

- Menger's theorems [167–169]: min-max characterizations of connectivity by number of pairwise internally-disjoint or edge-disjoint paths between pairs of vertices
- Meyniel graph [330]: any graph in which every odd cycle of length at least 5 has at least two chords
- Minimal imperfect graph [320]: imperfect graph where every proper induced subgraph is perfect
- Minimally 2-connected [175]: deleting any edge destroys 2-connectedness
- Minimum cut [178]: a source/sink cut having minimum value, or the value of such a cut
- Minimum degree  $\delta(G)$  [34]: minimum of the vertex degrees
- Minimum ( $P$ -object) [31]: for a property  $P$ , no smaller object of the same type also has property  $P$
- Minimum Spanning Tree (MST) [95]: spanning tree with minimum sum of edge weights
- Minor [251, 362]: graph (or matroid) obtained by deletions and contractions
- Mixed graph: a graph model allowing directed and undirected edges
- Möbius ladder: the graph obtained by adding to an even cycle the chords between vertex pairs at maximum distance on the cycle (can be drawn as a ladder with a twist)
- Möbius strip: the non-orientable surface obtained by identifying two opposite sides of a rectangle using opposite orientation
- Model A [430]: probability distribution generating simple graphs with vertex set  $[n]$  by letting each pair be an edge with probability  $p(n)$ , independently
- Model B [430]: probability distribution making the simple graphs with vertex set  $[n]$  and  $m$  edges equally likely
- $r$ th-moment [433]: expectation of  $X^r$
- Monochromatic [386]: in a coloring, a set having all elements the same color
- Monotone graph property [432]: preserved under deletion of edges or vertices
- Multigraph: used by many authors to mean graphs that allow (but don't require) multiple edges and loops (some authors forbid loops from multigraphs)
- Multinomial coefficient [489]: counts arrangements having fixed multiplicities of items; with  $k_i$  items of type  $i$ , there are  $(\sum k_i)! / \prod (k_i)!$  ways to arrange them in a list
- Multiple edges [2]: edges with the same endpoints
- Nearest-insertion [497]: TSP heuristic to grow a cycle
- Nearest-neighbor [496]: TSP heuristic to grow a path
- Neighborhood  $N(v)$  [34]: set of neighbors of  $v$  (*closed* neighborhood  $N[v]$  also includes  $v$ )
- Neighbors [2]: (noun) the vertices in the neighborhood; (verb) "is adjacent to"
- Net outflow [178]: at a vertex, the total exiting flow minus the total entering flow
- Network [176]: a directed graph with a distinguished initial vertex (source) and a distinguished terminal vertex (sink), in which each edge is assigned a flow capacity and possibly also a flow demand (lower bound)
- Node: vertex, especially in network flow problems
- Nondeterministic algorithm [494]: allowed to "guess" by having parallel computation paths
- Nondeterministic polynomial algorithm [494]: having a polynomial-time computation path for each guess of a polynomial number of bits
- Nonorientable surface: a surface with only one side
- Nontrivial graph [22]: having at least one edge
- Nonplanar [243]: having no embedding in the plane
- Nowhere-zero  $k$ -flow [207]: a  $k$ -flow in which all assigned weights are nonzero
- NP [495]: the class of problems solvable by nondeterministic polynomial algorithms
- NP-complete [495]: NP-hard and in NP
- NP-hard [495]: provides a polynomial algorithm for every problem in NP
- Null graph [3]: graph having no vertices
- Numbering: a bijection from  $V(G)$  to  $[n(G)]$
- Obstruction: forbidden substructure
- Odd antihole [340]: complement of an odd hole
- Odd component [136]: component with an odd number of vertices
- Odd cycle [24]: cycle with an odd number of edges (vertices)
- Odd graph: the disjointness graph of the  $k$ -subsets of  $[2k + 1]$
- Odd hole: chordless odd cycle
- Odd vertex [27]: vertex of odd degree

