Hybrid Network Coding for Unbalanced Slotted ALOHA Relay Networks

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***Abstract*—In this paper, we investigate the throughput performance of the network coding (NC) schemes under the slotted ALOHA protocol. We consider the *all-inclusive-interfering unbalanced* network in which two client groups with different numbers of nodes communicate with each other through a relay node. We derive the closed-form expressions of the network throughput under the physical-layer network coding (PNC), traditional high-layer network coding (HNC), and non-network-coding (NNC), respectively. We also show the necessary and sufficient condition to make the relay node unsaturated. From the analytical results, we find that although PNC has better transmission efficiency in the two-way relay channel (TWRC); it does not always have better network throughput when the network has multiple client nodes. To further improve the network throughput, we propose the hybrid NC scheme, which allows the relay node to turn to HNC scheme if it fails to explore the PNC transmission. We further obtain the closed-form expression of the network throughput and the necessary and sufficient condition to make the relay node unsaturated in the hybrid NC scheme. Simulation results show that the hybrid NC scheme has better throughput performance than the PNC, HNC, and NNC schemes. Moreover, we optimize the network throughput of the hybrid NC scheme in terms of the transmission probability of the relay node. Last but not least, we evaluate the throughput performance of hybrid NC scheme through simulations.**

***Index Terms*—Physical-layer network coding (PNC), high-layer network coding (HNC), hybrid network coding (NC), slotted ALOHA.**

I. Introduction

NETWORK coding (NC) [1], [2] has been proposed as a promising technique to improve the throughput of wireless network. Traditionally, NC operation is performed at the network layer, and three time slots are needed for two client nodes to exchange two packets through a relay node. We refer to this three-time-slot network coding scheme as high-layer network coding (HNC) scheme. The physical-layer network coding (PNC) [3], [4], further explores the superimposed electromagnetic (EM) waves as a natural way of network coding operation. In PNC scheme, only two time slots are needed for two client nodes to exchange two packets through a relay node, which significantly improves the network throughput.

In the literature, the study of PNC is mostly focused on the physical layer, and based on the two-way relay channel (TWRC), where two client nodes exchange packets through a relay node [5]–[7]. When PNC is applied to a general wireless network, the MAC protocol needs to be adjusted. In [8] and [9], two 802.11-like distributed MAC protocols were proposed to support PNC transmission. In [10] and [11], the authors proposed the multi-user PNC method for slotted ALOHA network. However, [8]–[11] do not provide analytical network throughput performance on the proposed MAC protocol with PNC.

Recently, the authors in [12] analyzed the throughput performance of PNC coordinated with an 802.11-like protocol in a simple balanced network in which all the client nodes have the same contention window size and transmission probability. The throughput performance of slotted ALOHA network with HNC scheme was investigated in [13]–[19]. In [13], Ibars et al. analyzed the performance of HNC in a multi-hop multicast slotted ALOHA network with game theoretic approach. In [14], Amerimehr et al. proposed a multi-class queuing model to analyze the maximum stable throughput of HNC in a slotted ALOHA wireless tandem network. In [15], Rajawat et al. optimized the transmission probabilities subject to flow conservation and rate constraints in an HNC slotted-ALOHA multihop wireless network. In [16], [17], the authors investigated the throughput gain of HNC versus non-network-coding (NNC) in a slotted ALOHA network with star topology. However, it is assumed that all the client nodes have the same transmission probability. Two recent papers, [18] and [19], analyzed the throughput performance of the NNC and HNC schemes in an unbalanced slotted ALOHA network with one relay node and two client groups. However, in [18], [19], the authors assumed that the two client groups are far apart such that the client nodes within different groups do not interfere with each other.

In this paper, we investigate the throughput performance of network coding schemes (both HNC and PNC) under the slotted ALOHA protocol in the relay system where two client groups communicate with each other through a relay node. We consider the *unbalanced* network, where the two groups have different numbers of nodes with different transmission probabilities. Furthermore, we consider the *all-inclusive-interfering* case where all the nodes in the network interfere with each other. This is because: 1) The unbalanced

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network is more practical and general. As in a wireless network, the number of client nodes in the two groups may not always be the same, and assigning the same transmission probability to all the nodes may not maximize the network throughput. The balanced network can be treated as a special case of unbalanced network. 2) In practice, the interference range is usually larger than the transmission range. The two client groups, which both are in the transmission range of the relay node, are highly likely to interfere with each other. However, both the modeling and the theoretical analysis of the all-inclusive-interfering unbalanced network are more complicated. We obtain the closed-form expressions of the network throughput of the PNC, HNC, and NNC schemes under the slotted ALOHA protocol in the all-inclusive-interfering unbalanced relay network. We further show the necessary and sufficient condition to ensure the relay node is unsaturated.

We find that the throughput performance of HNC is mainly affected by the balance of the total traffic between the two client groups; while the throughput performance of PNC is heavily affected by the value of α, which represents the probability that a transmitted NC packet contains the information of two packets. Thus, PNC may have lower throughput than HNC when α is small. This motivates us to further propose the hybrid NC scheme, which allows the relay node to turn to HNC scheme if it fails to explore the PNC transmission. We provide theoretical analysis on the throughput performance of hybrid NC scheme together with the necessary and sufficient condition to ensure the relay node is unsaturated. The hybrid NC scheme has better throughput performance than single HNC or PNC scheme. We further optimize the network throughput of the hybrid NC scheme in terms of the transmission probability of the relay node h_r. In particular, we show that the network throughput of hybrid NC scheme monotonically decreases with h_r. Therefore, the optimal h_r is the minimum value that ensures all the buffers at the relay node are unsaturated. Simulation results show that the throughput gain of hybrid NC scheme versus PNC, HNC, and NNC schemes can respectively reach 127.7%, 154.7%, and 188.9%.

The rest of this paper is organized as follows. Section II presents the network model and the transmission processes of the NNC, HNC and PNC schemes with the slotted ALOHA protocol. In section III, we derive the closed-form expressions of the network throughput for NNC, HNC, and PNC schemes. In section IV, we propose the hybrid NC scheme, derive the closed-form expression of the network throughput, and optimize the network throughput of hybrid NC scheme. In section V, we carry out extensive simulations to validate our model, discuss the relationship between the network throughput of hybrid NC scheme and system parameters, and compare the throughput of the NNC, HNC, PNC, and hybrid NC schemes. Section VI concludes this paper.

II. System Description

A. Network Model

We consider a bidirectional unbalanced relay network with two client groups of u_1 and u_2 nodes, as shown in Fig. 1. The relay node is within the transmission range of all the client nodes. The nodes in different groups are within the interference range but out of the transmission range of each other. Thus, two nodes in different groups can only exchange packets through the relay node. The relay node does not generate traffic, and there is no traffic within the same client group. We assume the saturated traffic load such that each client node always has packets to send. There is only one physical channel, and all the nodes contend for the channel according to the slotted ALOHA protocol. We assume half-duplex transmission such that a node cannot transmit and receive simultaneously.

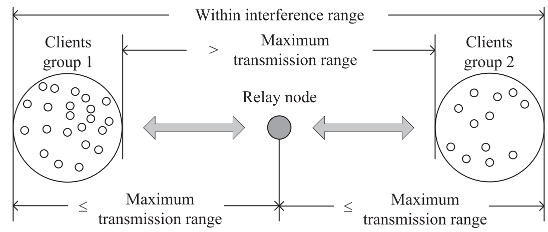


Fig. 1. Unbalanced relay network.

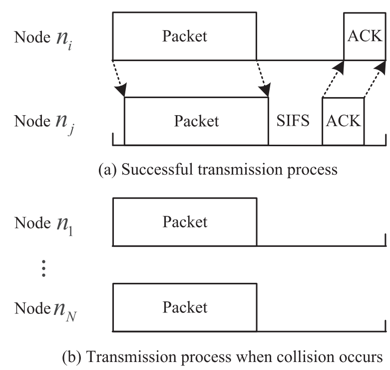
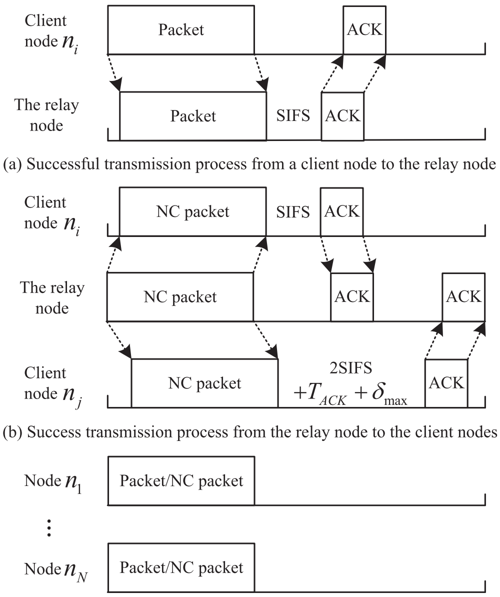


Fig. 2. Transmission process in NNC scheme.

B. The NNC, HNC and PNC Transmissions Coordinated With Slotted ALOHA Protocol

In the slotted ALOHA protocol, time is divided into a number of slots. Each packet transmission starts at the beginning of a time slot. To reduce mutual collisions, each node initiates a transmission with a given probability when it has packets to transmit. We assume that the transmission probability is the same for the nodes in the same group but different for the nodes in different groups. If only one node initiates a transmission in the current time slot, its destination node can successfully receive the packet. Otherwise, collision occurs, and the transmission in the current time slot is failed. Next we show the transmission processes in NNC, HNC and PNC schemes during a time slot, respectively. Note that since the overhead in each scheme is different, the slot time of each scheme is also different.

In NNC scheme, the successful transmission process from node n_i to n_j is shown in Fig. 2a. At the beginning of the time slot, n_i transmits a packet to n_j. After correctly receiving the packet from n_i, n_j waits for a short inter frame space (SIFS)

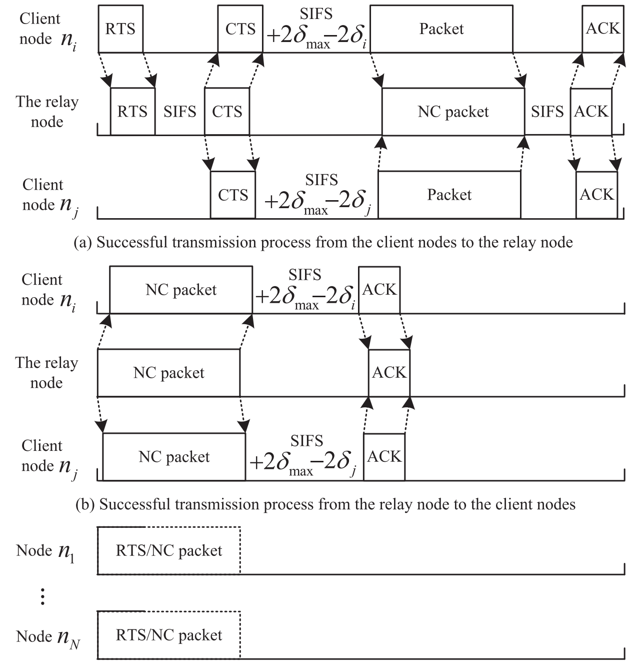


(a) Successful transmission process from a client node to the relay node

(b) Success transmission process from the relay node to the client nodes

(c) Transmission process when collision occurs

Fig. 3. Transmission process in HNC scheme.



