Multi-Leader Stackelberg Games in Multi-Channel Spatial Aloha Networks

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Abstract—This paper uses a multi-channel spatial Aloha model to describe a distributed autonomous wireless network where a group of transmit-receive pairs (users) share multiple collision channels via slotted-Aloha-like random access. The design objective is to enable each autonomous user i to select a channel c_i and decide a medium access probability (MAP) q_i to improve its throughput, while providing a certain degree of fairness among the users. Game theoretic approaches are applied, where each user i is a player who chooses the strategy (c_i, q_i) to improve its own throughput. To search for a Nash Equilibrium (NE), a *Multi-Leader Stackelberg Game (MLSG)* is formulated to iteratively obtain a solution on each dimension of the (c_i, q_i) strategy. Initially, multiple Stackelberg leaders are elected to manage the MAPs of all players. Then under the resulting MAP profile, each player iteratively chooses its channel to improve its throughput. An *Oscillation Resolving Mechanism (ORM)* is further proposed to stabilize the design in some special cases where the operating points of some players in a local region would oscillate between the two dimensions of the myopic search. Compared to existing methods of pre-allocating MAPs, the MLSG game further improves the overall network throughput by iteratively tuning the MAPs toward max-min throughput in each subnet. Simulation results show that the MLSG game gradually improves the total throughput until reaching a NE, which also provides good throughput fairness for the players.

I. INTRODUCTION

Game theoretic approaches have been widely used to design multiple access protocols in wireless networks. In [1], the authors provide a comprehensive review of the game models developed for different multiple access schemes. In particular, several channel access games in ALOHA-like protocols are presented. For example, an Aloha game model is proposed by Jin and Kesidis [2], whereby a group of heterogeneous users share a conventional collision channel and transmit via slotted Aloha. Each user in this game attempts to obtain a target rate by updating its MAP in response to observed activities. The conditions for the existence and stability of the equilibrium solutions have been well studied in these works. However, these studies are more suitably applied to the uplink random access channel in a fully connected centralized network.

On the other hand, *spatial reuse*, also known as frequency reuse, is a powerful technique to improve the area spectral efficiency of multi-user communication systems. In the context of *dynamic spectrum access* and *distributed open sharing networks* [3], we consider in [4] a group of Tx-Rx pairs (users) who share a common collision channel via slotted-Aloha-like random access. These users are allowed to reuse the channel if they receive negligible interference from others. We model

the competitions among the users as a *generalized Aloha game*, and prove the existence and uniqueness of the Nash Equilibrium (NE) in terms of the MAPs of all players. We further derive the stability condition of the NE and define the feasible throughput region for any given topology.

To further improve the network performance, multiple collision channels can be considered. Chen and Huang in [5] study the random access based distributed spectrum sharing problem with spatial reuse. Each user is allowed to access only one channel in each slot, under the assumption that the MAPs are fixed and pre-allocated. The problem is formulated as a spatial channel selection game, in which each user is a player who chooses one channel to access in order to maximize its own expected throughput. The game is shown to be a weighted potential game [6] and thus possesses a NE and the finite improvement property. However, this method relies on the assumption that the MAPs are fixed and pre-allocated, but the efficiency and fairness of such allocations are not discussed. A better way of the design is to let the users jointly optimize the MAPs and channel selection to maximize individual user throughput. This is a highly non-trivial problem since the setting of the MAPs would affect each user's expected throughput, intertwine with the channel selection decisions and thus affect the final location of the NE.

This challenging problem has been partially solved by Cohen *et al.* in [7], although no spatial reuse has been considered. The sub-game of channel selection under fixed MAP constraints is also shown to be a potential game, and thus the convergence to a NE is guaranteed. The authors then formulate the problem of choosing the MAPs as a single-leader Stackelberg game [8]. The classic Stackelberg games are a class of non-cooperative games in which a single "leader", who makes the first move in the game, anticipates the actions of the "follower" based on a model of how the follower would respond to the actions of the leader. Since no spatial reuse is considered, the users are homogeneous in the sense that each user's transmission will affect all remaining users in the network. Therefore, a single elected leader is sufficient to manage the network, which mandates all MAPs to be the same and sets this common MAP value to maximize the sum-rate at the NE of the sub-game. When spatial reuse is considered, heterogeneous MAPs are generally observed due to the *non-uniform* node degrees. Therefore, multiple leaders at different spatial locations might be a proper extension to better manage the network and handle the joint MAP tuning and channel selection problem in a local region.

