Advanced Macroeconomics: Het agent Model assignment 2

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A Stationary Equilibrium without a government

Definition of the stationary equilibrium

The stationary equilibrium is defined as follows:

$$H_{ss} = \begin{bmatrix} r_{ss}^K - \alpha\Gamma_{ss}^{\Upsilon}(K_{ss}/L_{ss}^{\Upsilon})^{\alpha-1} \\ r_{ss} - (r_{ss}^K - \delta) \\ w_{ss} - (1-\alpha)\Gamma_{ss}^{\Upsilon}(r_{ss}^K/L_{ss}^{\Upsilon})^{\alpha} \\ w_{ss}^t - w_{ss}(1-\tau_{ss}) \\ \mathbf{D}_{ss} - \prod_z' \underline{\mathbf{D}}_{ss} \\ \underline{\mathbf{D}}_{ss} - \Lambda_{ss}' \\ A_{ss} - K_{ss} \\ L_{ss}^{hh} - (L_{ss}^{\Upsilon} + L_{ss}^{G}) \\ A_{ss}^{hh} - A_{ss} \\ S_{ss} - min\{G_{ss}, \Gamma^G L_{ss}^G\} \\ G_{ss} + w_{ss}L^G + \chi_{ss} - (\tau w_{ss} L_{ss}^{hh}) \end{bmatrix} = 0$$

Values at the Stationary Equilibrium

Table 1 shows the steady-state values of the model with no government (Model A), as well as the models in the subsequent sections. These values result from the equation system H_{ss}, policy functions, which will be detailed below, and aggregation.

	Model A	Model B	Model C	Model D
K	3.389	2.897	3.479	3.731
L_Y	0.923	0.843	0.946	0.882
rK	0.121	0.126	0.121	0.120
w	1.034	1.014	1.034	1.187
Y	1.363	1.221	1.399	1.495
Gamma_Y	1.000	1.000	1.000	1.100
A	3.389	2.897	3.479	3.731
r	0.021	0.026	0.021	0.020
tau	0.000	0.655	0.477	0.482
wt	1.034	0.350	0.541	0.615
L_G	0.000	0.415	0.444	0.465
G	0.000	0.415	0.444	0.465
Chi	0.000	0.000	-0.218	-0.248
S	0.000	0.415	0.444	0.465
Gamma_G	1.000	1.000	1.000	1.000
A_hh	3.389	2.897	3.479	3.731
C_hh	1.024	0.517	0.606	0.657
ELL_hh	0.990	1.336	1.545	1.499
L_hh	0.923	1.258	1.391	1.347
INC_hh	1.024	0.517	0.606	0.657
U_hh	-100000001.618	-5.559	-5.476	-5.154
L	0.923	1.258	1.391	1.347
I	0.339	0.290	0.348	0.373
clearing_A	0.000	-0.000	-0.000	0.000
clearing_L	0.000	0.000	0.000	0.000
clearing_Y	0.000	0.000	0.000	0.000
clearing_G	0.000	-0.000	0.000	-0.000

Note: Each model correspond to the section in question.

Table 1: Steady state values

Policy Functions

The policy functions, as shown in 1, depict agents' decisions given their start-of-period assets in the steady state, for three different idiosyncratic productivity shocks.

In theright frame of figure 1, the endogenous labor supply is Depicted. This can be interpreted as the actual hours worked by households. Households with very low savings levels (effectively hand-to-mouth households) and a very low productivity shock exhibit the highest labor supply, compensating for their *bad luck* in productivity and lack of savings to augment their spending. Here, the income effect dominates the substitution effect. However, when the initial asset amount increases even slightly, workers with higher productivity shocks opt to supply more labor. With their higher productivity, they earn higher wages, making the cost of leisure more expensive, hence the substitution effect dominates.

The policy response for consumption indicates that higher productivity and a higher level of initial assets lead to increased consumption.

The middle frame illustrates the savings decisions of households. The function appears nearly linear, suggesting that once households have built a sufficient buffer stock, they spend the majority of their income.

