I. Quum singuli coëfficientes in Y' sint functiones tales radicum in periodo P' (quam $=$ (f, a') statuere licet) contentarum, quales functiones radicum in P sunt coëfficientes respondentes in Y, ex art. 347 manifestum est, Y' ex Y deriuari posse, si modo vbique in Y pro $(f, 1)$, (f, g), (f, gg) etc. resp. substituantur (f, a'), $(f, a'g)$, $(f, a'gg)$ etc. Et perinde Y'' ex Y deriuabitur substituendo vbique in Y pro $(f, 1)$, (f, g), (f, gg) etc. resp. (f, a''), $(f, a''g)$, $(f, a''gg)$ etc. etc. Simulatque igitur functio Y euoluta est, reliquae Y', Y'' etc. nullo negotio inde sequuntur.

II. Supponendo $Y = x^f - \alpha x^{f-1} + \beta x^{f-2}$ — etc., coëfficientes α, β etc. erunt resp. summa radicum aequ. $Y = 0$ i. e. quantitatum $\phi\omega$, $\phi a\omega$, $\phi b\omega$ etc., summa productorum e binis etc. At plerumque hi coëfficientes multo commodius eruntur per methodum ei quae art. 349 tradita est similem, computando summam radicum $\phi\omega$, $\phi a\omega$, $\phi b\omega$ etc., summam quadratorum, cuborum etc., atque hinc per theorema Newtonianum illos coëfficientes deducendo. — Quoties ϕ designat tangentem, secantem, cotangentem aut cosecantem adhuc alia compendia dantur, quae tamen silentio hic praeterimus.

III. Considerationem peculiarem meretur is casus vbi f est numerus par, adeoque quaeuis periodus P, P', P'' etc. ex $\frac{1}{2}f$ periodis binorum terminorum composita. Constet P ex his $(2, 1)$ $(2, \mathfrak{a})$, $(2, \mathfrak{b})$, $(2, \mathfrak{c})$ etc., conuenientque numeri 1, $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$ etc. atque $n - 1$, $n - \mathfrak{a}$, $n - \mathfrak{b}$, $n - \mathfrak{c}$ etc. simul sumti, cum his $1, a, b, c$

etc. aut saltem (quod hic eodem redit) his secundum modulum n congrui erunt. Sed est $\varphi(n-1)\omega = \pm\,\varphi\omega$, $\varphi(n-\mathfrak{a})\omega = \pm\,\varphi\mathfrak{a}\omega$ etc., signis superioribus valentibus quoties φ designat cosinum aut secantem, inferioribus quando φ exprimit sinum, tangentem, cotangentem aut cosecantem. Hinc colligitur, in duobus casibus prioribus inter factores e quibus compositus est Y binos semper aequales, adeoque Y quadratum esse, et quidem $Y = yy$, si y ponatur aequalis producto ex $x-\varphi\omega$, $x-\varphi\mathfrak{a}\omega$, $x-\varphi\mathfrak{b}\omega$ etc. Similiter in iisdem casibus functiones reliquae Y', Y'' etc. quadrata erunt, et quidem supponendo P' constare ex $(2, \mathfrak{a}')$, $(2, \mathfrak{b}')$, $(2, \mathfrak{c}')$ etc.; P'' ex $(2, \mathfrak{a}'')$, $(2, \mathfrak{b}'')$, $(2, \mathfrak{c}'')$ etc. etc., productum ex $x-\varphi\mathfrak{a}'\omega$, $x-\varphi\mathfrak{b}'\omega$, $x-\varphi\mathfrak{c}'\omega$ etc. esse $= y'$, productum ex $x-\varphi\mathfrak{a}''\omega$, $x-\varphi\mathfrak{b}''\omega$ etc., $= y''$ etc. erit $Y' = y'y'$, $Y'' = y''y''$ etc.; nec non etiam functio Z quadratum erit (conf. supra art. 337), et radix producto ex y, y', y'' etc. aequalis. Ceterum facile perspicietur, y', y'' etc. perinde ex y deriuari, vt Y', Y'' etc. ex Y sequi ante in I diximus; nec non singulos coëfficientes in y quoque ad formam $A + B(f, 1) + C(f, g) +$ etc. posse, quum summae singularum potestatum aequ. $y = 0$ manifesto sint semisses potestatum aequ. $Y = 0$, adeoque ad talem formam reducibiles. — In quatuor casibus posterioribus autem Y erit productum e factoribus $xx-(\varphi\omega)^2$, $xx-(\varphi\mathfrak{a}\omega)^2$, $xx-(\varphi\mathfrak{b}\omega)^2$ etc., adeoque formae $x^f - \lambda x^{f-2} + \mu x^{f-4} -$ etc., patetque coëfficientes λ, μ etc. e summis quadratorum, biquadratorum etc. radicum $\varphi\omega$, $\varphi\omega$, $\varphi\mathfrak{a}\omega$, $\varphi\mathfrak{b}\omega$ etc. deduci posse; et similiter se habebunt functiones Y', Y'' etc.

