



Figure 8.21: Pascal

the end of 1642, based on toothed wheels, but difficulties in the production of accurate parts delayed the appearance of the machine until 1645. This was the first working computer. The gear mechanism for addition seems rather obvious to us now, but in Pascal's day it already raised questions of the "Can a machine think?" kind. Pascal himself was sufficiently amazed by the mechanism to say that "the arithmetical machine produces effects which approach nearer to thought than all the actions of the animal. But it does nothing which would enable us to attribute will to it, as to the animals" (*Pascal, Pensées*, 340). The machine greatly impressed the French chancellor, and Pascal was granted exclusive rights to manufacture and sell it. Whether it was a commercial success is not known, but for a time, at least, Pascal was diverted by the opportunity to cash in on his ideas.

The direction of Pascal's life began to shift away from such worldly concerns in 1646, when his father was treated for a leg injury by two local bonesetters. The bonesetters were Jansenists, then a fast-growing sect within the Catholic church. Their influence resulted in the conversion of the whole family to Jansenism, and Pascal began to devote more time to religious thought. For some years, though, he continued with scientific work.

In 1647 he investigated the variation of barometric pressure with altitude, resulting in his *New Experiments Concerning the Vacuum*, published the same year; in 1651 he did pioneering work in hydrostatics, resulting in his *Great Experiment Concerning the Equilibrium of Fluids*, published in 1663; and in 1654 he investigated the so-called Pascal's triangle, making fundamental contributions to number theory, combinatorics, and probability theory (for more on this, see Chapter 11). In 1654 Pascal experienced a “second conversion,” which led to his almost complete withdrawal from the world and science and his increasing commitment to the Jansenist cause. Only in 1658 and 1659 did he concentrate at times on mathematics (on one occasion, so the story goes, to take his mind off the pain of a toothache). His favorite topic at this stage was the cycloid, the curve generated by a point on the circumference of a circle that rolls on a straight line. Later in the seventeenth century the cycloid became important in the development of mechanics and differential geometry (see Chapters 13 and 17).

Mathematicians are of course very sorry about Pascal's withdrawal from mathematics at an early age; however, it was not just religion that gained from Pascal's conversion. The *Provincial Letters*, which he wrote to promote Jansenist ideas, and his *Pensées*, which were edited by the Jansenists after his death, became classics of French literature. Undoubtedly Pascal is the only great mathematician whose standing is equally great among writers. Moreover, his devotion to the Jansenist ideal of serving the needy had one enduring practical consequence: his idea of a public transport system. Shortly before his death in 1662, Pascal saw the inauguration of the world's first omnibus service. Coaches could be taken from the Porte Sainte-Antoine to the Luxembourg in Paris for 5 sous, with profits being directed to the relief of the poor.

# 9

# Calculus

## 9.1 What Is Calculus?

Calculus emerged in the seventeenth century as a system of shortcuts to results obtained by the method of exhaustion and as a method for discovering such results. The types of problem for which calculus proved suitable were finding lengths, areas, and volumes of curved figures and determining local properties such as tangents, normals, and curvature—in short, what we now recognize as problems of integration and differentiation. Equivalent problems of course arise in mechanics, where one of the dimensions is time instead of distance, hence it was calculus that made mathematical physics possible—a development we shall consider in Chapter 13. In addition, calculus was intimately connected with the theory of infinite series, initiating developments that became fundamental in number theory, combinatorics, and probability theory.

The extraordinary success of calculus was possible, in the first instance, because it replaced long and subtle exhaustion arguments by short routine calculations. As the name suggests, calculus consists of *rules for calculating* results, not their logical justification. Mathematicians of the seventeenth century were familiar with the method of exhaustion and assumed they could always fall back on it if their results were challenged, but the flood of new results became so great that there was seldom time to do so. As Huygens (1659a), p. 337, wrote,

Mathematicians will never have enough time to read all the discoveries in Geometry (a quantity which is increasing from