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The Earth's eccentricity in Kepler's refutation of the Tychonic approach to the problem of Mars

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Abstract

In this paper I explain Kepler's procedure for refuting the Ptolemaic-Tychonic approach to the problem of Mars, by using latitudinal observations at Mars's opposition, from Chapter 19 of the *Astronomia Nova*. This critique is fundamental to his reformation of the foundations of astronomy with his first two laws. Moreover, as I argue, the strategy he follows is deeply rooted in certain rhetorical considerations that he employed during the composition of his work.

KEYWORDS

Astronomia Nova, history of astronomy, Johannes Kepler, modern science, Tycho Brahe

1 | INTRODUCTION

Chapter 16 of the *Astronomia Nova* shows us Kepler's rendition of how he arrived at a non-bisected model for Mars. To accomplish this, he used a set of achronycal observations in longitude, that is, the longitudes of Martian oppositions. At those moments, the heliocentric and geocentric longitudes of Mars are the same. Thus, by using observations at opposition, Kepler could isolate the first anomaly of Mars without worrying about the effects the Earth's motion has on the geocentric longitude of that planet. This is particularly important in the case of Mars, because given that the ratio between the orbits of Mars and Earth is about 1.53, those effects are very noticeable, and any flaws in the model of the Earth will cause much confusion in the theoretical interpretation of Martian observations. In this regard, we should note that Kepler had inherited Tycho's model for the Sun—or, in a heliocentric framework, for the Earth.[1] While Tycho's model is good enough to predict solar phenomena fairly accurately, it has a very negative effect on Martian longitude predictions, causing an error of up to about 30′, even if all other aspects of the model are correct.[2]

[1]Hereafter, I use the expressions "Earth's model" and "Sun's model" interchangeably, since the authors involved in the discussion were heliocentric (Kepler) and geocentric (Ptolemy and Tycho). Of course, Kepler was aware of this equivalence.
[2]Carman & Recio (2019, p. 102).

With these observations as data, and after devising an extremely time-consuming geometrical method, Kepler determined that for a radius of the orbit of 100,000 parts, the eccentricity of the Martian equant is 18,564^{p} and that of the deferent 11,332^{p}.[3]

After producing this Martian model from longitude observations, Kepler nevertheless recalculated the eccentricity of the deferent, this time using achronycal observations in latitude. Chapter 19 is the explanation of that procedure, which ends, in Kepler's words, with a "refutation" of the non-bisected model and his famous lines about the need for a full reformation of the foundations of mathematical astronomy. In what follows, I explain Kepler's refutation, and show what Kepler's strategy was regarding the nature of the relation of Earth's and Mars's models.

2 | KEPLER'S REFUTATION USING LATITUDE OBSERVATIONS

First of all, we should note that while the quality of a model for the Earth—such as Ptolemy's or Tycho's—has no relevance to the use of achronycal observations of longitude, this is not the case for the latitudinal aspect of Mars's motion.

Refer to Figure 1. Mars M, the Sun S and the Earth are all on a plane orthogonal to the plane of the ecliptic, meaning that it is an achronycal situation. Regardless of the eccentricity of the Earth's model, a Martian model that predicts correct heliocentric longitudes will also predict good geocentric longitudes at opposition, because, as the diagram shows, the two are the same: the synodic anomaly's influence is reduced to zero. However, this is only true regarding the longitudinal aspect of the geocentric position. It is clear that an Earth E_2 with a distance SE_2 to the Sun will correspond to an observed Martian latitude α that is greater than latitude β, which corresponds to an Earth E_1, with a distance to the Sun SE_1.

From these considerations, it is clear that Kepler's determination regarding the parameters for the Earth's model was of the utmost importance if he wanted to use Mars's latitude observations to check the Martian eccentricity. This is only discussed in Part III of the *Astronomia Nova*, a couple of chapters later.

