rorum in seriei 1, 2, 3 ... $n - 1$ qui tum ipsi tum simul numeri proximi vnitate maiores in $\mathfrak{K}$ continentur; similiter sit $(\mathfrak{K}\mathfrak{K}')$ multitudo numerorum in eadem serie qui ipsi in $\mathfrak{K}$ proxime sequentes vero in $\mathfrak{K}'$ continentur, vnde simul significatio signorum $(\mathfrak{K}\mathfrak{K}'')$, $(\mathfrak{K}'\mathfrak{K})$, $(\mathfrak{K}'\mathfrak{K}')$, $(\mathfrak{K}'\mathfrak{K}'')$, $(\mathfrak{K}''\mathfrak{K})$, $(\mathfrak{K}''\mathfrak{K}')$, $(\mathfrak{K}''\mathfrak{K}'')$ sponte innotescet. Quo facto dico *primo*, fieri $(\mathfrak{K}\mathfrak{K}') = (\mathfrak{K}'\mathfrak{K})$. Supponendo enim, h, h', h'' etc. esse omnes numeros seriei 1, 2, 3 ... $n - 1$ qui ipsi in $\mathfrak{K}$ proxime maiores $h + 1$, $h' + 1$, $h'' + 1$ etc. autem in $\mathfrak{K}'$ continentur, et quorum ideo multitudo $= (\mathfrak{K}\mathfrak{K}')$, manifestum est omnes numeros $n - h - 1$, $n - h' - 1$, $n - h'' - 1$ etc. in $\mathfrak{K}'$ contineri, proxime maiores vero $n - h$, $n - 1'$ etc. in $\mathfrak{K}$; quare quum tales numeri omnino dentur $(\mathfrak{K}'\mathfrak{K})$, certo nequit esse $(\mathfrak{K}'\mathfrak{K}) < (\mathfrak{K}\mathfrak{K}')$, et perinde demonstratur, esse non posse $(\mathfrak{K}\mathfrak{K}') < (\mathfrak{K}'\mathfrak{K})$, quocirca hi numeri necessario aequales erunt. Prorsus eodem modo probatur $(\mathfrak{K}\mathfrak{K}'') = (\mathfrak{K}''\mathfrak{K})$, $(\mathfrak{K}'\mathfrak{K}'') = (\mathfrak{K}''\mathfrak{K}')$. Secundo, quum necessario quemuis numerum ex $\mathfrak{K}$, maximo $n - 1$ excepto, sequi debeat proxime maior vel in $\mathfrak{K}$, vel in $\mathfrak{K}'$ vel in $\mathfrak{K}''$ contentus, summa $(\mathfrak{K}\mathfrak{K}) + (\mathfrak{K}\mathfrak{K}') + (\mathfrak{K}\mathfrak{K}'')$ fiet aequalis multitudini omnium numerorum in $\mathfrak{K}$ vnitate deminutae puta $= m - 1$, et simili ratione erit $(\mathfrak{K}'\mathfrak{K}) + (\mathfrak{K}'\mathfrak{K}') + (\mathfrak{K}'\mathfrak{K}'') = (\mathfrak{K}''\mathfrak{K}) + (\mathfrak{K}''\mathfrak{K}') + (\mathfrak{K}''\mathfrak{K}'') = m$.

His ita praeparatis euoluimus per praecepta art. 345 productum pp' in $(m, \mathfrak{A}' + 1) + (m, \mathfrak{B}' + 1) + (m, \mathfrak{C}' + 1) +$ etc., quam expressionem facile perspicietur reduci ad $(\mathfrak{K}'\mathfrak{K})p + (\mathfrak{K}'\mathfrak{K}')p' + (\mathfrak{K}'\mathfrak{K}'')p''$, et quum per art. 345 I productum

