Data 609

Mathematical Modeling Techniques for Data Analytics

Final Project Report

Title: Portfolio Optimization

Jim Lung

12/3/2018

Content:

Introduction:

Optimization is used to model the real world decision making problems. Mathematical models is used to make a decision for portfolio optimization Investigating portfolio optimization with expected return on investiment in risk control.

Portfolio optimization is to choose a combination of weight for each investment stock in strategies. The configuration is to find out the investment plan by maximizing the rate of return and minimizing the risk.

Data source:

Historical stock price data are readily accessible using functions in “Quantmod” package The filtered data for this application selects total 9 stock cases from 2017 to August 2018 The datasets include the date, daily market close price, market volumes, the closing price will be used to make for portfolio optimization.

1. Data

Construct a vector of tickers and gather prices for them using the getSymbols function within quantmod. We will next calculate returns and convert the data to a time series object.

```
# The symbols vector holds our tickers.
tickers <- c("AAPL", "AMD", "ADI",  "ABBV", "AET", "A",  "APD", "AA","CF")
# The prices object will hold our raw price data
close_price<-
  getSymbols(tickers,src="yahoo", from = "2017-01-01",
             auto.assign = TRUE, warnings = FALSE) %>%
  map(~Ad(get(.))) %>%   #Extract (transformed) data from a suitable OHLC obj
ect.
  reduce(merge) %>%   #reduce() combines from the left, reduce_right() combin
es from the right
  `colnames<-`(tickers )
```

```
## # A tibble: 6 x 10
##   index          AAPL      AMD      ADI     ABBV      AET        A      APD
##   <date>        <dbl>    <dbl>    <dbl>    <dbl>    <dbl>    <dbl>    <dbl>
## 1 2018-11-20 -0.0478   0.00523  0.0409  -0.0155  -0.0168  0.0784  -0.0146
## 2 2018-11-21 -0.00113 -0.0250   0.00741 -0.0220   0.00147 0.00296  0.0103
## 3 2018-11-23 -0.0254   0.0347  -0.0116  -0.00604  0.00519 0.00724 -0.0106
```

```
## 4 2018-11-26  0.0135   0.0361   0.0161   0.00712  0.0231  0.0152   0.00692
## 5 2018-11-27 -0.00218  0.0483  -0.00544  0.0167   0.00904 0.00159 -0.0146
## 6 2018-11-28  0.0385   0.0138   0.0182   0.0144   0.00330 0.0306   0.0153
## # ... with 2 more variables: AA <dbl>, CF <dbl>
```

Graphical Exploration

Use adjusted closed price to plot graph from 2017 until now:

According to the historial price movements, it indicate the future trends and it concern the maximum of expected return, it is difficult to find out any good investment stratege in plot graph. So we should calculate the return by daily, weekly or yearly to decide the short , long term investment.

2. Compute daily, monthly and yearly return

	ave_yearly_return
AAPL	0.6044175
AMD	0.8627081
ADI	0.3042133
ABBV	0.5339709
AET	0.7718944
A	0.5605331
APD	0.1653859
AA	0.1571870
CF	0.4239632

To compare the average yearly return, AMD and AET are the most increasing return yearly, but it can't indicate the risk only in average yearly return.

3.Linear Programming - Mean Variance model

Investors are risk averse in that they prefer higher return for a given level of risk (variance, standard deviation), or they want to minimize risk for a given level of returns, so we go to minimize the variance and maximize the return.

Suppose data are observed for N securities, over T time periods. Let

y_{jt} = Return on one dollar invested in security j in time period t.

