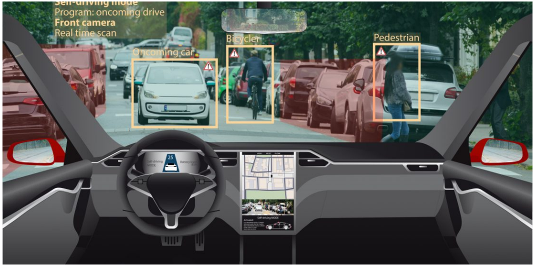
Supervised Learning II

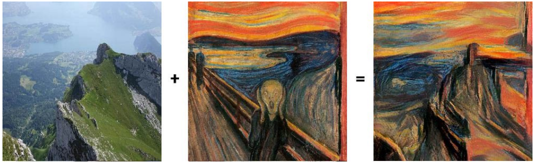
Deep Learning

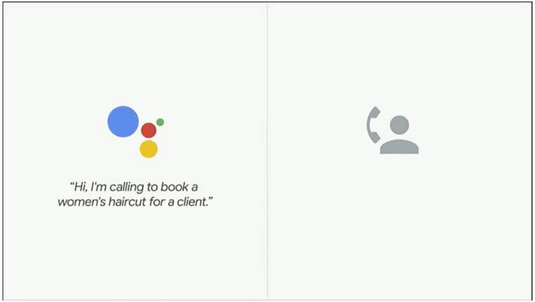
http://funkey.science/deep_learning.pdf

September 25, 2018

Deep Learning

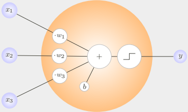




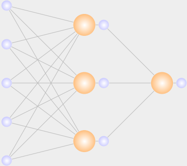


Outline

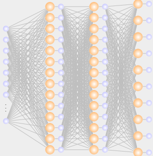
1. Perceptrons



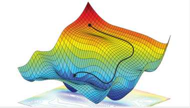
2. Networks



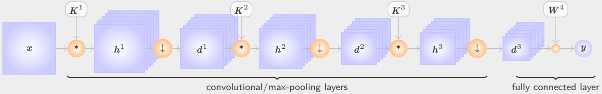
3. Deep Networks



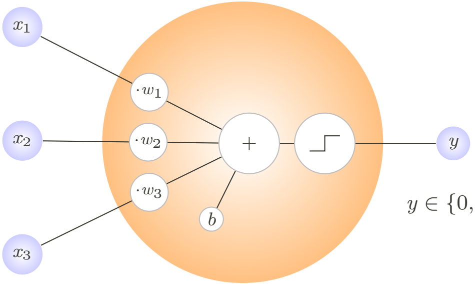
4. Learning



5. Convolutional Networks



The Building Block of Neural Networks



$y \in \{0, 1\}$

$x_i \in \mathbb{R}$

Perceptron

- invented by McCulloch and Pitts in 1943
- termed "perceptron" by Rosenblatt in 1958
- also known as: Threshold Logic Unit, Linear Threshold Unit, McCulloch-Pitts model

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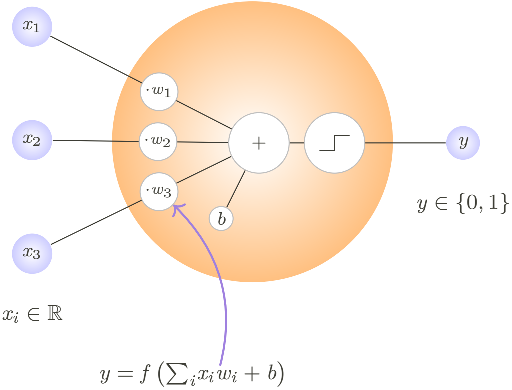

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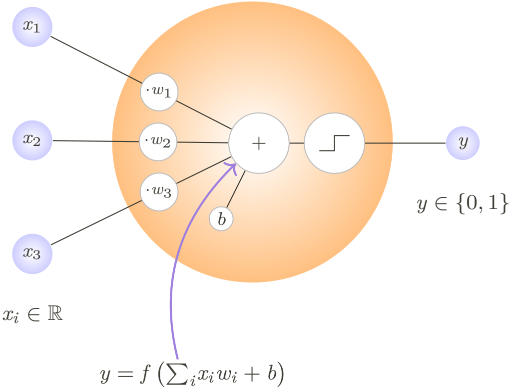

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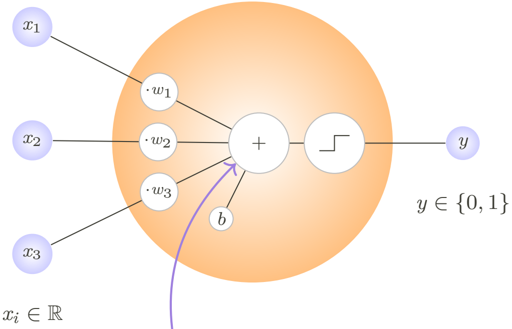

$$y = f\left(\sum_i x_i w_i + b\right) = f(\boldsymbol{x}^\intercal \boldsymbol{w} + b)$$

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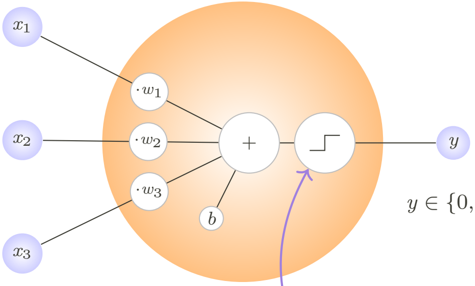

$y \in \{0, 1\}$

$x_i \in \mathbb{R}$

$$y = f\left(\sum_i x_i w_i + b\right) = f(\boldsymbol{x}^\mathsf{T} \boldsymbol{w} + b)$$

$$f(a) = \begin{cases} 0 & \text{if } a \leq 0 \\ 1 & \text{otherwise} \end{cases}$$

