$$\left.\begin{array}{l} +n^3a^3 + \beta n^2u^2t + \gamma nutt + \delta t^3 + \varepsilon n^2u^2 + \zeta nut + \eta tt + \theta nu + \iota t + \varkappa \\ -\beta mn^2u^3 - 2\gamma mnu^2t - 3\delta mutt \quad - \zeta mnuu - 2\eta mut - \iota mu \\ +\gamma m^2nu^3 + 3\delta m^2u^2t \qquad\qquad + \eta m^2uu \\ -\delta m^3u^3 \end{array}\right\} = 0$$

Cap X

Hinc pro Linea illa recta Diametri vicem sustinente, si ejus Applicata sub eodem angulo ad Abscissam t ducta vocetur $= v$, erit $3v = \dfrac{-\beta n^2t + 2\gamma mnt - 3\delta m^2t - \varepsilon nn + \zeta mn - \eta mm}{n^3 - \beta mn^2 + \gamma m^2n - \delta m^3}$.

243. Sit jam O intersectio harum duarum Diametrorum, unde ad Axem AZ primo prioribus Applicatis parallela ducatur OP, tum vero posterioribus parallela OQ, eritque $AP = x$, $PO = z$, $AQ = t$ & $OQ = v$. Tum vero erit $z = nv$ & $x = t - mv$, ideoque $v = \dfrac{z}{n}$, & $t = x + \dfrac{m}{n}z$. Primo itaque habetur $3z = -\beta x - \varepsilon$, porroque $3v = \dfrac{-\beta x}{n} - \dfrac{\varepsilon}{n}$ & $t = x - \dfrac{\beta mx}{3n} - \dfrac{\varepsilon m}{3n}$. Substituantur hi valores in æquatione ante inventa, & prodibit TAB. XII. Fig. 45.

$$\left.\begin{array}{l} -\beta nnx + \beta\beta mnx - \beta\gamma mmx + \dfrac{\beta\delta m^3x}{n} \\ -\varepsilon nn + \beta\varepsilon mn - \gamma\varepsilon mm + \dfrac{\delta\varepsilon m^3}{n} \\ +\beta nnx - \dfrac{\beta\beta mnx}{3} - \dfrac{\beta\varepsilon mn}{3} + \varepsilon nn \\ -2\gamma mnx + \dfrac{2\beta\gamma mmx}{3} + \dfrac{2\gamma\varepsilon mm}{3} - \zeta mm \\ +3\delta mmx - \dfrac{\beta\delta m^3x}{n} - \dfrac{\delta\varepsilon m^3}{n} + \eta mm \end{array}\right\} = 0$$

seu

$$\left.\begin{array}{l} \dfrac{2}{3}\beta\beta mnx - \dfrac{1}{3}\beta\gamma mmx - 2\gamma mnx + 3\delta mmx \\ + \dfrac{2}{3}\beta\varepsilon mn - \dfrac{1}{3}\gamma\varepsilon mm - \zeta mn + \eta mm \end{array}\right\} = 0.$$

 244. Pen-

244. Pendet ergo utique interſectio Diametrorum O ab inclinatione Applicatarum ad Axem, quæ litteris m & n continetur; neque idcirco, (ſi interſectionem Diametrorum *Centrum* vocare lubeat,) Lineæ tertii ordinis omnes Centro gaudent. Interim tamen caſus exhiberi poſſunt, quibus Diametrorum interſectio mutua in idem punctum fixum incidat. Fiet ſcilicet hoc, ſi termini per mn & mm affecti ſeorſim nihilo æquales ponantur, ac valores ipſius x inde orituri æquales ſtatuantur. Fiet autem ex his duabus æqualitatibus $x = \frac{3\zeta - 2\beta\varepsilon}{2\beta\beta - 6\gamma} = \frac{3\eta - \gamma\varepsilon}{\beta\gamma - 9\delta}$; qui duo valores ut congruant, neceſſe eſt ut ſit

$$6\beta\beta\eta - 2\beta\beta\gamma\varepsilon - 18\gamma\eta + 6\gamma\gamma\varepsilon = 3\beta\gamma\zeta - 2\beta\beta\gamma\varepsilon - 27\delta\zeta + 18\beta\delta\varepsilon,$$

