Undergraduate Texts in Mathematics

Undergraduate Texts in Mathematics

Abbott: Understanding Analysis.
Anglin: Mathematics: A Concise History and Philosophy.
Readings in Mathematics.
Anglin/Lambek: The Heritage of Thales.
Readings in Mathematics.
Apostol: Introduction to Analytic Number Theory. Second edition.
Armstrong: Basic Topology.
Armstrong: Groups and Symmetry.
Axler: Linear Algebra Done Right. Second edition.
Beardon: Limits: A New Approach to Real Analysis.
Bak/Newman: Complex Analysis. Second edition.
Banchoff/Wermer: Linear Algebra Through Geometry. Second edition.
Berberian: A First Course in Real Analysis.
Bix: Conics and Cubics: A Concrete Introduction to Algebraic Curves.
Brémaud: An Introduction to Probabilistic Modeling.
Bressoud: Factorization and Primality Testing.
Bressoud: Second Year Calculus.
Readings in Mathematics.
Brickman: Mathematical Introduction to Linear Programming and Game Theory.
Browder: Mathematical Analysis: An Introduction.
Buchmann: Introduction to Cryptography.
Buskes/van Rooij: Topological Spaces: From Distance to Neighborhood.
Callahan: The Geometry of Spacetime: An Introduction to Special and General Relavitity.
Carter/van Brunt: The Lebesgue–Stieltjes Integral: A Practical Introduction.
Cederberg: A Course in Modern Geometries. Second edition.
Childs: A Concrete Introduction to Higher Algebra. Second edition.
Chung/AitSahlia: Elementary Probability Theory: With Stochastic Processes and an Introduction to Mathematical Finance. Fourth edition.
Cox/Little/O'Shea: Ideals, Varieties, and Algorithms. Second edition.
Croom: Basic Concepts of Algebraic Topology.
Curtis: Linear Algebra: An Introductory Approach. Fourth edition.
Daepp/Gorkin: Reading, Writing, and Proving: A Closer Look at Mathematics.
Devlin: The Joy of Sets: Fundamentals of Contemporary Set Theory. Second edition.
Dixmier: General Topology.
Driver: Why Math?
Ebbinghaus/Flum/Thomas: Mathematical Logic. Second edition.
Edgar: Measure, Topology, and Fractal Geometry.
Elaydi: An Introduction to Difference Equations. Second edition.
Erdős/Surányi: Topics in the Theory of Numbers.
Estep: Practical Analysis in One Variable.
Exner: An Accompaniment to Higher Mathematics.
Exner: Inside Calculus.
Fine/Rosenberger: The Fundamental Theory of Algebra.
Fischer: Intermediate Real Analysis.
Flanigan/Kazdan: Calculus Two: Linear and Nonlinear Functions. Second edition.
Fleming: Functions of Several Variables. Second edition.
Foulds: Combinatorial Optimization for Undergraduates.
Foulds: Optimization Techniques: An Introduction.

(continued after index)

John Stillwell

Mathematics and Its History

Second Edition

Springer

John Stillwell
Department of Mathematics
University of San Francisco
San Francisco, CA 94117-1080
USA
stillwell@usfca.edu

Mathematics Subject Classification (2000): 01-01

Library of Congress Cataloging-in-Publication Data
Stillwell, John
Mathematics and its history / John Stillwell.—2nd ed.
p. cm.—(Undergraduate texts in mathematics)
Includes bibliographical references and index.

1. Mathematics—History. I. Title. II. Series.
QA21.S84 2001
510′.9—dc21 2001042958

Printed on acid-free paper.

ISBN 978-1-4419-2955-6 ISBN 978-1-4684-9281-1 (eBook)
DOI 10.1007/978-1-4684-9281-1

9 8 7 6 (Corrected printing, 2004)

springer.com

To Elaine, Michael, and Robert

Preface to the Second Edition

This edition has been completely retyped in LaTeX, and many of the figures redone using the PSTricks package, to improve accuracy and make revision easier in the future. In the process, several substantial additions have been made.

- There are three new chapters, on Chinese and Indian number theory, on hypercomplex numbers, and on algebraic number theory. These fill some gaps in the first edition and give more insight into later developments.
- There are many more exercises. This, I hope, corrects a weakness of the first edition, which had too few exercises, and some that were too hard. Some of the monster exercises in the first edition, such as the one in Section 2.2 comparing volume and surface area of the icosahedron and dodecahedron, have now been broken into manageable parts. Nevertheless, there are still a few challenging questions for those who want them.
- Commentary has been added to the exercises to explain how they relate to the preceding section, and also (when relevant) how they foreshadow later topics.
- The index has been given extra structure to make searching easier. To find Euler's work on Fermat's last theorem, for example, one no longer has to look at 41 different pages under "Euler." Instead, one can find the entry "Euler, and Fermat's last theorem" in the index.
- The bibliography has been redone, giving more complete publication data for many works previously listed with little or none. I have found the online catalogue of the Burndy Library of the Dibner Institute at MIT helpful in finding this information, particularly for

early printed works. For recent works I have made extensive use of MathSciNet, the online version of *Mathematical Reviews*.

