Sum-rate optimization for Device-to-Device communications over Rayleigh fading channel

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***Abstract*—In this paper, we investigate the sum-rate maximization in a Device-to-Device (D2D) underlay network, where many D2D pairs share the uplink resource with the cellular users (CUs). We show that the system sum-rate maximization problem can be formulated as a mixed integer non-linear programming problem, which is NP-hard in general. We circumvent this difficulty by applying the optimization decomposition: 1) Given a resource allocation policy, we derive the optimal Signal to Interference plus Noise Ratio (SINR) threshold to maximize the system sum-rate. 2) We then propose a coalition game approach to further optimize the resource allocation policy, and prove that the proposed coalition game approach can converge to the Nash-stable partition in finite time with probability 1. Simulation results show that 1) the performance of the coalition game is close to the exhaustive search; but its run-time is much shorter than the exhaustive search; 2) compared with several other resource allocation policies, the coalition game can achieve an average sum-rate improvement of 13%-173%, and has the best resource sharing fairness.**

***Index Terms*—Device-to-Device, resource allocation, SINR optimization, Rayleigh fading channel.**

I. Introduction

The mobile data traffic grows rapidly in recent years. To fulfill such rapid growing demand for mobile data access, the concept of Device-to-Device (D2D) communication, which allows the devices to communicate directly without traversing through the base station (BS), has been proposed to further enhance the capacity of the cellular network [1], [2].

Most works on D2D communication underlaying cellular network communication underlaying cellular network (referred to as D2D network) have been focused on how to maximize the system throughput [3]–[5]. In [3], [4], Yu and Wang et al. optimized the throughput of a simple D2D network with a single D2D pair while fulfilling prioritized cellular service constraints. In [5], Feng et al. studied a resource allocation problem to maximize the overall network throughput in the case that one CU shares its resource with only one D2D pair. The energy-efficient design of D2D network is investigated in [6], [7]. However, [3]–[7] consider the simple case that the channel gain is constant.

Recently, two papers [8] and [9] considered the case with time-varying channels. In [8], Peng et al. derived the success probability, spatial average rate, and area spectral efficiency performances for the D2D networks under Rician Fading Channels without considering the resource allocation problem. In [9], Shen et al. investigated the power minimization of D2D network under Rayleigh Fading Channels. However, they do not consider the optimization of the Signal to Interference plus Noise Ratio (SINR) threshold.

In this paper, we investigate the maximization of the sum-rate of all the CUs and D2D pairs in a D2D network, where many D2D pairs share the uplink resource of the CUs. We adopt the Rayleigh fading channel model, under which the calculation of the sum-rate is much more complicated. Furthermore, since the selection of the SINR threshold is critical to each transmission, we needs to jointly optimize the SINR threshold and the resource allocation policy. The main contributions of this paper are summarized as follows:

1) We give the closed-form sum-rate expression of the D2D network with many CUs and D2D pairs under Rayleigh fading channel. We show that the maximization problem of the system sum-rate can be formulated as a mixed integer non-linear programming problem.
2) We derive the optimal SINR thresholds that maximize the average rate of each transmission, which can be used to calculate the maximum system sum-rate when the resource allocation policy is given.
3) We propose a coalition game approach to further optimize the resource allocation policy. The coalition game approach is guaranteed to be feasible, albeit not necessarily optimal. Moreover, we prove that the coalition game approach converges to the Nash-stable partition in finite time with probability 1. Simulation results show that the performance of the coalition game approach is close to the optimal solution and can achieve an average sum-rate enhancement of 13%-173% without sacrificing the resource sharing fairness compared with several other resource allocation policies.

The rest of this paper is organized as follows. Section II presents the network model. In section III, we give the formulation of the system sum-rate maximization problem. In section IV, we derive the optimal SINR thresholds to maximize the average rate of each transmission, propose a coalition game approach to further optimize the resource allocation policy, and analyze the convergence and stability. In section V, we present the performance evaluation. Section VI concludes this paper.

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II. System description

We consider a D2D underlaying single cellular network which contains a BS, many CUs and, D2D communication pairs, as shown in Fig. 1. The D2D pairs share the uplink resource of the CUs. We adopt the Rayleigh fading channel model such that the channel gain g follows an exponential distribution [10]. Let $h(\cdot)$ denote the probability density function (PDF). Then, we have $h(g) = \frac{1}{E[g]}e^{-\frac{g}{E[g]}}$, where $E[g]$ is the expected value of g. Furthermore, we adopt the block fading model, in which the channel gain does not change during a packet transmission but independently varies in different packet transmissions [11]. Our goal is to maximize the sum-rate performance of all the CUs and D2D pairs by properly allocating the resource of CUs and choosing the SINR threshold. We further assume that each D2D pair at most shares the resource with one CU.

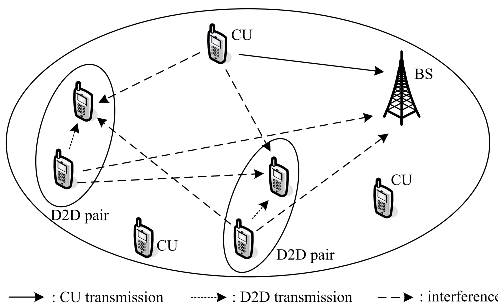

Fig. 1. The cellular relay network.

