III.

$$(Pq - Qp) + (Pr - Rp)y = 0.$$

Deinde multiplicetur prior æquatio per r, posterior vero per R, alteraque ab altera subtracta habebitur

IV.

$$(Pr - Rp) + (Qr - Rq)y = 0.$$

Cum igitur ex his duabus æquationibus sit

$$y = \frac{Qp - Pq}{Pr - Rp} = \frac{Rp - Pr}{Qr - Rq}$$

erit

$$(Qp - Pq)(Qr - Rq) + (Pr - Rp)^2 = 0,$$

seu

$$P^2r^2 - 2PRpr + R^2p^2 + Q^2pr - PQqr - QRpq + PRq^2 = 0.$$

Cujus æquationis singulæ radices reales ostendent totidem intersectiones veras, si quidem cuique valori ipsius x valor realis ipsius y convenit ex æquatione III. vel IV. Interim tamen fieri potest, ut intersectiones sint imaginariæ, quod evenit si æquationes III. & IV. habeant Factores; ita ut ex iis jam per divisionem æquatio ab y libera elici queat. Tum enim hæc æquatio in locum ultimæ substitui, atque ad valores ipsius x inde erutos ex primis æquationibus valores ipsius y respondentes quæri debebunt; qui si fuerint imaginarii, hoc erit indicio intersectiones esse imaginarias.

480. Sit porro in una Curva Applicata y Functio biformis, in altera autem triformis ipsius x; seu, sint ambæ Curvarum æquationes propositæ hæ

I.

$$P + Qy + Ryy = 0$$

II.

$$p + qy + ryy + sy^3 = 0.$$

Multiplicetur prior per p, posterior per P, alteraque ab altera subtracta remanebit

III.

III.

$$(Pq - Qp) + (Pr - Rp)y + Psyy = 0,$$

quæ cum prima conjuncta exhibet casum in præcedente paragrapho pertractatum; ita ut, quæ ibi erant p, q, r, hic sint $Pq - Qp$, $Pr - Rp$, & Ps: ideoque reperietur hic

$$y = \frac{PQq - QQp - PPr + PRp}{PPs - PRq + QRp},$$

&

$$y = \frac{PRq - QRp - PPs}{PQs - PRr + RRp};$$

unde fit

$$0 = (PRq - QRp - PPs)^2 + (PQs - PRr + RRp)(PQq - Q^2p - P^2r + PRp),$$

quæ æquatio evoluta dat

$$P^4s^2 \begin{matrix} -2P^3Rqs \\ -P^3Qrs \\ +P^3Rrr \end{matrix} \begin{matrix} +3P^2QRps \\ +P^2R^2qq \\ +P^2Q^2qs \\ -P^2QRqr \\ -2P^2R^2pr \end{matrix} \begin{matrix} -PQR^2pq \\ -PQ^3ps \\ +PQ^2Rpr \\ +PR^3pp \end{matrix} \begin{matrix} +Q^2R^2p^2 \\ -Q^2R^2p^2 \end{matrix} = 0$$

quæ, ob ultimum terminum evanescentem, divisibilis est per P, sicque prodibit hæc æquatio

$$\begin{matrix} +P^3s^2 - 2P^2Rqs - P^2Qrs + 3PQRps + PQ^2qs - Q^3ps + R^3p^2 \\ +P^2Rr^2 - PQRqr - 2PR^2pr + Q^2Rpr + PR^2q^2 - QR^2pq \end{matrix} = 0.$$

Ex cujus æquationis radicibus realibus intersectiones cognoscentur, si quidem ipsis valores reales ipsius y respondere deprehendantur.

