Power Controlled Scheduling with Consecutive Transmission Constraint: Complexity Analysis and Algorithm Design

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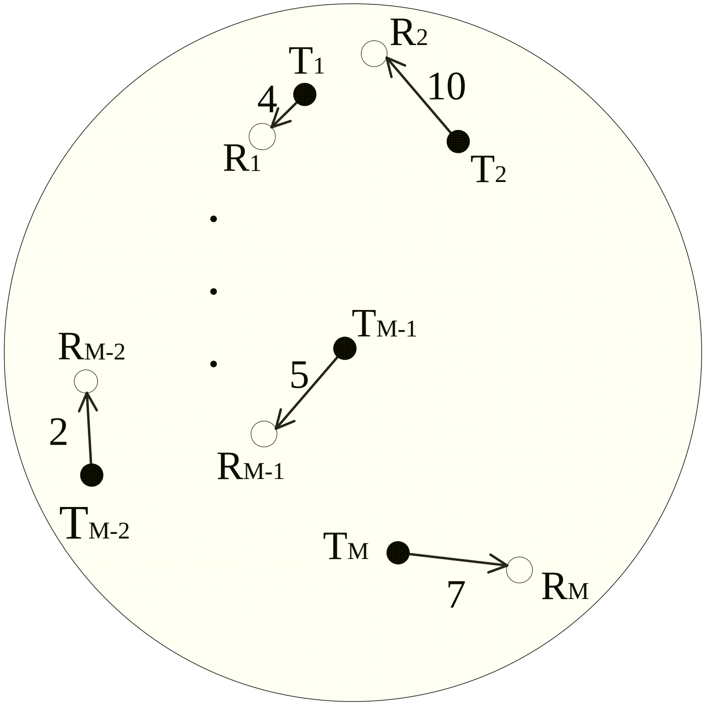
JPS-CC Problem:

Joint Power control and Scheduling with Consecutive Constraint

- Wireless scheduling
  - A set of transmitter-receiver pairs
  - Traffic demands
  - SINR constraints
  - Consecutive transmission constraints



- Objective
  - minimize the total number of time slots
- Power control
  - more links can be active simultaneously
- How to choose the active links in each time slot and the corresponding transmit power ?

$$SCH = \{ p(t), 1 \le t \le T \}$$

Outline

- Prove the JPS-CC problem is NP-complete
- Propose a polynomial-time approximation algorithm
  - Guaranteed and Greedy Scheduling (GGS)
- Prove the GGS algorithm has a bounded approximation ratio relative to the optimal scheduling algorithm
- Simulation Results

Complexity Study of JPS-CC

- The JPS-CC problem is NP-Complete.
  - Key Proof: The Partition Problem can be reduced to the JPS-CC Problem in polynomial time
    - The partition problem: Given a set of integers, is there a way to partition these integers into two disjoint subsets that has equal sums.
    - We construct a network such that any two links can be active simultaneously, however any three links can not be active simultaneously.

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The GGS Algorithm

- GGS algorithm consists of two parts
  - The guaranteed scheduling
  - The greedy scheduling

$$\frac{d_{\max}}{2^k} \le d_{ii} \le \frac{d_{\max}}{2^{k-1}}$$

  - Initialization Phase:
    - Divide the links into different groups according to their link lengths.
    - The lengths of the links belong to the same group differ by at most a factor of two.
    - There are total K groups:

$$K = \left\lfloor \log_2 \frac{d_{\max}}{d_{\min}} \right\rfloor + 1$$

Part 1:
The Guaranteed Scheduling

- The links belong to different groups are considered separately
- Divide the plane into hexagons and color them with 3 colors