(a) Successful transmission process from the client nodes to the relay node

(b) Successful transmission process from the relay node to the client nodes

(c) Transmission process when collision occurs

Fig. 4. Transmission process in PNC scheme.

time, and then sends back an acknowledgement (ACK) packet to n_i. The slot time T_{slot} in NNC scheme can be calculated as follows[1]:

$$T_{slot} = T_{packet} + T_{ACK} + \text{SIFS} + 2\delta_{\max}, \tag{1}$$

where T_{packet}, T_{ACK}, and $\delta_{\max}$ are the time used for transmitting the data packet, the time used for transmitting the ACK packet, and the maximum propagation delay, respectively.

In HNC scheme, the client node in each group transmits a packet to the relay node with the same process in NNC scheme, as shown in Fig. 3a. When a client node transmits, all the other client nodes in the same group perform "opportunistic listening" (receive the transmitted packet and store it in their temporary buffers). The transmitting client node also stores the transmitted packet in its temporary buffer. By doing so, the client nodes can decode all the NC packets from the relay node in the second step. After receiving both packets from two client groups, the relay node forms an NC packet, and then broadcasts it to the two destination nodes at the beginning of the next time slot (see Fig. 3b). Then, the two destination nodes respectively transmit an ACK packet to the relay node after a delay of SIFS and 2SIFS $+ T_{ACK} + \delta_{\max}$. If the relay node has a packet from only one client group, it transmits the packet to the destination node in the other group with the same process in NNC scheme (see Fig. 3a). From Fig. 3, we know the slot time T_{slot} in HNC scheme equals

$$T_{slot} = T_{packet} + 2T_{ACK} + 2\text{SIFS} + 3\delta_{\max}. \tag{2}$$

In PNC scheme, When two nodes in different groups concurrently transmit a packet to the relay node, Request to Send (RTS)/Clear to Send (CTS) mechanism is adopted to coordinate the transmissions. Considering that the propagation delay between the relay node and the client nodes may be different, we use the similar synchronization method proposed in [20] to make two packets from two client nodes in different groups can reach the relay node simultaneously. In practice, some technologies can be used to relax the synchronization requirement. For example, when Orthogonal Frequency Division Multiplexing (OFDM) technology is used, the relay node can successfully extract an NC packet from the superimposed EM wave if the difference of the arrival times is within the cyclic prefix (CP) of the OFDM system [20], [21]. The detailed transmission processes of PNC scheme are shown in Fig. 4. First, at the beginning of the time slot, a client node (i.e. n_i) sends an RTS packet to the relay node with the information indicating its destination client node (i.e. n_j) in the other group. After correctly receiving the RTS packet, the relay node first waits for an SIFS delay, and then broadcasts a CTS packet with a time stamp containing the transmit instant to both n_i and n_j. When n_i and n_j receive the CTS packet, they can respectively calculate the propagation delay between the relay node and themselves, δ_i and δ_j, by subtracting the time stamp in the CTS from the receiving time. Here, we assume that all the nodes have a common time reference, which can be obtained from the Global Positioning System (GPS) [20]. Then, n_i and n_j send a packet to the relay node with a delay of SIFS $+ 2\delta_{\max} - 2\delta_j$ and SIFS $+ 2\delta_{\max} - 2\delta_j$, respectively. In the case that n_j does not have a packet for n_i, it sends a dummy packet instead. Therefore, the relay node can receive the two packets simultaneously at the time "the CTS time stamp + SIFS $+ 2\delta_{\max}$", and extract an NC packet. The relay node then broadcasts an ACK packet to both n_i and n_j after an SIFS delay for confirmation. When a client node successfully transmits a packet, it stores the transmitted packet in its temporary buffer to make sure it can decode the NC packets from the relay node. The

[1]Since the propagation delay could be different for different transmissions, the slot time should is set according to the maximum propagation delay.

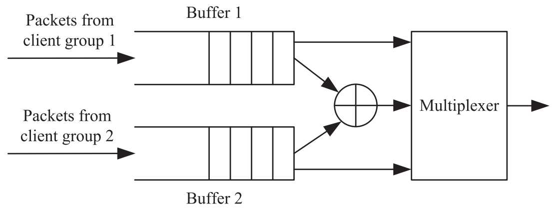


Fig. 5. The dataflow at the relay node in HNC scheme.

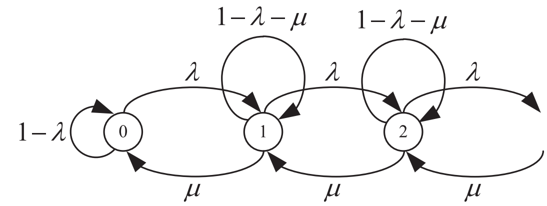


Fig. 6. The transition diagram of $S(t)$.

transmission process from the relay node to the client nodes is shown in Fig. 4b. At the beginning of the time slot, the relay node broadcasts an NC packet to n_i and n_j. After receiving the NC packet, n_i and n_j send back an ACK packet respectively after a delay of $\mathrm{SIFS} + 2\delta_{\max} - 2\delta_i$ and $\mathrm{SIFS} + 2\delta_{\max} - 2\delta_j$. Since the two ACK packets will arrive at the relay node simultaneously, the relay node can check the overlapped ACK packet and know whether the NC packet is correctly received or not. From Fig. 4, we know the slot time T_{slot} in PNC scheme equals

$$T_{slot} = T_{RTS} + T_{CTS} + T_{packet} + T_{ACK} + 3\mathrm{SIFS} + 4\delta_{\max}, \tag{3}$$

where T_{RTS} and T_{CTS} are the time for transmitting the RTS and the CTS packet, respectively.

In all schemes, when two or more nodes transmit, collision occurs and no node can successfully transmit a packet, as shown in Fig. 2b, Fig. 3c, and Fig. 4c, respectively. The collision time is the slot time in each scheme, no matter whether it is due to the concurrent transmissions of the data packets or RTS packets.

III. Throughput Analysis

In this section, we show the analytical saturated throughput of the unbalanced relay network with NNC, HNC, and PNC schemes, respectively. The key in the analysis is the calculation of the non-empty probability of the buffer(s) at the relay node. The closed-form expressions of the network throughput with NNC, HNC, and PNC schemes can then be obtained.

A. The Non-Empty Probability of the Buffer(s) at the Relay Node

In NNC and PNC schemes, all the packets from the client nodes are stored in one buffer. In HNC scheme, since the NC operation is performed at the network layer, the packets from the two client groups should be respectively stored in two different buffers, namely Buffer 1 and Buffer 2, as shown in Fig. 5. We focus on the case that the buffer at the relay node is unsaturated, i.e., the packet arrival probability λ is smaller than the packet departure probability μ when the buffer is not empty. The unsaturated buffer is necessary to ensure the average queue length, and thus the average queuing delay at the relay node does not go to infinity.

In the following, we present a common buffer model and derive its non-empty probability. Let t and $S(t)$ represent the beginning of a time slot and the number of packets in the considered buffer at time t, respectively. Since $S(t+1)$ is determined by $S(t)$ and the number of packets that arrive at and depart from the considered buffer during the time slot $[t, t+1]$, it can be modeled by a Markov chain. $S(t)$ may change because of the following two events: 1) One packet arrives at the considered buffer in current time slot, with the event probability of λ. 2) One packet departs from the considered buffer when it is not empty, with the event probability of μ. Considering that the relay node cannot transmit while receiving a packet, the above two events cannot concurrently occur. The transition diagram of $S(t)$ is shown in Fig. 6 with the one-step transition probabilities given by

$$\begin{cases} P\{S(t+1) = 0 \,|\, S(t) = 0\} = 1 - \lambda, \\ P\{S(t+1) = i | S(t) = i\} = 1 - \lambda - \mu, \quad i \geq 1, \\ P\{S(t+1) = i+1 \,|\, S(t) = i\} = \lambda, \quad i \geq 0, \\ P\{S(t+1) = i-1 | S(t) = i\} = \mu, \quad i \geq 1, \\ P\{S(t+1) = j \,|\, S(t) = i\} = 0, \quad |i-j| \geq 2. \end{cases}$$

The stationary probabilities of $S(t)$ can be calculated as follows. Let $\pi(i)$ denote the stationary probability of $S(t)$, given by $\lim_{t\to\infty} P\{S(t) = i\}$. According to the balance equations of the steady states $\sum_{i=0}^{+\infty} \pi(i) P\{S(t+1) = j | S(t) = i\} = \pi(j), \quad (j \geq 0)$, we have

$$\pi(i) = \pi(0)\left(\frac{\lambda}{\mu}\right)^i, \quad (i \geq 0).$$

When the considered buffer is unsaturated ($\lambda < \mu$), $\pi(i)$ exists. According to $\sum_{i=0}^{+\infty} \pi(i) = 1$, we have

$$\pi(0) = 1 - \frac{\lambda}{\mu}.$$

Thus, the non-empty probability of the buffer at the relay node, p_{ne}^{buf}, equals

$$p_{ne}^{buf} = \frac{\lambda}{\mu}. \tag{4}$$

The condition that the buffer is unsaturated is equivalent to $p_{ne}^{buf} < 1$.

Let h_{c1}, h_{c2}, and h_r denote the transmit probability of the client nodes in group 1, the transmit probability of the client nodes in group 2, and the transmit probability of the relay node when it has packets to transmit, respectively. Furthermore, let p_{c1}, p_{c2}, and p_r denote the collision probability when a client node in group 1, a client node in group 2, and the relay node transmit, respectively. The following proposition shows the non-empty probabilities of the sending buffer(s) at the relay node in NNC, HNC, and PNC schemes.

Proposition 1: In NNC and PNC schemes, the non-empty probability of the buffer at the relay node, $p_{ne}^{NNC,PNC}$, is

$$p_{ne}^{NNC,PNC} = \frac{u_1 h_{c1}(1-p_{c1}) + u_2 h_{c2}(1-p_{c2})}{h_r(1-p_r)}. \quad (5)$$

In HNC scheme, the non-empty probabilities of the Buffer 1 and Buffer 2 at the relay node, p_{ne1}^{HNC} and p_{ne2}^{HNC}, respectively equal

$$p_{ne1}^{HNC} = \frac{u_1 h_{c1}(1-p_{c1})}{h_r(1-p_r)}, \quad (6)$$

$$p_{ne2}^{HNC} = \frac{u_2 h_{c2}(1-p_{c2})}{h_r(1-p_r)}. \quad (7)$$

Proof: The probability that a client node in group 1 and group 2 successfully transmits a packet equals $h_{c1}(1-p_{c1})$ and $h_{c2}(1-p_{c2})$, respectively. Thus, in NNC and PNC schemes, the packet arrival probability at the relay node equals $u_1 h_{c1}(1-p_{c1}) + u_2 h_{c2}(1-p_{c2})$. Furthermore, the packet departure probability from the relay node when its buffer is not empty equals $h_r(1-p_r)$. According to equation (4), equation (5) can be obtained.