The main contributions of this paper are summarized as follows. We study the multi-channel Aloha networks with spatial reuse, with the objective to enable each autonomous user i to select a channel c_i and decide a MAP q_i to improve its throughput, while providing a certain degree of fairness among

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the users. We apply game theoretic approaches to the problem, where each user i is a player who chooses the strategy (c_i, q_i) to improve its own throughput. To search for a NE, a *Multi-Leader Stackelberg Game (MLSG)* [9] is formulated to iteratively obtain a solution on each dimension of the (c_i, q_i) strategy. First, multiple Stackelberg leaders are elected to manage the MAPs of all players. Then under the resulting MAP profile, each player iteratively chooses its channel to improve its throughput. Specifically, assume the current network consists of several single-channel sub-networks. These subnets are disconnected from others, either because they operate on different channels, or they are on the same channel but spatially disconnected. First of all, each subnet elects the player with the highest node degree to be the leader to manage this subnet. Within each subnet, the leader mandates the MAPs of all players to be the same, and sets the MAP value to provide localized max-min throughput fairness for the players in this subnet. Such a myopic best response update performed by the leader requires only local information within its subnet. Secondly, under the resulting MAP profile, each player iteratively chooses its channel to improve its own throughput. The iteration dynamics follow the formulation in [5], whose convergence is guaranteed under the theory of potential games. An *Oscillation Resolving Mechanism (ORM)* is further proposed to stabilize the design in some special cases where the operating points of some players in a local region would oscillate between the two dimensions of the myopic search. Compared to existing methods of pre-allocating MAPs as in [5], our iterative MAP management explicitly takes into account the localized fairness issue among the players, and is able to approximately achieve max-min throughput fairness in each subnet. Moreover, under the above fairness constraint, the MLSG game is able to further improve the overall network throughput iteratively compared to any pre-allocated MAPs. Simulation results show that, by playing the MLSG game, the overall network throughput is gradually improved until a NE is reached, and the resulting NE provides good throughput fairness for the players.

The rest of the paper is organized as follows. We introduce the multi-channel spatial Aloha model in Section II. We dedicate Section III to discuss the MLSG game in details. We evaluate the system performance through simulations in Section IV. We conclude the paper in Section V.

II. MULTI-CHANNEL SPATIAL ALOHA MODEL

Consider a distributed wireless network with N transmitters, where each transmitter has its unique designated receiver. Each Tx-Rx pair is a user who shares K orthogonal channels with other users, via slotted-Aloha-like random access. We make the following assumptions:

• In each time slot each user is allowed to access a single channel.

• Every user's transmission queue is continuously back-logged, i.e., the transmitter of every user always has a packet to transmit to its designated receiver.

• Each user perfectly estimates the load on all channels [7] (i.e., monitors the channel utilization for a sufficient time, or by enabling local information exchange about MAP).

• For simplicity, we assume that the channel conditions of all the channels at all the users remain stable for a relatively long period of time.
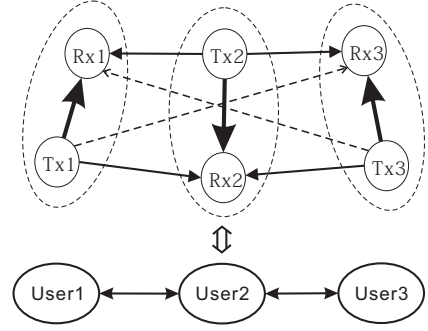

Fig. 1: 3 Tx-Rx Pairs and the corresponding Interference Graph [4]. In the upper part of the figure, the solid-thick arrow represents the transmission from a transmitter to its designated receiver; the solid-thin (dash-thin) arrow represents the non-negligible (negligible) interference from a transmitter to other unintended receivers.

If the users are located sufficiently far apart, then they can transmit in the same frequency band simultaneously without causing any performance degradation to each other. Such a spatial reuse model can be characterized by an "interference graph" as in [5], or similarly by a "contention graph" as in [10].

As an example, three Tx-Rx pairs and their equivalent interference graph are shown in Fig. 1, where users 1 and 3 can transmit concurrently without collisions but neither of them can transmit together with user 2, given that these users are sharing the same channel. Such interference relations can be characterized by an interference matrix $\mathbf{A}$. For the chain-like topology given in Fig. 1, $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ in which $a_{12} = 1$ means user 2 is a *neighbor* of user 1, $a_{13} = 0$ means user 3 is not a neighbor of user 1, etc. We further assume that the interference topology is an un-directed graph, then $\mathbf{A}$ is a symmetric matrix, i.e., $a_{ij} = a_{ji}, \forall i, j$. By default, $a_{ii} = 0, \forall i$.

