LIB. II. 107. Omnes ergo Diametri IG & ig se mutuo in eodem puncto C decussant: quod ergo si semel fuerit inventum, omnes Diametri per id transibunt; ac vicissim omnes rectæ per id ductæ erunt Diametri, quæ omnes Ordinatas sub certo quodam angulo ductas bisecent. Cum igitur hoc punctum in quavis Linea secundi ordinis sit unicum, in eoque omnes Diametri se mutuo decussent, hoc punctum vocari solet CENTRUM Sectionis conicæ. Quod ergo ex æquatione inter x & y proposita

$$0 = \alpha + \beta x + \gamma y + \delta xx + \varepsilon xy + \zeta yy$$

ita invenitur, ut sumta $AD = \frac{2\beta\zeta - \gamma\varepsilon}{\varepsilon\varepsilon - 4\delta\zeta}$, capiatur $CD = \frac{2\gamma\delta - \beta\varepsilon}{\varepsilon\varepsilon - 4\delta\zeta}$.

108. Supra autem invenimus esse $AK + AH = \frac{4\beta\zeta - 2\gamma\varepsilon}{\varepsilon\varepsilon - 4\delta\zeta}$: sunt autem IK & GH perpendicula ex terminis Diametri IG in Axem demissa; unde perspicitur esse $AD = \frac{AK + AH}{2}$, atque ideo punctum D erit medium inter puncta K & H. Quam ob rem Centrum quoque C in medio Diametri IG erit situm; quod cum de quavis alia Diametro æque valeat, consequens est non solum omnes Diametros se mutuo in eodem puncto C decussare, sed etiam se invicem bifariam secare.

TAB. VII. Fig. 26. 109. Sumamus nunc quamcunque Diametrum AI pro Axe ad quem Ordinatæ MN applicatæ sint sub angulo $APM = q$, cujus Sinus $= m$ & Cosinus $= n$. Ponatur Abscissa $AP = x$ & Applicata $PM = y$, cujus cum duo sint valores æquales alter alterius negativus eorumque adeo summa $= 0$, æquatio generalis pro Linea secundi Ordinis abibit in hanc formam $yy = \alpha + \beta x + \gamma xx$; quæ, si ponatur $y = 0$, dabit puncta G & I in Axe, ubi is a Curva trajicitur; æquationis scilicet

licet $xx + \frac{\beta}{\gamma} x + \frac{\alpha}{\gamma} = 0$ radices erunt $x = AG$ & $x =$ Cap. V.
AI; ideoque habebitur $AG + AI = \frac{-\beta}{\gamma}$, & $AG.AI = \frac{\alpha}{\gamma}$. Cum igitur Centrum C in medio Diametri GI sit positum, facile reperietur Centrum Sectionis conicæ C. Erit enim $AC = \frac{AG+AI}{2} = \frac{-\beta}{2\gamma}$.

110. Cognito jam Centro Sectionis conicæ C, in Axe AI, id convenientissime pro initio Abscissarum accipietur. Statuatur ergo $CP = t$, quia manet $PM = y$, ob $x = AC - CP = \frac{-\beta}{2\gamma} - t$, prodibit hæc æquatio inter Coordinatas t & y

$$yy = \alpha - \frac{\beta\beta}{2\gamma} + \frac{\beta\beta}{4\gamma} - \beta t + \beta t + \gamma tt$$

seu

$$yy = \alpha - \frac{\beta\beta}{4\gamma} + \gamma tt.$$

Posito igitur x loco t, habebitur æquatio generalis pro Lineis secundi ordinis, sumta Diametro quacunque pro Axe, & Centro pro Abscissarum initio, quæ, mutata constantium forma, erit $yy = \alpha - \beta xx$. Posito ergo $y = 0$ fiet $CG = CI = \sqrt{\frac{\alpha}{\beta}}$, ideoque tota Diameter GI erit $= 2\sqrt{\frac{\alpha}{\beta}}$.

111. Ponatur $x = 0$, ac reperietur Ordinata per Centrum transiens EF: fiet scilicet $CE = CF = \sqrt{\alpha}$; ideoque tota Ordinata $EF = 2\sqrt{\alpha}$; quæ, quia per Centrum transit, pariter erit Diameter, cum illa GI angulum faciens $ECG = q$. Hæc autem altera Diameter EF bisecabit omnes Ordinatas priori Diametro GI parallelas; facta enim Abscissa AP negativa, Applicata aC versus I cadens manebit priori PM æqualis; &, cum eidem sit parallela, puncta ambo M juncta dabunt Lineam Diametro GI parallelam, ideoque bisecandam

LIB. II. a Diametro EF. Hæ igitur ambæ Diametri GI & EF ita inter se sunt affectæ, ut altera bisecet omnes Ordinatas alteri parallelas, quam ob reciprocam proprietatem hæ duæ Diametri inter se CONJUGATÆ appellantur. Si igitur in terminis G & I Diametri GI ducantur rectæ alteri Diametro EF parallelæ, tangent hæ Lineam curvam, similique modo si per E & F ducantur rectæ Diametro GI parallelæ eæ tangent Curvam in punctis E & F.

