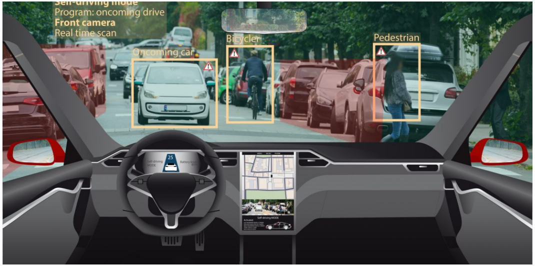
Introduction to Deep Learning

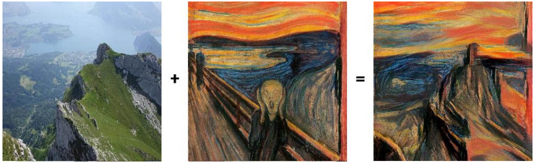
Deep Learning Workshop at MBL

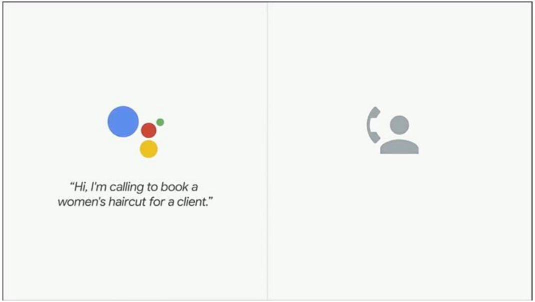
http://funkey.science/deep_learning_mbl.pdf

May 11, 2019


KE JIE
00:46:57

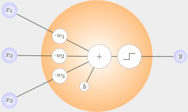

Program: oncoming drive
Front camera
Real time scan
Oncoming car
Bicycler
Pedestrian



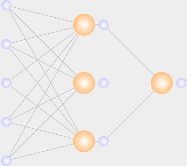

“Hi, I'm calling to book a women's haircut for a client.”

Outline

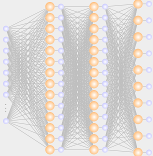
1. Perceptrons



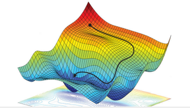
2. Networks



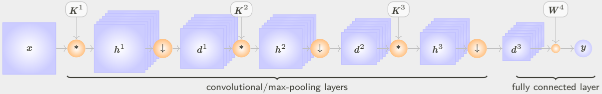
3. Deep Networks



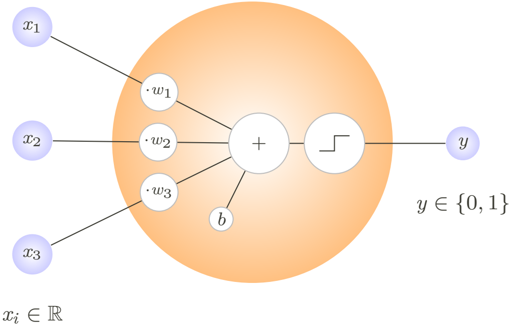
4. Learning



5. Convolutional Networks

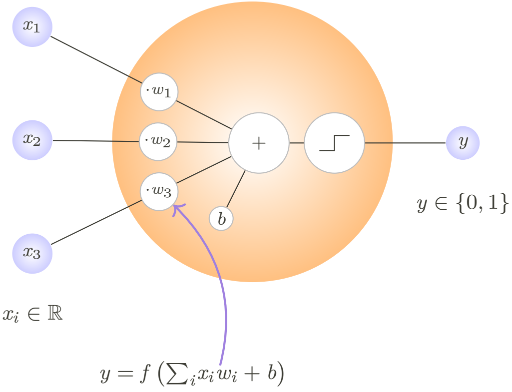


The Building Block of Neural Networks



Perceptron

- invented by McCulloch and Pitts in 1943
- termed "perceptron" by Rosenblatt in 1958
- also known as: Threshold Logic Unit, Linear Threshold Unit, McCulloch-Pitts model



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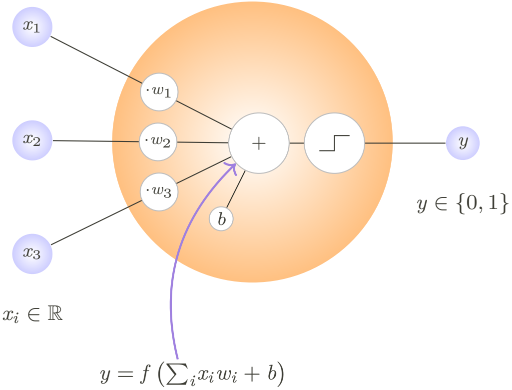

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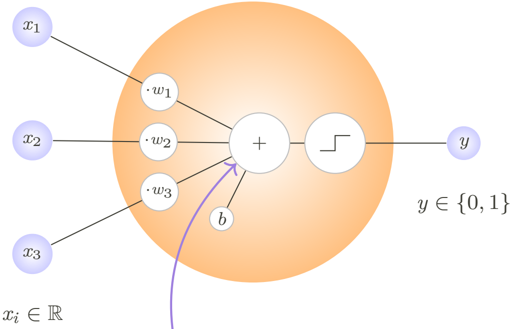

$$y = f\left(\sum_i x_i w_i + b\right) = f(\boldsymbol{x}^\intercal \boldsymbol{w} + b)$$

