Unit I

Unit I

One Dimensional Random Variable:

Random Experiment

Random Experiment is an experiment in which the result will not be known in advance is called Random Experiment.

Sample Space (S)

A sample space is a set of all possible outcome of an Random Experiment is called sample Space.

Eg:- Toss a Coin.

$S = \{H, T\}$

Probability

$$P(A) = \frac{n(A)}{n(S)}$$

Conditional Probability

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Types of Random Variable.

1) Discrete Random Variable
2) Continous Random Variable.

Discrete Random Variable

A random variable x which can take a finite number of values is called discrete random variable.

Continous Random Variables.

A random variable x which can take an infinite number of values is called discrete random variable.

PDF - Probability Density Function.
PMF - Probability Mass Function.

Probability Mass Function

Let 'X' be a discrete random variable then the function

~~P(X = x) =~~

$P(X = x_i) = P(x_i) = P_i$ is said to be probability mass function.

If
i) $P(x_i) \geq 0 \ \forall\ i$
ii) $\sum_{i=1}^{\infty} P(x_i) = 1$

or Distribution.

Probability Density fn.

Let 'X' be a continous random variable then the function $f(x)$ is said to be probability density fn

If
i) $f(x) \geq 0$
ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

1) A random variable 'x' has a probability function

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i) Find k

ii) Evaluate $P(x<6)$ $P(x \geq 6)$ $P(0<x<5)$

iii) Evaluate $P(1.5 < x < 4.5 / x > 2)$

iv) Find Mean & Variance.

v) Find the distribution fn of x

Soln:-

(i) $\sum P(x) = 1$.

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$= 9k + 10k^2 = 1$

$= 10k^2 + 9k - 1 = 0$.

$(k + \frac{10}{10})(k - \frac{1}{10}) = 0$

$P = 10$

$S = 9$

$N = \frac{10}{10}, \frac{-1}{10}$

~~$k = -10, k = 1$~~

$k = -1, k = 1/10$.

$k = -1$ is not possible. Since the probability cannot be negative value.

$\boxed{\therefore k = 1/10}$

x	0	1	2	3	4	5	6	7
$P(x)$	0	$1/10$	$2/10$	$2/10$	$3/10$	$(1/10)^2$	$2(1/10)^2$	$7(1/10)^2 + \frac{1}{10}$

(ii) ~~$P(x) = 0$~~

$P(x<6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$

$= \frac{8}{10} + \frac{1}{100}$

$= \frac{81}{100}$

$P(x \geq 6)$

$1 - P(x<6)$

$1 - \frac{81}{100} = \frac{100-81}{100}$.

$= \frac{19}{100}$

$P(0<x<5) = P(x=1) + P(x=2) + P(x=3) + P(X=4)$

$= 1/10 + \frac{2}{10} + \frac{2}{10} + \frac{3}{10}$

$= 8/10$.

(iii). $P(1.5 < X < 4.5 / X > 2)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{P(1.5 < X < 4.5) \cap P(X > 2)}{P(X > 2)}$$

$$\frac{P(X=3) + P(X=4)}{P(X>2)}$$

$$= \frac{\frac{2}{10} + \frac{3}{10}}{1 - P(X \leq 2)}$$

$$= \frac{\frac{5}{10}}{1 - P(X=0) + P(X=1) + P(X=2)}$$

$$= \frac{\frac{5}{10}}{1 - \frac{3}{10}}$$

$$= \frac{\frac{5}{10}}{7/10} = \frac{5}{10} \times \frac{10}{7}$$

$$= 5/7$$

(iv) Distribution fn of X

x	0	1	2	3	4	5	6	7
$p(x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100} + \frac{1}{10}$
$F(x)$	0	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{5}{10}$	$\frac{8}{10}$	$\frac{81}{100}$	$\frac{83}{100}$	$\frac{100}{100} = 1$

v) Mean and Variance

Mean

$\Sigma x . P(x)$

$$0 + \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{12}{10} + \frac{5}{100} + \frac{12}{100} + \frac{119}{100}$$

$$= \frac{23}{10} + \frac{136}{100}$$

$$= \frac{230 + 136}{100} = \frac{366}{100} = 3.66.$$

Variance

$$= \Sigma x^2 P(x) - (\text{Mean})^2$$

$$= 0 + \frac{1}{10} + \left(4 \times \frac{2}{10}\right) + \left(9 \times \frac{2}{10}\right) +$$

$$\left(16 \times \frac{3}{10}\right) + \left(25 \times \frac{1}{100}\right) + \left(36 \times \frac{2}{100}\right) +$$