In HNC scheme, the packet arrival probability of Buffer 1 and 2 at the relay node equals $u_1 h_{c1}(1-p_{c1})$ and $u_2 h_{c2}(1-p_{c2})$, respectively. Furthermore, in HNC scheme, the relay node has two transmission cases. When both Buffer 1 and 2 are not empty, the relay node forms an NC packet and broadcasts it to two client nodes in group 1 and 2. When only one of the two buffers is not empty, the relay node transmits the packet in the non-empty buffer to a client node in the other group. Thus, we know that when the relay node successfully transmits, the number of packets in all the non-empty buffers will decrease by one. Therefore, in HNC scheme, the packet departure probability of each non-empty buffer at the relay node equals $h_r(1-p_r)$. According to equation (4), equations (6) and (7) can be obtained. ■

B. Throughput Calculation

The network throughput, Q, is defined as the average number of information bits that are transmitted by the relay node per second. Let V denote the average number of equivalent NNC packets transmitted by the relay node in a time slot. Then, Q can be expressed as

$$Q = \frac{T_{payload} R_{phy} V}{T_{slot}}, \quad (8)$$

where $T_{payload}$ and R_{phy} denote the time for transmitting the payload in a data packet and the physical link rate.

To calculate Q, we first need to show the common relationship between the involved variables. Considering that a node can successfully transmit a packet only when all the other nodes do not transmit, we have

$$1 - p_r = (1-h_{c1})^{u_1}(1-h_{c2})^{u_2}, \quad (9)$$

$$1 - p_{c1} = (1-h_{c1})^{u_1-1}(1-h_{c2})^{u_2}(1-p_{ne}h_r), \quad (10)$$

$$1 - p_{c2} = (1-h_{c1})^{u_1}(1-h_{c2})^{u_2-1}(1-p_{ne}h_r), \quad (11)$$

where p_{ne} is the probability that the relay node has packets to transmit. The above three common eqations can be applied to all schemes. In NNC and PNC schemes, since there is only one buffer at the relay node, p_{ne} equals

$$p_{ne} = p_{ne}^{NNC,PNC} = \frac{u_1 h_{c1}(1-p_{c1}) + u_2 h_{c2}(1-p_{c2})}{h_r(1-p_r)}. \quad (12)$$

In HNC scheme, since the probability that the relay node does not have packets to transmit equals $\left(1-p_{ne1}^{HNC}\right)\left(1-p_{ne2}^{HNC}\right)$, p_{ne} equals

$$\begin{aligned} p_{ne} &= 1 - \left(1-p_{ne1}^{HNC}\right)\left(1-p_{ne2}^{HNC}\right) \\ &= p_{ne1}^{HNC} + p_{ne2}^{HNC} - p_{ne1}^{HNC} p_{ne2}^{HNC}. \end{aligned} \quad (13)$$

The network throughput Q in NNC, HNC, and PNC schemes is given in Theorem 1 and 2, respectively. To simplify notations, let $\beta_1 = \frac{u_1 h_{c1}}{(1-h_{c1})}$, and $\beta_2 = \frac{u_2 h_{c2}}{(1-h_{c2})}$. Furthermore, let

$$A_1 = \beta_1 + \beta_2, \text{ and } A_2 = \beta_1 \beta_2. \quad (14)$$

Theorem 1: In NNC and PNC schemes, the buffer at the relay node is unsaturated iff

$$h_r > \frac{A_1}{A_1+1}. \quad (15)$$

Furthermore, if condition (15) is satisfied, the network throughput in NNC and PNC schemes respectively equals

$$Q = \frac{A_1 T_{payload} R_{phy}}{(A_1+1)\, T_{slot}} (1-h_{c1})^{u_1}(1-h_{c2})^{u_2}, \quad (16)$$

$$Q = \frac{A_1 T_{payload} R_{phy}}{(A_1+1)\, T_{slot}} (1-h_{c1})^{u_1}(1-h_{c2})^{u_2}(1+\alpha), \quad (17)$$

where α is the probability that when a source client node occupies the channel, its destination node also has a packet for the source node.

Proof: We first derive the common closed-form expression of p_{ne} in NNC and PNC schemes. According to (9)–(12), we have

$$p_{ne}h_r = \frac{u_1 h_{c1}(1-h_{c2}) + u_2 h_{c2}(1-h_{c1})}{(1-h_{c1})(1-h_{c2})}(1-p_{ne}h_r). \quad (18)$$

Therefore,

$$\begin{aligned} p_{ne} &= \frac{u_1 h_{c1}(1-h_{c2}) + u_2 h_{c2}(1-h_{c1})}{h_r[u_1 h_{c1}(1-h_{c2}) + u_2 h_{c2}(1-h_{c1}) + (1-h_{c1})(1-h_{c2})]} \\ &= \frac{A_1}{h_r(A_1+1)}. \end{aligned} \quad (19)$$

The buffer at the relay node is unsaturated iff $p_{ne} < 1$, which is equivalent to condition (15).

In NNC scheme, the average number of equivalent NNC packets transmitted by the relay node in a time slot, V, can be expressed by

$$V = p_{ne} h_r (1-p_r). \quad (20)$$

In PNC scheme, with probability α, the NC packet extracted by the relay node contains the information of two packets;

with probability $1-\alpha$, it only contains the information of one packet. Therefore, in PNC scheme, V can be expressed by

$$\begin{aligned} V &= 2\alpha p_{ne} h_r (1-p_r) + (1-\alpha) p_{ne} h_r (1-p_r) \\ &= (1+\alpha) p_{ne} h_r (1-p_r). \end{aligned} \tag{21}$$

From (8), (9), (19), (20), and (21), equations (16) and (17) can be obtained. ■

Theorem 2: In HNC scheme, both the buffers at the relay node are unsaturated iff

$$h_r > \max\left\{\frac{\beta_1}{\beta_1+1}, \frac{\beta_2}{\beta_2+1}\right\}. \tag{22}$$

Furthermore, if condition (22) is satisfied, the network throughput in HNC scheme equals

$$Q = \frac{A_1 T_{payload} R_{phy}\left[h_r (A_1+1) - \sqrt{(A_1+1)^2 h_r^2 - 4A_2 h_r}\right]}{2A_2 T_{slot}} \cdot (1-h_{c1})^{u_1} (1-h_{c2})^{u_2}. \tag{23}$$

The proof of Theorem 2 is in Appendix A. From the analysis above, we know that the throughput performance of HNC scheme increases as the probability that the packet transmitted by the relay node is an NC packet increases. This implies that the HNC scheme is more beneficial in the case that the total traffic between the two groups is more or less the same. The PNC scheme can further increase the transmission efficiency. However, from Theorem 1, the network throughput of PNC scheme greatly depends on the value of α, which represents the traffic balance between each pair of nodes that communicate through the relay node. In the extreme case of "$\alpha = 0$", the network coding operation cannot be explored at all. This motivates us to propose the hybrid NC scheme that combines both advantages of HNC and PNC schemes.

IV. Hybrid NC Scheme

We propose the hybrid NC scheme, in which the relay node turns to the HNC scheme if it fails to explore the PNC transmission. As will be seen later, the hybrid NC scheme can further improve the network throughput.

A. Transmission Process of Hybrid NC Scheme

In hybrid NC scheme, the successful transmission process from the client nodes to the relay node is as follows. At the beginning of a time slot, client node n_i sends an RTS packet to the relay node with the information indicating its destination node n_j in the other client group. After correctly receiving the RTS packet, the relay node first waits for an SIFS delay, and then broadcasts a CTS packet with a time stamp containing the transmit instant to both n_i and n_j. When n_i and n_j receive the CTS packet, they can respectively calculate the propagation delay δ_i and δ_j. When n_j has a packet for n_i, the transmission process is the same as the one in PNC scheme (see Fig. 4a). Both n_i and n_j store its transmitted packet in its temporary buffer. When n_j does not have a packet for n_i, only n_i transmits

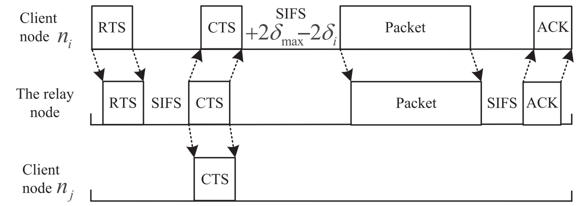


Fig. 7. Transmission process from the client nodes to the relay node in hybrid scheme when the destination node does not have a packet for the source node.

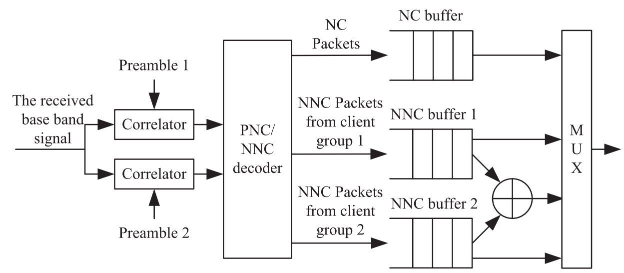


Fig. 8. The dataflow at the relay node in hybrid NC scheme.

a packet to the relay node after a delay of SIFS $+ 2\delta_{\max} - 2\delta_i$, and stores the transmitted packet in its temporary buffer, as shown in Fig. 7. All the other client nodes in the same group with n_i perform "opportunistic listening" to make sure they can decode the NC packet from the relay node. The relay node then sends back an ACK packet to n_i after an SIFS delay. Since the receiving process of NC packet is different from the NNC packet, to correctly decode the received packet, the relay node should first know whether one or two client nodes are transmitting. To solve this problem, the packets from client group 1 and client group 2 are assigned different orthogonal physical preambles, respectively termed as "Preamble 1" and "Preamble 2". At the relay node, the received base band signal is first traversed through two correlators to detect whether it contains two orthogonal physical preambles, as shown in Fig. 8. If two orthogonal physical preambles are detected, the received packet is an NC packet; otherwise, it is an NNC packet. Then, the relay node can use the corresponding receiving process to receive the packet.

There are three buffers at the relay node in hybrid NC scheme, as shown in Fig. 8. The first buffer is termed as "NC buffer" which stores the NC packets. The second and the third buffers are termed as "NNC buffer 1" and "NNC buffer 2", which are used to store the NNC packets from client group 1 and client group 2, respectively. If the NC buffer is not empty, the relay node broadcasts an NC packet to the two destination nodes with the same process shown in Fig. 3b. If the NC buffer is empty and both NNC buffer 1 and 2 are not empty, the relay node forms an NC packet from the two packets of the NNC buffer 1 and 2, and broadcasts it to the two destination nodes with the same process shown in Fig. 3b. If only NNC Buffer 1 or 2 is not empty, the relay node transmits the NNC packet in the non-empty buffer to its destination node with the same process shown in Fig. 3a. That is, the hybrid NC scheme first tries to explore PNC transmission. If the destination node does not have a packet for the source, it then uses the HNC scheme instead. When two or more nodes transmit, collision occurs and no node can successfully transmit a packet, as shown in Fig. 4c. The collision time is the slot time. From the above description, we know that the slot time in hybrid NC scheme is the same as the PNC scheme.

B. Throughput Analysis of Hybrid NC Scheme

In this section, we analyze the throughput performance of the hybrid NC scheme. We first show the non-empty probabilities of the three buffers at the relay node in Proposition 2. The closed-form expression of the network throughput can then be derived in Theorem 3.