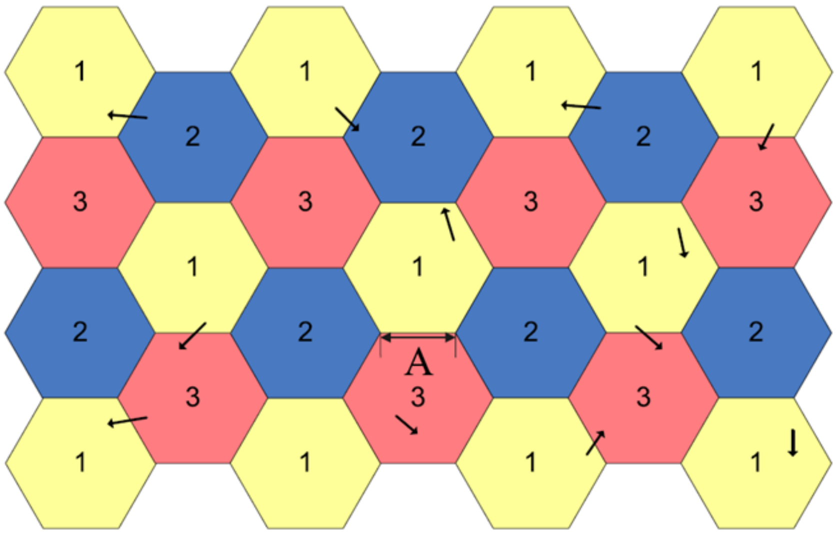

Fig. 1. Hexagon Coloring

$$\frac{d_{\max}}{2^k} \le d_{ii} \le \frac{d_{\max}}{2^{k-1}}$$

$$A = W \cdot \frac{d_{\max}}{2^{k-1}}$$

$$W = \left[6\gamma_0 \left(1 + \frac{2^\alpha}{\left(3\sqrt{3}-2\right)^\alpha (\alpha-2)} \right) \right]^{\frac{1}{\alpha}} + 1$$

- The selected links are guaranteed to be feasible.

Part 2:
The Greedy Scheduling

- “Squeeze” more links into the feasible matchings created in the guaranteed scheduling part in a greedy way
- Make full use of power control
- Check the Perron-Frobenius Eigenvalue condition

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Analysis of the GGS Algorithm

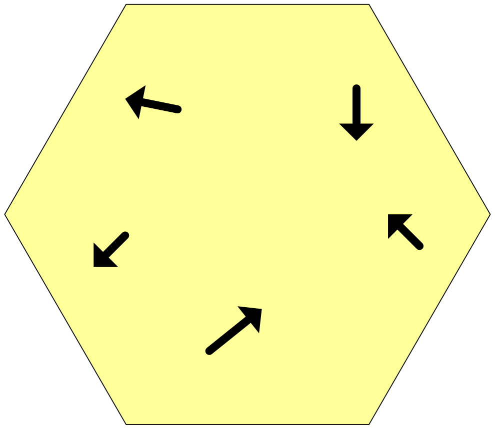
- The approximation ratio of the GGS algorithm is at most $3KN$.

$$N_1 = \left\lfloor \frac{1}{\gamma_0} \left(2\left(2W+1\right)\right)^{\alpha} + 1 \right\rfloor,$$

$$N_2 = \begin{cases} \left\lfloor 3\left(\frac{2W}{\gamma_0^{\frac{1}{\alpha}}-1}\right)^2 + 3\left(\frac{2W}{\gamma_0^{\frac{1}{\alpha}}-1}\right) + 1 \right\rfloor, & \text{if } \gamma_0 > 1 \\ \infty, & \text{otherwise} \end{cases}$$

and

$$N = \min\{N_1, N_2\}.$$



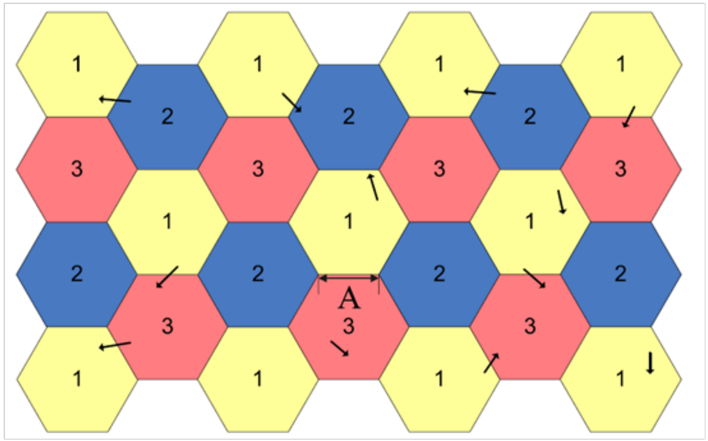
  - N is a constant dependent on the SINR γ_0 and path loss exponent α
  - N the upper bound of the maximum number of concurrent transmissions in one particular hexagon.

