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Energy conservation of bidirectional cellular relay network over Rayleigh fading channel

Shijun Lin
Department of Communication Engineering
Xiamen University, Xiamen, Fujian, China
Email: linsj@xmu.edu.cn

Liqun Fu
School of Information Science and Technology
ShanghaiTech University, Shanghai, China
Email: fulq@shanghaitech.edu.cn

***Abstract*—In this paper, we investigate the energy minimization problem while satisfying the transmission rate requirements in a bidirectional cellular relay network, where a group of mobile users communicate with a base station (BS) across a relay node. We consider a comprehensive power consumption model which includes the power of all components in the radio frequency (RF) transmission, and adopt the Rayleigh fading channel model. We show that the energy minimization problem of the bidirectional cellular relay network can be formulated as a sum of fractional programming problem. Furthermore, we derive the sufficient condition that guarantees the problem to be convex. In the case that the energy minimizing problem is not convex, we decompose it into several sub-problems, and propose an iterative algorithm to solve it. Simulation results show that the iterative algorithm is fast and can achieve an average energy reduction of 65%, compared with the maximum power transmission policy.**

***Index Terms*—Energy conservation, cellular relay network, Rayleigh fading channel.**

I. Introduction

Green wireless network design has become more and more important in the information technology (IT) industry. As reported, the energy consumption of wireless access network in 2012 has reached about 8.28 TWh, and will reach about 38.7 TWh by 2015, which has severe impact on the environment [1]. Furthermore, high energy consumption also results in short battery lifetime of mobile devices.

Since the radio frequency (RF) transmission is one of the main contributors to the energy consumption of wireless devices, we focus on minimizing the RF transmission energy that is consumed by both the power amplifier (PA) and the circuit blocks in this paper. According to the Shannon's capacity formula, the transmit power (the output power of a transmitting node) increases exponentially with the instantaneous transmission rate. Thus, for a given traffic requirement, although the needed active time fraction decreases linearly as the instantaneous transmission rate increases, the energy consumption of the PA still increases as the transmission rate requirement increases. Therefore, in order to minimize the PA energy consumption, we should reduce the instantaneous transmission rate. However, since the circuit energy consumption (the energy consumption of the circuit blocks except the PA) increases linearly with the active time fraction, we can not reduce the transmission rate arbitrarily. The question discussed in this paper is how to achieve a good tradeoff between the two components in order to minimize the overall energy consumption of a bidirectional cellular relay network while the traffic requirements are satisfied.

In the literature, several early papers discussed how to minimize the transmit power with delay constraints (e.g. [2]–[4]). Since they only consider the transmit power consumption, "lazy scheduling" is then proposed such that the packet transmission rates are set as slow as possible. The energy-efficient design when considering both transmit and circuit powers has been investigated in [5]–[13]. In [5], Lin et al. studied the scheduling strategy to minimize the energy consumed by data fusion in wireless sensor networks. In [6], Cui et al. investigated the optimal modulation strategy to minimize the total energy consumption required to send a given amount of traffic. In [7] and [8], the cross-layer approaches were discussed to obtain a better tradeoff between the energy consumption and delay. In [9] and [10], the authors considered maximizing the ratio of the effective capacity to the sum of transmit power and circuit power under delay-outage constraints over frequency-selective and Nakagami-m fading channels, respectively. In [11]- [12], the authors discussed the minimization of uplink power consumption in the single-cell and multi-cell networks while satisfying the rate requirements of all users. In [13], Zhong et al. considered minimizing the transmit and circuit energy consumption by adapting packet transmission rates with individual packet delay constraints in a network where a single node transmits packets to multiple receivers. However, [11]–[13] assume a simpler case that the channel gain is constant.

In this paper, we consider a bidirectional cellular relay network in which a group of mobile users communicate with a base station (BS) across a relay node, and focus on minimizing the overall energy consumption of the bidirectional cellular relay network by properly setting the transmit powers and signal-to-noise-ratio (SNR) thresholds while satisfying the transmission rate requirements. We adopt the Rayleigh fading channel model and a comprehensive RF power model that contains the transmit power, the circuit power, and the idle power. The main contributions of this paper are summarized as follows:

1) We show that the energy minimization problem of the bidirectional cellular relay network can be formulated as a sum of fractional programming problem, which is

very difficult to solve in general.

2) We derive the sufficient condition that guarantees the energy minimizing problem is convex. We find that the considered problem is more likely to be convex when the drain efficiency of the PA is big and the maximum transmit power is small.
3) In the case that the energy minimizing problem is not convex, we propose an iterative algorithm to solve it. The iterative algorithm is guaranteed to be feasible, albeit not necessarily optimal. Simulation results show that the iterative algorithm is fast and can achieve an average energy reduction of 65%, compared with the maximum power transmission policy.

