Throughput Capacity of IEEE 802.11 Many-to/From-One Bidirectional Networks With Physical-Layer Network Coding

Shijun Lin, Member, IEEE, and Liqun Fu, Member, IEEE

Abstract-In this paper, we investigate the throughput capacity of physical-layer network coding (PNC) in a non-all-inclusive carrier-sensing network with IEEE 802.11 distributed coordination function (DCF). In particular, we consider the manyto/from-one bidirectional networks in which a common center node exchanges packets with many other nodes through multihop transmissions. We first analyze the canonical networks with equal-link-length (ELL) and variable-link-length (VLL), respectively, and derive the corresponding analytical network capacity. Simulations show that the throughput capacities are reasonably tight. We further maximize the network capacity by properly selecting the signal-to-interference-plus-noise ratio (SINR) threshold/transmission rate through numerical calculation. Last but not least, we identify the optimal number of hops that has the maximum network throughput. In particular, the four-hop canonical networks have the maximum network throughput, which indicates that in a many-to/from-one network with five or more hops, it is preferable to transmit the packets across the four-hop nodes to make full use of the PNC scheme. Simulation results show that the throughput gain of PNC scheme with and without considering the synchronization cost can, respectively, reach upto 291.7% and 340.6%, compared with the traditional IEEE 802.11 multihop networks without network coding.

Index Terms—Physical-layer network coding, non-all-inclusive carrier-sensing network, IEEE 802.11 DCF, throughput capacity.

I. Introduction

D UE to the broadcast nature of wireless media, interference may occur when two or more wireless signals encounter in the same channel. A consequence of interference is transmission failure in traditional wireless systems. Recently, a new technique, namely, physical-layer network coding (PNC), turned the "interference" into good use. In particular, the network-coded (NC) packet can be extracted from

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- S. Lin is with the Department of Communication Engineering, Xiamen University, Xiamen 361000, China (e-mail: linsj@xmu.edu.cn).
- L. Fu is with the School of Information Science and Technology, ShanghaiTech University, Shanghai 200000, China (e-mail: fulq@shanghaitech.edu.cn).

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the superposition of wireless signals [1]–[3]. By doing so, the capacity of wireless networks can be significantly improved [4].

PNC has continuously attracted a lot of research interests since it was first proposed in 2006 [1], [2]. The physicallayer techniques of the PNC scheme have been extensively studied, mainly within the two-way relay network (TWRN), where two end users exchange packets through a relay node (e.g., see [5]-[11]). In [5]-[7], the authors studied the performance of different modulations with PNC. In [8], [9], the joint design of PNC with channel coding, i.e. convolutional code and low-density parity-check (LDPC) code, was discussed. The integration of multiple-input multiple-output (MIMO) system with PNC scheme was investigated in [10], [11]. In TWRN, only two time slots are needed when exchanging two packets between two end nodes across a relay node. The throughput gain of the PNC scheme in the TWRN can reach up to 200% [4]. An interesting question is what is the throughput gain of PNC scheme when it is applied to a general wireless network.

When PNC is applied to a general wireless network, the transmission process coordinated with media access control (MAC) protocols should be considered. The 802.11 protocol which coordinates the transmissions with carrier sense multiple access (CSMA) is the most widely used distributed MAC protocol in current wireless systems. The coordination of 802.11 protocol with traditional network-layer NC has been investigated in [12]. Recently, two distributed MAC protocols for PNC systems were proposed in [13], [14], which are evolved from traditional IEEE 802.11 protocol. The MAC protocol proposed in [13] solved the PHY-specific problems within the distributed MAC protocol, such as synchronization and the acquisition of the channel state information. In [14], the proposed protocol pro-actively enforces two independent packet transmissions to interfere in a controlled and cooperative manner with the assist of the relay node. However, none of them analyzes the network throughput performance of the proposed protocols. A very recent paper, [15], gave the analytical throughput performance of PNC coordinated with a 802.11-like MAC protocol in a network where two client groups communicate with each other across one relay node. The network considered in [15] is an all-inclusive carrier-sensing network where all the nodes in the network can carrier sense each other.

In this paper, we investigate the throughput capacity of the PNC scheme in a distributed wireless network coordinated with IEEE 802.11 distributed coordination function (DCF). The network considered here is a general many-to/from-one bidirectional network in which a common center end node exchanges

packets with many other end nodes by multi-hop transmissions. Therefore, it is a *non-all-inclusive* carrier-sensing network in which not all the nodes can carrier sense each other. When PNC is adopted, the transmission efficiency can be increased in the case where two nodes exchange packets through a relay node. However, since two nodes are allowed to concurrently transmit in the PNC scheme, a larger carrier-sensing range (CSR) is needed to prevent collisions. Therefore, the interplay of the CSR, Signal-to-Interference-plus-Noise Ratio (SINR) threshold, and the spatial reuse is more complicated in a *non-all-inclusive* carrier-sensing network with PNC scheme, which makes the throughput capacity analysis more difficult.

In the literature, there have been several studies of the throughput performance in the traditional CSMA networks without network coding scheme [16]–[19]. In [16], the throughput capacity of many-to-one multi-hop networks without network coding was discussed. In [17], Ma et al. optimized the CSR that achieves the trade-off between the hidden and exposed node problems in order to maximize the aggregate network throughput. In [18], Ye et al. derived an analytical model to calculate the successful transmission probability while considering the impact of CSR. In [19], Fu et al. considered the hidden-node free design in a general CSMA network under the physical interference model and proposed a new carrier-sensing mechanism to further improve the network throughput. Only recently, the authors in [20] and [21] studied the throughput maximization problem of distributed multi-hop networks while applying the network coding at the network layer. In [22], the authors studied the throughput and delay performance of oneto-many multi-hop networks with traditional linear network coding. Since the network coding operation considered in [20]-[22] is at the network layer, the MAC protocol design and the analysis method of network throughput for traditional CSMA networks remain valid. When PNC, the PHY technique, is considered, the transmission efficiency can be further improved. However, it involves a more complicated transmission process at MAC layer, and the interplay of CSR, SINR threshold, and spatial reuse is more sophisticated. The main contributions of this paper are summarized as follows:

- We derive the throughput capacities of canonical networks, in which all the chains are linear and all the *m*-hop nodes are located on a circle centered at the center node. We consider both the canonical networks with same or different link length. Simulation results show that the theoretical throughput capacities are reasonably tight.
- 2) Increasing the target SINR threshold will increase the transmission rate of a single link. However, it also decreases the network spatial reuse. We show that the overall network throughput does not monotonically increase with the target SINR threshold. Simulations show the overall network throughput with PNC scheme under all the rate choices in the IEEE 802.11a protocol, which gives the preferable rate/target SINR selection in the practical system.
- 3) We find the optimal number of hops that maximizes the overall network throughput. In particular, the four-hop canonical networks have the maximum network throughput. This indicates that in a many-to/from-one network

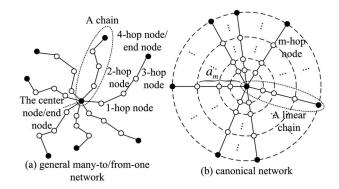


Fig. 1. Many-to/from-one network.

with five or more hops, it is preferable to transmit the packets across the 4-hop nodes to make full use of the PNC scheme. Simulation results show that the throughput gain of PNC scheme with and without considering the synchronization cost in the four-hop equal-link-length (ELL) canonical network can reach up to 291.7% and 340.6%, respectively.

The rest of this paper is organized as follows. In Section II, we introduce the network model, define the canonical network, and discuss the coordination of PNC with IEEE 802.11 DCF. In Sections III and IV, we analyze the throughput capacity of three-hop and four-hop canonical networks, respectively. In Section V, we carry out extensive simulations to validate our analysis, discuss the relation between the throughput capacity and the network parameters, and show the throughput gain of PNC scheme. Section VI concludes this paper.

II. SYSTEM DESCRIPTION

A. Network Model

We consider a many-to/from-one bidirectional network, which consists of multiple chains with a common center node, as shown in Fig. 1a. The center end node exchanges packets with all the other end nodes. In Fig. 1a, a black node is an end node which sends packets generated by itself to other nodes or receives packets destined for it; while a white node is a relay node which only relays packets but dose not generate packets itself. For simple expression, we refer to the nodes that are *m*-hop away from the center node as the *m*-hop nodes. We assume that the topology is fixed. The routing at the network layer is pre-determined in a way such that a node can only communicate with its one-hop neighbor node.

There is only one physical channel. All nodes contend for the channel according to the IEEE 802.11 DCF and work in the half-duplex mode. A node can successfully receive a packet if and only if the SINR is higher than a given threshold. That is, as long as the target SINR is satisfied, a link can be established. We further assume that: 1) Once the link is established, it has the same physical link rate regardless the distance between the transmitter and its destination node. 2) All nodes in the network use the same transmit power P_t , and all nodes have the same SINR threshold, γ_0 , for successful reception. We adopt the log-distance path-loss signal propagation model with a path-loss exponent $\alpha \in [2, 6]$ [23]. Let $\Gamma = \{n_s, 1 \le s \le |\Gamma|\}$ denote the

set of nodes that concurrently transmit with node n_x . Then, a successful transmission from n_x to n_y needs to satisfy

$$\frac{P_t d(n_x, n_y)^{-\alpha}}{N + \sum_{n_s \in \Gamma} P_t d(n_s, n_y)^{-\alpha}} \ge \gamma_0, \tag{1}$$

where $d(n_x, n_y)$ and N denote the distance between n_x and n_y , and the noise power, respectively.

