less secure. After receiving several offers from Italy and France, he accepted a position at the Paris Academy in 1787. The change of scene did not appreciably revive his general spirits or enthusiasm for mathematics. Though always welcome at social and scientific gatherings, he was always politely detached, sympathetic but uninvolved. At least it can be said of his detachment that it enabled him to survive the 1789 revolution, which took the lives of his more committed friends Condorcet and Lavoisier. The revolution did in fact stir some activity in Lagrange. In 1790 he became a member of the commission on weights and measures, which introduced the metric system now used universally in science. An interesting glimpse of mathematics during the revolution, in the form of a "panel discussion" between Lagrange, Laplace, and members of a student audience, may be found in Dedron and Itard (1973), pp. 302–310.

In 1792 Lagrange married Renée-Françoise-Adelaïde Le Monnier, the teenaged daughter of an astronomer colleague. His interest in life and mathematics revived, and even in his seventies he made some brilliant contributions to celestial mechanics, which he incorporated in a second edition of the *Mécanique analytique*. When he died in 1813 he was buried in the Pantheon in Paris.

Lagrange is known for an uncompromisingly formal approach to analysis and mechanics. He viewed all functions as power series and attempted to reduce all mechanics to the analysis of such functions, without use of geometry. He was proud of the fact that the *Mécanique analytique* contained no diagrams. His fear that mathematics would have to be abandoned "if new veins were not discovered" was of course unfounded, but understandable as an admission of the limitations of his own approach. The great advances of nineteenth-century analysis were due, more than anything else, to a revival of geometry. In particular, Lagrange's own view of functions as power series became intelligible only in the domain of complex functions, when it emerged from the geometric theory of complex integration discovered by Gauss and Cauchy.

Augustin-Louis Cauchy (Figure 16.5) was born in Paris in 1789, only weeks after the storming of the Bastille, but he was anything but a child of the revolution. His father, Louis-François, was a lawyer and government official who fled to Paris with his wife, Marie-Madeleine Desestre, during the Terror. Augustin-Louis was the first of their six children. Throughout his life Cauchy was to hold extreme antirevolutionary and proroyalist views. The family settled in the village of Arcueil, and Cauchy received his early education from his father. He also had the benefit of contact with famous scientists who came to visit Laplace, who was a neighbor. Lagrange is said to have predicted that Cauchy would become a scientific genius but advised his father not to show him a mathematics book before he was 17.

Figure 16.5: Augustin-Louis Cauchy

As Napoleon took power at the end of the eighteenth century, Cauchy's father returned to government service, and the family moved back to Paris. Cauchy concentrated on classics in secondary school, which he completed in 1804, but then gravitated toward a scientific career. He entered the École Polytechnique in 1805, transferred to the École des Ponts in 1807, and began working as an engineer around 1809. In 1810 he went to Cherbourg to help in the construction of Napoleon's naval base, carrying with him, so it was said, Laplace's *Mécanique céleste* and Lagrange's *Traité des fonctions analytiques*.

His first important mathematical work was the solution of a problem posed to him by Lagrange: to show that any convex polyhedron is rigid. (More precisely, to show that the dihedral angles of a convex polyhedron are uniquely determined by its faces.) An accessible proof of his result, which deserves to be better known, is in Lyusternik (1966). Cauchy's theorem partially settled a conjecture of Euler that any closed surface is rigid, and was in fact the best positive result obtainable, as Connelly (1977) has found a nonconvex polyhedron which is *not* rigid. Cauchy's second major discovery was his proof, in 1812, of Fermat's conjecture that every integer is the sum of at most n n-agonal numbers (see Section 3.2).

The Cauchy integral theorem, submitted to the French Academy in 1814, carried him into the mathematical mainstream. He also managed to catch the political tide, which was turning royalist again, and became member of the Academy on the expulsion of some republican members in 1816. At the same time he became professor at the École Polytechnique, where the 1820s saw the publication of his classic analysis texts and also one of his most important creations, the theory of elasticity. He also gained additional chairs at the Sorbonne and the Collège de France. He and Aloïse de Bure were married in 1818, and they had two daughters.

The mild revolution of 1830, which replaced the Bourbon King Charles X with the Orléans King Louis-Phillipe, was a catastrophe in Cauchy's view. From principles that were curious, though certainly firmly held, Cauchy refused to take the oath of allegiance to the new king. This meant he had to resign his chairs, but Cauchy went further than this—he left his family and followed the old king into exile. He did not return to Paris until 1838, and it was another 10 years before he regained one of his former chairs. Ironically, he had the revolution to thank for this, because it abolished the oath of allegiance. He returned to the Sorbonne and kept up a steady flow of mathematical papers until his death in 1857.

17

Differential Geometry

17.1 Transcendental Curves

We saw in Chapter 9 that the development of calculus in the seventeenth century was greatly stimulated by problems in the geometry of curves. Differentiation grew out of methods for the construction of tangents, and integration grew out of attempts to find areas and arc lengths. Calculus not only unlocked the secrets of the classical curves and of the algebraic curves defined by Descartes; it also extended the concept of curve itself. Once it became possible to handle slopes, lengths, and areas with precision, it also became possible to use these quantities to define new, nonalgebraic curves. These were the curves called "mechanical" by Descartes (Sections 7.3 and 13.3) and "transcendental" by Leibniz. In contrast to algebraic curves, which could be studied in some depth by purely algebraic methods, transcendental curves were inseparable from the methods of calculus. Hence it is not surprising that a new set of geometric ideas, the ideas of "infinitesimal" or *differential* geometry, first emerged from the investigation of transcendental curves.

A more surprising by-product of the investigation of transcendental curves was the first solution of the ancient problem of arc length. The problem was first posed for an algebraic curve, the circle, by the Greeks and in this case it is equivalent to an area problem ("squaring the circle"), since both area and arc length of the circle depend on the evaluation of π. As we now know, π is a transcendental number (Section 2.3), so the arc length problem for the circle has no solution by the elementary means allowed by the Greeks. The first curve whose arc length could be found by

J. Stillwell, *Mathematics and Its History*