1 Model

The model is in discrete time and indexed by $t = 0, 1, 2, \ldots$, with an infinite horizon.

We study the perfect foresight response to an energy price shock starting from a deterministic steady state at time 0, where no shocks were anticipated.

The foreign economy is exogenous.

The domestic economy is populated by a continuum of heterogeneous households indexed by $i \in [0, 1]$.

The domestic economy has two sectors, non-tradeable (NT) and tradeable (TH).

1.1 Foreign economy

The foreign economy sells a tradeable good at a fixed price P_F^* and energy at a time-varying price$P_{E,t}^*$.

The nominal exchange in domestic currency per foreign currency is $\mathcal{E}_t$, and therefore

$$P_{F,t} = \mathcal{E}_t P_F^* \tag{1}$$
$$P_{E,t} = \mathcal{E}_t P_{E,t}^* \tag{2}$$

The price of the domestically produced tradeable is $P_{TH,t}$, and therefore

$$P_{TH,t}^* = \frac{P_{TH,t}}{\mathcal{E}_t} \tag{3}$$

The foreign demand for the domestic tradeable good is

$$C_{TH,t}^* = \alpha^* \left(\frac{P_{TH,t}^*}{P_F^*} \right)^{-\eta^*} M^* \tag{4}$$

where $\alpha^* M^*$is the steady state demand.

There is free capital mobility implying the UIP condition

$$1 + i_t = \left(1 + i^f \right) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \tag{5}$$

where i^f is the constant nominal foreign interest rate.

1.2 Firms

A representative firm in each sector $s \in \{NT, TH\}$ hires labor $N_{s,t}$ and produce with

$$Y_{s,t} = Z_s N_{s,t} \tag{6}$$

where Z_s is the technology level. Profits are

$$\Pi_{s,t} = P_{s,t} Y_{s,t} - W_{s,t} N_{s,t} \tag{7}$$

where $P_{s,t}$ is the output price and $W_{s,t}$ is the wage level.

The first order condition for labor implies

$$P_{s,t} = \frac{W_{s,t}}{Z_s} \tag{8}$$

1.3 Demand system

From the household block below, we have consumption of the non-tradeable good and the tradeable good

$$C_{NT,t} = C_{NT,t}^{hh} \tag{9}$$

$$C_{T,t} = C_{T,t}^{hh} \tag{10}$$

A nested CES demand structure for the fundamental prices $P_{E,t}$, $P_{F,t}$ and $P_{TH,t}$ implies

$$C_{E,t} = \alpha_E \left(\frac{P_{E,t}}{P_{T,t}}\right)^{-\eta_E} C_{T,t} \tag{11}$$

$$C_{THF,t} = (1-\alpha_E)\left(\frac{P_{E,t}}{P_{T,t}}\right)^{-\eta_E} C_{T,t} \tag{12}$$

$$C_{TF,t} = \alpha_F \left(\frac{P_{F,t}}{P_{THF,t}}\right)^{-\eta_F} C_{THF,t} \tag{13}$$

$$C_{TH,t} = (1-\alpha_F)\left(\frac{P_{TH,t}}{P_{THF,t}}\right)^{-\eta_F} C_{THF,t} \tag{14}$$

where

$$P_{T,t} = \left[\alpha_E P_{E,t}^{1-\eta_E} + (1-\alpha_E)\, P_{THF,t}^{1-\eta_E}\right]^{\frac{1}{1-\eta_E}} \tag{15}$$

$$P_{THF,t} = \left[\alpha_F P_{F,t}^{1-\eta_F} + (1-\alpha_F)\, P_{TH,t}^{1-\eta_F}\right]^{\frac{1}{1-\eta_F}} \tag{16}$$

We define

$$1+\pi_{X,t} = P_{X,t}/P_{X,t-1} \tag{17}$$

for $X \in \{E, F, TN, TH, THF, T\}$.