Key Idea of the Proof

- Show an upper bound on the GGS algorithm
  - The GGS algorithm achieves a frame length $\leq 3KF_{\max}$

    ($F_{\max}$ is the maximum total traffic among all the hexagons)
- Show an lower bound on the optimal algorithm
  - The optimal algorithm achieves a frame length $\geq \frac{F_{\max}}{N}$
- Finally derive the approximation ratio:

$$\frac{T_{GGS}}{T_{opt}} \leq \frac{3KF_{\max}}{\frac{F_{\max}}{N}} \leq 3KN$$



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Simulation 1

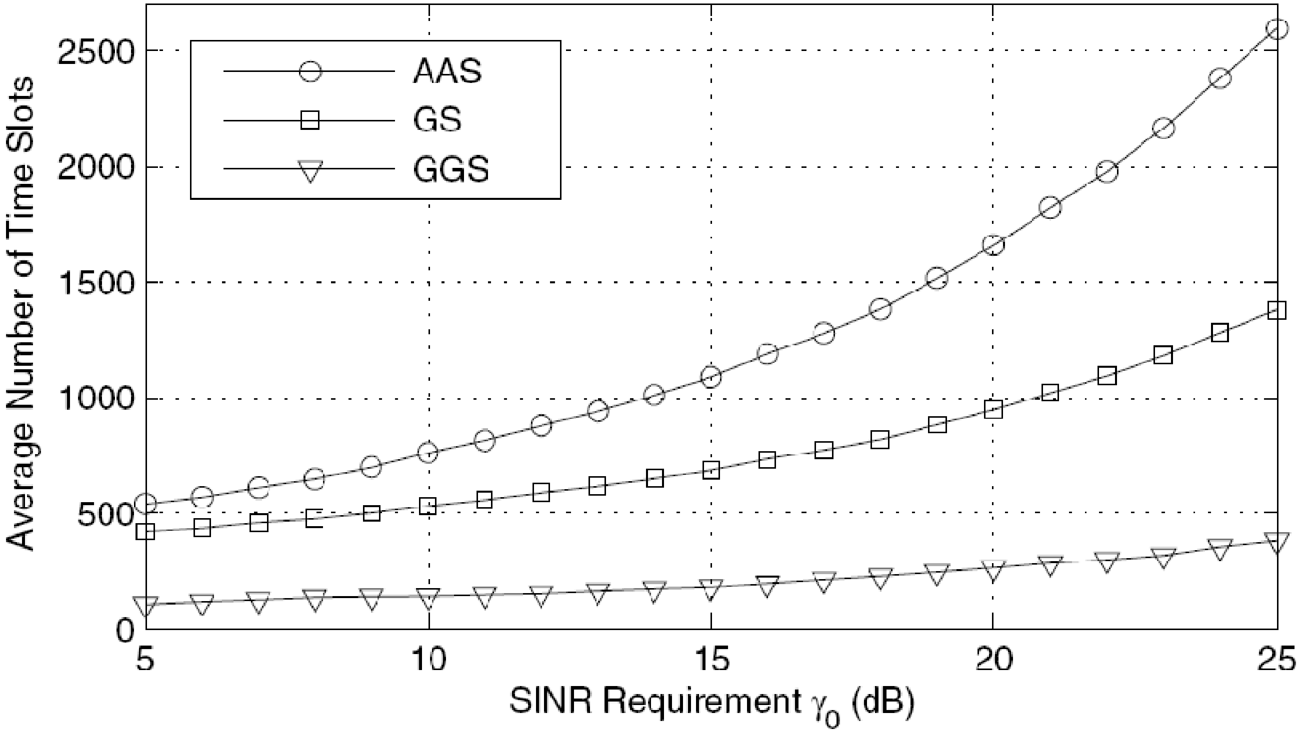

Fig. 3. Average Frame Lengths (the number of links= 500)