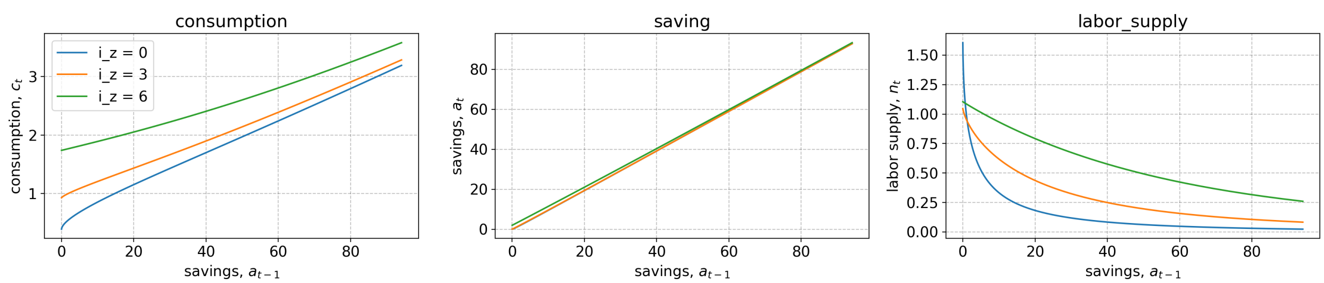

Figure 1: Policy functions

Distribution

Figure 2 shows the distribution of productivity shocks and assets. From the right frame, we can observe that almost 40 percent of households hold zero wealth, effectively acting as hand-to-mouth (H-t-M) households.

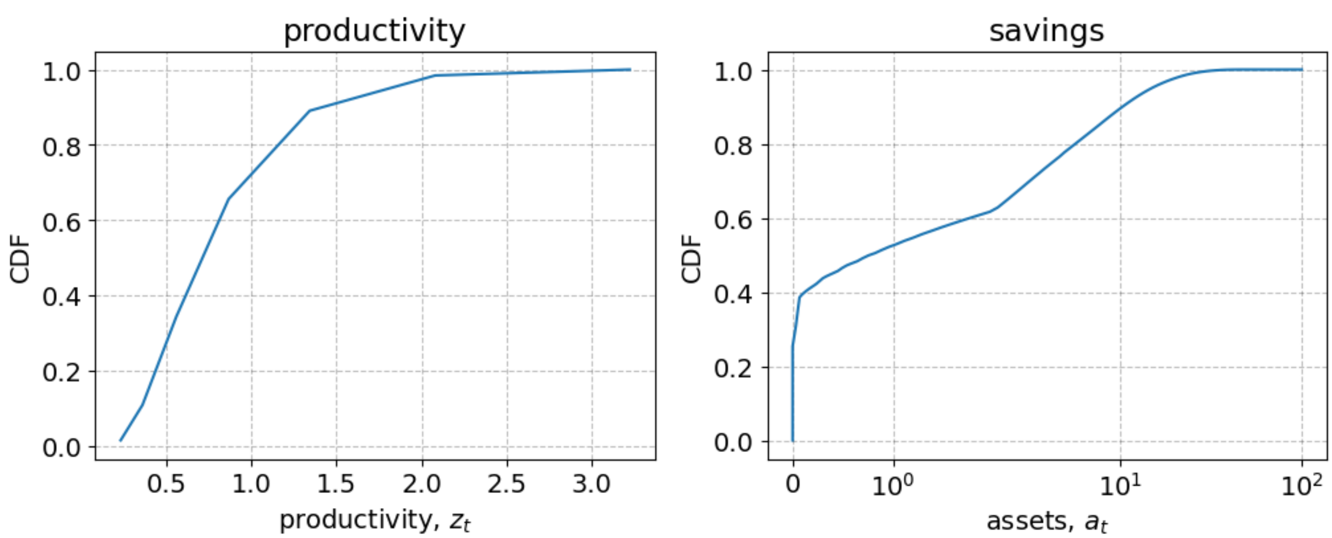

Figure 2: Distributions

Utility

In the absence of government, the expected discounted utility trends towards an exceedingly negative level (-

	B: Government, no transfers	B: Government and transfers	Higher productivity
Chi_val	0.000	-0.218	-0.248
Gov_opt	0.415	0.444	0.465
Gamma_Y	1.000	1.000	1.100
U_opt	-138.968	-136.889	-128.840
Y_G_relatio	0.340	0.318	0.311

Table 2: Additional calulations

25,000,000,037).[1] It does not approach $-\infty$ due to the presence of $\underline{S}$, which could be interpreted as representing a very minimal government intervention.

