγῳ εἰσὶ ταῖς ΑΕ, ΕΒ, καὶ σύμμετρόν ἐστι τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΕ, ΕΒ τετραγώνων τῷ συγκειμένῳ ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ τετραγώνων, τὸ δὲ ὑπὸ τῶν ΑΕ, ΕΒ τῷ ὑπὸ τῶν ΓΖ, ΖΔ· ὥστε καὶ αἱ ΓΖ, ΖΔ δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τὸ μὲν συγκείμενον ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ τετραγώνων μέσον, τὸ δʼ ὑπʼ αὐτῶν ῥητόν.

Ἡ ΓΔ ἄρα μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσά ἐστιν· ὅπερ ἔδει δεῖξαι.

## Proposition 106

A (straight-line) commensurable (in length) with a (straight-line) which with a rational (area) makes a medial whole is a (straight-line) which with a rational (area) makes a medial whole.



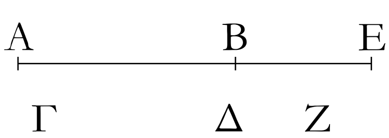

Let $AB$ be a (straight-line) which with a rational (area) makes a medial whole, and (let) $CD$ (be) commensurable (in length) with $AB$. I say that $CD$ is also a (straight-line) which with a rational (area) makes a medial (whole).

For let $BE$ be an attachment to $AB$. Thus, $AE$ and $EB$ are (straight-lines which are) incommensurable in square, making the sum of the squares on $AE$ and $EB$ medial, and the (rectangle contained) by them rational [Prop. 10.77]. And let the same construction have been made (as in the previous propositions). So, similarly to the previous (propositions), we can show that $CF$ and $FD$ are in the same ratio as $AE$ and $EB$, and the sum of the squares on $AE$ and $EB$ is commensurable with the sum of the squares on $CF$ and $FD$, and the (rectangle contained) by $AE$ and $EB$ with the (rectangle contained) by $CF$ and $FD$. Hence, $CF$ and $FD$ are also (straight-lines which are) incommensurable in square, making the sum of the squares on $CF$ and $FD$ medial, and the (rectangle contained) by them rational.

$CD$ is thus a (straight-line) which with a rational (area) makes a medial whole [Prop. 10.77]. (Which is) the very thing it was required to show.

## ρζʹ.

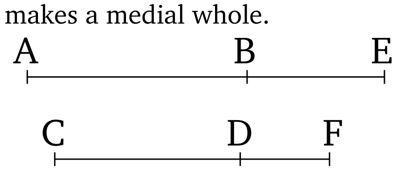
Ἡ τῇ μετὰ μέσου μέσον τὸ ὅλον ποιούσῃ σύμμετρος καὶ αὐτὴ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσά ἐστιν.




Ἔστω μετὰ μέσου μέσον τὸ ὅλον ποιοῦσα ἡ ΑΒ, καὶ τῇ

## Proposition 107

A (straight-line) commensurable (in length) with a (straight-line) which with a medial (area) makes a medial whole is itself also a (straight-line) which with a medial (area) makes a medial whole.




Let $AB$ be a (straight-line) which with a medial (area)

ΑΒ ἔστω σύμμετρος ἡ ΓΔ· λέγω, ὅτι καὶ ἡ ΓΔ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσά ἐστιν.

Ἔστω γὰρ τῇ ΑΒ προσαρμόζουσα ἡ ΒΕ, καὶ τὰ αὐτὰ κατεσκευάσθω· αἱ ΑΕ, ΕΒ ἄρα δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τό τε συγκείμενον ἐκ τῶν ἀπ᾿ αὐτῶν τετραγώνων μέσον καὶ τὸ ὑπ᾿ αὐτῶν μέσον καὶ ἔτι ἀσύμμετρον τὸ συγκείμενον ἐκ τῶν ἀπ᾿ αὐτῶν τετραγώνων τῷ ὑπ᾿ αὐτῶν. καί εἰσιν, ὡς ἐδείχθη, αἱ ΑΕ, ΕΒ σύμμετροι ταῖς ΓΖ, ΖΔ, καὶ τὸ συγκείμενον ἐκ τῶν ἀπὸ τῶν ΑΕ, ΕΒ τετραγώνων τῷ συγκειμένῳ ἐκ τῶν ἀπὸ τῶν ΓΖ, ΖΔ, τὸ δὲ ὑπὸ τῶν ΑΕ, ΕΒ τῷ ὑπὸ τῶν ΓΖ, ΖΔ· καὶ αἱ ΓΖ, ΖΔ ἄρα δυνάμει εἰσὶν ἀσύμμετροι ποιοῦσαι τό τε συγκείμενον ἐκ τῶν ἀπ᾿ αὐτῶν τετραγώνων μέσον καὶ τὸ ὑπ᾿ ἀὐτῶν μέσον καὶ ἔτι ἀσύμμετρον τὸ συγκείμενον ἐκ τῶν ἀπ᾿ αὐτῶν [τετραγώνων] τῷ ὑπ᾿ αὐτῶν.

Ἡ ΓΔ ἄρα μετὰ μέσου μέσον τὸ ὅλον ποιοῦσά ἐστιν· ὅπερ ἔδει δεῖξαι.

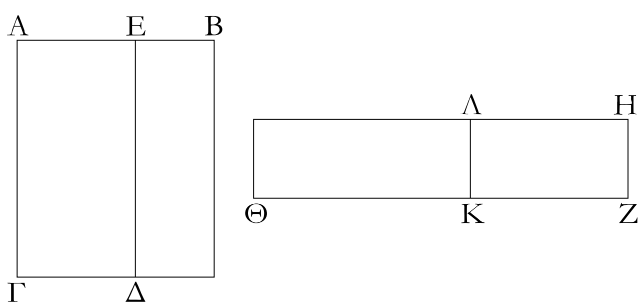
makes a medial whole, and let $CD$ be commensurable (in length) with $AB$. I say that $CD$ is also a (straight-line) which with a medial (area) makes a medial whole.

For let $BE$ be an attachment to $AB$. And let the same construction have been made (as in the previous propositions). Thus, $AE$ and $EB$ are (straight-lines which are) incommensurable in square, making the sum of the squares on them medial, and the (rectangle contained) by them medial, and, further, the sum of the squares on them incommensurable with the (rectangle contained) by them [Prop. 10.78]. And, as was shown (previously), $AE$ and $EB$ are commensurable (in length) with $CF$ and $FD$ (respectively), and the sum of the squares on $AE$ and $EB$ with the sum of the squares on $CF$ and $FD$, and the (rectangle contained) by $AE$ and $EB$ with the (rectangle contained) by $CF$ and $FD$. Thus, $CF$ and $FD$ are also (straight-lines which are) incommensurable in square, making the sum of the squares on them medial, and the (rectangle contained) by them medial, and, further, the sum of the [squares] on them incommensurable with the (rectangle contained) by them.

Thus, $CD$ is a (straight-line) which with a medial (area) makes a medial whole [Prop. 10.78]. (Which is) the very thing it was required to show.

## ρη΄.

Ἀπὸ ῥητοῦ μέσου ἀφαιρουμένου ἡ τὸ λοιπὸν χωρίον δυναμένη μία δύο ἀλόγων γίνεται ἤτοι ἀποτομὴ ἢ ἐλάσσων.

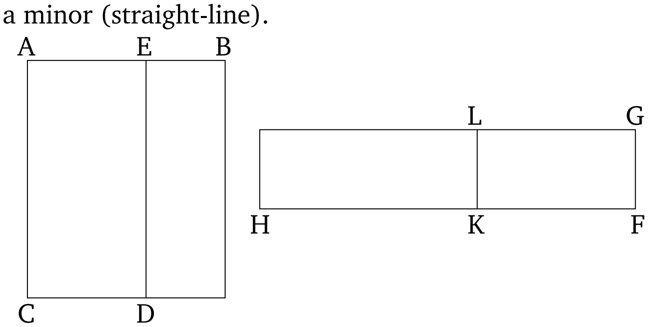



Ἀπὸ γὰρ ῥητοῦ τοῦ ΒΓ μέσον ἀφῃρήσθω τὸ ΒΔ· λέγω, ὅτι ἡ τὸ λοιπὸν δυναμένη τὸ ΕΓ μία δύο ἀλόγων γίνεται ἤτοι ἀποτομὴ ἢ ἐλάσσων.