112. Ducatur nunc Applicata quævis MQ obliquangula; sitque angulus $AQM = \Phi$, ejus Sinus $= \mu$ & Cos. $= \nu$. Ponatur Abscissa $CQ = t$, & Applicata $MQ = u$, eritque in triangulo PMQ ob ang. $PMQ = \Phi - q$ ac propterea *sin.* $PMQ = \mu n - \nu m$, $y : u : PQ = \mu : m : \mu n - \nu m$ hincque $y = \frac{\mu u}{m}$ & $PQ = \frac{(\mu n - \nu m)u}{m}$, unde $x = t - PQ = t - \frac{(\mu n - \nu m)u}{m}$. Substituantur hi valores in æquatione superiori $yy = \alpha - \beta xx$, seu $yy + \beta xx - \alpha = 0$, ac orietur

$$(\mu\mu + \beta(\mu n - \nu m)^2)uu - 2\beta m(\mu n - \nu m)tu + \beta m^2 tt - \alpha m^2 = 0,$$

ex qua Applicata u duos obtinet valores QM & $-Qn$ eritque $QM - Qn = \frac{2\beta m(\mu n - \nu m)t}{\mu\mu + \beta(un - \nu\mu)^2}$. Bisecetur Ordinata Mn in p, eritque recta Cpg nova Diameter secans omnes Ordinatas ipsi Mn parallelas bifariam, eritque $Qp = \frac{\beta m(\mu n - \nu m)t}{\mu\mu + \beta(\mu n - \nu m)^2}$.

113. Obtinetur autem hinc anguli GCg tangens $= \frac{\mu . Qp}{CQ + \nu . Qp}$, vel *tang.* $GCg = \frac{\beta m(\mu n - \nu m)}{\mu + n\beta(\mu n - \nu m)}$ & *tang.* $Mpg = \frac{\mu . CQ}{pQ + \nu . CQ} = \frac{\mu\mu + \beta(\mu n - \nu m)^2}{\mu\nu + \beta(\mu n - \nu m)(\nu n + \mu n)}$, qui est angulus sub quo novæ Ordinatæ Mn a Diametro gi bisecantur. Porro vero erit $Cp^2 = CQ^2 + Qp^2 + 2\nu . CQ \times Qp =$

$Qp = \frac{\mu^4 + 2\beta\mu^3 n(\mu n - \nu m) + \beta\beta\mu\mu(\mu n - \nu m)^2}{(\mu\mu + \beta(\mu n - \nu m)^2)^2}\, tt$: ideoque

$Cp = \frac{\mu t\sqrt{(\mu^2 + 2\beta\mu n(\mu n - \nu m) + \beta\beta(\mu n - \nu m)^2)}}{\mu\mu + \beta(\mu n - \nu m)^2}$

Ponatur $Cp = r$ & $pM = s$, eritque $t =$.

$\frac{(\mu\mu + \beta(\mu n - \nu m)^2)r}{\mu\sqrt{(\mu^2 + 2\beta\mu n(\mu n - \nu m) + \beta\beta(\mu n - \nu m)^2)}}$ & $u = s +$

$Qp = s + \frac{\beta m(\mu n - \nu m)r}{\mu\sqrt{(\mu^2 + 2\beta\mu n(\mu n - \nu m) + \beta\beta(\mu n - \nu m)^2)}}$,

qui valores porro dant

$$y = \frac{\mu s}{m} + \frac{\beta(\mu n - \nu m)r}{\sqrt{(\ldots\ldots\ldots\ldots)}}$$

$$x = -\frac{(\mu n - \nu m)s}{m} + \frac{\mu r}{\sqrt{(\ldots\ldots\ldots\ldots)}},$$

unde ex æquatione $yy + \beta xx - \alpha$ orietur

$\frac{(\mu\mu + \beta(\mu n - \nu m)^2)ss}{mm} + \frac{\beta(\mu\mu + \beta(\mu n - \nu m)^2)rr}{\mu\mu + 2\beta\mu n(\mu n - \nu m) + \beta\beta(\mu n - \nu m)^2} -$

$\alpha = 0$.

