1 Household problem

1.1 Nominal

There is a continuum of households indexed by $i \in [0, 1]$.

The beginning of period states are:

1. $s_i \in \{NT, HT\}$ sector
2. a_{it-1} nominal lagged assets
3. z_{it} idiosyncratic productivity following a Markov process

The household chooses $e_{it} \geq 0$ and is allocated an amount of labor by their union

$$n_{it} = n_{s_i,t} = \frac{N_{s_i}}{S_{s_i}} \tag{1}$$

where N_s is the total amount labor in each sector and $S_s \in (0, 1)$ is the share of households working in the sector with $S_{NT} + S_{HT} = 1$.

The nominal budget constraint is

$$e_{it} + a_{it} = (1 + i_{t-1})a_{it-1} + (1 - \tau) W_{s_i,t} n_{s_i,t} z_{it} \tag{2}$$

where i_{t-1} is the nominal interest rate from period $t - 1$ to t, and $(1 - \tau) W_{s_i,t}$ is the real wage. The household is not allowed to borrow

$$a_{it} \geq 0 \tag{3}$$

Utility is given by

$$U_0 = \sum_{k=0}^{\infty} \beta^k \left[u\left(e_{it}, P_{NT,t}, P_{T,t}\right) - \xi\left(n_{it}\right)\right] \tag{4}$$

where

$$u\left(e_{it}, P_{NT,t}, P_{T,t}\right) = \frac{1}{\epsilon}\left[\left(\frac{e_t}{P_{NT,t}}\right)^{\epsilon} - 1\right] - \frac{\nu}{\gamma}\left[\left(\frac{P_{T,t}}{P_{NT,t}}\right)^{\gamma} - 1\right] \tag{5}$$

$$\xi\left(N_t\right) = \varphi \frac{n_{it}^{1+\nu}}{1+\nu} \tag{6}$$

and $P_{NT,t}$ and $P_{T,t}$ are aggregate prices.

Roys identity implies

$$c_{T,it} = \frac{e_{it}}{P_{T,t}}\left[\nu\left(\frac{P_{NT,t}}{e_{it}}\right)^{\epsilon} - \left(\frac{P_{T,t}}{P_{NT,t}}\right)^{\gamma}\right] \tag{7}$$

$$c_{NT,it} = \frac{e_{it}}{P_{NT,t}}\left[1 - \nu\left(\frac{P_{NT,t}}{e_{it}}\right)^{\epsilon} - \left(\frac{P_{T,t}}{P_{NT,t}}\right)^{\gamma}\right] \tag{8}$$

1.2 Real

We now define the following variables

$$p_t \equiv \frac{P_{T,t}}{P_{NT,t}} \tag{9}$$

$$\tilde{w}_t \equiv (1-\tau)\frac{\tilde{W}_t}{P_{NT,t}} \tag{10}$$

$$1+\tilde{r}_t \equiv \frac{1+i_{t-1}}{P_{NT,t}/P_{NT,t-1}} \tag{11}$$

$$\tilde{e}_{it} \equiv \frac{e_t}{P_{NT,t}} \tag{12}$$

$$\tilde{a}_{it} \equiv \frac{a_t}{P_{NT,t}} \tag{13}$$

and

$$\tilde{u}\left(\tilde{e}_{it}, p_t\right) = \frac{1}{\epsilon}\left[\left(\tilde{e}_{it}\right)^{\epsilon} - 1\right] - \frac{\nu}{\gamma}\left[\left(p_t\right)^{\gamma} - 1\right] \tag{14}$$

In recursive form, the household problem now is

$$v_t\left(s_i, z_{it}, \tilde{a}_{it-1}\right) = \max_{\tilde{e}_{it}} \tilde{u}\left(\tilde{e}_{it}, p_t\right) - \xi\left(n_{it}\right) + \beta \underline{v}_{t+1}\left(s_i, z_{it}, \tilde{a}_{it}\right) \tag{15}$$

$$\text{s.t.}$$

$$\tilde{e}_{it} + \tilde{a}_{it} = \left(1+\tilde{r}_t\right)\tilde{a}_{t-1} + \tilde{w}_t n_{s_i t} z_{it}$$

where

$$\underline{v}_t\left(s_i, z_{it-1}, \tilde{a}_{it-1}\right) \equiv \mathbb{E}_t\left[v_t\left(s_i, z_{it}, \tilde{a}_{it-1}\right)\right] \tag{16}$$

The distribution of households over states s_i, z_{it} and $\tilde{a}_{it-1}$ is denoted $\boldsymbol{D}_t$.