Ἐκκείσθω γὰρ ῥητὴ ἡ ΖΗ, καὶ τῷ μὲν ΒΓ ἴσον παρὰ τὴν ΖΗ παραβεβλήσθω ὀρθογώνιον παραλληλόγραμμον τὸ ΗΘ, τῷ δὲ ΔΒ ἴσον ἀφῃρήσθω τὸ ΗΚ· λοιπὸν ἄρα τὸ ΕΓ ἴσον ἐστὶ τῷ ΛΘ. ἐπεὶ οὖν ῥητὸν μέν ἐστι τὸ ΒΓ, μέσον δὲ τὸ ΒΔ, ἴσον δὲ τὸ μὲν ΒΓ τῷ ΗΘ, τὸ δὲ ΒΔ τῷ ΗΚ, ῥητὸν μὲν ἄρα ἐστὶ τὸ ΗΘ, μέσον δὲ τὸ ΗΚ. καὶ παρὰ ῥητὴν τὴν ΖΗ παράκειται· ῥητὴ μὲν ἄρα ἡ ΖΘ καὶ σύμμετρος τῇ ΖΗ μήκει, ῥητὴ δὲ ἡ ΖΚ καὶ ἀσύμμετρος τῇ ΖΗ μήκει· ἀσύμμετρος ἄρα

## Proposition 108

A medial (area) being subtracted from a rational (area), one of two irrational (straight-lines) arise (as) the square-root of the remaining area—either an apotome, or a minor (straight-line).




For let the medial (area) $BD$ have been subtracted from the rational (area) $BC$. I say that one of two irrational (straight-lines) arise (as) the square-root of the remaining (area), $EC$—either an apotome, or a minor (straight-line).

For let the rational (straight-line) $FG$ have been laid out, and let the right-angled parallelogram $GH$, equal to $BC$, have been applied to $FG$, and let $GK$, equal to $DB$, have been subtracted (from $GH$). Thus, the remainder $EC$ is equal to $LH$. Therefore, since $BC$ is a rational (area), and $BD$ a medial (area), and $BC$ (is) equal to ἐστὶν ἡ ΖΘ τῇ ΖΚ μήκει. αἱ ΖΘ, ΖΚ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΚΘ, προσαρμόζουσα δὲ αὐτῇ ἡ ΚΖ. ἤτοι δὴ ἡ ΘΖ τῆς ΖΚ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἢ οὔ.

Δυνάσθω πρότερον τῷ ἀπὸ συμμέτρου. καί ἐστιν ὅλη ἡ ΘΖ σύμμετρος τῇ ἐκκειμένῃ ῥητῇ μήκει τῇ ΖΗ· ἀποτομὴ ἄρα πρώτη ἐστὶν ἡ ΚΘ. τὸ δ᾿ ὑπὸ ῥητῆς καὶ ἀποτομῆς πρώτης περιεχόμενον ἡ δυναμένη ἀποτομή ἐστιν. ἡ ἄρα τὸ ΛΘ, τουτέστι τὸ ΕΓ, δυναμένη ἀποτομή ἐστιν.

Εἰ δὲ ἡ ΘΖ τῆς ΖΚ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ, καί ἐστιν ὅλη ἡ ΖΘ σύμμετρος τῇ ἐκκειμένῃ ῥητῇ μήκει τῇ ΖΗ, ἀποτομὴ τετάρτη ἐστὶν ἡ ΚΘ. τὸ δ᾿ ὑπὸ ῥητῆς καὶ ἀποτομῆς τετάρτης περιεχόμενον ἡ δυναμένη ἐλάσσων ἐστίν· ὅπερ ἔδει δεῖξαι.

$GH$, and $BD$ to $GK$, $GH$ is thus a rational (area), and $GK$ a medial (area). And they are applied to the rational (straight-line) $FG$. Thus, $FH$ (is) rational, and commensurable in length with $FG$ [Prop. 10.20], and $FK$ (is) also rational, and incommensurable in length with $FG$ [Prop. 10.22]. Thus, $FH$ is incommensurable in length with $FK$ [Prop. 10.13]. $FH$ and $FK$ are thus rational (straight-lines which are) commensurable in square only. Thus, $KH$ is an apotome [Prop. 10.73], and $KF$ an attachment to it. So, the square on $HF$ is greater than (the square on) $FK$ by the (square) on (some straight-line which is) either commensurable, or not (commensurable), (in length with $HF$).

First, let the square (on it) be (greater) by the (square) on (some straight-line which is) commensurable (in length with $HF$). And the whole of $HF$ is commensurable in length with the (previously) laid down rational (straight-line) $FG$. Thus, $KH$ is a first apotome [Def. 10.1]. And the square-root of an (area) contained by a rational (straight-line) and a first apotome is an apotome [Prop. 10.91]. Thus, the square-root of $LH$—that is to say, (of) $EC$—is an apotome.

And if the square on $HF$ is greater than (the square on) $FK$ by the (square) on (some straight-line which is) incommensurable (in length) with ($HF$), and (since) the whole of $FH$ is commensurable in length with the (previously) laid down rational (straight-line) $FG$, $KH$ is a fourth apotome [Prop. 10.14]. And the square-root of an (area) contained by a rational (straight-line) and a fourth apotome is a minor (straight-line) [Prop. 10.94]. (Which is) the very thing it was required to show.

## ρθʹ.

Ἀπὸ μέσου ῥητοῦ ἀφαιρουμένου ἄλλαι δύο ἄλογοι γίνονται ἤτοι μέσης ἀποτομὴ πρώτη ἢ μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσα.

Ἀπὸ γὰρ μέσου τοῦ ΒΓ ῥητὸν ἀφῃρήσθω τὸ ΒΔ. λέγω, ὅτι ἡ τὸ λοιπὸν τὸ ΕΓ δυναμένη μία δύο ἀλόγων γίνεται ἤτοι μέσης ἀποτομὴ πρώτη ἢ μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσα.

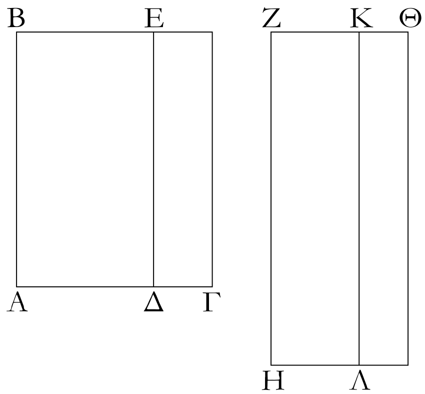
Ἐκκείσθω γὰρ ῥητὴ ἡ ΖΗ, καὶ παραβεβλήσθω ὁμοίως τὰ χωρία. ἔστι δὴ ἀκολούθως ῥητὴ μὲν ἡ ΖΘ καὶ ἀσύμμετρος τῇ ΖΗ μήκει, ῥητὴ δὲ ἡ ΚΖ καὶ σύμμετρος τῇ ΖΗ μήκει· αἱ ΖΘ, ΖΚ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΚΘ, προσαρμόζουσα δὲ ταύτῃ ἡ ΖΚ. ἤτοι δὴ ἡ ΘΖ τῆς ΖΚ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ ἢ τῷ ἀπὸ ἀσυμμέτρου.

## Proposition 109

A rational (area) being subtracted from a medial (area), two other irrational (straight-lines) arise (as the square-root of the remaining area)—either a first apotome of a medial (straight-line), or that (straight-line) which with a rational (area) makes a medial whole.

For let the rational (area) $BD$ have been subtracted from the medial (area) $BC$. I say that one of two irrational (straight-lines) arise (as) the square-root of the remaining (area), $EC$—either a first apotome of a medial (straight-line), or that (straight-line) which with a rational (area) makes a medial whole.

For let the rational (straight-line) $FG$ be laid down, and let similar areas (to the preceding proposition) have been applied (to it). So, accordingly, $FH$ is rational, and incommensurable in length with $FG$, and $KF$ (is) also rational, and commensurable in length with $FG$. Thus, $FH$ and $FK$ are rational (straight-lines which are) com-




Εἰ μὲν οὖν ἡ ΘΖ τῆς ΖΚ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ, καί ἐστιν ἡ προσαρμόζουσα ἡ ΖΚ σύμμετρος τῇ ἐκκειμένη ῥητῇ μήκει τῇ ΖΗ, ἀποτομὴ δευτέρα ἐστὶν ἡ ΚΘ. ῥητὴ δὲ ἡ ΖΗ· ὥστε ἡ τὸ ΛΘ, τουτέστι τὸ ΕΓ, δυναμένη μέσης ἀποτομὴ πρώτη ἐστίν.