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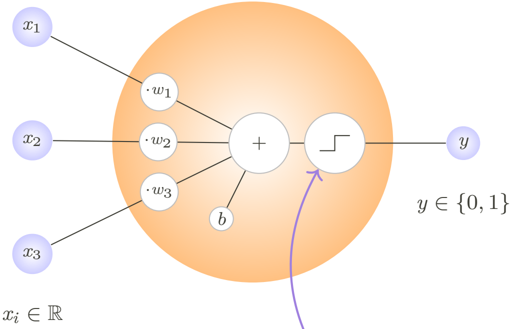

$$y = f\left(\sum_i x_i w_i + b\right) = f(\boldsymbol{x}^\intercal \boldsymbol{w} + b)$$

$$f(a) = \begin{cases} 0 & \text{if } a \leq 0 \\ 1 & \text{otherwise} \end{cases}$$

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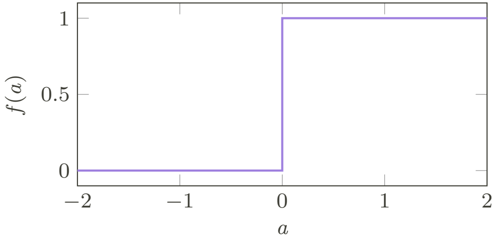
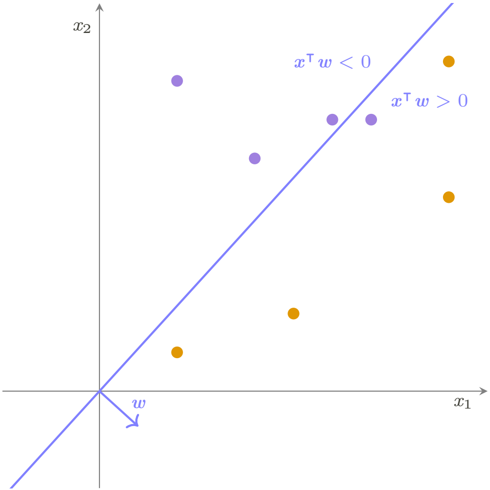
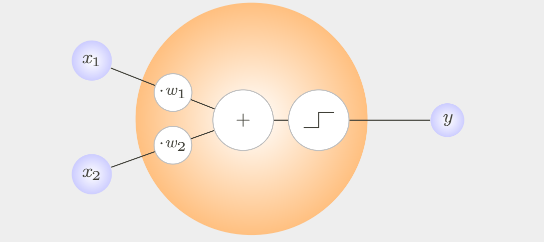


Just a Linear Classifier



Example with 2 inputs



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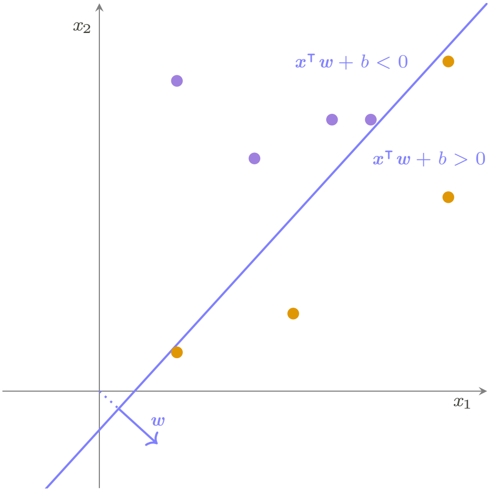

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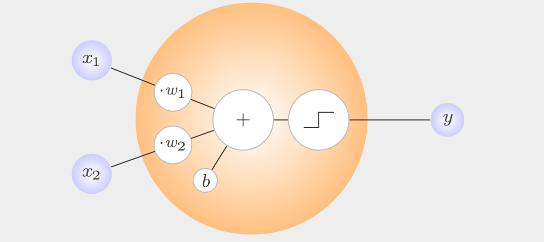

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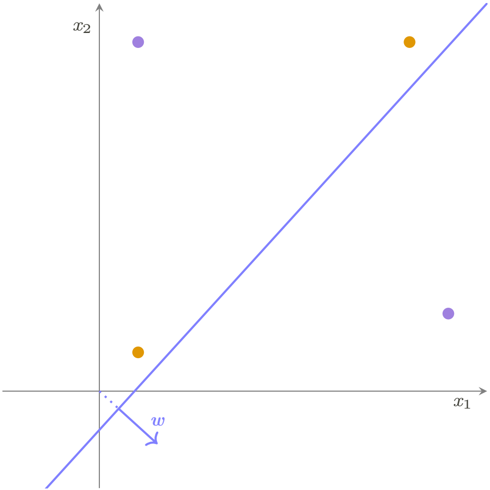

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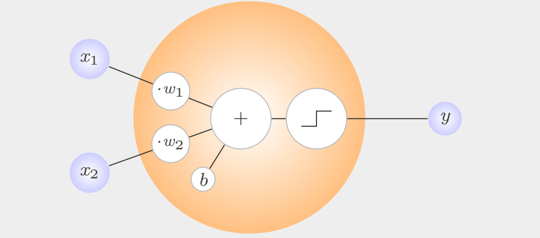
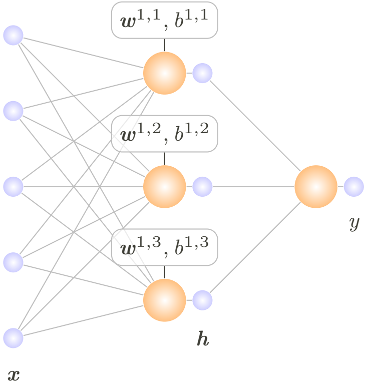