- Odd walk [24]: walk of odd length  
 Open walk [20]: walk in which the first and last vertex are different  
 Optimal tour: a solution to the traveling salesman problem or Chinese postman problem  
 Order of graph [34]: the number of vertices  
 Ordered graph [406]: a graph with an order relation (usually linear) on the edges  
 Order-preserving property [358]: for a function  $\sigma$  on the set of subsets of a set, the requirement that  $X \subseteq Y$  implies  $\sigma(X) \subseteq \sigma(Y)$   
 Orientable surface: a surface with two distinct sides  
 Orientation of graph [62]: a digraph obtained by designating a head and tail for each edge  
 Outdegree [58]: for a vertex, the number of edges of which it is the tail  
 Outerplanar graph [239]: a planar graph embeddable in the plane so that all the vertices are on the boundary of the exterior region  
 Outerplane graph [239]: a particular embedding of an outerplanar graph
- Parallel elements [351]: non-loops in a matroid that form a set of rank 1  
 Parent [100]: the neighbor of a vertex along the path to the root in a rooted tree  
 Parity [473]: odd or even  
 Parity subgraph of  $G$  [312]: subgraph  $H$  such that  $d_H(v) \equiv d_G(v) \pmod{2}$  for all  $v \in V(G)$   
 $k$ -partite [5]: same as  $k$ -colorable  
 Partite set [4]: a set in a vertex partition into independent sets (color class)  
 Partition matroid [357]: a matroid induced by a partition of the ground set in which a set is independent if and only if it has at most one element from each block of the partition  
 Partitionable graph [335]: a graph with  $aw + 1$  vertices where each vertex-deleted subgraph is colorable by  $w$  stable sets of size  $a$  and coverable by  $a$  cliques of size  $w$   
 Path [5]: a simple graph whose vertices can be listed so that vertices are adjacent if and only if they are consecutive in the list  
 $u, v$ -path [20]: a path with  $u$  and  $v$  as endpoints  
 Path addition [163]: a step in an ear decomposition  
 Path decomposition [414]: expression of a graph as a union of pairwise edge-disjoint paths  
 Paw [12]: simple 4-vertex graph obtained by adding one edge to a claw  
 $p$ -critical graph [334]: an imperfect graph whose proper induced subgraphs are all perfect  
 Pendant edge [67]: edge incident with a vertex of degree 1  
 Pendant vertex [67]: a vertex of degree 1  
 $\alpha$ -perfect [319]:  $\alpha(H) = \theta(H)$  for every induced subgraph  $H$   
 $\beta$ -perfect: [335]  $\alpha(H)\omega(H) \geq n(H)$  for every induced subgraph  $H$   
 $\gamma$ -perfect: [319]  $\chi(H) = \omega(H)$  for every induced subgraph  $H$   
 Perfect elimination ordering [224]: deletion order such that when each vertex is deleted, its neighborhood in what remains is a clique (same as *simplicial elimination ordering*)  
 Perfect graph [226]: graph such that  $\chi(H) = \omega(H)$  for every induced subgraph  $H$   
 Perfect Graph Theorem (PGT) [226, 320]: a graph is perfect if and only if its complement is perfect  
 Perfect order [331]: a vertex order yielding optimal greedy colorings for all subgraphs  
 Perfectly orderable graph [331]: having a perfect order  
 Perfect matching [107]: a set of edges such that each vertex belongs to exactly one of them  
 Peripheral vertex [70]: a vertex of maximum eccentricity  
 Permutation [486]: a bijection from a finite set to itself  
 Permutation graph: representable by a permutation  $\sigma$  by  $v_i \leftrightarrow v_j$  if and only if  $\sigma$  reverses the order of  $i$  and  $j$   
 Permutation matrix [120]: a 0,1-matrix having exactly one 1 in each row and column  
 Petersen graph [12]: the disjointness graph of the 2-sets in a 5-element set  
 Pigeonhole principle [491]: every set of numbers has one at least as large as the average  
 Pigeonhole property [427]: a finite probability space has an element where the value of a random variable is at least as large as its expectation  
 Planar graph [5, 235]: a graph embeddable in the plane  
 Plane graph [235]: a particular planar embedding of a planar graph  
 Plane tree [101]: tree with fixed cyclic embedding order of edges at each vertex  
 Planted tree [101]: rooted plane tree  
 Platonic solid [242]: bounded regular polyhedron

**Point:** vertex

Polygonal curve [234]: concatenation of segments

Polyhedron [242]: an intersection of half-spaces

Polytope: the convex hull of a set of vertices

Positional game [120]: a game in which the objective is seizing the positions of a winning set

$k$ th-power ( $G^k$ ): the graph with vertex set  $V(G)$  in which  $u \leftrightarrow v$  if and only if  $d_G(u, v) \leq k$

Predecessor [54]: for  $v$  in a digraph, a vertex  $u$  with  $u \rightarrow v$

Predecessor set [58]: for  $v$  in a digraph, the set of predecessors

Prefix-free code [101]: no code word is a prefix of another

Prim's Algorithm [104]: grows a minimum spanning tree by adding a leaf to the current tree in the cheapest way

Principal submatrix: square submatrix using rows and columns with the same indices

Product dimension [398]: minimum number of coordinate in a product representation of  $G$

Product representation [398]: encoding of graph such that vertices are adjacent if and only if their codes differ in every coordinate

Proper coloring [192]: (1) for vertices, a coloring in which no edge is monochromatic; (2) for edges, a coloring in which edges sharing an endpoint have distinct colors

Proper subgraph of  $G$  [192]: a subgraph not equal to  $G$

Proper subset of  $S$  [472]: a subset not equal to  $S$

Proposal Algorithm [131]: procedure for creating a stable matching

Prüfer code [81]: for a labeled tree, a sequence of length  $n - 2$  obtained by successively deleting the leaf with smallest label and recording its neighbor's label

Pseudograph: graph model that allows loops and multiple edges, used by authors who define multigraphs not to have loops

