APPEND. 75. Si fuerit $m^2.\mathit{fin}.\Phi^2$ major quam $\mathit{cof}.\Phi^2$, seu $\mathit{tang}.\Phi$ major quam $\frac{1}{m}$; ita ut recta BC cum latere Coni opposito Oa sursum divergat, sectio erit Hyperbola, cujus semilatus transversum erit $= \frac{mf.\mathit{fin}.\Phi}{-\mathit{cof}.\Phi^2 + m^2.\mathit{fin}.\Phi^2}$, & semilatus conjugatum $= \frac{nf.\mathit{fin}.\Phi}{\sqrt{(m^2.\mathit{fin}.\Phi^2 - \mathit{cof}.\Phi^2)}}$; ac semilatus rectum $= \frac{nn}{m}f.\mathit{fin}.\Phi$, & anguli, sub quo Asymtotæ Axem in Centro G decussant, tangens erit $= \frac{n}{m}\sqrt{(m^2.\mathit{fin}.\Phi^2 - \mathit{cof}.\Phi^2)}$. Quare Hyperbola erit æquilatera si fuerit $m^2n^2.\mathit{fin}.\Phi^2 - n^2.\mathit{cof}.\Phi^2 = m^2 = (mm+1)nn.\mathit{fin}.\Phi^2 - nn = mm$, seu $\mathit{fin}.\Phi = \frac{\sqrt{(mm+nn)}}{n\sqrt{(1+mm)}}$, & $\mathit{cof}.\Phi = \frac{m\sqrt{(nn-1)}}{n\sqrt{(1+mm)}}$. Ad hoc ergo necesse est ut sit n major unitate, alioquin Hyperbola æquilatera per sectionem hujusmodi produci nequit.

76. Si Conus est rectus, seu $m = n$, tum omnes sectiones, ad has, quas evolvimus referri possunt, quia positio rectæ AB ab arbitrio nostro pendet. At pro Cono scaleno superest, ut investigemus sectiones quæ a plano utcunque oblique ad rectam AB posito formantur. Sit igitur BR intersectio plani secantis cum plano Basis $AEBF$. Ponatur $OB = f$, angulus $OBR = \theta$, & angulus inclinationis secantis ad Basin $= \Phi$; erit, demisso ex O in BR perpendiculo OR, $OR = f.\mathit{fin}.\theta$ & $BR = f.\mathit{cof}.\theta$. Tum, ducta in plano secante recta RC, erit, ob angulum $ORC = \Phi$, $RC = \frac{f.\mathit{fin}.\theta}{\mathit{cof}.\Phi}$ & $OC = \frac{f.\mathit{fin}.\theta.\mathit{fin}.\Phi}{\mathit{cof}.\Phi}$. Si jam sectio ad Axem Coni OC normalis in planum Basis projiciatur, erunt ejus Axes principales secundum rectas AB & EF dispositi, alterque erit ut m alter ut n.

TAB. XXXVI. Fig. 137.

77. In hac sectione projecta ducatur Diameter ef parallela ipsi BR: erit angulus $BOe = \theta$; sitque aOb positio Diametri

metri ejus conjugatæ. Ponatur ſemidiameter $Oa = \mu$, $Oe = \nu$, erit CAP. III.

$$\mu = \frac{\sqrt{(m^4.\,\text{ſin}.\theta^2 + n^4.\,\text{coſ}.\theta^2)}}{\sqrt{(m^2.\,\text{ſin}.\theta^2 + n^2.\text{coſ}.\theta^2)}}$$

&

$$\nu = \frac{mn}{\sqrt{(m^2.\text{ſin}.\theta^2 + n^2.\text{coſ}.\theta^2)}},$$

atque

$$\text{tang}.\ BOb = \frac{mn.\,\text{coſ}.\theta}{mm.\,\text{ſin}.\,\theta},$$

cujus anguli propterea erit

$$\text{Sinus} = \frac{nn.\,\text{coſ}.\,\theta}{\sqrt{(m^4.\text{ſin}.\theta^2 + n^4.\text{coſ}\,\theta^2)}}$$

&

$$\text{Coſinus} = \frac{mm.\,\text{ſin}.\,\theta}{\sqrt{(m^4.\text{ſin}.\theta^2 + n^4.\text{coſ}.\theta^2)}}.$$

Jam eſt angulus $ObR = \theta + BOb$: ergo

$$\text{ſin}.\,ObR = \frac{m^2.\text{ſin}.\theta^2 + n^2.\text{coſ}\,\theta^2}{\sqrt{(m^4.\text{ſin}.\theta^2 + n^4.\text{coſ}.\theta^2)}},$$

&

$$\text{coſ}.\,ObR = \frac{(mm - nn).\,\text{ſin}.\,\theta\ \text{coſ}.\theta}{\sqrt{(m^4.\,\text{ſin}.\theta^2 + n^4.\,\text{coſ}.\theta^2)}}.$$