The XOR Affair and AI Winter

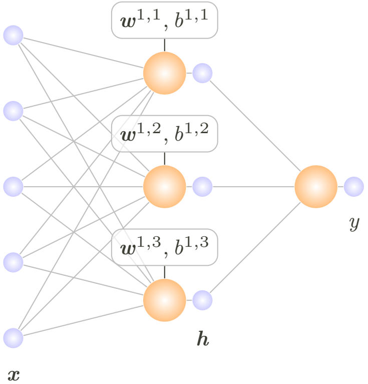
- "The perceptron may eventually be able to learn, make decisions, and translate languages." –Rosenblatt, 1958
- "Some functions are tricky." –Minsky/Papert, 1969
- interest in neural networks declined until the 80s

[1]



Output Function with one Hidden Layer

$$h = \left(f(w^{\mathbf{1,1}}x + b^{1,1}), f(w^{\mathbf{1,2}}x + b^{1,2}), f(w^{\mathbf{1,3}}x + b^{1,3})\right)$$



Output Function with one Hidden Layer

$$
\begin{aligned}
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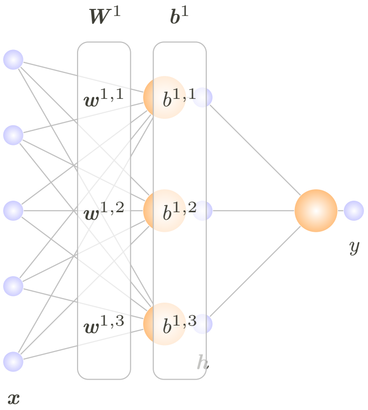
\end{aligned}
$$



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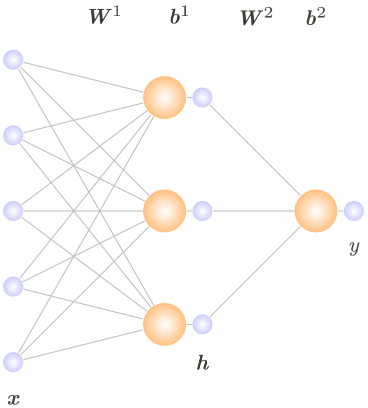

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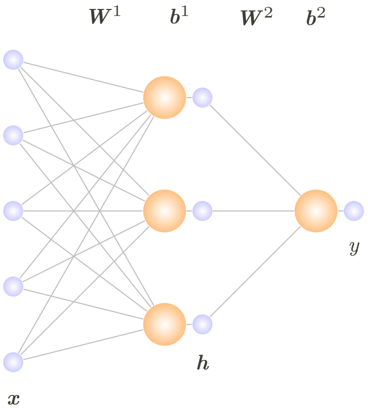
$$y = f(\boldsymbol{W}^2\boldsymbol{h} + \boldsymbol{b}^2) = f(\boldsymbol{W}^2 f(\boldsymbol{W}^1\boldsymbol{x} + \boldsymbol{b}^1) + \boldsymbol{b}^2)$$

Neural Networks



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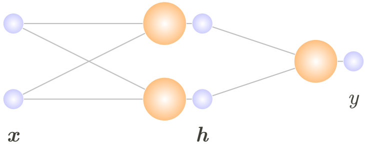
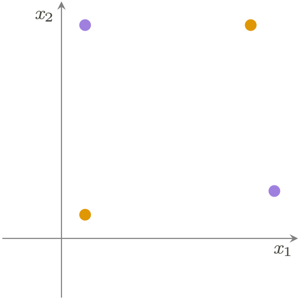
$$y = f(\boldsymbol{W}^2\boldsymbol{h} + \boldsymbol{b}^2) = f(\boldsymbol{W}^2 f(\boldsymbol{W}^1\boldsymbol{x} + \boldsymbol{b}^1) + \boldsymbol{b}^2)$$

- just matrix multiplications and element-wise non-linearities

XOR Revisited





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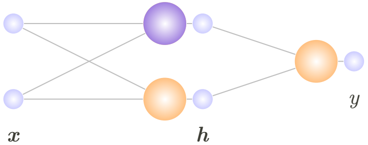

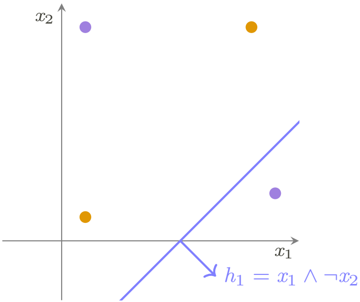

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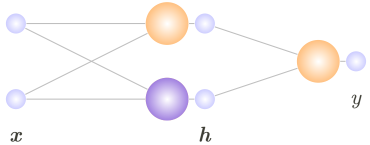
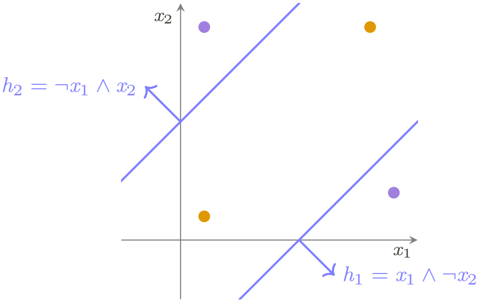
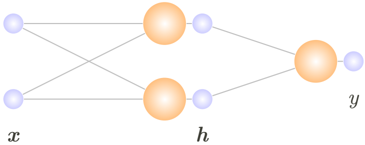
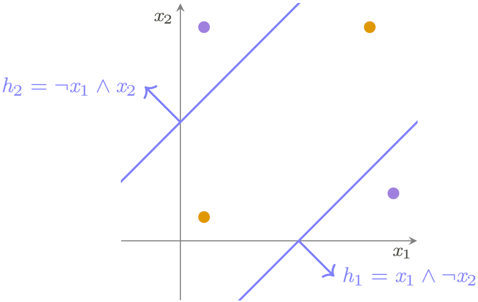
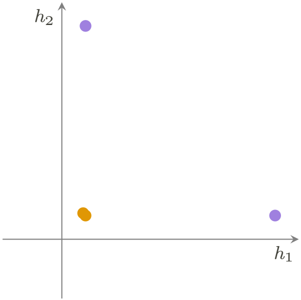