There are also many small changes, some prompted by recent mathematical events, such as the proof of Fermat's last theorem. (Fortunately, this one did not force a major rewrite, because the background theory of elliptic curves was covered in the first edition.)

I thank the many friends, colleagues, and reviewers who drew my attention to faults in the first edition, and helped me in the process of revision. Special thanks go to the following people.

- My sons Michael and Robert, who did most of the typing, and my wife, Elaine, who did a great deal of the proofreading.
- My students in Math 310 at the University of San Francisco, who tried out many of the exercises, and to Tristan Needham, who invited me to USF in the first place.
- Mark Aarons, David Cox, Duane DeTemple, Wes Hughes, Christine Muldoon, Martin Muldoon, and Abe Shenitzer, for corrections and suggestions.

JOHN STILLWELL
Monash University
Victoria, Australia
2001

Preface to the First Edition

One of the disappointments experienced by most mathematics students is that they never get a course on mathematics. They get courses in calculus, algebra, topology, and so on, but the division of labor in teaching seems to prevent these different topics from being combined into a whole. In fact, some of the most important and natural questions are stifled because they fall on the wrong side of topic boundary lines. Algebraists do not discuss the fundamental theorem of algebra because "that's analysis" and analysts do not discuss Riemann surfaces because "that's topology," for example. Thus if students are to feel they really know mathematics by the time they graduate, there is a need to unify the subject.

This book aims to give a unified view of undergraduate mathematics by approaching the subject through its history. Since readers should have had some mathematical experience, certain basics are assumed and the mathematics is not developed formally as in a standard text. On the other hand, the mathematics is pursued more thoroughly than in most general histories of mathematics, because mathematics is our main goal and history only the means of approaching it. Readers are assumed to know basic calculus, algebra, and geometry, to understand the language of set theory, and to have met some more advanced topics such as group theory, topology, and differential equations. I have tried to pick out the dominant themes of this body of mathematics, and to weave them together as strongly as possible by tracing their historical development.

In doing so, I have also tried to tie up some traditional loose ends. For example, undergraduates can solve quadratic equations. Why not cubics? They can integrate $1/\sqrt{1-x^2}$ but are told not to worry about $1/\sqrt{1-x^4}$. Why? Pursuing the history of these questions turns out to be very fruitful, leading to a deeper understanding of complex analysis and algebraic geometry, among other things. Thus I hope that the book will be not only a bird's-eye view of undergraduate mathematics but also a glimpse of wider horizons.

Some historians of mathematics may object to my anachronistic use of modern notation and (fairly) modern interpretations of classical mathematics. This has certain risks, such as making the mathematics look simpler than it really was in its time, but the risk of obscuring ideas by cumbersome, unfamiliar notation is greater, in my opinion. Indeed, it is practically a truism that mathematical ideas generally arise before there is notation or language to express them clearly, and that ideas are implicit before they become explicit. Thus the historian, who is presumably trying to be both clear and explicit, often has no choice but to be anachronistic when tracing the origins of ideas.

Mathematicians may object to my choice of topics, since a book of this size is necessarily incomplete. My preference has been for topics with elementary roots and strong interconnections. The major themes are the concepts of number and space: their initial separation in Greek mathematics, their union in the geometry of Fermat and Descartes, and the fruits of this union in calculus and analytic geometry. Certain important topics of today, such as Lie groups and functional analysis, are omitted on the grounds of their comparative remoteness from elementary roots. Others, such as probability theory, are mentioned only briefly, as most of their development seems to have occurred outside the mainstream. For any other omissions or slights I can only plead personal taste and a desire to keep the book within the bounds of a one- or two-semester course.

The book has grown from notes for a course given to senior undergraduates at Monash University over the past few years. The course was of half-semester length and a little over half the book was covered (Chapters 1–11 one year and Chapters 5–15 another year). Naturally I will be delighted if other universities decide to base a course on the book. There is plenty of scope for custom course design by varying the periods or topics discussed. However, the book should serve equally well as general reading for the student or professional mathematician.

Biographical notes have been inserted at the end of each chapter, partly to add human interest but also to help trace the transmission of ideas from one mathematician to another. These notes have been distilled mainly from secondary sources, the *Dictionary of Scientific Biography* (DSB) normally being used in addition to the sources cited explicitly. I have followed the DSB's practice of describing the subject's mother by her maiden name.

References are cited in the name (year) form, for example, Newton (1687) refers to the *Principia*, and the references are collected at the end of the book.

The manuscript has been read carefully and critically by John Crossley, Jeremy Gray, George Odifreddi, and Abe Shenitzer. Their comments have resulted in innumerable improvements, and any flaws remaining may be due to my failure to follow all their advice. To them, and to Anne-Marie Vandenberg for her usual excellent typing, I offer my sincere thanks.

JOHN STILLWELL
Monash University
Victoria, Australia
1989

Contents