$\bar{y_j}$ =Average Return on security j

$w_j =$ Portfolio allocation to security j.

y_{pt} = Return on portfolio in time period t

E_p =Average Return on portfolio

M_p = Minimum return on portfolio

The objective function:

$$min \sum_{j=k}^{N} \sum_{j=1}^{N} w_j w_k s_{jk}$$

subject to:

$$\sum_{j=1}^{N} w_j \bar{y_j} \geq G$$

with:

$$s_{jk} = \frac{1}{T-N} \sum_{t=1}^{T} (y_{jt} - \bar{y_j})(y_{kt} - \bar{y_k})$$

```
##      Weight    Ave.Return      Stdev         Sharp
## AAPL 0.0283  0.0138927012 0.10329584  0.134494296
## AMD  0.0000  0.0729485380 0.23835453  0.306050566
## ADI  0.2142  0.0025407452 0.06428978  0.039520204
## ABBV 0.1270 -0.0153324794 0.09888394 -0.155055300
## AET  0.0001  0.0148118327 0.04606748  0.321524677
## A    0.2655 -0.0004756023 0.06037735 -0.007877164
## APD  0.0436 -0.0032581455 0.03793639 -0.085884435
## AA   0.1120 -0.0403532553 0.08293752 -0.486550045
## CF   0.2093  0.0068655083 0.09323691  0.073635089
```

The average monthly return of the portofolio at the evenly distributed allocation is 6.8 %. After optimization, the average monthly return of portfolio is -0.436 % when the global variance is at minimum 0.048. The maximized monthly return of portfolio is 1.904 % when the global variance is 0.0915.

4.Linear Programming - Minimax Model

The minimax model will maximize return with respect to one of these prior distributions providing valuable insight regarding an investor's risk attitude and decision behavior.

Suppose data are observed for N securities, over T time periods. Let

y_{jt} = Return on one dollar invested in security j in time period t.

$\bar{y_j}$ =Average Return on security j

$w_j =$ Portfolio allocation to security j.

y_{pt} = Return on portfolio in time period t

E_p =Average Return on portfolio

M_p = Minimum return on portfolio

The objective function:

$$max\, M_p$$

subject to:

$$\sum_{j=1}^{N} w_j y_{jt} \geq M_p$$

t = 1,…,T

$$\sum_{j=1}^{N} w_j \bar{y_j} \geq G$$

```
##      Weight Worst.Return    Ave.Return      Stdev         Sharp
## AAPL 0.0435  -0.08940314  0.0138927012 0.10329584  0.134494296
## AMD  0.0000  -0.16540599  0.0729485380 0.23835453  0.306050566
## ADI  0.2236  -0.06174904  0.0025407452 0.06428978  0.039520204
## ABBV 0.0001  -0.11421642 -0.0153324794 0.09888394 -0.155055300
## AET  0.1569  -0.03125565  0.0148118327 0.04606748  0.321524677
## A    0.2356  -0.06085295 -0.0004756023 0.06037735 -0.007877164
## APD  0.0318  -0.04119454 -0.0032581455 0.03793639 -0.085884435
## AA   0.0005  -0.12329078 -0.0403532553 0.08293752 -0.486550045
## CF   0.3079  -0.08637140  0.0068655083 0.09323691  0.073635089
```

Average monthly return is 8.6%, After optimization, mininum average loss is 6.49 % when variance is 1e+07.

5. linear programming vs log returns:

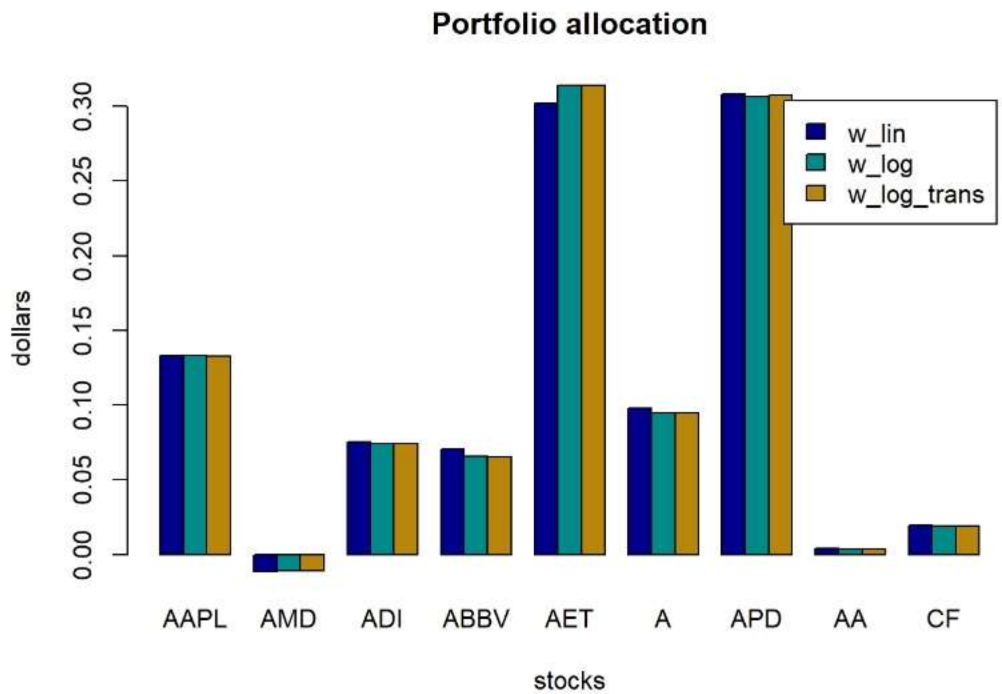
Modeling linear vs log returns: Now we are ready to obtain the sample estimates from the returns$\mathbf{x}_t$

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t$$

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{T-1}\sum_{t=1}^{T}(\mathbf{x}_t - \hat{\mu})(\mathbf{x}_t - \hat{\mu})^T$$

Daily rebalancing We will start with a daily rebalancing since we already have the daily returns readily available.

```
##             w_lin        w_log  w_log_trans
## AAPL  0.133266875  0.133093599  0.132663160
## AMD  -0.011305201 -0.010945478 -0.010885562
## ADI   0.075478176  0.074269407  0.074486872
## ABBV  0.070315726  0.065447418  0.065214751
## AET   0.302006519  0.313954359  0.313825748
## A     0.098051485  0.094692532  0.094554219
## APD   0.308204376  0.306635367  0.307459326
## AA    0.004160171  0.003564821  0.003389748
## CF    0.019821873  0.019287975  0.019291738
```



By portfolio allocation, AAPL, AET and APD are shown the most positive in investing value, but it is not significate in difference between log and tranformation.

6. Qudratic programming

Modelling portfolio optimization as quadratic program

Uniform portfolio

The uniform portfolio allocates equal weight to each stock: $\mathbf{w} = \frac{1}{N}\mathbf{1}$