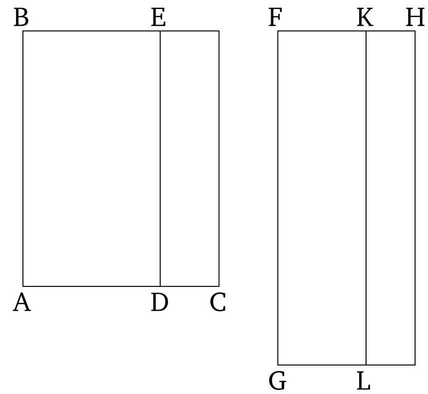
Εἰ δὲ ἡ ΘΖ τῆς ΖΚ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου, καί ἐστιν ἡ προσαρμόζουσα ἡ ΖΚ σύμμετρος τῇ ἐκκειμένῃ ῥητῇ μήκει τῇ ΖΗ, ἀποτομὴ πέμπτη ἐστὶν ἡ ΚΘ· ὥστε ἡ τὸ ΕΓ δυναμένη μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσά ἐστιν· ὅπερ ἔδει δεῖξαι.

## ριʹ.

Ἀπὸ μέσου μέσου ἀφαιρουμένου ἀσυμμέτρου τῷ ὅλῳ αἱ λοιπαὶ δύο ἄλογοι γίνονται ἤτοι μέσης ἀποτομὴ δευτέρα ἢ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσα.

Ἀφῃρήσθω γὰρ ὡς ἐπὶ τῶν προκειμένων καταγραφῶν ἀπὸ μέσου τοῦ ΒΓ μέσον τὸ ΒΔ ἀσύμμετρον τῷ ὅλῳ· λέγω, ὅτι ἡ τὸ ΕΓ δυναμένη μία ἐστὶ δύο ἀλόγων ἤτοι μέσης ἀποτομὴ δευτέρα ἢ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσα.

mensurable in square only [Prop. 10.13]. $KH$ is thus an apotome [Prop. 10.73], and $FK$ an attachment to it. So, the square on $HF$ is greater than (the square on) $FK$ either by the (square) on (some straight-line) commensurable (in length) with ($HF$), or by the (square) on (some straight-line) incommensurable (in length with $HF$).



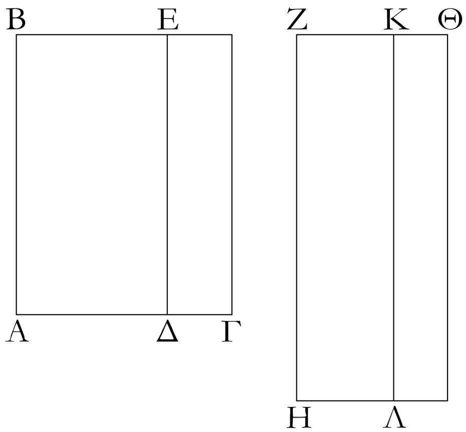

Therefore, if the square on $HF$ is greater than (the square on) $FK$ by the (square) on (some straight-line) commensurable (in length) with ($HF$), and (since) the attachment $FK$ is commensurable in length with the (previously) laid down rational (straight-line) $FG$, $KH$ is a second apotome [Def. 10.12]. And $FG$ (is) rational. Hence, the square-root of $LH$—that is to say, (of) $EC$—is a first apotome of a medial (straight-line) [Prop. 10.92].

And if the square on $HF$ is greater than (the square on) $FK$ by the (square) on (some straight-line) incommensurable (in length with $HF$), and (since) the attachment $FK$ is commensurable in length with the (previously) laid down rational (straight-line) $FG$, $KH$ is a fifth apotome [Def. 10.15]. Hence, the square-root of $EC$ is that (straight-line) which with a rational (area) makes a medial whole [Prop. 10.95]. (Which is) the very thing it was required to show.

## Proposition 110

A medial (area), incommensurable with the whole, being subtracted from a medial (area), the two remaining irrational (straight-lines) arise (as) the (square-root of the area)—either a second apotome of a medial (straight-line), or that (straight-line) which with a medial (area) makes a medial whole.

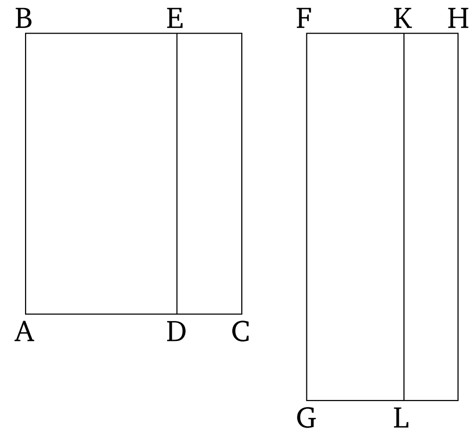
For, as in the previous figures, let the medial (area) $BD$, incommensurable with the whole, have been subtracted from the medial (area) $BC$. I say that the square-root of $EC$ is one of two irrational (straight-lines)—either a second apotome of a medial (straight-line), or that (straight-line) which with a medial (area) makes a medial whole.




Ἐπεὶ γὰρ μέσον ἐστὶν ἑκάτερον τῶν ΒΓ, ΒΔ, καὶ ἀσύμμετρον τὸ ΒΓ τῷ ΒΔ, ἔσται ἀκολούθως ῥητὴ ἑκατέρα τῶν ΖΘ, ΖΚ καὶ ἀσύμμετρος τῇ ΖΗ μήκει. καὶ ἐπεὶ ἀσύμμετρόν ἐστι τὸ ΒΓ τῷ ΒΔ, τουτέστι τὸ ΗΘ τῷ ΗΚ, ἀσύμμετρος καὶ ἡ ΘΖ τῇ ΖΚ· αἱ ΖΘ, ΖΚ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΚΘ [προσαρμόζουσα δὲ ἡ ΖΚ. ἤτοι δὴ ἡ ΖΘ τῆς ΖΚ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἢ τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ].

Εἰ μὲν δὴ ἡ ΖΘ τῆς ΖΚ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ, καὶ οὐδετέρα τῶν ΖΘ, ΖΚ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ μήκει τῇ ΖΗ, ἀποτομὴ τρίτη ἐστὶν ἡ ΚΘ. ῥητὴ δὲ ἡ ΚΛ, τὸ δ᾽ ὑπὸ ῥητῆς καὶ ἀποτομῆς τρίτης περιεχόμενον ὀρθογώνιον ἄλογόν ἐστιν, καὶ ἡ δυναμένη αὐτὸ ἄλογός ἐστιν, καλεῖται δὲ μέσης ἀποτομὴ δευτέρα· ὥστε ἡ τὸ ΛΘ, τουτέστι τὸ ΕΓ, δυναμένη μέσης ἀποτομή ἐστι δευτερά.

Εἰ δὲ ἡ ΖΘ τῆς ΖΚ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ [μήκει], καὶ οὐδετέρα τῶν ΘΖ, ΖΚ σύμμετρός ἐστι τῇ ΖΗ μήκει, ἀποτομὴ ἕκτη ἐστὶν ἡ ΚΘ. τὸ δ᾽ ὑπὸ ῥητῆς καὶ ἀποτομῆς ἕκτης ἡ δυναμένη ἐστὶ μετὰ μέσου μέσον τὸ ὅλον ποιοῦσα. ἡ τὸ ΛΘ ἄρα, τουτέστι τὸ ΕΓ, δυναμένη μετὰ μέσου μέσον τὸ ὅλον ποιοῦσά ἐστιν· ὅπερ ἔδει δεῖξαι.



