Quum is casus vbi ambae formae componendae compositionem directe ingrediuntur simplicissimus sit, ad ipsumque reliqui facile reducantur, illum solum in sequentibus contemplabimur, ita vt si forma aliqua simpliciter dicatur e duabus aliis composita, semper subintelligere oporteat, ex vtraque illam proprie esse compositam *). Eadem restrictio valebit, quoties forma in productum e duabus aliis transformabilis dicetur.

240. Theorema. *Si e formis f, f' composita est forma F; ex F et f'' forma $\mathfrak{F}$; ex f, f'' forma F'; ex F' et f' forma $\mathfrak{F}'$: formae $\mathfrak{F}$, $\mathfrak{F}'$ proprie aequiualentes erunt.*

Dem. I. Sit $f = axx + 2bxy + cyy$, $f' = a'x'x' + 2b'x'y' + c'y'y'$, $f'' = a''x''x'' + 2b''x''y'' + c''y''y''$, $F = AXX + 2BXY + CYY$, $F' = A'X'X' + 2B'X'Y' + C'Y'Y'$, $\mathfrak{F} = \mathfrak{A}\mathfrak{X}\mathfrak{X} + 2\mathfrak{B}\mathfrak{X}\mathfrak{Y} + \mathfrak{C}\mathfrak{Y}\mathfrak{Y}$, $\mathfrak{F}' = \mathfrak{A}'\mathfrak{X}'\mathfrak{X}' + 2\mathfrak{B}'\mathfrak{X}'\mathfrak{Y}' + \mathfrak{C}'\mathfrak{Y}'\mathfrak{Y}'$; determinantes harum septem formarum resp. d, d', d'', D, D', $\mathfrak{D}$, $\mathfrak{D}'$, qui omnes eadem signa et rationem quadratorum inter se habebunt. Porro sit m diuisor communis maximus numerorum a, $2b$, c, similemque significationem habeant m', m'', M relatiue ad formas f', f'', F. Tum ex concl. 4 art. 235, D erit diu. comm. max. numerorum $dm'm'$, $d'mm$, adeoque $Dm''m''$ diu. comm. max. numerorum $dm'm'm''m''$,

*) Similiter vt in compositione rationum (quae cum compositione formarum magnam analogiam habet) subintelligi solet, rationes componendas directe accipiendas esse nisi vbi contrarium monetur.

$d'mmm''m''$; $M = mm'$; $\mathfrak{D}$ diu. comm. max. num. $Dm''m''$, $d''MM$, siue numerorum $Dm''m''$, $d''mmm'm'$. Hinc concluditur, $\mathfrak{D}$ esse diu. comm. max. trium numerorum $dm'm'm''m''$, $d'mmm''m''$, $d''mmm'm'$; ex simili autem ratione $\mathfrak{D}'$ eorundem trium numerorum diuisor communis maximus erit; quare quum $\mathfrak{D}$, $\mathfrak{D}'$ eadem signa habeant, erit $\mathfrak{D} = \mathfrak{D}'$, siue formae $\mathfrak{F}$, $\mathfrak{F}'$ eundem determinantem habebunt.

II. Iam transeat F in ff' per substitutionem $X = pxx' + p'xy' + p''yx' + p'''yy'$, $Y = qxx' + q'xy' + q''yx' + q'''yy'$, atque $\mathfrak{F}$ in Ff'' per substitutionem $\mathfrak{X} = \mathfrak{p}Xx'' + \mathfrak{p}'Xy'' + \mathfrak{p}''Yx'' + \mathfrak{p}'''Yy''$, $\mathfrak{Y} = \mathfrak{q}Xx'' + \mathfrak{q}'Xy'' + \mathfrak{q}''Yx'' + \mathfrak{q}'''Yy''$, designenturque radices quadratae positiuae ex $\frac{d}{D}$, $\frac{d'}{D}$, $\frac{D}{\mathfrak{D}}$, $\frac{d''}{\mathfrak{D}}$ per n, n', $\mathfrak{N}$, $\mathfrak{n}''$. Tunc per art. 233 habebuntur decem et octo aequationes, quarum semissis altera ad transformationem formae F in ff' pertinebit, altera ad transformationem formae $\mathfrak{F}$ in Ff''. Prima erit $pq' - qp' = an'$, ad cuius instar facile formari poterunt reliquae breuitatis gratia hic omittendae. Ceterum quantitates n, n', $\mathfrak{N}$, $\mathfrak{n}''$ rationales quidem erunt, sed non necessario numeri integri.