GMVP

The Global Minimum Variance Portfolio (GMVP) is formulated as

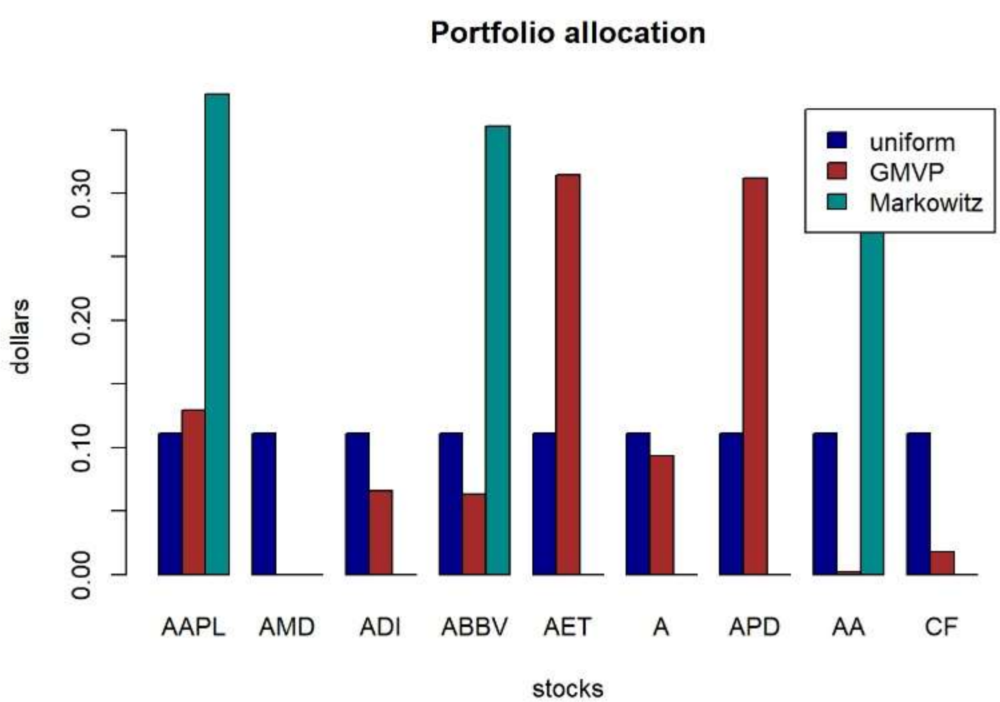
$$\begin{array}{ll} \underset{\mathbf{w}}{\text{minimize}} & \mathbf{w}^T\mathbf{\Sigma}\mathbf{w} \\ \text{subject to} & \mathbf{1}^T\mathbf{w} = 1 \\ & \mathbf{w} \geq \mathbf{0} \end{array}$$

Markowitz portfolio

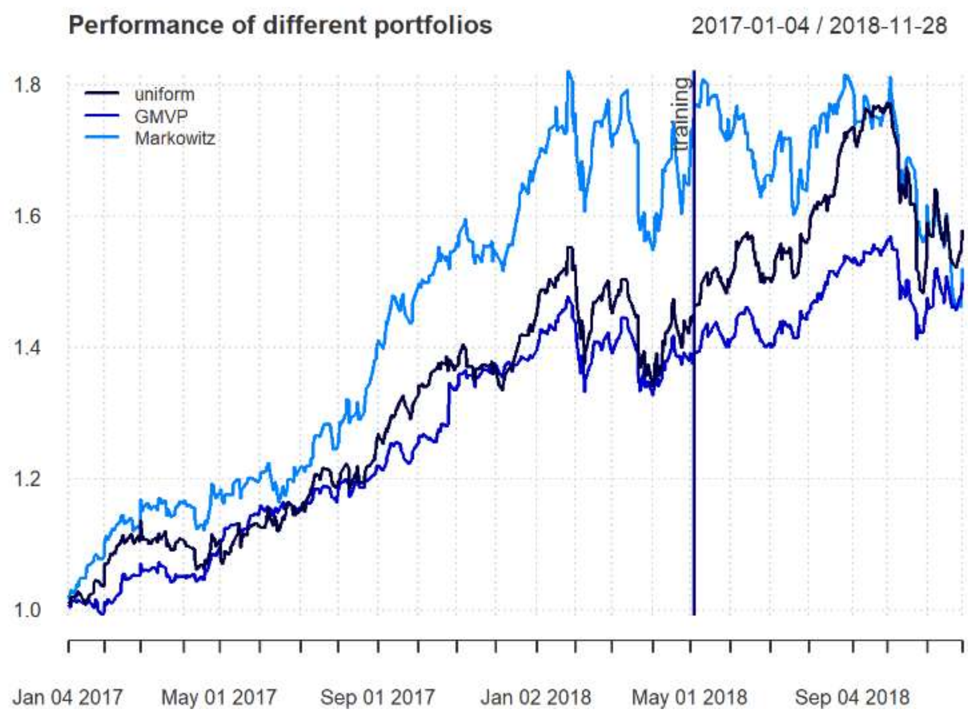
The mean-variance Markowitz portfolio with no shorting is formulated as:

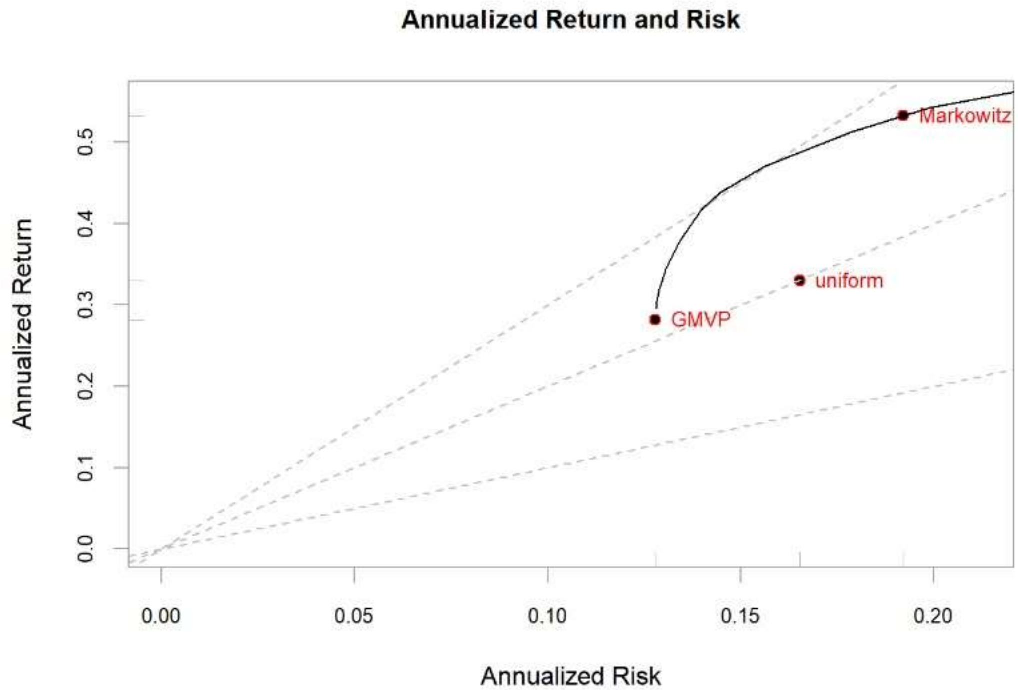
$$\begin{array}{ll} \underset{\mathbf{w}}{\text{maximize}} & \boldsymbol{\mu}^T\mathbf{w} - \lambda\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w} \\ \text{subject to} & \mathbf{1}^T\mathbf{w} = 1 \\ & \mathbf{w} \geq \mathbf{0} \end{array}$$

```
##      uniform GMVP Markowitz
## AAPL    0.11 0.13      0.38
## AMD     0.11 0.00      0.00
## ADI     0.11 0.07      0.00
## ABBV    0.11 0.06      0.35
## AET     0.11 0.31      0.00
## A       0.11 0.09      0.00
## APD     0.11 0.31      0.00
## AA      0.11 0.00      0.27
## CF      0.11 0.02      0.00
```



```
##                              uniform   GMVP Markowitz
## Annualized Return             0.3301 0.2820    0.5328
## Annualized Std Dev            0.1653 0.1279    0.1921
## Annualized Sharpe (Rf=0%)   1.9970 2.2045    2.7741
```





From the graph of annualized Return and Risk, Markowitz optimization has larger annualized return and risk.

Conclusion

We can conclude with the following points:

To compare the average yearly return, AMD and AET are the most increasing return yearly.

It seems that using linear returns or log-returns does not make any significant difference for daily, weekly, and monthly returns.

Mean-variance Markowitz portfolio has larger annualized return and risk, and Global Minimum Variance Portfolio (GMVP) has lower annualized return and risk.