The rest of this paper is organized as follows. Section II presents the network model and problem formulation. In section III, we derive the sufficient condition that guarantees the problem to be convex, and further propose a fast iterative algorithm to minimize the energy consumption when the considered problem is not convex. In section IV, we present the results of performance evaluation. Section V concludes this paper.

II. System description and problem formulation

A. System description

We consider a bidirectional cellular relay network in which a group of mobile users communicate with a BS across a relay node, as shown in Fig. 1. The mobile users and the BS are out of the transmission range of each other and thus they can only communicate through the relay node. The relay node does not generate traffic and there is no communication among the mobile users. What's more, we assume that the centralized Time Division Multiple Access (TDMA) mechanism is adopted and the relay node is used to allocate the time resource by assigning the transmit power and the SNR threshold for each device. The time is divided into fixed length frames. In each frame, each device is allocated a dedicated time period. Our goal is to minimize the energy consumption of the relay network by properly allocating the time resource while satisfying the transmission rate requirements.

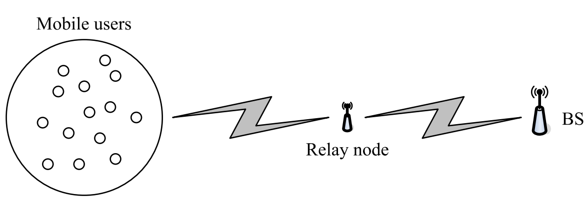

Fig. 1. The cellular relay network.

B. Problem Formulation

The energy consumption of the cellular relay network is determined by both the power consumption of each transmission and the time fraction in which each transmitter is active. Since the time fraction of each transmission is determined by its average transmission rate, we need to show the average transmission rate first. In this paper, the Rayleigh fading channel is adopted. Then, the channel gain g follows an exponential distribution. Let $E\left[g\right]$ denote the expected value of g. The probability density function (PDF) of g, $f(g)$, equals $\frac{1}{E[g]}e^{-\frac{g}{E[g]}}$. Let P_{tr}, N_0, and W denote the transmit power, the power spectral density of additive Gaussian white noise, and the channel bandwidth, respectively. Then, the SNR at the receiver γ equals $\frac{P_{tr}g}{N_0W}$, which follows an exponential distribution with the expected value $\frac{P_{tr}E[g]}{N_0W}$. That is, the PDF of γ, $f(\gamma)$, equals $\frac{N_0W}{P_{tr}E[g]}e^{-\frac{N_0W}{P_{tr}E[g]}\gamma}$. Let γ_0 denote the SNR threshold for successful reception. Then, the successful probability of a transmission equals

$$Q_s = \int_{\gamma_0}^{+\infty} f(\gamma)d\gamma = e^{-\frac{\gamma_0 N_0 W}{P_{tr}E[g]}}. \quad (1)$$

Furthermore, according to Shannon's capacity formula, the maximum instantaneous transmission rate $x_{\max}$ equals $W\log\left(1+\gamma_0\right)$ with the unit nats/s. Thus, the average transmission rate x_{avg} equals

$$x_{avg} = x_{\max}Q_s = W\log\left(1+\gamma_0\right)e^{-\frac{\gamma_0 N_0 W}{P_{tr}E[g]}}. \quad (2)$$

Next, we formulate the energy minimization problem of the cellular relay network. Let $\mathcal{N} = \{n_i, 1 \le i \le |\mathcal{N}|\}$ denote a set of mobile users. Let r_i^u and r_i^d denote the uplink and the downlink rate requirement of n_i, respectively. Let n_R and n_B denote the relay node and the BS, respectively. We assume that the call admission control is adopted and the network can satisfy the rate requirement of all the mobile users. The total energy consumption of the bidirectional network can be divided into three parts: the first part is the energy consumption of the transmissions from the mobile users and the BS to the relay node; the second part is the energy consumption of the transmissions from the relay node to the mobile users and the BS; the third part is the system idle energy consumption, which is the energy consumption of the system when all the devices do not transmit.

Let $P_{tr}\left(v_1,v_2\right)$ and $\gamma_0\left(v_1,v_2\right)$ denote the transmit power and the corresponding SNR threshold for successful reception when n_{v_1} transmits packets to n_{v_2}, respectively. According to (2), the average transmission rate from n_i to n_R equals $W\log\left(1+\gamma_0\left(i,R\right)\right)e^{-\frac{\gamma_0(i,R)N_0W}{P_{tr}(i,R)E\left[g_{i,R}\right]}}$, where g_{v_1,v_2} is the channel gain between n_{v_1} and n_{v_2}. Then, the fraction of time that needs to satisfy the rate requirement from n_i to n_R equals $\frac{r_i^u}{W\log(1+\gamma_0(i,R))e^{-\frac{\gamma_0(i,R)N_0W}{P_{tr}(i,R)E\left[g_{i,R}\right]}}}$. Similarly, since the total rate requirement from n_B to n_R is $r_B^u = \sum_{n_i\in\mathcal{N}} r_i^d$, the fraction of time that needs to satisfy the rate requirement from n_B to n_R equals $\frac{r_B^u}{W\log(1+\gamma_0(B,R))e^{-\frac{\gamma_0(B,R)N_0W}{P_{tr}(B,R)E\left[g_{B,R}\right]}}}$. Then, the energy consumption of the transmissions from the mobile users and the BS to the relay node, E_1, can be calculated as follows:

$$E_1 = \sum_{n_i \in \mathcal{N}} \left(\frac{r_i^u}{W \log(1+\gamma_0(i,R)) e^{-\frac{\gamma_0(i,R) N_0 W}{P_{tr}(i,R) E\left[g_{i,R}\right]}}} \cdot \left(\frac{P_{tr}(i,R)}{\theta} + P_{ct}^i + P_{cr}^R + \sum_{n_j \in \mathcal{N} \setminus \{n_i\}} P_{id}^j + P_{id}^B \right) \right) + \frac{r_B^u}{W \log(1+\gamma_0(B,R)) e^{-\frac{\gamma_0(B,R) N_0 W}{P_{tr}(B,R) E\left[g_{B,R}\right]}}} \cdot \left(\frac{P_{tr}(B,R)}{\theta} + P_{ct}^B + P_{cr}^R + \sum_{n_j \in \mathcal{N}} P_{id}^j \right). \quad (3)$$

Here, θ, P_{ct}, P_{cr}, and P_{id} are the drain efficiency of the PA, the power consumption of the circuit blocks in the transmitter except the PA, the power consumption of the circuit blocks in the receiver, and the power consumption when a device does not transmit or receive, respectively. We put the superscripts i, R and B for P_{ct}, P_{cr}, and P_{id} to indicate the different parameters of n_i, n_R, and n_B.

The energy consumptions of the transmissions from the relay node to the mobile users and the BS, E_2, can similarly be calculated as follows:

$$E_2 = \sum_{n_i \in \mathcal{N}} \left(\frac{r_i^d}{W \log(1+\gamma_0(R,i)) e^{-\frac{\gamma_0(R,i) N_0 W}{P_{tr}(R,i) E\left[g_{R,i}\right]}}} \cdot \left(\frac{P_{tr}(R,i)}{\theta} + P_{ct}^R + P_{cr}^i + \sum_{n_j \in \mathcal{N} \setminus \{n_i\}} P_{id}^j + P_{id}^B \right) \right) + \frac{r_B^d}{W \log(1+\gamma_0(R,B)) e^{-\frac{\gamma_0(R,B) N_0 W}{P_{tr}(R,B) E\left[g_{R,B}\right]}}} \cdot \left(\frac{P_{tr}(R,B)}{\theta} + P_{ct}^R + P_{cr}^B + \sum_{n_j \in \mathcal{N}} P_{id}^j \right), \quad (4)$$

where r_B^d is the total rate requirement from n_R to n_B, which equals $\sum_{n_i \in \mathcal{N}} r_i^u$.

Let T_{act} denote the active time of the system, which equals

$$T_{act} = \sum_{n_i \in \mathcal{N}} \frac{r_i^u}{W \log(1+\gamma_0(i,R)) e^{-\frac{\gamma_0(i,R) N_0 W}{P_{tr}(i,R) E\left[g_{i,R}\right]}}} + \frac{r_B^u}{W \log(1+\gamma_0(B,R)) e^{-\frac{\gamma_0(B,R) N_0 W}{P_{tr}(B,R) E\left[g_{B,R}\right]}}} + \sum_{n_i \in \mathcal{N}} \frac{r_i^d}{W \log(1+\gamma_0(R,i)) e^{-\frac{\gamma_0(R,i) N_0 W}{P_{tr}(R,i) E\left[g_{R,i}\right]}}} + \frac{r_B^d}{W \log(1+\gamma_0(R,B)) e^{-\frac{\gamma_0(R,B) N_0 W}{P_{tr}(R,B) E\left[g_{R,B}\right]}}}. \quad (5)$$

Then, the system idle energy consumption, E_3, equals

$$E_3 = (1 - T_{act}) \left(\sum_{n_i \in \mathcal{N}} P_{id}^i + P_{id}^B + P_{id}^R \right). \quad (6)$$