The interference matrix characterizes the spatial distribution and frequency reuse capability of the users. Each user has different neighboring users who directly affect its transmission. In the single channel scenario, for a successful transmission of user i, $i \in \mathcal{N} = \{1, 2, \cdots, N\}$, all of user i's neighbors (user j with $a_{ij} = 1$), should not transmit. Therefore, assuming that each user i chooses a MAP q_i, then the throughput θ_i can be obtained as:

$$\theta_i = q_i \prod_{a_{ij}=1}(1 - q_j), \forall i \in \mathcal{N}. \tag{1}$$

Then we generalize to the case with K orthogonal channels. Hereafter, we assume the game begins with all users transmitting in one of the channels, where the interference matrix $\mathbf{A}$ is applied. Only those neighboring users transmitting on the same channel can interfere with each other. The objective of the game is to distribute these users among all channels. We shall adopt the following notations. Denote $\mathcal{K} = \{1, 2, \cdots, K\}$ as the available channel set, and $c_i \in \mathcal{K}$ as the channel selected by user $i, \forall i \in \mathcal{N}$. Since we assume that all users experience the same channel conditions on all channels, the neighboring relationship remains. After the channel allocation, all users are partitioned into G groups, where each group $\mathcal{M}_g(g \in \mathcal{G} = \{1, \cdots, G\})$ corresponds to a connected sub-network, and $\cup_{g=1}^{G} \mathcal{M}_g = \mathcal{N}$. These subnets are disconnected from others, i.e., $\mathcal{M}_{g_1} \cap \mathcal{M}_{g_2} = \emptyset, \forall g_1, g_2 \in \mathcal{G}, g_1 \neq g_2$, either

(a) K=1, G=1 [5]   (b) K=2, G=3

Fig. 2: 4 Users Topology

because they operate on different channels, or they are on the same channel but spatially disconnected. Then the throughput for user i is

$$\theta_i = q_i \prod_{j:a_{ij}=1,c_j=c_i}(1-q_j), \forall i \in \mathcal{N}. \quad (2)$$

For example, consider the interference graph in Fig. 2a, with the corresponding interference matrix $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$.

Now assume that two channels are available and the channel allocation is $\underline{c} = (1, 2, 1, 1)$ as indicated in Fig. 2b. Then there are 3 subnets, i.e., $\mathcal{M}_1 = \{1\}$, $\mathcal{M}_2 = \{2\}$ and $\mathcal{M}_3 = \{3, 4\}$. The throughput for each user, according to (2), is $\theta_1 = q_1$, $\theta_2 = q_2$, $\theta_3 = q_3(1 - q_4)$, $\theta_4 = q_4(1 - q_3)$, respectively.

We further observe that, under a certain channel allocation profile, we can improve the users' throughputs and provide a certain degree of fairness by properly managing the MAPs of the users. For the example in Fig. 2b, we can tune up the MAPs as $\underline{q} = (1, 1, 1/2, 1/2)$ so that max-min throughput is achieved in each subnet, i.e., $\underline{\theta} = (1, 1, 1/4, 1/4)$.

Therefore, the motivation of this paper is to enable the autonomous users to properly choose their channels and tune their MAPs so that the network throughput is improved while providing a certain degree of fairness for the users. In the above distributed wireless network model where users make autonomous decisions based on local information, game theory is a natural framework to analyze such strategic interactions.

III. MULTI-LEADER STACKELBERG GAMES

In this section we design a *Multi-Leader Stackelberg Game (MLSG)*, where each user i is a player who chooses the strategy (c_i, q_i) to improve its own throughput. First, multiple Stackelberg leaders are elected to manage the MAPs of all players. Then under the resulting MAP profile, each player iteratively chooses its channel to improve its own throughput. The process will repeat until a stable solution is obtained.

