Remarks. (1) Fractions can also be expanded in any base, i.e., they can be represented in the form $(d_{k-1}d_{k-2}\cdots d_1 d_0.d_{-1}d_{-2}\cdots)_b$. (2) When $b > 10$ it is customary to use letters for the digits beyond 9. One could also use letters for *all* of the digits.

Example 1. (a) $(11001001)_2 = 201$.

(b) When $b = 26$ let us use the letters A—Z for the digits 0—25, respectively. Then $(BAD)_{26}$=679, whereas $(B.AD)_{26} = 1\frac{3}{676}$.

Example 2. Multiply 160 and 199 in the base 7. **Solution:**

$$\begin{array}{r} 316 \\ \underline{403} \\ 1254 \\ \underline{16030} \\ 161554 \end{array}$$

Example 3. Divide $(11001001)_2$ by $(100111)_2$, and divide $(HAPPY)_{26}$ by $(SAD)_{26}$.

Solution:

$$\begin{array}{r} 101\ \frac{110}{100111} \\ 100111\,|\overline{11001001} \\ \underline{100111} \\ 101101 \\ \underline{100111} \\ 110 \end{array} \qquad \begin{array}{r} KD\ \frac{MLP}{SAD} \\ SAD\,|\overline{HAPPY} \\ \underline{GYBE} \\ COLY \\ \underline{CCAJ} \\ MLP \end{array}$$

Example 4. Convert 10^6 to the bases 2, 7 and 26 (using the letters A—Z as digits in the latter case).

Solution. To convert a number n to the base b, one first gets the last digit (the ones' place) by dividing n by b and taking the remainder. Then replace n by the quotient and repeat the process to get the second-to-last digit d_1, and so on. Here we find that

$$10^6 = (11110100001001000000)_2 = (11333311)_7 = (CEXHO)_{26}.$$

Example 5. Convert $\pi = 3.1415926\cdots$ to the base 2 (carrying out the computation 15 places to the right of the point) and to the base 26 (carrying out 3 places to the right of the point).

Solution. After taking care of the integer part, the fractional part is converted to the base b by multiplying by b, taking the integer part of the result as d_{-1}, then starting over again with the fractional part of what you now have, successively finding d_{-2}, d_{-3}, ... In this way one obtains:

$$3.1415926\cdots = (11.001001000011111\cdots)_2 = (D.DRS\cdots)_{26}.$$