Therefore, the energy consumption minimizing problem of the bidirectional cellular relay network can be formulated as follows:

$$\begin{array}{ll} \text{Minimize} & E_1 + E_2 + E_3. \\ \text{Subject to} & T_{act} \le 1. \\ \text{Variables} & P_{tr}(i,R) > 0, P_{tr}(R,i) > 0, \forall n_i \in \mathcal{N}, \\ & \gamma_0(i,R) > 0, \gamma_0(R,i) > 0, \forall n_i \in \mathcal{N}, \\ & P_{tr}(B,R) > 0, P_{tr}(R,B) > 0, \\ & \gamma_0(B,R) > 0, \gamma_0(R,B) > 0. \end{array} \quad (7)$$

Applying (3)-(6), Problem (7) can be equivalent to

$$\begin{array}{ll} \text{Minimize} & \sum_{n_i \in \mathcal{N}} \left(\frac{r_i^u}{W \log(1+\gamma_0(i,R)) e^{-\frac{\gamma_0(i,R) N_0 W}{P_{tr}(i,R) E\left[g_{i,R}\right]}}} \cdot \left(\frac{P_{tr}(i,R)}{\theta} + P_{ct}^i + P_{cr}^R - P_{id}^i - P_{id}^R \right) \right) \\ & + \sum_{n_i \in \mathcal{N}} \left(\frac{r_i^d}{W \log(1+\gamma_0(R,i)) e^{-\frac{\gamma_0(R,i) N_0 W}{P_{tr}(R,i) E\left[g_{R,i}\right]}}} \cdot \left(\frac{P_{tr}(R,i)}{\theta} + P_{ct}^R + P_{cr}^i - P_{id}^R - P_{id}^i \right) \right) \\ & + \frac{r_B^u}{W \log(1+\gamma_0(B,R)) e^{-\frac{\gamma_0(B,R) N_0 W}{P_{tr}(B,R) E\left[g_{B,R}\right]}}} \cdot \left(\frac{P_{tr}(B,R)}{\theta} + P_{ct}^B + P_{cr}^R - P_{id}^B - P_{id}^R \right) \\ & + \frac{r_B^d}{W \log(1+\gamma_0(R,B)) e^{-\frac{\gamma_0(R,B) N_0 W}{P_{tr}(R,B) E\left[g_{R,B}\right]}}} \cdot \left(\frac{P_{tr}(R,B)}{\theta} + P_{ct}^R + P_{cr}^B - P_{id}^R - P_{id}^B \right). \\ \text{Subject to} & T_{act} - 1 \le 0 \\ \text{Variables} & P_{tr}(i,R) > 0, P_{tr}(R,i) > 0, \forall n_i \in \mathcal{N}, \\ & \gamma_0(i,R) > 0, \gamma_0(R,i) > 0, \forall n_i \in \mathcal{N}, \\ & P_{tr}(B,R) > 0, P_{tr}(R,B) > 0, \\ & \gamma_0(B,R) > 0, \gamma_0(R,B) > 0. \end{array} \quad (8)$$

To further simplify notations, let $F = 2|\mathcal{N}| + 2$,

$$s_k = \begin{cases} P_{tr}(k,R), 1 \le k \le |\mathcal{N}|, \\ P_{tr}(R, k - |\mathcal{N}|), |\mathcal{N}| + 1 \le k \le 2|\mathcal{N}|, \\ P_{tr}(B,R), k = 2|\mathcal{N}| + 1, \\ P_{tr}(R,B), k = 2|\mathcal{N}| + 2, \end{cases}$$

$$t_k = \begin{cases} \gamma_0(k,R), 1 \le k \le |\mathcal{N}|, \\ \gamma_0(R, k - |\mathcal{N}|), |\mathcal{N}| + 1 \le k \le 2|\mathcal{N}|, \\ \gamma_0(B,R), k = 2|\mathcal{N}| + 1, \\ \gamma_0(R,B), k = 2|\mathcal{N}| + 2, \end{cases}$$

$$D_k = \begin{cases} P_{ct}^k + P_{cr}^R - P_{id}^k - P_{id}^R, 1 \le k \le |\mathcal{N}|, \\ P_{ct}^R + P_{cr}^{k-|\mathcal{N}|} - P_{id}^R - P_{id}^{k-|\mathcal{N}|}, |\mathcal{N}| + 1 \le k \\ \le 2|\mathcal{N}|, \\ P_{ct}^B + P_{cr}^R - P_{id}^B - P_{id}^R, k = 2|\mathcal{N}| + 1, \\ P_{ct}^R + P_{cr}^B - P_{id}^R - P_{id}^B, k = 2|\mathcal{N}| + 2, \end{cases}$$

$$M_k = \begin{cases} \frac{1}{E[g_{k,R}]}, 1 \le k \le |\mathcal{N}|, \\ \frac{1}{E\left[g_{R,k-|\mathcal{N}|}\right]}, |\mathcal{N}| + 1 \le k \le 2|\mathcal{N}|, \\ \frac{1}{E[g_{B,R}]}, k = 2|\mathcal{N}| + 1, \\ \frac{1}{E[g_{R,B}]}, k = 2|\mathcal{N}| + 2, \end{cases}$$

$$L_k = \begin{cases} r_k^u, 1 \le k \le |\mathcal{N}|, \\ r_{k-|\mathcal{N}|}^d, |\mathcal{N}|+1 \le k \le 2|\mathcal{N}|, \\ r_B^u, k = 2|\mathcal{N}|+1, \\ r_B^d, k = 2|\mathcal{N}|+2. \end{cases}$$