481. Exprimatur nunc utraque Applicata per æquationem cubicam, sintque ambæ æquationes propositæ hæ

I.

$$P + Qy + Ryy + Sy^3 = 0$$

II.

$$p + qy + ryy + sy^3 = 0.$$

Multiplicetur prior per p, poſterior per P, factaque ſubtractione alterius ab altera, remanebit

III.

$$(Pq - Qp) + (Pr - Rp)y + (Ps - Sp)yy = 0.$$

Deinde multiplicetur prior per s, poſteriorque per S, factaque ſubtractione remanebit

IV.

$$(Sp - Ps) + (Sq - Qs)y + (Sr - Rs)yy = 0.$$

Hæ æquationes III. & IV. ſi comparentur cum binis æquationibus §. 479. tractatis, fiet ut ſequitur

$$\begin{array}{l|l} P = Pq - Qp & p = Sp - Ps \\ Q = Pr - Rp & q = Sq - Qs \\ R = Ps - Sp & r = Sr - Rs \end{array}$$

Quibus in æquatione finali ſubſtitutis, emerget

$$\begin{aligned} &+ (Pq - Qp)^2(Sr - Rs)^2 - 2(Pq - Qp)(Ps - Sp)(Sp - Ps)(Sr - Rs) \\ &+ (Ps - Sp)^2(Sp - Ps)^2 + (Pr - Rp)^2(Sp - Ps)(Sr - Rs) - \\ &\quad (Pq - Qp)(Pr - Rp)(Sq - Qs)(Sr - Rs) - (Pr - Rp)(Ps - Sp) \\ &\quad (Sp - Ps)(Sq - Qs) + (Pq - Qp)(Ps - Sp)(Sq - Qs)^2 = 0. \end{aligned}$$

In hac æquatione ſeptem ſunt termini, qui omnes ſunt diviſibiles per $Sp - Ps$, præter primum & quintum: qui autem, ſi conjungantur, duos habebunt Factores, alterum $(Pq - Qp)(Sr - Rs)$, alterum vero $Pq - Qp)(Sr - Rs) - (Pr - Rp)(Sq - Qs)$, qui poſterior reſolutus fit $= PQrs + RSpq - PRqs - QSpr$ ideoque, $= (Sp - Ps)(Rq - Qr)$: unde termini I. & V. coaleſcent in hanc formam $(Pq - Qp)(Sr - Rs)(Sp - Ps)(Rq - Qr)$ quoque per $Sp - Ps$ diviſibilem. Quocirca orietur hæc æquatio

o =

$$0 = (Pq - Qp)(Sr - Rs)(Rq - Qr) + 2(Pq - Qp)(Sp - Ps)(Sr - Rs) + (Sp - Ps)^3 + (Pr - Rp)^2(Sr - Rs) + (Pr - Rp)(Sp - Ps)(Sq - Qs) - (Pq - Qp)(Sq - Qs)^2$$

quæ evoluta dabit

$$\begin{aligned} &+ S^3p^3 - 3PS^2p^2s + P^2Sr^3 + 2PR^2prs - P^2Rr^2s + P^2Qrss + PRSqqr \\ &- P^3s^3 + 3P^2Sps^2 - R^3p^2s - 2PRSpr^2 + R^2Sp^2r - RSSp^2q - QQRprs \\ &- PR^2qqs - PQSqrr + PQRqrs + 3PSSpqr - 3PPSqrs + PQSprs \\ &+ Q^2Sprr + QRRpqs - QRSpqr - 3PQRpss + 3QRSpps - PRSpqs \\ &+ 2P^2Rqss + 2PQSqqs - PSSq^3 - PQ^2qss - \\ &2QS^2ppr - 2QQSpqs + Q^3pss + QS^2pqq = 0. \end{aligned}$$

482. Quo methodus ista eliminandi y ex duabus æquationibus altiorum graduum clarius percipiatur, ponamus utramque æquationem propositam esse quarti ordinis

I.

$$P + Qy + Ry^2 + Sy^3 + Ty^4 = 0$$

II.

$$p + qy + ry^2 + sy^3 + ty^4 = 0,$$

multiplicetur æquatio prior per p, posterior per P, atque post subtractionem relinquetur

III.

$$(Pq - Qp) + (Pr - Rp)y + (Ps - Sp)y^2 + (Pt - Tp)y^3 = 0.$$

Deinde multiplicetur æquatio I. per t, posterior II. per T, &, facta subtractione, remanebit

