using both sines and cosines. In algebra he wrote 'res' for $x$, and 'census' for the square. Regiomontanus probably died of the plague, but there was a rumour that he was poisoned by the sons of a rival scholar.

Columbus took a copy of Regiomontanus's *Ephemerides* on his fourth voyage, and used its prediction of the lunar eclipse of February 29, 1504 to intimidate some hostile Indians in Jamaica.

Johannes Widman (1462–1500) of Eger, now in the Czech Republic, published a book, *Mercantile Arithmetic*, in 1489, in which the modern symbols $+$ and $-$ appeared for the first time.

Luca Pacioli of Italy (1445–1517) was a Franciscan monk. He used the 'res' and 'census' terminology of Regiomontanus. In 1509 he published the *Divina Proportione* (*Divine (or Golden) Ratio*), a book that was illustrated by none other than Leonardo da Vinci. There is a famous painting of Pacioli by Jacopo de' Barbari, which is now in the Museum of Naples. It shows the friar with his friend Guidebaldo standing in the presence of a dodecahedron. One of the problems solved by Pacioli was the following:

> The radius of the inscribed circle of a triangle is 4, and the segments into which one side is divided by the point of contact are 6 and 8. Determine the other sides.

Robert Recorde of England (1510–1558) was the first person to use the symbol $=$ for equality, asserting that 'noe 2 thynges can be more equalle'. Recorde got into a tangle with the Earl of Pembroke and died in jail.

Christoff Rudolff of Germany used $\sqrt{\ }$ for 'radix' in 1525.

Adam Riese of Bavaria (1492–1559) published arithmetic books that went through more than a hundred editions, and established the use of the signs $+$ and $-$.

Michael Stifel of Germany (1487–1567) was a monk who became an early follower of Luther. He introduced $1A$, $1AAA$, $1AAA$ for $A$, $A^2$ and $A^3$. He was the first to use negative integers as exponents and had a way of applying mathematics to the Bible which led him to conclude that Pope Leo X was the beast of the *Book of Revelation,* and also to prophesy the end of the world for 18 October, 1533. The peasants of the village where he was pastor believed this prophecy and spent all their money. When the world failed to end, Stifel found himself, not in heaven, but in a jail in Wittenberg.

Thomas Harriot of England (1560–1621) wrote $a$, $aa$, $aaa$ for $a$, $a^2$, $a^3$ and introduced the signs $>$ and $<$ for strict inequality. He went to America with Sir Walter Raleigh and became a tobacco addict. In 1603 Harriot computed the area of a spherical triangle:

> Take the sum of all three angles and subtract 180 degrees. Set the remainder as numerator of a fraction with denominator 360

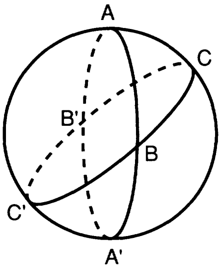


FIGURE 24.1. Area of a spherical triangle

degrees. This fraction tells us how great a portion of the hemisphere is occupied by the triangle.

To prove this, first note that, if a sphere has unit radius, then its surface is $4\pi$ or $720°$. Hence, the area between two great circles, e.g., between two meridians on the earth, one degree apart, on one hemisphere, is $720°/360 = 2°$. It follows, moreover, that the area between two great circles, on one hemisphere, separated by an angle $A$ is $2A$.

The spherical triangle $ABC$ is bounded by three great circles, as in Figure 24.1, where $A'$, $B'$ and $C'$ are the antipodes of $A$, $B$ and $C$ respectively. We note that

$$\triangle ABC + \underline{\triangle A'BC} = 2A,$$

$$\triangle A'B'C' + \underline{\triangle A'BC'} = 2B,$$

$$\underline{\triangle ABC} + \underline{\triangle ABC'} = 2C.$$

The four underlined triangles make up the visible hemisphere (Figure 24.1), namely, $360°$; hence adding the three equations we get

$$\triangle ABC + \triangle A'B'C' + 360° = 2A + 2B + 2C.$$

Now $\triangle A'B'C' = \triangle ABC$, the two being antipodal triangles. Dividing by 2, we obtain

$$\triangle ABC + 180° = A + B + C.$$

Thus the area of the spherical triangle, measured in degrees, is its *spherical excess* $A + B + C - 180°$.

## (2) **The theory of equations**

Ever since the ancient Babylonians, people knew how to find positive, real solutions to any linear or quadratic equation. This they could do arith-