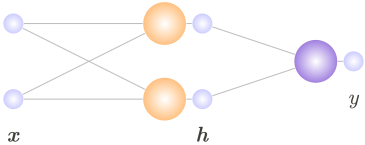
XOR Revisited

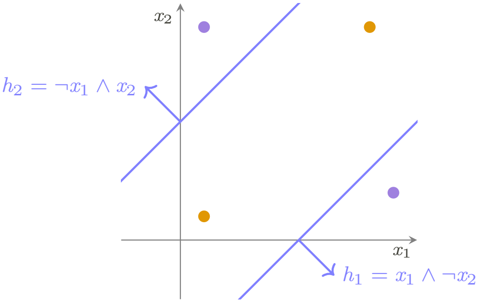


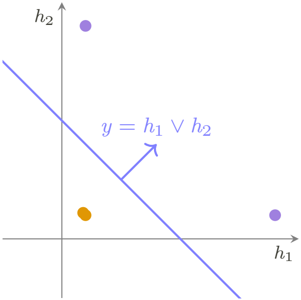




XOR Revisited







How many layers do we need?

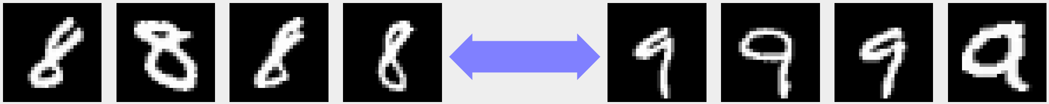
Theoretically...

...one hidden layer is sufficient to model any function:

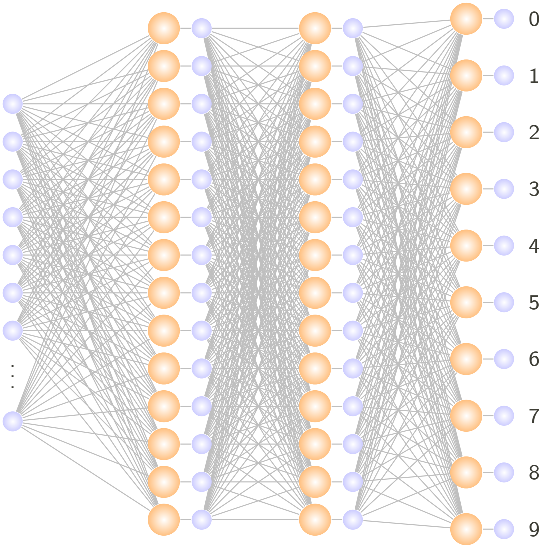
- every boolean function can be transformed into disjunctive normal form [2]
- every continuous function can be approximated with one hidden layer [3]

Practically...

...more than one layer will be more efficient:



Example: Digit Recognition



[4]

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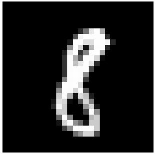

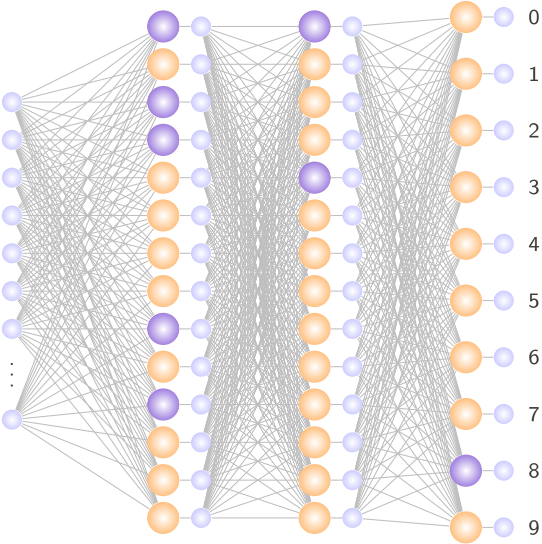

[4]