Then, Problem (8) can be simply expressed by

$$\begin{array}{ll} \text{Minimize} & \sum\limits_{1\le k\le F} \frac{L_k}{W\log(1+t_k)e^{-\frac{t_k N_0 W M_k}{s_k}}}\left(\frac{s_k}{\theta}+D_k\right). \\ \text{Subject to} & \sum\limits_{1\le k\le F} \frac{L_k}{W\log(1+t_k)e^{-\frac{t_k N_0 W M_k}{s_k}}} - 1 \le 0. \\ \text{Variables} & s_k > 0, t_k > 0, 1 \le k \le F. \end{array} \quad (9)$$

Obviously, Problem (9) is a sum of fractional programming problem, which is usually difficult to solve in general. Fortunately, we find that this problem is conditionally convex. Therefore, the solution method to this problem can be classified into two cases: 1) When the convex condition is satisfied, we can use the typical methods for convex optimization problems [14] to obtain the optimal s_k and t_k. 2) When the convex condition can not be satisfied, we then propose an iterative algorithm to solve it.

III. Convex Condition and Solutions

A. Convex condition

Let $P_{tr}^{\max}$, $\gamma_0^{\min}$, and $\gamma_0^{\max}$ denote the maximum transmit power, the minimum and the maximum SNR threshold of the system. The convex condition of Problem (9) is shown in the following theorem.

Theorem 1: Problem (9) is convex if

$$D_k \ge A\frac{P_{tr}^{\max}}{\theta}, \forall k \in [1, F], \quad (10)$$

where A is the minimum value that satisfies

$$\begin{array}{l} -\frac{2t_k}{1+t_k} + \frac{2(A+1)t_k^2}{(1+t_k)^2\log(1+t_k)} + \frac{(A+1)t_k^2}{(1+t_k)^2} \ge 0, \\ \forall t_k \in \left[\gamma_0^{\min}, \gamma_0^{\max}\right]. \end{array} \quad (11)$$

Proof: We first show that the object function of Problem (9) is convex if $D_k \ge A\frac{P_{tr}^{\max}}{\theta}, \forall k \in [1, F]$. Let $H_k^1(s_k, t_k) \triangleq \frac{L_k}{W\log(1+t_k)e^{-\frac{t_k N_0 W M_k}{s_k}}}\left(\frac{s_k}{\theta}+D_k\right)$. We need to prove that $H_k^1(s_k, t_k)$ is convex if $D_k \ge A\frac{P_{tr}^{\max}}{\theta}$.

Next we show that the Hessian Matrix of $H_k^1(s_k, t_k)$ is positive semi-definite if $D_k \ge A\frac{P_{tr}^{\max}}{\theta}$. We have

$$\begin{array}{l} (s_k \quad t_k)\left[\begin{array}{cc} \frac{\partial^2 H_k^1(s_k,t_k)}{\partial^2 s_k} & \frac{\partial^2 H_k^1(s_k,t_k)}{\partial s_k \partial t_k} \\ \frac{\partial^2 H_k^1(s_k,t_k)}{\partial t_k \partial s_k} & \frac{\partial^2 H_k^1(s_k,t_k)}{\partial^2 t_k} \end{array}\right]\left(\begin{array}{c} s_k \\ t_k \end{array}\right) \\ = \frac{L_k}{W(\log(1+t_k))^2 e^{-\frac{t_k N_0 W M_k}{s_k}}} \cdot \\ \left(-\frac{2s_k t_k}{\theta(1+t_k)} + \frac{2s_k t_k^2}{(1+t_k)^2\theta\log(1+t_k)}\right. \\ \left. + \frac{2D_k t_k^2}{(1+t_k)^2\log(1+t_k)} + \frac{s_k t_k^2}{\theta(1+t_k)^2} + \frac{D_k t_k^2}{(1+t_k)^2}\right). \end{array} \quad (12)$$