IV.

$$(Pt - Tp) + (Qt - Tq)y + (Rt - Tr)y^2 + (St - Ts)y^3 = 0.$$

Ponatur nunc brevitatis gratia

$Pq - Qp = A$	$Pt - Tp = a$	$Sq - Qs = \alpha$
$Pr - Rp = B$	$Qt - Tq = b$	$Rq - Qr = \beta$
$Ps - Sp = C$	$Rt - Tr = c$	
$Pt - Tp = D$	$St - Ts = d$	

ubi notandum est esse non solum $a = D$; sed esse quoque

$Ad -$

$$Ad - Cb = (Pt - Tp)(Sp - Qs) = D\alpha$$
$$Ac - Bb = (Pt - Tp)(Rq - Qr) = D\beta.$$

His ergo ſubſtitutionibus æquationes III. & IV. induent has formas

III.

$$A + By + Cyy + Dy^3 = 0$$

IV.

$$a + by + cyy + dy^3 = 0.$$

Nunc porro æquationes hæ multiplicentur reſpective per d & D, & a ſe invicem ſubtrahantur, prodibitque

V.

$$(Ad - Da) + (Bd - Db)y + (Cd - Dc)y^2 = 0.$$

Tum eædem illæ æquationes multiplicentur per a & A, & poſt ſubtractionem relinquetur

VI.

$$(Ab - Ba) + (Ac - Ca)y + (Ad - Da)y^2 = 0.$$

Jam ſtatuatur iterum brevitatis gratia

$Ab - Ba = E$	$Ad - Da = e$	$Cb - Bc = \zeta$
$Ac - Ca = F$	$Bd - Db = f$	
$Ad - Da = G$	$Cd - Dc = g$	

eritque $G = e$; & $Eg - Ff = G\zeta$; ita ut & $Eg - Ff$ ſit diviſibile per G. Hinc ſequentes habebimus æquationes

V.

$$E + Fy + Gyy = 0$$

VI.

$$e + fy + gyy = 0.$$

Ex quibus per ſimilem operationem eliciuntur iſtæ

VII.

$$(Ef - Fe) + (Eg - Ge)y = 0$$

VIII.

$$(Eg - Ge) + (Fg - Gf)y = 0.$$

Denique

Denique iterum ponatur brevitatis gratia

$$\begin{array}{l|l} Ef - Fe = H & Eg - Ge = h \\ Eg - Ge = I & Fg - Gf = i \end{array}$$

ita ut ſit $I = h$, habebiturque

VII.

$$H + Iy = 0$$

VIII.

$$h + iy = 0,$$

ex quibus tandem colligitur hæc æquatio ab y libera

$$Hi - Ih = 0.$$

In qua ſi valores præcedentes ſucceſſive reſtituantur, obtinebitur æquatio quam ſolæ Functiones P, Q, R, &c. p, q, r, &c., primarum æquationum ingredientur. Æquatio vero inter E, F, G, e, f, g diviſibilis erit per $G = e$; atque, ſi procedatur ad litteras A, B, C, D, a, b, c, d, æquatio reſultans diviſionem admittet per $D^2 = a^2$, ita ut in æquatione ultima quivis terminus octo tantum complexurus ſit litteras, quatuor majuſculas, totidemque minuſculas. Hoc itaque modo in genere, quotcunque dimenſiones ipſius y utraque æquatio propoſita contineat, ſemper incognita y poterit eliminari, atque æquatio, quæ ſolam incognitam x involvat, inveniri.

483. Etſi hujus methodi ex duabus æquationibus unam incognitam eliminandi uſus latiſſime patet, tamen aliam adhuc methodum ſubjungam, quæ tot repetitis ſubſtitutionibus non indigeat. Sint igitur propoſitæ duæ æquationes quotcunque dimenſionum

I.

$$Py^m + Qy^{m-1} + Ry^{m-2} + Sy^{m-3} + \&c. = 0$$

II.

$$py^n + qy^{n-1} + ry^{n-2} + sy^{n-3} + \&c. = 0,$$