At eſt

$$\mu\nu = \frac{mn.\sqrt{(m^4.\text{ſin}.\,\theta^2 + n^4\ \text{coſ}.\theta^2)}}{mm.\,\text{ſin}.\,\theta^2 + nn.\text{coſ}.\,\theta^2}.$$

78. Cum igitur ſit $OR = f.\,\text{ſin}.\,\theta$, erit

$$Ob = \frac{OR}{\text{ſin}.\,ObR} = \frac{f.\,\text{ſin}.\,\theta\sqrt{(m^4.\,\text{ſin}.\,\theta^2 + n^4.\,\text{coſ}.\theta^2)}}{m^2.\text{ſin}.\theta^2 + n^2.\text{coſ}.\theta^2}$$

&

$$Rb = \frac{(mm - nn)\,f.\,\text{ſin}.\,\theta.\ \text{coſ}.\,\theta}{m^2.\text{ſin}.\theta^2 + n^2.\text{coſ}.\theta^2}.$$

Hinc, ex Triangulo RbC ad R rectangulo, erit anguli CbR tangens $= \frac{m^2.\,\text{ſin}.\,\theta^2 + n^2.\,\text{coſ}.\,\theta^2}{(m^2 - n^2).\text{ſin}.\theta.\text{coſ}.\,\theta.\text{coſ}.\phi}$: unde, angulus CbR erit cognitus. Jam, ex puncto ſectionis quovis M ad rectam RT ducatur MT parallela ipſi Cb, atque ex M ad Cb pa-

APPEND. rallela MS ipſi RT: vocenturque $bT = MS = t$; $bS = TM = u$; quæ, tanquam Coordinatæ obliquangulæ ſectionis quæſitæ ſpectentur, exiſtente anguli bSM tangente $= \frac{m^2 \; ſin\; \theta^2 + n^2 \; coſ.\; \theta^2}{(m^2 . - n^2)\, ſin.\; \theta.\; coſ\; \theta.\; coſ. \Phi}$. Patet ergo has Coordinatas fieri orthogonales in Cono recto, propterea quia fit $m = n$.

79. Ex puncto ſectionis M ad planum $AEBF$ demittatur perpendiculum MQ; junctaque TQ erit parallela Diametro ab; tum ex Q ducatur ordinata QP alteri Diametro ef parallela. Atque, vocatis $OP = x$; $PQ = y$ & $QM = z$; erit, ex natura Coni

$$\mu^2 \nu^2 z^2 = \mu^2 y^2 + \nu^2 x^2.$$

Namque, ſi per punctum M concipiatur Coni ſectio Baſi parallela, erunt ejus ſemidiametri rectis ab & ef parallelæ μz & νz. At, cum inventa ſint Trianguli rectanguli COb latera OC & Ob, erit Hypothenuſa

$$Cb = \frac{f. ſin. \theta \sqrt{(m^4 . ſin. \theta^2 + n^4 . coſ. \theta^2 - (m^2 - n^2)^2 ſin. \theta^2 . coſ\, \theta^2 . ſin. \Phi^2)}}{(m^2 . ſin. \theta^2 + n^2 .\; coſ.\; \theta^2).\; coſ. \Phi}$$

& ob Triangula TMQ, bCO ſimilia, erit

$$TM(u) : TQ(Ob - x) : QM(z) = bC : Ob : OC$$

ergo $x = Ob - \frac{Ob.\, u}{Cb}$; $z = \frac{OC.\, u}{Cb}$; & $y = t$; ideoque

$$\mu^2 \nu^2 . OC^2 . u^2 = \mu^2 . Cb^2 . t^2 + \nu^2 . Ob^2 \,(Cb - u)^2$$

80. Æquatio hæc evoluta dabit hanc

$$= \mu^2 .\; Cb^2 .\; tt + \nu^2 (Ob^2 - \mu^2 .\; Oc^2)\, uu - 2\nu^2 .\; Ob^2 .\; Cb. u + \nu^2 .\; Ob^2 .\; Cb^2,$$

n qua ſi ponatur $u - \frac{Ob^2 . Cb}{Ob^2 - \mu^2 .\; Oc^2} = s$: ſeu, ſumta $bG = \frac{Ob^2 . Cb}{Ob^2 - \mu^2 .\; Oc^2} = \frac{Cb}{1 - (m^2 . ſin.\; \theta^2 + n .\; coſ.\; \theta^2)\, tang.\; \Phi^2}$, & vocata

cata $GS = s$; erit G Centrum sectionis conicæ cujus æquatio inter Coordinatas t & s erit CAP. III.