III. Si valores ipsorum X, Y in valoribus ipsorum $\mathfrak{X}$, $\mathfrak{Y}$ substituuntur, prodit substitutio talis: $\mathfrak{X} = (1)\,xx'x'' + (2)\,xx'y'' + (3)\,xy'x'' + (4)\,xy'y'' + (5)\,yx'x'' + (6)\,yx'y'' + (7)\,yy'x'' + (8)\,yy'y''$; $\mathfrak{Y} = (9)\,xx'x'' + (10)\,xx'y'' + (11)\,xy'x'' + (12)\,xy'y'' + (13)\,yx'x'' + (14)\,yx'y'' + (15)\,yy'x'' + (16)\,yy'y''$, per quam manifesto

$\mathfrak{F}$ transibit in productum $ff'f''$. Coefficiens (1) erit $= p\mathfrak{p} + q\mathfrak{p}''$; valores quindecim reliquorum non apponimus, quippe quos quisque nullo negotio euoluet. Designemus numerum (1) (10) — (2) (9) per (1, 2), numerum (1) (11) — (3) (9) per (1, 3), et generaliter $(g)(8+h) - (h)(8+g)$ per (g, h), supponendo g, h esse integros inaequales inter 1 et 16 quorum maior h*); hoc modo omnino viginti et octo signa habebuntur. Iam denotatis radicibus quadratis positiuis ex $\frac{d}{\mathfrak{D}}$, $\frac{d'}{\mathfrak{D}}$ per $\mathfrak{n}, \mathfrak{n}'$ (quae erunt $= \mathfrak{n}\mathfrak{M}$, $\mathfrak{n}'\mathfrak{M}$), eruentur sequentes 28 aequationes: $(1, 2) = aa'\mathfrak{n}''$, $(1, 3) = aa''\mathfrak{n}'$, $(1, 4) = ab'\mathfrak{n}'' + ab''\mathfrak{n}'$, $(1, 5) = a'a''\mathfrak{n}$, $(1, 6) = a'b\mathfrak{n}'' + a'b''\mathfrak{n}$, $(1, 7) = a''b\mathfrak{n}' + a''b'\mathfrak{n}$, $(1, 8) = bb'\mathfrak{n}'' + bb''\mathfrak{n}' + b'b''\mathfrak{n} + \mathfrak{D}\mathfrak{n}\mathfrak{n}'\mathfrak{n}''$, $(2, 3) = ab''\mathfrak{n}' - ab'\mathfrak{n}''$, $(2, 4) = ac''\mathfrak{n}'$, $(2, 5) = a'b''\mathfrak{n} - a'b\mathfrak{n}''$, $(2, 6) = a'c''\mathfrak{n}$, $(2, 7) = bb''\mathfrak{n}' + b'b''\mathfrak{n} - bb'\mathfrak{n}'' - \mathfrak{D}\mathfrak{n}\mathfrak{n}'\mathfrak{n}''$, $(2, 8) = bc''\mathfrak{n}' + b'c''\mathfrak{n}$, $(3, 4) = ac'\mathfrak{n}''$, $(3, 5) = a''b'\mathfrak{n} - a''b\mathfrak{n}'$, $(3, 6) = bb'\mathfrak{n}'' + b'b''\mathfrak{n} - bb''\mathfrak{n}' - \mathfrak{D}\mathfrak{n}\mathfrak{n}'\mathfrak{n}''$, $(3, 7) = a''c'\mathfrak{n}$, $(3, 8) = bc'\mathfrak{n}'' + b''c'\mathfrak{n}$, $(4, 5) = b'b''\mathfrak{n} - bb'\mathfrak{n}'' - bb''\mathfrak{n}' + \mathfrak{D}\mathfrak{n}\mathfrak{n}'\mathfrak{n}''$, $(4, 6) = b'c''\mathfrak{n} - bc''\mathfrak{n}'$, $(4, 7) = b''c'\mathfrak{n} - bc'\mathfrak{n}''$, $(4, 8) = c'c''\mathfrak{n}$, $(5, 6) = ca'\mathfrak{n}''$, $(5, 7) = ca''\mathfrak{n}'$, $(5, 8) = b'c\mathfrak{n}'' - b''c\mathfrak{n}'$, $(6, 7) = b''c\mathfrak{n}' - b'c\mathfrak{n}''$, $(6, 8) = cc''\mathfrak{n}'$, $(7, 8) = cc'\mathfrak{n}''$, quas per Φ designabimus, nouemque aliae: $(10)(11) - (9)(12) = a\mathfrak{n}'\mathfrak{n}''\mathfrak{A}$, $(1)(12) -$

*) Horum signorum significatio praesens non est confundenda cum ea in qua in art. 234 accepta erant; nam numeri per haec signa *hic* expressi apprime respondent iis, qui in art. 234 per numeros similibus signis *illic* denotatos multiplicabantur.