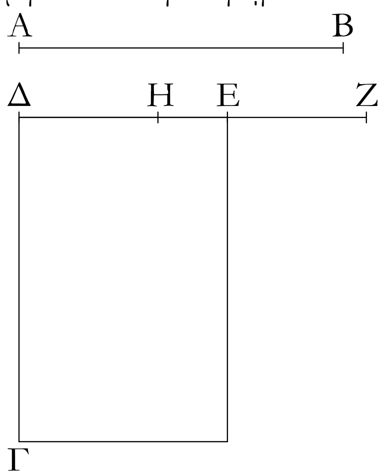

For since $BC$ and $BD$ are each medial (areas), and $BC$ (is) incommensurable with $BD$, accordingly, $FH$ and $FK$ will each be rational (straight-lines), and incommensurable in length with $FG$ [Prop. 10.22]. And since $BC$ is incommensurable with $BD$—that is to say, $GH$ with $GK$—$HF$ (is) also incommensurable (in length) with $FK$ [Props. 6.1, 10.11]. Thus, $FH$ and $FK$ are rational (straight-lines which are) commensurable in square only. $KH$ is thus as apotome [Prop. 10.73], [and $FK$ an attachment (to it). So, the square on $FH$ is greater than (the square on) $FK$ either by the (square) on (some straight-line) commensurable, or by the (square) on (some straight-line) incommensurable, (in length) with ($FH$).]

So, if the square on $FH$ is greater than (the square on) $FK$ by the (square) on (some straight-line) commensurable (in length) with ($FH$), and (since) neither of $FH$ and $FK$ is commensurable in length with the (previously) laid down rational (straight-line) $FG$, $KH$ is a third apotome [Def. 10.3]. And $KL$ (is) rational. And the rectangle contained by a rational (straight-line) and a third apotome is irrational, and the square-root of it is that irrational (straight-line) called a second apotome of a medial (straight-line) [Prop. 10.93]. Hence, the square-root of $LH$—that is to say, (of) $EC$—is a second apotome of a medial (straight-line).

And if the square on $FH$ is greater than (the square on) $FK$ by the (square) on (some straight-line) incommensurable [in length] with ($FH$), and (since) neither of $HF$ and $FK$ is commensurable in length with $FG$, $KH$ is a sixth apotome [Def. 10.16]. And the square-root of the (rectangle contained) by a rational (straight-line) and a sixth apotome is that (straight-line) which with a medial (area) makes a medial whole [Prop. 10.96]. Thus, the square-root of $LH$—that is to say, (of) $EC$—is that (straight-line) which with a medial (area) makes a medial whole. (Which is) the very thing it was required to show.

## ριαʹ.

Ἡ ἀποτομὴ οὐκ ἔστιν ἡ αὐτὴ τῇ ἐκ δύο ὀνομάτων.



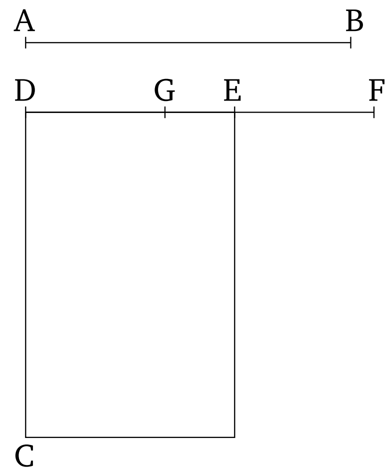

Ἔστω ἀποτομὴ ἡ ΑΒ· λέγω, ὅτι ἡ ΑΒ οὐκ ἔστιν ἡ αὐτὴ τῇ ἐκ δύο ὀνομάτων.

Εἰ γὰρ δυνατόν, ἔστω· καὶ ἐκκείσθω ῥητὴ ἡ ΔΓ, καὶ τῷ ἀπὸ τῆς ΑΒ ἴσον παρὰ τὴν ΓΔ παραβεβλήσθω ὀρθογώνιον τὸ ΓΕ πλάτος ποιοῦν τὴν ΔΕ. ἐπεὶ οὖν ἀποτομή ἐστιν ἡ ΑΒ, ἀποτομὴ πρώτη ἐστὶν ἡ ΔΕ. ἔστω αὐτῇ προσαρμόζουσα ἡ ΕΖ· αἱ ΔΖ, ΖΕ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι, καὶ ἡ ΔΖ τῆς ΖΕ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ, καὶ ἡ ΔΖ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ μήκει τῇ ΔΓ. πάλιν, ἐπεὶ ἐκ δύο ὀνομάτων ἐστὶν ἡ ΑΒ, ἐκ δύο ἄρα ὀνομάτων πρώτη ἐστὶν ἡ ΔΕ. διῃρήσθω εἰς τὰ ὀνόματα κατὰ τὸ Η, καὶ ἔστω μεῖζον ὄνομα τὸ ΔΗ· αἱ ΔΗ, ΗΕ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι, καὶ ἡ ΔΗ τῆς ΗΕ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ, καὶ τὸ μεῖζον ἡ ΔΗ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ μήκει τῇ ΔΓ. καὶ ἡ ΔΖ ἄρα τῇ ΔΗ σύμμετρός ἐστι μήκει· καὶ λοιπὴ ἄρα ἡ ΗΖ σύμμετρός ἐστι τῇ ΔΖ μήκει. [ἐπεὶ οὖν σύμμετρός ἐστιν ἡ ΔΖ τῇ ΗΖ, ῥητὴ δέ ἐστιν ἡ ΔΖ, ῥητὴ ἄρα ἐστὶ καὶ ἡ ΗΖ. ἐπεὶ οὖν σύμμετρός ἐστιν ἡ ΔΖ τῇ ΗΖ μήκει] ἀσύμμετρος δὲ ἡ ΔΖ τῇ ΕΖ μήκει. ἀσύμμετρος ἄρα ἐστὶ καὶ ἡ ΖΗ τῇ ΕΖ μήκει. αἱ ΗΖ, ΖΕ ἄρα ῥηταί [εἰσι] δυνάμει μόνον σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΕΗ. ἀλλὰ καὶ ῥητή· ὅπερ ἐστὶν ἀδύνατον.

Ἡ ἄρα ἀποτομὴ οὐκ ἔστιν ἡ αὐτὴ τῇ ἐκ δύο ὀνομάτων· ὅπερ ἔδει δεῖξαι.

## Proposition 111

An apotome is not the same as a binomial.




Let $AB$ be an apotome. I say that $AB$ is not the same as a binomial.

For, if possible, let it be (the same). And let a rational (straight-line) $DC$ be laid down. And let the rectangle $CE$, equal to the (square) on $AB$, have been applied to $CD$, producing $DE$ as breadth. Therefore, since $AB$ is an apotome, $DE$ is a first apotome [Prop. 10.97]. Let $EF$ be an attachment to it. Thus, $DF$ and $FE$ are rational (straight-lines which are) commensurable in square only, and the square on $DF$ is greater than (the square on) $FE$ by the (square) on (some straight-line) commensurable (in length) with ($DF$), and $DF$ is commensurable in length with the (previously) laid down rational (straight-line) $DC$ [Def. 10.10]. Again, since $AB$ is a binomial, $DE$ is thus a first binomial [Prop. 10.60]. Let ($DE$) have been divided into its (component) terms at $G$, and let $DG$ be the greater term. Thus, $DG$ and $GE$ are rational (straight-lines which are) commensurable in square only, and the square on $DG$ is greater than (the square on) $GE$ by the (square) on (some straight-line) commensurable (in length) with ($DG$), and the greater (term) $DG$ is commensurable in length with the (previously) laid down rational (straight-line) $DC$ [Def. 10.5]. Thus, $DF$ is also commensurable in length with $DG$ [Prop. 10.12]. The remainder $GF$ is thus commensurable in length with $DF$ [Prop. 10.15]. [Therefore, since $DF$ is commensurable with $GF$, and $DF$ is rational, $GF$ is thus also rational. Therefore, since $DF$ is commensurable in length with $GF$,] $DF$ (is) incommensurable in length with $EF$. Thus, $FG$ is also incommensurable in length with $EF$ [Prop. 10.13]. $GF$ and $FE$ [are] thus rational (straight-lines which are) commensurable in square only. Thus,

$EG$ is an apotome [Prop. 10.73]. But, (it is) also rational. The very thing is impossible.

Thus, an apotome is not the same as a binomial. (Which is) the very thing it was required to show.

## [Πόρισμα.]

Ἡ ἀποτομὴ καὶ αἱ μετʼ αὐτὴν ἄλογοι οὔτε τῇ μέσῃ οὔτε ἀλλήλαις εἰσὶν αἱ αὐταί.