III. Problem formulation

Let $\mathcal{C} = \left\{c_1, c_2, \cdots, c_{|\mathcal{C}|}\right\}$ and $\mathcal{D} = \left\{d_1, d_2, \cdots, d_{|\mathcal{D}|}\right\}$ denote the set of CUs and D2D pairs in the cellular network. Let d_j^1 and d_j^2 denote the source node and the destination node of the D2D pair d_j, respectively. We define $y_{i,j}^{cd} \in \{0,1\}$ as the indicators which indicate whether CU c_i shares the uplink resource with D2D pair d_j. That is, $y_{i,j}^{cd} = 1$ when CU c_i shares the uplink resource with D2D pair d_j; otherwise, $y_{i,j}^{cd} = 0$. Considering that each D2D pair is allowed to share the resource with only one CU, we have $\sum\limits_{c_i \in \mathcal{C}} y_{i,j}^{cd} = 1, \forall d_j \in \mathcal{D}$. Next we present the calculation of the average rates of a CU and a D2D pair, respectively.

A. The average rate of CU c_i

Let I_u^v and g_u^v denote the interference and the channel gain from node u to node v, respectively. Let $\gamma^{u,v}$, P_{tr}^u, N_0, and W denote the SINR at node v when node u transmits, the transmit power of node u, the power spectral density of additive Gaussian white noise, and the channel bandwidth, respectively. Then, we have

$$\gamma^{c_i,BS} = \frac{P_{tr}^{c_i} g_{c_i}^{BS}}{N_0 W + \sum\limits_{d_j \in \mathcal{D}} y_{i,j}^{cd} I_{d_j^1}^{BS}}.$$

Let $\mathcal{I}_{all}^{u,v}$ denote the set of all the interferences when node u transmits packets to node v. Then, given $\mathcal{I}_{all}^{c_i,BS}$, $\gamma^{c_i,BS}$ follows an exponential distribution with the expected value $\frac{P_{tr}^{c_i} E\left[g_{c_i}^{BS}\right]}{N_0 W + \sum\limits_{d_j \in \mathcal{D}} y_{i,j}^{cd} I_{d_j^1}^{BS}}$, and we have

$$h\left(\gamma^{c_i,BS}\right) = \frac{N_0 W + \sum\limits_{d_j \in \mathcal{D}} y_{i,j}^{cd} I_{d_j^1}^{BS}}{P_{tr}^{c_i} E\left[g_{c_i}^{BS}\right]} e^{-\frac{\gamma^{c_i,BS}\left(N_0 W + \sum\limits_{d_j \in \mathcal{D}} y_{i,j}^{cd} I_{d_j^1}^{BS}\right)}{P_{tr}^{c_i} E\left[g_{c_i}^{BS}\right]}}.$$

Let $\gamma_0^{u,v}$ denote the SINR threshold for the successful transmission from node u to node v. Then, the successful transmission probability from c_i to BS when $\mathcal{I}_{all}^{c_i,BS}$ is given, $s_{c_i}^{BS}\left(\mathcal{I}_{all}^{c_i,BS}\right)$, can be calculated as follows:

$$\begin{aligned} & s_{c_i}^{BS}\left(\mathcal{I}_{all}^{c_i,BS}\right) \\ & = \Pr(\gamma^{c_i,BS} \geq \gamma_0^{c_i,BS}) = \int_{\gamma_0^{c_i,BS}}^{+\infty} h\left(\gamma^{c_i,BS}\right) d\gamma^{c_i,BS} \\ & = e^{-\frac{\gamma_0^{c_i,BS}\left(N_0 W + \sum\limits_{d_j \in \mathcal{D}} y_{i,j}^{cd} I_{d_j^1}^{BS}\right)}{P_{tr}^{c_i} E\left[g_{c_i}^{BS}\right]}}. \end{aligned}$$

Let $x_{\max}^{u,v}$ denote the maximum instantaneous transmission rate from node u to node v. According to Shannon's capacity formula, $x_{\max}^{c_i,BS}$ equals $W \log\left(1 + \gamma_0^{c_i,BS}\right)$, where the unit of $x_{\max}^{c_i,BS}$ is nats/s. Then, the average transmission rate from c_i to BS when $\mathcal{I}_{all}^{c_i,BS}$ is given, $x_{avg}^{c_i,BS}\left(\mathcal{I}_{all}^{c_i,BS}\right)$, equals

$$\begin{aligned} & x_{avg}^{c_i,BS}\left(\mathcal{I}_{all}^{c_i,BS}\right) = x_{\max}^{c_i,BS} s_{c_i}^{BS}\left(\mathcal{I}_{all}^{c_i,BS}\right) \\ & = W \log\left(1 + \gamma_0^{c_i,BS}\right) e^{-\frac{\gamma_0^{c_i,BS}\left(N_0 W + \sum\limits_{d_j \in \mathcal{D}} y_{i,j}^{cd} I_{d_j^1}^{BS}\right)}{P_{tr}^{c_i} E\left[g_{c_i}^{BS}\right]}}. \end{aligned}$$

Since $I_{d_j^1}^{BS} = P_{tr}^{d_j^1} g_{d_j^1}^{BS}$, $I_{d_j^1}^{BS}$ follows an exponential distribution with the expected value $P_{tr}^{d_j^1} E\left[g_{d_j^1}^{BS}\right]$. That is,

$$h\left(I_{d_j^1}^{BS}\right) = \frac{1}{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{BS}\right]} e^{-\frac{I_{d_j^1}^{BS}}{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{BS}\right]}}.$$