$$\mu^2\, Cb^2.tt + \nu^2 (Ob^2 - \mu^2.Oc^2)ss = \frac{\mu^2\nu^2\, Ob^2.\, Oc^2.\, Cb^2}{Ob^2 - \mu^2.Oc^2},$$

cujus semidiameter transversus erit $= \frac{\mu.\, Ob.\, Oc.\, Cb}{Ob^2 - \mu^2.\, Oc^2}$, & semidiameter conjugatus $= \frac{\nu.\, Ob.\, Oc}{\sqrt{(Ob^2 - \mu^2.\, Oc^2)}}$, & semilatus rectum $= \frac{\nu\nu.\, Ob.\, Oc}{\mu.\, Cb}$. Ceterum apparet si sit $tang.\, \Phi$ minor quam $\frac{1}{\sqrt{(m^2.fin.\theta^2 + n^2.cof.\theta^2)}}$, seu $tang.\, \Phi$ minor quam $\frac{\nu}{mn}$, Curvam fore Ellipsin; si sit $tang.\, \Phi = \frac{\nu}{mn}$, Parabolam; & si $tang.\, \Phi$ major quam $\frac{\nu}{mn}$, Hyperbolam.

81. Tertium Corpus, cujus sectiones plano factas hic investigare constituimus, est Globus, cujus quidem omnes sectiones planas Circulos esse ex Geometria elementari constat. Interim tamen quo methodus clarius perspiciatur, quemadmodum ex data æquatione pro Solido quocunque ejus sectiones quævis erui debeant, idem negotium hic analytice absolvam quod vulgo synthetice tradi solet. Sit igitur C Centrum Globi, per quod planum tabulæ transire concipiatur, ita ut sectio hoc plano facta sit Circulus maximus, cujus radius $CA = CB$; ponatur $= a$, qui simul erit radius Globi. Sit porro recta DT intersectio plani secantis cum isto plano tabulæ, ad quam ex C ducatur normalis CD, quæ sit $= f$, angulus autem inclinationis sit $= \varphi$. TAB. XXXVII. *Fig.* 138.

82. Sit M punctum sectionis quæsitæ quodcunque; unde ad planum tabulæ demittatur perpendiculum MQ hincque ad rectam CD pro Axe assumtam perpendicularis QP. Quod si jam vocentur Coordinatæ $CP = x$, $PQ = y$ & $QM = z$; erit, ex natura Globi, $xx + yy + zz = aa$. Ducatur ex M pariter ad rectam DT normalis MT; & juncta QT, ob ambas QT & MT ad DT normales, metietur angulus MTQ

 incli-

APPEND. inclinationem plani ſecantis ad planum Baſis, quæ eſt $= \phi$. Quare ſi DT & MT tanquam Coordinatæ ſectionis quæſitæ ſpectentur, vocenturque $DT = t$, $TM = u$, fiet $MQ = u.\sin.\phi$, & $TQ = u.\cos.\phi$. Erit ergo $CP = x = f - u.\cos.\phi$; $PQ = y = t$; & $QM = z = u.\sin.\phi$. Quibus valoribus ſubſtitutis emerget æquatio pro ſectione Globi quæſita hæc

$$ff - 2fu.\cos.\phi + uu + tt = aa.$$

83. Perſpicuum jam eſt hanc æquationem eſſe pro Circulo. Namque ſi ponatur $u - f.\cos.\phi = s$, fiet

$$ff.\sin.\phi^2 + ss + tt = aa.$$

unde radius ſectionis erit $= \surd(aa - ff.\sin.\phi^2)$. Quare, ſi ex D Applicatæ TM parallela ducatur Dc, in eamque ex Centro C perpendiculum demittatur Cc, ob $CD = f$ & angulum $CDc = \phi$, erit $Dc = f\cos.\phi$ & $Cc = f.\sin.\phi$. Hinc, cum Coordinatæ s & t ad Centrum referantur, erit punctum c Centrum ſectionis, & $\surd(CB^2 - Cc^2)$ radius iſtius Circuli, uti ex Elementis eſt manifeſtum. Simili autem modo omnium aliorum Solidorum, dummodo eorum natura ſit æquatione inter tres variabiles expreſſa, ſectiones quæcunque planis factæ inveſtigari poterunt.

TAB. XXXVII. Fig. 139. 84. Quo tamen tota operatio melius perſpiciatur, proponatur Solidum quodcunque, cujus natura ſit expreſſa æquatione inter ternas Coordinatas $AP = x$, $PQ = y$ & $QM = z$; quarum illæ poſitæ ſint in plano tabulæ hæc vero z ſit ad planum normalis. Secetur jam hoc Solidum plano quocunque, cujus cum plano tabulæ interſectio ſit recta DT, & inclinationis angulus $= \phi$. Ponatur recta $AD = f$, angulus $ADE = \theta$; eritque, demiſſo ex A in DE perpendiculo AE, $AE = f.\sin.\theta$ & $DE = f.\cos.\theta$. Tum, ex ſectionis quæſitæ puncto M ad DT ducatur perpendicularis MT; junctaque QT, æquabitur angulus MTQ inclinationi datæ ϕ. Quare, ſi