Ex. I. Sit $n = 17$, $f = 8$ atque designet φ cosinum. Hinc fit $Z = (x^8 + \frac{1}{2}x^7 - \frac{7}{4}x^6 - \frac{3}{4}x^5 + \frac{15}{16}x + \frac{5}{16}x^3 - \frac{5}{32}xx - \frac{1}{32}x + \frac{1}{256})^2$, oportetque adeo $\sqrt{Z}$ in duos factores quaternorum dimensionum y, y' resoluere. Periodus $P = (8, 1)$ constat ex $(2, 1)$, $(2, 9)$, $(2, 13)$, $(2, 15)$ vnde y erit productum e factoribus $x - \varphi\omega$ $x - \varphi 9\omega$, $x - \varphi 13\omega$, $x - \varphi 15\omega$. Substituendo $\frac{1}{2}[k] + \frac{1}{2}[n - k]$ pro $\varphi k\omega$, inuenitur $\varphi\omega + \varphi 9\omega + \varphi 13\omega + \varphi 15\omega = \frac{1}{2}(8, 1)$; $(\varphi\omega)^2 + (\varphi 9\omega)^2 + (\varphi 13\omega)^2 + (\varphi 15\omega)^2 = 2 + \frac{1}{4}(8, 1)$; perinde summa cuborum $= \frac{3}{8}(8, 1) + \frac{1}{8}(8, 3)$, summa biquadratorum $= 1\frac{1}{2} + \frac{5}{16}(8, 1)$; hinc per theorema Newtonianum coëfficientibus in y determinatis prodit $y = x^4 - \frac{1}{2}(8, 1)x^3 + \frac{1}{4}((8, 1) + 2(8, 3))xx - \frac{1}{8}((8, 1) + 3(8, 3))x + \frac{1}{16}((8, 1) + (8, 3))$; y' vero ex y deriuatur commutando $(8, 1)$ cum $(8, 3)$; substituendo itaque pro $(8, 1)$, $(8, 3)$ valores $-\frac{1}{2} + \frac{1}{2}\sqrt{17}$, $-\frac{1}{2} - \frac{1}{2}\sqrt{17}$ fit

$$y = x^4 + (\tfrac{1}{4} - \tfrac{1}{4}\sqrt{17})x^3 - (\tfrac{3}{8} + \tfrac{1}{8}\sqrt{17})xx + (\tfrac{1}{4} + \tfrac{1}{8}\sqrt{17})x - \tfrac{1}{16}$$

$$y' = x^4 + (\tfrac{1}{4} + \tfrac{1}{4}\sqrt{17})x^3 - (\tfrac{3}{8} - \tfrac{1}{8}\sqrt{17})xx + (\tfrac{1}{4} - \tfrac{1}{8}\sqrt{17})x - \tfrac{1}{16}$$

Simili modo $\sqrt{Z}$ in quatuor factores binarum dimensionum resolui potest, quorum primus erit $(x - \varphi\omega)(x - \varphi 13\omega)$, secundus $(x - \varphi 9\omega)(x - \varphi 15\omega)$, tertius $(x - \varphi 3)$ $(x\ \omega - \varphi 5\omega)$, quartus $(x - \varphi 10\omega)$ $(x - \varphi 11\omega)$, omnesque coëfficientes in his factoribus per quatuor aggregata $(4, 1)$ $(4, 9)$, $(4, 3)$, $(4, 10)$ exprimi poterunt. Manifesto autem productum e factore