Kepler himself tells us that the *Astronomia Nova* is not a literal depiction of the path his research actually took. That would have been "boring and pointless to recount."[4] Instead, Kepler gives us his personal reconstruction of how he studied two intertwined problems: those of the motions of Mars and the Earth. As we saw, Kepler's choice in the book was to deal with Mars's non-bisected model first, and only then move on to discussing the Earth's orbit. In reality, though, Kepler was fighting on several fronts at the same time, and by the time he refuted the non-bisected model, he had already developed a revised model for the Earth, with an equant and a bisected eccentricity. In turn, this revised model would give way to a final model with an elliptic orbit and a motion according to the area

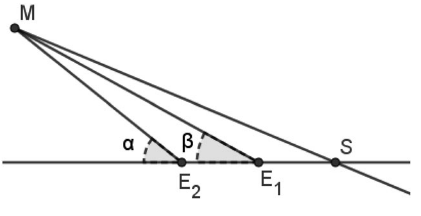

FIGURE 1 Heliocentric diagram of an achronycal moment. The Sun S, Mars M, and the Earth are all on the same plane, orthogonal to the ecliptic SE_1E_2. While a good model for oppositions will give good longitude predictions regardless of the eccentricity of the Earth, this will not be the case for latitude predictions: The predicted latitudes α and β for the two corresponding distances to the Sun SE_2 and SE_1 will be different

[3]See Gingerich (1964/1993a; 1973/1993b) for a study of Kepler's calculation. The values are given in Kepler (2015, p. 200). On the following page, he gives slightly different values. The difference is due to different trigonometric approaches to the computation.

[4]Kepler (2015, p. 135). See Voelkel (2001, Ch. 9) for a more general study about the rhetorical character of the book.

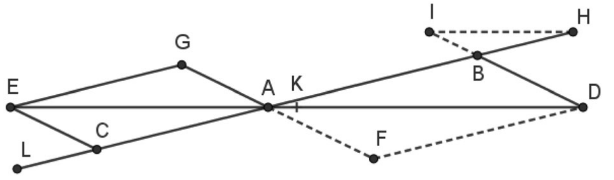

FIGURE 2 Kepler's diagram at the beginning of Chapter 19, Part II of *Astronomia Nova*

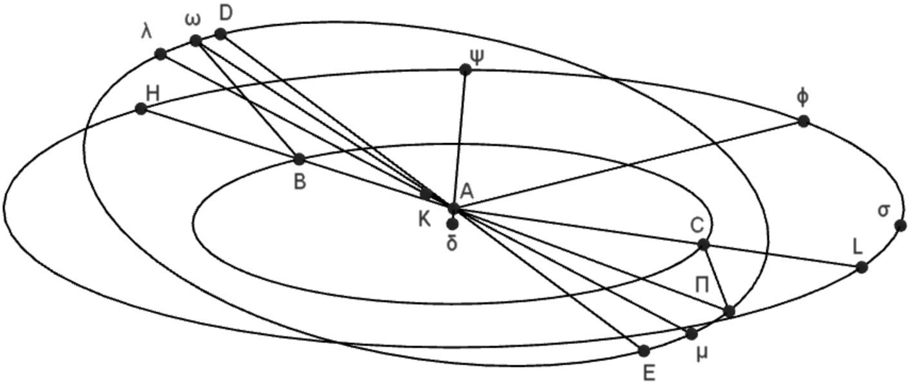

FIGURE 3 Heliocentric rendition of Kepler's Figure 2

law. But, given that he had still not explained to the reader what the structure and parameters of this revised model were, and most importantly, what his reasons were for this revision, he had to avoid assuming them. Instead, he chose a more cautious path, one whose assumptions could be accepted by his readers, and one that could also prepare the way for the modifications to come.

Refer to Figure 2. This is Kepler's diagram representing the situation for the Martian oppositions of 1585 and 1593.[5] Kepler's representation may not be easily understood by a modern reader, so I have provided my own heliocentric rendition of Kepler's diagram (Figure 3). I did not include points I and F, because those points are only necessary in the geocentric version of the argument, which I do not consider.

The three circles are, from the exterior to the interior, the ecliptic, the orbit of Mars, and the orbit of the Earth. The plane of the Martian orbit is inclined 1;50° with respect to the ecliptic.