Utility in a given period:

$$v_t(c_{it}, S_t, \ell_{i,t}) = \underbrace{\frac{c_{i,t}^{1-\sigma}}{1-\sigma}}_{\text{Utility of consumption}} + \underbrace{\frac{\left(S_t + \underline{S}\right)^{1-\omega}}{1-\omega}}_{\text{Utility government production}} + \varphi \underbrace{\frac{\left(\ell_{i,t}^{1+\nu}\right)^{1+\nu}}{1+\nu}}_{\text{Dis-utility of labor}} \tag{1}$$

For given parameters and $S_t \to 0$, utility from government production $\to \frac{(10^{-8})^{-1}}{-1}$, A large negative value reflecting very low utility. The model is truncated at 500. To ensure that the utility is an approximation for an infinite period, the discounted sum of utility is plotted in Figure3, from which it is evident that the truncation at time $t = 500$ is justified.

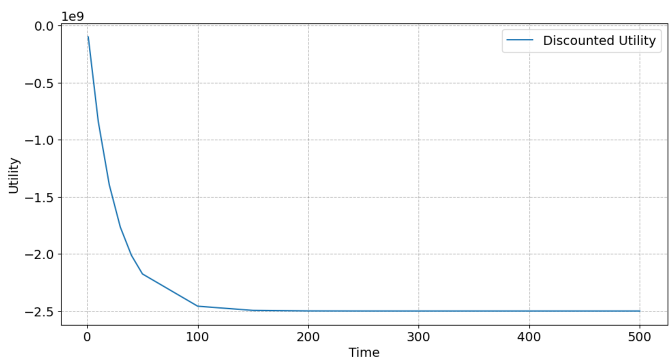

Figure 3: Accumulation discounted utility

B Optimal welfare policy I: Without transfer payments

Figure 4 plots the expected discounted utility for a given level of government production. The optimal level of government production, assuming no transfers, is 0.415, which corresponds to roughly one-third of total private production allocated towards government production (see Table 3). The expected discounted utility at this level is -138.968, in contrast to the -2.5 $\times 10^9$ value observed in the absence of government production.

[1] We cannot conclude that this utility is low merely because it is negative, as utility is ordinal. This is indicated by the utility function and calculations in question, suggesting that this is indeed a low utility level.

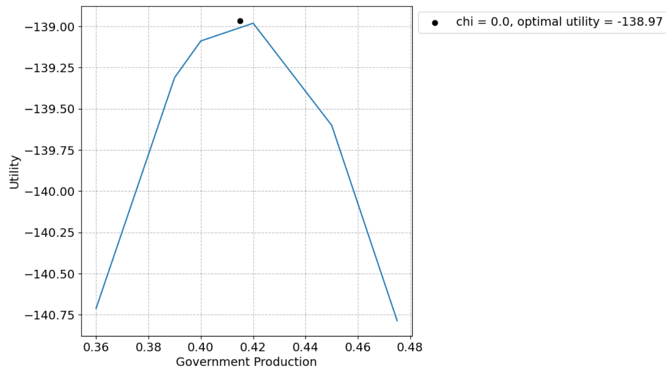

Figure 4: Optimal Government Production

C Optimal welfare policy II: With transfer payments

Positive or negative transfer's

It is preferable for the government to have a negative transfer, that is, a lump sum tax from the household's (figure 5).

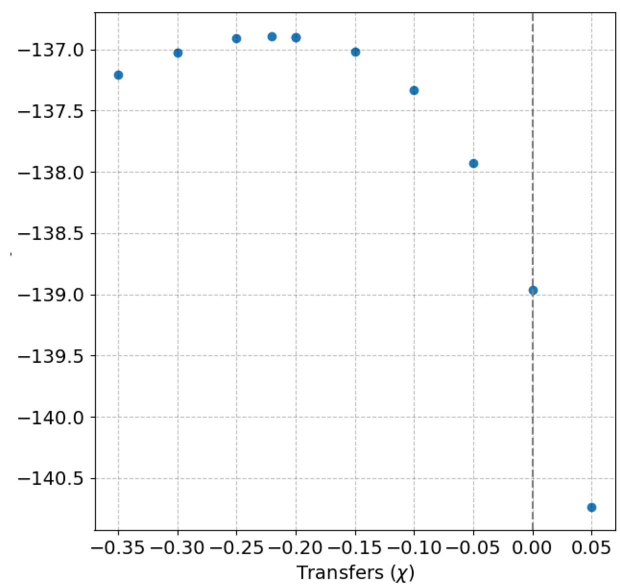

Figure 5: Optimal Government Production

Optimal transfer payments and government production

The optimal level of transfer payments is -0.218 (lump-sum tax), with government production at 0.444, resulting in a utility of -136.889. Figure 6 shows the utility associated with government production levels, for various χ