Radius [70]: the minimum of the vertex eccentricities

Ramsey number [380]: the minimum number of vertices such that assigning colors to all pairs of those vertices produces a monochromatic clique of specified size (or a specified graph) in one of the colors

Random graph [430]: a graph from a probability space, most often the space in which each labeled pair of vertices independently has probability  $p$  of adjacency; typically,  $p = 1/2$  or  $p$  is a function of  $n$

Random variable [427]: a variable that takes on a value at each point in a probability space

Rank (matroids) [349]: for a set of elements, the largest size of an independent set it contains

Reconstructible [38]: a graph determined (up to isomorphism) by the list of subgraphs obtainable by deleting a single vertex

Reconstruction Conjecture [38]: claim that all graphs with at least 3 vertices are reconstructible

Rectilinear crossing number: the minimum number of crossings in a drawing of the graph in the plane in which all edges appear as straight line segments

Reducible configuration [258]: forbidden from purported minimal 5-chromatic planar graph

Reflexive [490]: (1) a digraph with a loop at every vertex; (2) a binary relation  $R$  with  $xRx$  for all  $x$

Region [235]: for an embedding of a graph on a surface, a maximal connected subset of the surface that does not contain any part of the graph

Regular [34]: having all vertex degrees equal

Regular matroid [351]: representable over every field

$k$ -regular [34]: having all vertex degrees equal to  $k$

Representable matroid [351]: linear matroid

Restriction martingale [445]: martingale in which the value of successive variables is an expectation over a shrinking subset of the probability space

Rigid circuit graph: chordal graph

Robbins' Theorem [166]: every 2-edge-connected graph has a strong orientation

Root [100]: (1) a distinguished vertex; (2) in a branching, the vertex with indegree 0

Rooted plane tree [100]: a tree with a distinguished root vertex so that children of each non-leaf have a specified left-to-right ordering in the plane

Rotation scheme: a description of a 2-cell embedding; a circular permutation of the edges appearing at each vertex, giving their counter-clockwise order around the vertex

- SATISFIABILITY** [499]: the problem of finding truth values for variables to make a logical input formula true
- Satisfiable** [499]: formula having a “yes” answer in the SATISFIABILITY problem
- Saturated vertex** [107]: for a matching, a matched vertex
- Score sequence** [62]: the sequence of outdegrees in a tournament
- Second moment method** [433]: method for obtaining threshold functions
- Self-complementary** [11]: isomorphic to the complement
- Self-converse**: isomorphic to the converse
- Self-dual**: isomorphic to the dual
- Semi-strong perfect graph theorem** [344]: if  $V(G) = V(H)$  and a set of vertices induces  $P_4$  in  $G$  if and only if it induces  $P_4$  in  $H$ , then  $G$  is perfect if and only if  $H$  is perfect
- Semipath**: an semiwalk in which each vertex appears at most once
- Semiwalk**: a sequence of edges (or adjacent vertices) in a directed graph such that each successive pair of edges are adjacent, without regard to the orientation of the edges
- Separable**: having a cut-vertex
- Separating set**: a vertex set whose deletion increases the number of components
- $k$ -set** [380]: set of size  $k$
- Shannon Switching Game** [365]: a game played on a matroid by the Spanner and the Cutter, one trying to seize a set of elements spanning a specified element, the other trying to prevent this
- Shift graph** [202]: graph on the 2-subsets of  $[n]$  having  $\{i, j\}$  adjacent to  $\{j, k\}$  when  $i < j < k$
- Signed (di)graph**: special case of weighted (di)graph, assigning + or - to each edge
- Simple** [2]: (1) a graph with no loops or multiple edges; (2) a digraph having at most one edge with each ordered pair of endpoints; (3) a matroid having no loops or parallel elements
- Simplicial vertex** [224]: (1) a vertex whose neighbors induce a clique;
- Sink** [176]: a distinguished terminal vertex, or any vertex with outdegree 0
- Size** [35, 473]: (1) the number of edges; (2) the number of elements
- Skew partition** [347]: a partition  $X, Y$  of  $V(G)$  such that  $G[X]$  and  $\bar{G}[Y]$  are disconnected
- $f$ -soluble** [148]: having an edge weighting so that the sum of the weights incident to  $v$  is  $f(v)$
- Source** [176]: a distinguished initial vertex, or any vertex with indegree 0
- Source/sink cut** [178]: a partition of the vertices of a network into sets  $S, T$  such that  $S$  contains the source and  $T$  contains the sink
- Span function** [358]: the span of a set  $X$  in a hereditary system consists of  $X$  and the elements not in  $X$  that complete circuits with subsets of  $X$
- Spanning subgraph**: a subgraph containing each vertex
- Spanning set** [67]: a set whose span (in a hereditary system on  $E$ ) is  $E$
- Spanning tree** [67]: a spanning, connected, acyclic subgraph
- Spectrum** [453]: the list of eigenvalues with multiplicities
- Split graph** [345]: a graph whose vertices can be covered by a clique and an independent set
- Splittance**: minimum number of edges to be added or deleted to obtain a split graph
- Square of a graph**: the second power
- Squashed-cube dimension** [401]: minimum length of the vectors in a squashed cube embedding
- Squashed-cube embedding** [401]: encodes vertices by 0, 1, \*-vectors such that distance between two vertices is the number of coordinates where one has 0 and the other has 1
- Stability number** [319]: independence number
- Stable matching** [130]: a matching having no instance of  $x$  and  $y$  each preferring the other to their current partner in the matching
- $r$ -staset** [447]: stable set of size  $r$
- Stable set** [3, 319]: a set of pairwise nonadjacent vertices (same as *independent set*)
- Star** [67]: the tree  $K_{1,n-1}$  with at most one non-leaf
- Star-cutset** [333]: separating set inducing a subgraph having a vertex adjacent to all others
- Star-cutset Lemma** [334]: no p-critical graph has a star-cutset
- Steinitz exchange property** [358]: the property of span functions that if  $e$  is in the span of  $X \cup f$  but not in the span of  $X$ , then  $f$  is in the span of  $X \cup e$
- Steinitz's Theorem**: 3-connected planar graphs have only one embedding in the plane (more precisely, only one dual graph)
- Strength** [440]: of a theorem, the fraction of the time when the conclusion holds that the hypothesis also holds