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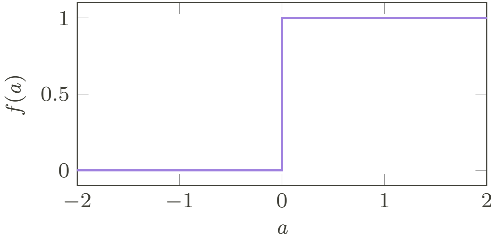
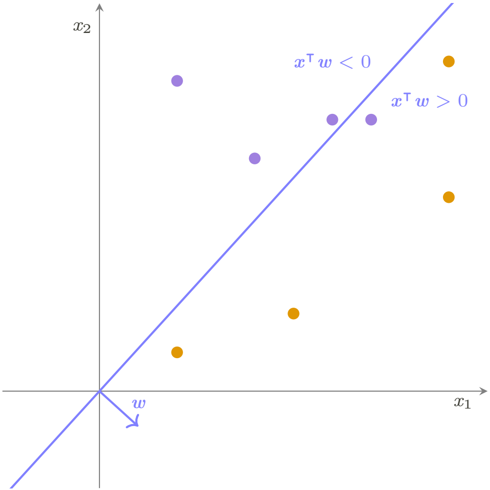
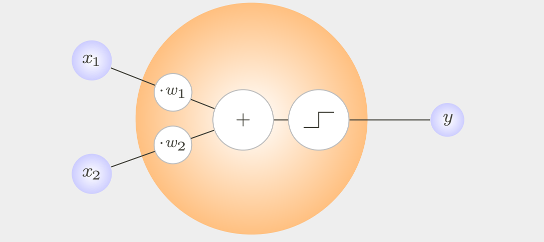


Just a Linear Classifier



Example with 2 inputs



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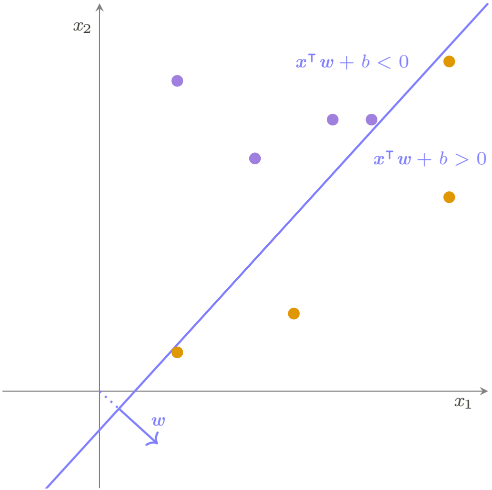

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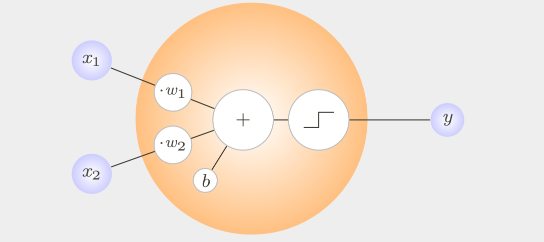

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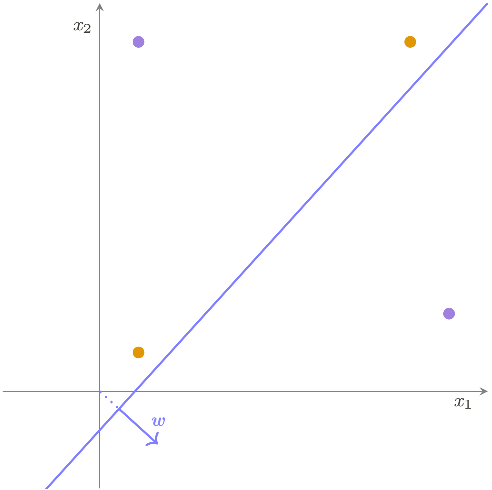

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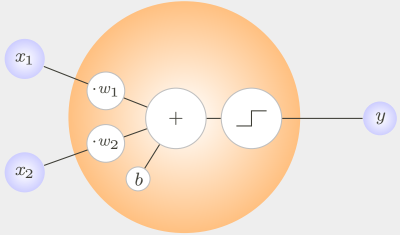
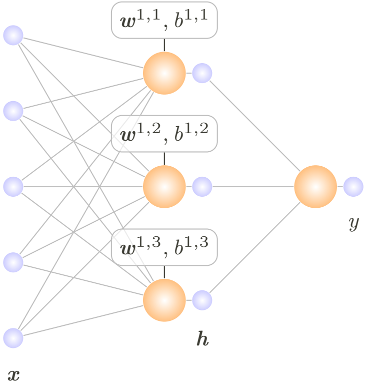


The XOR Affair and AI Winter

- "The perceptron may eventually be able to learn, make decisions, and translate languages." –Rosenblatt, 1958
- "Some functions are tricky." –Minsky/Papert, 1969
- interest in neural networks declined until the 80s

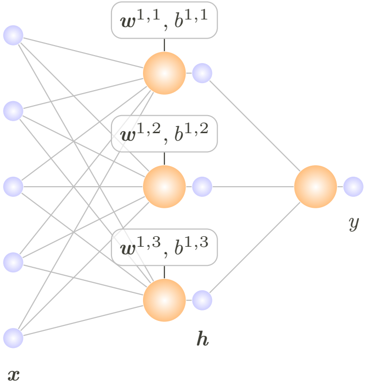
[1]

Neural Networks



Output Function with one Hidden Layer

$$\boldsymbol{h} = \left(f(\boldsymbol{w^{1,1}}\boldsymbol{x} + b^{1,1}), f(\boldsymbol{w^{1,2}}\boldsymbol{x} + b^{1,2}), f(\boldsymbol{w^{1,3}}\boldsymbol{x} + b^{1,3})\right)$$



