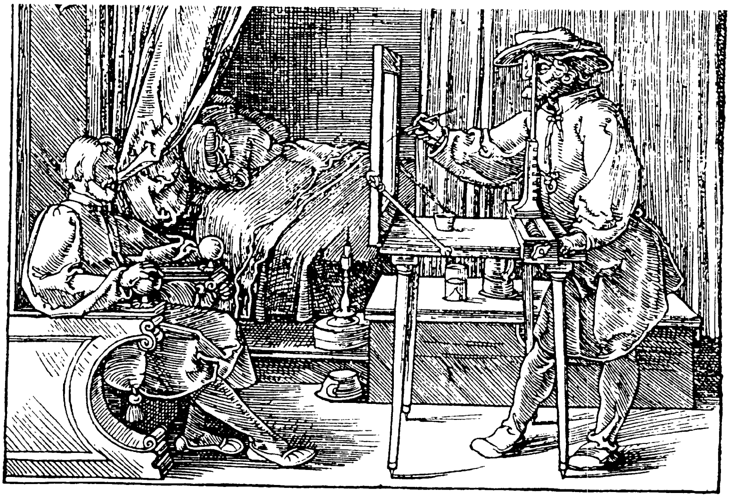

FIGURE 8.10 Dürer woodcut showing how to use Alberti's veil.

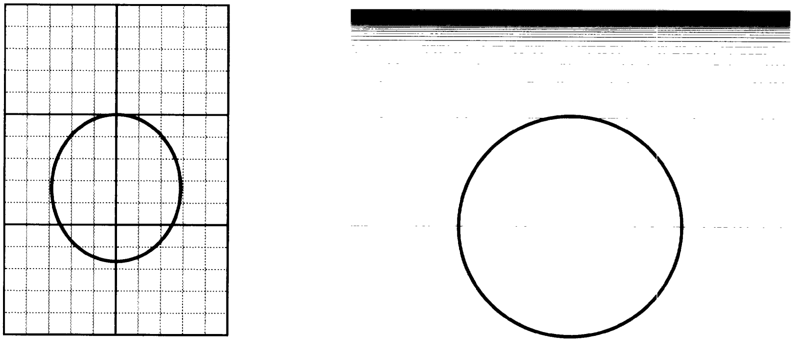

FIGURE 8.11 Ellipse and its perspective view.

Each figure shows two views of the conic section in question. Looking down on the horizontal plane, they appear as they normally do; looking toward the horizon, each looks like a circle, because of the peculiar choice of viewpoint P. The views toward the horizon also include parallel lines in the horizontal plane to establish where the

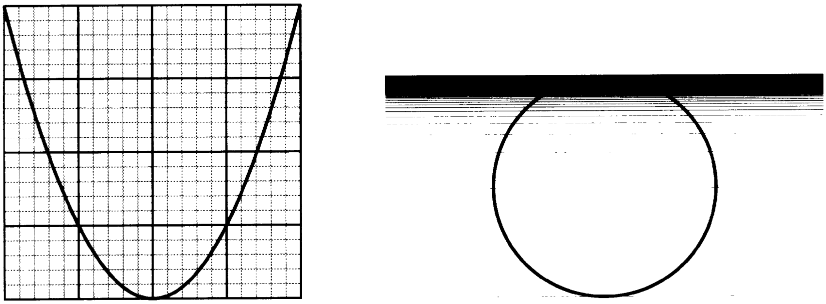

FIGURE 8.12 Parabola and its perspective view.

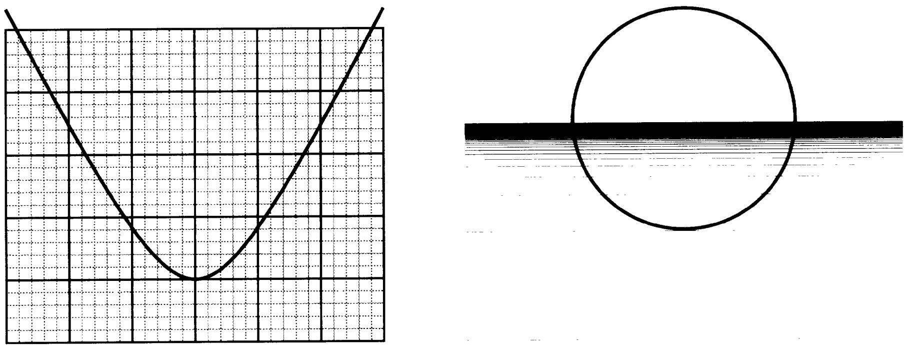

FIGURE 8.13 Hyperbola and its perspective view.

horizon is. These lines are actually equally spaced in the horizontal plane, but of course they appear to "accumulate" at the horizon.

(In the view of the hyperbola, the portion above the horizon corresponds to the other branch of the hyperbola, behind the eye. A real eye, of course, can't see this, but we can imagine it seen by a "mathematically ideal" eye, which can see in all directions.)

The interesting thing about the horizon is that it represents points that *do not belong* to the horizontal plane. These points are aptly called *points at infinity* because they are where actual points appear to go as they move far away. The horizon itself is called the *line at infinity,* and the horizontal plane together with its line at infinity is called the *projective plane. Projective geometry* is the study of the projective plane and the transformations of it (such as projection) that map lines to lines. This geometry is much less discriminating than Euclidean geometry, because it thinks every conic section looks like a circle, but it can distinguish between them very simply once the line at infinity is given:

- an ellipse is a conic section with no points at infinity,
- a parabola is a conic section with one point at infinity,
- a hyperbola is a conic section with two points at infinity.

This classification, which goes back to Kepler, gives another sense in which the hyperbola, ellipse, and parabola are "too much", "too little," and "just right." There is also a neat connection with the other discovery of Kepler (and Newton), that a celestial body travels on a conic section under the influence of the sun's gravity. The conic is a hyperbola if the body has more than enough energy to go to infinity, an ellipse if it has too little energy, and a parabola if the energy is just right.