1.4 Household problem

1.4.1 Nominal

The beginning of period idiosyncratic states are:

1. $s_i \in \{NT, HT\}$ sector
2. a_{it-1} nominal lagged assets
3. z_{it} idiosyncratic productivity following a Markov process

The household chooses $e_{it} \geq 0$ and is allocated an amount of labor by their union

$$n_{it} = n_{s_i,t} = \frac{N_{s_i}}{S_{s_i}} \tag{18}$$

where N_s is the total amount labor in each sector and $S_s \in (0,1)$ is the share of households working in the sector with $S_{NT} + S_{HT} = 1$.

The nominal budget constraint is

$$e_{it} + a_{it} = (1+i_{t-1})a_{it-1} + (1-\tau_t)\, W_{s_i,t} n_{s_i,t} z_{it} \tag{19}$$

where i_{t-1} is the nominal interest rate from period $t-1$ to t, and $(1-\tau_t) W_{s_i,t}$ is the real wage. The household is not allowed to borrow

$$a_{it} \geq 0 \tag{20}$$

Utility is given by

$$U_0 = \sum_{k=0}^{\infty} \beta^k \left[u\left(e_{it}, P_{NT,t}, P_{T,t}\right) - \xi\left(n_{it}\right) \right] \tag{21}$$

where

$$u\left(e_{it}, P_{NT,t}, P_{T,t}\right) = \frac{1}{\epsilon}\left[\left(\frac{e_t}{P_{NT,t}}\right)^{\epsilon} - 1\right] - \frac{\nu}{\gamma}\left[\left(\frac{P_{T,t}}{P_{NT,t}}\right)^{\gamma} - 1\right] \tag{22}$$

$$\xi\left(N_t\right) = \varphi \frac{n_{it}^{1+\kappa}}{1+\kappa} \tag{23}$$

and $P_{NT,t}$ and $P_{T,t}$ are aggregate prices.

Roy's identity implies

$$c_{T,it} = \frac{e_{it}}{P_{T,t}}\left[\nu\left(\frac{P_{NT,t}}{e_{it}}\right)^{\epsilon}\left(\frac{P_{T,t}}{P_{NT,t}}\right)^{\gamma}\right] \tag{24}$$

$$c_{NT,it} = \frac{e_{it}}{P_{NT,t}}\left[1 - \nu\left(\frac{P_{NT,t}}{e_{it}}\right)^{\epsilon}\left(\frac{P_{T,t}}{P_{NT,t}}\right)^{\gamma}\right] \tag{25}$$

1.4.2 Real

We now define the following variables

$$\tilde{p}_t \equiv \frac{P_{T,t}}{P_{NT,t}} \tag{26}$$

$$\tilde{w}_t \equiv \left(1-\tau_t\right)\frac{W_t}{P_{NT,t}} \tag{27}$$

$$1 + \tilde{r}_t \equiv \frac{1 + i_{t-1}}{1 + \pi_{NT,t}} \tag{28}$$

$$\tilde{e}_{it} \equiv \frac{e_t}{P_{NT,t}} \tag{29}$$

$$\tilde{a}_{it} \equiv \frac{a_t}{P_{NT,t}} \tag{30}$$

and

$$\tilde{u}\left(\tilde{e}_{it}, p_t\right) = \frac{1}{\epsilon}\left[\tilde{e}_{it}^{\epsilon} - 1\right] - \frac{\nu}{\gamma}\left[\tilde{p}_t^{\gamma} - 1\right] \tag{31}$$

In recursive form, the household problem now is

$$v_t\left(s_i, z_{it}, \tilde{a}_{it-1}\right) = \max_{\tilde{e}_{it}} \tilde{u}\left(\tilde{e}_{it}, p_t\right) - \xi\left(n_{it}\right) + \beta \underline{v}_{t+1}\left(s_i, z_{it}, \tilde{a}_{it}\right) \tag{32}$$

$$\text{s.t.}$$

$$\tilde{e}_{it} + \tilde{a}_{it} = \left(1 + \tilde{r}_t\right)\tilde{a}_{t-1} + \tilde{w}_t n_{s_i t} z_{it}$$

where

$$\underline{v}_t\left(s_i, z_{it-1}, \tilde{a}_{it-1}\right) \equiv \mathbb{E}_t\left[v_t\left(s_i, z_{it}, \tilde{a}_{it-1}\right)\right] \tag{33}$$

The distribution of households over states s_i, z_{it} and $\tilde{a}_{it-1}$ is denoted $\boldsymbol{D}_t$.