σ is Aries 0°, A is the Sun, and δ is the center of the Earth´s orbit. Thus, ψ is the Sun's apogee at 95;30°—that is, when the Sun is viewed from the Earth as having the longitude ψ, at which point the distance between the Earth and the Sun is the greatest. D is the northernmost point of Mars's orbit, and E is the southernmost. λ is the aphelion of Mars, and μ is the perihelion, with K being the center of Mars's orbit. B is the position of the Earth in the 1585 opposition, and C is the position in the 1593 opposition. ω is the position of Mars for the 1585 opposition, and π is its position in the 1,93 opposition. H and L are the projections of AB and AC to the ecliptic, respectively. Φ is the ascending node.

In order to simplify his diagram, Kepler unified points D, ω, and λ, and points E, π, and μ, so that D sometimes is taken to be the northernmost latitude, but at other times the position of Mars at opposition, and at others the aphelion. The same applies with the other three. So, one must interpret Kepler's exposition to understand his derivation. In my version, I have distinguished all points, leaving the original names for Kepler's points (in the cases of D and E, giving them the first function Kepler chooses for them), and adding minuscule Greek letters to my additions.

[5]Kepler (2015, p. 209).

As I said, ω is Mars's location on the 1,585 opposition at 141°. Therefore, the Earth is on B, where A, B, and ω lie on the same plane, orthogonal to the ecliptic. Since Kepler knew that the distance between D at 136° and ω was about 5°, and that the inclination of Mars's orbit $\angle BAD = 1;50°$, he knew that

$$\sin\angle BA\omega = \sin 85° \times \sin 1;50°$$

which gives $\angle BA\omega = 1;49,30°$.[6]

He knew from observation that $\angle HB\omega = 4;32,10°$.[7] Given that $\angle B\omega A = \angle BA\omega - \angle HB\omega$, he obtained $\angle B\omega A = 2;42,40°$. Therefore, $\angle \omega BA = 180° - \angle BA\omega - \angle B\omega A = 175;27,50°$.

Then he assumed that the ratio between AB and AC is 97,500 to 101,400. This was the ratio given by Tycho's solar tables. So, assuming AB = 97,500, we can solve triangle ωAB

$$\frac{\sin\angle B\omega A}{BA} = \frac{\sin\angle \omega BA}{DA}$$

and get ω A = 163,000. He also calculates that if we assume AB = 100,000^p, we get ω A = 167,200^p.

Kepler then repeated the calculation for the 1593 opposition, and got Aπ = 139,300^p for an assumed AC = 101,400^p. Again, he also calculated that Aπ = 137,380^p if we assume AC = 100,000^p.

As I mentioned, in the 1585 opposition Mars was observed at 141°. Its observed longitude for the 1593 opposition was 342°. Kepler also knew that the aphelion of Mars λ is at 149°, with a perihelion μ at 329°. So, in the first opposition Mars was 8° before the line of apsides, and in the second it was 13° past that line.

Refer to Figure 4. The diagram is a representation of the plane of Mars's deferent, with an aphelion λ and a perihelion μ. The Sun is point A, and the center of the Martian deferent is K. ω is the longitude of Mars at the 1585 opposition, and π is the one at the 1593 opposition. So far, Kepler knows that $\angle \omega A\lambda = 8°$, and $\angle \pi A\mu = 13°$. Also, he

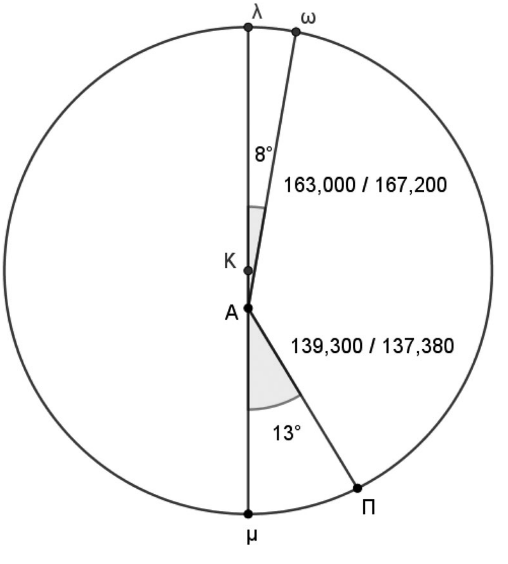

FIGURE 4 Diagram for the two oppositions. Line λKAμ is the line of apsides, with K being the center of Mars's deferent, and A the Sun. Points Ω and Π and the longitudes for the 1585 and 1593 oppositions, respectively

[6]Actually, it is 1;49,35°. Hereafter, I will not point out errors in Kepler's computations. Suffice it to say that they are always negligible.