ſeu

$$\beta\gamma\zeta - 2\beta\beta\eta - 9\delta\zeta + 6\gamma\eta + 6\beta\delta\varepsilon - 2\gamma\gamma\varepsilon = 0,$$

unde fit $\eta = \frac{\beta\gamma\zeta - 9\delta\zeta + 6\beta\delta\varepsilon - 2\gamma\gamma\varepsilon}{2\beta\beta - 6\gamma}$. Quoties ergo η hujuſmodi habuerit valorem, toties omnes Diametri ſe mutuo in uno eodemque puncto interſecant; ideoque hæ Lineæ tertii ordinis Centro gaudebunt, quod reperietur ſumendo in Axe.

$$AP = \frac{3\zeta - 2\beta\varepsilon}{2\beta\beta - 6\gamma}, \text{ \&}$$

$$PO = \frac{-3\beta\zeta + 6\gamma\varepsilon}{2\beta\beta - 6\gamma}.$$

245. Hæc eadem Centri determinatio, ſi quod datur, locum habet ſi pro primo coëfficiente α non ponatur unitas. Si enim propoſita fuerit æquatio generaliſſima pro Lineis tertii ordinis

$$\alpha y^3 + \beta y^2 x + \gamma y x^2 + \delta x^3 + \varepsilon yy + \zeta xy + \eta xx + \theta y + \iota x + \kappa = 0,$$

hæ Curvæ Centro erunt præditæ, ſi fuerit

$\eta = \frac{\beta\gamma\zeta - 9\alpha\delta\zeta + 6\beta\delta\varepsilon - 2\gamma\gamma\varepsilon}{2\beta\beta - 6\alpha\gamma}$. Tum vero Centrum

erit

erit in O, existente $AP = \frac{3\alpha\zeta - 2\beta\varepsilon}{2\beta\beta - 6\alpha\gamma}$ & $PO = \frac{6\gamma\varepsilon - 3\beta\zeta}{2\beta\beta - 6\alpha\gamma}$. CAP. X

Quare, si unica Ordinata Curvam in tribus punctis secans ita dividatur, ut binæ Applicatæ ad unam partem sitæ æquentur tertiæ ad alteram partem jacenti, tum recta per Centrum & hoc divisionis punctum ducta, omnes alias Ordinatas illi parallelas similiter secabit.

246. Si hæc ad æquationes Specierum supra enumeratarum accommodentur, patebit Species primam, secundam, tertiam, quartam & quintam Centro gaudere, si modo sit $a = 0$; hocque casu Centrum in ipso Abscissarum initio esse positum. Species sexta & septima Centro prorsus carent, quia coëfficiens a abesse nequit. Species vero octava, nona, decima, undecima, duodecima & decima-tertia Centrum habent, semper in Abscissarum initio positum. In Speciebus decima-quarta, decima-quinta & decima-sexta Centrum infinite distat, ideoque omnes illæ Lineæ Triametri inter se erunt parallelæ.

247. His de summa trium cujusque Applicatæ valorum notatis, contemplemur eorundem productum, quoniam de rectangulorum aggregato nihil admodum notatu dignum reperitur. Erit ergo ex æquatione generali §.239. $-PM.PL.PN = -\delta x^3 - \eta xx - \iota x - \kappa$: ad quam expressionem explicandam ad hoc attendamus, quod si ponatur $y = 0$, fiat $\delta x^3 + \eta xx + \iota x + \kappa = 0$, cujus propterea æquationis radices dabunt Axis AZ & Curvæ intersectiones. Quæ si sint in punctis B, C, & D erit $\delta x^3 + \eta xx + \iota x + \kappa = \delta(x - AB)(x - AC)(x - AD)$; quapropter erit $PL.PM.PN = \delta.PB.PC.PD$; ideoque, sumta alia quacunque Ordinata lmn priori parallela, erit $PL.PM.PN : PB.PC.PD = pl.pm.pn : pB.pC.pD$; quæ proprietas omnino similis est illi, quam supra pro Lineis secundi ordinis ratione rectangulorum invenimus; atque similis proprietas in Lineas quarti, quinti, & superiorum ordinum competet.