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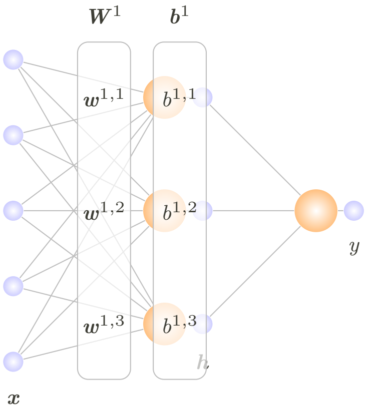

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$$
\begin{aligned}
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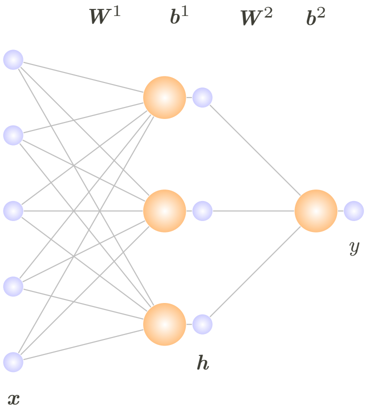
&= f(\boldsymbol{W}^1\boldsymbol{x} + \boldsymbol{b}^1)
\end{aligned}
$$



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\end{aligned}
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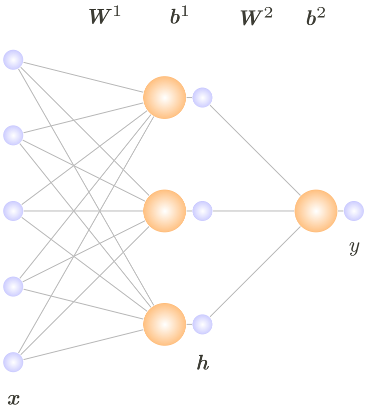
$$
y = f(\boldsymbol{W}^2\boldsymbol{h} + \boldsymbol{b}^2) = f(\boldsymbol{W}^2 f(\boldsymbol{W}^1\boldsymbol{x} + \boldsymbol{b}^1) + \boldsymbol{b}^2)
$$



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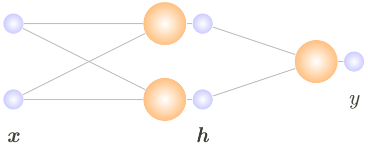
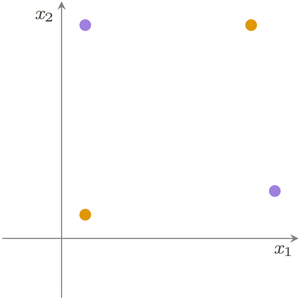
$$y = f(\boldsymbol{W}^2\boldsymbol{h} + \boldsymbol{b}^2) = f(\boldsymbol{W}^2 f(\boldsymbol{W}^1\boldsymbol{x} + \boldsymbol{b}^1) + \boldsymbol{b}^2)$$

- just matrix multiplications and element-wise non-linearities

XOR Revisited





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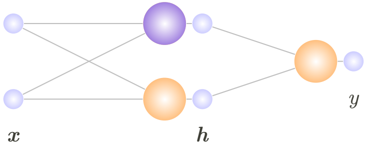

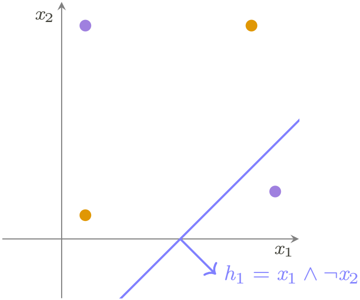

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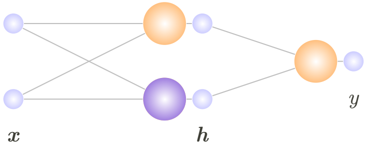

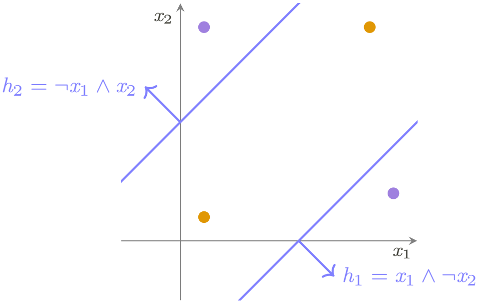

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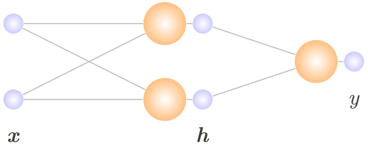

