An Autonomous Pareto Optimality Achieving Algorithm beyond Aloha Games with Spatial Reuse

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***Abstract*—Aloha games with spatial reuse study the interactions among a group of selfish transmit-receive pairs which share a common collision channel using slotted-Aloha-like protocols. These Tx-Rx pairs are allowed to reuse the channel if they cause negligible interference to each other. Our work in [1] has proved the existence of a Least Fixed Point (LFP) which is the most energy-efficient operating point as well as the unique Nash Equilibrium (NE) in such games. Based on the earlier derived conditions for the stability of this NE and the way to converge to this NE, it is possible to design a self-adaptive algorithm for the players to self-adjust their target rates based on a set of pre-installed rules so that the network always achieves Pareto optimal bandwidth utilization. In this paper, we implement such an algorithm in a fully distributed manner, which requires no information exchange among the players. Each player repeatedly measures its current throughput and uses the measured value to make myopic best response to the current channel idle rate. Our simulations show that the system indeed achieves close to Pareto optimal performance while guaranteeing a certain degree of fairness. The algorithm is robust and can handle various practical issues such as the dynamic arrival/departure of players, parameter estimation errors, etc.**

I. Introduction

Game theoretic approaches have been widely used to design multiple access protocols in wireless networks. Reference [2] provides a comprehensive review of the game models developed for different multiple access schemes. In particular, several channel access games in ALOHA-like protocols are presented. For example, in [3] [4], MacKenzie and Wicker consider the slotted Aloha protocol as a game between users contending for a conventional collision channel where no two or more users are allowed to transmit simultaneously. A strategy in this game is a mapping from the number of backlogged users (assumed to be known to all users) to a transmission probability. The authors conclude that, for optimal value of the cost parameter, the throughput of a slotted-Aloha system with non-cooperative users can be as high as the throughput of a centrally controlled system. This result is generalized in [5] to show that the same result holds for multi-packet reception channels that allow more than one packet to be successfully received simultaneously.

An alternative Aloha game model is proposed by Jin and Kesidis [6], whereby a group of heterogeneous users share a conventional collision channel and transmit via slotted Aloha. Each user in this game attempts to obtain a target rate by updating its transmission probability in response to observed activities. The authors further assume in [7] that, for users with inelastic bandwidth requirements, each user's target rate depends on its utility function and its willingness to pay, and they propose a pricing strategy to control the behavior of the users (in order to bring their target rates within the feasible region). This Aloha game model is further investigated in [8]–[10]. In [8], the authors investigate the effects of altruistic behavior on the stability of equilibrium points in a two-player game. In [9], the authors generalize the model and propose a generic networking game with applications to circuit-switched networks. In [10], Menache and Shimkin extend the model by incorporating time-varying channel conditions to the channel model. The conditions for the existence and stability of the equilibrium solutions have been well studied in these works. However, the results of these studies are more suitably applied to the uplink random access channel.

Spatial reuse is a powerful technique to improve the area spectral efficiency of multi-user communication systems. In a distributed wireless network, different transmit-receive pairs at a distance away are allowed to transmit simultaneously, with the objective to achieve higher system capacity whilst still meeting all the transmission quality requirements [11]. In our previous work [1], the Aloha game model in [6] is generalized to include spatial reuse capability, named as a *generalized Aloha game*. We have proved the existence of a LFP which is the most energy-efficient operating point and the unique NE in such games. We also provide the conditions for the stability of this NE, and the way to reach this NE in game iterations.

In this paper, we go beyond the generalized Aloha game defined in [1], and study how future autonomous radios can make use of the developed theory to improve the overall system performance. For example, consider the scenarios where cognitive radio pairs are competing among themselves to transmit over the channel. If some or all the players are over-demanding (total target rate beyond the network capacity), the resulting network is unstable and all pairs will suffer from network congestions. On the other hand, if the players set a low target rate, the network is stable but the bandwidth is not fully exploited. We therefore call for a set of target rate adjusting rules which are commonly agreed by the players, and enable the players to improve their throughputs without affecting the network stability. This will result in a win-win situation for all transmission pairs. In order to accomplish such goals, we therefore develop an autonomous Pareto optimality achieving algorithm beyond the generalized Aloha game.