[7]The observational basis for the calculations can be found in Kepler (2015, p. 183), which is a summary of Tycho's observations of oppositions.

TABLE 1 Final parameters of Figure 4

Aλ	163,150	or	167,350
Aμ	139,000	or	137,080
λμ (Aλ+ Aμ)	302,150	or	304,430
λK (λμ/2)	151,075	or	152,215
AK	12,075	or	151,35

Note: The values in the left column are derived assuming a ratio 97,500^p to 101,400^p between AB and AC, and those in the right column assume a value of 100,000^p for both lines.

knows that the ratio between ωA and πA is 163,000^p to 139,300^p (assuming the values for AB and AC from Tycho's solar tables) or 167,200^p to 137,380^p (assuming AB = AC = 100,000^p). It is clear, for a given pair of values of the angles and segments, that there is only one possible value for the eccentricity AK, and therefore for distances Aλ and Aμ. After some rounding, Kepler says that, if we assume Aω and Aπ to be 163,000 and 139,300, respectively, then Aλ = 163,150^p and Aμ = 139,000^p. If one assumes Aω and Aπ to be 167,200^p and 137,380^p, respectively, then Aλ = 167,350^p and Aμ = 137,080^p.

From this he can obtain the rest of the parameters. He built the following Table 1 (I have modified the references so it is coherent with my Figure 4).

Kepler then assumed a value of 100,000^p for the radius of Mars's orbit λK, and found that AK is 8,000^p (again, assuming the values for AB and AC from Tycho's tables) or 9,943^p (again, assuming AB = AC = 100,000^p). These values for the eccentricity are significantly different from the 11,332^p he had obtained using longitude observations.

So, through his use of latitude observations, Kepler has shown that Tycho's model for the Earth is incompatible with the Martian eccentricity he had derived from longitude observations. Moreover, he has shown that a zero-eccentricity model for the Earth is also incompatible with it. This is what was entailed in the second calculation, where AB = AC = 100,000^p. Why present this double calculation, especially when it was clear that a zero-eccentricity model had nothing to do with any accepted solar model at the time? Stephenson's modern study does not mention this important aspect, and merely states that “Kepler solved such triangles for the oppositions of 1,585 and 1,593, when Mars was respectively near aphelion and perihelion.”[8] Small's 1804 study on Kepler's astronomy, however, briefly points out this aspect of the procedure and connects it to Kepler's thoughts about the size of the Earth's eccentricity.[9]

In this line, I think that the answer is directly linked to the rhetorical strategy I mentioned earlier regarding Kepler's way of presenting, sequentially, the solution to a set of problems that are essentially intertwined. At this stage, it was most likely clear to him that any final answer to the Martian problem needed to assume a model for the Earth that approximated a bisected eccentricity, with the point of (quasi-) uniform motion at the Tychonic center of the deferent, and a center of the deferent located midway between the Sun and the equant. The development of such a model, as I said above, would only come some chapters later.

Kepler could have told the reader what his bisected solution for the Earth's motion was going to be and proceeded from there. After all, such provisional use of results, with their proofs promised later, is a perfectly legitimate mode of argumentation, provided that those promises are not left unfulfilled. However, Kepler knew that what he had on his hands was a profound challenge to the foundations of astronomy, foundations which could be traced at least back to the times of Hipparchus, 18 centuries in the past. The path down which he was leading his readers was one that had already demanded much from them. Because of that, it was better for Kepler to take one step at a

[8]Stephenson (1987, p. 45).

[9]“If the eccentricity of the earth's orbit should not amount to 3,584 [as Tycho said], but, as Kepler suspected, to little more than one-half of it, the results for AD and AE [our Aλ and Aμ, respectively] would be different; greater, to wit, than the first, and less than the second.” Small (1804/1963, p. 190).

time, calmly arguing his way through what would ultimately be his refutation of some of the basic tenets of mathematical astronomy.