The envelope condition implies

$$\partial \underline{v}_t / \partial \tilde{a}_{it-1} = (1+\tilde{r}_t)\,\mathbb{E}_t\left[\partial \tilde{u}\left(\tilde{e}_{it}, \tilde{p}_t\right)/\partial \tilde{e}_{it}\right] = (1+r_t^a)\,\mathbb{E}_t\left[\tilde{e}_{it}^{\epsilon-1}\right] \tag{34}$$

The first order condition is

$$\tilde{e}_{it}^{\epsilon-1} = \beta\left(\underline{v}_{t+1}/\partial \tilde{a}_{it}\right) \Leftrightarrow \tilde{e}_{it} = \left(\beta\left(\underline{v}_{t+1}/\partial \tilde{a}_{it}\right)\right)^{\frac{1}{\epsilon-1}} \tag{35}$$

1.4.3 Sequence space

The inputs to the household block is

$$\left\{\tilde{r}_t, \tilde{w}_{NT,t}, \tilde{w}_{TH,t}, n_{NT,t}, n_{TH,t}, \tilde{p}_t\right\}_{t=0}^{\infty} \tag{36}$$

The idiosyncratic outputs are $\tilde{e}_{it}$ and

$$c_{T,it} = \tilde{e}_{it}\tilde{p}_t^{-1}\left[\nu \tilde{e}_{it}^{-\epsilon}\tilde{p}_t^{\gamma}\right] \tag{37}$$

$$c_{NT,it} = \tilde{e}_{it}\left[1-\nu \tilde{e}_{it}^{-\epsilon}\tilde{p}_t^{\gamma}\right] \tag{38}$$

$$\delta_{it} \equiv \tilde{e}_{it}^{\epsilon-1} z_{it} \tag{39}$$

$$\overline{u}_{it} \equiv \tilde{u}\left(\tilde{e}_{it}, p_t\right) - \xi\left(n_{it}\right) \tag{40}$$

Aggregates are

$$\tilde{A}_t^{hh} \int \tilde{a}_{it} d\boldsymbol{D}_t \tag{41}$$

$$\tilde{E}_t^{hh} \equiv \int \tilde{e}_{it} d\boldsymbol{D}_t \tag{42}$$

$$C_{T,t}^{hh} = \int c_{T,it} d\boldsymbol{D}_t \tag{43}$$

$$C_{NT,t}^{hh} = \int c_{NT,it} d\boldsymbol{D}_t \tag{44}$$

$$\Delta_{s,t}^{hh} \equiv \frac{1}{S_s}\int 1_{s_i=s}\delta_{it} d\boldsymbol{D}_t \tag{45}$$

$$\overline{U}_{s,t}^{hh} \equiv \frac{1}{S_s}\int 1_{s_i=s}\overline{u}_{it} d\boldsymbol{D}_t \tag{46}$$

1.5 CPI

The expenditure share of the tradeable good is

$$\omega_T\left(\tilde{e}_{it}, \tilde{p}_t\right) = \nu \tilde{e}_{it}^{-\epsilon}\tilde{p}_t^{\gamma} \tag{47}$$

Lemma 3 in Boppart (2014) states the intra-temporal elasticity between the two goods are

$$\eta_T\left(\tilde{e}_{it}, \tilde{p}_t\right) = 1-\gamma-\frac{\nu \tilde{p}_t^{\gamma}}{\tilde{e}_{it}^{\epsilon}-\nu \tilde{p}_t^{\gamma}}\left[\gamma-\epsilon\right] \tag{48}$$