Our main contributions in this paper are as follows. First of all, we implement the algorithm in a fully distributed manner, which requires no information exchange among the players. As is commented in [2], the cost of information gathering needs to be considered when designing games in communication networks. In our algorithm, each player measures its current throughput and uses it to make myopic best response to the current channel idle rate. Therefore, there is no communication overheads in performing information exchange among the players. Moreover, channel sensing is necessary and needs to be performed only once for a player to set its initial target rate when it joins the network. Secondly, we design a set of target rate adjusting rules to control

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the players' behaviors in a distributed manner. Each player uses its measured throughput to dynamically adjust its target rate so that all players have their throughputs adaptively approaching the Pareto optimal bandwidth utilization. The predefined rules can be set to guarantee certain criteria of fairness. Finally, the algorithm is robust and can handle various practical issues such as dynamic arrival/departure of players, parameter estimation errors, etc.

In Section II, we introduce the generalized Aloha game and summarize our analytical results from [1]. We present the details of our algorithm in Section III, then we describe how the measured throughput can be obtained from practical channel collision scenarios in Section IV. We test our algorithm through extensive simulations in Section V and conclude the paper in Section VI.

II. Aloha Games with Spatial Reuse

Consider a distributed network with N transmitters, where each transmitter has its unique designated receiver. Each Tx-Rx pair is a player who competes for the channel to transmit. The conventional Aloha games are generalized to the scenarios where there exists spatial reuse among a group of non-cooperative players, i.e., those players who will not interfere each other can transmit concurrently. Here, only a connected network is considered (If the network is not connected, then it can be divided into several independent connected sub-networks, and then be dealt with separately). We assume that every player's transmission queue is continuously backlogged, i.e., the transmitter of every player always has a packet to transmit to its designated receiver.

As an example, three Tx-Rx pairs and their equivalent chain-like topology are shown in Fig. 1, where players 1 and 3 can transmit concurrently without collisions but neither of them can transmit together with player 2. Such interference relations can be characterized by an interference matrix $\mathbf{A}$. For the chain-like topology given in Fig.1,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

in which $a_{12} = 1$ means player 2 is a one-hop neighbor of player 1, $a_{13} = 0$ means player 3 is not a one-hop neighbor of player 1, etc. Notice that in this example $\mathbf{A}$ is a symmetric matrix. However, $a_{ij} = a_{ji}$ is not necessarily true, i.e., the interference topology is a directed graph.

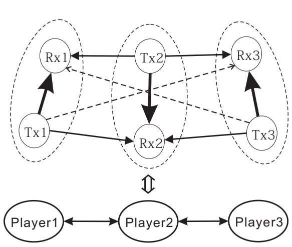

Fig. 1: 3 transmit-receive pairs

The interference matrix characterizes the spatial distribution and frequency reuse capability of the players. Each player has different neighboring players which directly affect its transmission. For a successful transmission of player i, $i \in \mathcal{N} = \{1, 2, \cdots, N\}$, all of player i's one-hop neighbors (player j where $a_{ij} = 1$), should not transmit. Therefore, assuming that each player i chooses a transmission probability q_i, then the measured throughput θ_i can be obtained as:

$$\theta_i = q_i \prod_{a_{ij}=1}(1 - q_j), \forall i \in \mathcal{N}. \tag{1}$$

With the interference matrix, we can now study the behavior of the generalized Aloha game. The objective of each player i is to select a suitable transmission probability q_i so that it achieves its target rate y_i, with the lowest possible energy consumption, i.e., each player uses the smallest transmission probability as it could to attain its target rate. We now formally state the generalized Aloha game as follows:

Players: Distributed Tx-Rx pairs, $i \in \mathcal{N}$, who compete for a single collision channel to transmit via slotted-Aloha-like random access scheme.

Actions: Each player *i* chooses a transmission probability $q_i \in [0, 1]$, $\forall i \in \mathcal{N}$.