Τὸ μὲν γὰρ ἀπὸ μέσης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ῥητὴν καὶ ἀσύμμετρον τῇ, παρʼ ἣν παράκειται, μήκει, τὸ δὲ ἀπὸ ἀποτομῆς παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν πρώτην, τὸ δὲ ἀπὸ μέσης ἀποτομῆς πρώτης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν δευτέραν, τὸ δὲ ἀπὸ μέσης ἀποτομῆς δευτέρας παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν τρίτην, τὸ δὲ ἀπὸ ἐλάσσονος παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν τετάρτην, τὸ δὲ ἀπὸ τῆς μετὰ ῥητοῦ μέσον τὸ ὅλον ποιούσης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν πέμπτην, τὸ δὲ ἀπὸ τῆς μετὰ μέσου μέσον τὸ ὅλον ποιούσης παρὰ ῥητὴν παραβαλλόμενον πλάτος ποιεῖ ἀποτομὴν ἕκτην. ἐπεὶ οὖν τὰ εἰρημένα πλάτη διαφέρει τοῦ τε πρώτου καὶ ἀλλήλων, τοῦ μὲν πρώτου, ὅτι ῥητή ἐστιν, ἀλλήλων δὲ, ἐπεὶ τῇ τάξει οὐκ εἰσὶν αἱ αὐταί, δῆλον, ὡς καὶ αὐταὶ αἱ ἄλογοι διαφέρουσιν ἀλλήλων. καὶ ἐπεὶ δέδεικται ἡ ἀποτομὴ οὐκ οὖσα ἡ αὐτὴ τῇ ἐκ δύο ὀνομάτων, ποιοῦσι δὲ πλάτη παρὰ ῥητὴν παραβαλλόμεναι αἱ μετὰ τὴν ἀποτομὴν ἀποτομὰς ἀκολούθως ἑκάστη τῇ τάξει τῇ καθʼ αὑτήν, αἱ δὲ μετὰ τὴν ἐκ δύο ὀνομάτων τὰς ἐκ δύο ὀνομάτων καὶ αὐταὶ τῇ τάξει ἀκολούθως, ἕτεραι ἄρα εἰσὶν αἱ μετὰ τὴν ἀποτομὴν καὶ ἕτεραι αἱ μετὰ τὴν ἐκ δύο ὀνομάτων, ὡς εἶναι τῇ τάξει πάσας ἀλόγους $\overline{\iota\gamma}$,

## [Corollary]

The apotome and the irrational (straight-lines) after it are neither the same as a medial (straight-line) nor (the same) as one another.

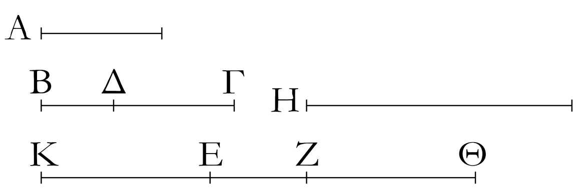
For the (square) on a medial (straight-line), applied to a rational (straight-line), produces as breadth a rational (straight-line which is) incommensurable in length with the (straight-line) to which (the area) is applied [Prop. 10.22]. And the (square) on an apotome, applied to a rational (straight-line), produces as breadth a first apotome [Prop. 10.97]. And the (square) on a first apotome of a medial (straight-line), applied to a rational (straight-line), produces as breadth a second apotome [Prop. 10.98]. And the (square) on a second apotome of a medial (straight-line), applied to a rational (straight-line), produces as breadth a third apotome [Prop. 10.99]. And (square) on a minor (straight-line), applied to a rational (straight-line), produces as breadth a fourth apotome [Prop. 10.100]. And (square) on that (straight-line) which with a rational (area) produces a medial whole, applied to a rational (straight-line), produces as breadth a fifth apotome [Prop. 10.101]. And (square) on that (straight-line) which with a medial (area) produces a medial whole, applied to a rational (straight-line), produces as breadth a sixth apotome [Prop. 10.102]. Therefore, since the aforementioned breadths differ from the first (breadth), and from one another—from the first, because it is rational, and from one another since they are not the same in order—clearly, the irrational (straight-lines) themselves also differ from one another. And since it has been shown that an apotome is not the same as a binomial [Prop. 10.111], and (that) the (irrational straight-lines) after the apotome, being applied to a rational (straight-line), produce as breadth, each according to its own (order), apotomes, and (that) the (irrational straight-lines) after the binomial themselves also (produce as breadth), according (to their) order, binomials, the (irrational straight-lines) after the apotome are thus different, and the (irrational straight-lines) after the binomial (are also) different, so that there are, in order, 13 irrational (straight-lines) in all:

Μέσην,
Ἐκ δύο ὀνομάτων,
Ἐκ δύο μέσων πρώτην,
Ἐκ δύο μέσων δευτέραν,
Μείζονα,
Ῥητὸν καὶ μέσον δυναμένην,
Δύο μέσα δυναμένην,
Ἀποτομήν,
Μέσης ἀποτομὴν πρώτην,
Μέσης ἀποτομὴν δευτέραν,
Ἐλάσσονα,
Μετὰ ῥητοῦ μέσον τὸ ὅλον ποιοῦσαν,
Μετὰ μέσου μέσον τὸ ὅλον ποιοῦσαν.

Medial,
Binomial,
First bimedial,
Second bimedial,
Major,
Square-root of a rational plus a medial (area),
Square-root of (the sum of) two medial (areas),
Apotome,
First apotome of a medial,
Second apotome of a medial,
Minor,
That which with a rational (area) produces a medial whole,
That which with a medial (area) produces a medial whole.

## ριβ΄.

Τὸ ἀπὸ ῥητῆς παρὰ τὴν ἐκ δύο ὀνομάτων παραβαλλόμενον πλάτος ποιεῖ ἀποτομήν, ἧς τὰ ὀνόματα σύμμετρά ἐστι τοῖς τῆς ἐκ δύο ὀνομάτων ὀνόμασι καὶ ἔτι ἐν τῷ αὐτῷ λόγῳ, καὶ ἔτι ἡ γινομένη ἀποτομὴ τὴν αὐτὴν ἕξει τάξιν τῇ ἐκ δύο ὀνομάτων.

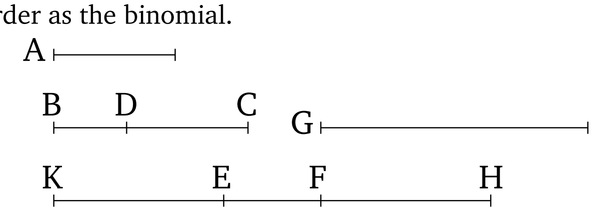



Ἔστω ῥητὴ μὲν ἡ Α, ἐκ δύο ὀνομάτων δὲ ἡ ΒΓ, ἧς μεῖζον ὄνομα ἔστω ἡ ΔΓ, καὶ τῷ ἀπὸ τῆς Α ἴσον ἔστω τὸ ὑπὸ τῶν ΒΓ, ΕΖ· λέγω, ὅτι ἡ ΕΖ ἀποτομή ἐστιν, ἧς τὰ ὀνόματα σύμμετρά ἐστι τοῖς ΓΔ, ΔΒ, καὶ ἐν τῷ αὐτῷ λόγῳ, καὶ ἔτι ἡ ΕΖ τὴν αὐτὴν ἕξει τάξιν τῇ ΒΓ.