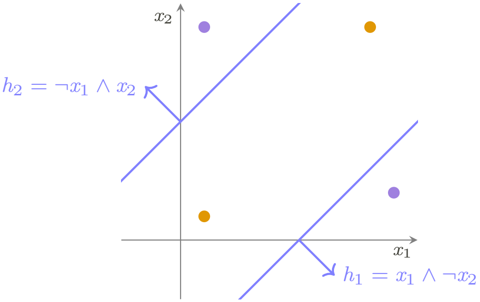

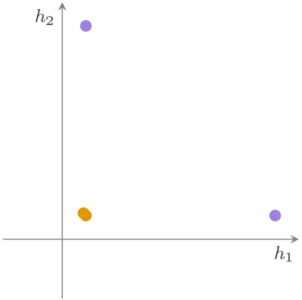

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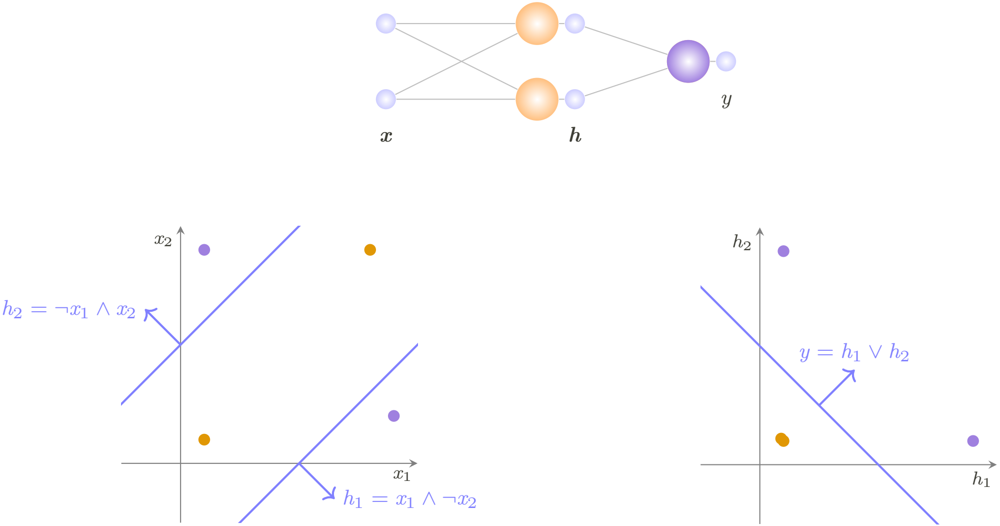


How many layers do we need?

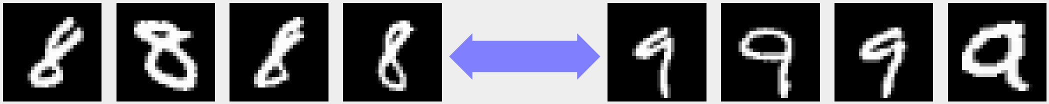
Theoretically...

...one hidden layer is sufficient to model any function:

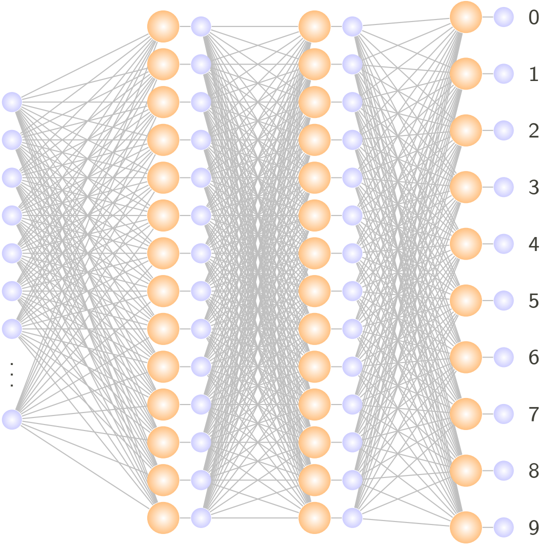
- every boolean function can be transformed into disjunctive normal form [2]
- every continuous function can be approximated with one hidden layer [3]

Practically...

...more than one layer will be more efficient:

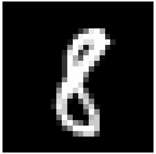


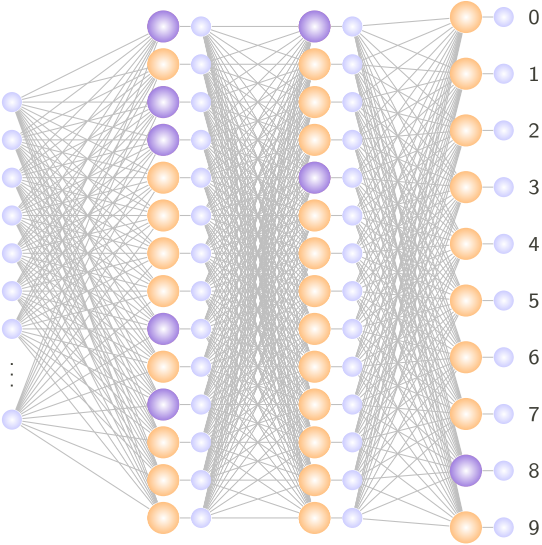
Example: Digit Recognition



[4]

Example: Digit Recognition





[4]

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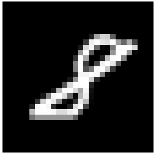

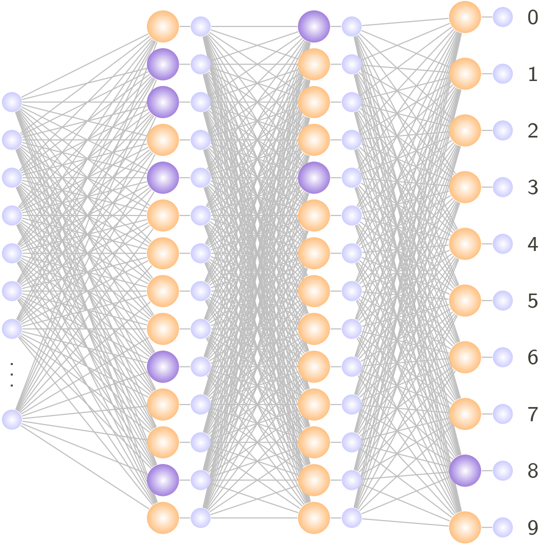

[4]

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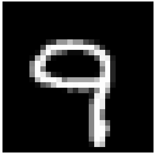

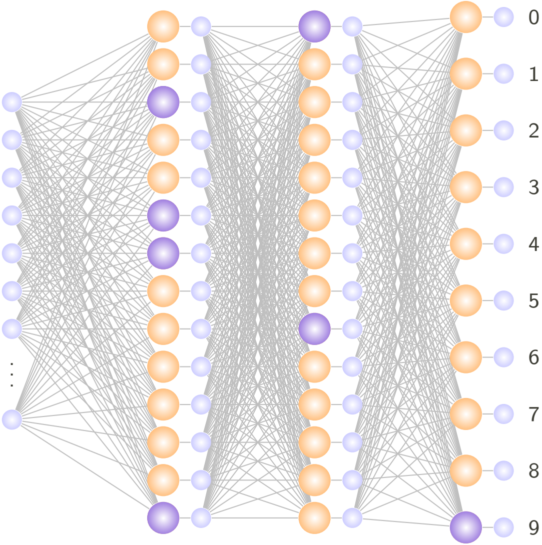

[4]

