0.1 Model Blocks

Variables and Target Equations

Exogenous Variables (Shocks)

The exogenous variables/external shocks to the equation system are productivity levels, the size of the foreign economy, real foreign interest rates, foreign prices in foreign currency, and government consumption:

$$\{Z_{TH}, Z_{NT}, M_s^*, r^f, PF_s^*, G\}$$

Endogenous Variables

The endogenous variables are the nominal interest rate or the nominal exchange rate, depending on whether the exchange rate is fixed, labor supply in each sector, and wage inflation in each sector:

$$\{N_{NT}, N_{TH}, \pi_{W,TH}, \pi_{W,NT}, E/i\}$$

Target Equations

The target equations must hold for the system to be in equilibrium. These include the New Keynesian Wage Phillips Curves, the Uncovered Interest Parity (UIP), and market clearing conditions for the markets for non-tradable and domestic tradable goods:

$$\{\text{NKWC}_T, \text{NKWC}_{NT}, \text{clearing}_{YTH}, \text{clearing}_{YNT}, \text{UIP}\}$$

To simplify, I drop the sector subscript and use $s \in \{NT, HT\}$ to denote the domestic sectors: non-tradable goods (NT) and home tradable goods (HT), when there is no confusion.

The blocks are calculated for each $t \in T$. I drop the time subscripts for within-period variables and, when necessary, use subscript $+$ to indicate the next period and $-$ to indicate the preceding period.

1. **Firm Block**
   Inputs: $Z_s, N_s, \pi_{w,s}, W_s^{SE}$
   Outputs: Y_s, W_s, P_s

   This block calculates, for sectors $s \in \{NT, T\}$:

   - **Production:**
     Based on the guesses for labor supply (N_s) and the exogenous productivity (Z_s) for each sector s. Production is linear and given by:
     $$Y_s = Z_s N_s$$
   - **Wages:**
     Based on the guesses for wage inflation and the preceding nominal wage:
     $$W_s = \begin{cases} W_s^{SS}(1+\pi_{w,s}), & \text{if } t = 0, \\ W_{s-}(1+\pi_{w,s}), & \text{if } t > 0. \end{cases}$$
   - **Prices:**
     Given perfect competition, prices are equal to the marginal cost:
     $$P_s = \frac{W_s}{Z_s}$$

2. **Price Block**
   Inputs: $PF^*, E, PTH, PNT, WTH, WNT, \alpha_F, \eta_F, \alpha_T, \eta_T$
   Outputs: $PF, PTH_s, PT, \pi_{F^*}, \pi_F, \pi_{NT}, \pi_{TH}, \pi_T, \pi_{TH_s}$

   This block calculates:

- **Currency Conversion:**
  The nominal exchange rate in home currency units per foreign currency is denoted E. Taking the foreign price level P_F^* as exogenously given, in home currency, the foreign price level is:

$$PF = PF^* E$$

  The price of home tradable goods in the foreign currency is:

$$PTH^* = \frac{PTH}{E}$$

  Where E is either an endogenous variable determined by a Taylor rule and the UIP or fixed $E = \bar{E}$, depending on the exchange rate regime.

- **Price Index for Tradable Goods:**
  Because preferences over domestic and foreign tradable goods are CES, prices of tradable goods are:

$$PT = \left(\alpha_F PF^{1-\eta_F} + (1-\alpha_F) PTH^{1-\eta_F}\right)^{\frac{1}{1-\eta_F}}$$

- **Cost of Living price index**
  In my baseline model, I use the representative agent Konüs cost-of-living index consistent with PIGL preferences. PIGL preferences maintain tractable aggregation properties that eliminate the conflict between heterogeneous indices at the individual level and the existence of welfare-relevant group cost-of-living indices. This means unions can look at alternative aggregations of individual cost-of-living indices (discussed more in Section XX).

  From the PIGL preferences, the representative agent Konüs cost-of-living index is:

$$P^{RA} = (\tilde{P}^{RA})^{\frac{\gamma}{\epsilon}} P_{NT}^{-\frac{\gamma}{\epsilon}} \tag{1}$$

  where

$$\tilde{P}^{RA} = \left[\left(1 - \frac{\epsilon\bar{\omega}_T}{\gamma}\right) P_{NT}^{\gamma} + \frac{\epsilon\bar{\omega}_T}{\gamma} P_T^{\gamma}\right]^{\frac{1}{\gamma}}$$

  and $\bar{\omega}_T$ is the initial average expenditure share on tradable goods, to get the representative agent Cost-Of-Living index.