Objectives: Each player *i* ($i \in \mathcal{N}$) aims to minimize the energy consumption in attaining its target rate y_i, i.e.,

$$\begin{array}{ll} \min & q_i \\ \text{s.t.} & y_i = \theta_i = q_i \prod_{a_{ij}=1}(1 - q_j). \end{array} \tag{2}$$

In order to make the intermediately measured throughput θ_i approach the target rate y_i, player i' myopic best response strategy in the $(m + 1)$th iteration is given as:

$$q_i^{(m+1)} = \min\{\frac{y_i}{\prod_{a_{ij}=1}(1 - q_j^{(m)})}, 1\}, \quad \forall i \in \mathcal{N}. \tag{3}$$

Notice that we explicitly include the bound $q_i = 1$ in (3) to ensure that the mapping is within the compact domain $[0, 1]^N$. This would introduce an extraneous solution $\underline{q}^* = \underline{1}$, which happens when the system diverges to a dead-end situation with $\underline{q} = \underline{1}$ and $\underline{\theta} = \underline{0}$. Despite this undesirable situation, a stable NE solution would satisfy $q_i^* = y_i / \prod_{a_{ij}=1}(1 - q_j^*), \forall i \in \mathcal{N}$. Therefore, at such an operating point $\underline{q}^*$, the measured throughput θ_i is strictly equal to the target rate y_i, i.e., $\theta_i = q_i^* \prod_{a_{ij}=1}(1 - q_j^*) = y_i, \forall i \in \mathcal{N}$. Besides satisfying the equality constraints in (2), we also prove in [1] that there exists a least fixed point (LFP) which enables each player to operate with the minimal transmission probability concurrently. This optimal solution $\underline{q}^*$ is then the unique NE of the generalized Aloha game defined in (2).

We then study the convergence method and stability of the NE. We derive in [1] that the initial transmission probabilities $\underline{q}^{(0)}$ can be set equal to the target rates $\underline{y}$ to reach the NE through game iterations. Moreover, the stability conditions of the NE is given in Proposition 3 in [1], which can be used to find the feasible region for the target rate combination $\underline{y}$. The word "feasible" here means that, at the stabilized operating point $\underline{q}^*$, the measured throughput $\underline{\theta} = \underline{y}$, i.e., the target rate combination $\underline{y}$ is achievable.

We illustrate the feasible target rate region (the region under the mesh surface) for the 3-player chain-like topology in Fig. 2. Each point on the surface is a target rate combination that achieves the Pareto optimal bandwidth utilization.

III. Fully Distributed Algorithm

From the analytical results, we know that there exists a feasible region of the target rate combination $\underline{y}$. The upper boundary of this feasible region is the Pareto front [12]. The overall design objective is to achieve Pareto optimal bandwidth utilization for all players. While all the players self-adjust their target rates to approach the Pareto front, it is necessary to guarantee system stability with a predefined way of maintaining certain fairness

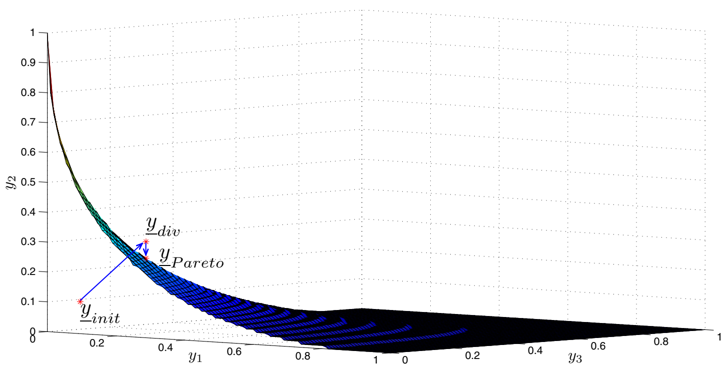

Fig. 2: Feasible target rate region, 3-player chain-like topology

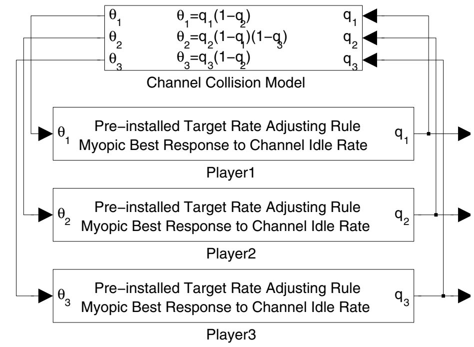

Fig. 3: System diagram for 3-player chain-like topology

among the players. The general ideas behind such target rate adjustment are as follows:

1) Suppose the initial target rate combination $\underline{y}_{init}$ is within the feasible region (as illustrated in Fig. 2). After the system is stabilized and $\underline{y}_{init}$ is achieved, the players find that there is a room to increase their transmission rates, thus repeatedly increasing the target rates in some predefined manner until reaching a point $\underline{y}_{div}$ beyond the Pareto front.