Thus, in this chapter Kepler limited himself to showing how two boundary values for the Earth's eccentricity are inadequate to producing a Martian model that could account for the observations in longitude. As I mentioned at the beginning, those observations indicated a Martian eccentricity of the deferent of $11{,}332^p$. The value for the Earth's eccentricity was irrelevant to that calculation. However, when he turned to the latitude observations—where it was very relevant—he obtained a Martian eccentricity of the deferent of $8{,}000^p$, assuming the maximum eccentricity of the Earth that his readers would accept: that of Tycho. The minimum conceivable eccentricity, one that equals zero, gave him a value of $9{,}943^p$ for the eccentricity of the deferent of Mars.

In this way, he pointed to the existence of a fundamental flaw in the consensus approach to the whole problem. Given the Tychonic eccentricity of the Earth, in order to obtain a good fit with the latitude data one was forced to obliterate the parameters that had allowed one to obtain a good fit with the longitude data. A possible way out was to discard Tycho's value for the eccentricity of the Earth and try different options. This would of course produce many additional problems regarding the prediction of solar phenomena, so it would be difficult to accept a variation in that parameter. But, to leave no doubt in anyone's mind, Kepler showed that not even if one assumed a zero-eccentricity model for the Earth could one obtain the Martian deferent from the initial $8{,}000^p$ to the desired $11{,}332^p$. That extreme move would only get you up to $9{,}943^p$. Something was rotten in the astronomy of Denmark.

It is at this point that Kepler made his famous declaration that, given the difficulties evidenced by his study of the latitude data provided by Tycho himself, it was necessary to review not the parameters of the models, but rather the theoretical assumptions within which those models were being developed: circularity and uniformity.[10] In no part of the refutation did Kepler directly address the subject of the correct parameters for the Earth, even though they would have shed a clear light on the limitations of thinking about the Martian problem within an essentially Ptolemaic framework. Despite the potential that option had for providing a more accurate numerical description of the inconsistencies he was finding, it would have been a detrimental decision in rhetorical terms. After reading the chapter, and given the clear description of those inconsistencies, one is not only convinced of the impossibility of solving them within the inherited framework, but also eager to find out how Kepler will propose a way out. Thus, he has prepared and prompted his readers to follow him on to the arguments which will result in an *Astronomia Nova*.

3 | CONCLUSION

In this paper, I have explained Kepler's procedure for refuting the Ptolemaic-Tychonic approach to the problem of Mars, by using latitudinal observations at Mars's opposition. I have argued that the reason for the double calculation he presented there was a strategy of refuting not a particular set of parameters for Earth and Mars, but rather all possible parameters for those planets that are framed within the Ptolemaic-Tychonic foundations of circularity and uniformity.

This section of *Astronomia Nova* is not part of the more famous chapters in which Kepler developed what are now known as his first two laws. Therefore, its technical details have not, in comparison, received much scholarly attention. Nevertheless, Kepler's method should be of interest to most than those reader concerned with technical aspects of his work, because it reveals what Voelkel called "the rhetorical character of the *Astronomia Nova*"—that is, the fact that Kepler's presentation is one where he is trying to convey, in the clearest way he can, the series of steps he had to take to go from the essentially Ptolemaic framework in which he (as Tycho and Copernicus before him) had been working, to the reformed a*stronomia* he would end up presenting.[11] This implied a "tidying up" of all the intertwined problems and solutions he had to face and solve during his research, and the creation of an ordered and

[10]Kepler (2015, pp. 210–211).
[11]Voelkel (2001, pp. 211–216.).

coherent narrative that would allow the reader to tackle, along with Kepler-the-author, one discussion at a time, even if this meant he had to hide the real way in which Kepler-the-astronomer had carried out his struggles with planetary motions. So, even if the *Astronomia Nova* is a book that could only be approached by the few mathematicians who were very well acquainted with Ptolemaic and Copernican astronomy, it was still the product of a great teacher who made the effort to tell a story of a long and arduous battle.

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