A. MAP Management by Multiple Stackelberg Leaders

Assume the current network settles with G subnets, as described in Section II. Each subnet $\mathcal{M}_g, g \in \mathcal{G}$ elects a player $l_g \in \mathcal{M}_g$ as the leader within this subnet. Further assume that each leader's management goal is to provide localized max-min throughput fairness for the players in its subnet, by mandating a proper common MAP value for the players in this subnet to follow. Hence, the leader l_g solves the following optimization problem:

$$\begin{aligned} \max_{q_{l_g}} \quad & \min_{i \in \mathcal{M}_g}\{\theta_i\} \\ \text{s.t.} \quad & \begin{cases} \theta_i = q_i \prod_{j:a_{ij}=1,c_j=c_i}(1-q_j), \forall i \in \mathcal{M}_g, \\ q_i = q_{l_g}, \forall i \in \mathcal{M}_g, \\ 0 \leq q_{l_g} \leq 1. \end{cases} \end{aligned} \quad (3)$$

The optimization problem in (3) can be equivalently reduced to:

$$\begin{aligned} \max_{q_{l_g}} \quad & \min_{i \in \mathcal{M}_g}\{\theta_i\} \\ \text{s.t.} \quad & \begin{cases} \theta_i = q_{l_g}(1-q_{l_g})^{N_{i,g}}, \\ 0 \leq q_{l_g} \leq 1, \end{cases} \end{aligned} \quad (4)$$

where $N_{i,g}$ is the number of neighbors for user i (excluding user i itself) in subnet $\mathcal{M}_g$, which is also known in graph theory [11] as the *node degree* of user i in this subnet. It is now obvious that the minimum throughput always happens at the player with the highest node degree in this subnet under the common MAP constraint. Since the leader l_g is responsible for solving the max-min problem in (4), it is natural to elect the player with the highest node degree as the leader l_g. For simplicity, in cases where there are multiple players in this subnet who have the same highest node degree, the one with the smallest identity number (ID) is elected as the leader. The leader l_g then solves the max-min problem in (4) by actually maximizing its own throughput:

$$\begin{aligned} \max_{q_{l_g}} \quad & \theta_{l_g} = q_{l_g}(1-q_{l_g})^{N_{l_g,g}} \\ \text{s.t.} \quad & 0 \leq q_{l_g} \leq 1. \end{aligned} \quad (5)$$

which yields the optimal solution to (4) as:

$$q_{l_g} = 1/(N_{l_g,g} + 1). \quad (6)$$

We thus formulate the problem of choosing the MAPs and subsequent spatial channel selection as a multi-leader Stackelberg game [9], where the player l_g with the highest node degree $N_{l_g,g}$ in each subnet $\mathcal{M}_g, g \in \mathcal{G}$ is elected to be the leader to manage the MAPs in this subnet. Within each subnet $\mathcal{M}_g$, the leader l_g mandates the MAPs of all players to be the same, and sets the MAP value as $1/(N_{l_g,g} + 1)$ to provide max-min throughput fairness for the players in this subnet. Such a myopic best response update by the leader requires only local information within its subnet. As a result, a MAP profile of all the players would be generated.

B. Spatial Channel Selection Process

Under the MAP profile resulting from the above MAP management process, each player then iteratively chooses its channel to improve its own throughput using the iteration dynamics formulated in [5]. Here we briefly rewrite the spatial channel selection subgame using our settings and notations:

Players: Distributed Tx-Rx pairs, $i \in \mathcal{N}$, who compete for K orthogonal channels to transmit via slotted-Aloha-like random access scheme.

Strategies: Each player i chooses a single channel $c_i \in \mathcal{K}$ to access. Its MAP $q_i \in [0, 1]$ is assumed to be fixed and given, $\forall i \in \mathcal{N}$. We denote $\underline{c} = [c_1, c_2, \cdots, c_N]$ as the channel profile and $\underline{q} = [q_1, q_2, \cdots, q_N]$ as the MAP profile of all players.

Objectives: Each player i ($i \in \mathcal{N}$) aims to maximize its utility function $u_i(\underline{c}, \underline{q}, \mathbf{A})$, which is defined as the logarithm of its throughput θ_i in its transmitting channel:

$$\begin{aligned} u_i(\underline{c}, \underline{q}, \mathbf{A}) &= \log \theta_i = \log[q_i \prod_{j:a_{ij}=1,c_j=c_i}(1-q_j)] \\ &= \log q_i + \sum_{j:a_{ij}=1,c_j=c_i} \log(1-q_j), \forall i \in \mathcal{N}. \quad (7) \end{aligned}$$