Supervised Learning

Model

- function $M_{\boldsymbol{\theta}} : \mathbb{R}^n \mapsto \mathbb{R}^m$, $\hat{\boldsymbol{y}} = M_{\boldsymbol{\theta}}(x)$
- parameterized by $\boldsymbol{\theta}$

Data

- $T = \{\boldsymbol{x}^i, \boldsymbol{y}^i\}_{i=1,\ldots,k}$
- input $\boldsymbol{x}$
- desired output $\boldsymbol{y}$

Objective

- $l : \mathbb{R}^m \times \mathbb{R}^m \mapsto \mathbb{R}$, e.g., $l(\hat{\boldsymbol{y}}, \boldsymbol{y}) = \frac{1}{2}|\hat{\boldsymbol{y}} - \boldsymbol{y}|^2$
- $L(\boldsymbol{\theta}) = \frac{1}{k}\sum_{i=1}^{k} l(\hat{\boldsymbol{y}}^i, \boldsymbol{y}^i)$

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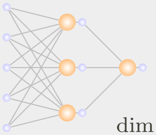

$\dim(\boldsymbol{\theta}) \gg 1$

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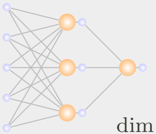

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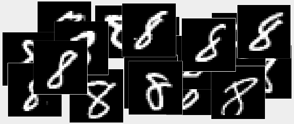

lots of it

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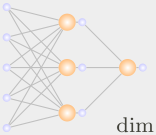

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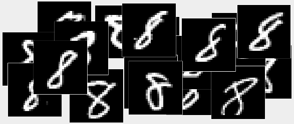

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smooth, differentiable

Learning by Gradient Descent

Optimization Problem

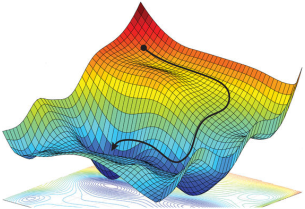
Learning objective:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \; L(\theta)$$

- in general no closed-from solution
- probably not even wanted

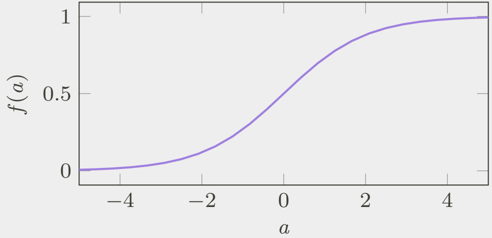
$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^t - \alpha \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta})$$

- fallback to gradient descent

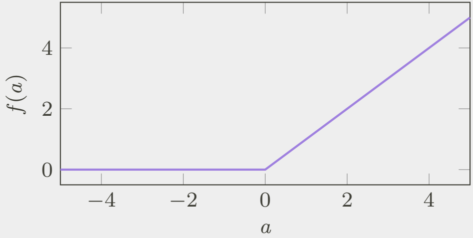


Differentiable Non-linearities

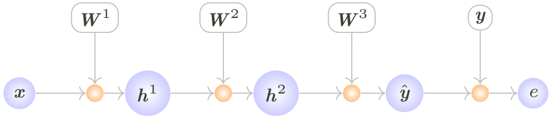
Sigmoid activation function:



Rectified linear unit (ReLU) activation function:



Error Back Propagation



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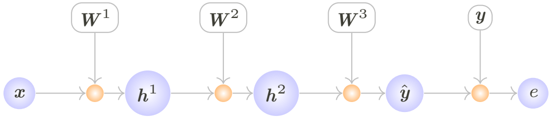
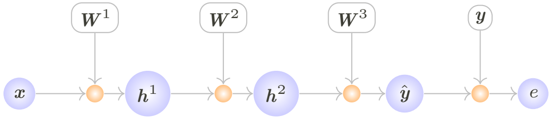


$$
\begin{aligned}
\hat{\boldsymbol{y}} &= M_{\boldsymbol{\theta}}(\boldsymbol{x}) \;\; (\text{with } \boldsymbol{\theta} = \boldsymbol{W}^1, \boldsymbol{W}^2, \boldsymbol{W}^3) \\
&= f(\boldsymbol{W}^3 \boldsymbol{h}^2) \\
&= f(\boldsymbol{W}^3 f(\boldsymbol{W}^2 \boldsymbol{h}^1)) \\
&= f(\boldsymbol{W}^3 f(\boldsymbol{W}^2 f(\boldsymbol{W}^1 \boldsymbol{x})))
\end{aligned}
$$

$$
e = L(\boldsymbol{\theta}) = \frac{1}{k} \sum_{i=1}^{k} l(\hat{\boldsymbol{y}}, \boldsymbol{y})
$$

How to compute $\nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}) = \frac{\mathrm{d}e}{\mathrm{d}\boldsymbol{\theta}}$?

Error Back Propagation



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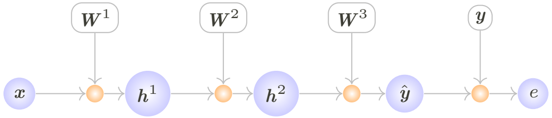
Chain Rule

$$\frac{\mathrm{d}f(g(x))}{\mathrm{d}x} = \frac{\mathrm{d}f(g(x))}{\mathrm{d}g(x)} \frac{\mathrm{d}g(x)}{\mathrm{d}x}$$

Error Back Propagation



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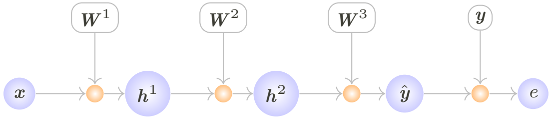
Error Back Propagation