In order to eliminate collisions by properly setting the CSR, we need to assume: 1) The back-off counters of different nodes would not countdown to zero at the same time [16]. 2) The "RS (Re-Start) mode" is enabled at the receiver to avoid the transmission failures caused by the "Receiver-Capture effect" [24]. With the RS mode, the receiver will switch to capture the signal with the highest power. It has been shown in [19] and [24] that no matter how large the CSR is set, transmission failures can still occur if without the RS mode. Furthermore, there is perfect synchronization for PNC system to guarantee that the relay node can extract an NC packet from the superimposed electromagnetic (EM) waves. In practice, high-precision synchronization can be implemented using phase-locked loop (PLL) or maximum likelihood estimation (MLE) methods [25]. Moreover, some technologies, such as Orthogonal Frequency Division Multiplexing (OFDM), can be used so that the synchronization requirement can be relaxed [4].

B. Canonical network

The throughput capacity analysis of general many-to/fromone bidirectional networks with arbitrary topology is really difficult even without PNC scheme [16]. In this paper, we focus on the throughput capacity analysis of canonical networks with regular topology structure. In the canonical network, all the chains are linear, and all the m-hop nodes are located on a circle centered at the center node with radius a_m , as shown in Fig. 1b. We focus on two kinds of canonical networks: ELL canonical network and variable-link-length (VLL) canonical network. In the ELL canonical network, the distance between two adjacent nodes in a chain is the same, denoted by l. Then, a_m equals ml. In the VLL canonical network, the distance between adjacent nodes in a chain can be different.

C. The coordination of PNC with IEEE 802.11 DCF

In [1], Zhang et al. showed the optimal scheduling for the PNC scheme to achieve the maximum throughput in a bidirectional multi-hop linear network. However, it requires a centralized controller to coordinate the transmissions of all the nodes on the chain. This is not amendable to the CSMA network which coordinates the transmissions in a distributed manner. Therefore, we assume that in a PNC network coordinated with IEEE 802.11 DCF, the transmission of a packet with three or more hops is replaced by the transmissions of several one-hop and two-hop packets. What's more, to coordinate the transmissions of PNC scheme easily, we consider the Request to Send (RTS)/Clear to Send (CTS) mechanism. In the following, we describe the details of the transmission process of the

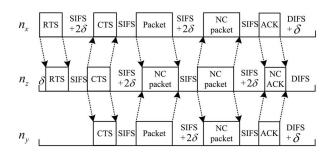


Fig. 2. Transmission process of the PNC scheme.

PNC scheme. We assume that the propagation delay δ between any two adjacent nodes is the same.

When nodes n_x and n_y exchange two two-hop packets across a relay node n_z , adopting PNC scheme can improve the transmission efficiency. The process that node n_x initiates the transmission is shown in Fig. 2. First, n_x sends an RTS packet to the relay node n_z with the information indicating that n_y is its destination node. After correctly receiving the RTS packet for a short inter frame space (SIFS) time, the relay node n_z broadcasts a CTS packet to both n_x and n_y . After correctly receiving the CTS packet, both n_x and n_y wait for an SIFS delay first, and then send a data packet to n_z simultaneously. If n_y does not have a data packet for n_x , it sends a dummy packet instead. Under the assumption of perfect synchronization, node n_z can deduce an NC data packet from the received superimposed EM waves from n_x and n_y . After waiting for an SIFS delay, node n_z broadcasts the NC data packet to both n_x and n_y . When correctly receiving the NC data packet, n_x and n_y can extract the packet they need and then know whether their data packets have been transmitted successfully. After waiting for another SIFS time, both n_x and n_y send back an ACK (acknowledgment) packet to n_z simultaneously. By checking the overlapped ACK packets, node n_z knows whether the NC data packet is correctly received or not. After a distributed inter frame space (DIFS) time, the channel occupied by node n_x is released.

From the above description, there are two kinds of links in the 802.11 PNC network. The first one is the traditional one-hop link, in which a node transmits a packet to its one-hop neighbor node; the second one is the two-hop link, in which two nodes that are two hops away exchange two packets across a relay node with PNC scheme.

III. THROUGHPUT CAPACITY OF THREE-HOP CANONICAL NETWORKS

The throughput capacity of canonical networks with three-hop and four-hop is the key in the capacity analysis of general multi-hop canonical networks. The throughput capacity of one-hop and two-hop canonical network is equal to S_1 and S_2 , respectively, which are the throughput of a single one-hop link and a single two-hop link with PNC scheme. The corresponding schedule to achieve the capacity is to set a sufficiently large CSR such that only one end node can initiate a one-hop (two-hop) link in each time slot. For the canonical networks with five or more hops, all the packets exchanged between the center end

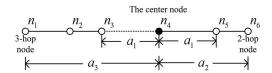


Fig. 3. A 3-hop node and a 2-hop node with separation of $(a_2 + a_3)$ concurrently initiate a two-hop link.

node and the other end nodes need to traverse through the 3hop or 4-hop nodes first. Then, the packet exchange between the end nodes can be divided into two sub-transmissions: 1) the packet exchange between the center end node and the 3-hop (or 4-hop) nodes; 2) the packet exchange between the 3-hop (or 4-hop) nodes and the other end nodes. Since these two sub-transmissions can not always work concurrently, the total time used for the packet exchange between the center end node and the other end nodes is no shorter than the time used for the packet exchange between the center end node and the 3hop (or 4-hop) nodes. Thus, the throughput capacity of the canonical networks with five or more hops is no more than $\max(C_3, C_4)$, where C_3 and C_4 denote the throughput capacity of the three-hop and four-hop canonical networks, respectively. In the following discussion, the noise power is assumed to be zero for simplicity.

With PNC scheme, the packets from the four-hop node may not need to traverse through the three-hop node (it can reach the two-hop node directly with a PNC link). Therefore, C_3 and C_4 should be derived separately. In this section, we focus on the throughput capacity of three-hop canonical networks. In the discussion, we do not restrict the actions of nodes and thus each node can select to initiate a two-hop link or a one-hop link if they have packets to transmit. In the following, we first show the requirement on the CSR for three-hop canonical networks to ensure successful transmissions in Lemma 1. Given the required CSR, we then show the best possible spatial reuse in the three-hop ELL and VLL canonical networks in Lemma 2 and Lemma 3, respectively. The throughput capacities of the three-hop canonical networks are finally derived following Lemmas 1, 2, and 3.

Lemma 1: In the three-hop canonical network, the CSR should be no less than $(a_2 + a_3)$ to ensure successful transmissions.

Proof: When "RS mode" is adopted, a receiver will switch to capture the signal with the highest power. Thus, to guarantee a successful reception, the power of the intended signal should be larger than all the interferences. If CSR is less than $(a_2 + a_3)$, a 3-hop node and a 2-hop node that are $(a_2 + a_3)$ away can concurrently initiate a two-hop link, as shown in Fig. 3. If n_4 receives a CTS/NC packet from n_5 during the transmission of n_3 , transmission failure will occur since the power of the received signal at n_4 is not larger than the interference power from n_3 . Therefore, the CSR should be no less than $(a_2 + a_3)$ to prevent collisions.

A. Throughput capacity of three-hop ELL canonical networks

The best possible spatial reuse depends on the targeted SINR threshold, which is given in the following Lemma.

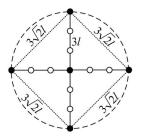


Fig. 4. Four 3-hop nodes concurrently initiate a link in the three-hop ELL canonical network.

Lemma 2: In the three-hop ELL canonical network, at most *K* 3-hop nodes can concurrently initiate a link (one-hop link or two-hop PNC link), where

$$\begin{cases} K = 3, & \text{if } \gamma_0 \le \frac{0.5}{3^{-\frac{\alpha}{2}} + 13^{-\frac{\alpha}{2}}}, \\ K = 2, & \text{if } \frac{0.5}{3^{-\frac{\alpha}{2}} + 13^{-\frac{\alpha}{2}}} < \gamma_0 \le \frac{1}{2^{-\alpha} + 4^{-\alpha}}, \\ K = 1, & \text{if } \gamma_0 > \frac{1}{2^{-\alpha} + 4^{-\alpha}}. \end{cases}$$

Proof: We first show that four or more 3-hop nodes can not concurrently initiate a link given any γ_0 . According to Lemma 1, the CSR in the three-hop ELL canonical network should be no less than 5l. However, as shown in Fig. 4, if four 3-hop nodes concurrently initiate a link, the minimum distance between two adjacent 3-hop nodes at most equals $3\sqrt{2}l$, which is smaller than 5l. Thus, it is impossible for more than three 3-hop nodes to concurrently initiate a link.