114. Vocemus jam ſemidiametrum $CG = f$ & ſemiconjugatam $CE = CF = g$, eritque $f = \sqrt{\frac{\alpha}{\beta}}$ & $g = \sqrt{\alpha}$, ſeu $\alpha = gg$ & $\beta = \frac{gg}{ff}$: unde fit $yy + \frac{ggxx}{ff} = gg$. Ponamus porro angulum $GCg = p$, erit $tang.\, p = \frac{\beta m(\mu n - \nu m)}{\mu + n\beta(\mu n - \nu m)}$. At, ob angulum $GCE = q$, ſi ponatur angulus $ECe = \varpi$, erit $AQM = \Phi = q + \varpi$; ideoque $\mu = ſin.\,(q + \pi)$; $\nu = coſ.\,(q + \varpi)$, $m = ſin.\, q$ & $n = coſ.\, q$. Ergo $tang.\, p = \frac{\beta.\, ſin.\, q.\, ſin.\, \varpi}{ſin.\,(q + \varpi) + \beta.\, coſ.\, q.\, ſin.\, \pi} =$

$\frac{\beta.\, tang\, q.\, tang.\, \varpi}{tang.\, q + tang.\, \varpi + \beta\, tang.\, \varpi}$, &

$ſin.\, p = \frac{\beta.\, ſin.\, q.\, ſin.\, \pi}{\sqrt{(\mu\mu + 2\beta\mu n(\mu n - \nu m) + \beta\beta(\mu n - \nu m)^2)}}$,

$\mu\mu +$

atque

LIB. II. $\mu\mu + \beta(\mu n - \nu m)^2 = (\sin.(q+\varpi)^2) + \beta(\sin.\varpi)^2$, quibus valoribus in subsidium vocatis prodit ista æquatio inter. r & s $\frac{((\sin.\overline{q+\varpi})^2 + \beta(\sin.\pi)^2)ss}{(\sin.q)^2} + \frac{\beta((\sin.\overline{q+\pi})^2 + \beta(\sin.\pi)^2)rr}{\beta\beta(\sin q)^2(\sin.\varpi)^2}$

$(\sin.p)^2 - \alpha = 0$. At est $\beta = \frac{tang\ p.\sin.(q+\pi)}{(\sin.q - \cos q.tang.p)\sin.\pi} =$

$\frac{tang.p.(tang.q + tang.\varpi)}{tang.\pi(tang.q - tang.p)} = \frac{gg}{ff} = \frac{cot.\varpi.tang.q+1}{cot.p.tang.q-1}$, seu

$tang.q = \frac{ff \pm gg}{gg.cot.p - ff.cot.\pi}$, unde plurima consectaria deduci possunt. Erit vero $\frac{gg}{ff} = \frac{\sin.p.\sin.(q+\pi)}{\sin.\pi\sin.(q-p)}$.

115. Sit semidiameter $Cg = a$, cjusque semidiameter conjugata $C = b$; erit ex æquatione ante inventa,

$a = \frac{\sin.q.\sin.\pi\sqrt{\alpha\beta}}{\sin.p\sqrt{((\sin.\overline{q+\pi})^2 + \beta(\sin.\pi)^2)}} =$

$\frac{gg.\sin.q.\sin.\pi}{\sin.p\sqrt{(ff(\sin.(q+\pi))^2 + gg(\sin.\varpi)^2)}}$, & $b =$

$\frac{fg.\sin.q}{\sqrt{(ff(\sin.(q+\pi)^2) + gg(\sin.\pi)^2)}}$, hinc erit $a:b =$

$g.\sin.\pi : f.\sin.p$. Est vero porro $(\sin.(q+\pi))^2 +$

$\frac{gg}{ff}(\sin.\pi)^2 = \frac{\sin.(q+\pi)}{\sin.(q-p)}(\sin.(q-p).\sin.(q+\pi) + \sin.p\sin.\pi)$

$= \frac{\sin q.(\sin(q+\pi).\sin.(q+\pi-p)}{\sin(q-p)}$, unde fiet $a =$

$\frac{gg.\sin.\pi}{f.\sin.p}\sqrt{\frac{\sin.q.\sin.(q-p)}{\sin.(q+\pi)\sin.(q+\pi-p)}}$; seu, ob $\frac{gg}{ff} =$

$\frac{\sin.p\ \sin.(q+\varpi)}{\sin.\pi.\sin.(q-p)}$, erit $a = f\sqrt{\frac{\sin q\ \sin.(q+\pi)}{\sin.(q-p).\sin.(q+\pi-p)}}$

& $b = g\sqrt{\frac{\sin.q\ \sin.(q-p)}{\sin.(q+\pi).\sin.(q+\pi-p)}}$, ergo erit

$a : b = f.\sin.(q+\pi) : g.\sin.(q-p)$ & $ab =$

$\frac{fg.\sin.q}{\sin(q+\pi-p)}$.

116. S