For a hypothetical household with average expenditure we define

$$\overline{\omega}_T \equiv \omega_T\left(\tilde{E}_{ss}^{hh}, \tilde{p}_{ss}\right)$$

$$\overline{\eta}_T \equiv \eta_T\left(\tilde{E}_{ss}^{hh}, \tilde{p}_{ss}\right)$$

Next, we define the CPI index as

$$P_t = \left(\overline{\omega}_T P_{T,t}^{1-\overline{\eta}_T} + (1-\overline{\omega}_T)\, P_{NT}^{1-\overline{\eta}_T}\right)^{\frac{1}{1-\overline{\eta}_T}} \tag{49}$$

and CPI inflation is

$$1 + \pi_t = P_t / P_{t-1} \tag{50}$$

1.6 Union

From the point of view of the union in sector s, the marginal cost of supplying more labor is average of marginal disutility of its members

$$\frac{1}{S_s} \int \frac{\partial \xi(n_{it})}{\partial N_{s,t}} d\boldsymbol{D}_t = \varphi n_{s,t}^{\kappa} \tag{51}$$

The marginal benefit of supplying more labor is the average marginal utility of the additional consumption

$$\begin{aligned}\frac{1}{S_s} \int \frac{\partial u(e_{it}, P_{NT,t} P_{TH,t})}{\partial N_{s,t}} d\boldsymbol{D}_t =& \frac{1}{S_s}\,(1-\tau)\, W_{s,t} \int \frac{\partial u(e_{it}, P_{NT,t} P_{TH,t})}{\partial e_{it}} z_{it} d\boldsymbol{D}_t \\ =& \frac{1}{S_s}\,(1-\tau)\, W_{s,t} \int e_{it}^{\epsilon-1} P_{NT,t}^{-\epsilon} z_{it} d\boldsymbol{D}_t \\ =& \frac{1}{S_s}\,(1-\tau)\, \frac{W_{s,t}}{P_{NT,t}} \int \tilde{e}_{it}^{\epsilon-1} z_{it} d\boldsymbol{D}_t \\ =& \tilde{w}_{s,t} \Delta_{s,t}^{hh} \end{aligned} \tag{52}$$

The NKWPC therefore becomes

$$\pi_{s,t}^{w} = \kappa_w \left[\varphi N_{s,t}^{\nu} - \frac{1}{\mu} \tilde{w}_{s,t} \Delta_t^{hh}\right] + \beta \pi_{s,t+1}^{w} \tag{53}$$

where

$$1 + \pi_{s,t}^{w} = W_{s,t} / W_{s,t-1} \tag{54}$$

1.7 Government

The interest rate is set according to a Taylor rule

$$i_t = i^f + \phi_\pi \pi_{t+1} \tag{55}$$

The government in nominal terms is

$$B_t = (1 + i_{t-1})\, B_{t-1} - \tau_t \sum_s W_{s,t} N_{s,t} \tag{56}$$

We define real government bonds and the real wage as as

$$b_t \equiv \frac{B_t}{P_t} \tag{57}$$

$$w_t = \frac{W_t}{P_t} \tag{58}$$

such that

$$b_t = (1 + r_t)\, b_{t-1} - \tau_t \sum w_{s,t} N_{s,t} \tag{59}$$

where

$$1 + r_t = \frac{1 + i_{t-1}}{P_t / P_{t-1}} \tag{60}$$

We assume the tax rule

$$\tau_t = \tau_{ss} + \omega \frac{b_{t-1} - b_{ss}}{\sum_s Y_{s,ss}} \tag{61}$$

1.8 Market clearing

The market clearing conditions are

$$\begin{aligned} Y_{T,t} &= C_{TH,t} + C^*_{TH,t} \\ Y_{NT,t} &= C_{NT,t} \end{aligned}$$

2 Equilibrium

Given a sequence of foreign energy prices $P^*_{E,t}$, a sequence of

1. Prices, and
2. Quantities

must satisfy all optimality conditions and accounting equations and clear the goods markets.