Ἔστω γὰρ πάλιν τῷ ἀπὸ τῆς Α ἴσον τὸ ὑπὸ τῶν ΒΔ, Η. ἐπεὶ οὖν τὸ ὑπὸ τῶν ΒΓ, ΕΖ ἴσον ἐστὶ τῷ ὑπὸ τῶν ΒΔ, Η, ἔστιν ἄρα ὡς ἡ ΓΒ πρὸς τὴν ΒΔ, οὕτως ἡ Η πρὸς τὴν ΕΖ. μείζων δὲ ἡ ΓΒ τῆς ΒΔ· μείζων ἄρα ἐστὶ καὶ ἡ Η τῆς ΕΖ. ἔστω τῇ Η ἴση ἡ ΕΘ· ἔστιν ἄρα ὡς ἡ ΓΒ πρὸς τὴν ΒΔ, οὕτως ἡ ΘΕ πρὸς τὴν ΕΖ· διελόντι ἄρα ἐστὶν ὡς ἡ ΓΔ πρὸς τὴν ΒΔ, οὕτως ἡ ΘΖ πρὸς τὴν ΖΕ. γεγονέτω ὡς ἡ ΘΖ πρὸς τὴν ΖΕ, οὕτως ἡ ΖΕ πρὸς τὴν ΚΕ· καὶ ὅλη ἄρα ἡ ΘΚ πρὸς ὅλην τὴν ΚΖ ἐστιν, ὡς ἡ ΖΚ πρὸς ΚΕ· ὡς γὰρ ἓν τῶν ἡγουμένων πρὸς ἓν τῶν ἑπομένων, οὕτως ἅπαντα τὰ ἡγούμενα πρὸς ἅπαντα τὰ ἑπόμενα. ὡς δὲ ἡ ΖΚ πρὸς ΚΕ, οὕτως ἐστὶν ἡ ΓΔ πρὸς τὴν ΔΒ· καὶ ὡς ἄρα ἡ ΘΚ πρὸς ΚΖ, οὕτως ἡ ΓΔ πρὸς τὴν ΔΒ. σύμμετρον δὲ τὸ ἀπὸ τῆς ΓΔ τῷ ἀπὸ τῆς ΔΒ· σύμμετρον ἄρα ἐστὶ καὶ τὸ ἀπὸ τῆς ΘΚ τῷ

## Proposition 112[†]

The (square) on a rational (straight-line), applied to a binomial (straight-line), produces as breadth an apotome whose terms are commensurable (in length) with the terms of the binomial, and, furthermore, in the same ratio. Moreover, the created apotome will have the same order as the binomial.




Let $A$ be a rational (straight-line), and $BC$ a binomial (straight-line), of which let $DC$ be the greater term. And let the (rectangle contained) by $BC$ and $EF$ be equal to the (square) on $A$. I say that $EF$ is an apotome whose terms are commensurable (in length) with $CD$ and $DB$, and in the same ratio, and, moreover, that $EF$ will have the same order as $BC$.

For, again, let the (rectangle contained) by $BD$ and $G$ be equal to the (square) on $A$. Therefore, since the (rectangle contained) by $BC$ and $EF$ is equal to the (rectangle contained) by $BD$ and $G$, thus as $CB$ is to $BD$, so $G$ (is) to $EF$ [Prop. 6.16]. And $CB$ (is) greater than $BD$. Thus, $G$ is also greater than $EF$ [Props. 5.16, 5.14]. Let $EH$ be equal to $G$. Thus, as $CB$ is to $BD$, so $HE$ (is) to $EF$. Thus, via separation, as $CD$ is to $BD$, so $HF$ (is) to $FE$ [Prop. 5.17]. Let it have been contrived that as $HF$ (is) to $FE$, so $FK$ (is) to $KE$. And, thus, the whole $HK$ is to the whole $KF$, as $FK$ (is) to $KE$. For as one of the leading (proportional magnitudes is) to one of the ἀπὸ τῆς ΚΖ. καί ἐστιν ὡς τὸ ἀπὸ τῆς ΘΚ πρὸς τὸ ἀπὸ τῆς ΚΖ, οὕτως ἡ ΘΚ πρὸς τὴν ΚΕ, ἐπεὶ αἱ τρεῖς αἱ ΘΚ, ΚΖ, ΚΕ ἀνάλογόν εἰσιν. σύμμετρος ἄρα ἡ ΘΚ τῇ ΚΕ μήκει. ὥστε καὶ ἡ ΘΕ τῇ ΕΚ σύμμετρός ἐστι μήκει. καὶ ἐπεὶ τὸ ἀπὸ τῆς Α ἴσον ἐστὶ τῷ ὑπὸ τῶν ΕΘ, ΒΔ, ῥητὸν δέ ἐστι τὸ ἀπὸ τῆς Α, ῥητὸν ἄρα ἐστὶ καὶ τὸ ὑπὸ τῶν ΕΘ, ΒΔ. καὶ παρὰ ῥητὴν τὴν ΒΔ παράκειται· ῥητὴ ἄρα ἐστὶν ἡ ΕΘ καὶ σύμμετρος τῇ ΒΔ μήκει· ὥστε καὶ ἡ σύμμετρος αὐτῇ ἡ ΕΚ ῥητή ἐστι καὶ σύμμετρος τῇ ΒΔ μήκει. ἐπεὶ οὖν ἐστιν ὡς ἡ ΓΔ πρὸς ΔΒ, οὕτως ἡ ΖΚ πρὸς ΚΕ, αἱ δὲ ΓΔ, ΔΒ δυνάμει μόνον εἰσὶ σύμμετροι, καὶ αἱ ΖΚ, ΚΕ δυνάμει μόνον εἰσὶ σύμμετροι. ῥητὴ δέ ἐστιν ἡ ΚΕ· ῥητὴ ἄρα ἐστὶ καὶ ἡ ΖΚ. αἱ ΖΚ, ΚΕ ἄρα ῥηταὶ δυνάμει μόνον εἰσὶ σύμμετροι· ἀποτομὴ ἄρα ἐστὶν ἡ ΕΖ.

Ἤτοι δὲ ἡ ΓΔ τῆς ΔΒ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ ἢ τῷ ἀπὸ ἀσυμμέτρου.

Εἰ μὲν οὖν ἡ ΓΔ τῆς ΔΒ μεῖζον δύναται τῷ ἀπὸ συμμέτρου [ἑαυτῇ], καὶ ἡ ΖΚ τῆς ΚΕ μεῖζον δυνήσεται τῷ ἀπὸ συμμέτρου ἑαυτῇ. καὶ εἰ μὲν σύμμετρός ἐστιν ἡ ΓΔ τῇ ἐκκειμένῃ ῥητῇ μήκει, καὶ ἡ ΖΚ· εἰ δὲ ἡ ΒΔ, καὶ ἡ ΚΕ· εἰ δὲ οὐδετέρα τῶν ΓΔ, ΔΒ, καὶ οὐδετέρα τῶν ΖΚ, ΚΕ.

Εἰ δὲ ἡ ΓΔ τῆς ΔΒ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ, καὶ ἡ ΖΚ τῆς ΚΕ μεῖζον δυνήσεται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. καὶ εἰ μὲν ἡ ΓΔ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ μήκει, καὶ ἡ ΖΚ· εἰ δὲ ἡ ΒΔ, καὶ ἡ ΚΕ· εἰ δὲ οὐδετέρα τῶν ΓΔ, ΔΒ, καὶ οὐδετέρα τῶν ΖΚ, ΚΕ· ὥστε ἀποτομή ἐστιν ἡ ΖΕ, ἧς τὰ ὀνόματα τὰ ΖΚ, ΚΕ σύμμετρά ἐστι τοῖς τῆς ἐκ δύο ὀνομάτων ὀνόμασι τοῖς ΓΔ, ΔΒ καὶ ἐν τῷ αὐτῷ λόγῳ, καὶ τὴν αὐτὴν τάξιν ἔχει τῇ ΒΓ· ὅπερ ἔδει δεῖξαι.

following, so all of the leading (magnitudes) are to all of the following [Prop. 5.12]. And as $FK$ (is) to $KE$, so $CD$ is to $DB$ [Prop. 5.11]. And, thus, as $HK$ (is) to $KF$, so $CD$ is to $DB$ [Prop. 5.11]. And the (square) on $CD$ (is) commensurable with the (square) on $DB$ [Prop. 10.36]. The (square) on $HK$ is thus also commensurable with the (square) on $KF$ [Props. 6.22, 10.11]. And as the (square) on $HK$ is to the (square) on $KF$, so $HK$ (is) to $KE$, since the three (straight-lines) $HK$, $KF$, and $KE$ are proportional [Def. 5.9]. $HK$ is thus commensurable in length with $KE$ [Prop. 10.11]. Hence, $HE$ is also commensurable in length with $EK$ [Prop. 10.15]. And since the (square) on $A$ is equal to the (rectangle contained) by $EH$ and $BD$, and the (square) on $A$ is rational, the (rectangle contained) by $EH$ and $BD$ is thus also rational. And it is applied to the rational (straight-line) $BD$. Thus, $EH$ is rational, and commensurable in length with $BD$ [Prop. 10.20]. And, hence, the (straight-line) commensurable (in length) with it, $EK$, is also rational [Def. 10.3], and commensurable in length with $BD$ [Prop. 10.12]. Therefore, since as $CD$ is to $DB$, so $FK$ (is) to $KE$, and $CD$ and $DB$ are (straight-lines which are) commensurable in square only, $FK$ and $KE$ are also commensurable in square only [Prop. 10.11]. And $KE$ is rational. Thus, $FK$ is also rational. $FK$ and $KE$ are thus rational (straight-lines which are) commensurable in square only. Thus, $EF$ is an apotome [Prop. 10.73].