Considering that all the interferences to BS when c_i transmits are independent, the average transmission rate of c_i, $x_{avg}^{c_i}$, can be calculated as follows:

$$\begin{aligned} & x_{avg}^{c_i} = \int\int ... \int_0^{+\infty} x_{avg}^{c_i,BS}\left(\mathcal{I}_{all}^{c_i,BS}\right) \prod_{d_j \in \mathcal{D}} \left(h\left(I_{d_j^1}^{BS}\right) dI_{d_j^1}^{BS}\right) \\ & = W \log\left(1 + \gamma_0^{c_i,BS}\right) e^{-\frac{\gamma_0^{c_i,BS} N_0 W}{P_{tr}^{c_i} E\left[g_{c_i}^{BS}\right]}} \cdot \\ & \prod_{d_j \in \mathcal{D}} \frac{P_{tr}^{c_i} E\left[g_{c_i}^{BS}\right]}{y_{i,j}^{cd} P_{tr}^{d_j^1} E\left[g_{d_j^1}^{BS}\right] \gamma_0^{c_i,BS} + P_{tr}^{c_i} E\left[g_{c_i}^{BS}\right]}. \end{aligned} \tag{1}$$

B. The average rate of D2D pair d_j

When a D2D pair d_j transmits, the interference comes from the concurrent transmissions of the CU and the other D2D pairs. Thus, $\gamma^{d_j^1,d_j^2}$ equals

$$\gamma^{d_j^1,d_j^2} = \frac{P_{tr}^{d_j^1} g_{d_j^1}^{d_j^2}}{N_0 W + \sum\limits_{c_i \in \mathcal{C}} y_{i,j}^{cd} I_{c_i}^{d_j^2} + \sum\limits_{c_i \in \mathcal{C}} \sum\limits_{d_{j'} \in \mathcal{D}\setminus\{d_j\}} y_{i,j}^{cd} y_{i,j'}^{cd} I_{d_{j'}^1}^{d_j^2}}.$$

Given $\mathcal{I}_{all}^{d_j^1,d_j^2}$, $\gamma^{d_j^1,d_j^2}$ follows an exponential distribution with the expected value $\frac{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]}{N_0 W + \sum\limits_{c_i \in \mathcal{C}} y_{i,j}^{cd} I_{c_i}^{d_j^2} + \sum\limits_{c_i \in \mathcal{C}} \sum\limits_{d_{j'} \in \mathcal{D}\setminus\{d_j\}} y_{i,j}^{cd} y_{i,j'}^{cd} I_{d_{j'}^1}^{d_j^2}}$. Thus, we have

$$h\left(\gamma^{d_j^1,d_j^2}\right) = \frac{N_0 W + \sum\limits_{c_i \in \mathcal{C}} y_{i,j}^{cd} I_{c_i}^{d_j^2} + \sum\limits_{c_i \in \mathcal{C}} \sum\limits_{d_{j'} \in \mathcal{D}\setminus\{d_j\}} y_{i,j}^{cd} y_{i,j'}^{cd} I_{d_{j'}^1}^{d_j^2}}{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]} \cdot e^{-\frac{\gamma^{d_j^1,d_j^2}\left(N_0 W + \sum\limits_{c_i \in \mathcal{C}} y_{i,j}^{cd} I_{c_i}^{d_j^2} + \sum\limits_{c_i \in \mathcal{C}} \sum\limits_{d_{j'} \in \mathcal{D}\setminus\{d_j\}} y_{i,j}^{cd} y_{i,j'}^{cd} I_{d_{j'}^1}^{d_j^2}\right)}{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]}}.$$

Then, the successful transmission probability from d_j^1 to d_j^2 when $\mathcal{I}_{all}^{d_j^1,d_j^2}$ is given, $s_{d_j^1}^{d_j^2}\left(\mathcal{I}_{all}^{d_j^1,d_j^2}\right)$, can be calculated as follows:

$$\begin{aligned} s_{d_j^1}^{d_j^2}\left(\mathcal{I}_{all}^{d_j^1,d_j^2}\right) &= \int_{\gamma_0^{d_j^1,d_j^2}}^{+\infty} h\left(\gamma^{d_j^1,d_j^2}\right) d\gamma^{d_j^1,d_j^2} \\ &= e^{-\frac{\gamma_0^{d_j^1,d_j^2}\left(N_0 W + \sum\limits_{c_i \in \mathcal{C}} y_{i,j}^{cd} I_{c_i}^{d_j^2} + \sum\limits_{c_i \in \mathcal{C}} \sum\limits_{d_{j'} \in \mathcal{D}\setminus\{d_j\}} y_{i,j}^{cd} y_{i,j'}^{cd} I_{d_{j'}^1}^{d_j^2}\right)}{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]}}. \end{aligned}$$