When three 3-hop nodes concurrently initiate a link, the worst case of interference happens when each 3-hop node initiates a two-hop link. To reduce the mutual interference, the three two-hop links should be separated as far away as possible, as shown in Fig. 5. From Fig. 5, we can see that the separation between any two adjacent 3-hop nodes is $3\sqrt{3}l$, which satisfies the CSR requirement in Lemma 1. Moreover, among all the possible transmissions, the minimum SINR happens when n_3 receives a packet from n_2 , under the interference from nodes n_4 , n_5 , n_6 , and n_7 . Therefore, successful transmissions on all the links can be ensured if

$$\frac{P_t l^{-\alpha}}{2(P_t d_{53}^{-\alpha} + P_t d_{43}^{-\alpha})} \ge \gamma_0, \tag{2}$$

where d_{xy} is the simplified notation of $d(n_x, n_y)$. From Fig. 5, we know that $d_{53} = 2l \sin(\frac{\pi}{3})$ and $d_{43} = \sqrt{l^2 + (3l)^2 - 6l^2 \cos(\frac{2\pi}{3})}$. By simplifying inequality (2), we have $\gamma_0 \leq \frac{0.5}{3^{-\frac{\alpha}{2}} + 13^{-\frac{\alpha}{2}}}$.

Similarly, when K = 2, the worst case of interference happens when each 3-hop node initiates a two-hop link. Furthermore, to reduce the mutual interference, the two two-hop links should be placed in a line, as shown in Fig. 6. The minimum SINR happens when n_3 receives a packet from n_2 under the interference from both n_4 and n_6 . Therefore, successful transmissions on all the links can be ensured if

$$\frac{P_t l^{-\alpha}}{P_t(2l)^{-\alpha} + P_t(4l)^{-\alpha}} \ge \gamma_0,\tag{3}$$

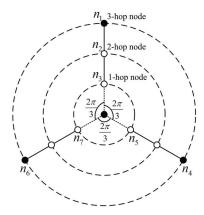


Fig. 5. Three two-hop links with the maximum separation in the three-hop ELL canonical network.

3-hop node node node node node node node
$$n_1$$
 node n_2 node n_3 node node node node node node n_4 n_5 n_6 n_6

Fig. 6. Two two-hop links with the maximum separation in the three-hop ELL canonical network.

which is equivalent to $\gamma_0 \leq \frac{1}{2^{-\alpha}+4^{-\alpha}}$. From (3), we can further obtain that only one 3-hop node can initiate a link when $\gamma_0 > \frac{1}{2^{-\alpha}+4^{-\alpha}}$.

Theorem 1: The throughput capacity of the three-hop ELL canonical network is

$$\begin{cases} C_3 = \max\left(\frac{1}{1 + \frac{T_2}{6T_1}}S_1, \frac{1}{1 + \frac{4T_1}{3T_2}}S_2\right), & \text{if } \gamma_0 \le \frac{0.5}{3^{-\frac{\alpha}{2}} + 13^{-\frac{\alpha}{2}}}, \\ C_3 = \max\left(\frac{1}{1 + \frac{T_2}{4T_1}}S_1, \frac{1}{1 + \frac{3T_1}{2T_2}}S_2\right), & \text{if } \frac{0.5}{3^{-\frac{\alpha}{2}} + 13^{-\frac{\alpha}{2}}} < \gamma_0 \le \frac{1}{2^{-\alpha} + 4^{-\alpha}}, \\ C_3 = \max\left(\frac{1}{1 + \frac{T_2}{2T_1}}S_1, \frac{1}{1 + \frac{2T_1}{T_2}}S_2\right), & \text{if } \gamma_0 > \frac{1}{2^{-\alpha} + 4^{-\alpha}}. \end{cases}$$

Here, T_1 and T_2 respectively denote the time that a successful one-hop link and a successful two-hop PNC link experience¹.

Proof: To improve the transmission efficiency, we should use the two-hop link with PNC scheme as much as possible. Thus, when a 3-hop end node exchanges two packets with the center end node, two one-hop links and one two-hop link should be initiated. There are two transmission choices. The first one is that the center end node first exchanges packets with the 2hop node by a two-hop PNC link, and then the 2-hop node exchanges packets with the 3-hop end node by two one-hop links. The other one is the center end node first exchanges packets with the 1-hop node by two one-hop links, and then the 1-hop node exchanges packets with the 3-hop end node by a two-hop PNC link. Assume that there are W chains using the first choice and Z chains using the second choice. Then, when all the (W + Z) 3-hop end nodes each exchange two packets with the center end node, there are totally W one-hop links initiated by the 3-hop nodes, W one-hop links initiated by the 2-hop nodes, W two-hop PNC links initiated by the 2-hop nodes (or the center end node), Z one-hop links initiated by the center end node, Z one-hop links initiated by the 1-hop nodes, and Z two-hop PNC links initiated by the 1-hop nodes (or the 3-hop end nodes).

When $\gamma_0 \leq \frac{0.5}{3^{-\frac{\alpha}{2}}+13^{-\frac{\alpha}{2}}}$, according to Lemma 2, we know that at most three 3-hop end nodes can concurrently initiate a link. Furthermore, since the CSR should be no less than 5l, we have: 1) when a 3-hop node has initiated a link, all the 2-hop nodes, 1-hop nodes, and the center node can not initiate a link; 2) all the 2-hop nodes, 1-hop nodes and the center node can not initiate a link concurrently. Thus, the minimum total time used for all the 3-hop end nodes to exchange two packets with the center end node is

$$T_{total}^{\min} = \frac{W}{3}T_1 + WT_1 + WT_2 + ZT_1 + ZT_1 + \frac{Z}{3}T_2$$
$$= W\left(\frac{4}{3}T_1 + T_2\right) + Z\left(2T_1 + \frac{T_2}{3}\right).$$

The proportion of time for the center end node to transmit or receive packets by two-hop PNC links and one-hop links equals $\frac{WT_2}{T_{total}^{\min}}$ and $\frac{2ZT_1}{T_{total}^{\min}}$, respectively. Therefore, the network throughput S is given by

$$S = \frac{WT_2}{T_{total}^{\min}} S_2 + \frac{2ZT_1}{T_{total}^{\min}} S_1.$$

By combining the preceding two relations, and after some simplifications, we have

$$S = \frac{2T_1S_1}{2T_1 + \frac{T_2}{3}} \left(1 + \frac{\frac{T_2S_2}{2T_1S_1} - \frac{\frac{4}{3}T_1 + T_2}{2T_1 + \frac{T_2}{3}}}{\frac{\frac{4}{3}T_1 + T_2}{2T_1 + \frac{T_2}{3}} + \frac{Z}{W}} \right). \tag{4}$$

We can find that under the condition $\frac{T_2S_2}{2T_1S_1} \leq \frac{\frac{4}{3}T_1 + T_2}{2T_1 + \frac{T_2}{3}}$ (or equivalently $\frac{1}{1 + \frac{4T_1}{3T_2}}S_2 \leq \frac{1}{1 + \frac{T_2}{6T_1}}S_1$), S reaches the maximum value of $\frac{1}{1 + \frac{T_2}{6T_1}}S_1$ when W equals zero; while under the condition $\frac{T_2S_2}{2T_1S_1} > \frac{\frac{4}{3}T_1 + T_2}{2T_1 + \frac{T_2}{3}}$ (or equivalently $\frac{1}{1 + \frac{4T_1}{3T_2}}S_2 > \frac{1}{1 + \frac{T_2}{6T_1}}S_1$), S reaches the maximum value of $\frac{1}{1 + \frac{4T_1}{3T_2}}S_2$ when Z equals zero. Therefore, $C_3 = \max\left(\frac{1}{1 + \frac{T_2}{6T_1}}S_1, \frac{1}{1 + \frac{4T_1}{3T_2}}S_2\right)$ when $\gamma_0 \leq \frac{0.5}{3^{-\frac{\alpha}{2}} + 13^{-\frac{\alpha}{2}}}$.

Similarly, we can prove that
$$C_3 = \max\left(\frac{1}{1 + \frac{T_2}{4T_1}}S_1, \frac{1}{1 + \frac{3T_1}{2T_2}}S_2\right)$$
 when $\frac{0.5}{3^{-\frac{\alpha}{2}} + 13^{-\frac{\alpha}{2}}} < \gamma_0 \le \frac{1}{2^{-\alpha} + 4^{-\alpha}},$ and $C_3 = \max\left(\frac{1}{1 + \frac{T_2}{2T_1}}S_1, \frac{1}{1 + \frac{2T_1}{T_2}}S_2\right)$ when $\gamma_0 > \frac{1}{2^{-\alpha} + 4^{-\alpha}}$.

B. Throughput capacity of three-hop VLL canonical networks

In the three-hop VLL canonical network, the distance between two adjacent nodes in a chain can be different. In

 $^{{}^{1}}T_{1}$ is given in [15] and T_{2} can be calculated according to Fig. 2.

order to obtain its throughput capacity, we need to identify the network topology that has the best possible spatial reuse.