Proposition 2: In hybrid NC scheme, the non-empty probabilities of the NC buffer, the NNC buffer 1, and the NNC buffer 2, $p_{ne\_NC}^{hyb}$, $p_{ne\_NNC1}^{hyb}$, and $p_{ne\_NNC2}^{hyb}$, respectively equal

$$p_{ne\_NC}^{hyb} = \frac{\alpha\left[u_1 h_{c1}(1-p_{c1}) + u_2 h_{c2}(1-p_{c2})\right]}{h_r\,(1-p_r)}, \tag{24}$$

$$p_{ne\_NNC1}^{hyb} = \frac{u_1 h_{c1}(1-p_{c1})\,(1-\alpha)}{h_r\,(1-p_r)\left(1-p_{ne\_NC}^{hyb}\right)}, \tag{25}$$

$$p_{ne\_NNC2}^{hyb} = \frac{u_2 h_{c2}(1-p_{c2})\,(1-\alpha)}{h_r\,(1-p_r)\left(1-p_{ne\_NC}^{hyb}\right)}. \tag{26}$$

Proof: In hybrid NC scheme, when a source node occupies the channel and transmits a packet to the relay node, with probability α, its destination node also has a packet for it. Thus, the packet arrival probability of the NC buffer at the relay node equals $\alpha[u_1 h_{c1}(1-p_{c1}) + u_2 h_{c2}(1-p_{c2})]$. Furthermore, since the relay node will first transmit the packets in the NC buffer if it is not empty, the packet departure probability from the NC buffer equals $h_r\,(1-p_r)$. Therefore, according to equation (4), equation (24) can be obtained.

When one of the nodes in group 1 occupies the channel, the NNC buffer 1 will receive a packet with probability $(1-\alpha)$. Thus, the packet arrival probability of the NNC buffer 1 equals $u_1 h_{c1}(1-p_{c1})\,(1-\alpha)$. Furthermore, when the NNC buffer 1 is not empty, one packet in it will be sent when the NC buffer is empty and the relay node successfully transmits a packet. Thus, the packet departure probability of the NNC buffer 1 equals $h_r\,(1-p_r)\left(1-p_{ne\_NC}^{hyb}\right)$. Therefore, according to equation (4), equation (25) is obtained. Similarly, the non-empty probability of the NNC buffer 2, $p_{ne\_NNC2}^{hyb}$, can be derived. ∎

Theorem 3: In hybrid NC scheme, all the buffers at the relay node are unsaturated iff

$$h_r > \frac{1-K\alpha A_1}{K\,(A_1+1) - K^2\left[\alpha A_1\,(A_1+1) + A_2\,(1-\alpha)^2\right]}, \tag{27}$$

where $K = \min\left\{\frac{1}{\beta_1(1-\alpha)+\alpha A_1}, \frac{1}{\beta_2(1-\alpha)+\alpha A_1}\right\}$. Furthermore, if condition (27) is satisfied, the network throughput in hybrid NC scheme equals

$$Q = T_{payload} R_{phy} A_1 (1+\alpha)(1-h_{c1})^{u_1}(1-h_{c2})^{u_2} \cdot \frac{h_r(A_1+1)+\alpha A_1 - \sqrt{[h_r(A_1+1)-\alpha A_1]^2 - 4h_r A_2(1-\alpha)^2}}{2T_{slot}[\alpha A_1(A_1+1) + A_2(1-\alpha)^2]}. \tag{28}$$

The proof of Theorem 3 is in Appendix B. In HNC, PNC, and hybrid NC schemes, each source node should maintain an additional temporary buffer to store the transmitted packets of itself or from the client nodes in the same group in order to decode the received NC packets. In fact, the stability of these temporary buffers at the source nodes can be ensured as long as the buffer(s) at the relay node is(are) unsaturated. The reasons are as follows. Each packet in the temporary buffer of a source node will be dropped if the packet (or its formed NC packet) in the buffer of the relay node is successfully transmitted. Then, we have $D_x^c \leq D_x^r + 2\delta_{\max}$, where D_x^c and D_x^r are the queuing time of packet x at the temporary buffer of the source node and the buffer of the relay node, respectively. Hence we know that the corresponding average queuing times $\overline{D^c}$ and $\overline{D^r}$ satisfy $\overline{D^c} \leq \overline{D^r} + 2\delta_{\max}$. When the buffer(s) at the relay node is(are) unsaturated, $\overline{D^r}$ is a limited value. Thus, $\overline{D^c}$ is a limited value. Furthermore, we know that for M/M/1 system, $\overline{D^c}$ will not go to infinity only if the temporary buffer is stable. Therefore, the stability of the temporary buffers at the source nodes is ensured.

C. Throughput Optimization of Hybrid NC Scheme

The transmission probability of the relay node, h_r, is crucial to the throughput performance of hybrid NC scheme. The optimal h_r that maximizes the network throughput can be derived from the following theorem.

Theorem 4: In hybrid NC scheme, the network throughput monotonically decreases with respect to h_r.

Proof: In hybrid NC scheme, the first and second derivatives of the network throughput with respect to h_r respectively equal

$$\begin{aligned} Q'(h_r) = {} & (A_1+1)\,A_3 - A_3\Big\{(A_1+1)\,[(A_1+1)\,h_r - A_1\alpha] \\ & -2\,(1-\alpha)^2\,A_2\Big\} \\ & \cdot\Big\{[(A_1+1)\,h_r - A_1\alpha]^2 - 4\,(1-\alpha)^2\,A_2 h_r\Big\}^{-\frac{1}{2}}, \end{aligned}$$

$$\begin{aligned} Q''(h_r) = {} & 4A_3\left[(A_1 h_r + h_r - A_1\alpha)^2 - 4\,(1-\alpha)^2\,A_2 h_r\right]^{-\frac{3}{2}} \\ & \cdot\left[(1-\alpha)^4 A_2^2 + A_1\,(A_1+1)\,A_2\alpha\,(1-\alpha)^2\right], \end{aligned}$$

where $A_3 = \frac{T_{payload} R_{phy} A_1 (1+\alpha)(1-h_{c1})^{u_1}(1-h_{c2})^{u_2}}{2T_{slot}\left[\alpha A_1(A_1+1)+A_2(1-\alpha)^2\right]}$. Since $Q''(h_r) \leq 0$, $Q'(h_r)$ increases with respect to h_r. Thus, we have $Q'(h_r) \leq Q'(1)$. Next, we show that $Q'(1) \leq 0$. We know

$$Q'(1) = (A_1+1)\,A_3\left\{1 - \frac{[(A_1+1) - A_1\alpha] - \frac{2(1-\alpha)^2 A_2}{(A_1+1)}}{\sqrt{[(A_1+1)-A_1\alpha]^2 - 4(1-\alpha)^2 A_2}}\right\}.$$

Furthermore, we have

$$\begin{aligned} & \left\{[(A_1+1) - A_1\alpha] - \frac{2\,(1-\alpha)^2\,A_2}{(A_1+1)}\right\}^2 \\ & \quad - \left\{[(A_1+1) - A_1\alpha]^2 - 4\,(1-\alpha)^2\,A_2\right\} \\ & \quad = \frac{4A_2^2\,(1-\alpha)^4}{(A_1+1)^2} + \frac{4A_1 A_2\alpha\,(1-\alpha)^2}{A_1+1} \geq 0. \end{aligned}$$

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
RTS	160 bits	CTS	112 bits
ACK	112 bits	Packet size	8472 bits
Payload size	8184 bits	Physical preamble	20 μs
SIFS	10 μs	Physical link rate	11 Mb/s
Maximum propagation delay	1 μs		

Let $W(\alpha) \triangleq [(A_1+1) - A_1\alpha] - \frac{2(1-\alpha)^2 A_2}{(A_1+1)}$. We know that $Q'(1) \leq 0$ if $W(\alpha) > 0$. In the following, we show that $W(\alpha) > 0$ for any $0 \leq \alpha \leq 1$.

The first and second derivatives of $W(\alpha)$ respectively equal

$$W'(\alpha) = -A_1 + \frac{4(1-\alpha)A_2}{A_1+1},$$
$$W''(\alpha) = -\frac{4A_2}{A_1+1}.$$

Since $W''(\alpha) \leq 0$, $W'(\alpha)$ is a decreasing function of α. Furthermore, we have

$$\begin{aligned} W'(0) &= -A_1 + \frac{4A_2}{A_1+1} \\ &= -\frac{1}{(\beta_1+\beta_2+1)}\left[(\beta_1-\beta_2)^2 + (\beta_1+\beta_2)\right] \leq 0. \end{aligned}$$

Thus, for any value of α, we have $W'(\alpha) \leq 0$. Then, $W(\alpha)$ is a decreasing function of α, and

$$W(\alpha) \geq W(1) = 1 > 0.$$

Therefore, we have $Q'(h_r) \leq 0$. This means that the network throughput monotonically decreases with respect to h_r. ■

From Theorem 4, we know that the maximum network throughput is achieved when h_r is set to the minimum value that ensures all the buffers at the relay node are unsaturated (satisfying condition (27)). In hybrid NC scheme, as h_r decreases, the probability that both the NNC buffers are non-empty increases, which increases the probability that the relay performs HNC. Therefore, the network throughput increases as h_r decreases. Since the hybrid NC scheme is reduced to HNC scheme when $\alpha = 0$, Theorem 4 is also valid in HNC scheme. In PNC and NNC schemes, as long as the buffer at the relay node is unsaturated, the total number of packets successfully transmitted by the relay node is only determined by the total number of packets successfully transmitted by the client nodes. Thus, the network throughputs of PNC and NNC schemes are independent of h_r.

V. SIMULATION RESULTS

In this section, we carry out simulations to validate the analytical models, discuss the relationship between the network throughput of hybrid NC scheme and system parameters, and compare the throughput of the NNC, HNC, PNC, and hybrid NC schemes. The simulation parameters are given in Table I.