Since $x_{\max}^{d_j^1,d_j^2}$ equals $W\log\left(1+\gamma_0^{d_j^1,d_j^2}\right)$, the average transmission rate from d_j^1 to d_j^2 when $\mathcal{I}_{all}^{d_j^1,d_j^2}$ is given, $x_{avg}^{d_j^1,d_j^2}\left(\mathcal{I}_{all}^{d_j^1,d_j^2}\right)$, equals

$$\begin{aligned} x_{avg}^{d_j^1,d_j^2}\left(\mathcal{I}_{all}^{d_j^1,d_j^2}\right) &= x_{\max}^{d_j^1,d_j^2} s_{d_j^1}^{d_j^2}\left(\mathcal{I}_{all}^{d_j^1,d_j^2}\right) \\ &= W\log\left(1+\gamma_0^{d_j^1,d_j^2}\right) \cdot \\ & e^{-\frac{\gamma_0^{d_j^1,d_j^2}\left(N_0 W + \sum\limits_{c_i \in \mathcal{C}} y_{i,j}^{cd} I_{c_i}^{d_j^2} + \sum\limits_{c_i \in \mathcal{C}} \sum\limits_{d_{j'} \in \mathcal{D}\setminus\{d_j\}} y_{i,j}^{cd} y_{i,j'}^{cd} I_{d_{j'}^1}^{d_j^2}\right)}{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]}}. \end{aligned}$$

Furthermore, $I_{c_i}^{d_j^2} = P_{tr}^{c_i} g_{c_i}^{d_j^2}$, and $I_{d_{j'}^1}^{d_j^2} = P_{tr}^{d_{j'}^1} g_{d_{j'}^1}^{d_j^2}$. Then, $I_{c_i}^{d_j^2}$ and $I_{d_{j'}^1}^{d_j^2}$ follow an exponential distribution with the expected value $P_{tr}^{c_i} E\left[g_{c_i}^{d_j^2}\right]$ and $P_{tr}^{d_{j'}^1} E\left[g_{d_{j'}^1}^{d_j^2}\right]$, respectively. Thus, we have $h\left(I_{c_i}^{d_j^2}\right) = \frac{1}{P_{tr}^{c_i} E\left[g_{c_i}^{d_j^2}\right]} e^{-\frac{I_{c_i}^{d_j^2}}{P_{tr}^{c_i} E\left[g_{c_i}^{d_j^2}\right]}}$, and $h\left(I_{d_{j'}^1}^{d_j^2}\right) = \frac{1}{P_{tr}^{d_{j'}^1} E\left[g_{d_{j'}^1}^{d_j^2}\right]} e^{-\frac{I_{d_{j'}^1}^{d_j^2}}{P_{tr}^{d_{j'}^1} E\left[g_{d_{j'}^1}^{d_j^2}\right]}}$.

Since all the interferences are independent, the average transmission rate of d_j, $x_{avg}^{d_j}$, can be calculated as follows:

$$\begin{aligned} & x_{avg}^{d_j} = \int\int \ldots \int_0^{+\infty} x_{avg}^{d_j^1,d_j^2}\left(\mathcal{I}_{all}^{d_j^1,d_j^2}\right) \cdot \\ & \prod_{c_i \in \mathcal{C}} \left(h\left(I_{c_i}^{d_j^2}\right) dI_{c_i}^{d_j^2}\right) \prod_{d_{j'} \in \mathcal{D}\setminus\{d_j\}} \left(h\left(I_{d_{j'}^1}^{d_j^2}\right) dI_{d_{j'}^1}^{d_j^2}\right) \\ & = W\log\left(1+\gamma_0^{d_j^1,d_j^2}\right) e^{-\frac{\gamma_0^{d_j^1,d_j^2} N_0 W}{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]}} \\ & \prod_{c_i \in \mathcal{C}} \frac{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]}{\left(y_{i,j}^{cd} P_{tr}^{c_i} E\left[g_{c_i}^{d_j^2}\right] \gamma_0^{d_j^1,d_j^2} + P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]\right)} \cdot \\ & \prod_{c_i \in \mathcal{C}} \prod_{d_{j'} \in \mathcal{D}\setminus\{d_j\}} \frac{P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]}{\left(y_{i,j}^{cd} y_{i,j'}^{cd} P_{tr}^{d_{j'}^1} E\left[g_{d_{j'}^1}^{d_j^2}\right] \gamma_0^{d_j^1,d_j^2} + P_{tr}^{d_j^1} E\left[g_{d_j^1}^{d_j^2}\right]\right)}. \end{aligned} \tag{2}$$

After deriving the average transmission rate of c_i and d_j, the system sum-rate, R_{sum}, can be expressed as:

$$R_{sum} = \sum_{c_i \in \mathcal{C}} \left(x_{avg}^{c_i} + \sum_{d_j \in \mathcal{D}} y_{i,j}^{cd} x_{avg}^{d_j}\right). \tag{3}$$

Thus, the system sum-rate maximization problem can be formulated as follows:

$$\begin{aligned} &\text{Maximize} \quad R_{sum} \\ &\text{s.t.} \quad \left\{ \begin{array}{l} \sum\limits_{c_i \in \mathcal{C}} y_{i,j}^{cd} = 1, \forall d_j \in \mathcal{D}, \\ \gamma_0^{\min} \le \gamma_0^{c_i,BS} \le \gamma_0^{\max}, \forall c_i \in \mathcal{C}, \\ \gamma_0^{\min} \le \gamma_0^{d_j^1,d_j^2} \le \gamma_0^{\max}, \forall d_j \in \mathcal{D}, \\ y_{i,j}^{cd} \in \{0,1\}, \forall c_i \in \mathcal{C}, \forall d_j \in \mathcal{D}. \end{array} \right. \end{aligned} \tag{4}$$

Here, $\gamma_0^{\min}$ and $\gamma_0^{\max}$ are the given minimum and maximum values of the SINR threshold of the system. Obviously, Problem (4) is a mixed integer non-linear programming problem, which is difficult to solve in general. Therefore, we propose a decomposition method to alternately optimize SINR threshold and the resource allocation policy.