The envelope condition implies

$$\partial \underline{v}_t / \partial \tilde{a}_{it-1} = \left(1+\tilde{r}_t\right)\mathbb{E}_t\left[\partial \tilde{u}\left(\tilde{e}_{it}, p_t\right)/\partial \tilde{e}_{it}\right] = \left(1+r_t^a\right)\mathbb{E}_t\left[\tilde{e}_{it}^{\epsilon-1}\right] \tag{17}$$

The first order condition is

$$\tilde{e}_{it}^{\epsilon-1} = \beta\left(\underline{v}_{t+1}/\partial \tilde{a}_{it}\right) \Leftrightarrow \tilde{e}_{it} = \left(\beta\left(\underline{v}_{t+1}/\partial \tilde{a}_{it}\right)\right)^{\frac{1}{\epsilon-1}} \tag{18}$$

1.3 Sequence space

The inputs to the household block is

$$\left\{\tilde{r}_t, \tilde{w}_{NT,t}, \tilde{w}_{TH,t}, n_{NT,t}, n_{TH,t}, p_t\right\}_{t=0}^{\infty} \tag{19}$$

The idiosyncratic outputs are $\tilde{e}_{it}$ and

$$c_{T,it} = \tilde{e}_{it} p_t^{-1}\left[\nu \tilde{e}_{it}^{-\epsilon} - p_t^{\gamma}\right] \tag{20}$$

$$c_{NT,it} = \tilde{e}_{it}\left[1 - \nu \tilde{e}_{it}^{-\epsilon} - p_t^{\gamma}\right] \tag{21}$$

$$\delta_{it} \equiv \tilde{e}_{it}^{\epsilon-1} z_{it} \tag{22}$$

$$\overline{u}_{it} \equiv \tilde{u}\left(\tilde{e}_{it}, p_t\right) - \xi\left(n_{it}\right) \tag{23}$$

Aggregates are

$$\tilde{E}_t^{hh} \equiv \int \tilde{e}_{it} d\boldsymbol{D}_t \tag{24}$$

$$C_{T,t}^{hh} = \int c_{T,it} d\boldsymbol{D}_t \tag{25}$$

$$C_{NT,t}^{hh} = \int c_{NT,it} d\boldsymbol{D}_t \tag{26}$$

$$\Delta_{s,t}^{hh} \equiv \frac{1}{S_s} \int 1_{s_i=s} \delta_{it} d\boldsymbol{D}_t \tag{27}$$

$$\overline{U}_{s,t}^{hh} \equiv \frac{1}{S_s} \int 1_{s_i=s} \overline{u}_{it} d\boldsymbol{D}_t \tag{28}$$

Note: If $\overline{U}_{s,t}^{hh}$ are not needed, then the only required inputs are

$$\{r_t^a, \tilde{w}_{NT,t}, \tilde{w}_{TH,t}\}_{t=0}^{\infty}$$

2 Union

From the point of view of the union in sector s, the marginal cost of supplying more labor is average of marginal disutility of its members

$$\frac{1}{S_s} \int \frac{\partial \varepsilon(n_{it})}{\partial N_t} d\boldsymbol{D}_t = \varphi n_{s,t}^{\nu} \tag{29}$$

The marginal benefit of supplying more labor is the average marginal utility of the additional consumption

$$\begin{aligned}
\frac{1}{S_s} \int \frac{\partial u(e_{it}, P_{NT,t} P_{TH,t})}{\partial N_{s,t}} d\boldsymbol{D}_t =& \frac{1}{S_s} (1-\tau) W_{s,t} \int \frac{\partial u(e_{it}, P_{NT,t} P_{TH,t})}{\partial e_{it}} z_{it} d\boldsymbol{D}_t \\
=& \frac{1}{S_s} (1-\tau) W_{s,t} \int e_{it}^{\epsilon-1} P_{NT,t}^{-\epsilon} z_{it} d\boldsymbol{D}_t \\
=& \frac{1}{S_s} (1-\tau) \frac{W_{s,t}}{P_{NT,t}} \int \tilde{e}_{it}^{\epsilon-1} z_{it} d\boldsymbol{D}_t \\
=& \tilde{w}_{s,t} \Delta_{s,t}^{hh}
\end{aligned} \tag{30}$$

The NKWPC therefore becomes

$$\pi_{s,t}^{w} = \kappa \left[\varphi N_{s,t}^{\nu} - \frac{1}{\mu} \tilde{w}_{s,t} \Delta_t^{hh} \right] + \beta \pi_{s,t+1}^{w} \tag{31}$$

3 Demand

From the household block we have

$$C_{NT,t} = C_{NT,t}^{hh} \tag{32}$$

$$C_{T,t} = C_{T,t}^{hh} \tag{33}$$

A CES demand structure for fundamental prices $P_{E,t}$, $P_{TF,t}$ and $P_{TH,t}$ implies

$$C_{E,t} = \alpha_E \left(\frac{P_{E,t}}{P_{T,t}} \right)^{-\eta_E} C_{T,t} \tag{34}$$

$$C_{THF,t} = (1 - \alpha_E) \left(\frac{P_{E,t}}{P_{T,t}} \right)^{-\eta_E} C_{T,t} \tag{35}$$

$$C_{TF,t} = \alpha_F \left(\frac{P_{TF,t}}{P_{THF,t}} \right)^{-\eta_F} C_{THF,t} \tag{36}$$

$$C_{TH,t} = (1 - \alpha_F) \left(\frac{P_{TH,t}}{P_{THF,t}} \right)^{-\eta_F} C_{THF,t} \tag{37}$$

where

$$P_{T,t} = \left[\alpha_E P_{E,t}^{1-\eta_E} + (1 - \alpha_E) P_{THF,t}^{1-\eta_E} \right]^{\frac{1}{1-\eta_E}} \tag{38}$$

$$P_{THF,t} = \left[\alpha_F P_{TF,t}^{1-\eta_E} + (1 - \alpha_F) P_{TH,t}^{1-\eta_E} \right]^{\frac{1}{1-\eta_E}} \tag{39}$$