If $D_k \ge A\frac{P_{tr}^{\max}}{\theta}$, then, $D_k \ge A\frac{s_k}{\theta}$. Thus, we have

$$\begin{array}{l} (s_k t_k)\left[\begin{array}{cc} \frac{\partial^2 H_k^1(s_k,t_k)}{\partial^2 s_k} & \frac{\partial^2 H_k^1(s_k,t_k)}{\partial s_k \partial t_k} \\ \frac{\partial^2 H_k^1(s_k,t_k)}{\partial t_k \partial s_k} & \frac{\partial^2 H_k^1(s_k,t_k)}{\partial^2 t_k} \end{array}\right]\left(\begin{array}{c} P_{tr} \\ t_k \end{array}\right) \\ \ge \frac{L_k s_k}{\theta W(\log(1+t_k))^2 e^{-\frac{t_k N_0 W M_k}{s_k}}} \cdot \\ \left(-\frac{2t_k}{(1+t_k)} + \frac{2(A+1)t_k^2}{(1+t_k)^2\log(1+t_k)} + \frac{(A+1)t_k^2}{(1+t_k)^2}\right), \end{array} \quad (13)$$

which is non-negative when A is the minimum value that satisfies condition (11). That is, the object function of Problem (9) is convex if $D_k \ge A\frac{P_{tr}^{\max}}{\theta}, \forall k \in [1, F]$.

In the following, we show that the constraint set of Problem (9) is a convex set. Let $H_k^2(s_k, t_k) \triangleq \frac{L_k}{W\log(1+t_k)e^{-\frac{t_k N_0 W M_k}{s_k}}}$. We need to prove that $H_k^2(s_k, t_k)$ is convex. Since

$$\begin{array}{l} (s_k t_k)\left[\begin{array}{cc} \frac{\partial^2 H_k^2(s_k,t_k)}{\partial^2 s_k} & \frac{\partial^2 H_k^2(s_k,t_k)}{\partial s_k \partial t_k} \\ \frac{\partial^2 H_k^2(s_k,t_k)}{\partial t_k \partial s_k} & \frac{\partial^2 H_k^2(s_k,t_k)}{\partial^2 t_k} \end{array}\right]\left(\begin{array}{c} s_k \\ t_k \end{array}\right) \\ = \frac{L_k}{W(\log(1+t_k))^2 e^{-\frac{t_k N_0 W M_k}{s_k}}} \cdot \left(\frac{2t_k^2}{(1+t_k)^2\log(1+t_k)} + \frac{t_k^2}{(1+t_k)^2}\right) \end{array}$$

is always non-negative, $H_k^2(s_k, t_k)$ is convex. Therefore, Problem (9) is convex if condition (10) is satisfied. ■

From Theorem 1, we find that Problem (9) is more likely to be convex when the drain efficiency of the PA is big and the maximum transmit power is small. For example, when $\gamma_0^{\min}$ and $\gamma_0^{\max}$ respectively equal 0.5 and 500, we can obtain $A = 0.52$ from numerical search. Then, with the typical power parameters given in [11] and [15], if the drain efficiency of the PA can reach 0.6 [16], Problem (9) is convex when the maximum transmit power is smaller than 205 mW.

B. The iterative algorithm when Problem (9) is not convex

When Problem (9) is not convex, it is difficult to find the optimal solution to Problem (9). In this case, we apply decomposition method that alternatively optimizes s_k and t_k in two sub-processes. First, given s_k, we obtain the optimal t_k that minimizes $H_k^1(s_k, t_k)$, denoted by $t_k^{opt}(s_k)$. Second, we further optimize s_k by an iterative algorithm.

Theorem 2: Given s_k, the optimal t_k that minimizes $H_k^1(s_k, t_k)$, equals

$$t_k^{opt}(s_k) = \begin{cases} \gamma_0^{\min}, & if\ t_k^0 < \gamma_0^{\min}, \\ t_k^0, & if\ \gamma_0^{\min} \le t_k^0 \le \gamma_0^{\max}, \\ \gamma_0^{\max}, & if\ t_k^0 > \gamma_0^{\max}, \end{cases} \quad (14)$$

where t_k^0 is the value of t_k which satisfies

$$N_0 W M_k(1+t_k)\log(1+t_k) - s_k = 0. \quad (15)$$

Proof: The second partial derivative of $H_k^1(s_k, t_k)$, $\frac{\partial^2 H_k^1(s_k,t_k)}{\partial^2 t_k}$, equals

$$\begin{array}{l} \frac{\partial^2 H_k^1(s_k,t_k)}{\partial^2 t_k} = \frac{L_k\left(\frac{s_k}{\theta}+D_k\right)}{W(\log(1+t_k))^3 e^{-\frac{t_k N_0 W M_k}{s_k}}} \cdot \left(\frac{1}{(1+t_k)^2}\right. \\ \left. + \left(\frac{1}{1+t_k} - \frac{N_0 W M_k \log(1+t_k)}{s_k}\right)^2 + \frac{1}{(1+t_k)^2}\log(1+t_k)\right), \end{array}$$

which is always non-negative. Thus, given s_k, the optimal t_k that minimizes $H_k^1(s_k, t_k)$ can be obtained by imposing $\frac{\partial H_k^1(s_k,t_k)}{\partial t_k}$ equal to 0. Then, we have

$$\begin{array}{l} \frac{L_k}{W(\log(1+t_k))^2 e^{-\frac{t_k N_0 W M_k}{s_k}}}\left(\frac{s_k}{\theta}+D_k\right) \cdot \\ \left(\frac{N_0 W M_k}{s_k}\log(1+t_k) - \frac{1}{(1+t_k)}\right) = 0. \end{array}$$