Lemma 3: In the three-hop VLL canonical network, at most *K* 3-hop nodes can concurrently initiate a link, where

$$\begin{cases} K = 3, & \text{if } \gamma_0 < \frac{0.366^{-\alpha}}{2\left(3^{-\frac{\alpha}{2}} + 4.232^{-\frac{\alpha}{2}}\right)}, \\ K = 2, & \text{if } \gamma_0 \ge \frac{0.366^{-\alpha}}{2\left(3^{-\frac{\alpha}{2}} + 4.232^{-\frac{\alpha}{2}}\right)}. \end{cases}$$

Proof: Let θ be the minimum angle between two adjacent chains in which the 3-hop nodes n_1 and n_2 concurrently initiate a link. Then, we have

$$CSR < d_{12} = 2a_3 \sin\left(\frac{\theta}{2}\right).$$

According to the preceding relation and Lemma 1, we have

$$a_2 + a_3 < 2a_3 \sin\left(\frac{\theta}{2}\right). \tag{5}$$

Let a_3 equal βa_2 . It follows

$$\sin\left(\frac{\theta}{2}\right) > \frac{1+\beta}{2\beta}.\tag{6}$$

In the following, we show that four 3-hop nodes can not concurrently initiate a link given $2 \le \alpha \le 6$. When four 3-hop nodes concurrently initiate a two-hop link (the worst case), θ at most equals $\frac{\pi}{2}$. Then, we have

$$\frac{1+\beta}{2\beta} < \sin\left(\frac{\theta}{2}\right) \le \frac{\sqrt{2}}{2} \Rightarrow \beta > 2.4142. \tag{7}$$

To minimize the mutual interference, the four two-hop links are placed as far away as possible. What's more, when fixing the positions of the 3-hop nodes and moving the 2-hop and 1-hop nodes towards the corresponding 3-hop nodes, the SINRs at all the nodes when all the 3-hop nodes concurrently initiate a two-hop link increase. Then, we can use the best settings $a_2 = a_1$ and $a_3 = 2.4142a_2$ to further reduce the mutual interference, as shown in Fig. 7. The minimum SINR happens when a 2-hop node receives a packet from the 3-hop node in the same chain under the interference from the 1-hop nodes and 3-hop nodes of all the other chains. Therefore, successful transmissions can be ensured if

$$\frac{P_t(2.4142a_2 - a_2)^{-\alpha}}{2P_t d_{72}^{-\alpha} + 2P_t d_{92}^{-\alpha} + P_t d_{42}^{-\alpha} + P_t d_{62}^{-\alpha}} \ge \gamma_0, \tag{8}$$

where d_{72} , d_{92} , d_{42} , and d_{62} are equal to $2a_2\sin(\frac{\pi}{4})$, $\sqrt{(2.4142a_2)^2 + a_2^2}$, $2a_2$, and $a_2 + 2.4142a_2$, respectively. The left-hand side of (8) reaches its maximum value when $\alpha = 6$. Thus, we have $\gamma_0 \leq 0.4587$, which is smaller than all the possible target SINR requirements for all the transmission-rate levels defined in the current 802.11 systems [26], [27]. Therefore, it is impossible for four 3-hop nodes to concurrently initiate a

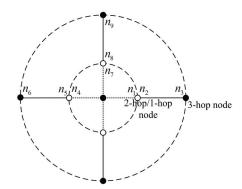


Fig. 7. Four two-hop links with the maximum separation under the best setting of a_1 , a_2 , and a_3 in the three-hop VLL canonical network.

link. Next we show the condition for three 3-hop nodes to concurrently initiate a link. When three 3-hop nodes concurrently initiate a two-hop link (the worst case), θ at most equals $\frac{2\pi}{3}$, and $\sin\left(\frac{\theta}{2}\right) \leq \frac{\sqrt{3}}{2}$. Then, according to (6), we have

$$\frac{1+\beta}{2\beta} < \frac{\sqrt{3}}{2} \Rightarrow \beta > 1.3660. \tag{9}$$

Similarly, we separate the three two-hop links as far away as possible and use the best settings $a_2 = a_1$ and $a_3 = 1.3660a_2$ to minimize the mutual interference, as shown in Fig. 8. When a 2-hop node receives a packet from the 3-hop node in the same chain under the interference from the 1-hop nodes and 3-hop nodes of all the other chains, its SINR reaches the minimum value. Thus, successful transmissions can be ensured if

$$\frac{P_t(1.3660a_2 - a_2)^{-\alpha}}{2P_t d_{48}^{-\alpha} + 2P_t d_{68}^{-\alpha}} \ge \gamma_0,\tag{10}$$

where d_{48} and d_{68} are equal to $2a_2\sin(\frac{\pi}{3})$ and $\sqrt{a_2^2+(1.3660a_2)^2-2a_2(1.3660a_2)\cos(\frac{2\pi}{3})}$, respectively. Inequality (10) can be simplified to $\gamma_0 \leq \frac{0.366^{-\alpha}}{2\left(3^{-\frac{\alpha}{2}}+4.232^{-\frac{\alpha}{2}}\right)}$. Since a_3 is strictly larger than $1.3660a_2$ and a_2 is strictly larger than a_1 , it follows that at most three 3-hop nodes can concurrently initiate a link when $\gamma_0 < \frac{0.366^{-\alpha}}{2\left(3^{-\frac{\alpha}{2}}+4.232^{-\frac{\alpha}{2}}\right)}$.

Next we show that in a three-hop VLL canonical network, by setting appropriate a_1 , a_2 , and a_3 , there always exists the case that two 3-hop nodes can concurrently initiate a link when $\gamma_0 \ge \frac{0.366^{-\alpha}}{2\left(3^{-\frac{\alpha}{2}}+4.232^{-\frac{\alpha}{2}}\right)}$. To minimize the mutual interference, the

two two-hop links are placed in a line, as shown in Fig. 9, where $\theta = \pi$. Then, according to inequality (6), we have

$$\frac{1+\beta}{2\beta} < 1 \Rightarrow \beta > 1. \tag{11}$$

This means that when two 3-hop nodes concurrently initiate a link, a_2 can be infinitely close to a_3 . Furthermore, when a_2 is infinitely close to a_3 , and a_1 is infinitely close to a_2 , the SINR at all the nodes can be infinitely large. This means that for any value of γ_0 , the successful transmissions of the two two-hop links can be ensured by setting appropriate a_1 , a_2 , and a_3 .

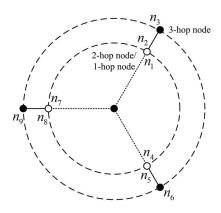


Fig. 8. Three two-hop links with the maximum separation under the best setting of a_1 , a_2 , and a_3 in the three-hop VLL canonical network.

Fig. 9. Two two-hop links with the maximum separation in the three-hop VLL canonical network.

Theorem 2: The throughput capacity of the three-hop VLL canonical network is

$$\begin{cases} C_3 = \max\left(\frac{1}{1 + \frac{T_2}{6T_1}}S_1, \frac{1}{1 + \frac{4T_1}{3T_2}}S_2\right), \\ if \ \gamma_0 < \frac{0.366^{-\alpha}}{2\left(3^{-\frac{\alpha}{2}} + 4.232^{-\frac{\alpha}{2}}\right)}, \\ C_3 = \max\left(\frac{1}{1 + \frac{T_2}{4T_1}}S_1, \frac{1}{1 + \frac{3T_1}{2T_2}}S_2\right), \\ if \ \gamma_0 \ge \frac{0.366^{-\alpha}}{2\left(3^{-\frac{\alpha}{2}} + 4.232^{-\frac{\alpha}{2}}\right)}. \end{cases}$$

The proof of Theorem 2 is similar to the proof of Theorem 1, and is omitted here.

IV. THROUGHPUT CAPACITY OF FOUR-HOP CANONICAL NETWORKS

In the four-hop canonical networks, each 4-hop node exchanges packets with the center node by two two-hop links to make full use of the PNC scheme. The throughput capacity of any four-hop canonical network is upper-bounded by S_2 , which is the throughput of a two-hop PNC link. We first show the CSR setting and the condition of the SINR threshold under which this upper-bound can be achieved. For the remaining cases, we show the best possible spatial reuse and obtain the throughput capacity accordingly.

In order to ensure the upper-bound, S_2 , can be achieved, we need to make sure that a 2-hop node and a 4-hop node in different chains can concurrently and successfully initiate a two-hop link. Thus, the CSR should be smaller than $(a_2 + a_4)$. On the other hand, the CSR should be as large as possible to minimize the mutual interference. Therefore, the CSR should be set to $(a_2 + a_4) - \varepsilon$, where ε is an arbitrary small positive number. The following lemma shows the possible concurrent transmissions when the CSR is set to $(a_2 + a_4) - \varepsilon$.

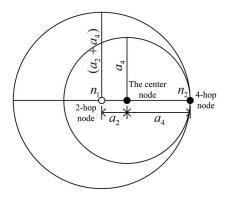


Fig. 10. The maximum distance between a 2-hop node and a 4-hop node in the four-hop canonical network.

Lemma 4: In the four-hop (ELL/VLL) canonical network, when the CSR is set to $(a_2 + a_4) - \varepsilon$, if a 2-hop node has initiated a link, none of the other nodes can initiate a link except the 4-hop node that is $(a_2 + a_4)$ away from the 2-hop node.