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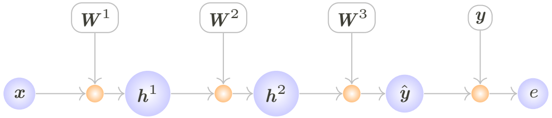
Error Back Propagation



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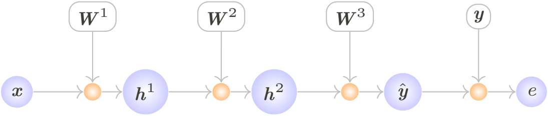
$$\frac{\mathrm{d}e}{\mathrm{d}\boldsymbol{W}^1} = \frac{\mathrm{d}e}{\mathrm{d}\hat{\boldsymbol{y}}} \frac{\mathrm{d}\hat{\boldsymbol{y}}}{\mathrm{d}\boldsymbol{h}^2} \frac{\mathrm{d}\boldsymbol{h}^2}{\mathrm{d}\boldsymbol{h}^1} \frac{\mathrm{d}\boldsymbol{h}^1}{\mathrm{d}\boldsymbol{W}^1}$$



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$$
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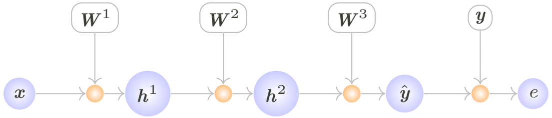
$$

$$
\frac{\mathrm{d}e}{\mathrm{d}\boldsymbol{W}^i} = \frac{\mathrm{d}e}{\mathrm{d}\boldsymbol{h}^i} f'(\boldsymbol{W}^i \boldsymbol{h}^{i-1}) \boldsymbol{h}^{i-1}
$$

Error Back Propagation



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$$= f(W^3 h^2)$$
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$$\frac{de}{dW^3} = \frac{de}{d\hat{y}}\frac{d\hat{y}}{dW^3} = \frac{de}{d\hat{y}} f'(W^3 h^2) h^2$$

$$\frac{de}{dW^2} = \frac{de}{d\hat{y}}\frac{d\hat{y}}{dh^2}\frac{dh^2}{dW^2}$$

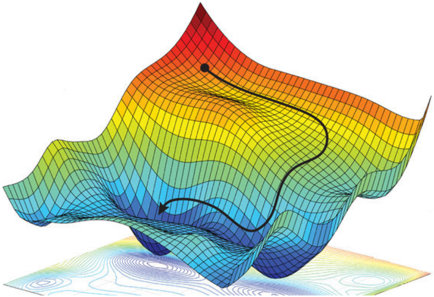
$$\frac{de}{dW^1} = \frac{de}{d\hat{y}}\frac{d\hat{y}}{dh^2}\frac{dh^2}{dh^1}\frac{dh^1}{dW^1}$$

$$\frac{de}{dW^i} = \frac{de}{dh^i} f'(W^i h^{i-1}) h^{i-1}$$

$$\frac{de}{dh^i} = \frac{de}{dh^{i+1}}\frac{dh^{i+1}}{dh^i} = \frac{de}{dh^{i+1}}\left(f'(W^{i+1}h^i)\, W^{i+1}\right)$$

Sampling Strategies

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^{t} - \alpha \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta})$$



Gradient Descent

$$L(\boldsymbol{\theta}) = \frac{1}{k} \sum_{i=1}^{k} l(\hat{\boldsymbol{y}}, \boldsymbol{y})$$

- impractical for large k

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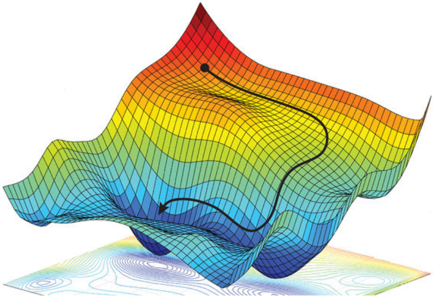

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Stochastic Gradient Descent

$$L(\boldsymbol{\theta}) = l(\hat{\boldsymbol{y}}^i, \boldsymbol{y}^i) \;\; \text{with } i \text{ random in } [1, \ldots, k]$$

- randomly sampled training example

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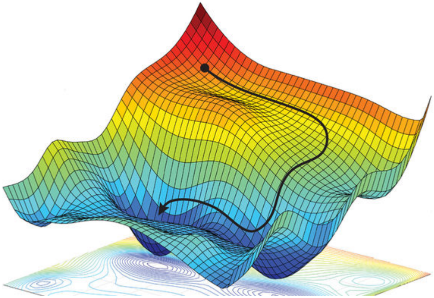

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$$L(\boldsymbol{\theta}) = \frac{1}{k} \sum_{i=1}^{k} l(\hat{\boldsymbol{y}}, \boldsymbol{y})$$

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Stochastic Gradient Descent

$$L(\boldsymbol{\theta}) = l(\hat{\boldsymbol{y}}^{i}, \boldsymbol{y}^{i}) \text{ with } i \text{ random in } [1, \ldots, k]$$

- randomly sampled training example

Batch Gradient Descent

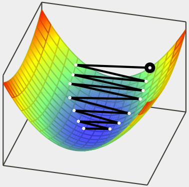
$$L(\boldsymbol{\theta}) = \frac{1}{b} \sum_{i=1}^{b} l(\hat{\boldsymbol{y}}^{p_i}, \boldsymbol{y}^{p_i}) \text{ with } p_i \text{ random in } [1, \ldots, k]$$

- randomly sampled set of training examples

Optimization Algorithms

Plain Gradient Descent

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^{t} - \alpha \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}^{t})$$



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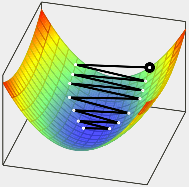
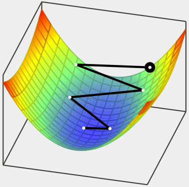