The solution of the spatial channel selection subgame is a NE of the subgame, which is defined as a strategy profile (in our case, $\underline{c}^* = [c_1^*, \cdots, c_N^*]$) in which each player i's channel

selection strategy c_i^* is a best response to the strategies of all the other players c_{-i}^* [12], i.e.,

$$c_i^* = \arg\max_{c_i \in \mathcal{K}} u_i(c_i, c_{-i}^*, \underline{q}, \mathbf{A}), \forall i \in \mathcal{N}. \quad (8)$$

Assume that asynchronous myopic best response is adopted, i.e., at any given time, only one player could update its channel selection, which aims to maximize its own utility defined in (7). To make a best response, each player needs to estimate the load on all channels and choose the one with the highest channel availability. If we assume that there exists one-hop information exchange about MAPs among the players, then on each channel $k \in \mathcal{K}$ player i observes:

$$v_i(k) := \prod_{j:a_{ij}=1, c_j=k}(1 - q_j), \quad (9)$$

which is the probability that the kth channel is available. The myopic best response by player i is therefore given by

$$\tilde{c}_i = \arg\max_{c_i \in \mathcal{K}} u_i(c_i, c_{-i}, \underline{q}, \mathbf{A}) = \arg\max_{k \in \mathcal{K}} v_i(k), \forall i \in \mathcal{N}. \quad (10)$$

Under the above asynchronous myopic best response update, the convergence to a NE $\underline{c}^*$ of the subgame is guaranteed under the theory of potential games [5].

C. Iterative Play of the MLSG game

The MLSG game is then played iteratively based on the procedure given in the above two subsections until a NE is reached. Compared to existing methods of pre-allocating MAPs, the MLSG game further improves the overall network throughput by iteratively tuning the MAPs toward max-min throughput in each subnet. We summarize the iteration process of the MLSG game in Algorithm 1.

Algorithm 1 Iteration Process of the MLSG Game

1: **Initialize**:
2: Player i stays on channel $c_i = 1$ (channel 1), for all $i \in \mathcal{N}$ (initially all players are in the same subnet);
3: Elect the player L with the highest node degree N_L as the leader;
4: Player i sets MAP $q_i = 1/(N_L + 1)$, for all $i \in \mathcal{N}$.

5: **repeat**:
6:    **repeat**:
7:       **for** $i = 1, \cdots, N$ players **do**:
8:          Estimate the channel availability $v_i(k)$ on each channel $k \in \mathcal{K}$;
9:          Choose the channel $c_i = \arg\max_{k \in \mathcal{K}} v_i(k)$;
10:       **end for**
11:    **until** No player changes its channel.
12:
13:    **for** each connected subnet $\mathcal{M}_g, g \in \mathcal{G}$ **do**:
14:       Elect the player $l_g \in \mathcal{M}_g$ with the highest node degree N_{l_g} as the leader, which broadcasts $q_{l_g} = 1/(N_{l_g} + 1)$ to all other players in $\mathcal{M}_g$;
15:       Player i sets MAP $q_i = q_{l_g}$, for all $i \in \mathcal{M}_g$;
16:    **end for**
17:    (Oscillation Resolving Mechanism)
18: **until** No player changes its MAP.

D. Oscillation Resolving Mechanism

Our simulation results show that convergence to a NE can always be achieved after a finite number of game plays, except in some cases the operating points of some players in a local region exhibit oscillation between the two dimensions of the
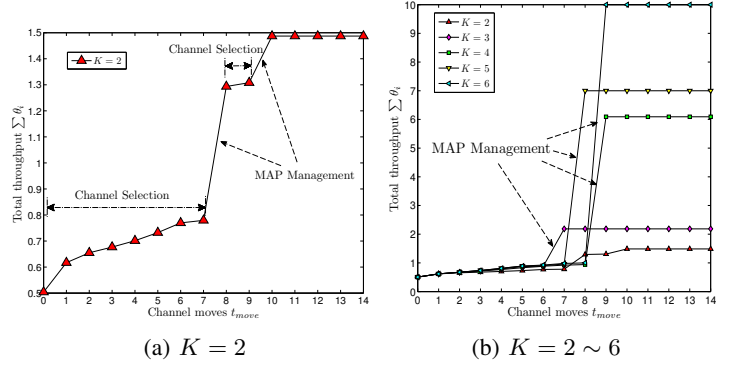

Fig. 3: Improvement over total throughput, $N = 10, K = 2 \sim 6$

myopic search. In order to resolve this problem, we introduce an *Oscillation Resolving Mechanism (ORM)* to stabilize the design.