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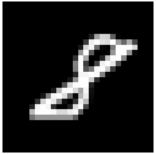
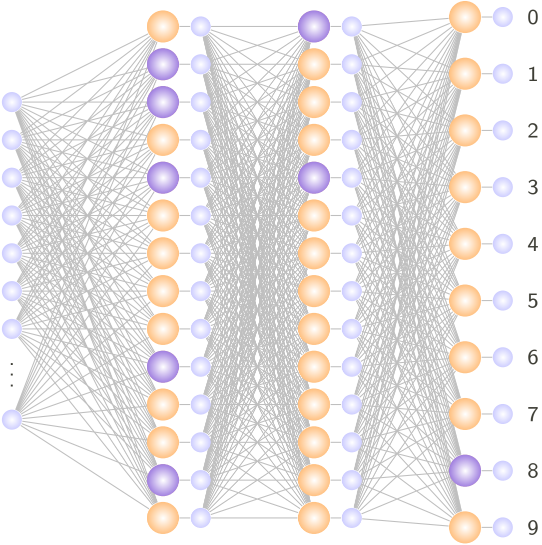
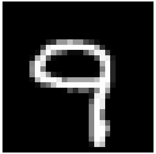
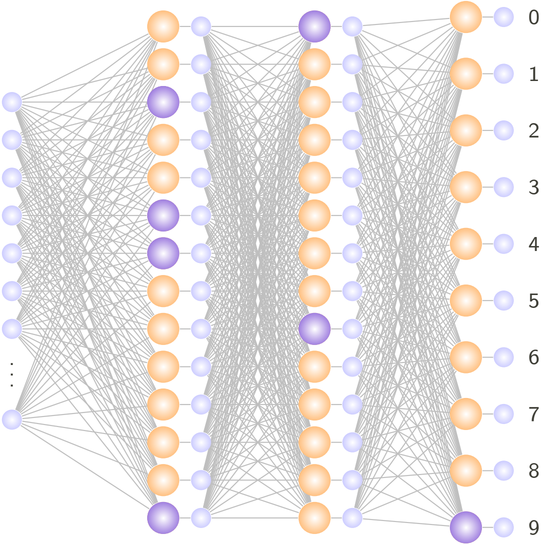




[4]





[4]

Supervised Learning

Model

- function $M_{\boldsymbol{\theta}} : \mathbb{R}^n \mapsto \mathbb{R}^m$, $\hat{\boldsymbol{y}} = M_{\boldsymbol{\theta}}(x)$
- parameterized by $\boldsymbol{\theta}$

Data

- $T = \left\{\boldsymbol{x}^i, \boldsymbol{y}^i\right\}_{i=1,\dots,k}$
- input $\boldsymbol{x}$
- desired output $\boldsymbol{y}$

Objective

- $l : \mathbb{R}^m \times \mathbb{R}^m \mapsto \mathbb{R}$, e.g., $l(\hat{\boldsymbol{y}}, \boldsymbol{y}) = \frac{1}{2}|\hat{\boldsymbol{y}} - \boldsymbol{y}|^2$
- $L(\boldsymbol{\theta}) = \frac{1}{k}\sum_{i=1}^{k} l(\hat{\boldsymbol{y}}^i, \boldsymbol{y}^i)$

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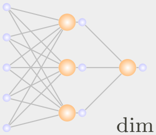

$\dim(\boldsymbol{\theta}) \gg 1$

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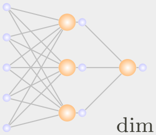

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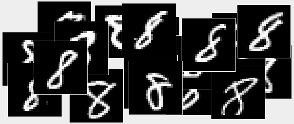

lots of it

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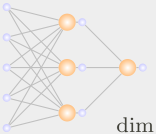

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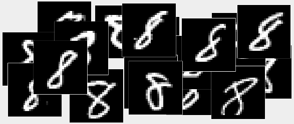

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smooth, differentiable

Learning by Gradient Descent

Optimization Problem

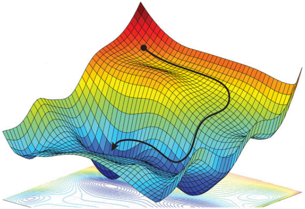
Learning objective:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \; L(\theta)$$

- in general no closed-from solution
- probably not even wanted

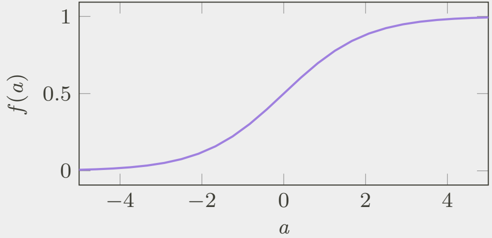
$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^t - \alpha \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta})$$

- fallback to gradient descent

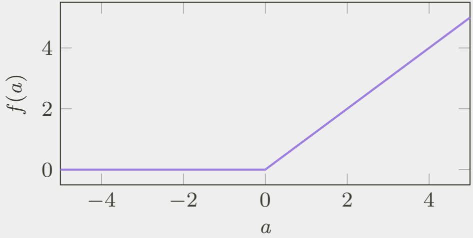


Differentiable Non-linearities

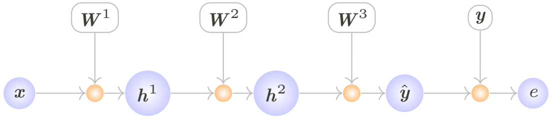
Sigmoid activation function:



Rectified linear unit (ReLU) activation function:



Error Back Propagation



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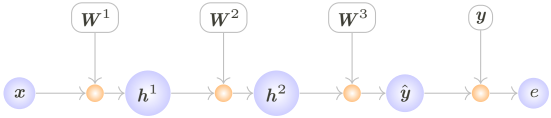
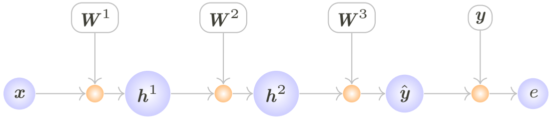


$$\begin{aligned}\hat{\boldsymbol{y}} &= M_{\boldsymbol{\theta}}(\boldsymbol{x}) \;\; (\text{with } \boldsymbol{\theta} = \boldsymbol{W}^1, \boldsymbol{W}^2, \boldsymbol{W}^3) \\ &= f(\boldsymbol{W}^3 \boldsymbol{h}^2) \\ &= f(\boldsymbol{W}^3 f(\boldsymbol{W}^2 \boldsymbol{h}^1)) \\ &= f(\boldsymbol{W}^3 f(\boldsymbol{W}^2 f(\boldsymbol{W}^1 \boldsymbol{x})))\end{aligned}$$

$$e = L(\boldsymbol{\theta}) = \frac{1}{k}\sum_{i=1}^{k} l(\hat{\boldsymbol{y}}, \boldsymbol{y})$$

How to compute $\nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}) = \frac{\mathrm{d}e}{\mathrm{d}\boldsymbol{\theta}}$?