After some simplifications, equation (15) and t_k^0 can be obtained. Considering that t_k^0 may not be always in the range of $\left[\gamma_0^{\min}, \gamma_0^{\max}\right]$, the optimal t_k that minimizes $H_k^1(s_k, t_k)$ respectively equals $\gamma_0^{\min}$ and $\gamma_0^{\max}$ when $t_k^0 < \gamma_0^{\min}$ and $t_k^0 > \gamma_0^{\max}$. ■

Based on Theorem 2, Problem (9) can then be simplified to

$$\begin{aligned}
&\text{Minimize} && \sum_{1\le k\le F} \frac{L_k}{W\log\left(1+t_k^{opt}(s_k)\right)e^{-\frac{N_0 W M_k t_k^{opt}(s_k)}{s_k}}}\left(\frac{s_k}{\theta}+D_k\right).\\
&\text{Subject to} && \sum_{1\le k\le F} \frac{L_k}{W\log\left(1+t_k^{opt}(s_k)\right)e^{-\frac{N_0 W M_k t_k^{opt}(s_k)}{s_k}}}-1\le 0.\\
&\text{Variables} && s_k>0, 1\le k\le F.
\end{aligned}\tag{16}$$

Let $Z(s_k) \triangleq \frac{L_k}{W\log\left(1+t_k^{opt}(s_k)\right)e^{-\frac{N_0 W M_k t_k^{opt}(s_k)}{s_k}}}\left(\frac{s_k}{\theta}+D_k\right)$.

We next propose an iterative algorithm to solve Problem (16). The iterative algorithm contains two main steps, as shown in Fig. 2. In Step 1, for each $k \in [1, F]$, we first relax the constraint of s_k to $\frac{L_k}{W\log\left(1+t_k^{opt}(s_k)\right)e^{-\frac{N_0 W M_k t_k^{opt}(s_k)}{s_k}}} \le 1$, and find the optimal s_k that minimizes $Z(s_k)$ through numerical search. That is, the obtained s_k $(1 \le k \le F)$ is optimal but it may not satisfy the constraint of Problem (16). If the constraint of Problem (16) is not satisfied, we execute Step 2 repeatedly until the constraint of Problem (16) is satisfied with minimum energy consumption increase.

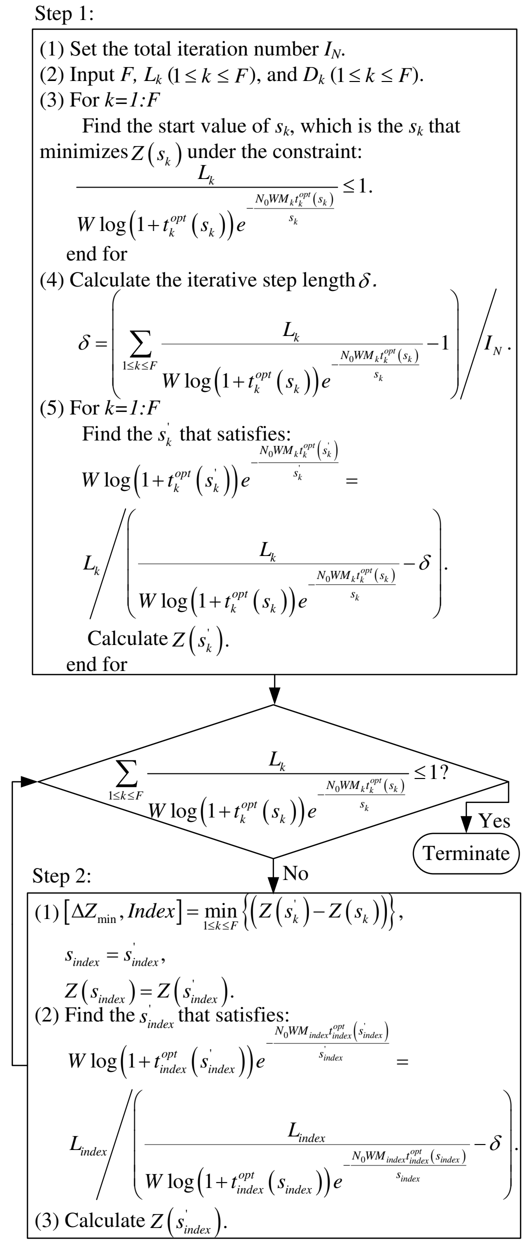

Fig. 2. Flowchart of the iterative algorithm.