2) Since $\underline{y}_{div}$ is outside the feasible region, the system will diverge to $\underline{q} = \underline{1}$, and $\underline{\theta} = \underline{0}$. We can then reduce $\underline{y}_{div}$ in some predefined manner to bring it back to the feasible region. We can refine the steps of decrement so that the target rate combination will come to a point $\underline{y}_{Pareto}$ on the Pareto front.

3) We should guarantee certain criteria of fairness among the players. E.g., a player with a larger target rate should increase less when the target rate combination goes from $\underline{y}_{init}$ to $\underline{y}_{div}$, and decrease more when it goes from $\underline{y}_{div}$ to $\underline{y}_{Pareto}$.

The above target rate adjusting mechanism is designed to provide best effort transmissions for users with elastic traffic. This is different from [7] which is designed for users with inelastic bandwidth requirements, though they both try to adjust the target rates to maintain system stability and achieve better bandwidth utilization. Another difference is that the target rate adjustment in [7] is centrally controlled by network pricing strategies. As a result, our model can find its applications when distributed cognitive radio pairs are competing among themselves to transmit over the channel. If everyone wants to transmit more, the interference level will be high and all pairs will suffer, whilst if everyone transmits at a low probability, the bandwidth is not fully exploited. Therefore, we would like each device to equip with intelligence so that while competing to transmit, each transmission pair is also governed by the underlying rules so that maximum throughput can be achieved without affecting the network stability.

A. System Diagram

In the channel collision model, each player i transmits with a certain probability q_i. Simultaneously transmitted packets are either successful or in collision according to the relationship specified by the interference matrix. Then each player i can monitor its successful packet rate (throughput) θ_i, and use it to 1) make myopic best response to the current channel idle rate; 2) dynamically adjust its target rate from $y_{i,init}$ according some pre-installed target rate adjusting rules. For the example of 3-player chain-like topology, the general implementation scheme is shown in Fig. 3.

B. Myopic Best Response to Channel Idle Rate

The channel idle rate of player i's neighborhood is defined as the probability that none of player i's neighbors are transmitting in a slot, i.e., $x_i := \prod_{a_{ij}=1}(1-q_j)$. Given a target rate y_i, player i's myopic best response to the channel idle rate in the $(m+1)$th iteration is given in (3). Since a fully distributed algorithm requires no information exchange among the players, x_i should be estimated by player i itself.

For the initial channel idle rate $x_i^{(0)}$, a one-time channel sensing is needed, i.e., player i listens to the channel for some time and counts the number of idle slots to obtain $x_i^{(0)}$. Afterwards, player i starts transmitting, and $x_i^{(m)}, (m \geq 1)$ can be obtained from the measured throughput $\theta_i^{(m)}$, which requires no channel sensing. Specifically, since the measured throughput in the mth iteration is $\theta_i^{(m)} = q_i^{(m)} \prod_{a_{ij}=1}(1 - q_j^{(m)}) = q_i^{(m)} x_i^{(m)}$, we then have $x_i^{(m)} = \theta_i^{(m)} / q_i^{(m)}$.

C. Pre-Installed Target Rate Adjusting Rules

1) Initializing Target Rates: When player i powers on, it first synchronizes to its neighbors. Then player i sets its initial target rate $y_{i,init}$ based on $x_i^{(0)}$. During the one-time sensing process, player i also finds out the number of active neighbors, i.e., $N_i = \sum_j a_{ij}$. Then player i uses the geometric mean of $1 - q_j$ (i.e., x_i^{1/N_i}) and assumes the transmission probability of every of its neighbors to be $\bar{q}_{-i} := 1 - x_i^{1/N_i}$.

In terms of fairness, we set player i's initial target rate in approximation to the average target rate of its neighbors. Since the actually achieved throughput of a player is no larger than its transmission probability due to collisions, therefore, under the current $\bar{q}_{-i}$, we guess the average target rate of player i's neighbors $\bar{y}_{-i} \leq \bar{q}_{-i}$. Therefore, we design $y_{i,init} = \bar{q}_{-i}^{\gamma}$, in which the exponent parameter $\gamma \geq 1$ so that $y_{i,init} \leq \bar{q}_{-i}$.