Algorithm 2 Oscillation Resolving Mechanism

1: **for** $i = 1, \cdots, N$ players **do**:
2:    **for** $t = 2, 3, \cdots, T_{max}$ **do**:
3:       $C_i(t-1) = C_i(t)$;
4:       $Q_i(t-1) = Q_i(t)$;
5:    **end for**
6:    $C_i(t) = c_i$;
7:    $Q_i(t) = q_i$.

8:    **for** $T = 2, 3, \cdots, T_{max}/2$ **do**:
9:       **for** $h = 1, \cdots, T$ **do**:
10:          **if** $C_i(T_{max} - h + 1) = C_i(T_{max} - h + 1 - T)$
11: **and** $Q_i(T_{max} - h + 1) = Q_i(T_{max} - h + 1 - T)$
12: **and** $\exists t, s \in \{T_{max} - T + 1, \cdots, T_{max}\}$,s.t. $C_i(t) \neq C_i(s)$
13: **and** $\exists t, s \in \{T_{max} - T + 1, \cdots, T_{max}\}$,s.t. $Q_i(t) \neq Q_i(s)$ **then**
14:          Oscillation detected, with period $T_i = T$.
15:       **end if**
16:    **end for**
17:    **end for**

18:    **if** Oscillation detected **then**
19:       freeze operating point (c_i, q_i).
20:    **end if**
21: **end for**

The ORM mechanism is presented in Algorithm 2, which should be inserted in line 17 of Algorithm 1. Specifically, each player i keeps the history of its operating point (c_i, q_i) for the recent T_{max} rounds of the MLSG game play, denoted as $(C_i(t), Q_i(t)), t = 1, \cdots, T_{max}$. If the operating points oscillate with a certain period T_i, then player i would freeze its operating point at the first detected oscillation point (c_i, q_i). This simple mechanism stabilizes the whole network.

IV. SIMULATION STUDIES

A. Illustration of the MLSG Game: 10 Users Case

In this subsection we apply the MLSG game to the interference graph in Fig. 4a to illustrate the iteration process and the improvement over total network throughput. We first consider the case with $K = 2$ available channels, and the improvement steps of the MLSG game are plotted in Fig. 3a.

The game converges to a NE after $t_{move} = 10$ channel moves (for $t_{move} > 10$, no user is changing its channel and the total throughput remains unchanged thereafter). It can be
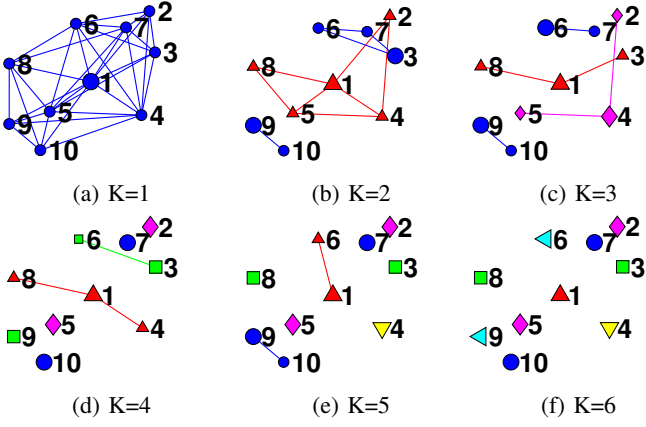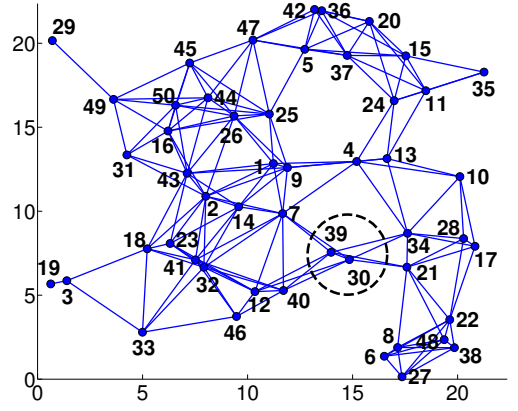
Fig. 4: 10 Users Topology, with 1∼6 Channels