IV. SIMULATION RESULTS

In the simulation, the mobile users are randomly located within the circular area centered at (0, 0) (the origin of coordinates) with radius 100 m. The X coordinates of the relay node and the BS are set to 600 m and 1200 m, respectively. The Y coordinates of the relay node and the BS are both set to 0 m. The expected value of channel gain $E[g]$ is obtained from log-distance path-loss model with path-loss exponent of 4. The power related parameters are set as the same as in [11] and [15]. The drain efficiency of the PA is set to 0.2. The maximum transmit power, the noise power density, and the bandwidth are set to 27.5 dBm, -174 dBm/Hz, and 1 MHz, respectively. The uplink and downlink rate requirements of the mobile users are uniformly distributed from 30 kbps to 70 kbps.

A. Energy consumption improvement

We compare the system energy consumption of the proposed iterative algorithm with the maximum power transmission policy, in which all the devices transmit with the same maximum power. Furthermore, we evaluate the running time of the proposed iterative algorithm on a laptop with i7 CPU working at the frequency of 2.8 GHz and 4 GB RAM. The iteration number I_N is set to 1000. The energy consumption and the corresponding running time of the iterative algorithm versus the number of mobile users are shown in Fig. 3. From Fig. 3, we can find that the iterative algorithm achieves an energy reduction up to 65%, compared with the maximum power transmission policy. Furthermore, the running time of the iterative algorithm is within several seconds. This implies that the iterative algorithm can be used in some scenarios with dynamic rate requirements.

B. The effect of the iteration number I_N

The energy consumption of the relay system and the running time of the iterative algorithm versus the iteration number are shown in Fig. 4. The number of mobile users is set to 20. From Fig. 4, we can find that 1) the energy consumption of the relay system decreases as the iteration number I_N increases and finally converges to a constant value; 2) the running time of the iterative algorithm does not increase greatly as the iteration number I_N increases. Therefore, we can select a large enough

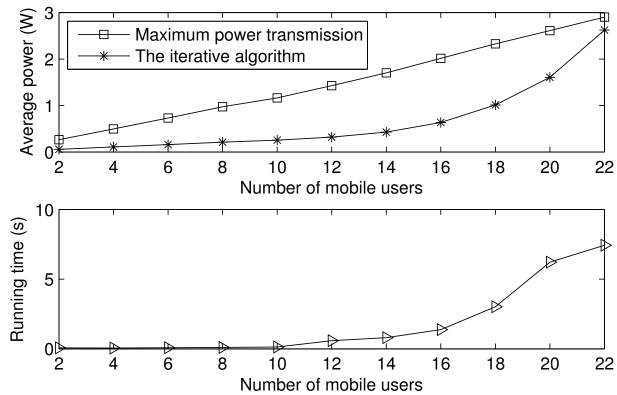

Fig. 3. The energy consumption and the running time of the iterative algorithm v.s. the number of mobile users.

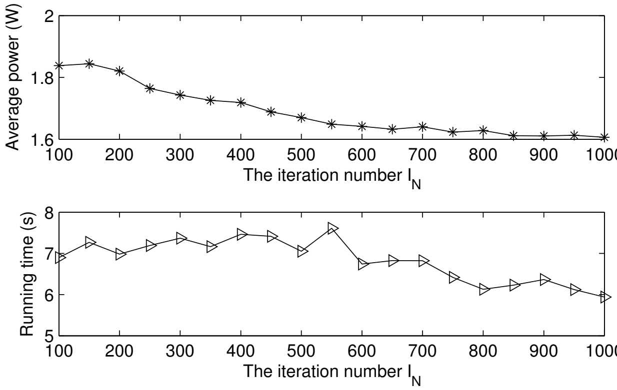

Fig. 4. The energy consumption of the relay system and the running time of the iterative algorithm v.s. the iteration number.

I_N to reduce the energy consumption.

V. CONCLUSIONS

In this paper, we have considered minimizing the energy consumption of the cellular relay network over the Rayleigh fading channel while satisfying the transmission rate requirements. We adopt a comprehensive RF power model that contains the transmit power, the circuit power at the transmitter and the receiver, and the idle power. We have showed that it can be formulated as a sum of fractional programming problem. Furthermore, we have obtained a sufficient condition that ensures the problem to be convex. When the convex condition is not satisfied, we propose a decomposition method that solves the problem with two sub-processes: 1) Given the transmit power, we derive the optimal SNR threshold. 2) Based on the optimal SNR threshold, we then use an iterative algorithm to find the power consumption of the network. Simulation results show that the iterative algorithm is fast and can achieve an energy reduction up to 65%, compared with the maximum power transmission policy.

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