$p'p''$ ex illo oriatur, substituendo pro $(m, 1)$, (m, g), (m, gg) resp. (m, g), (m, gg), (mg^3) i. e. pro p, p', p'' resp. p', p'', p, fiet $p'p'' = (\mathfrak{K}'\mathfrak{K})\,p' + (\mathfrak{K}'\mathfrak{K}')\,p'' + (\mathfrak{K}'\mathfrak{K}'')\,p$, et prorsus simili modo $p''p = (\mathfrak{K}'\mathfrak{K})\,p'' + (\mathfrak{K}'\mathfrak{K}')\,p + (\mathfrak{K}'\mathfrak{K}'')\,p'$. Hinc protinus sequitur *primo* $B = m\,(p + p' + p'') = -m$; *secundo* quum simili ratione, vt antea pp' euolutum est, etiam pp'' ad $(\mathfrak{K}''\mathfrak{K})\,p + (\mathfrak{K}''\mathfrak{K}')\,p' + (\mathfrak{K}''\mathfrak{K}'')\,p''$ reducatur, atque haec expressio cum praecedente identica esse debeat, necessario erit $(\mathfrak{K}''\mathfrak{K}) = (\mathfrak{K}'\mathfrak{K}')$ et $(\mathfrak{K}''\mathfrak{K}'') = (\mathfrak{K}'\mathfrak{K})$. Hinc colligitur, statuendo $(\mathfrak{K}'\mathfrak{K}'') = (\mathfrak{K}''\mathfrak{K}') = a$, $(\mathfrak{K}''\mathfrak{K}'') = (\mathfrak{K}'\mathfrak{K}) = (\mathfrak{K}\mathfrak{K}') = b$, $(\mathfrak{K}'\mathfrak{K}') = (\mathfrak{K}''\mathfrak{K}) = (\mathfrak{K}\mathfrak{K}'') = c$, fieri $m - 1 = (\mathfrak{K}\mathfrak{K}) + (\mathfrak{K}\mathfrak{K}') + (\mathfrak{K}\mathfrak{K}'') = (\mathfrak{K}\mathfrak{K}) + b + c$, atque $a + b + c = m$, vnde $(\mathfrak{K}\mathfrak{K}) = a - 1$, ita vt illae nouem quantitates incognitae ad tres, a, b, c siue potius propter aequationem $a + b + c = m$ ad duas reductae sint. Denique patet, quadratum pp euolui in $(m, 1 + 1) + (m, \mathfrak{A} + 1) + (m, \mathfrak{B} + 1) + (m, \mathfrak{C} + 1) +$ etc.; inter partes huius expressionis reperietur (m, n) quae reducitur ad $(m, 0)$ siue ad m, reliquas vero facile perspicietur reduci ad $(\mathfrak{K}\mathfrak{K})\,p + (\mathfrak{K}\mathfrak{K}')\,p' + (\mathfrak{K}\mathfrak{K}'')\,p''$, vnde habetur $pp = m + (a - 1)\,p + bp' + cp''$.

Hoc itaque modo per disquisitiones praecedentes quatuor hasce reductiones nacti sumus:

$$\begin{aligned}
pp &= m + (a - 1)\,p + bp' + cp''\\
pp' &= bp + cp' + ap''\\
pp'' &= cp + ap' + bp''\\
p'p'' &= ap + bp' + cp''
\end{aligned}$$

vbi inter tres incognitas a, b, c aequatio conditionalis $a + b + c = m \ldots$(I) intercedit, insuperque certum est ipsas esse numeros integros. Hinc colligitur $C = p \times p'p'' = app + bpp' + cpp'' = am + (aa + bb + cc - a)p + (ab + bc + ac)p' + (ab + bc + ac)p''$. At quum $pp'p''$ sit functio inuariabilis aggregatorum p, p', p'', coëfficientes per quos haec in expr. praec. multiplicata sunt necessario aequales erunt (art. 350), vnde habetur aequatio noua $aa + bb + cc - a = ab + bc + ac \ldots$(II), atque hinc $C = am + (ab + bc + ac)(p + p' + p'')$, siue (propter I, et $p + p' + p'' = -1$), $C = aa - bc \ldots$(III). Iam etsi C hic a tribus incognitis pendeat, inter quas duae tantum aequationes habentur, tamen hae, adiumento conditionis ex qua a, b, c sunt integri, ad plenam determinationem ipsius C sufficiunt. Quod vt ostendamus, aequationem II ita exhibemus $12a + 12b + 12c + 4 = 36aa + 36bb + 36cc - 36ab - 36ac - 36bc - 24a + 12b + 12c + 4$; pars prior, per I, fit $= 12m + 4 = 4n$; posterior vero reducitur ad $(6a - 3b - 3c - 2)^2 + 27(b - c)^2$, aut scribendo k pro $2a - b - c$, ad $(3k - 2)^2 + 27(b - c)^2$. Hinc patet, numerum $4n$ (i. e. generaliter quadruplum cuiuslibet primi formae $3n + 1$) per formam $xx + 27yy$ repraesentari posse, quod quidem sine difficultate e theoria generali formarum binariarum deduci potest, attamen satis mirum est, talem discerptionem cum valoribus ipsarum a, b, c cohaerere. At numerus $4n$ semper vnico tantum modo in quadratum et quadratum 27^{plex} discerpi potest,