248. Habeat nunc Linea tertii ordinis tres quoque Asymtotas rectas FBf, GDg, HCh. Quoniam ipsa Linea tertii TAB. XII. Fig. 46.

 ordinis

Lib. II. ordinis in has tres Aſymtotas abit, ſi æquatio pro Curva reſolubilis fiat in tres Factores ſimplices formæ $py + qx + r$; pro Aſymtotis, tanquam Linea complexa, peculiaris æquatio exhiberi poterit, cujus ſupremum membrum conveniet cum ſupremo membro pro Curva. Deinde vero, quia Aſymtotarum poſitio ex ſecundo æquationis membro determinatur, æquatio pro Aſymtotis & æquatio pro Curva ſecundum quoque membrum commune habebunt. Quare, ſi pro Curva ad Axem AP relata hæc fuerit æquatio inter Abſciſſam $AP = x$, & Applicatam $PM = y$,

$$y^3 + (\beta x + \varepsilon)y^2 + (\gamma xx + \zeta x + \theta)y + \delta x^3 + \eta xx + \iota x + \kappa = 0.$$

Pro Aſymtotis ad eundem Axem AP relatis ſequens habebitur æqu tio inter Abſciſſam $AP = x$ & Applicatam $PG = z$

$$z^3 + (\beta x + \varepsilon) z^2 + (\gamma xx + \zeta x + B) z + \delta x^3 + \eta x^2 + Cx + D = 0,$$

in qua coëfficiens ζ, B, C, D ita ſunt comparati, ut æquatio in tres Factores ſimplices reſolubilis evadat.

249. Quod ſi ergo ducatur Applicata quæcunque PN, cum Curvam ſecans in tribus punctis L, M, N, tum etiam Aſymtotas in tribus punctis F, G, H ſecans, erit ex æquatione pro Curva $PL + PM + PN = -\beta x - \varepsilon$. At ex æquatione pro Aſymtotis erit pari modo $PF + PG + PH = -\beta x - \varepsilon$. Hanc ob rem erit $PL + PM + PN = PF + PG + PH$, ſeu $FL - GM + HN = 0$. Atque, ſi alia quæcunque Applicata pf ducatur, erit eodem modo $fn - gm + hl = 0$. Si igitur recta quæcunque cum Curvam tum tres Aſymtotas ſecet in tribus punctis, binæ partes Lineæ inter Aſymtotas & Curvam contentæ quæ ad eandem regionem vergunt, æquales erunt parti in regionem oppoſitam vergenti.

250. In Linea igitur tertii ordinis, quæ tres habet Aſymtotas rectas, tria crura ad has Aſymtotas convergentia non omnia ad eaſdem Aſymtotarum partes poſſunt eſſe diſpoſita : ſed,

ſed, ſi duo ad eandem partem vergant, tertium neceſſario ad oppoſitas tendet. Hanc ob rem hujuſmodi Linea tertii ordinis, qualem figura repræſentat, eſt impoſſibilis, quoniam recta ſecans Aſymtotas in punctis f, g, h, Curvam vero in l, m, n, præbet partes fn, gm, hl in eandem plagam vergentes, quarum ſumma nihilo æqualis eſſe nequit. Partes enim in eandem plagam vergentes obtinent idem ſignum, puta $+$; quæ vero in contrariam plagam tendunt ſignum $-$: unde patet ſummam trium harum partium evaneſcere non poſſe niſi ſignis diverſis ſint præditæ. Cap. X. Tab. XIII. *Fig.* 47.

251. Hinc jam clare perſpicitur ratio cur in Linea tertii ordinis dari nequeant duæ Aſymtotæ rectæ ſpeciei $u = \frac{A}{tt}$, dum tertia Aſymtota ſit ſpeciei $u = \frac{A}{t}$, propterea quod illa crura hyperbolica infinities magis ad ſuam Aſymtotam convergant, quam crus hyperbolicum ſpeciei $u = \frac{A}{t}$. Ponamus enim rectam fl in infinitum removeri, fientque intervalla fn, gm, hl infinite parva. At, ſi rami duo nx, my ponantur ſpeciei $u = \frac{A}{tt}$, tertius vero ramus lz ſpeciei $u = \frac{A}{t}$, tum intervalla fn & gm infinities erunt minora quam intervallum hl, ideoque eſſe nequit $gm = fn + hl$. Tab. XII. *Fig.* 46.

252. In Lineis ergo ſuperiorum ordinum, quæ tot habent Aſymtotas quot dimenſiones, unica Aſymtota ſpeciei $u = \frac{A}{t}$ adeſſe nequit, dum reliquæ ſint ſpecierum ſuperiorum $u = \frac{A}{tt}$, $u = \frac{A}{t^3}$ &c.; ſed, ſi una adſit ſpeciei $u = \frac{A}{t}$, neceſſario & altera adeſſe debet. Ob eandem rationem, ſi Aſymtota ſpeciei $u = \frac{A}{t}$ nulla adſit, fieri non poteſt ut una tantum ſpeciei $u = \frac{A}{tt}$ adſit, ſed ad minimum duæ ad-