We also apply a minimum and a maximum initial target rate m_{init} and M_{init} $(0 < m_{init} < M_{init} \leq 1)$. m_{init} is needed in the case that none of player i's neighbors are active when player i powers on, and hence player i attempts to transmit with a small initial target rate. M_{init} is used to proportionally control the size of the increment. Therefore, the Target Rate Initializing Curve is designed as:

$$y_{i,init} = \max\{m_{init}, M_{init} \cdot \bar{q}_{-i}^{\gamma}\}, \gamma \geq 1. \tag{4}$$

2) Increase Target Rates from $\underline{y}_{init}$ to $\underline{y}_{div}$: After the system is stabilized and $\underline{y}_{init}$ is achieved, we can increase the target rates by a step size, and wait for the system to be stabilized again. Then we increase the target rates again until the system diverges (with target rate combination $\underline{y}_{div}$).

Based on fairness criteria, the player who currently has a larger target rate should increase less in the above process. Conversely, the player who has a smaller target rate deserves a larger step size. The Target Rate Increment Curve is given in (5). M_{inc} is used to proportionally control the increment size, while $(1-y_i)^{\alpha}$ is used so that the increment steps are refined and gradually decrease as the target rate increases.

$$\Delta_{i,inc} = M_{inc} \cdot (1-y_i)^{\alpha}, \alpha \geq 1, 0 < M_{inc} \leq 1. \tag{5}$$

3) Reduce Target Rates from $\underline{y}_{div}$ to $\underline{y}_{Pareto}$: Since the system diverges under $\underline{y}_{div}$, we reduce the target rates by a step size and wait for the system to be stabilized. If the system still diverges, we reduce the target rates again until the system becomes stabilized (with target rate combination $\underline{y}_{Pareto}$).

Based on fairness criteria, the player who currently has a larger target rate should decrease more. The Target Rate Decrement Curve is given in (6). M_{dec} is used to proportionally control the size of decrement, while y_i^{β} is used so that the decrement steps are refined and gradually decrease as the target rate decreases.

$$\Delta_{i,dec} = M_{dec} \cdot y_i^{\beta}, \beta \geq 1, 0 < M_{dec} \leq 1. \tag{6}$$

When player i detects divergence and reduces its target rate from y_i to y_i', it will lock the transmission probability $q_i = y_i'$ for some time, so that the previously high contention it caused to other players will be removed.

4) Increase Target Rates again when Channel Idle Rate Significantly Rises: The system finally settles down and achieves maximum bandwidth utilization $\underline{y}_{Pareto}$. Later when some of player i's neighbors leave the collision channel, player i might detect significant rise of the channel idle rate. In such cases, it will attempt to increase its target rate again. The curve in (5) can be used again to increase the target rates. An alternative way is to predict the amount of increment currently available for player i, and set the increment accordingly.

The amount of increment available for player i is upper bounded by $(\frac{x_i}{x_{i,inf}}-1)y_i$. $x_{i,inf}$ is the lowest value of x_i recorded whenever the system is stabilized. In other words, $x_{i,inf}$ stands for the highest level of contention under which the system is still stable. ($x_{i,inf}$ should be reset to 1 and recorded again when divergence happens.) If we observe that $x_i = b \cdot x_{i,inf}, (b \geq M_{rise} > 1)$, i.e., there is a significant rise of the channel idle rate, and we keep q_i unchanged, then ideally we can achieve a higher throughput $\theta_i' = q_i \cdot x_i = q_i \cdot b \cdot x_{i,inf} = b \cdot \theta_i$. Therefore, the amount of increment available for player i is upper bounded by $(\frac{x_i}{x_{i,inf}}-1)y_i$.

Another design principle is based on fairness criteria. The player who has a larger target rate should increase less aggressively, although it might have a larger amount of increment available. Therefore, we apply a factor of $(1-y_i)^{\rho}, \rho \geq 0$ to the target rate increment. Finally, the Target Rate Increment Curve (Channel Idle Rate Rises) is given in (7).

$$\Delta_{i,inc2} = (x_i/x_{i,inf}-1)y_i \cdot (1-y_i)^{\rho}, \rho \geq 0 \tag{7}$$

D. Measured Throughput Characteristics

The measured signal is the throughput θ_i that player i achieved during one iteration time. We need to characterize it to judge the convergence or divergence of the system.