Simulation 2

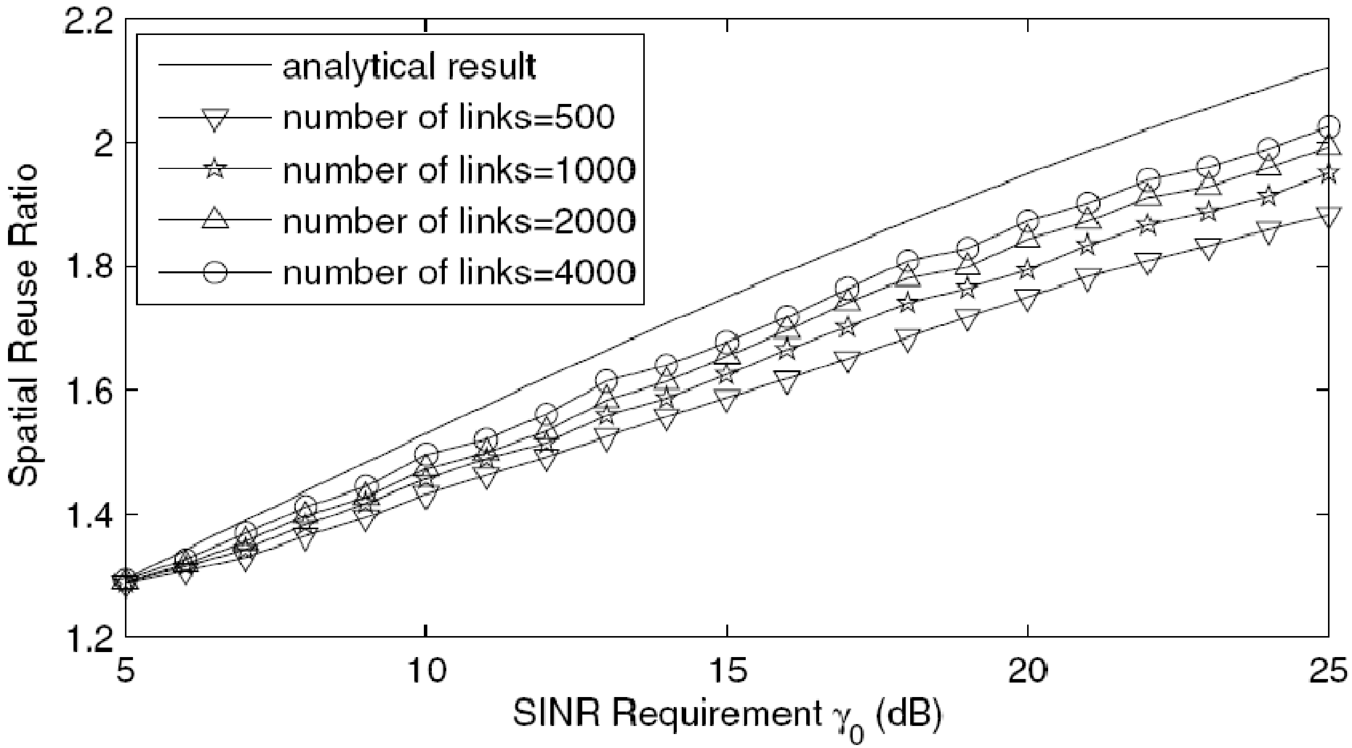

Fig. 4. Spatial Reuse Ratio (Simulations v.s. Analysis) with $\alpha = 4$

Conclusion

- Prove the Joint Power control and Scheduling problem with Consecutive transmission Constraint (JPS-CC) is NP-complete

- Propose a polynomial-time approximation algorithm: Guaranteed and Greedy Scheduling (GGS)

- Prove the GGS algorithm has a bounded approximation ratio $3KN$ relative to the optimal scheduling algorithm

Thanks!