Proof: Fig. 10 shows two circles. One is centered at the center end node with radius a_4 ; the other one is centered at the 2-hop node, n_1 , with radius $(a_2 + a_4)$. The two circles are tangent. That is, only the 4-hop node that locates at n_2 can concurrently initiate a link with n_1 .

A. Throughput Capacity of four-hop ELL canonical networks

In the four-hop ELL canonical networks, according to the previous discussion, we know that if the CSR is set to $6l - \varepsilon$, a 2-hop node and a 4-hop node in different chains can concurrently initiate a two-hop link with minimum mutual interference. In Lemma 5 and Lemma 6, we show all the possible concurrent transmissions under this CSR setting and the corresponding SINR requirements to ensure successful transmissions, respectively. In Lemma 7, we show the best possible spatial reuse and the corresponding SINR requirement when CSR is larger than $6l - \varepsilon$. The throughput capacity of the four-hop ELL canonical networks can be obtained following Lemmas 5, 6, and 7.

Lemma 5: In the four-hop ELL canonical network, when the CSR is set to $6l - \varepsilon$, at most three 4-hop nodes can concurrently initiate a link.

Proof: As shown in Fig. 11a, if four 4-hop nodes concurrently initiate a link, the minimum distance between two adjacent 4-hop nodes at most equals $4\sqrt{2}l$, which is smaller than $6l - \varepsilon$. Fig. 11b shows a network with three 4-hop nodes. The separation between any two adjacent 4-hop nodes is $4\sqrt{3}l$, which satisfies the CSR requirement. Thus, these three 4-hop nodes can concurrently initiate a link.

Lemma 6: In the four-hop ELL canonical network, if the CSR is set to $6l - \varepsilon$, all the possible current transmissions are ensured to be successful when $\gamma_0 \leq \min(\frac{0.5}{9^{-\frac{\alpha}{2}} + 22^{-\frac{\alpha}{2}}}, \frac{1}{2^{-\alpha} + 4^{-\alpha}})$.

Proof: According to Lemma 4 and Lemma 5, when CSR is set to $6l - \varepsilon$, there are only two possible cases of concurrent transmissions in the four-hop ELL canonical network. One case

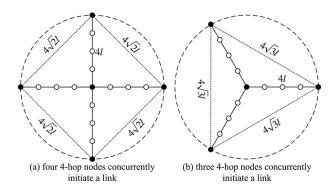


Fig. 11. The maximum separation between the end nodes in the four-hop ELL canonical networks

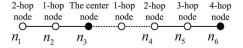


Fig. 12. A 2-hop node and a 4-hop node with maximum separation concurrently initiate a two-hop link in the four-hop ELL canonical network.

is that a 2-hop node and a 4-hop node that are 6l away concurrently initiate a two-hop link; the other one is that three 4-hop nodes concurrently initiate a two-hop link.

In the first case, the minimum SINR happens when n_4 receives a packet from n_5 under the interference from n_1 and n_3 , as shown in Fig. 12. Therefore, successful transmissions can be ensured if

$$\frac{P_t l^{-\alpha}}{P_t(2l)^{-\alpha} + P_t(4l)^{-\alpha}} \ge \gamma_0, \tag{12}$$

which can be simplified to $\gamma_0 \le \frac{1}{2^{-\alpha} + 4^{-\alpha}}$. In the second case, since $CSR = 6l - \varepsilon$, the distance between two simultaneous active 4-hop end nodes at least equals 6l, as shown in Fig. 13. The minimum SINR happens when n_4 receives a packet from n_5 under the interference from n_1 , n_3 , n_7 , and n_9 . Therefore, successful transmissions of the three two-hop links can be ensured if

$$\frac{P_t l^{-\alpha}}{2(P_t d_{74}^{-\alpha} + P_t d_{94}^{-\alpha})} \ge \gamma_0, \tag{13}$$

where $d_{74} = 3l$ and $\sqrt{(4l)^2 + (2l)^2 - 16l^2 \cos\left[2\arcsin(\frac{3}{4})\right]}$. By ing inequality (13), we have $\gamma_0 \leq \frac{0.5}{9^{-\frac{\alpha}{2}} + 22^{-\frac{\alpha}{2}}}$. Thus, if $\gamma_0 \leq \min(\frac{0.5}{9^{-\frac{\alpha}{2}} + 22^{-\frac{\alpha}{2}}}, \frac{1}{2^{-\alpha} + 4^{-\alpha}})$, successful transmissions can

Lemma 7: In the four-hop ELL canonical network, if the CSR is no less than 6l, at most K 4-hop nodes can concurrently initiate a two-hop PNC link, where

$$\begin{cases} K = 3, & \text{if } \gamma_0 \le \frac{0.5}{12^{-\frac{\alpha}{2}} + 28^{-\frac{\alpha}{2}}}, \\ K = 2, & \text{if } \frac{0.5}{12^{-\frac{\alpha}{2}} + 28^{-\frac{\alpha}{2}}} < \gamma_0 \le \frac{1}{4^{-\alpha} + 6^{-\alpha}}, \\ K = 1, & \text{if } \gamma_0 > \frac{1}{4^{-\alpha} + 6^{-\alpha}}. \end{cases}$$

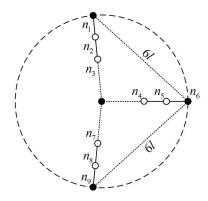


Fig. 13. The topology of three concurrently transmitting links in which the minimum SINR occurs when $CSR = 6l - \varepsilon$ in the four-hop ELL canonical network.

Proof: From Lemma 5, we can also find that it is impossible for four or more 4-hop nodes to concurrently initiate a link if the CSR is no less than 6l.

When K = 3, the CSR should be increased to separate the three concurrently transmitting two-hop links as far away as possible in order to minimize the mutual interference, as shown in Fig. 14. The minimum SINR happens when n_3 receives a packet from n_2 under the interference from n_4 , n_6 , n_7 , and n_9 . Therefore, successful transmissions can be ensured if

$$\frac{P_t l^{-\alpha}}{2P_t d_{03}^{-\alpha} + 2P_t d_{73}^{-\alpha}} \ge \gamma_0,\tag{14}$$

where $d_{93} = 4l \sin(\frac{\pi}{3})$ and $d_{73} = \sqrt{(2l)^2 + (4l)^2 - 16l^2 \cos(\frac{2\pi}{3})}$. By simplifying inequality (14), we have $\gamma_0 \le \frac{0.5}{12^{-\frac{\alpha}{2}} + 28^{-\frac{\alpha}{2}}}$.

When K = 2, similarly, the CSR should be set to $8l - \varepsilon$ to separate the two concurrently transmitting two-hop links as far away as possible (in a line) in order to minimize the mutual interference, as shown in Fig. 15. The minimum SINR happens when n_3 receives a packet from n_2 under the interference from n_4 and n_6 . Therefore, successful transmissions can be ensured if

$$\frac{P_t l^{-\alpha}}{P_t(4l)^{-\alpha} + P_t(6l)^{-\alpha}} \ge \gamma_0,\tag{15}$$

which is simplified to $\gamma_0 \leq \frac{1}{4^{-\alpha}+6^{-\alpha}}$. Moreover, we can find that only one 4-hop node can initiate a two-hop link when $\gamma_0 > \frac{1}{4^{-\alpha}+6^{-\alpha}}$, if the CSR is no less than 6l in the four-hop ELL canonical network.

Theorem 3: The throughput capacity of the four-hop ELL canonical network is

$$\begin{cases} C_4 = S_2, & \text{if } \gamma_0 \leq \min\left(\frac{0.5}{9^{-\frac{\alpha}{2}} + 22^{-\frac{\alpha}{2}}}, \frac{1}{2^{-\alpha} + 4^{-\alpha}}\right), \\ C_4 = \frac{3}{4}S_2, & \text{if } \min\left(\frac{0.5}{9^{-\frac{\alpha}{2}} + 22^{-\frac{\alpha}{2}}}, \frac{1}{2^{-\alpha} + 4^{-\alpha}}\right) < \gamma_0 \leq \frac{0.5}{12^{-\frac{\alpha}{2}} + 28^{-\frac{\alpha}{2}}}, \\ C_4 = \frac{2}{3}S_2, & \text{if } \frac{0.5}{12^{-\frac{\alpha}{2}} + 28^{-\frac{\alpha}{2}}} < \gamma_0 \leq \frac{1}{4^{-\alpha} + 6^{-\alpha}}, \\ C_4 = \frac{1}{2}S_2, & \text{if } \gamma_0 > \frac{1}{4^{-\alpha} + 6^{-\alpha}}. \end{cases}$$

Proof: We first show that the throughput capacity of a four-hop ELL canonical network can reach S_2 when $\gamma_0 \le$

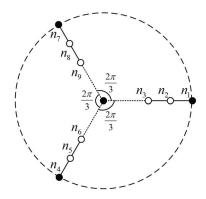


Fig. 14. Three concurrently transmitting two-hop links with the maximum separation in the four-hop ELL canonical network.



Fig. 15. Two concurrently transmitting two-hop links with the maximum separation in the four-hop ELL canonical network.