- Strict digraph [294]: a digraph having no loops and at most one edge with each ordered pair of endpoints
- Strictly balanced: average vertex degree in subgraphs is maximized only by the full graph
- Strong absorption property (matroids) [355]: if  $r(X \cup e) = r(X)$  for all  $e \in Y$ , then  $r(X \cup Y) = r(X)$
- Strong component [56]: maximal strongly connected subdigraph
- Strong orientation [165]: orientation of  $G$  in which each vertex is reachable from every other
- Strong Perfect Graph Conjecture (SPGC) [320]: the conjecture that a graph is perfect if and only if it has no odd hole or odd antihole
- Strong product  $G_1 \cdot G_2$ : a graph product with vertex set  $V(G_1) \times V(G_2)$  and edge set  $(u_1, v_1) \leftrightarrow (u_2, v_2)$  if  $u_1 = u_2$  or  $u_1 \leftrightarrow u_2$  and  $v_1 = v_2$  or  $v_1 \leftrightarrow v_2$
- Strongly connected (or strong) digraph [56]: a digraph with each vertex reachable from all others
- Strongly perfect [330]: a graph in which some stable set meets every maximal clique
- Strongly regular [464]: a  $k$ -regular graph whose adjacent pairs have  $\lambda$  common neighbors, and whose nonadjacent pair have  $\mu$  common neighbors
- Subconstituent [470]: the subgraph induced by a vertex neighborhood or by a vertex non-neighborhood
- Subdigraph [56]: a subgraph of a directed graph
- Subdivision [212]: (1) the operation of replacing an edge by a path of two edges through a new vertex; (2) a graph obtained by a sequences of subdivisions
- $H$ -subdivision [212]: a graph obtained from  $H$  by subdivisions
- Subgraph [5]: a graph whose vertices and edges all belong to  $G$
- Submodular function [354]: a function such that  $r(X \cup Y) + r(X \cap Y) \leq r(X) + r(Y)$  for all sets  $X, Y$
- Submodularity property (matroids) [354]: having a submodular rank function
- $k$ -subset [471]: subset with  $k$  elements
- Subtree representation [324]: assigns subtrees of a host tree to each vertex of a chordal graph so that vertices are adjacent if and only if the corresponding subtrees intersect
- Successor [54]: for  $u$  in a digraph, a vertex  $v$  with  $u \rightarrow v$
- Successor set [58]: for  $u$  in a digraph, the set of successors
- Sum [39]: (1) for cycles and cocycles, same as symmetric difference; (2) for a graph, the disjoint union; (3) for matroids on disjoint sets, the matroid on their union whose independent sets are all unions of an independent set from each
- Supergraph of  $G$ : a graph containing  $G$
- Superregular [470]: a regular graph that is null or whose subconstituents are all superregular
- Supply [184]: source constraint in a transportation network
- 2-switch [46]: a degree-preserving switch of two disjoint edges for two others not present
- Symmetric [490]: (1) for a graph, having a non-trivial automorphism; (2) for a simple digraph,  $u \rightarrow v \Leftrightarrow v \rightarrow u$ ; (3) for a binary relation  $R$ ,  $xRy \Leftrightarrow yRx$
- Symmetric difference  $A \Delta B$  [109, 473]: the set of elements in exactly one of  $A$  and  $B$
- System of distinct representatives (SDR) [119]: from a collection of sets, a choice of one member from each set so that all the representatives are distinct
- Szekeres-Wilf Theorem [231]:  $\chi(G) \leq 1 + \max_{H \subseteq G} \delta(H)$
- Tail [53]: the first vertex of an edge in a digraph
- Tait coloring [301]: for a planar cubic graph, a proper 3-edge-coloring
- Tarry's Algorithm [95]: procedure for exploring a maze
- Telegraph problem [423]: directed version of gossip problem with one-way transmissions
- Telephone problem [422]: gossip problem
- Tensor product: weak product
- Ternary matroid [357]: representable over the field with three elements
- Thickness [261]: the minimum number of planar graphs whose union is  $G$
- Threshold dimension: minimum number of threshold graphs whose union is  $G$
- Threshold function for  $Q$  [433]: a function  $t$  such that  $Q$  almost always or almost never occurs, depending on whether the parameter in the model belongs to  $\sigma(t)$  or to  $\omega(t)$ .
- Threshold graph: having a threshold  $t$  and a vertex weighting  $w$  such that  $u \not\leftrightarrow v$  iff  $w(u) + w(v) \leq t$ ; many other characterizations, including absence of a 2-switch and existence of a construction ordering by adding isolated or dominating vertices
- Topological graph theory: the study of drawings of graphs on surfaces