Gradient Descent with Momemtum

$$\boldsymbol{\delta}^t = \alpha \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}^t) + \beta \boldsymbol{\delta}^{t-1}$$
$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^t - \boldsymbol{\delta}^t$$



Optimization Algorithms

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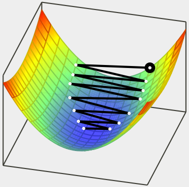

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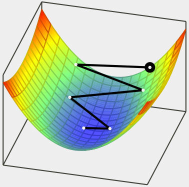
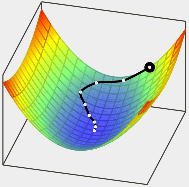


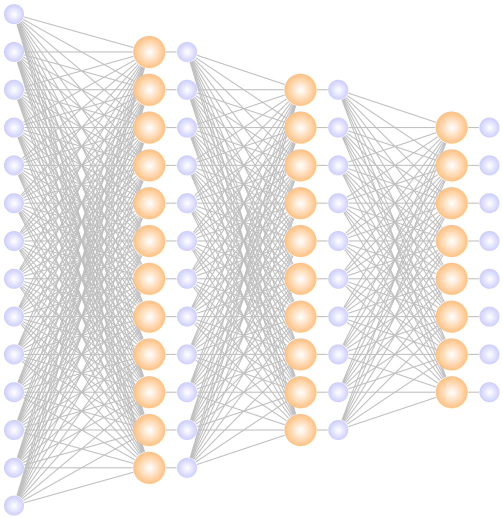
Adaptive Moment Estimation (Adam)

$$\begin{aligned}
\boldsymbol{g}^{t} &= \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}^{t}) \\
\boldsymbol{m}^{t} &= \beta_1 \boldsymbol{m}^{t-1} + (1-\beta_1) \boldsymbol{g}^{t} \\
\boldsymbol{v}^{t} &= \beta_2 \boldsymbol{v}^{t-1} + (1-\beta_2) \left(\boldsymbol{g}^{t} \odot \boldsymbol{g}^{t}\right) \\
\boldsymbol{\theta}^{t+1} &= \boldsymbol{\theta}^{t} - \alpha \frac{\boldsymbol{m}^{t}}{\sqrt{\boldsymbol{v}^{t}} + \epsilon}
\end{aligned}$$

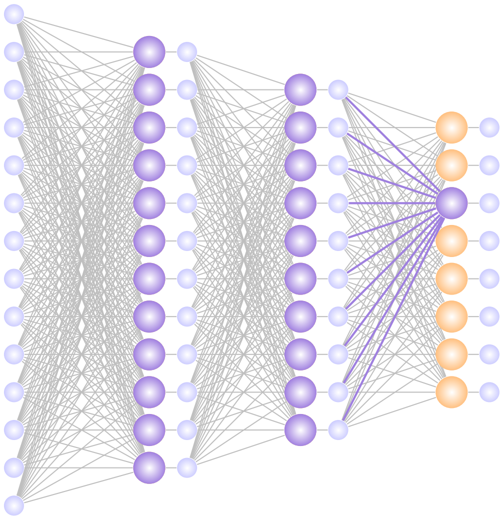


[5]

Receptive Fields

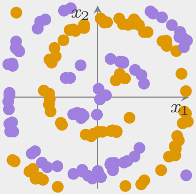


Receptive Fields

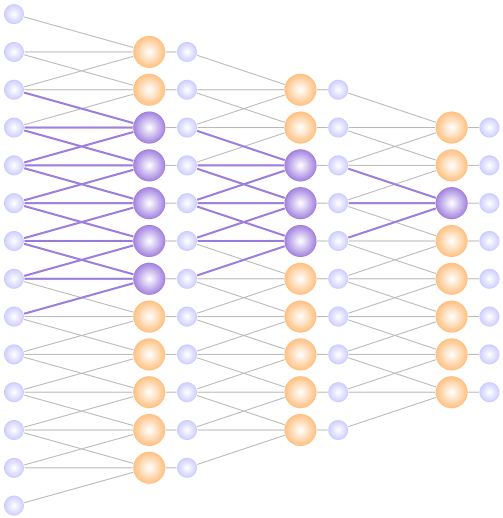


Fully Connected

- if inputs are unstructured



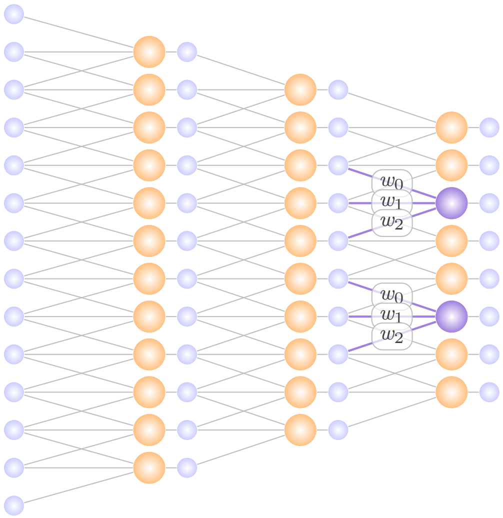
Receptive Fields



Locally Connected and Shared Weights

- if inputs have spatial/temporal structure

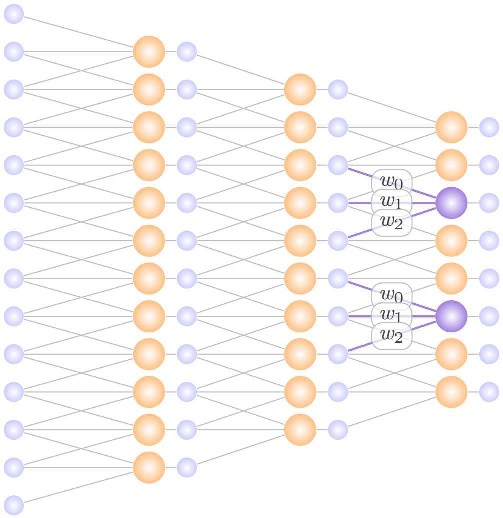
Receptive Fields



Locally Connected and Shared Weights

- if inputs have spatial/temporal structure
- if outputs are equivariant to inputs