 $\min(\frac{0.5}{9^{-\frac{\alpha}{2}}+22^{-\frac{\alpha}{2}}},\frac{1}{2^{-\alpha}+4^{-\alpha}})$. Consider a four-hop ELL canonical network with U pairs of chains. Each pair of chains consists of two chains that are in a line but with different directions. According to Lemma 6, when $\gamma_0 \leq \min(\frac{0.5}{9^{-\frac{\alpha}{2}}+22^{-\frac{\alpha}{2}}},\frac{1}{2^{-\alpha}+4^{-\alpha}})$, the CSR can be set to $6l-\varepsilon$. Then, for each pair of chains, a 2-hop node of a chain and the 4-hop node of the other chain in the same line can concurrently initiate a two-hop link. Thus, the minimum time for each 4-hop end node in a pair of chains to exchange two packets with the center end node equals $2T_2$. Then, the minimum total time for each 4-hop end node in the network to exchange two packets with the center end node equals $2UT_2$. What's more, during the period of time $2UT_2$, the center end node is transmitting or receiving packets all the time. Therefore, the throughput capacity can reach S_2 in this case.

When $\min(\frac{0.5}{9^{-\frac{\alpha}{2}}+22^{-\frac{\alpha}{2}}},\frac{1}{2^{-\alpha}+4^{-\alpha}}) < \gamma_0 \le \frac{0.5}{12^{-\frac{\alpha}{2}}+28^{-\frac{\alpha}{2}}}$, from Lemma 6, we know that the CSR should be no less than 6l. Then, we have: 1) all the 2-hop nodes and the center node can not initiate a link concurrently with a 4-hop node; 2) all the 2-hop nodes and the center node can not initiate a link concurrently. Furthermore, according to Lemma 7, at most three 4-hop end nodes can concurrently initiate a two-hop link when $\gamma_0 \le \frac{0.5}{12^{-\frac{\alpha}{2}}+28^{-\frac{\alpha}{2}}}$. Thus, in a four-hop ELL canonical network with W chains, the minimum total time for each 4-hop end node to exchange two packets with the center end node equals

$$T_{total}^{\min} = \frac{W}{3}T_2 + WT_2. \tag{16}$$

That is, the maximum time proportion for the center end node to transmit or receive packets is $\frac{WT_2}{T_{total}^{min}} = \frac{3}{4}$. Therefore, the network throughput capacity in this case equals $\frac{3}{4}S_2$.

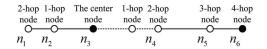


Fig. 16. A 2-hop node and a 4-hop node with maximum separation concurrently transmit in the four-hop VLL canonical network.

TABLE I THE NUMBER OF SIMULTANEOUS ACTIVE 4-HOP NODES WHEN $CSR = (a_2 + a_4) - \varepsilon$

Conditions	The number of simultaneous active 4-hop nodes
$CSR \geq 2a_4 \sin(\frac{\pi}{3})$	2
$2a_4\sin(\frac{\pi}{4}) \le CSR < 2a_4\sin(\frac{\pi}{3})$	3
$2a_4\sin(\frac{\pi}{5}) \le CSR < 2a_4\sin(\frac{\pi}{4})$	4
$CSR < 2a_4\sin(\frac{\pi}{5})$	5

The throughput capacity when $\frac{0.5}{12^{-\frac{\alpha}{2}}+28^{-\frac{\alpha}{2}}} < \gamma_0 \le \frac{1}{4^{-\alpha}+6^{-\alpha}}$ and $\gamma_0 > \frac{1}{4^{-\alpha}+6^{-\alpha}}$ can be calculated in a similar way, which equals $\frac{2}{3}S_2$, and $\frac{1}{2}S_2$, respectively.

B. Throughput Capacity of four-hop VLL canonical networks

In the four-hop VLL canonical networks, the distance between adjacent nodes in a chain can be different. It is difficult to derive the best settings of a_1 , a_2 , a_3 , and a_4 that maximize the network throughput. In this case, we resort to numerical method to find the throughput capacity. In particular, we first compute the network throughput capacity given a_1 , a_2 , a_3 , a_4 , α , and γ_0 . Then we find out the best settings of a_1 , a_2 , a_3 , and a_4 that maximize the network throughput capacity for different choices of γ_0 and α by numerical search.

The calculation of the network throughput capacity with given network parameters is as follows. We first determine whether a 2-hop node and a 4-hop node in different chains can concurrently initiate a two-hop link without interfering each other. If yes, the throughput capacity can reach the upper-bound S_2 ; otherwise, we need to find out the best possible spatial reuse, i.e., the maximum number of 4-hop nodes that can concurrently initiate a two-hop link, K, while ensuring successful transmissions. Then, according to the proof of Theorem 3, the throughput capacity is given by $\frac{K}{K+1}S_2$.

1) Determine whether a 2-hop node and a 4-hop node in different chains can concurrently initiate a two-hop link without interfering each other: As discussed, the CSR should be set to $(a_2+a_4)-\varepsilon$ in order to allow the concurrent transmissions of a 2-hop node and a 4-hop node in different chains with minimum mutual interference. Furthermore, we need to ensure that all the possible transmissions under this CSR are successful. According to Lemma 4, when $CSR = (a_2+a_4)-\varepsilon$, there are two cases of possible concurrent transmissions: one case is that a 2-hop node and a 4-hop node with separation of (a_2+a_4) concurrently initiate a two-hop link; the other one is several 4-hop nodes that are outside the CSR of each other concurrently initiate a two-hop link. Given the values of a_1 , a_2 , a_3 , a_4 , α , and γ_0 , we need to check whether the SINR conditions in both cases can be satisfied.

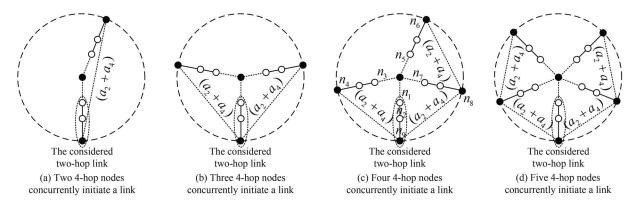


Fig. 17. The network topologies that cause maximum interference to the considered two-hop link when $CSR = (a_2 + a_4) - \varepsilon$ in the four-hop VLL canonical networks.

In the first case, successful transmissions can be ensured if the following SINR conditions are satisfied, as shown in Fig. 16.

$$\begin{cases}
\frac{P_{t}d_{23}^{-\alpha}}{P_{t}d_{43}^{-\alpha} + P_{t}d_{63}^{-\alpha}} \geq \gamma_{0}, \\
\frac{P_{t}d_{12}^{-\alpha}}{P_{t}d_{42}^{-\alpha} + P_{t}d_{62}^{-\alpha}} \geq \gamma_{0}, \\
\frac{P_{t}d_{54}^{-\alpha}}{P_{t}d_{34}^{-\alpha} + P_{t}d_{14}^{-\alpha}} \geq \gamma_{0}, \\
\frac{P_{t}d_{65}^{-\alpha}}{P_{t}d_{35}^{-\alpha} + P_{t}d_{15}^{-\alpha}} \geq \gamma_{0},
\end{cases}$$
(17)

In the second case, when $CSR = (a_2 + a_4) - \varepsilon$, the distance between two simultaneously active 4-hop nodes should be no less than $(a_2 + a_4)$. Since $0 < a_2 < a_4$, we then have $a_4 \le CSR < 2a_4$. Thus, the maximum number of 4-hop nodes that can concurrently initiate a two-hop link is between 2 and 5. Table I shows all the possible numbers of the simultaneous active 4-hop nodes, and the condition on CSR. The corresponding network topologies in which the minimum SINR occurs are shown in Fig. 17. Similarly, we need to check if the SINR conditions in each of the topologies in Fig. 17 can be satisfied. Here we give the SINR conditions in Fig. 17c where four 4hop nodes concurrently initiate a two-hop link as an example. The SINR conditions in the other network topologies in Fig. 17 can be found with the same method. In Fig. 17c, the minimum SINR happens when n_1 receives a packet from n_2 or n_2 receives a packet from n_9 under the interference from the 2-hop nodes and the 4-hop nodes of the other three links. Thus, successful transmissions can be ensured if the following inequalities are satisfied.

$$\begin{cases}
\frac{P_{t}d_{21}^{-\alpha}}{2(P_{t}d_{31}^{-\alpha} + P_{t}d_{41}^{-\alpha}) + P_{t}d_{51}^{-\alpha} + P_{t}d_{61}^{-\alpha}} \geq \gamma_{0}, \\
\frac{P_{t}d_{92}^{-\alpha}}{2(P_{t}d_{32}^{-\alpha} + P_{t}d_{42}^{-\alpha}) + P_{t}d_{52}^{-\alpha} + P_{t}d_{62}^{-\alpha}} \geq \gamma_{0}.
\end{cases} (18)$$

2) Determine the maximum number of 4-hop nodes that can concurrently initiate a two-hop link, K, when CSR is larger than $(a_2 + a_4) - \varepsilon$: We first show that K is not larger than 5. When $K \ge 6$, the minimum distance between two adjacent concurrently active 4-hop nodes at most equals a_4 , which is smaller

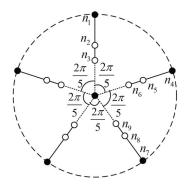


Fig. 18. Five concurrently transmitting two-hop links with maximum separation in the four-hop VLL canonical network.

than the CSR. Thus, we know $1 \le K \le 5$. Starting from 5, K is reduced by 1 in each step until the CSR condition and the SINR requirements of all the possible concurrent transmissions can be satisfied. Here we show the CSR condition and the SINR requirements when K = 5 as an example. The CSR condition and the SINR requirements for the other values of K can be found with the same method.