- Toroidal [266]: graph having a 2-cell embedding on the torus  
 Torus [266]: the (orientable) surface with one handle  
 Total coloring [411]: a labeling of both the vertices and edges so that elements that are adjacent or incident receive different colors  
 Total Coloring Conjecture [411]: every graph  $G$  has a total coloring using at most  $\Delta(G) + 2$  colors  
 Total domination number [117]: minimum number of vertices in a set  $S$  such that every vertex has a neighbor in  $S$   
 Total interval number: minimum of the total number of intervals used to represent  $G$  as the intersection graph of unions of intervals on the real line  
 Totally unimodular [469]: a matrix in which all square submatrices have determinant 0 or  $\pm 1$   
 Toughness [288]: the minimum  $t$  such that  $|S| \geq t \cdot c(G - S)$  for every separating set  $S$ , where  $c(G - S)$  is the number of components of the subgraph obtained by deleting  $S$   
 Tournament [61]: an orientation of the complete graph  
 Trace [453]: sum of the diagonal elements of a matrix  
 Traceable: having a Hamiltonian path  
 Trail [20, 59]: a walk in which no edge appears more than once  
 Transitive digraph [228]:  $u \rightarrow v$  and  $v \rightarrow w$  together imply  $u \rightarrow w$   
 Transitive closure: (1) for a digraph  $D$ , the digraph with  $u \rightarrow w$  whenever there is a path from  $u$  to  $w$  in  $D$ ; (2) for a relation  $R$ , the relation  $S$  with  $xSy$  whenever there is a sequence  $x_0, \dots, x_k$  with  $x = x_0Rx_1R\dots Rx_k = y$   
 Transitivity of dependence (matroids) [359]:  $e \in \sigma(X)$  and  $X \subseteq \sigma(Y)$  imply  $e \in \sigma(Y)$   
 Transportation constraints [184]: supplies and demands  
 Transportation Problem [185]: generalization of the assignment problem with supplies at each source and demands at each destination  
 Transversal [125]: a system of distinct representatives (this is the word used when the concept is generalized); also used for a system of representatives not necessarily distinct  
 Transversal matroid [352]: a matroid whose elements are one partite set of a bipartite graph and whose independent sets are the subsets saturated by matchings  
 Traveling Salesman Problem (TSP) [493]: problem of finding a minimum-weight spanning cycle  
 Tree [67]: a connected graph with no cycles  
 $k$ -ary tree [101]: rooted tree with at most  $k$  children at each non-leaf vertex  
 $k$ -tree [345]: a chordal graph obtained from a  $k$ -clique by iteratively adding a vertex whose neighborhood when added is a  $k$ -clique  
 Triangle [12]: a cycle of length 3  
 Triangle-free [41]: not having  $K_3$  as a subgraph  
 Triangle inequality:  $d(x, y) + d(y, z) \geq d(x, z)$   
 Triangular chord: chord of length two along a path or cycle  
 Triangulated graph [225]: a graph with no chordless cycle  
 Triangulation [242]: a graph embedding on a surface such that every region is a 3-gon  
 Trivalent: having degree 3  
 Trivial graph [22]: graph with no edges (some authors restrict to one vertex)  
 $k$ -tuple [474]: a list of length  $k$   
 Turán graph [207]: an equipartite complete multipartite graph  
 Turán's theorem [208]: characterization of the complete equipartite  $r$ -partite graphs as the largest graph of a given order with no  $r + 1$ -clique  
 Tutte polynomial: a generalization of the chromatic polynomial and of other polynomials  
 Tutte's Theorem [146, 174, 250]: (1) for matchings, characterization of graphs with 1-factors; (2) for connectivity, characterization of 3-connected graphs by contractions to wheels; (3) for planar graphs, 3-connected planar graphs have embeddings with all bounded faces convex.  
 Twins [348]: vertices having the same neighborhood (false twins are adjacent vertices with the same closed neighborhoods)  
 Unavoidable set [258]: a collection of configurations such that every graph in a specified class contains some configuration in the collection  
 Underlying graph [56]: the graph obtained from a digraph by treated edges as unordered pairs  
 Unicyclic: having exactly one cycle  
 $k$ -uniform hypergraph [449]: having only edges of size  $k$