$$\left(49 \times \frac{17}{100}\right)$$

$$= 0 + \frac{1}{10} + \frac{8}{10} + \frac{18}{10} + \frac{48}{10} + \frac{25}{100} + \frac{72}{100} +$$

$$= \frac{1680}{100} - \left(\frac{366}{100}\right)^2$$

$$= \frac{84}{5} - \left(\frac{366}{100}\right)^2$$

$$= 3.4044.$$

2) The probability fn of an infinite discrete distribution is gn by

$$P[x=j] = \frac{1}{2^j}, \quad j = 1, 2 \ldots \infty. \text{ Find.}$$

i) $P(X$ is even)

ii) $P(X \geq 5)$ $\quad 1/2^5$

iii) $P(X$ is divisible by $3)$ $\quad 1/2^3$

iv) Find Mean & Variance.

Soln:-

(i) $P(X)$ is even

$$P(X=1) + P(X=2) + P(X=4) + P(X=6) + \ldots$$

$$= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \ldots$$

$$= \frac{1}{2^2}\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \ldots\right)$$

$$= \frac{1}{2^2}\left[1 - \frac{1}{2^2}\right]^{-1}$$

$$= \frac{1}{4}\left[1 - \frac{1}{4}\right]^{-1}$$

$$= \frac{1}{4}\left(\frac{3}{4}\right)^{-1} \qquad \left[[1-x]^{-1} = 1 + x + x^2 + x^3 + \ldots\right]$$

$$= \left(\frac{1}{4}\right)\left(\frac{4}{3}\right)$$

$$= 1/3.$$

(ii) $P(X \geq 5) = P(X=5) + P(X=6) + P(X=7) + \ldots$

$$= \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \ldots$$

$$= \frac{1}{2^5}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\right]$$

$$= \frac{1}{2^5}\left[1 - \frac{1}{2}\right]^{-1}$$

$$= \frac{1}{2^5}\left(\frac{1}{2}\right)^{-1} = \frac{2}{2^5} = \frac{1}{2^4} = \frac{1}{16}.$$

iii) $P(x \text{ is divisible by } 3) =$

$$\frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} + \cdots$$

$$= \frac{1}{2^3}\left[1 + \frac{1}{2^3} + \frac{1}{2^6} + \cdots\right]$$

$$= \frac{1}{2^3}\left[1 - \frac{1}{2^3}\right]^{-1}$$

$$= \frac{1}{2^3}\left(7/8\right)^{-1}$$

$$= \frac{1}{8}\left(8/7\right)$$

$$= 1/7$$

iv) Mean $\qquad P(x = j) = \frac{1}{2^j}$

$\Sigma x \, . \, P(x)$

x	0	1	2	3	4	5	6	...
$P(x)$	1	1/2	1/4	1/8	1/16	1/32	1/64	...

$$0 + 1/2 + 1/2 + \frac{3}{8} + 1/4 + \frac{5}{32} + \frac{6}{64} + \cdots$$

$$= 1 + \frac{3}{8} + \frac{1}{4} + \frac{5}{32} + \frac{6}{64}$$

$$= 1 + 0.375 + 0.25 + 0.15625 + 0.09375$$

$$= 1.875$$

Variance

$$\Sigma x^2 \, P(x) - (\text{Mean})^2$$

$$0 + \frac{1}{2} + 1 + \frac{9}{8} + 1 + \frac{25}{32} + \frac{36}{64} + \cdots - (1.875)^2$$

$$0 + 0.5 + 1 + 1.125 + 1 + 0.78125 + 0.5625 - (1.875)^2$$

$$4.96875 - 3.515625$$

$$= 1.453125$$

iv) Mean

$$\Sigma x \, . \, P(x)$$

$$= 1 . \, P(x=1) + 2 \, P(x=2) + 3 \, P(x=3)$$

$$= 1 \times \frac{1}{2} + 2 \times \frac{1}{2^2} + 3 \times \frac{1}{2^3} + \cdots$$

$$= \frac{1}{2}\left[1 + 2(1/2) + 3(1/2)^2 + \cdots\right]$$

$$= 1/2\left[1 - 1/2\right]^{-2} \qquad (1-x)^2 = 1 + 2x + 3x^2$$

$$= \frac{1}{2}\left[1/2\right]^{-2}$$

$$= 1/2 \times 4$$

$$= 2$$

Variance :

$\Sigma x^2 P(x) - (\text{Mean})^2$

$1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{2^2} + 3^2 \times \frac{1}{2^3} + \ldots$

$- (2)^2$

$\frac{1}{2} + 4 \times \frac{1}{2^2} + 3^2 \times \frac{1}{2^3} + \ldots - (2)^2$

$= \frac{1}{2}\left[1 + 4 \times \frac{1}{2} + 9 \times \frac{1}{2^2} + \ldots\right] - (2)^2$

=

3) Let X be a Continous Random Variable with Pdf

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax + 3a & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

i) Find a

ii) Compute $P(x \le 1.5)$

iii) The CDF of X.