A. Model Validation

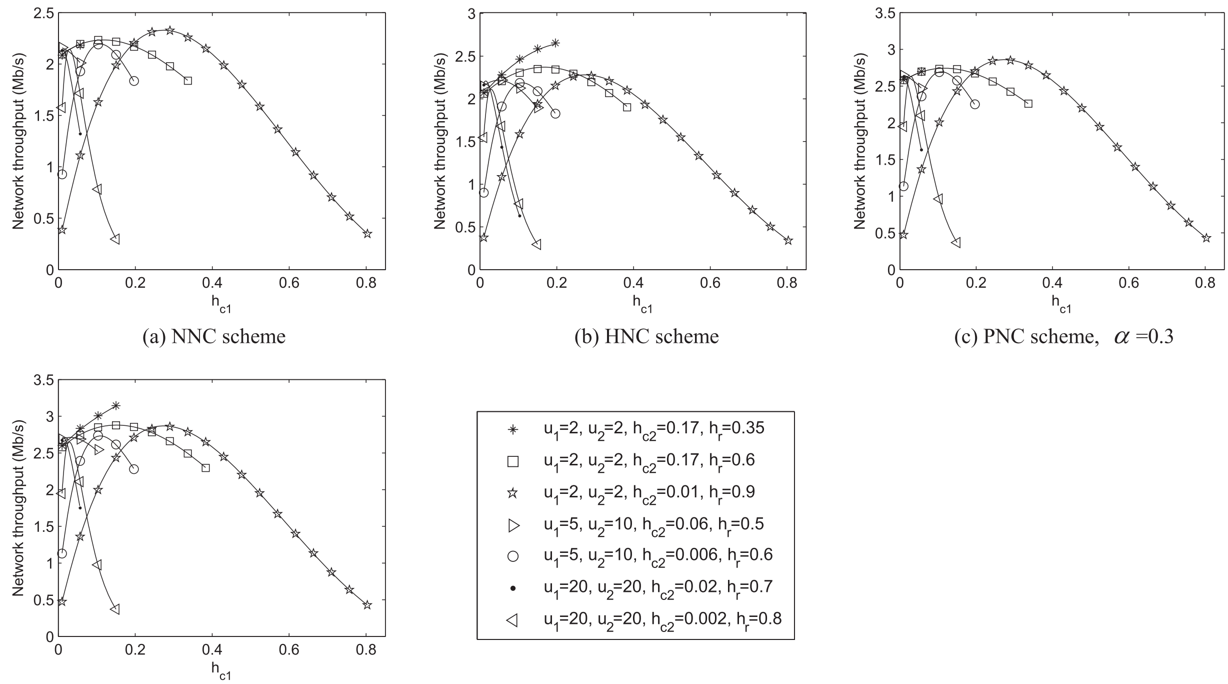
We calculate the network throughputs of these schemes with different numbers of client nodes and transmit probabilities. Given each h_{c2} and h_r, the value of h_{c1} varies from 0 to its maximum value which makes the relay node nearly saturated (the non-empty probability of one buffer equals 0.99). Fig. 9 shows the network throughputs obtained from both analysis (solid curves) and simulations (symbols). We can see that our models are extremely accurate. The simulation results of all the schemes precisely match with the analytical results.

B. Network Throughput in Hybrid NC Scheme

Fig. 10 shows the network throughput of hybrid NC scheme v.s. h_r, given α, u_1, u_2, h_{c1}, and h_{c2}. From Fig. 10, we can see that in hybrid NC scheme, the network throughput decreases as h_r increases, which matches Theorem 4. Another observation from Fig. 10 is that the effect of h_r on the network throughput becomes small when α is big or the total traffic between the two groups is unbalanced. The reason is that the value of h_r mainly affects the probability of the relay node performing HNC. Since the proportion that the relay node uses HNC scheme to transmit packets becomes small, the effect of h_r becomes small accordingly.

Fig. 11 shows the maximum network throughput versus the transmission probability of the relay node h_r when h_{c1} and h_{c2} are optimized. Given each h_r, the maximum network throughput is obtained by finding the optimal values of h_{c1} and h_{c2} with numerical search. From Fig. 11, we can see that: 1) When α is not close to 1, the maximum network throughput first increases and then decreases, and converges to a nearly constant value with respect to h_r. 2) When α is close to 1, the maximum network throughput monotonically increases, and converges to a nearly constant value as h_r increases. When h_r is small, the system bottleneck is at the relay node since it does not have enough opportunity to transmit the packets in its buffers. Thus, the maximum network throughput increases as h_r increases. However, as h_r further increases, the probability that both the NNC buffers are nonempty decreases. Thus, the probability that the relay node performs HNC decreases, which reduces the network throughput. When α is close to 1, since most of the received packets at the relay node are PNC packets, the increase of h_r will not decrease the maximum network throughput.

Fig. 12 shows the maximum network throughput versus the value of α with different client node numbers. The maximum network throughput is obtained by finding the optimal values of h_{c1}, h_{c2}, and h_r with numerical search. We can see that approximately, the maximum network throughput increases linearly as α increases. This is because that PNC has higher transmission efficiency than HNC, and an increase in α will increase the probability of using PNC to transmit packets. Another observation from Fig. 12 is that for each given α, the maximum network throughput decreases as the total number of client nodes increases. As the total number of client nodes increases, there will be more collisions, and thus the maximum network throughput is reduced.



(a) NNC scheme (b) HNC scheme (c) PNC scheme, α =0.3

(d) Hybrid NC scheme, α =0.3

Fig. 9. Throughput comparison: analysis v.s. simulation.

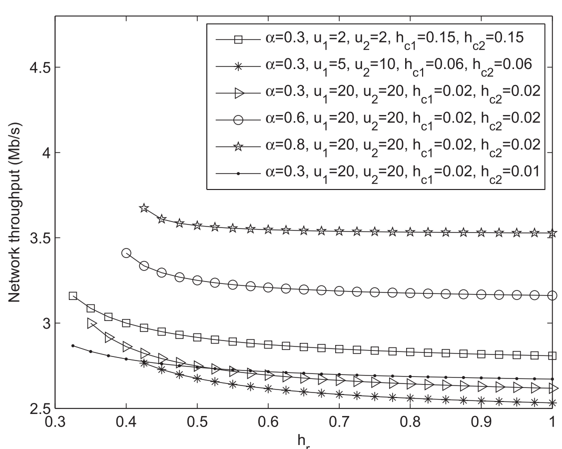


Fig. 10. The network throughput v.s. h_r.

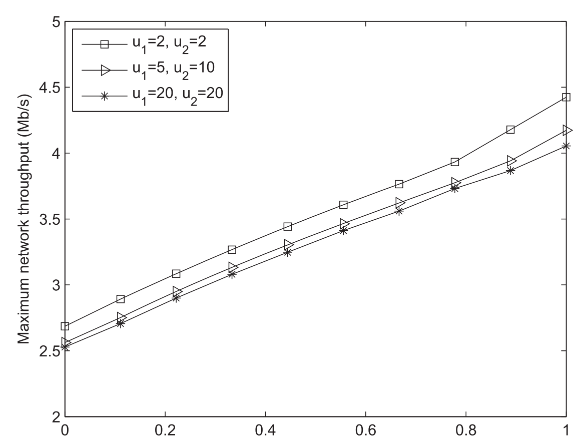


Fig. 12. The maximum network throughput v.s. α.

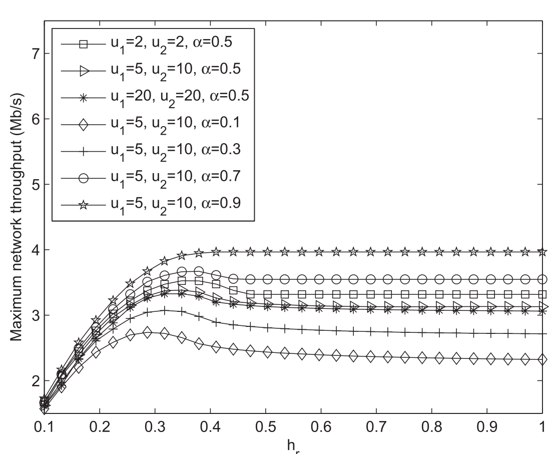


Fig. 11. The maximum network throughput v.s. h_r.

As discussed, h_{c1} and h_{c2} can be regarded as the average rate that a client node from client group 1 and client group 2 injects packets into the network, respectively. Therefore, the total injection rate of the network, G, can be defined as $u_1h_{c1} + u_2h_{c2}$. Fig. 13 shows the network throughput versus G with balanced traffic ($h_{c1} = 0.5G/u_1$) and unbalanced traffic ($h_{c1} = 0.1G/u_1$). The value of G varies from 0 to its maximum value, which makes one of the buffers at the relay node nearly saturated. From Fig. 13, we can see that in all cases, the network throughput first increases and then decreases with G. When G is small, the channel is idle in most of the time slots. An increase of G will increase the channel utilization. As G further increases, the collision probability increases, which decreases the network throughput. Another observation from Fig. 13 is

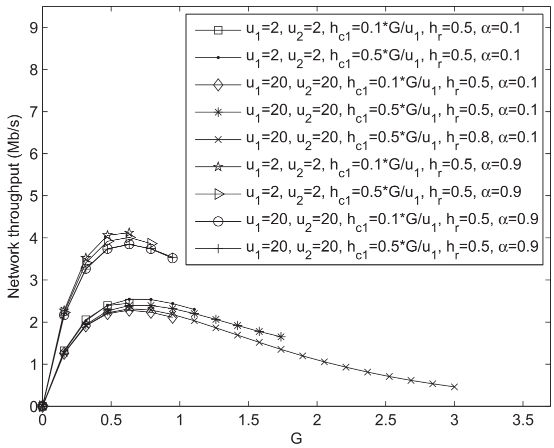


Fig. 13. The network throughput v.s. G.

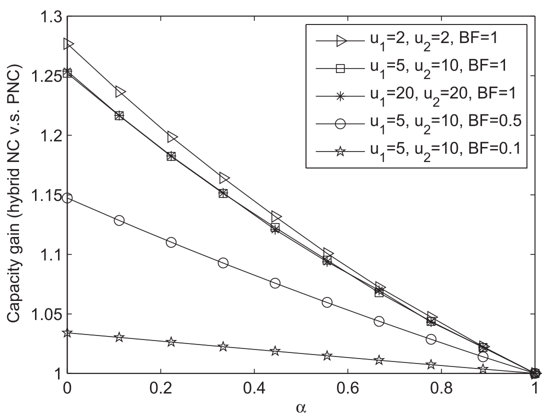


Fig. 14. Capacity gain: hybrid NC v.s. PNC.

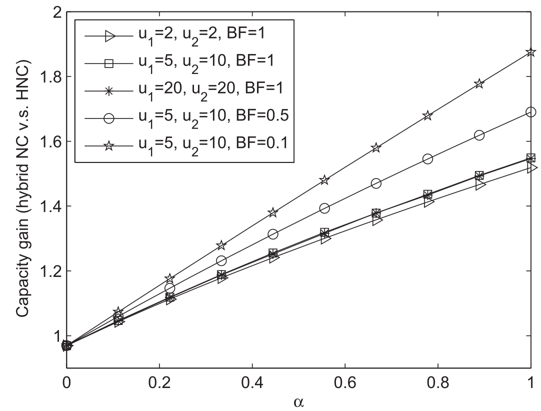


Fig. 15. Capacity gain: hybrid NC v.s. HNC.

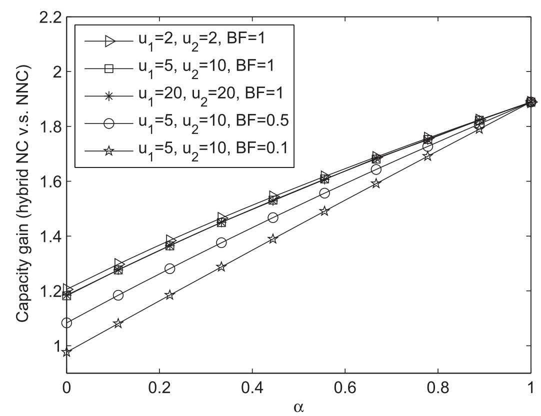


Fig. 16. Capacity gain: hybrid NC v.s. NNC.

that the optimal G that maximizes the network throughput in all cases is approximately the same (≈ 0.63), which could be useful in setting the values of h_{c1} and h_{c2} in a practical system.