**Uniform matroid**  $U_{k,n}$  [357]: matroid on  $[n]$  whose independent sets are the sets of size at most  $k$   
**Uniformity property (matroids)** [354]: for all  $X \subseteq E$ , the maximal independent subsets of  $X$  have the same size

**Union** ( $G_1 \cup G_2$ ) [25]: a graph whose vertex set is the union of the vertices in  $G_1$  and  $G_2$  and whose edge set is the union of the edges in  $G_1$  and  $G_2$  (written  $G_1 + G_2$  if the vertex sets are disjoint)

**Union of matroids** [369]: the union of matroids  $M_1, \dots, M_k$  is the hereditary system whose independent sets are  $\{I_1 \cup \dots \cup I_k : I_i \in \mathbf{I}_i\}$

**Unit-distance graph** [201]: the graph with vertex set  $\mathbb{R}^2$  in which points are adjacent if the distance between them is 1

**Unlabeled graph** [9]: informal term for isomorphism class

**$M$ -unsaturated** [107]: vertex not belonging to an edge of  $M$

**Upper embeddable**: having a 2-cell embedding on a surface of genus  $\lfloor (e(G) - n(G) + 1)/2 \rfloor$

**Valence**: vertex degree

**Value of a flow** [176]: the net flow out of the source or into the sink

**Variance** [433]: expected squared deviation from the mean

**Vectorial matroid** [351]: linear matroid

**Vertex** [2]: element of  $V(G)$ , the vertex set

**Vertex chromatic number** [191]: chromatic number

**Vertex connectivity** [149]: connectivity

**Vertex cover** [112]: a set of vertices containing at least one endpoint of every edge

**Vertex-critical**: deletion of any vertex changes the parameter

**Vertex cut** [149, 164]: a separating set of vertices

**Vertex-deleted subgraph** [37]: a subgraph obtained by deleting one vertex

**Vertex multiplication** [320]: a replacement of vertices of  $G$  by independent sets such that copies of  $x$  and  $y$  are adjacent if and only if  $xy \in E(G)$

**Vertex partition**: a partition of the vertex set

**Vertex set**  $V(G)$  [2]: the set of elements on which the graph is defined

**Vertex-transitive** [14]: for each pair  $x, y \in V(G)$ , some automorphism of  $G$  maps  $x$  to  $y$

**Vizing's Theorem** [275]: upper bound on edge-chromatic number in terms of maximum degree and maximum edge multiplicity

**Walk** [20, 59]: an alternating list of vertices and edges in a graph such that each vertex belongs to the edge before and after it (in a digraph, must follow arrows)

**$u, v$ -walk** [20]: a walk from  $u$  to  $v$ .

**Weak elimination property** [352]: property of matrices that the union of distinct intersecting circuits contains a circuit that avoids a specified point in the intersection

**Weak product**  $G_1 \otimes G_2$ : a graph product with vertices  $V(G_1) \times V(G_2)$ , and edges  $(u_1, v_1) \leftrightarrow (u_2, v_2)$  iff  $u_1 \leftrightarrow u_2$  and  $v_1 \leftrightarrow v_2$

**Weakly chordal** [330]: having no chordless cycle of length at least 5 in  $G$  or  $\overline{G}$

**Weakly connected** [56]: a directed graph whose underlying graph is connected

**Weight**: a real number

**Weighted**: having an assignment of weights (to edges and/or vertices)

**Well Ordering Property** [19]: every nonempty set (of natural numbers) has a least element

**Wheel** [174]: a graph obtained by taking the join of a cycle and a single vertex

**Whitney's 2-isomorphism Theorem** [376]: a characterization of the pairs of graphs whose cycle matroids are isomorphic

**Wiener index** [72]: the sum of the pairwise distances between vertices

**Zero flow**: a flow in a network with flow 0 on every edge

# Appendix E

## Supplemental Reading

Many books have been published about graph theory. Here we list a few for the interested reader who seeks an alternative presentation or more detailed material on special topics. We list several general textbooks grouped approximately into three levels. Specialized texts and monographs follow, listed by the relevant chapter in this book. Finally, we list some books that present additional topics in graph theory.

### *General / elementary:*

- Chartrand, G. *Graphs as Mathematical Models*. Prindle--Weber--Schmidt, 1977. Reprinted as *Introductory Graph Theory*, Dover, 1985.  
Clark J. and D.A. Holton, *A first look at graph theory*. World Scientific, 1991.  
Trudeau R.J., *Introduction to graph theory* (originally *Dots and Lines*, 1976). Dover, 1993.  
Wilson R.J. *Introduction to graph theory*. Academic Press, 1979, 1972; Longman, 1985.  
Wilson R.J. and J.J. Watkins, *Graphs: An introductory approach*. John Wiley & Sons, 1990.