In order to allow five 4-hop nodes to concurrently initiate a link, we know $CSR < 2a_4 \sin(\frac{\pi}{5})$. Since $CSR \ge (a_2 + a_4)$, we then have

$$(a_2 + a_4) < 2a_4 \sin\left(\frac{\pi}{5}\right).$$
 (19)

To minimize mutual interference, the five chains should be separated as far away as possible, as shown in Fig. 18. Since the five chains are symmetric to each other, successful transmissions can be ensured if the following SINR requirements are satisfied.

$$\begin{cases}
\frac{P_{t}d_{23}^{-\alpha}}{2(P_{t}d_{63}^{-\alpha} + P_{t}d_{43}^{-\alpha} + P_{t}d_{93}^{-\alpha} + P_{t}d_{73}^{-\alpha})} \geq \gamma_{0}, \\
\frac{P_{t}d_{12}^{-\alpha}}{2(P_{t}d_{62}^{-\alpha} + P_{t}d_{42}^{-\alpha} + P_{t}d_{92}^{-\alpha} + P_{t}d_{72}^{-\alpha})} \geq \gamma_{0}.
\end{cases} (20)$$

Given the values of a_1 , a_2 , a_3 , a_4 , α , and γ_0 , we can then check if both (19) and (20) can be satisfied or not.

Finally, we vary the values of a_1 , a_2 , a_3 , and a_4 to find the maximum throughput capacity through numerical search. The

 $\alpha = 2$ $\alpha = 3$ $\alpha = 4$ $\alpha = 5$ $\alpha = 6$ C_4 C_4 γ_0 C_4 γ_0 γ_0 γ_0 γ_0 S_2 S_2 ≤ 12 S_2 ≤ 31 S_2 $\gamma_0 \le 192$
 $\gamma_0 \le 1000$ γ_0 γ_0 γ_0 γ_0 $\frac{13 \le \gamma_0}{13 \le 298}$ $\frac{3}{4}S_2$ $\frac{3}{4}S_2$ $\frac{3}{4}S_2$ $\frac{3}{4}S_{2}$ $\gamma_0 \le 28$ $32 \le \gamma_0 \le 1000$ $78 \le \gamma_0 \le 1000$ $193 \le$ $\gamma_0 \le 1000$ $299 \le \gamma_0 \le 1000$ S_2

TABLE II
THE MAXIMUM THROUGHPUT CAPACITIES OF FOUR-HOP VLL CANONICAL NETWORKS

maximum throughput capacities with typical values of γ_0 and α are shown in Table II.

In four-hop canonical network, we derive the throughput capacity by focusing on the case where each 4-hop node exchanges packets with the center node by two two-hop links. We know that without the above restriction, there are more possible concurrent transmission cases. Thus, there needs a no smaller CSR to guarantee all the possible concurrent transmissions are successful. Furthermore, using two two-hop PNC links to exchange packets between the 4-hop nodes and the center node is the optimal transmission way. Therefore, the derived throughput capacity can still be used as the throughput upper-bound of the case without restricting that each 4-hop node exchanges packets with the center node by two two-hop links. In this paper, we only focus on the throughput analysis at the MAC layer and assume that the physical link rate is a given parameter which only depends on its target SINR at the receiver. Therefore, if S_1 and S_2 are given, the throughput analysis results are independent of the link length.

V. SIMULATIONS AND DISCUSSIONS

We use Matlab to simulate the 802.11a many-to/from-one network with the parameters given in Table III. Since the topology is fixed, the transport and network layer protocols are neglected, and we directly generate the packets at data link layer with a given probability.

A. Capacity validation

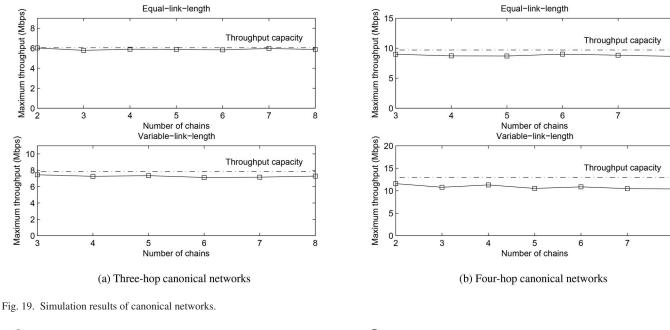
We first validate the theoretical throughput capacities of the many-to/from-one networks. The SINR threshold γ_0 is set to 20, and the corresponding physical link rate equals 18 Mbps [26]. The path-loss exponent α is set to 4. In the ELL networks, the link-length is set to 25 m. In the three-hop VLL network, a_1 , a_2 , and a_3 are set to 26 m, 36 m, and 56 m, respectively. In the four-hop VLL network, a_1 , a_2 , a_3 , and a_4 are set to 15 m, 36 m, 48 m, and 66 m, respectively. The chains are placed in the optimal positions and the CSR is set according to the following two principles: 1) it can prevent the hidden-node (HN) problem; 2) it is set as small as possible to maximize the network throughput. Furthermore, to maximize the network throughput, we select an optimal transmission probability for each node by carefully tuning the minimum contention window size and search for an optimal packet generation probability to prevent the end nodes from injecting too much packets into the network. In particular, since the center node and the 1-hop and 2-hop nodes can not concurrently transmit, their contention window size is set to the same value of 2^{I_0} , where I_0 is a positive integer. For the m-hop nodes $(m \ge 3)$, the contention window size should be no smaller since concurrent transmission is possible. Thus, the

TABLE III SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
Slot time	$9 \mu s$	CTS	112 bits
SIFS	$16 \ \mu s$	ACK	112 bits
DIFS	$34~\mu s$	Packet size	8496 bits
Physical preamble	$20~\mu s$	Payload size	8184 bits
RTS	160 bits	Propagation delay	$1 \ \mu s$
Transmit power	200 mW	Channel bandwidth	20 MHz
Noise power level	dBm/Hz		

contention window size of the m-hop nodes ($m \geq 3$) is set to $2^{I_0 + I_m}$, where I_m is a non-negative integer. The packet generation probability of all the end nodes is set to g. We then search for the best values of I_0 , I_m , and g that maximize the network throughput through simulations. The network throughput performances of the canonical networks are shown in Fig. 19. We can find that the theoretical capacities are reasonably tight for the three-hop and four-hop ELL/VLL canonical networks, even when the countdown collisions and noise power are considered.

Next we generate 100 random many-to/from-one networks. In particular, the distance from the i-hop nodes to the center node is uniformly distributed between (55i - 50) meters and 55i meters. The angle of the nodes in the j^{th} chain is uniformly distributed between $\frac{2\pi}{B}(j-1)$ and $\frac{2\pi}{B}(j-0.1)$, where B is the total number of chains. The throughput results of 3hop and 4-hop random networks are shown in Fig. 20a and Fig. 20b respectively. We further investigate the throughput performance of random networks with random chain numbers and hop numbers, as shown in Fig. 20c. In particular, the chain number and the hop number are chosen with equal probability from the sets of integers $\{2, \ldots, 8\}$ and $\{3, \ldots, 8\}$, respectively. From Fig. 20, we find that the throughput capacity derived from the canonical networks may be used to approximate the capacity upper-bound of general many-to/from-one networks. Another observation from Fig. 20 is that the throughput variance in three-hop random network is smaller than the four-hop random network. The reasons are as follows. First, from the analysis in Section III and IV, the range of the theoretical throughput capacity of three-hop network is much smaller than the four-hop network. For example, under the parameters given in Table III, the theoretical throughput capacity of threehop network ranges from 6.0512 Mbps to 7.8485 Mbps; while the theoretical throughput capacity of four-hop network ranges from 6.4701 Mbps to 12.9402 Mbps. Second, in the simulation, a successful two-hop PNC transmission may not always contain two packets since the destination node may not always have packets for the source node. That is, there is an additional uncertainty in the two-hop PNC transmission in the simulation. Since the two-hop PNC transmission is used more frequently



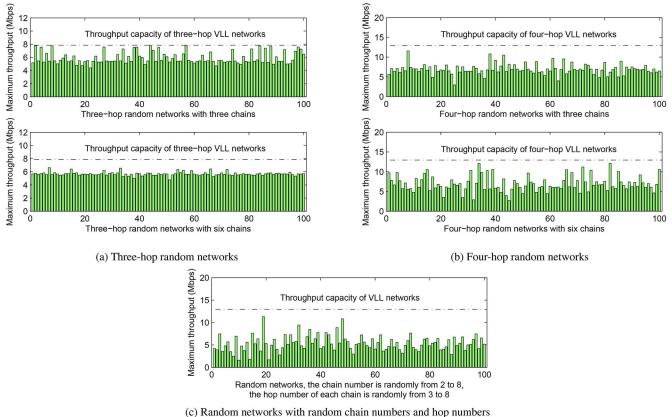


Fig. 20. Simulation results of random networks.

in the four-hop network than in the three-hop network, such uncertainty may cause a higher variance in the throughput of four-hop networks.