C. Throughput Gain

We compare the throughput of NNC, HNC, PNC, and hybrid NC schemes. We use the "balance factor (BF)" which equals $u_2 h_{c2}/u_1 h_{c1}$ to describe the degree of traffic balance between the two client groups. The total traffic is balanced when $BF = 1$. Fig. 14 shows the capacity gain of hybrid NC scheme versus PNC scheme under different values of u_1, u_2, BF, and α. From Fig. 14, we can see that the capacity gain of hybrid NC scheme versus PNC scheme almost linearly decreases as α increases. Furthermore, the capacity gain is larger when $BF = 1$, which can reach 127.7%. In fact, when $0 < \alpha < 1$ and $0 < BF < 1$, PNC scheme can be regarded as the combination of NNC and PNC schemes; while hybrid NC scheme can be regarded as the combination of PNC, HNC, and NNC schemes. The value of α affects the proportion of PNC transmissions; while the value of BF affects the proportion of HNC transmissions. For each given BF, as α increases, the proportion of HNC transmissions in hybrid NC scheme becomes smaller, and thus the capacity gain becomes smaller. Similarly, for a given α, as BF decreases, the capacity gain decreases since the proportion of HNC transmissions in hybrid NC scheme decreases.

Fig. 15 and 16 show the capacity gain of hybrid NC scheme versus HNC and NNC schemes under different values of u_1, u_2, BF, and α, respectively. We can see that the capacity gain of hybrid NC scheme versus HNC and NNC schemes increases as α increases, and can respectively reach 154.7% and 188.9% when $BF = 1$. The reason is that the network throughput of hybrid NC scheme increases as α increases; while the network throughput of HNC and NNC schemes is independent of α. Another observation from Fig. 15 and 16 is that the capacity gain of hybrid NC scheme versus HNC scheme increases as BF decreases; while the capacity gain of hybrid NC scheme versus NNC scheme decreases as BF decreases. In hybrid NC scheme, the PNC transmission is independent of BF. Thus, the value of BF has more impact on the network throughput in HNC scheme than in the hybrid NC scheme. As BF decreases, the network throughput of HNC scheme decreases faster than the hybrid NC scheme. Therefore, the capacity gain of hybrid NC scheme versus HNC scheme increases as BF decreases. Furthermore, since BF has little effect on the throughput of NNC scheme, the capacity gain of hybrid NC scheme versus NNC scheme decreases as BF decreases.

The PNC scheme can be generalized to the multi-user case [10], [11]. We further compare the maximum network throughput of hybrid NC scheme and multi-user PNC scheme under different values of u_1, u_2, and α. The values of h_{c1}, h_{c2}, and h_r

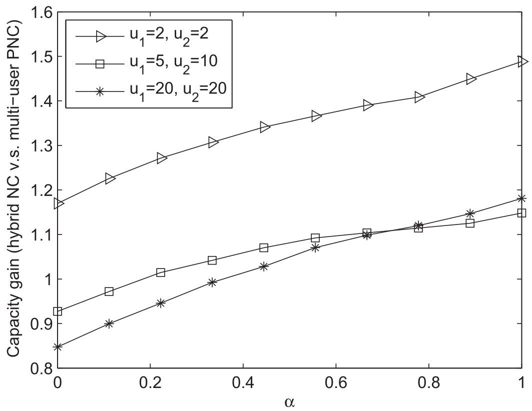


Fig. 17. Capacity gain: hybrid NC v.s. multi-user PNC.

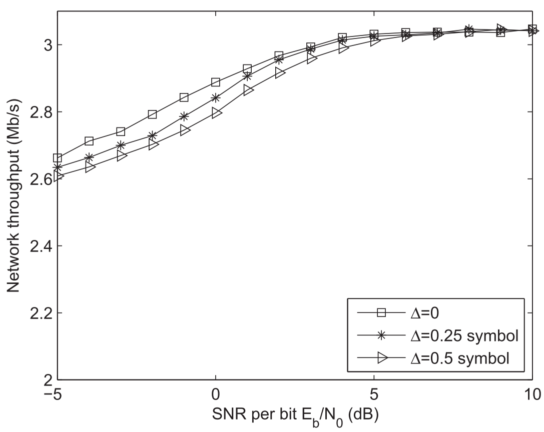


Fig. 18. The network throughput loss due to asynchronous arrival.

are tuned to maximize the network throughput through simulations. The slot time in multi-user PNC scheme equals $T_{RTS} + T_{CTS} + T_{packet} + 2\text{SIFS} + 3\delta_{\max}$ since the ACK packet is not needed. From Fig. 17, we can see that the hybrid NC scheme outperforms the multi-user PNC scheme when the value of α is big or when the number of client nodes is small. This is because in hybrid NC scheme, the value of α affects both the packet transmission processes from client nodes to the relay node and from the relay node to the client nodes; while in multi-user PNC scheme, it only affects the packet transmission process from the relay node to the client nodes. As α increases, the throughput of hybrid NC scheme increases faster than the multi-user PNC scheme. Furthermore, when the number of client nodes is large, the network throughput performance of the hybrid NC scheme and the multi-user PNC scheme are comparable. As expected, the benefit of the multi-user PNC scheme is more pronounced when the number of client nodes in each group is large. On the other hand, when the number of client nodes in each group is large, there are more collisions in the hybrid NC scheme, which reduces the network throughput. As shown in Fig. 17, we can find that when $u_1 = u_2 = 20$, the hybrid NC scheme still has better network throughput when α is greater than 0.4.

D. The Performance Loss of Asynchrony

In this section, we show the performance loss in terms of the network throughput, which is due to the asynchronous arrival of the packets from different nodes. As shown in [7], the bit error rate (BER) at the relay node will increase if the two packets from two client groups do not arrive synchronously. Let Δ denote the difference between the arrival times of the two packets at the relay node in hybrid NC scheme. We use the BER data given in [7] and show the performance of network throughput of hybrid NC scheme versus signal-to-noise-ratio (SNR) under different values of Δ. The values of u_1, u_2, h_{c1}, h_{c2}, h_r, and α are set to 20, 20, 0.02, 0.02, 0.6, and 0.5, respectively. From Fig. 18, we can see that when the SNR is greater than 6 dB, the network throughput loss is quite small, i.e., less than 0.34%. However, the impact of the asynchrony to the network throughput performance of the hybrid NC scheme is more severe in the low SNR regime. When the SNR is –5 dB, the network throughput is decreased by 1.07% and 2.02% for $\Delta = 0.25$ symbol and $\Delta = 0.5$ symbol, respectively.

VI. Conclusions

In this paper, we studied the throughput performance of the network coding schemes in the unbalanced slotted ALOHA network. In particular, we derived the closed-form expressions of the network throughput for the PNC, HNC, and NNC schemes, together with the corresponding necessary and sufficient condition to make the relay node unsaturated. We found that the throughput performance of PNC significantly deteriorates when the traffic between two client nodes in different groups is not balanced. Therefore, we proposed the hybrid NC scheme that combines the advantages of PNC and HNC schemes to further improve the network throughput. We then derived the closed-form expression of network throughput for hybrid NC scheme together with the necessary and sufficient condition to make the relay node unsaturated. Simulations showed that the throughput gain of hybrid NC scheme versus PNC, HNC, and NNC schemes can respectively reach 127.7%, 154.7%, and 188.9%. Moreover, we optimized the network throughput of the hybrid NC scheme in terms of the transmission probability of the relay node. In addition, we evaluated the throughput performance of the hybrid NC scheme with simulations, and showed some interesting observations: 1) When the value of α is not close to 1, the value of h_r should not be too large since it has significant effect on the probability of the relay node performing HNC. 2) The maximum network throughput decreases as the number of client nodes increases, but increases almost linearly as α increases. 3) The optimal G that maximizes the network throughput in different system parameters is approximately the same, which could be useful when setting h_{c1} and h_{c2} in a practical slotted ALOHA system.

Appendix A
Proof of Theorem 2

Proof: In HNC scheme, when both Buffer 1 and Buffer 2 are not empty, the relay node successfully transmits an

NC packet. In this case, the average number of equivalent NNC packets transmitted by the relay node in a time slot is $2p_{ne1}^{HNC}p_{ne2}^{HNC}h_r(1-p_r)$. When only one of the two buffers is not empty, the relay node successfully transmits one NNC packet. In this case, the average number of equivalent NNC packets transmitted by the relay node in a time slot is $p_{ne1}^{HNC}\left(1-p_{ne2}^{HNC}\right)h_r(1-p_r)+\left(1-p_{ne1}^{HNC}\right)p_{ne2}^{HNC}h_r(1-p_r)$. Thus, the average number of equivalent NNC packets transmitted by the relay node in a time slot, V, equals

$$\begin{aligned}V &= 2p_{ne1}^{HNC}p_{ne2}^{HNC}h_r(1-p_r)+\left[p_{ne1}^{HNC}\left(1-p_{ne2}^{HNC}\right)\right.\\&\quad\left.+\left(1-p_{ne1}^{HNC}\right)p_{ne2}^{HNC}\right]h_r(1-p_r)\\&=\left(p_{ne1}^{HNC}+p_{ne2}^{HNC}\right)h_r(1-p_r).\end{aligned}\tag{29}$$

Furthermore, according to (6), (7), (9), (10), and (11), and let $z=1-p_{ne}h_r$, we have

$$p_{ne1}^{HNC}=\frac{u_1h_{c1}(1-p_{ne}h_r)}{h_r(1-h_{c1})}=\frac{\beta_1}{h_r}z,\tag{30}$$

$$p_{ne2}^{HNC}=\frac{u_2h_{c2}(1-p_{ne}h_r)}{h_r(1-h_{c2})}=\frac{\beta_2}{h_r}z.\tag{31}$$

Applying equations (30) and (31) to equation (13), and replacing p_{ne} with $\frac{1-z}{h_r}$, we have

$$A_2z^2-h_r(A_1+1)z+h_r=0.\tag{32}$$

The solution to (32) exists when $(A_1+1)^2h_r^2-4A_2h_r\geq 0$, which is equivalent to

$$h_r\geq\frac{4A_2}{(A_1+1)^2}.\tag{33}$$

When condition (33) is satisfied, z has two possible solutions:

$$z_1=\frac{h_r(A_1+1)-\sqrt{(A_1+1)^2h_r^2-4A_2h_r}}{2A_2},\text{ or}\tag{34}$$

$$z_2=\frac{h_r(A_1+1)+\sqrt{(A_1+1)^2h_r^2-4A_2h_r}}{2A_2}.\tag{35}$$

Then, according to (8), (9), (29), (30), and (31), we have

$$Q=\frac{T_{payload}R_{phy}}{T_{slot}}zA_1(1-h_{c1})^{u_1}(1-h_{c2})^{u_2}.\tag{36}$$

Next we show that in HNC scheme, z_2 is not feasible. We know that when $h_{c2}=0$, the HNC scheme is reduced to NNC scheme. When $h_{c2}=0$, we have $\beta_2=0$. Then, z_1 equals

$$\begin{aligned}\lim_{\beta_2\to 0}z_1&=\lim_{\beta_2\to 0}\frac{h_r(A_1+1)-\sqrt{(A_1+1)^2h_r^2-4A_2h_r}}{2A_2}\\&=\lim_{\beta_2\to 0}\frac{h_r(\beta_1+\beta_2+1)-\sqrt{(\beta_1+\beta_2+1)^2h_r^2-4\beta_1\beta_2h_r}}{2\beta_1\beta_2}\\&=\frac{1}{\beta_1+1}.\end{aligned}$$

Similarly, we have $\lim_{\beta_2\to 0}z_2=\frac{h_r}{\beta_1}-\frac{1}{\beta_1+1}$. Thus, according to equation (36), when $z=z_1$, we have $Q=\frac{\beta_1T_{payload}R_{phy}}{(\beta_1+1)T_{slot}}(1-h_{c1})^{u_1}$, which is the same as the throughput of the NNC scheme, i.e., equation (16) with $h_{c2}=0$. However, when $z=z_2$, we have $Q=\frac{T_{payload}R_{phy}}{T_{slot}}\left(\frac{h_r}{\beta_1}-\frac{1}{\beta_1+1}\right)\beta_1(1-h_{c1})^{u_1}$, which is not equal to the throughput of NNC scheme with $h_{c2}=0$. Therefore, z_2 is not feasible. Applying (34) to (36), equation (23) can be obtained.