### *General / intermediate:*

- Bondy J.A. and U.S.R. Murty, *Graph Theory with Applications*. Elsevier, 1976.  
Chartrand G. and L. Lesniak, *Graphs and Digraphs*. PWS Publishers, 1979; Wadsworth-Brooks/Cole, 1986; Chapman & Hall, 1996.  
Gould R., *Graph Theory*. Benjamin/Cummings, 1988.  
Gross J. and J. Yellen, *Graph Theory*. CRC Press, 1999.  
Harary F., *Graph Theory*. Addison-Wesley, 1969.  
Ore O., *Theory of Graphs*. AMS Colloq. **38**, Amer. Math. Soc., 1962.

### *General / advanced:*

- Berge, C. *Graphs*. North-Holland 1973, 1976, 1985. (1970, 1983 in French.)  
Bollobás B., *Graph Theory: An Introductory Course*. Grad. Texts in Math. **63**; Springer-Verlag, 1979.  
Bollobás B., *Modern Graph Theory*. Grad. Texts Math. **184**; Springer, 1998.  
Diestel R., *Graph Theory* Grad. Texts Math. **173**; Springer-Verlag, 1996, 2000.  
Zykov A.A. *Fundamentals of graph theory* Nauka, 1987 (Russian). Transl. by L. Boron, C. Christenson, and B. Smith, BCS Associates, 1990.

*Chapter 1:*

- Asratian A.S., T.M.J. Denley, and R. Häggkvist, *Bipartite graphs and their applications*. Cambridge Tracts in Math., **131**; Cambridge Univ. Press, 1998.
- Fleischner H., *Eulerian Graphs and Related Topics*, Vols 1 & 2. Ann. Discrete Math. **45** & **50**, North-Holland, 1990 & 1991.
- Harary F., R.Z. Norman, and D. Cartwright, *Structural Models: An Introduction to the Theory of Directed Graphs*. John Wiley & Sons, 1965.

*Chapter 2:*

- Buckley F. and F. Harary, *Distance in Graphs*. Addison-Wesley, 1990
- Moon J., *Counting Labelled Trees*. Canadian Math. Congress, 1970.

*Chapter 3:*

- Gusfield D. and R.W. Irving, *The Stable Marriage Problem: Structure and Algorithms*. MIT Press, 1989.
- Haynes T.W., S.T. Hedetniemi, and P.J. Slater, *Fundamentals of Domination in Graphs*. Pure and Applied Math. **208**; Marcel Dekker, 1998.
- Lovász L. and M.D. Plummer, *Matching Theory*. North-Holland, 1986.

*Chapter 4:*

- Ahuja R.K., T.L. Magnanti, and J. Orlin, *Network Flows*. Prentice-Hall, 1993.
- Ford L.R. and D.R. Fulkerson, *Flows in Networks*. Princeton Univ. Press, 1962.
- Tutte W.T., *Connectivity in Graphs*. Univ. Toronto Press, 1966.

*Chapter 5:*

- Jensen T.R. and B. Toft, *Graph coloring problems*. Wiley-Interscience, 1995.

*Chapter 6:*

- Aigner M., *Graph Theory: A Development from the 4-Color Problem*. Teubner, 1984 (German). Transl. by BCS Associates, 1987.
- Bonnington C.P. and C.H.C. Little, *The Foundations of Topological Graph Theory*. Springer-Verlag, 1995.
- Fritsch R. and G. Fritsch, *The Four-Color Theorem*. Springer, 1994, 1998.
- Gross, J.L. & T.W. Tucker, *Topological Graph Theory*. Wiley-Interscience, 1987.
- Nishizeki T. and N. Chiba, *Planar Graphs: Theory and Algorithms*. North-Holland Math. Studies **140**, Annals Disc. Math. **32**; North-Holland 1988.
- Saaty T.L. and P.C. Kainen, *The Four-Color Problem: Assaults and Conquests*. McGraw-Hill, 1977; reprinted Dover, 1986.
- White A.T., *Graphs, Groups and Surfaces*. North-Holland Math. Studies **8**; North-Holland 1973, 1984.

*Chapter 7:*

- Fiorini S. and R.J. Wilson, *Edge-colourings of Graphs*. Res. Notes in Math. **16**; Pitman, 1977.
- Voss H.-J., *Cycles and Bridges in Graphs*. Kluwer Academic, 1991.
- Zhang C.-Q., *Integer Flows and Cycle Covers of Graphs*. Pure and Applied Math. **205**; Marcel Dekker, 1997.

*Section 8.1:*

- Golumbic M.C., *Algorithmic Graph Theory & Perfect Graphs*. Acad. Press, 1980.
- Brandstädt A., V.B. Le, and J.P. Spinrad, *Graph Classes: A Survey*. Soc. Ind. Appl. Math., 1999.

***Section 8.2:***

Oxley J., *Matroid Theory*. Clarendon Press, Oxford Univ. Press 1992.  
 Welsh D.J., *Matroid Theory*. Academic Press, 1976.