IV. The Decomposition Optimization Method

In this section, we decompose the sum-rate maximization problem into two sub-problems: 1) Given $y_{i,j}^{cd}, \forall c_i \in \mathcal{C}, \forall d_j \in \mathcal{D}$, we derive the optimal SINR threshold for each transmission that maximizes the system sum-rate. 2) We then propose a coalition game approach to further optimize $y_{i,j}^{cd}$.

A. The optimization of SINR thresholds given $y_{i,j}^{cd}$

From equation (3), we know that when $y_{i,j}^{cd}$ is given, the system sum-rate is maximized when $x_{avg}^{c_i}$ and $x_{avg}^{d_j}$ are respectively maximized, which is given in the following theorem.

Theorem 1: The average rate $x_{avg}^{c_i}$ is maximized when

$$\gamma_0^{c_i,BS}=\begin{cases}\gamma_0^{\min}, & \gamma_0^*\left(c_i,BS\right)<\gamma_0^{\min},\\ \gamma_0^*\left(c_i,BS\right), & \gamma_0^{\min}\le\gamma_0^*\left(c_i,BS\right)\le\gamma_0^{\max},\\ \gamma_0^{\max}, & \gamma_0^*\left(c_i,BS\right)>\gamma_0^{\max},\end{cases}$$

where $\gamma_0^*\left(c_i,BS\right)$ is the value of $\gamma_0^{c_i,BS}$ that satisfies

$$1-\frac{N_0W\left(1+\gamma_0^{c_i,BS}\right)\log\left(1+\gamma_0^{c_i,BS}\right)}{P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}-\left(\log\left(1+\gamma_0^{c_i,BS}\right)\right)\cdot$$
$$\sum_{d_j\in\mathcal{D}}\frac{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\left(1+\gamma_0^{c_i,BS}\right)}{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\gamma_0^{c_i,BS}+P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}=0.$$

The average rate $x_{avg}^{d_j}$ is maximized when

$$\gamma_0^{d_j^1,d_j^2}=\begin{cases}\gamma_0^{\min}, & \gamma_0^*\left(d_j^1,d_j^2\right)<\gamma_0^{\min},\\ \gamma_0^*\left(d_j^1,d_j^2\right), & \gamma_0^{\min}\le\gamma_0^*\left(d_j^1,d_j^2\right)\le\gamma_0^{\max},\\ \gamma_0^{\max}, & \gamma_0^*\left(d_j^1,d_j^2\right)>\gamma_0^{\max},\end{cases}$$

where $\gamma_0^*\left(d_j^1,d_j^2\right)$ is the value of $\gamma_0^{d_j^1,d_j^2}$ that satisfies

$$1-\frac{N_0W}{P_{tr}^{d_j^1}E\left[g_{d_j^1}^{d_j^2}\right]}\left(1+\gamma_0^{d_j^1,d_j^2}\right)\log\left(1+\gamma_0^{d_j^1,d_j^2}\right)-$$
$$\left(\log\left(1+\gamma_0^{d_j^1,d_j^2}\right)\right)\sum_{c_i\in\mathcal{C}}\frac{y_{i,j}^{cd}P_{tr}^{c_i}E\left[g_{c_i}^{d_j^2}\right]\left(1+\gamma_0^{d_j^1,d_j^2}\right)}{\left(\gamma_0^{d_j^1,d_j^2}y_{i,j}^{cd}P_{tr}^{c_i}E\left[g_{c_i}^{d_j^2}\right]+P_{tr}^{d_j^1}E\left[g_{d_j^1}^{d_j^2}\right]\right)}$$
$$-\left(\log\left(1+\gamma_0^{d_j^1,d_j^2}\right)\right)\cdot$$
$$\sum_{c_i\in\mathcal{C}}\sum_{d_{j'}\in\mathcal{D}\setminus\{d_j\}}\frac{y_{i,j}^{cd}y_{i,j'}^{cd}P_{tr}^{d_{j'}^1}E\left[g_{d_{j'}^1}^{d_j^2}\right]\left(1+\gamma_0^{d_j^1,d_j^2}\right)}{\left(\gamma_0^{d_j^1,d_j^2}y_{i,j}^{cd}y_{i,j'}^{cd}P_{tr}^{d_{j'}^1}E\left[g_{d_{j'}^1}^{d_j^2}\right]+P_{tr}^{d_j^1}E\left[g_{d_j^1}^{d_j^2}\right]\right)}=0.$$

Proof: According to equations (1), the first order derivative of $x_{avg}^{c_i}$ equals

$$\frac{\mathrm{d}x_{avg}^{c_i}}{\mathrm{d}\gamma_0^{c_i,BS}}=\frac{W}{1+\gamma_0^{c_i,BS}}e^{-\frac{\gamma_0^{c_i,BS}N_0W}{P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}}\cdot$$
$$\prod_{d_j\in\mathcal{D}}\frac{P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\gamma_0^{c_i,BS}+P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}\cdot$$
$$\left(1-\frac{N_0W\left(1+\gamma_0^{c_i,BS}\right)\log\left(1+\gamma_0^{c_i,BS}\right)}{P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}-\left(\log\left(1+\gamma_0^{c_i,BS}\right)\right)\cdot\right.$$
$$\left.\sum_{d_j\in\mathcal{D}}\frac{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\left(1+\gamma_0^{c_i,BS}\right)}{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\gamma_0^{c_i,BS}+P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}\right).$$