Error Back Propagation



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Chain Rule

$$\frac{\mathrm{d}f(g(x))}{\mathrm{d}x} = \frac{\mathrm{d}f(g(x))}{\mathrm{d}g(x)} \frac{\mathrm{d}g(x)}{\mathrm{d}x}$$

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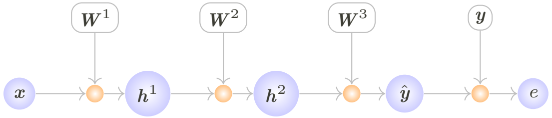

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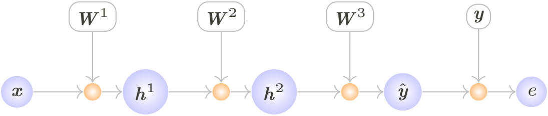

$$\hat{y} = M_{\theta}(x) \ \ (\text{with } \theta = W^1, W^2, W^3)$$
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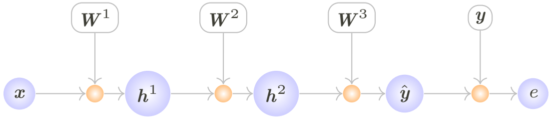
Error Back Propagation



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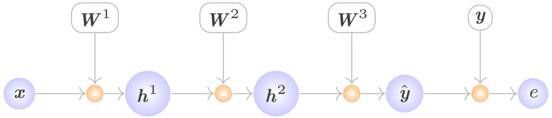
Error Back Propagation



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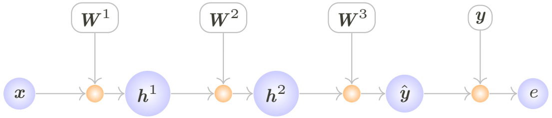

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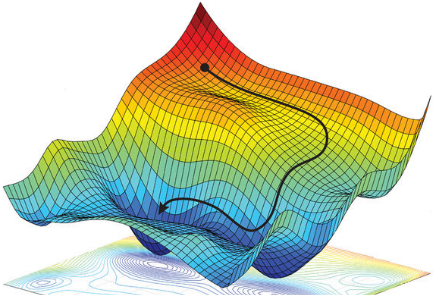
Sampling Strategies

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^{t} - \alpha \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta})$$



Gradient Descent

$$L(\boldsymbol{\theta}) = \frac{1}{k} \sum_{i=1}^{k} l(\hat{\boldsymbol{y}}, \boldsymbol{y})$$

- impractical for large k

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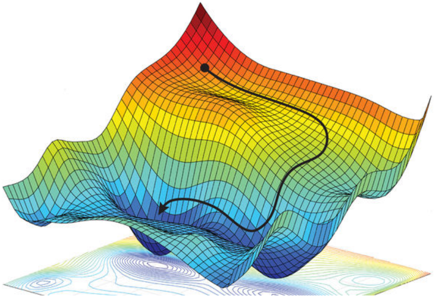

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Stochastic Gradient Descent

$$L(\boldsymbol{\theta}) = l(\hat{\boldsymbol{y}}^{i}, \boldsymbol{y}^{i}) \quad \text{with } i \text{ random in } [1, \dots, k]$$

- randomly sampled training example

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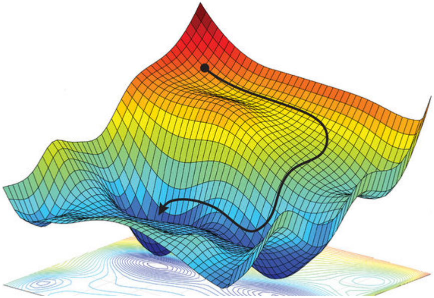

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- randomly sampled training example

Batch Gradient Descent

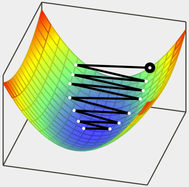
$$L(\boldsymbol{\theta}) = \frac{1}{b} \sum_{i=1}^{b} l(\hat{\boldsymbol{y}}^{p_i}, \boldsymbol{y}^{p_i}) \quad \text{with } p_i \text{ random in } [1, \ldots, k]$$

- randomly sampled set of training examples

Optimization Algorithms

Plain Gradient Descent

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^{t} - \alpha \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}^{t})$$



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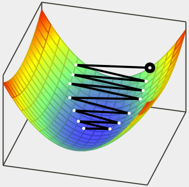
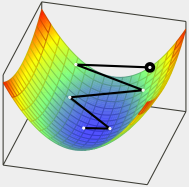


Gradient Descent with Momemtum

$$\boldsymbol{\delta}^{t} = \alpha \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}^{t}) + \beta \boldsymbol{\delta}^{t-1}$$
$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^{t} - \boldsymbol{\delta}^{t}$$



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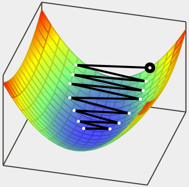

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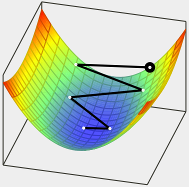
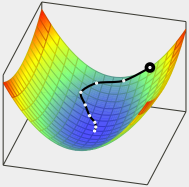