We focus on equilibria, where the real exchange, defined as

$$Q_t = \frac{P_{F,t}}{P_t} = \frac{\mathcal{E}_t P^*_F}{P_t} \tag{62}$$

returns to steady state, i.e. $Q_\infty = Q_{ss}$. This implies that we must have

$$P^*_{E,\infty} = 1 \tag{63}$$

The proof is as follows:

1. Eq. (1) requires $\frac{\mathcal{E}_t}{P_{F,t}} = 1$.
2. Eq. (62) using $Q_\infty = 1$ requires $\frac{\mathcal{E}_\infty}{P_\infty} = 1$.
3. Unchanged $\boldsymbol{D}_\infty$ requires $\frac{P_{T,\infty}}{P_{NT,\infty}} = 1$ and $\frac{W_{TH,\infty}}{W_{NT,\infty}} = 1$.
4. Eq. (8) requires $\frac{P_{NT,\infty}}{P_{TH,\infty}} = 1$ using point 3.
5. Eq. (49) requires $\frac{\mathcal{E}_\infty}{P_{T,\infty}} = \frac{\mathcal{E}_\infty}{P_{NT,\infty}} = 1$ using point 2 and 3.
6. Point 4+5 implies $\frac{\mathcal{E}_\infty}{P_{TH,\infty}} = 1$.
7. Point 1 and 6 together with eq. (16) implies $\frac{\mathcal{E}_\infty}{P_{THF,\infty}} = 1$.
8. Point 5 and 7 with eq. (15) now requires $\frac{\mathcal{E}_\infty}{P_{E,\infty}} = 1 \Leftrightarrow P^*_{E,\infty} = 1$ using eq. (2).

3 Accounting

We define the following variables,

$$\text{Nominal Gross Domestic Product: } GDP_t = P_{TH,t}Y_{TH,t} + P_{NT,t}Y_{NT} \tag{64}$$

$$\text{Nominal net exports: } NX_t = GDP_t - E_t \tag{65}$$

$$\text{Nominal net foreign assets: } NFA_t = A_t^{hh} - B_t \tag{66}$$

$$\text{NominalCurrent account: } CA_t = NX_t + i_{t-1}NFA_{t-1} \tag{67}$$

Walras' law then implies

$$NFA_t - NFA_{t-1} = CA_t, \tag{68}$$

as shown by

$$\begin{aligned}
\int \tilde{a}_{it} d\boldsymbol{D}_t &= \int (1+\tilde{r}_t)\tilde{a}_{it-1} + \tilde{w}_{s_i,t} n_{s_i,t} z_{it} - \tilde{e}_{it} d\boldsymbol{D}_t \Leftrightarrow \\
\tilde{A}_t^{hh} &= (1+\tilde{r}_t)\tilde{A}_{t-1}^{hh} + (1-\tau)\frac{1}{P_{NT,t}}\sum_s W_{s,t}N_{s,t} - \tilde{E}_t \Leftrightarrow \\
A_t^{hh} &= (1+i_{t-1})A_{t-1}^{hh} + GDP_t - E_t - \tau_t \sum_s W_{s,t}, N_{s,t} \\
&= (1+i_{t-1})A_{t-1}^{hh} + GDP_t - E_t + (B_t - (1+i_{t-1})B_{t-1}) \\
&= (1+i_{t-1})NFA_{t-1} + NX_t + B_t \Leftrightarrow \\
NFA_t - NFA_{t-1} &= i_{t-1}NFA_{t-1} + NX_t.
\end{aligned}$$

4 Analysis

Question: How does increase in foreign energy price lower domestic demand?

1. With PIGL $\text{corr}(\omega_T, \text{MPC}) > 0$ such that demand effects will be larger
2. Understand Jacobians $\tilde{w}_{NT,t}, \tilde{w}_{TH,t}, \tilde{p}_t$
3. Understanding how $dP_{E,t}^*$ affect $\tilde{w}_{NT,t}, \tilde{w}_{TH,t}$ and $\tilde{p}_t$

Next: What is good monetary policy for different households?