B. Maximum throughput of canonical networks under Rayleigh fading channel

Fig. 21 shows the maximum throughput of the four canonical networks in Section V-A under the Rayleigh fading channel when $y_0 = 20$. The expected value of the channel gain is obtained from the log-distance path-loss model with $\alpha = 4$. We search for an optimal CSR that maximizes the network throughput. Under the Rayleigh fading channel, the HN problem cannot be completely eliminated. When HN problem occurs, if the transmitted packet is failure, it needs to be retransmitted. From Fig. 21, we can see that the throughput performance of all the canonical networks degrades except the three-hop ELL one. For example, under Rayleigh fading

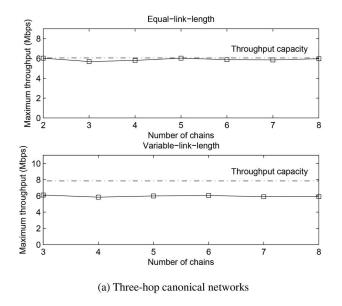
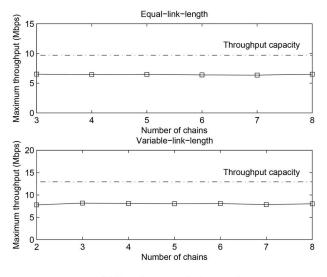


Fig. 21. Simulation results of canonical networks under Rayleigh fading channel.

channel, the throughput of the three-hop VLL, four-hop ELL, four-hop VLL networks is about 78%, 68%, and 63% of the corresponding throughput capacity, respectively. The reason is that we may need to set a bigger CSR to compensate the effect of HN problem caused by the randomness of the Rayleigh fading channel, which decreases the spatial reuse. In the three-hop ELL scenario, the capacity is reached when setting a sufficiently large CSR to allow only one node to initiate a link at a time. Thus, the HN problem can still be eliminated by setting an appropriate CSR even under the Rayleigh fading channel. Therefore, the maximum throughput of the three-hop ELL network is close to its theoretical throughput capacity.

C. Throughput capacity vs. the SINR threshold γ_0

According to 802.11 protocol, the physical link rate increases as γ_0 increases. However, as shown in Sections III and IV, the spatial reuse decreases as γ_0 increases. Therefore, it is interesting to see the relation between the network throughput capacity and γ_0 . Fig. 22 shows the network throughput capacities of the three-hop and four-hop ELL/VLL canonical networks as a function of γ_0 based on the eight physical link rates defined in the 802.11a protocol [26] when $\alpha = 4$. From Fig. 22, we can see that the throughput capacity does not always increase as γ_0 increases. Another observation from Fig. 22 is that the throughput capacities of four-hop networks are always bigger than the one of three-hop networks. The main reason is that in four-hop networks, the 4-hop end nodes only need to use two-hop PNC links to exchange packets with the center end node; while in three-hop networks, the 3-hop end nodes should use both onehop links and two-hop PNC links to exchange packets with the center end node. That is, two-hop PNC links, which has have higher transmission efficiency, are used more frequently in the four-hop networks.





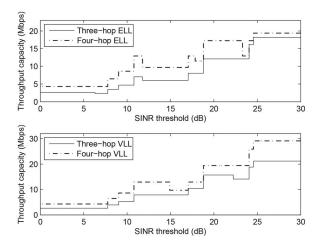


Fig. 22. Throughput capacity vs. the SINR threshold γ_0 .

D. The maximum throughput vs. the number of hops

Fig. 23 shows the maximum throughput of ELL canonical networks and random networks with 6 chains when $\alpha = 4$ and $\gamma_0 = 20$. The hop number m in each chain ranges from 3 to 8. When m = 5 or 7, we use two kinds of transmission methods. In the first transmission method, the 5-hop (or 7-hop) end nodes first exchange packets with 4-hop (or 6-hop) nodes by one-hop links, and then the 4-hop (or 6-hop) nodes exchange packets with the center end node by several two-hop links. In the second transmission method, the 5-hop (or 7-hop) end nodes first exchange packets with 1-hop nodes by several two-hop links, and the 1-hop nodes exchange packets with the center end node by one-hop links. From Fig. 23, we can find that the four-hop many-to/from-one networks have the best throughput performance. Furthermore, we can observe that when m = 5 or 7, the throughput in the first transmission method is larger than in the second one. This implies that for a many-to/from-one network

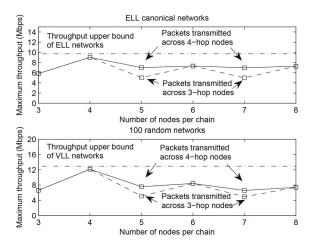


Fig. 23. The maximum throughput vs. the number of hops.

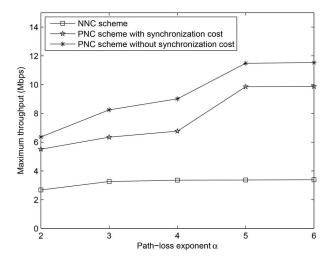


Fig. 24. Throughput gain of PNC scheme.

with five or more hops, it is preferable to transmit the packets across the 4-hop nodes than the 3-hop nodes.

E. Throughput gain of PNC scheme

We compare the maximum throughput of non-network-coding (NNC) scheme and PNC scheme in four-hop ELL canonical networks with and without considering the PNC synchronization cost when $\gamma_0=20$. We use the parameter given in [25] such that the synchronization time of PNC is about 15.7% of the data transmission time. From Fig. 24, we can see that the throughput gain of the four-hop ELL canonical network brought by PNC scheme with and without considering the synchronization cost respectively ranges from 205.8% to 291.7% and from 237.8% to 340.6%, depending on different values of α .

VI. CONCLUSIONS

In this paper, we investigated the throughput capacity of the many-to/from-one network with PNC scheme coordinated with the IEEE 802.11 DCF. In particular, we derived the throughput capacities of the ELL and VLL canonical networks.

Although PNC scheme requires a larger CSR to prevent collisions (i.e., the spatial reuse is reduced), the overall network throughput can be significantly improved if the CSR and the target SINR/transmission rate are properly selected. Simulation results show that the throughput gain of PNC scheme with and without considering the synchronization cost in the four-hop ELL canonical networks can reach up to 291.7% and 340.6%, respectively. Furthermore, the theoretical throughput capacities derived under the canonical networks are reasonably tight, and may be used to approximate the maximum throughput of general many-to/from-one networks. Another key result is that the throughput capacities of four-hop networks are always bigger than the one of three-hop networks. This indicates that in a many-to/from-one network with five or more hops, it is preferable to transmit the packets first to 4-hop nodes other than to 3-hop nodes in order to fully exploit the transmission efficiency of PNC scheme.

In this paper, we mainly focus on the case where all the links have the same physical link rate. One interesting direction in the future work is to further explore the impact of the node location on the network throughput and discuss how to place the relay nodes to maximize the network throughput. In this case, different links may have different physical link rate, which makes the network throughput analysis much more difficult and requires a joint optimization approach between the physical layer, MAC layer, and the network layer. Another interesting direction is to consider the adaptive CSR setting which allows more desired concurrent transmissions and avoids the concurrent transmissions that lead to collision. Thus the network throughput can be further improved. The theoretical network throughput analysis with adaptive CSR is an interesting yet challenging topic for future study.

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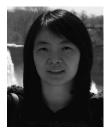
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Shijun Lin (M'12) received the B.S. degree (Hons.) in electrical engineering from Xiamen University, Xiamen, China, in 2005, and the Ph.D. degree in electrical engineering from Tsinghua University, Beijing, China, in 2010. He is currently an Assistant Professor with the Department of Communication Engineering, Xiamen University. His research interests include wireless communications and networking, with focus on physical-layer network coding, wireless greening, resource allocation, and distributed protocol design. He has served as a TPC member for ChinaCom and

Globecom Workshop SDRANCAN.



Liqun Fu (S'08–M'11) received the Ph.D. degree in information engineering from the Chinese University, Hong Kong, China. She is currently an Assistant Professor with the School of Information Science and Technology, ShanghaiTech University, Shanghai, China. During 2009, she was a Visiting Scholar at the Department of Electrical Engineering, Princeton University, Princeton, NJ, USA. Before joining ShanghaiTech University, she was a Postdoctoral Research Fellow with the Institute of Network Coding of the Chinese University, Hong Kong, China,

and the ACCESS Linnaeus Centre, KTH Royal Institute of Technology. Her research interests include wireless communication and networking, with current focus on wireless greening, resource allocation, distributed protocol design, and signal processing. She has served as a TPC Co-Chair of INFOCOM workshop on GCCCN 2014 and the TPC member for ICC, Globecom, PIMRC, WCNC, and VTC. She was the recipient of CUHK Global Scholarship for Research Excellence in 2009.