to within 6 minutes, the angle between the ecliptic and the equator, the annual precession of the equinoxes, the lunar parallax, the eccentricity of the solar orbit, etc. He knew that the moon moves only approximately in a circle with center at the earth, a better approximation being an 'epicycle'. The same was true of the sun. Hipparchus suggested that epicycles of higher orders were necessary to describe the motions of the planets.

In mathematics his great contribution was the founding of trigonometry. He drew up a table giving for each angle with vertex at the center of a circle of radius 1 the length of the chord it cuts off in the circle. For example, suppose $\angle AOB$ is 30°, with O the center of the circle, and $OA = OB = 1$ two radii of the circle. Then the chord in question is the segment AB. This has length 31.06/60, so, in the table of Hipparchus, we would find

$$\text{chord } (30^\circ) = 31.06/60.$$

In modern terms, $\text{chord}(x) = 2\sin(x/2)$.

To construct this table, Hipparchus made use of formulas which we would express as follows:

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

and

$$2\sin^2(x/2) = 1 - \cos x.$$

Heron is known for the formula, probably discovered by Archimedes, which expresses the area of a triangle in terms of the lengths a, b and c of its sides. If $s = (a+b+c)/2$, this formula gives the area of the triangle as

$$\sqrt{s(s-a)(s-b)(s-c)}.$$

We shall meet this formula again when we discuss the mathematics of India.

Menelaus was the first to study spherical trigonometry. He is also known for the following theorem, which is found in his *Spherica.*

Menelaus's Theorem:

> Let ABC be a triangle. Suppose D is on the line through B and C, E is on the line through A and C, and F is on the line through A and B. Suppose that either two or none of D, E, F are on sides of the triangle. Then D, E, F are collinear if and only if $BD \cdot CE \cdot AF = CD \cdot AE \cdot BF$.

Proof: Suppose D, E, F are collinear, in line l. We may suppose l does not pass through A, B or C. Let A', B', C' be points in l such that AA', BB', CC' are all perpendicular to l. Then $BD/CD = BB'/CC'$, $CE/AE = CC'/AA'$

1	α	10	ι	100	ρ
2	β	20	κ	200	σ
3	γ	30	λ	300	τ
4	δ	40	μ	400	υ
5	ϵ	50	ν	500	ϕ
6	f	60	ξ	600	χ
7	ζ	70	o	700	ψ
8	η	80	π	800	ω
9	θ	90	q	900	

TABLE 20.1. Ptolomaic notation

and $AF/BF = AA'/BB'$. Multiplying the three equations together we obtain the result.

The converse is easy.

As his *Tetrabiblos* shows, Ptolemy was a keen believer in the superstition called astrology. In spite of this, he did for astronomy what Euclid had done for geometry and arithmetic: he wrote the definitive textbook, known by its Arabic name, the 'Almagest' or 'Greatest'. Like Hipparchus, Ptolemy gave a table of chords.

Book I of the 'Almagest' contains 'Ptolemy's Theorem': in a cyclic quadrilateral, the product of the diagonals is equal to the sum of the products of the two pairs of opposite sides. He used Greek letters to denote numbers. Curiously, however, he retained two Phoenician letters, corresponding to Latin f and q, which had actually disappeared in Greek. He also added one symbol at the end, giving him a total of 27 symbols, which allowed him to represent the numbers 1 to 9, 10 to 90 and 100 to 900. He also made use of a small circle to denote zero. In his tables, he employed the Babylonian system to denote not only angles, as we still do, but also lengths as had Hipparchus before him. Thus he wrote

$$120°0'0'' = \rho\kappa|\circ|\circ,$$

$$\text{chord}\, 1° = 1°2'50'' = \alpha|\beta|\nu.$$

So he took the radius to be 1.

His underlying decimal notation was based on the alphabetic code in Table 20.1. We have substituted the Latin letters for 6 and 90 and omitted the symbol for 900.

In 250 AD, in Rome, Plotinus was teaching his version of Platonism. At the same time, in Alexandria, Diophantus was writing the *Arithmetica*. This originally contained 13 books. Until 1973 we had only six of these,