When the system is converging, the absolute gap between the target rate and the measured throughput will diminish with time, and finally go to 0 when the system becomes stabilized. The dynamic characteristics is $|y_i-\theta_i^{(m)}| \leq |y_i-\theta_i^{(m-1)}|$. The steady-state characteristics is $|y_i-\theta_i| \leq m_{gap}$, where m_{gap} is a small positive threshold value.

When the system is diverging, the measured throughput θ_i is less than the target rate y_i, and θ_i keeps decreasing until it becomes 0. During the diverging process, the dynamic characteristics is $y_i - \theta_i^{(m)} > y_i-\theta_i^{(m-1)} > 0$. If θ_i drops below a certain threshold, i.e., $y_i-\theta_i \geq M_{gap} > 0$, player i will judge the system as diverging. Another scenario for player i to judge system divergence is that player i has reached the dead-end situation, i.e., $q_i = 1, \theta_i = 0$.

IV. Modelling Practical Packet Collisions

A. Estimating Throughput

So far we have been using the mathematical model in (1) to model the channel collisions. We now describe how player i's throughput can be estimated from practical packet collision scenarios. Suppose each iteration consists of L_I slots. In the *m*th iteration, player i generates $\lfloor q_i^{(m)} \cdot L_I \rfloor$ packets, and randomly scatter these packets in the L_I slots. We then use the interference matrix to judge the status of each player's packets, i.e., successful or in collision. Then player i counts the number of successful packets $N_{suc,i}^{(m)}$ in the *m*th iteration. Its throughput is then estimated by $\hat{\theta}_i^{(m)} = N_{suc,i}^{(m)}/L_I$. This $\hat{\theta}_i$ is used as the measured throughput in our algorithm.

B. Measures Taken to Handle Estimation Error

1) The estimation error of $\hat{\theta}_i$ compared to its genuine value θ_i is mainly affected by the iteration length L_I. The larger L_I is, the smaller the estimation error. However, a larger L_I also means that it takes longer time (even with the same number of iterations) for the system to converge.

2) We apply a L_q-taps mean-value filter to player i's transmission probability q_i, so as to smooth out the trembling effect introduced by the estimation error of $\hat{\theta}_i$.

3) We compose every L_B iterations as a block, and assume the players to be block synchronized. Player i would make a judgement about system convergence or divergence at the end of each block. The measured throughput is averaged over each block to reduce the effects of the estimation error. In this way, the judgements made would have smaller errors.

V. Simulation Studies

We implement our scheme for $N = 11$ players with an interference topology given in Fig. 4. The parameters for the target rate adjusting curves are: γ=1.5, m_{init}=0.01, M_{init}=0.8; α=25, M_{inc}=0.02; β=1.2, M_{dec}=0.2; ρ=0.35, M_{rise}=1.1. The threshold parameters for the measured throughput characteristics are: m_{gap}=0.005, M_{gap}=0.025. The parameters for the practical packet collision model are: L_I=2000, L_q=10, L_B=4. Suppose the channel bandwidth is 20MHz and the packet length is 100 bits, then the slot time is $5\mu s$. In this case, L_I=2000 means that iterations take place every 10 ms.

The simulation consists of 3 phases. *Phase 1*: Player $1 \sim 10$ power on sequentially at iterations $= 1, 10, 20, 30, \cdots, 90$, while player 11 remains at power-off. *Phase 2*: Player 11 powers on at iterations $=$ 2000. *Phase 3*: Player 1 powers off at iterations $=$ 4000. The results are plotted in Fig. 5.

Phase 1: Player $1 \sim 10$ power on sequentially, and the system settles down within 1000 iterations. Upon starting up, player