According to (30), (31), and (34), both the buffers at the relay node are unsaturated iff

$$\frac{h_r(A_1+1)-\sqrt{(A_1+1)^2h_r^2-4A_2h_r}}{2h_r\beta_2}<1,\text{ and}$$
$$\frac{h_r(A_1+1)-\sqrt{(A_1+1)^2h_r^2-4A_2h_r}}{2h_r\beta_1}<1,$$

which are equivalent to

$$\sqrt{(A_1+1)^2-\frac{4A_2}{h_r}}>\max\{\beta_1-\beta_2+1,\beta_2-\beta_1+1\}>0.\tag{37}$$

When $\beta_1\geq\beta_2$ and $\beta_1<\beta_2$, condition (37) can be equivalent to $h_r>\frac{\beta_1}{\beta_1+1}$ and $h_r>\frac{\beta_2}{\beta_2+1}$, respectively. Therefore, condition (22) is obtained.

Next, we show that condition (33) always holds if condition (22) is satisfied. We need to show

$$\frac{4A_2}{(A_1+1)^2}=\frac{4\beta_1\beta_2}{(\beta_1+\beta_2+1)^2}\leq\max\left\{\frac{\beta_1}{\beta_1+1},\frac{\beta_2}{\beta_2+1}\right\}.\tag{38}$$

Since both sides of inequality (38) are symmetric functions of β_1 and β_2, without loss of generality, it suffices to consider the case $\beta_1\geq\beta_2$ only. When $\beta_1\geq\beta_2$, we have $\max\left\{\frac{\beta_1}{\beta_1+1},\frac{\beta_2}{\beta_2+1}\right\}=\frac{\beta_1}{\beta_1+1}$. Furthermore, we have

$$\frac{4\beta_1\beta_2}{(\beta_1+\beta_2+1)^2}-\frac{\beta_1}{\beta_1+1}=\frac{-\beta_1(\beta_1-\beta_2+1)^2}{(\beta_1+\beta_2+1)^2(\beta_1+1)}\leq 0.$$

Thus, we know inequality (38) is satisfied. Therefore, condition (33) always holds when both the buffers at the relay node are unsaturated. ■

APPENDIX B
PROOF OF THEOREM 3

Proof: In hybrid NC scheme, the relay node transmits an NC packet that contains the information of two packets in two cases: 1) the NC buffer is not empty; 2) the NC buffer is empty but both the NNC buffers are not empty. Thus, the probability that the transmitted packet contains the information of two packets equals $p_{ne\_NC}^{hyb}+p_{ne\_NNC1}^{hyb}p_{ne\_NNC2}^{hyb}\left(1-p_{ne\_NC}^{hyb}\right)$. The relay node transmits an NNC packet only when the NC buffer is empty and one of the two NNC buffers is not empty, the probability of which equals $p_{ne\_NNC1}^{hyb}\left(1-p_{ne\_NNC2}^{hyb}\right)\left(1-p_{ne\_NC}^{hyb}\right)+$

$p^{hyb}_{ne\_NNC2}\left(1-p^{hyb}_{ne\_NNC1}\right)\left(1-p^{hyb}_{ne\_NC}\right)$. Therefore, the average number of equivalent NNC packets transmitted in a successful transmission of the relay node equals

$$\begin{aligned}M_r^{hyb} &= 2\left[p^{hyb}_{ne\_NC}+p^{hyb}_{ne\_NNC1}p^{hyb}_{ne\_NNC2}\left(1-p^{hyb}_{ne\_NC}\right)\right]\\&+p^{hyb}_{ne\_NNC1}\left(1-p^{hyb}_{ne\_NNC2}\right)\left(1-p^{hyb}_{ne\_NC}\right)\\&+p^{hyb}_{ne\_NNC2}\left(1-p^{hyb}_{ne\_NNC1}\right)\left(1-p^{hyb}_{ne\_NC}\right)\\&=2p^{hyb}_{ne\_NC}+\left(1-p^{hyb}_{ne\_NC}\right)\left(p^{hyb}_{ne\_NNC1}+p^{hyb}_{ne\_NNC2}\right).\end{aligned}$$

Therefore, in hybrid NC scheme, the average number of equivalent NNC packets transmitted by the relay node in a time slot, V, equals

$$\begin{aligned}V &= h_r\left(1-p_r\right)M_r^{hyb}=h_r\left(1-p_r\right)\\&\cdot\left[2p^{hyb}_{ne\_NC}+\left(1-p^{hyb}_{ne\_NC}\right)\left(p^{hyb}_{ne\_NNC1}+p^{hyb}_{ne\_NNC2}\right)\right].\end{aligned}\tag{39}$$

Next, we derive the closed-form expression of the network throughput. Applying (9)–(11) to (24)–(26), and let $z=1-p_{ne}h_r$, we have

$$\begin{aligned}p^{hyb}_{ne\_NC} &= \frac{\alpha(1-p_{ne}h_r)\left[u_1h_{c1}(1-h_{c2})+u_2h_{c2}(1-h_{c1})\right]}{h_r(1-h_{c1})(1-h_{c2})}\\&=\frac{\alpha}{h_r}z\left(\beta_1+\beta_2\right),\end{aligned}\tag{40}$$

$$p^{hyb}_{ne\_NNC1}=\frac{u_1h_{c1}\left(1-p_{ne}h_r\right)(1-\alpha)}{h_r\left(1-h_{c1}\right)\left(1-p^{hyb}_{ne\_NC}\right)}=\frac{\beta_1(1-\alpha)z}{h_r\left(1-p^{hyb}_{ne\_NC}\right)},\tag{41}$$

$$p^{hyb}_{ne\_NNC2}=\frac{u_2h_{c2}\left(1-p_{ne}h_r\right)(1-\alpha)}{h_r\left(1-h_{c2}\right)\left(1-p^{hyb}_{ne\_NC}\right)}=\frac{\beta_2(1-\alpha)z}{h_r\left(1-p^{hyb}_{ne\_NC}\right)}.\tag{42}$$

Furthermore, in hybrid NC scheme, the probability that the relay node has packets to transmit equals

$$p_{ne}=1-\left(1-p^{hyb}_{ne\_NC}\right)\left(1-p^{hyb}_{ne\_NNC1}\right)\left(1-p^{hyb}_{ne\_NNC2}\right).\tag{43}$$

Applying (40)–(42) and after some simplification, we have

$$p_{ne}=\frac{A_1}{h_r}z-\frac{A_2(1-\alpha)^2}{h_r^2\left(1-\frac{\alpha}{h_r}zA_1\right)}z^2.\tag{44}$$

Since $p_{ne}=\frac{1-z}{h_r}$, equation (44) can be further expressed as

$$\begin{aligned}&\left[\alpha A_1\left(A_1+1\right)+A_2\left(1-\alpha\right)^2\right]z^2\\&-\left[h_r\left(A_1+1\right)+\alpha A_1\right]z+h_r=0.\end{aligned}\tag{45}$$

Thus, the solution of z exists when $\left[h_r\left(A_1+1\right)+\alpha A_1\right]^2-4h_r\left[\alpha A_1\left(A_1+1\right)+A_2\left(1-\alpha\right)^2\right]\geq 0$, which can be simplified to

$$\left[h_r\left(A_1+1\right)-\alpha A_1\right]^2-4h_rA_2\left(1-\alpha\right)^2\geq 0.\tag{46}$$

In this case, z has two possible solutions:

$$z_1=\frac{h_r(A_1+1)+\alpha A_1-\sqrt{[h_r(A_1+1)-\alpha A_1]^2-4h_rA_2(1-\alpha)^2}}{2[\alpha A_1(A_1+1)+A_2(1-\alpha)^2]},\text{ or}\tag{47}$$

$$z_2=\frac{h_r(A_1+1)+\alpha A_1+\sqrt{[h_r(A_1+1)-\alpha A_1]^2-4h_rA_2(1-\alpha)^2}}{2\left[\alpha A_1\left(A_1+1\right)+A_2\left(1-\alpha\right)^2\right]}.\tag{48}$$

Then, according to (8), (9), (39), (40), (41), and (42), we have

$$Q=\frac{T_{payload}R_{phy}}{T_{slot}}A_1\left(1+\alpha\right)\left(1-h_{c1}\right)^{u_1}\left(1-h_{c2}\right)^{u_2}z.\tag{49}$$

Next, we show that the solution "$z=z_2$" is not feasible. When $\alpha=0$, the hybrid NC scheme is reduced to HNC scheme. In the following, we show that when $z=z_1$, the throughput of hybrid NC scheme with "$\alpha=0$" is the same as the one in HNC scheme. However, when $z=z_2$, there is no such equivalence.

When $\alpha=0$, z_1 and z_2 equal $\frac{h_r(A_1+1)-\sqrt{[h_r(A_1+1)]^2-4h_rA_2}}{2A_2}$ and $\frac{h_r(A_1+1)+\sqrt{[h_r(A_1+1)]^2-4h_rA_2}}{2A_2}$, respectively. Furthermore, according to equation (49), if $z=z_1$, we have

$$\begin{aligned}Q&=T_{payload}R_{phy}A_1\left(1-h_{c1}\right)^{u_1}\left(1-h_{c2}\right)^{u_2}\\&\cdot\frac{h_r\left(A_1+1\right)-\sqrt{\left[h_r\left(A_1+1\right)\right]^2-4h_rA_2}}{2A_2T_{slot}},\end{aligned}$$

which is the same as (23). If $z=z_2$, we have

$$\begin{aligned}Q&=T_{payload}R_{phy}A_1\left(1-h_{c1}\right)^{u_1}\left(1-h_{c2}\right)^{u_2}\\&\cdot\frac{h_r\left(A_1+1\right)+\sqrt{\left[h_r\left(A_1+1\right)\right]^2-4h_rA_2}}{2A_2T_{slot}},\end{aligned}$$

which is different from (23).

Therefore, the solution "$z=z_2$" is not feasible. Applying (47) to (49), equation (28) can be obtained.