By implementing a lump-sum tax, the government can balance its budget without making work less attractive through increased taxes. As a result, private production increases more than government spending, leading to a decrease in government expenditure as a fraction of total production.

	B: Government, no transfers	B: Government and transfers	Higher productivity
Chi_val	0.000	-0.218	-0.248
Gov_opt	0.415	0.444	0.465
Gamma_Y	1.000	1.000	1.100
U_opt	-138.968	-136.889	-128.840
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Table 3: Additional calulations

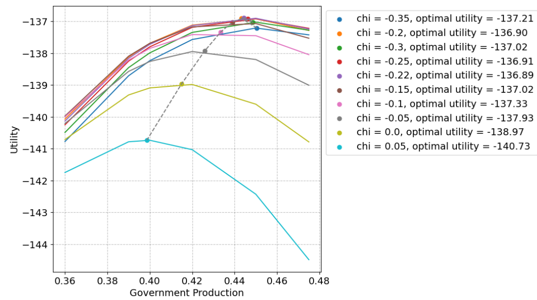

Figure 6: Utility given optimal government production, for transfer levels

D Increased TFP

Increasing productivity in the private sector has several effects. From the household perspective, higher productivity implies increased wages, which in turn boosts the labor supply. This aligns with findings from the policy functions: households with higher productivity shocks opt to supply more labor (Table 1 and figure 1). On the production side, firms increase output, both due to the larger labor supply and the enhanced productivity of this labor. The government's demand for labor also rises.
Although the government's productivity has not increased, the households' preference for both government and private production in their utility means a higher optimal level of government production. The government share of total private production falls.(see Table 3).

E Transition path

As the Total Factor Productivity (TFP) increases, there is a new optimal level of government production. However, capital takes time to accumulate, and therefore private production does not immediately adjust to the new steady-state levels. This delay might result in the government consuming ”too much” of the private production and employing ”too much” of the labor force and a potentially suboptimal transfer level. Figure 7 shows the transition paths for two government policies: one where government production immediately jumps to the new steady-state level, and another where government production linearly transitions from the previous steady-state level to the new one over 30 periods. Transfers are adjusted immediately to the new steady-state levels. The associated utility values are -129.446 for the model with the immediate increase in government production, and -135.24 for the model with linear implementation of government production. Of these two, the immediate implementation is preferable. In the linear implementation model, private production increases excessively (see Figure 7, bottom frame).

Note: More options for government policie responses will be explored

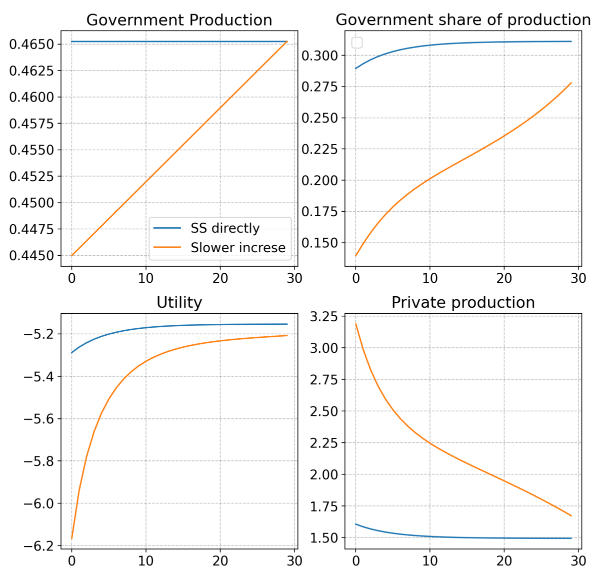

Figure 7: Transition path