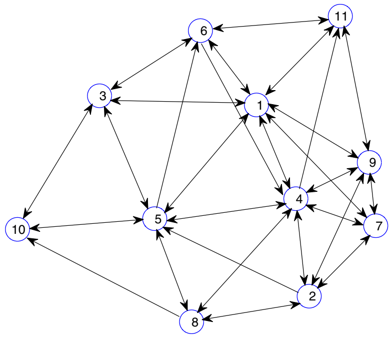

Fig. 4: Interference topology for 11 players

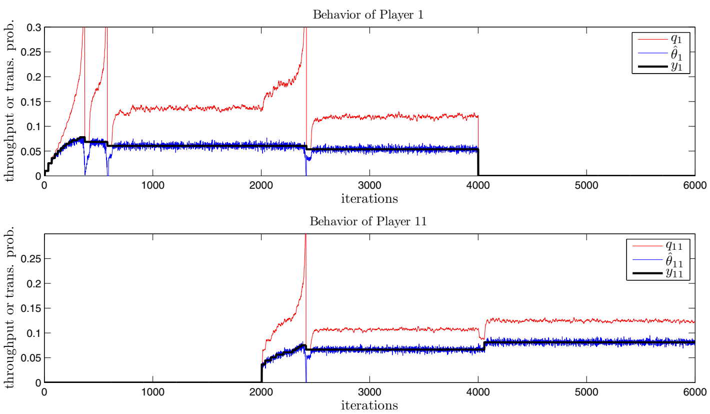

Fig. 5: Behaviors of player 1 and player 11

1 attempts transmitting with a small initial target rate. Then it gradually increases the target rate in diminishing steps until the system diverges. When divergence is detected, player 1 reduces its target rate in diminishing steps. Then the system becomes stabilized and achieves $\underline{\hat{y}}_{Pareto}$ close to the Pareto front. Notice that there exists a trade-off between the convergence time and the distance to the Pareto front. If we decrease the target rate in very small steps, then $\underline{\hat{y}}_{Pareto}$ will be very close to the Pareto front. However, the system will experience more times of divergence before it settles down.

We can use the analytical method in [1] to find out the "distance" between $\underline{\hat{y}}_{Pareto}$ and the Pareto front, i.e., we proportionally increase $\underline{\hat{y}}_{Pareto}$ until it is not achievable according to the analytical method. The increased ratio $k(k \geq 1)$ then indicates such distance. For phase 1, $k = 1.05$. In addition, the mean value and standard deviation of $\underline{\hat{y}}_{Pareto}$ is 0.0689 and 0.0093 respectively. The standard deviation is small compared to the mean value, thus suggesting a certain degree of fairness among the players. Finally, the estimation error of $\hat{\theta}_i$ is well handled, and the steady state error between the average $\hat{\theta}_i$ and y_i is close to 0.

Phase 2: Player 11 sets $y_{11,init}$ close to the current target rates of its neighbors. As player 11 gradually increases its target rate and raises the contention level of its neighbors, the system diverges some time later. Both player 1 and 11 detect divergence and reduce their target rates. Then the system becomes stabilized with $k = 1.08$. The mean value and standard deviation of $\underline{\hat{y}}_{Pareto}$ is 0.0653 and 0.0131 respectively.

Phase 3: Player 1 powers off and player 11 detects significant rise of the channel idle rate, thus increasing its target rate based on the prediction given in (7). The system then becomes stabilized again with $k = 1.09$. The mean value and standard deviation of $\underline{\hat{y}}_{Pareto}$ is 0.0770 and 0.0100 respectively.

Brief summary: Phase 1 illustrates the target rate adjusting rules envisioned at the beginning of Section III. The target rate increment process and decrement process are illustrated. After the adjustment, the system becomes stabilized and achieves close to Pareto optimal bandwidth utilization (distance to Pareto front $k = 1.05$, close to 1). Phase 2 demonstrates the dynamics in the players' throughputs when a new player enters the system. The newly entered player chooses a proper initial target rate close to the average value of its neighbors. Moreover, after dynamic target rate adjustment, the system becomes stabilized again and achieves close to Pareto optimal performance ($k = 1.08$). Phase 3 demonstrates the case when an existing player leaves the system. The remaining players are able to detect the bandwidth opportunity left over by the leaving player, and adjust their target rates accordingly so that the system still operates at a point close to the Pareto front ($k = 1.09$). Finally, the standard deviation of $\underline{\hat{y}}_{Pareto}$ is small compared to its mean value in Phase $1 \sim 3$ respectively, suggesting a certain degree of fairness among the players.

VI. Conclusions

This paper goes beyond the Aloha games with spatial reuse in [1], and develops an autonomous Pareto optimality achieving algorithm that enables the players to maximize their throughputs without affecting the network stability. The algorithm is implemented in a fully distributed manner, which requires no information exchange among the players. Our simulations show that the system indeed achieves close to Pareto optimal performance while guaranteeing a certain degree of fairness. The algorithm is robust and can handle various practical issues such as the dynamic arrival/departure of players, parameter estimation errors, etc.

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