Receptive Fields



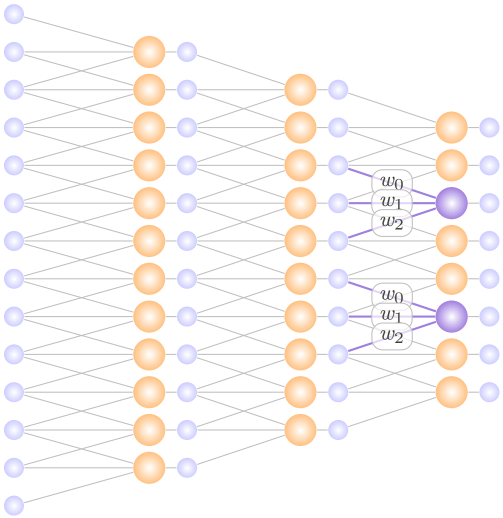
Locally Connected and Shared Weights

- if inputs have spatial/temporal structure
- if outputs are equivariant to inputs

$$y_i = f(\boldsymbol{W}\boldsymbol{h} + \boldsymbol{b})_i$$

$$= f\left(\sum_{j=0}^{2} h_{i+j} w_j + b_i\right)$$

$$\boldsymbol{y} = f(\boldsymbol{h} * \boldsymbol{k} + \boldsymbol{b}) \ \text{ with } \boldsymbol{k} = (w_j)_{j=0,\ldots,2}$$



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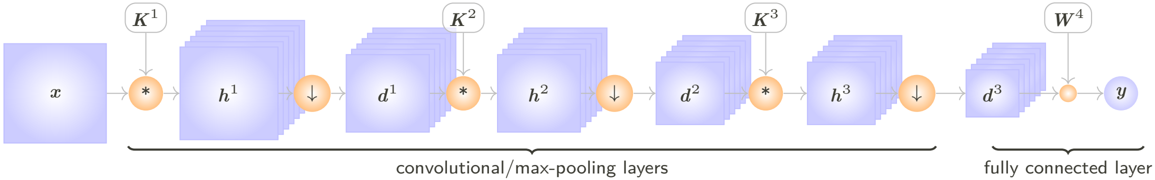
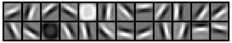
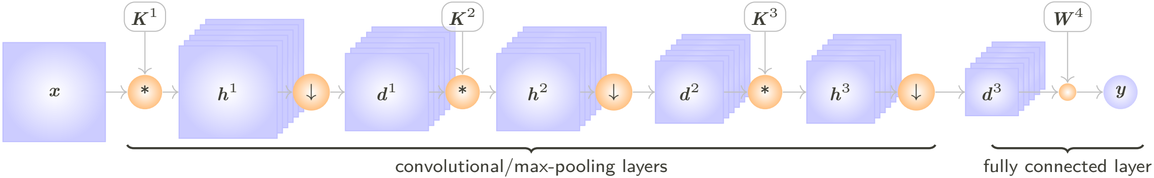
$\rightarrow$ convolutional neural networks (CNNs)

Convolutional Neural Networks

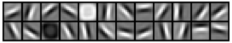


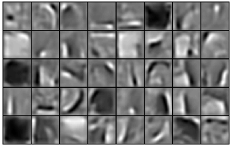
Convolutional Neural Networks

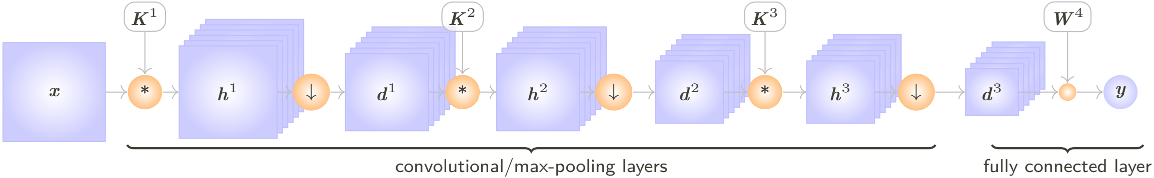




Convolutional Neural Networks







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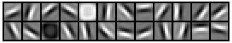
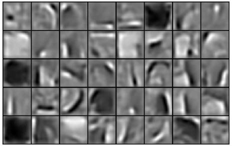
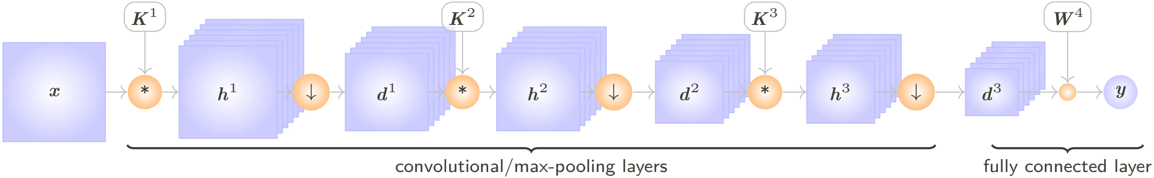
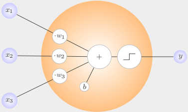




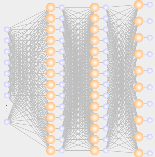


Summary

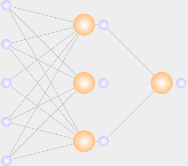
1. Perceptrons



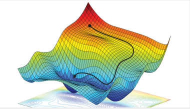
3. Deep Networks



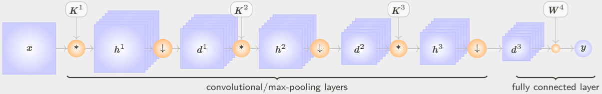
2. Networks



4. Learning



5. Convolutional Networks



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