And the square on $CD$ is greater than (the square on) $DB$ either by the (square) on (some straight-line) commensurable, or by the (square) on (some straight-line) incommensurable, (in length) with ($CD$).

Therefore, if the square on $CD$ is greater than (the square on) $DB$ by the (square) on (some straight-line) commensurable (in length) with [$CD$] then the square on $FK$ will also be greater than (the square on) $KE$ by the (square) on (some straight-line) commensurable (in length) with ($FK$) [Prop. 10.14]. And if $CD$ is commensurable in length with a (previously) laid down rational (straight-line), (so) also (is) $FK$ [Props. 10.11, 10.12]. And if $BD$ (is commensurable), (so) also (is) $KE$ [Prop. 10.12]. And if neither of $CD$ or $DB$ (is commensurable), neither also (are) either of $FK$ or $KE$.

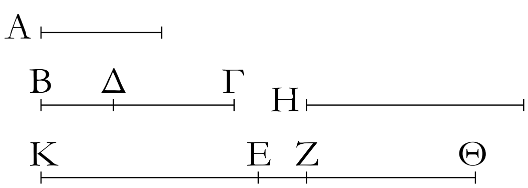
And if the square on $CD$ is greater than (the square on) $DB$ by the (square) on (some straight-line) incommensurable (in length) with ($CD$) then the square on $FK$ will also be greater than (the square on) $KE$ by the (square) on (some straight-line) incommensurable (in length) with ($FK$) [Prop. 10.14]. And if $CD$ is commensurable in length with a (previously) laid down rational (straight-line), (so) also (is) $FK$ [Props. 10.11, 10.12]. And if $BD$ (is commensurable), (so) also (is) $KE$

[Prop. 10.12]. And if neither of $CD$ or $DB$ (is commensurable), neither also (are) either of $FK$ or $KE$. Hence, $FE$ is an apotome whose terms, $FK$ and $KE$, are commensurable (in length) with the terms, $CD$ and $DB$, of the binomial, and in the same ratio. And ($FE$) has the same order as $BC$ [Defs. 10.5—10.10]. (Which is) the very thing it was required to show.

† Heiberg considers this proposition, and the succeeding ones, to be relatively early interpolations into the original text.

## ριγʹ.

Τὸ ἀπὸ ῥητῆς παρὰ ἀποτομὴν παραβαλλόμενον πλάτος ποιεῖ τὴν ἐκ δύο ὀνομάτων, ἧς τὰ ὀνόματα σύμμετρά ἐστι τοῖς τῆς ἀποτομῆς ὀνόμασι καὶ ἐν τῷ αὐτῷ λόγῳ, ἔτι δὲ ἡ γινομένη ἐκ δύο ὀνομάτων τὴν αὐτὴν τάξιν ἔχει τῇ ἀποτομῇ.

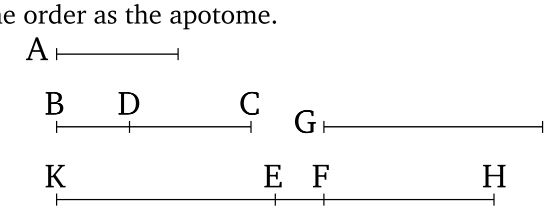



Ἔστω ῥητὴ μὲν ἡ Α, ἀποτομὴ δὲ ἡ ΒΔ, καὶ τῷ ἀπὸ τῆς Α ἴσον ἔστω τὸ ὑπὸ τῶν ΒΔ, ΚΘ, ὥστε τὸ ἀπὸ τῆς Α ῥητῆς παρὰ τὴν ΒΔ ἀποτομὴν παραβαλλόμενον πλάτος ποιεῖ τὴν ΚΘ· λέγω, ὅτι ἐκ δύο ὀνομάτων ἐστὶν ἡ ΚΘ, ἧς τὰ ὀνόματα σύμμετρά ἐστι τοῖς τῆς ΒΔ ὀνόμασι καὶ ἐν τῷ αὐτῷ λόγῳ, καὶ ἔτι ἡ ΚΘ τὴν αὐτὴν ἔχει τάξιν τῇ ΒΔ.

Ἔστω γὰρ τῇ ΒΔ προσαρμόζουσα ἡ ΔΓ· αἱ ΒΓ, ΓΔ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι. καὶ τῷ ἀπὸ τῆς Α ἴσον ἔστω καὶ τὸ ὑπὸ τῶν ΒΓ, Η. ῥητὸν δὲ τὸ ἀπὸ τῆς Α· ῥητὸν ἄρα καὶ τὸ ὑπὸ τῶν ΒΓ, Η. καὶ παρὰ ῥητὴν τὴν ΒΓ παραβέβληται· ῥητὴ ἄρα ἐστὶν ἡ Η καὶ σύμμετρος τῇ ΒΓ μήκει. ἐπεὶ οὖν τὸ ὑπὸ τῶν ΒΓ, Η ἴσον ἐστὶ τῷ ὑπὸ τῶν ΒΔ, ΚΘ, ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΓΒ πρὸς ΒΔ, οὕτως ἡ ΚΘ πρὸς Η. μείζων δὲ ἡ ΒΓ τῆς ΒΔ· μείζων ἄρα καὶ ἡ ΚΘ τῆς Η. κείσθω τῇ Η ἴση ἡ ΚΕ· σύμμετρος ἄρα ἐστὶν ἡ ΚΕ τῇ ΒΓ μήκει. καὶ ἐπεί ἐστιν ὡς ἡ ΓΒ πρὸς ΒΔ, οὕτως ἡ ΘΚ πρὸς ΚΕ, ἀναστρέψαντι ἄρα ἐστὶν ὡς ἡ ΒΓ πρὸς τὴν ΓΔ, οὕτως ἡ ΚΘ πρὸς ΘΕ. γεγονέτω ὡς ἡ ΚΘ πρὸς ΘΕ, οὕτως ἡ ΘΖ πρὸς ΖΕ· καὶ λοιπὴ ἄρα ἡ ΚΖ πρὸς ΖΘ ἐστιν, ὡς ἡ ΚΘ πρὸς ΘΕ, τουτέστιν [ὡς] ἡ ΒΓ πρὸς ΓΔ. αἱ δὲ ΒΓ, ΓΔ δυνάμει μόνον [εἰσὶ] σύμμετροι· καὶ αἱ ΚΖ, ΖΘ ἄρα δυνάμει μόνον εἰσὶ σύμμετροι· καὶ ἐπεί ἐστιν ὡς ἡ ΚΘ πρὸς ΘΕ, ἡ ΚΖ πρὸς ΖΘ, ἀλλ᾽ ὡς ἡ ΚΘ πρὸς ΘΕ, ἡ ΘΖ πρὸς ΖΕ, καὶ ὡς ἄρα ἡ ΚΖ πρὸς ΖΘ, ἡ ΘΖ πρὸς ΖΕ· ὥστε καὶ ὡς ἡ πρώτη πρὸς τὴν τρίτην, τὸ ἀπὸ τῆς πρώτης πρὸς τὸ ἀπὸ τῆς δευτέρας· καὶ ὡς ἄρα ἡ ΚΖ πρὸς ΖΕ, οὕτως τὸ ἀπὸ τῆς ΚΖ πρὸς τὸ ἀπὸ τῆς ΖΘ. σύμμετρον δέ ἐστι τὸ ἀπὸ τῆς ΚΖ τῷ ἀπὸ τῆς ΖΘ· αἱ γὰρ ΚΖ, ΖΘ δυνάμει εἰσὶ σύμμετροι· σύμμετρος ἄρα ἐστὶ καὶ ἡ ΚΖ τῇ ΖΕ μήκει· ὥστε ἡ ΚΖ καὶ

## Proposition 113

The (square) on a rational (straight-line), applied to an apotome, produces as breadth a binomial whose terms are commensurable with the terms of the apotome, and in the same ratio. Moreover, the created binomial has the same order as the apotome.