Next we derive the condition that ensures all the buffers at the relay node are unsaturated in hybrid NC scheme. According to (40)–(42), all the buffers at the relay node are unsaturated iff $\frac{\alpha}{h_r}A_1z<1$, $\frac{\beta_1(1-\alpha)z}{h_r\left(1-\frac{\alpha}{h_r}A_1z\right)}<1$, and $\frac{\beta_2(1-\alpha)z}{h_r\left(1-\frac{\alpha}{h_r}A_1z\right)}<1$, which can be simplified to $\frac{z}{h_r}<\frac{1}{\alpha A_1}$, $\frac{z}{h_r}<\frac{1}{\beta_1(1-\alpha)+\alpha A_1}$, and $\frac{z}{h_r}<\frac{1}{\beta_2(1-\alpha)+\alpha A_1}$. Since $\frac{1}{\alpha A_1}\geq\frac{1}{\beta_1(1-\alpha)+\alpha A_1}$ and $\frac{1}{\alpha A_1}\geq\frac{1}{\beta_2(1-\alpha)+\alpha A_1}$, the conditions that ensure all the buffers at the relay node are unsaturated can be expressed as

$$\frac{z}{h_r}<\min\left\{\frac{1}{\beta_1\left(1-\alpha\right)+\alpha A_1},\frac{1}{\beta_2\left(1-\alpha\right)+\alpha A_1}\right\}=K.\tag{50}$$

Applying (47) and after some simplification, we have

$$\begin{aligned}&\sqrt{\left[h_r\left(A_1+1\right)-\alpha A_1\right]^2-4h_rA_2\left(1-\alpha\right)^2}>h_r\left(A_1+1\right)\\&+\alpha A_1-2h_rK\left[\alpha A_1\left(A_1+1\right)+A_2\left(1-\alpha\right)^2\right].\end{aligned}\tag{51}$$

Next we show that

$$h_r\,(A_1+1)+\alpha A_1-2h_r K\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]> -\sqrt{[h_r\,(A_1+1)-\alpha A_1]^2-4h_r A_2\,(1-\alpha)^2}. \tag{52}$$

Obviously, inequality (52) holds when $2K\left[\alpha A_1(A_1+1)+A_2\,(1-\alpha)^2\right]-(A_1+1)<0$. Thus, we only need to consider the case $2K\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]-(A_1+1)\geq 0$. We need to prove that the following inequalities cannot be both satisfied.

$$h_r\,(A_1+1)+\alpha A_1-2h_r K\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right] < 0, \tag{53}$$

$$\{h_r\,(A_1+1)+\alpha A_1-2h_r K[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2]\}^2 \geq [h_r\,(A_1+1)-\alpha A_1]^2-4h_r A_2\,(1-\alpha)^2. \tag{54}$$

From (53), we have

$$h_r>\frac{\alpha A_1}{2K\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]-(A_1+1)}. \tag{55}$$

From (54), we have

$$\left\{K\,(A_1+1)-K^2\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]\right\}h_r \leq 1-K\alpha A_1. \tag{56}$$

Next we show that

$$(A_1+1)-K\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]\geq 0. \tag{57}$$

Since the left side of inequality (57) is a symmetric function of β_1 and β_2, without loss of generality, it suffices to consider the case $\beta_1\geq\beta_2$ only. When $\beta_1\geq\beta_2$,

$$K=\frac{1}{\beta_1\,(1-\alpha)+\alpha A_1}. \tag{58}$$

Substituting (58) into (57) and applying (14), condition (57) can be simplified to

$$1-\frac{\beta_2\,(1-\alpha)}{\beta_1+\beta_2+1}\geq 0. \tag{59}$$

Since $\beta_2\,(1-\alpha)\leq\beta_2$, condition (59) always holds. Thus, condition (57) is satisfied. Therefore, inequality (56) is equivalent to

$$h_r\leq\frac{1-K\alpha A_1}{K\,(A_1+1)-K^2\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]}. \tag{60}$$

From (55) and (60), we know that (53) and (54) cannot be both satisfied if

$$\frac{1-K\alpha A_1}{K\,(A_1+1)-K^2\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]} \leq\frac{\alpha A_1}{2K\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]-(A_1+1)}. \tag{61}$$

Since both sides of inequality (61) are symmetric functions of β_1 and β_2, without loss of generality, it suffices to consider the case $\beta_1\geq\beta_2$ only. When $\beta_1\geq\beta_2$, substituting (58) into (61) and applying (14), (61) can be simplified to

$$\alpha\,(\beta_1+\beta_2)\,\beta_2+2\beta_1\beta_2\,(1-\alpha)-\beta_1\,(\beta_1+\beta_2+1)\leq 0. \tag{62}$$

Let $P(\alpha)\triangleq\alpha(\beta_1+\beta_2)\beta_2+2\beta_1\beta_2(1-\alpha)-\beta_1(\beta_1+\beta_2+1)$. The derivative of $P\,(\alpha)$ equals $\beta_2\,(\beta_2-\beta_1)$, which is non-positive when $\beta_1\geq\beta_2$. Thus, we have

$$\begin{aligned}P\,(\alpha)\leq P\,(0)&=2\beta_1\beta_2-\beta_1\,(\beta_1+\beta_2+1)\\&=-\left[\beta_1\,(\beta_1-\beta_2)+\beta_1\right]\leq 0.\end{aligned}$$

Thus, inequality (61) holds. Therefore, (53) and (54) cannot be both satisfied and (52) holds.

Then, (51) is equivalent to

$$\{h_r\,(A_1+1)+\alpha A_1-2h_r K[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2]\}^2 < [h_r\,(A_1+1)-\alpha A_1]^2-4h_r A_2\,(1-\alpha)^2,$$

which can be simplified to (27).

Next, we show that condition (46) always holds when all the buffers at the relay node are unsaturated. Condition (46) can be simply expressed as

$$(A_1+1)^2h_r^2-[2\alpha A_1(A_1+1)+4A_2(1-\alpha)^2]h_r+\alpha^2A_1^2\geq 0,$$

which is equivalent to

$$h_r\leq\frac{[\alpha A_1(A_1+1)+2A_2(1-\alpha)^2]-\sqrt{4A_2^2(1-\alpha)^4+4\alpha A_1(A_1+1)A_2(1-\alpha)^2}}{(A_1+1)^2},$$

or

$$h_r\geq\frac{[\alpha A_1(A_1+1)+2A_2(1-\alpha)^2]+\sqrt{4A_2^2(1-\alpha)^4+4\alpha A_1(A_1+1)A_2(1-\alpha)^2}}{(A_1+1)^2}.$$

Thus, we need to show that

$$\frac{[\alpha A_1(A_1+1)+2A_2(1-\alpha)^2]+\sqrt{4A_2^2(1-\alpha)^4+4\alpha A_1(A_1+1)A_2(1-\alpha)^2}}{(A_1+1)^2} \leq\frac{1-K\alpha A_1}{K\,(A_1+1)-K^2\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]},$$

which is equivalent to

$$\sqrt{4A_2^2\,(1-\alpha)^4+4\alpha A_1\,(A_1+1)\,A_2\,(1-\alpha)^2} \leq\frac{(1-K\alpha A_1)\,(A_1+1)^2}{K(A_1+1)-K^2\left[\alpha A_1\,(A_1+1)+A_2\,(1-\alpha)^2\right]} -\left[\alpha A_1\,(A_1+1)+2A_2\,(1-\alpha)^2\right]. \tag{63}$$

To prove (63), we need to show that

$$4A_2^2(1-\alpha)^4 + 4\alpha A_1(A_1+1)A_2(1-\alpha)^2 \le \left\{\frac{(1-K\alpha A_1)(A_1+1)^2}{K(A_1+1)-K^2\left[\alpha A_1(A_1+1)+A_2(1-\alpha)^2\right]} - \left[\alpha A_1(A_1+1)+2A_2(1-\alpha)^2\right]\right\}^2. \tag{64}$$

$$\frac{(1-K\alpha A_1)(A_1+1)^2}{K(A_1+1)-K^2\left[\alpha A_1(A_1+1)+A_2(1-\alpha)^2\right]} - \left[\alpha A_1(A_1+1)+2A_2(1-\alpha)^2\right] \ge 0. \tag{65}$$

Since both sides of inequalities (64) and (65) are symmetric functions of β_1 and β_2, without loss of generality, it suffices to consider the case $\beta_1 \ge \beta_2$ only. When $\beta_1 \ge \beta_2$, substituting (58) and applying (14), condition (64) can be simplified to

$$\left[\beta_1(\beta_1+\beta_2+1)-\alpha\beta_2(\beta_1+\beta_2)\right]^2 + 4(1-\alpha)\beta_1\beta_2\left(-\beta_1^2-\beta_1+\alpha\beta_2^2\right) \ge 0. \tag{66}$$

Let $F(\alpha) \triangleq \left[\beta_1(\beta_1+\beta_2+1)-\alpha\beta_2(\beta_1+\beta_2)\right]^2 + 4(1-\alpha)\beta_1\beta_2\left(-\beta_1^2-\beta_1+\alpha\beta_2^2\right)$. The first and second derivatives of $F(\alpha)$ respectively equal

$$\begin{aligned} F'(\alpha) = &-2\beta_2(\beta_1+\beta_2)\left[\beta_1(\beta_1+\beta_2+1)-\alpha\beta_2(\beta_1+\beta_2)\right] \\ &-4\beta_1\beta_2\left(-\beta_1^2-\beta_1+\alpha\beta_2^2\right)+4(1-\alpha)\beta_1\beta_2^3, \\ F''(\alpha) = &\ 2\beta_2^2(\beta_1-\beta_2)^2. \end{aligned}$$

Since $F''(\alpha) \ge 0$, we have $F'(\alpha) \ge F'(0)$. Furthermore,

$$\begin{aligned} F'(0) &= -2\beta_1\beta_2(\beta_1+\beta_2)(\beta_1+\beta_2+1) + 4\beta_1\beta_2\left(\beta_1^2+\beta_1\right)+4\beta_1\beta_2^3 \\ &= 2\beta_1\beta_2\left[(\beta_1-\beta_2)^2+(\beta_1-\beta_2)\right], \end{aligned}$$

which is non-negative when $\beta_1 \ge \beta_2$. Thus, we have $F'(\alpha) \ge 0$. Then, $F(\alpha)$ is an increasing function of α. Moreover,

$$\begin{aligned} F(0) &= \left[\beta_1(\beta_1+\beta_2+1)\right]^2 - 4\beta_1\beta_2\left(\beta_1^2+\beta_1\right) \\ &= \beta_1^2(\beta_1-\beta_2+1)^2, \end{aligned}$$

which is non-negative. Then we have $F(\alpha) \ge 0$. Thus, condition (64) is satisfied.

Similarly, when $\beta_1 \ge \beta_2$, applying (58), condition (65) can be simplified to

$$\left[\beta_1(A_1+1)-A_2(1-\alpha)\right]^2 + \alpha A_1(A_1+1)A_2 + A_2^2(1-\alpha)^2 \ge 0. \tag{67}$$

Since (67) always holds, (65) holds. Therefore, condition (46) always holds when all the buffers at the relay node are unsaturated. ■

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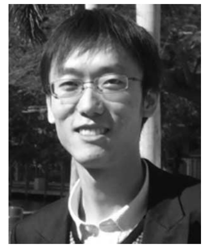

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