Fig. 5: 50 Users Topology

Fig. 6: Oscillation States of the MLSG Game

seen from Fig. 3a that the MLSG game gradually improves the overall network throughput until reaching a NE. Specifically, there are two sudden rises in the total throughput, which are brought by two rounds of MAP management that occur at $t_{move} = 7 \sim 8$ and $t_{move} = 9 \sim 10$ respectively. When $t_{move} \leq 7$, the users are asynchronously updating their channels to improve their own throughputs, and the total throughput gradually improves as well. Then the channel selection subgame converges after $t_{move} = 7$, and the MAP management process improves the total throughput by tuning the MAPs towards max-min throughput in each subnet. As a result, the total throughput is significantly improved under the fairness constraint. Similarly, there is another round of spatial channel selection subgame and MAP tuning between $t_{move} = 8 \sim 10$, and the total throughput is further improved until converging to a NE.

Notice that the game could actually start with any pre-allocated MAPs as in [5]. However, the pre-allocated MAPs might be unfair if some users are allocated much larger MAPs than other users in the same subnet. If the game starts with such unfairly pre-allocated MAPs, then it is possible that the total throughput would decrease at first, as a result of the MAP management which re-allocates the MAPs to approximately provide max-min throughput fairness for the users in each subnet. After that, our MAP management process is able to further improve the total network throughput iteratively under the fairness constraint.

We then apply the MLSG game to the same interference graph with more available channels. The improvement steps of total network throughput with $K = 2 \sim 6$ channels are plotted in Fig. 3b. Similar to the $K = 2$ case, the MLSG game gradually improves the total network throughput until reaching a NE. In particular, the sudden rise(s) of the total throughput in each case is a result brought by the MAP management process.

The final channel allocation results for $K = 1, \cdots, 6$ are plotted in Fig. 4, where we use different colors and shapes to denote different channels, and the leader in each subnet is plotted using a bigger icon than other players. With K increasing from 1 to 6, the network is partitioned into more subnets with smaller sizes (the number of subnets G gradually increases from 1 to 10), and the total throughput of all players gradually increases as well. When $K = 6$, each player occupies a single channel without interfering other players, and the total throughput reaches 10 (each player has a throughput of 1).
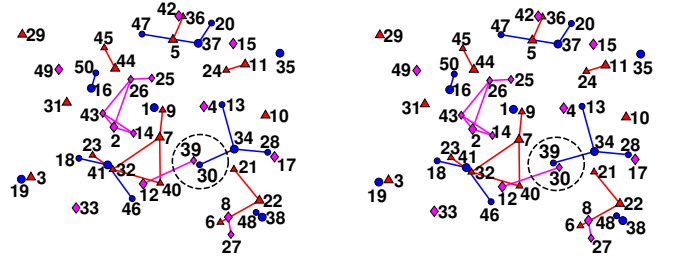
B. 50 Users Case

Consider a distributed network with N users, which are randomly placed in a square region of a given area. We assume that all users have a transmission range of 5 unit length, and that all the distances between any transmitter and its designated receiver are much smaller than the distances between any two transmitters. We further assume that in the single-channel case, those users who are in each other's transmission range will have significant interference on each other, and the two users are said to be connected. Based on the above assumptions, we can generate a random connected topology with 50 users in a square region of area 500 (units), as plotted in Fig. 5.

1) ORM Mechanism: It happens that player 30 and player 39 in this particular topology would have oscillating operating points when $K = 3$ in the MLSG game. As indicated in Fig. 6a, the operating points of player 30 and player 39 are $(c_{30}, q_{30}) = (1, 1/4)$ and $(c_{39}, q_{39}) = (3, 1/2)$ respectively. Given such a MAP profile, player 39 would jump from channel 3 to channel 1 since $v_{39}(1) = 1 - q_{30} = 1 - 1/4 = 3/4 > v_{39}(3) = 1 - q_{12} = 1 - 1/2 = 1/2$. After player 39 switched its channel, player 30 would jump from channel 1 to channel 3 since $v_{30}(3) = 1 - q_{12} = 1 - 1/2 = 1/2 > v_{30}(1) = (1 - q_{39})(1 - q_{34}) = (1 - 1/2)(1 - 1/4) = 3/8$. Then the channel selection subgame converges, with players 30 and 39 swapping their channels. After that, the MAPs of players 30 and 39 are swapped as well, as a result of the MAP management process. Now the subgame restarts with a new state as indicated in Fig. 6b. The only difference with Fig. 6a is that players 30 and 39 have swapped their operating points. In this way, there are $N_{osc} = 2$ players (players 30 and 39) whose operating points are oscillating with a period T_{osc} of 2 rounds of the MLSG
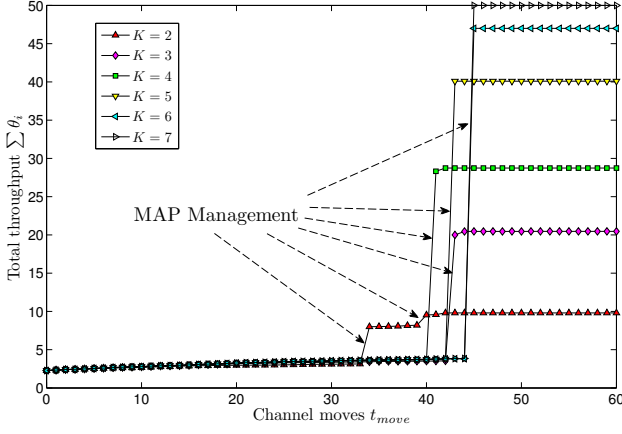
Fig. 7: Improvement over total throughput, $N = 50, K = 2 \sim 7$