Let $Q_1\triangleq 1-\frac{N_0W\left(1+\gamma_0^{c_i,BS}\right)\log\left(1+\gamma_0^{c_i,BS}\right)}{P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}-\left(\log\left(1+\gamma_0^{c_i,BS}\right)\right)\sum_{d_j\in\mathcal{D}}\frac{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\left(1+\gamma_0^{c_i,BS}\right)}{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\gamma_0^{c_i,BS}+P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}$.
We next show that Q_1 decreases from a positive value to a negative value when $\gamma_0^{c_i,BS}$ increases from 0. Let $U_1\triangleq\frac{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\left(1+\gamma_0^{c_i,BS}\right)}{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\gamma_0^{c_i,BS}+P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}$, which can be expressed as $1-\frac{P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]-y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]}{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\gamma_0^{c_i,BS}+P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}$. Considering that the average power of the signal at a receiver should be bigger than the one of any interference, we have $P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]-y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]>0$. Thus, U_1 increases as $\gamma_0^{c_i,BS}$ increases. Therefore, Q_1 decreases as $\gamma_0^{c_i,BS}$ increases. Furthermore, we have $\lim\limits_{\gamma_0^{c_i,BS}\to 0}Q_1=1$, and $\lim\limits_{x\to+\infty}Q_1=-\infty$. Thus, Q_1 decreases from a positive value to a negative value when $\gamma_0^{c_i,BS}$ increases from 0. Since $Q_1=0$ when $\gamma_0^{c_i,BS}=\gamma_0^*\left(d_j^1,d_j^2\right)$, then, $Q_1>0$ when $0<\gamma_0^{c_i,BS}<\gamma_0^*\left(d_j^1,d_j^2\right)$, and $Q_1<0$ when $\gamma_0^{c_i,BS}>\gamma_0^*\left(d_j^1,d_j^2\right)$. Furthermore, we have $\frac{W}{1+\gamma_0^{c_i,BS}}e^{-\frac{\gamma_0^{c_i,BS}N_0W}{P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}}\prod_{d_j\in\mathcal{D}}\frac{P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}{y_{i,j}^{cd}P_{tr}^{d_j^1}E\left[g_{d_j^1}^{BS}\right]\gamma_0^{c_i,BS}+P_{tr}^{c_i}E\left[g_{c_i}^{BS}\right]}>0$. Thus, $\frac{\mathrm{d}x_{avg}^{c_i}}{\mathrm{d}\gamma_0^{c_i,BS}}>0$ when $0<\gamma_0^{c_i,BS}<\gamma_0^*\left(d_j^1,d_j^2\right)$; $\frac{\mathrm{d}x_{avg}^{c_i}}{\mathrm{d}\gamma_0^{c_i,BS}}=0$ when $\gamma_0^{c_i,BS}=\gamma_0^*\left(d_j^1,d_j^2\right)$; $\frac{\mathrm{d}x_{avg}^{c_i}}{\mathrm{d}\gamma_0^{c_i,BS}}<0$ when $\gamma_0^{c_i,BS}>\gamma_0^*\left(d_j^1,d_j^2\right)$. Therefore, $x_{avg}^{c_i}$ first increases and then decreases as $\gamma_0^{c_i,BS}$ increases. It reaches the maximum value when $\gamma_0^{c_i,BS}=\gamma_0^*\left(c_i,BS\right)$. Considering that $\gamma_0^*\left(c_i,BS\right)$ is not always in the range $\left[\gamma_0^{\min},\gamma_0^{\max}\right]$, we have that $x_{avg}^{c_i}$ is maximized when $\gamma_0^{c_i,BS}=\gamma_0^{\min}$ if $\gamma_0^*\left(c_i,BS\right)<\gamma_0^{\min}$, and when $\gamma_0^{c_i,BS}=\gamma_0^{\max}$ if $\gamma_0^*\left(c_i,BS\right)>\gamma_0^{\max}$. The proof of the maximization of $x_{avg}^{d_j}$ is similar and thus is omitted here. ■

Next, we propose a coalition game approach to further optimize $y_{i,j}^{cd}$.

B. Coalition game formulation

In the coalition game, the players form coalitions to maximize the system utility. For the considered problem, the D2D pairs are treated as players. They form $|\mathcal{C}|$ coalitions $\mathcal{L}=\left\{\mathcal{L}_1,\mathcal{L}_2...\mathcal{L}_i...\mathcal{L}_{|\mathcal{C}|}\right\}$ to respectively share the resource of different CUs. That is, CU c_i shares its resource with the D2D pair(s) in $\mathcal{L}_i$. If $\mathcal{L}_i$ is $\emptyset$, c_i does not share its resource with any D2D pair. Obviously, we have $\mathcal{L}_i\cap\mathcal{L}_{i'}=\emptyset$ for any $i\neq i'$, and $\bigcup\limits_{c_i\in\mathcal{C}}\mathcal{L}_i=\mathcal{D}$. The sum-rate of the CU c_i and its sharing D2D pairs in $\mathcal{L}_i$, $R_{\mathcal{L}_i}$, can be calculated as follows:

$$R_{\mathcal{L}_i}=x_{avg}^{c_i}+\sum_{d_j\in\mathcal{L}_i}x_{avg}^{d_j}. \tag{5}$$

Thus, the transferable utility of the coalition $\mathcal{L}_i$ can be defined by the maximum value of $R_{\mathcal{L}_i}$, denoted by $R_{\mathcal{L}_i}^{\max}$, which can be obtained from Section IV-A.