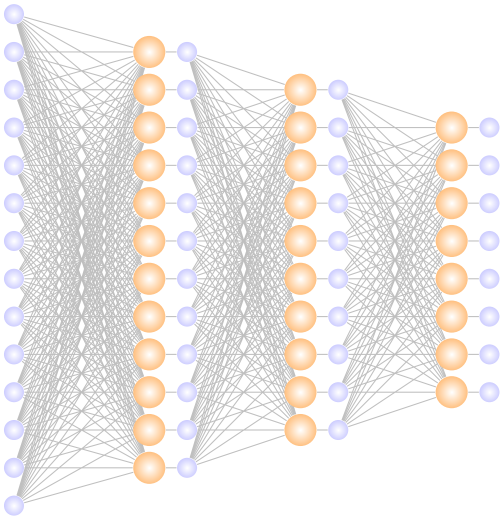
Adaptive Moment Estimation (Adam)

$$\begin{aligned}
\boldsymbol{g}^t &= \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}^t) \\
\boldsymbol{m}^t &= \beta_1 \boldsymbol{m}^{t-1} + (1 - \beta_1) \boldsymbol{g}^t \\
\boldsymbol{v}^t &= \beta_2 \boldsymbol{v}^{t-1} + (1 - \beta_2) \left(\boldsymbol{g}^t \odot \boldsymbol{g}^t\right) \\
\boldsymbol{\theta}^{t+1} &= \boldsymbol{\theta}^t - \alpha \frac{\boldsymbol{m}^t}{\sqrt{\boldsymbol{v}^t} + \epsilon}
\end{aligned}$$



[5]

Receptive Fields



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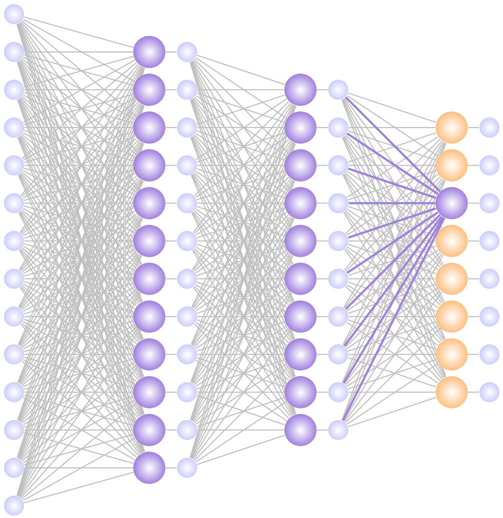
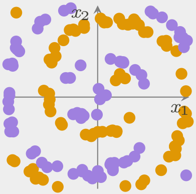


Fully Connected

- if inputs are unstructured



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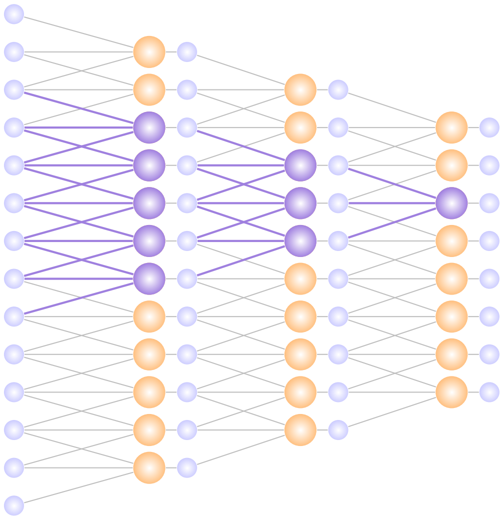
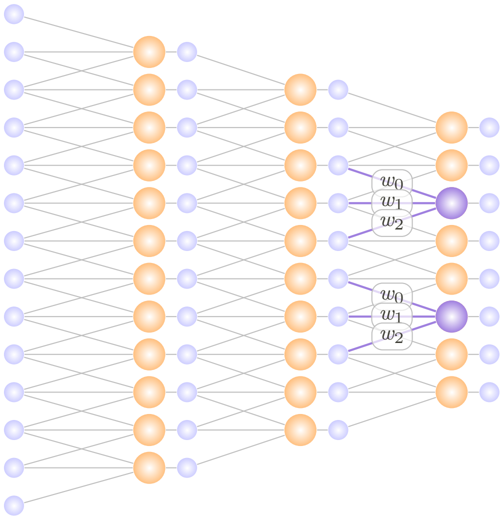


Locally Connected and Shared Weights

- if inputs have spatial/temporal structure

Receptive Fields



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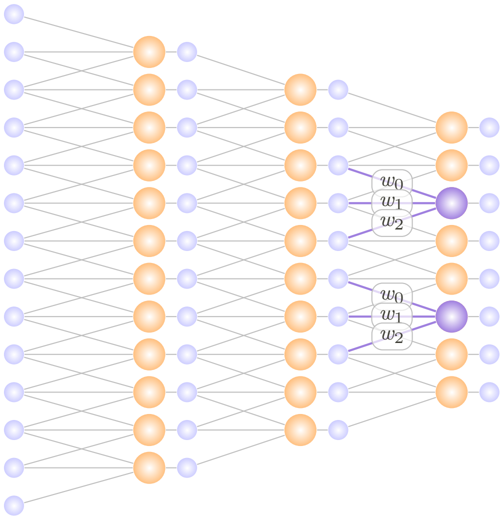
- if inputs have spatial/temporal structure
- if outputs are equivariant to inputs

