MATH 4410
Nate Stott A02386053

Quiz #1; Due 11:59 pm, 1/16/2024

1. Write n^4 as a sum of falling powers: $n^4 = c_0 \cdot n^{\underline{0}} + c_1 n^{\underline{1}} + c_2 n^{\underline{2}} + c_3 n^{\underline{3}} + c_4 n^{\underline{4}}$.

I used a python program I made to make the difference table. Here is the code on github if you want to see:

`https://github.com/funkybooboo/MATH4110_DifferanceTable`

The program produced the numbers in the following table. However I don't know Latex that well so I copied the table you made and checked that our numbers matched then edited it to look how I like.

n	0	1	2	3	4	5	6	7
$\Delta^{(0)}n^4$	0	1	16	81	256	625	1296	2401
$\Delta^{(1)}n^4$	1	15	65	175	369	671	1105	
$\Delta^{(2)}n^4$	14	50	110	194	302	434		
$\Delta^{(3)}n^4$	36	60	84	108	132			
$\Delta^{(4)}n^4$	24	24	24	24				
$\Delta^{(5)}n^4$	0	0	0					

Then I wanted to prove that this statement is true to make sure you weren't pulling any trickery.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n^{\underline{k}}}{k!}$$

Start proof.
Lets focus in.

$$\frac{n!}{k!(n-k)!} = \frac{n^{\underline{k}}}{k!}$$

We can eliminate k!.

$$\frac{n!}{(n-k)!} = n^{\underline{k}}$$

Wait $\frac{n!}{(n-k)!} = n^{\underline{k}}$ is the definition of $n^{\underline{k}}$ so I guess you weren't pulling anything there.
End proof.
Lets use The Discrete Taylor Theorem to find a falling power representation for n^4.

$$f(n) = \sum_{k\geq 0} \Delta^{(k)} f(a) \binom{n-a}{k}$$

Let $a = 0$ so that we can get the first item of every row on the difference table.

$$f(n) = n^4 = \sum_{k\geq 0} \Delta^{(k)} f(0) \binom{n}{k} = 0\binom{n}{0} + 1\binom{n}{1} + 14\binom{n}{2} + 36\binom{n}{3} + 24\binom{n}{4} + 0\binom{n}{5} + 0\binom{n}{6} + 0\binom{n}{7} + \ldots$$

We can through out the terms with 0 as the leading coefficient.

$$= 1\binom{n}{1} + 14\binom{n}{2} + 36\binom{n}{3} + 24\binom{n}{4}$$

OK so now I can use $\binom{n}{k} = \frac{n^{\underline{k}}}{k!}$ to my advantage.

$$= 1(\frac{n^{\underline{1}}}{1!}) + 14(\frac{n^{\underline{2}}}{2!}) + 36(\frac{n^{\underline{3}}}{3!}) + 24(\frac{n^{\underline{4}}}{4!})$$

Simplifying.

$$= n^{\underline{1}} + 14(\frac{n^{\underline{2}}}{2}) + 36(\frac{n^{\underline{3}}}{6}) + 24(\frac{n^{\underline{4}}}{24})$$
$$= n^{\underline{1}} + 7n^{\underline{2}} + 6n^{\underline{3}} + n^{\underline{4}}$$

There you have it.

$$n^4 = n^{\underline{1}} + 7n^{\underline{2}} + 6n^{\underline{3}} + n^{\underline{4}}$$

2. Create a function that is a polynomial in n (ideally with no falling powers, but simplifying is not necessary) for the $\sum_{i=0}^{n} i^4$.

$$\sum_{i=0}^{n} i^4 = \sum_{i=0}^{n} (i^{\underline{1}} + 7i^{\underline{2}} + 6i^{\underline{3}} + i^{\underline{4}})$$

I know the summation doesn't distribute but you know what I'm doing

$$= \sum_{i=0}^{n} i^{\underline{1}} + \sum_{i=0}^{n} 7i^{\underline{2}} + \sum_{i=0}^{n} 6i^{\underline{3}} + \sum_{i=0}^{n} i^{\underline{4}}$$

$$= \frac{i^{\underline{2}}}{2}\bigg|_{i=0}^{i=n+1} + 7(\frac{i^{\underline{3}}}{3}\bigg|_{i=0}^{i=n+1}) + 6(\frac{i^{\underline{4}}}{4}\bigg|_{i=0}^{i=n+1}) + \frac{i^{\underline{5}}}{5}\bigg|_{i=0}^{i=n+1}$$

$$= (\frac{(n+1)^{\underline{2}}}{2} - \frac{(0)^{\underline{3}}}{3}) + 7(\frac{(n+1)^{\underline{3}}}{3} - \frac{(0)^{\underline{3}}}{3}) + 6(\frac{(n+1)^{\underline{4}}}{4} - \frac{(0)^{\underline{4}}}{4}) + (\frac{(n+1)^{\underline{5}}}{5} - \frac{(0)^{\underline{5}}}{5})$$

$$= \frac{(n+1)^{\underline{2}}}{2} + 7(\frac{(n+1)^{\underline{3}}}{3}) + 6(\frac{(n+1)^{\underline{4}}}{4}) + \frac{(n+1)^{\underline{5}}}{5}$$

$$= \frac{(n+1)(n)}{2} + 7(\frac{(n+1)(n)(n-1)}{3}) + 6(\frac{(n+1)(n)(n-1)(n-2)}{4}) + \frac{(n+1)(n)(n-1)(n-2)(n-3)}{5}$$

$$= \frac{n^2+n}{2} + 7(\frac{n^3-n}{3}) + 6(\frac{n^4-2n^3-n+2}{4}) + \frac{n^5-5n^4+5n^3+5n^2-6n}{5}$$

