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New Beginnings in Europe

Europeans only began to rouse themselves from the intellectual slumber of the Dark Ages as they came in contact with Arab civilization, mostly in Spain.

Gerbert (940–1003) had studied in Spain, where he learned the Indian numerals (but not zero). He wrote on arithmetic and geometry, which so overawed his contemporaries that they believed he had a pact with the devil. In spite of this, he became Pope and was known as Sylvester II from 999 to 1003.

One of the most difficult problems in his *Geometry* was the following: find x and y such that $x^2 + y^2 = a^2$ and $\frac{1}{2}xy = b$. This would have been an easy exercise for a Babylonian scribe!

Contemporary with Gerbert was another mathematician and churchperson, Hrotsvitha of Saxony (932–1002), who had an interest in perfect numbers.

The Englishman Adelhard of Bath (1075–1160) attended lectures at Cordova in Spain, about 1120, disguising himself as a Moslem. There he obtained a copy of Euclid's *Elements* in Arabic, which he translated into Latin. All European editions of the *Elements* were based on this translation until 1533, when the Greek original finally became available.

Abraham Ben Ezra (ca. 1095–1167), while based in Spain, travelled widely between Egypt and England. His book *Sefer ha-Mispar* explained the Hindu arithmetic, using Hebrew letters for numerals, with a zero added.

He wrote poetry of a pessimistic nature, asserting that, if he were to trade in candles, it would always be noon.

Jordanus Nemorarius (early 13th century) wrote about triangles, circles, regular polygons, Arabic numerals, primes, perfect numbers, polygonal numbers, ratios, powers and progressions. Like Diophantus, he used letters to denote the unknowns in equations.

In his *Tractatus de numeris datis*, Jordanus discusses problems of the following sort: find x and y such that $x + y = 10$ and $x^2 + y^2 = 58$. This is the sort of problem the ancient Mesopotamians were good at.

Nicole Oresme (1320–1382) was a French bishop who wrote extensively on mathematics and gave the first proof of the divergence of the harmonic series.

In the thousand years from 400 to 1400 AD there was exactly one outstanding European mathematician, namely, Leonardo of Pisa (1180–1250), who was also known as Fibonacci. A contemporary of St. Francis of Assisi, he learned his mathematics in Algeria, where his father was a custom house official. In 1202 he published the *Liber abaci*, in which he explained the Indian system of numerals and introduced the famous ‘Fibonacci sequence’

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots,$$

where each number is the sum of the preceding two. This sequence has important applications in science and in advanced number theory.

Another book by Fibonacci is his *Liber quadratorum*. This original work in indeterminate analysis gave the first proof of the identity

$$(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (bc + ad)^2.$$

This is equivalent to the theorem that the product of the norms of two complex numbers equals the norm of their product. (The *norm* is the square of the absolute value.) The identity implies that, if each of two integers is a sum of two squares, then so is their product.

We recall that a numerical instance of Fibonacci’s identity had already been mentioned by Diophantus (Problem 19, Book III, *Arithmetica*). An explicit statement of the identity first occurred in a commentary on Diophantus by al-Khazin. (See Anbonba [1979].)

In 1225, emperor Frederick II delayed his departure on a crusade to organize a mathematical contest. Leonardo answered all the challenges with flying colours. Two of the problems were the following:

1. Find a fraction a/b such that $(a/b)^2 \pm 5$ are both squares of fractions.
2. Solve $x^3 + 2x^2 + 10x = 20$.

Fibonacci understood negative numbers, interpreting them, in one place, as losses.