- **Inflation:**
  Inflation is calculated for all relevant variables so they can be used to inform monetary policy:

$$\pi_{F_s} = \frac{PF^*}{PF_-^*} - 1, \quad \pi_F = \frac{PF}{PF_-} - 1, \quad \pi_{NT} = \frac{PNT}{PNT_-} - 1$$

$$\pi_{TH} = \frac{PTH}{PTH_-} - 1, \quad \pi_T = \frac{PT}{PT_-} - 1 \quad \pi^{PIGL} = \frac{P^{PIGL}}{P_-^{PIGL}} - 1$$

3. **Monetary Policy Block**
   Inputs: $\pi, \pi^{ss}, i^{ss}, i_{\text{shock}}, E/i$
   Outputs: E/i

   Monetary policy controls domestic nominal interest rates i. It is set either following a Taylor rule or to fix the exchange rate.

   - **Floating Exchange Rate (Taylor Rule):**
     i is based on a Taylor rule, and E is endogenous:

$$i = (1 + i^{ss}) \left(\frac{1 + \pi_+}{1 + \pi^{ss}}\right)^{\phi} - 1 + i_{\text{shock}}$$

     The inflation π_+ is the inflation type that the central bank targets. In the baseline I use the PIGL representative agent inflation.

- **Fixed Exchange Rate:**
  E is exogenous, and i is endogenously determined:

$$E = \bar{E}$$

4. **Government Budget Block**
   Inputs: $PNT, WTH, NTH, WNT, NNT, i_-, G, \tau^{ss}, B^{ss}, YTH^{ss}, YNT^{ss}$
   Outputs: B, τ

   The government budget constraint, in nominal terms, is:

$$B = (1 + i_-)B_- + P_{NT}G - \tau \sum_{s=1}^{2} W_s N_s$$

   Where B represents nominal government bonds, G represents the real government size.

   Taxes adjust dynamically:

$$\tau_t = \tau^{ss} + \omega \left(\frac{\frac{B_-}{P_{NT}} - \frac{B^{ss}}{P_{NT}}}{Y_{TH}^{ss} + Y_{NT}^{ss}} \right)$$

5. **Household Block**
   Inputs: $w_{\tau,TH}, w_{\tau,NT}, r_a$
   Outputs: $C, A, C_{NT}, C_T, C_{TH}, C_{TF}$

   - **Preparing**

     Household nominal income per effective labor supply (before idiosyncratic productivity):

$$W_{\tau,TH} = (1 - \tau)W_{TH}N_{TH}, \quad W_{\tau,NT} = (1 - \tau)W_{NT}N_{NT}$$

     I calculate the inputs, deflated with the price of non-tradable goods:

$$\tilde{e} = \frac{e}{P_{NT}}, \quad \tilde{a} = \frac{a}{P_{NT}}, \quad p = \frac{P_T}{P_{NT}}, \quad \tilde{w}_{NT} = \frac{W_{NT}}{P_{NT}}, \quad \tilde{w}_{HT} = \frac{W_{HT}}{P_{NT}}, \quad (1 + \tilde{r}^a) = (1 + i_-)\frac{P_{NT,-}}{P_{NT}} \tag{2}$$

     where $(1 + r^a)$ is the return on beginning-of-period assets, deflated with prices $(\tilde{a}_-)$.

     This gives me all inputs for the household problem, as described in Section XX

   - **Household problem**

     Household problem

   - **Post calculations**

     The households problem outputs the aggregate household values for expenditure, assets measured in units of non-tradable goods, and consumption of: non-tradable goods, tradable goods, home tradable goods, and foreign tradable goods. Multiplied by P_{NT} to get nominal values for expenditure for assets:

$$E = P_{NT}E^{hh}, \quad A = P_{NT}A^{hh}$$

     The real aggregate consumption levels are given directly

$$\{C_{NT} = C_{NT}^{hh}, \quad C_T = C_T^{hh}, \quad C_{TH} = C_{TH}^{hh}, \quad C_{TF} = C_{TF}^{hh}\}$$

   - **Foreign Consumption Block**
     Inputs: P_{TH}^*, P_F^*, M^*

Outputs: C^*_{HT}
The foreign demand for home tradable goods is:

$$C^*_{HT} = \left(\frac{P^*_{TH}}{P^*_F}\right)^{-\eta^*} M^*$$

Where M^* is the size of the foreign market, calibrated in steady state, and η^* is the elasticity of foreign demand.