The coalition game contains two sub-processes: the initial process and the adjusting process, as shown in Algorithm 1. In the initial process, the coalition game generates a random coalition partition $\mathcal{L}_{ini}$ such that for each receiver, the average received power is bigger than any interference power. In the adjusting process, the coalition game tries to move a D2D pair from a coalition to another coalition. The details are as follows. At each step, the coalition game randomly selects a D2D pair (i.e. d_j) in a coalition (i.e. $\mathcal{L}_i$) and moves it to another randomly selected coalition (i.e. $\mathcal{L}_{i'}$). If for each receiver, the average received power is bigger than any interference power, and

$$R_{\mathcal{L}_i}^{\max} + R_{\mathcal{L}_{i'}}^{\max} < R_{\mathcal{L}_i \setminus \{d_j\}}^{\max} + R_{\mathcal{L}_{i'} \cup \{d_j\}}^{\max},$$

then, the movement is executed; otherwise, the movement is withdrawn. After a limited number of movements, the coalition partition converges to the final Nash-stable partition $\mathcal{L}_{final}$.

Algorithm 1: The algorithm of the coalition game

Initialization: Generate a random coalition partition $\mathcal{L}_{ini}$ such that for each receiver, the average received power is bigger than any interference power;
Set the current partition $\mathcal{L}_{cur}$ as $\mathcal{L}_{ini}$;
repeat
  Randomly select a D2D pair (i.e. d_j) in a coalition (i.e. $\mathcal{L}_i$);
  Randomly select a coalition $\mathcal{L}_{i'} \in \mathcal{L}_{cur} \setminus \{\mathcal{L}_i\}$;
  if for each receiver, the average received power is bigger than any interference power, and $R_{\mathcal{L}_i}^{\max} + R_{\mathcal{L}_{i'}}^{\max} < R_{\mathcal{L}_i \setminus \{d_j\}}^{\max} + R_{\mathcal{L}_{i'} \cup \{d_j\}}^{\max}$ **then**
    Move D2D pair d_j from $\mathcal{L}_i$ to $\mathcal{L}_{i'}$;
    Update the current partition set as follows: $(\mathcal{L}_{cur} \setminus \{\mathcal{L}_i, \mathcal{L}_{i'}\}) \cup \{\mathcal{L}_i \setminus \{d_j\}, \mathcal{L}_{i'} \cup \{d_j\}\} \to \mathcal{L}_{cur}$;
  end if
until The coalition partition converges to the final Nash-stable partition $\mathcal{L}_{final}$.

C. Convergence and stability

In this part, we discuss the convergence and stability of the proposed coalition game. We first give the definition of the Nash-stable coalition partition as follows.

Definition 1: A coalition partition $\mathcal{L} = \{\mathcal{L}_1, \mathcal{L}_2 ... \mathcal{L}_i ... \mathcal{L}_{|\mathcal{C}|}\}$ is Nash-stable if $\forall d_j \in \mathcal{L}_i \in \mathcal{L}, R_{\mathcal{L}_i}^{\max} + R_{\mathcal{L}_{i'}}^{\max} > R_{\mathcal{L}_i \setminus \{d_j\}}^{\max} + R_{\mathcal{L}_{i'} \cup \{d_j\}}^{\max}$, for all $\mathcal{L}_{i'} \in \mathcal{L} \setminus \{\mathcal{L}_i\}$.

Based on Definition 1, we have the following Theorem.

Theorem 2: Algorithm 1 converges to the Nash-stable partition $\mathcal{L}_{final}$ in finite time with probability 1.

Proof: Suppose that the final coalition partition $\mathcal{L}_{final}$ obtained from Algorithm 1 is not Nash-stable. According to Definition 1, we can find a $d_j \in \mathcal{L}_i \in \mathcal{L}$ and a $\mathcal{L}_{i'} \in \mathcal{L} \setminus \{\mathcal{L}_i\}$ that satisfy $R_{\mathcal{L}_i}^{\max} + R_{\mathcal{L}_{i'}}^{\max} < R_{\mathcal{L}_i \setminus \{d_j\}}^{\max} + R_{\mathcal{L}_{i'} \cup \{d_j\}}^{\max}$. In this case, we can move d_j from $\mathcal{L}_i$ to $\mathcal{L}_{i'}$ to further improve the total system utility, which contradicts the supposition that $\mathcal{L}_{final}$ is the final coalition partition. Thus, Algorithm 1 converges to the Nash-stable partition $\mathcal{L}_{final}$ with probability 1. Furthermore, since the number of coalitions is $|\mathcal{C}|$, the number of possible coalition partitions is the Bell number [12]. Therefore, Algorithm 1 converges in finite time. ■

V. Simulation results

In the simulation, the CUs and D2D pairs are randomly located within the circular area centered at the BS with radius 500 m. For each D2D pair, we first randomly generate the position of the transmitter, and its receiver randomly locates on a circle centered at the transmitter with radius 10 m. The expected value of the channel gain $E[g]$ is obtained from log-distance path-loss model with a path-loss exponent of 4. The transmit power of CUs and the transmitter of the D2D pairs, the noise power density, and the channel bandwidth are set to 33 dBm, 13 dBm, -174 dBm/Hz, and 1 MHz, respectively. The values of $\gamma_0^{\min}$ and $\gamma_0^{\max}$ are set to 0.1 and 1000, respectively.