Let $A$ be a rational (straight-line), and $BD$ an apotome. And let the (rectangle contained) by $BD$ and $KH$ be equal to the (square) on $A$, such that the square on the rational (straight-line) $A$, applied to the apotome $BD$, produces $KH$ as breadth. I say that $KH$ is a binomial whose terms are commensurable with the terms of $BD$, and in the same ratio, and, moreover, that $KH$ has the same order as $BD$.

For let $DC$ be an attachment to $BD$. Thus, $BC$ and $CD$ are rational (straight-lines which are) commensurable in square only [Prop. 10.73]. And let the (rectangle contained) by $BC$ and $G$ also be equal to the (square) on $A$. And the (square) on $A$ (is) rational. The (rectangle contained) by $BC$ and $G$ (is) thus also rational. And it has been applied to the rational (straight-line) $BC$. Thus, $G$ is rational, and commensurable in length with $BC$ [Prop. 10.20]. Therefore, since the (rectangle contained) by $BC$ and $G$ is equal to the (rectangle contained) by $BD$ and $KH$, thus, proportionally, as $CB$ is to $BD$, so $KH$ (is) to $G$ [Prop. 6.16]. And $BC$ (is) greater than $BD$. Thus, $KH$ (is) also greater than $G$ [Prop. 5.16, 5.14]. Let $KE$ be made equal to $G$. $KE$ is thus commensurable in length with $BC$. And since as $CB$ is to $BD$, so $HK$ (is) to $KE$, thus, via conversion, as $BC$ (is) to $CD$, so $KH$ (is) to $HE$ [Prop. 5.19 corr.]. Let it have been contrived that as $KH$ (is) to $HE$, so $HF$ (is) to $FE$. And thus the remainder $KF$ is to $FH$, as $KH$ (is) to $HE$—that is to say, [as] $BC$ (is) to $CD$ [Prop. 5.19]. And $BC$ and $CD$ [are] commensurable in square only.

τῇ ΚΕ σύμμετρός [ἐστι] μήκει. ῥητὴ δέ ἐστιν ἡ ΚΕ καὶ σύμμετρος τῇ ΒΓ μήκει. ῥητὴ ἄρα καὶ ἡ ΚΖ καὶ σύμμετρος τῇ ΒΓ μήκει. καὶ ἐπεί ἐστιν ὡς ἡ ΒΓ πρὸς ΓΔ, οὕτως ἡ ΚΖ πρὸς ΖΘ, ἐναλλὰξ ὡς ἡ ΒΓ πρὸς ΚΖ, οὕτως ἡ ΔΓ πρὸς ΖΘ. σύμμετρος δὲ ἡ ΒΓ τῇ ΚΖ· σύμμετρος ἄρα καὶ ἡ ΖΘ τῇ ΓΔ μήκει. αἱ ΒΓ, ΓΔ δὲ ῥηταί εἰσι δυνάμει μόνον σύμμετροι· καὶ αἱ ΚΖ, ΖΘ ἄρα ῥηταί εἰσι δυνάμει μόνον σύμμετροι· ἐκ δύο ὀνομάτων ἐστὶν ἄρα ἡ ΚΘ.

Εἰ μὲν οὖν ἡ ΒΓ τῆς ΓΔ μεῖζον δύναται τῷ ἀπὸ συμμέτρου ἑαυτῇ, καὶ ἡ ΚΖ τῆς ΖΘ μεῖζον δυνήσεται τῷ ἀπὸ συμμέτρου ἑαυτῇ. καὶ εἰ μὲν σύμμετρός ἐστιν ἡ ΒΓ τῇ ἐκκειμένῃ ῥητῇ μήκει, καὶ ἡ ΚΖ, εἰ δὲ ἡ ΓΔ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ μήκει, καὶ ἡ ΖΘ, εἰ δὲ οὐδετέρα τῶν ΒΓ, ΓΔ, οὐδετέρα τῶν ΚΖ, ΖΘ.

Εἰ δὲ ἡ ΒΓ τῆς ΓΔ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ, καὶ ἡ ΚΖ τῆς ΖΘ μεῖζον δυνήσεται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. καὶ εἰ μὲν σύμμετρός ἐστιν ἡ ΒΓ τῇ ἐκκειμένῃ ῥητῇ μήκει, καὶ ἡ ΚΖ, εἰ δὲ ἡ ΓΔ, καὶ ἡ ΖΘ, εἰ δὲ οὐδετέρα τῶν ΒΓ, ΓΔ, οὐδετέρα τῶν ΚΖ, ΖΘ.

Ἐκ δύο ἄρα ὀνομάτων ἐστὶν ἡ ΚΘ, ἧς τὰ ὀνόματα τὰ ΚΖ, ΖΘ σύμμετρά [ἐστι] τοῖς τῆς ἀποτομῆς ὀνόμασι τοῖς ΒΓ, ΓΔ καὶ ἐν τῷ αὐτῷ λόγῳ, καὶ ἔτι ἡ ΚΘ τῇ ΒΓ τὴν αὐτὴν ἕξει τάξιν· ὅπερ ἔδει δεῖξαι.

$KF$ and $FH$ are thus also commensurable in square only [Prop. 10.11]. And since as $KH$ is to $HE$, (so) $KF$ (is) to $FH$, but as $KH$ (is) to $HE$, (so) $HF$ (is) to $FE$, thus, also as $KF$ (is) to $FH$, (so) $HF$ (is) to $FE$ [Prop. 5.11]. And hence as the first (is) to the third, so the (square) on the first (is) to the (square) on the second [Def. 5.9]. And thus as $KF$ (is) to $FE$, so the (square) on $KF$ (is) to the (square) on $FH$. And the (square) on $KF$ is commensurable with the (square) on $FH$. For $KF$ and $FH$ are commensurable in square. Thus, $KF$ is also commensurable in length with $FE$ [Prop. 10.11]. Hence, $KF$ [is] also commensurable in length with $KE$ [Prop. 10.15]. And $KE$ is rational, and commensurable in length with $BC$. Thus, $KF$ (is) also rational, and commensurable in length with $BC$ [Prop. 10.12]. And since as $BC$ is to $CD$, (so) $KF$ (is) to $FH$, alternately, as $BC$ (is) to $KF$, so $DC$ (is) to $FH$ [Prop. 5.16]. And $BC$ (is) commensurable (in length) with $KF$. Thus, $FH$ (is) also commensurable in length with $CD$ [Prop. 10.11]. And $BC$ and $CD$ are rational (straight-lines which are) commensurable in square only. $KF$ and $FH$ are thus also rational (straight-lines which are) commensurable in square only [Def. 10.3, Prop. 10.13]. Thus, $KH$ is a binomial [Prop. 10.36].

Therefore, if the square on $BC$ is greater than (the square on) $CD$ by the (square) on (some straight-line) commensurable (in length) with ($BC$), then the square on $KF$ will also be greater than (the square on) $FH$ by the (square) on (some straight-line) commensurable (in length) with ($KF$) [Prop. 10.14]. And if $BC$ is commensurable in length with a (previously) laid down rational (straight-line), (so) also (is) $KF$ [Prop. 10.12]. And if $CD$ is commensurable in length with a (previously) laid down rational (straight-line), (so) also (is) $FH$ [Prop. 10.12]. And if neither of $BC$ or $CD$ (are commensurable), neither also (are) either of $KF$ or $FH$ [Prop. 10.13].

And if the square on $BC$ is greater than (the square on) $CD$ by the (square) on (some straight-line) incommensurable (in length) with ($BC$) then the square on $KF$ will also be greater than (the square on) $FH$ by the (square) on (some straight-line) incommensurable (in length) with ($KF$) [Prop. 10.14]. And if $BC$ is commensurable in length with a (previously) laid down rational (straight-line), (so) also (is) $KF$ [Prop. 10.12]. And if $CD$ is commensurable, (so) also (is) $FH$ [Prop. 10.12]. And if neither of $BC$ or $CD$ (are commensurable), neither also (are) either of $KF$ or $FH$ [Prop. 10.13].

$KH$ is thus a binomial whose terms, $KF$ and $FH$, [are] commensurable (in length) with the terms, $BC$ and $CD$, of the apotome, and in the same ratio. Moreover,