TABLE I: MLSG Game ($N = 100, K = 5$)

area	D	$\Sigma\theta_i$	G	t_{move}	N_{osc}	T_{osc}
12.5	8	1.89	5	80	0	-
25	4	2.10	5	102	0	-
50	2	3.10	5	142	0	-
100	1	5.34	5	160	0	-
200	0.5	16.96	21	195	3	2
400	0.25	32.02	42	111	2	3
1000	0.1	83.50	89	81	0	-

game play.

Fortunately, we can apply the ORM mechanism to stabilize the network. In the above scenario, player 30 and player 39 both detect oscillation of their operating points, thus both of them freeze their operating points at the state in Fig. 6b, and the network is stabilized. The total number of channel moves before convergence is $t_{move} = 51$.

2) Improvement over total throughput: Similar to Section IV-A, we apply the MLSG game to the interference graph in Fig. 5 with more available channels. The improvement steps of total network throughput with $K = 2 \sim 7$ channels are plotted in Fig. 7, which shows that the MLSG game converges after $t_{move} \approx 50$ channel moves (no oscillation is detected except the $K = 3$ case, which is stabilized by ORM). Similar to the 10 users case in Fig. 3b, the MLSG game gradually improves the total network throughput until reaching a NE. In particular, the sudden rise(s) of the total throughput in each case is a result of the MAP management process which iteratively tunes the MAPs towards max-min throughput in each subnet. As a result, the total throughput is significantly improved under the fairness constraint.

C. 100 Users with Various User Density

We define the *user density* D as the number of users per unit area. For the 50 users case above, the user density is D=50/500=0.1. Now we generate a sequence of random connected topologies with various user density, by randomly scattering 100 users in a square region of various areas. In particular, when the square region has an area of 12.5, the network would become fully connected (the diagonal line length is equal to the transmission range of 5). We apply the MLSG

games to these topologies with K=5 and summarize the results in TABLE I.

It can be seen that the MLSG game converges after a finite number of channel moves. Oscillation is detected in two cases ($D = 0.25$ and $D = 0.5$) in the MLSG game, and the ORM mechanism is applied to stabilize the network. TABLE I also shows that as the user density decreases from 8 to 0.1 (from a fully connected network to a network with lower connectivity), the network is partitioned into more subnets with smaller sizes (the number of subnets G gradually increases). As a result of spatial reuse, the total throughput increases due to decreased interference from fewer neighbors experienced by each user.

V. CONCLUSIONS

We study the multi-channel spatial Aloha network with the objective to enable each autonomous user i to select a channel c_i and decide a MAP q_i to improve its throughput, while providing a certain degree of fairness among the users. Game theoretic approaches are applied, where each user i is a player who chooses the strategy (c_i, q_i) to improve its own throughput. To search for a NE, a MLSG game is formulated to iteratively obtain a solution on each dimension of the (c_i, q_i) strategy. An ORM mechanism is proposed to stabilize the design in some special cases where the operating points of some players in a local region would oscillate between the two dimensions of the myopic search. Compared to existing methods of pre-allocating MAPs, the MLSG game further improves the overall network throughput by iteratively tuning the MAPs toward max-min throughput in each subnet. Simulation results show that the MLSG game gradually improves the total throughput until reaching a NE, which also provides good throughput fairness for the players.

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