Receptive Fields



Locally Connected and Shared Weights

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$$y_i = f(\boldsymbol{W}\boldsymbol{h} + \boldsymbol{b})_i$$

$$= f(\sum_{j=0}^{2} h_{i+j} w_j + b_i)$$

$$\boldsymbol{y} = f(\boldsymbol{h} * \boldsymbol{k} + \boldsymbol{b}) \text{ with } \boldsymbol{k} = (w_j)_{j=0,\dots,2}$$

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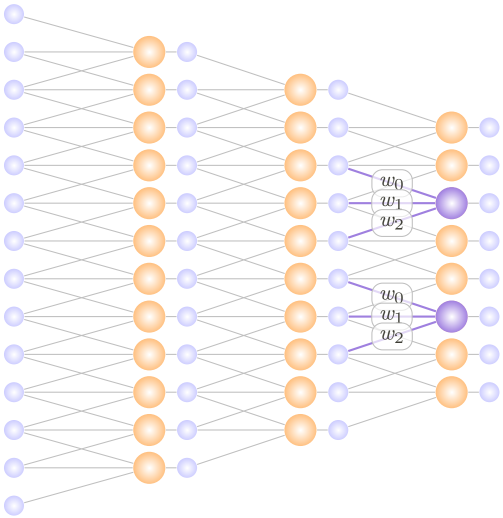

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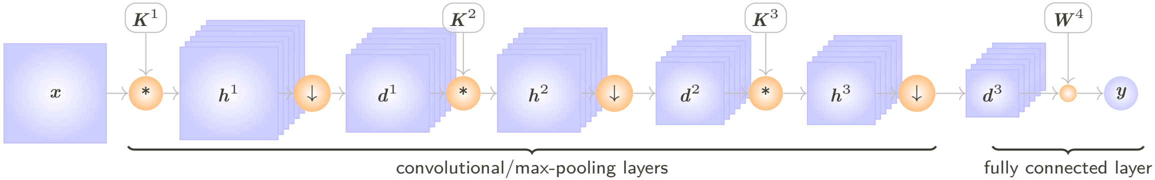
$\rightarrow$ convolutional neural networks (CNNs)

Convolutional Neural Networks



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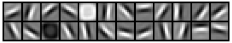

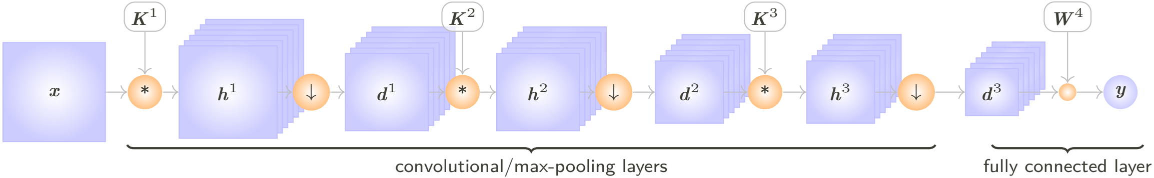

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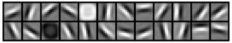
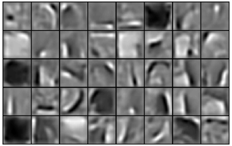
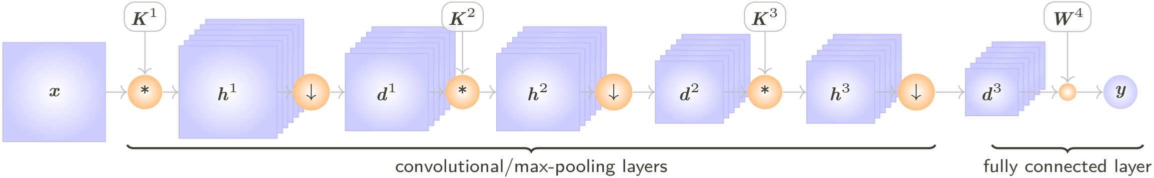
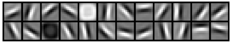
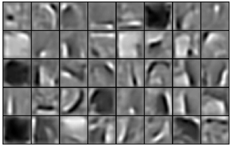


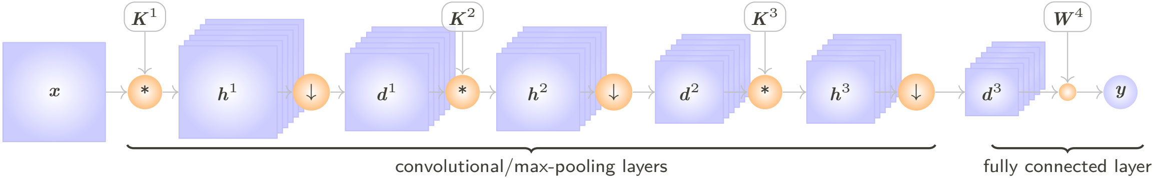




Convolutional Neural Networks

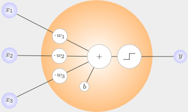




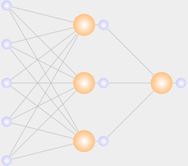


Summary

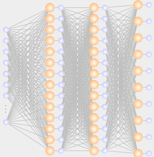
1. Perceptrons



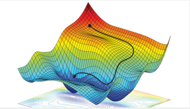
2. Networks



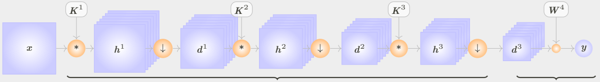
3. Deep Networks



4. Learning



5. Convolutional Networks



References

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