6. **New Keynesian Wage Phillips Curve Block**
Inputs: $\pi_{w,TH}, \pi_{w,NT}, N_{TH}, N_{NT}, w_{TH}, w_{NT}, \tau, E_{TH,hh}, E_{NT,hh}$
Outputs: Phillips Curves Targets

The unions calculate wages deflated with a PIGL cost-of-living index of their choosing.
$E_{s,hh}$ is the total expenditure of households working in sector $s \in \{TH, NT\}$.

The wage deflated with the cost-of-living index is:

$$w_{NT} = \frac{W_{NT}}{P}, \quad w_{TH} = \frac{W_{TH}}{P}$$

The wage inflation dynamics are as follows:

- **Tradable Sector:**

$$\pi_{w,TH} = \kappa \left[\varphi_{TH} \left(\frac{N_{TH}}{s_T}\right)^{-\nu} - \frac{1}{\mu_w}(1-\tau) w_{TH} U'(E^{hh}_{TH})\right] + \beta \pi_{w,TH+} \quad \text{(Target)}$$

- **Non-Tradable Sector:**

$$\pi_{w,NT} = \kappa \left[\varphi_{NT} \left(\frac{N_{NT}}{1-s_T}\right)^{-\nu} - \frac{1}{\mu_w}(1-\tau) w_{NT} U'(E^{hh}_{NT})\right] + \beta \pi_{w,NT+} \quad \text{(Target)}$$

Where $U'(E^{hh}_s)$ is the marginal utility of (average) expenditure, in sector s deflated with the price of non-tradable goods. This deflation insures a return to a steady state independent of prices(Correct?).

7. **Uncovered Interest Parity (and Mutual Fund)**
Inputs: Q, r, r_F
Outputs: UIP Target

Foreign real interest rates are given by

$$i + r^f = \frac{1 + i^f}{1 + \pi^f_+} \tag{3}$$

Where the foreign real interest rate and the foreign inflation are exogenous.
Rearranging to get the foreign nominal interest rate

$$i^f = (1 + r^f)(1 + \pi^f_+) - 1 \tag{4}$$

Which can be inserted in the nominal UIP

$$1 + i = (1 + i^f)\frac{E_+}{E} \quad \text{(Target)}$$

A target which pins down the (endogenous) nominal exchange rate or interest rate, depending on the monetary regime.

If households save in nominal bonds:
Do i have a problem here?

When there is an unexpected shock, the arbitrage condition will be violated in period $t = 1$.
Solution if there is a problem:
Therefore, I could model a mutual fund that aggregates all household assets and invests in government and foreign nominal bonds. They distribute the returns they generate back to the households.

8. **Market Clearing Block**
Inputs: $YTH, CTH, CTH_s, YNT, CNT, G$
Outputs: Targets and Clearing Conditions

   - **Non-Tradable Goods:**
   $$YNT = CNT + G \quad \text{(Target)}$$
   - **Tradable Goods:**
   $$YTH = CTH + CTH^* \quad \text{(Non-targeted clearing)}$$

9. **Accounting Block**
Inputs: $PTH, YTH, PNT, YNT, P, C_{hh}, G, A_{hh}, B, r_a$
Outputs: GDP, NX, CA, NFA, Walras

   This block includes:

   - **Nominal GDP:**
   $$GDP = P_{TH}Y_{TH} + P_{NT}Y_{NT}$$
   - **Net Exports (NX):**
   Nominal production minus total expenditure (private (E) and public ($P_{NT}G$) is the nominal net exports
   $$NX = GDP - E - P_{NT}G$$
   - **Current Account (CA):**
   (Nominal)
   $$CA = NX + i^f_{-}NFA_{-}\frac{E}{E_{-}}$$
   - **Net Foreign Assets (NFA):**
   (Nominal)
   $$NFA = A - B$$
   - **Walras' Law:**
   $$\text{Walras} = (NFA - NFA_{-}) - CA$$