A. The SINR threshold

Fig. 2 shows the performance of the derived optimal SINR threshold (Opt) versus the following four SINR threshold setting policies under 15 random cases when the number of CUs and D2D pairs varies from 20 to 30:

1) Average SINR policy (AS), in which the SINR threshold is set to the ratio of the average signal power to the average sum of all the interference powers;
2) Twice Average SINR policy (TwAS), in which the SINR threshold is set to twice of the one in the AS policy;
3) Half Average SINR policy (HaAS), in which the SINR threshold is set to half of the one in the AS policy;
4) Single SINR policy (SinS), in which the SINR threshold for all the transmissions is the same and optimized by numerical search.

From Fig. 2, we can see that the optimal SINR threshold is larger than the other four SINR threshold setting policies.

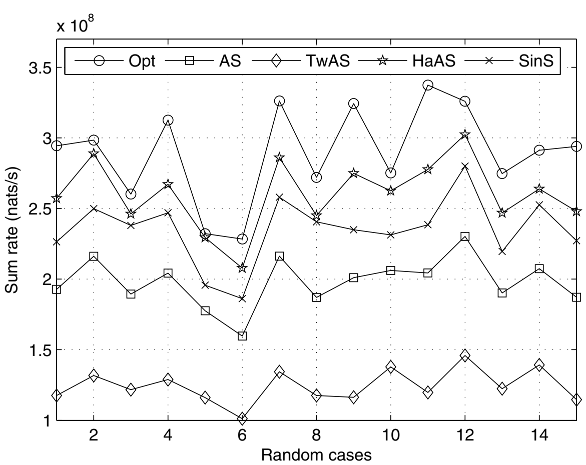

Fig. 2. The optimal SINR threshold versus several other SINR threshold setting policies.

B. Performance of the coalition game under the optimal SINR thresholds

We compare the system sum-rate and the run-time of the coalition game approach (CG) with the optimal solution obtained from exhaustive search (ES), as shown in Fig. 3. Since the computation complexity of exhaustive search increases exponentially with the network size, we can only obtain the optimal solution for small networks. From Fig. 3, we can see that the performance of the coalition game is close to the exhaustive search; but its run-time is much shorter than the exhaustive search.

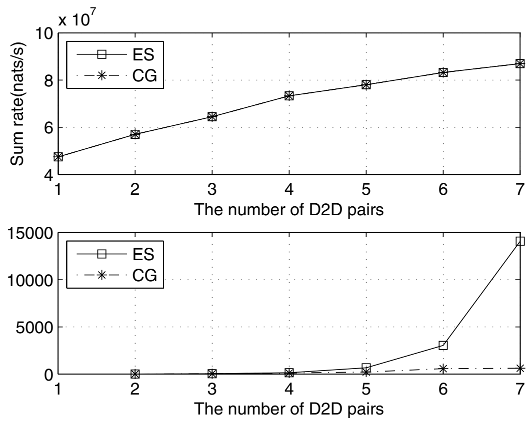

Fig. 3. The coalition game versus the exhaustive search under different number of D2D pairs when the number of CUs is set to 4.

We further compare the coalition game with the following three resource allocation policies in terms of system sum-rate and the fairness under the optimal SINR threshold:

1) Nearest First (NF), in which each D2D pair shares the resource with the nearest CU;
2) Farthest First (FF), in which each D2D pair shares the resource with the farthest CU;
3) Random Selection (RS), in which each D2D pair shares the resource with the randomly selected CU.

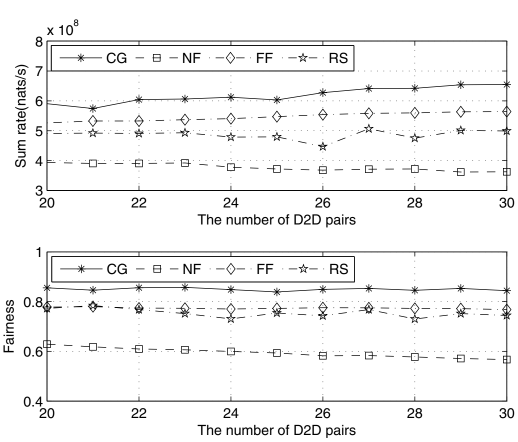

Fig. 4. The coalition game versus the other resource allocation policies under different number of D2D pairs when the number of CUs is set to 50.

We use the Jain's fairness (JF) [13] to as the fairness metric. The fairness is better if JF is more closer to 1. From Fig. 4, we can see that compared with the other policies, the proposed coalition game achieves an average sum-rate improvement of 13%-173%. Furthermore, it has the best resource sharing fairness.

VI. Conclusions

In this paper, we have considered maximizing the system sum-rate of a D2D communication underlaying cellular network over the Rayleigh fading channel. The problem formulation turns out to be a mixed integer non-linear programming problem. To solve the problem, we propose a decomposition method that alternately optimizes the SINR threshold and the resource allocation policy. Simulation results show that the proposed scheme achieves close-to-optimal solution with short run-time, and has the best resource sharing fairness compared with several other resource allocation policies.

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