Simplify

$$= \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} + \frac{3n^2}{2} - \frac{68n}{15} + 3$$

There you have it

$$\sum_{i=0}^{n} i^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} + \frac{3n^2}{2} - \frac{68n}{15} + 3$$

3. Show that $\sum_{i=0}^{n}(4i^3+6i^2+4i+1)=(n+1)^4$.

Start proof

$$\sum_{i=0}^{n}(4i^3+6i^2+4i+1)=(n+1)^4$$

$$\sum_{i=0}^{n}(4i^3+6i^2+4i+1)=n^4+4n^3+6n^2+4n+1$$

focus in on left side

$$=\sum_{i=0}^{n}(4(i^{\underline{3}}+3i^{\underline{2}}+i^{\underline{1}})+6(i^{\underline{2}}+i^{\underline{1}})+4(i^{\underline{1}})+1)$$

I know the summation doesn't distribute but you know

$$=\sum_{i=0}^{n}4(i^{\underline{3}}+3i^{\underline{2}}+i^{\underline{1}})+\sum_{i=0}^{n}6(i^{\underline{2}}+i^{\underline{1}})+\sum_{i=0}^{n}4(i^{\underline{1}})+\sum_{i=0}^{n}1$$

$$=4\sum_{i=0}^{n}(i^{\underline{3}}+3i^{\underline{2}}+i^{\underline{1}})+6\sum_{i=0}^{n}(i^{\underline{2}}+i^{\underline{1}})+4\sum_{i=0}^{n}i^{\underline{1}}+n+1$$

$$=4(\sum_{i=0}^{n}i^{\underline{3}}+3\sum_{i=0}^{n}i^{\underline{2}}+\sum_{i=0}^{n}i^{\underline{1}})+6(\sum_{i=0}^{n}i^{\underline{2}}+\sum_{i=0}^{n}i^{\underline{1}})+4\sum_{i=0}^{n}i^{\underline{1}}+n+1$$

$$=4(\left.\frac{i^{\underline{4}}}{4}\right|_{i=0}^{i=n+1}+3(\left.\frac{i^{\underline{3}}}{3}\right|_{i=0}^{i=n+1})+\left.\frac{i^{\underline{2}}}{2}\right|_{i=0}^{i=n+1})+6(\left.\frac{i^{\underline{3}}}{3}\right|_{i=0}^{i=n+1}+\left.\frac{i^{\underline{2}}}{2}\right|_{i=0}^{i=n+1})+4(\left.\frac{i^{\underline{2}}}{2}\right|_{i=0}^{i=n+1})+n+1$$

$$=4(\frac{(n+1)^{\underline{4}}}{4}+3(\frac{(n+1)^{\underline{3}}}{3})+\frac{(n+1)^{\underline{2}}}{2})+6(\frac{(n+1)^{\underline{3}}}{3}+\frac{(n+1)^{\underline{2}}}{2}+4(\frac{(n+1)^{\underline{2}}}{2})+n+1$$

$$=4(\frac{(n+1)n(n-1)(n-2)}{4}+3(\frac{(n+1)n(n-1)}{3})+\frac{(n+1)(n)}{2})+6(\frac{(n+1)n(n-1)}{3}+\frac{(n+1)n}{2})+4(\frac{(n+1)n}{2})+n+1$$

$$=n^4+4n^3+6n^2+4n+1$$

There you go

$$\sum_{i=0}^{n}(4i^3+6i^2+4i+1)=(n+1)^4$$

End proof

4. Compute $\left\{ {7 \atop 3} \right\}$ using the difference table for n^7.

n	0	1	2	3	4	5	6	7	8
$\Delta^{(0)}n^7$	0	1	128	2187	16384	78125	279936	823543	2097152
$\Delta^{(1)}n^7$	1	127	2059	14197	61741	201811	543607	1273609	
$\Delta^{(2)}n^7$	126	1932	12138	47544	140070	341796	730002		
$\Delta^{(3)}n^7$	1806	10206	35406	92526	201726	388206			
$\Delta^{(4)}n^7$	8400	25200	57120	109200	186480				
$\Delta^{(5)}n^7$	16800	31920	52080	77280					
$\Delta^{(6)}n^7$	15120	20160	25200						
$\Delta^{(7)}n^7$	5040	5040							
$\Delta^{(8)}n^7$	0								

Given

$$\left\{ {k \atop j} \right\} = \frac{\Delta^{(j)}[(0)^k]}{j!}$$

Compute

$$\left\{ {7 \atop 3} \right\} = \frac{\Delta^{(3)}[(0)^7]}{3!} = \frac{1806}{3!} = 301$$