***Section 8.3:***

Graham R.L., B.L. Rothschild, and J.H. Spencer, *Ramsey Theory*. Wiley-Interscience, John Wiley & Sons, 1980, 1990.

***Section 8.4:***

Bollobás B., *Extremal graph theory*. London Math. Soc. Monographs **11**; Academic Press, 1978. (Also treats material of Chapter 5.)

***Section 8.5:***

Alon N. and J. Spencer, *The Probabilistic Method*.

Bollobás B., *Random graphs*. Academic Press, 1985.

Janson S., T. Łuczak, and A. Ruciński, *Random Graphs*. Wiley-Interscience, John Wiley & Sons, 2000.

Palmer E.M., *Graphical Evolution*. John Wiley & Sons, 1985.

***Section 8.6:***

Biggs N., *Algebraic graph theory*. Cambridge Tracts in Math. **67**, Cambridge Univ. Press, 1974, 1993.

Chung F.R.K. *Spectral graph theory*. CBMS Reg. Conf. Series in Math. **92**; Amer. Math. Soc. 1997.

Cvetković D.M., M. Doob, and H. Sachs, *Spectra of graphs: Theory and Applications*. Pure and Appl. Math. **87**, Academic Press, 1980; 1985; Johann Ambrosius Barth, 1995.

***Algorithms and Applications:***

Chartrand G. and O.R. Oellermann, *Applied and Algorithmic Graph Theory*. McGraw-Hill, 1993.

Chen W.K. *Applied Graph Theory: Graphs and Electrical Networks*. Series in Appl. Math. & Mechanics **13**, North-Holland, 1976 (2nd ed.).

Christofides N., *Graph Theory: An Algorithmic Approach*. Acad. Press, 1975.

Even S., *Graph algorithms*. Computer Science Press, 1979.

Foulds L.R., *Graph Theory Applications*. Universitext; Springer-Verlag, 1992.

Gibbons A., *Algorithmic Graph Theory*. Cambridge Univ. Press, 1985.

Gondran M. and M. Minoux, *Graphs and algorithms*, (translated by Steven Vajda). Wiley-Interscience, John Wiley & Sons, 1984.

Lawler E., J.K. Lenstra, A.H.G. Rinnooy-Kan, and D.B. Shmoys, *The Traveling Salesman Problem*. Wiley-Interscience, John Wiley & Sons, 1985, 1990.

McHugh J.A., *Algorithmic Graph Theory*. Prentice-Hall, 1990.

Swamy M.N.S. and K. Thulasiraman, *Graphs, Networks, and Algorithms*. Wiley-Interscience, John Wiley & Sons, 1981.

Temperley H.N.V., *Graph Theory and Applications*. Halstead Press, 1981.

Wilson R.J. and L.W. Beineke (eds.), *Applications of Graph Theory*. Academic Press, 1979.

***Additional Topics:***

Beineke L.W. and R.J. Wilson (eds.), *Selected topics in graph theory*, Vols. 1 & 2 & 3. Academic Press, 1978 & 1983 & 1988.

- Cameron P.J and J.H. van Lint, *Designs, Graphs, Codes and Their Links*. Lond. Math. Soc. Student Texts **22**, Cambridge Univ. Press, 1991.
- Capobianco M. and J.C. Molluzzo, *Examples and counterexamples in graph theory*. North-Holland, 1978.
- Berge C., *Hypergraphs*. N.-H. Math. Lib. **45**, North-Holland, 1987, 1989.
- Biggs, N.L., K.E. Lloyd, and R.J. Wilson, *Graph Theory: 1736–1936*. Clarendon Press, Oxford Univ. Press, 1976, 1986.
- Bosák J., *Decompositions of graphs*. Math. & Its Appl. (East European Series) **47**, Kluwer Academic Publishers, 1990.
- Brouwer A.E., A.M. Cohen, and A. Neumaier, *Distance-regular graphs*. Springer-Verlag, York, 1989.
- Chung F.R.K. and R.L. Graham, *Erdős on Graphs: His Legacy of Unsolved Problems*. A.K. Peters, 1998.
- Fulkerson D.R. (ed.), *Studies in graph theory*, Parts I & II. Studies in Math. **11** & **12**. Math. Assoc. Amer., 1975.
- Harary F. and E.M. Palmer, *Graphical Enumeration*.
- Hartsfield N. and G. Ringel, *Pearls in graph theory*. Academic Press, 1990, 1994.
- Holton D.A. and J. Sheehan, *The Petersen graph*. Australian Math. Soc. Lect. Series **7**, Cambridge University Press, 1993.
- Imrich W. and S. Klavžar, *Product Graphs: Structure and Recognition*. Wiley-Interscience, John Wiley & Sons, 2000.
- Lovász L., R.L. Graham, and M. Grötschel (eds.), *Handbook of Combinatorics*, Vol. I. Elsevier, 1995.
- Mahadev N.V.R. and U.N. Peled, *Threshold Graphs and Related Topics*. Ann. Disc. Math. **56**; North-Holland, 1995.
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# Appendix F

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