Cummulative density fn.

Soln:-

$\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\int_{0}^{3} f(x)\, dx = 1$

$= \int_0^1 ax\, dx + \int_1^2 a\, dx + \int_2^3 (-ax + 3a)\, dx = 1.$

$= \left[\frac{ax^2}{2}\right]_0^1 + \left[ax\right]_1^2 + \left[-\frac{ax^2}{2} + 3ax\right]_2^3 = 1$

$= \frac{a}{2} + (2a - a) + \left[\left(-\frac{9a}{2} + 9a\right) - \left(-\frac{4a}{2} + 6a\right)\right] = 1$

$= \frac{a}{2} + a + \left(\frac{9a}{2} - \frac{8a}{2}\right) = 1$

$= \frac{a}{2} + a + \frac{a}{2} = 1$

$= \frac{2a}{2} + a = 1$

$= 2a = 1 \quad a = 1/2$

ii) $P(\cancel{x \le} 1.5) \cancel{=} \int_0^{1.5} \cancel{f(x)} \cancel{dx}$.

$$f(x) = \begin{cases} x/2 & 0 \le x \le 1 \\ 1/2 & 1 \le x \le 2 \\ -1/2 x + 3/2 & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

ii) $P(X \le 1.5) = \int_0^{1.5} f(x)\,dx$.

$$= \int_0^1 ax\,dx + \int_1^{1.5} a\,dx.$$

$$= \int_0^1 \frac{x}{2}\,dx + \int_1^{1.5} 1/2\,dx.$$

$$= \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^{1.5}$$

$$= \frac{1}{4} + \frac{1.5}{2} - \frac{1}{2}$$

$$= \frac{1}{4} + \frac{0.5}{2} = \frac{1+1}{4} = \frac{2}{4}$$

$$= 1/2$$

iii) Cummulative density fn.

$0 \le x < 1$; $F(x) = \int_0^x f(x)\,dx$.

$$= \int_0^x \frac{x}{2}\,dx$$

$$= \left[\frac{x^2}{4}\right]_0^x$$

$$= \frac{x^2}{4}.$$

$1 \le x < 2$; $F(x) = \int_0^1 f(x)\,dx + \int_1^x f(x)\,dx$.

$$= \int_0^1 \frac{x}{2}\,dx + \int_1^x 1/2\,dx$$

$$= \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^x$$

$$= \frac{1}{4} + \frac{x}{2} - \frac{1}{2}$$

$$= \frac{x}{2} + \frac{1-2}{4}$$

$$= \frac{x}{2} - \frac{1}{4}.$$

$2 \le x < 3$; $F(x) = \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^x f(x)dx$

$= \int_0^1 \frac{x}{2} dx + \int_1^2 \frac{1}{2} dx + \int_2^x -\frac{1}{2}x + \frac{3}{2} dx$

$= \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^2 + \left[-\frac{x^2}{4} + \frac{3x}{2}\right]_2^x$

$= \frac{1}{4} + \left(1 - \frac{1}{2}\right) + \left[\left(-\frac{x^2}{4} + \frac{3x}{2}\right) - \left(\frac{-4}{4} + \frac{6}{2}\right)\right]$

$= \frac{1}{4} + \frac{1}{2} + \left(\left(-\frac{x^2}{4} + \frac{3x}{2}\right) - (-1 + 3)\right)$

$= \frac{3}{4} - 2 + \left(-\frac{x^2}{4} + \frac{3x}{2}\right)$

$= \frac{-5}{4} - \frac{x^2}{4} + \frac{3x}{2}$

$= \frac{-x^2}{4} + \frac{3x}{2} - \frac{5}{4}$.

4) A random variable X has the P.D.F

$f(x) = \begin{cases} 2x & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$

i) $P(X < 1/2)$

$= \int_0^{1/2} f(x)\, dx$

ii) $P(1/4 < x < 1/2)$

iii) $P(x \ge 3/4 \mid x > 1/2)$

i) $P(X < 1/2)$

$= \int_0^{1/2} f(x)\, dx$

$= \int_0^{1/2} 2x\, dx = \left[\frac{2x^2}{2}\right]_0^{1/2} = \frac{1}{4}$.

ii) $P(1/4 < x < 1/2)$

$= \int_{1/4}^{1/2} f(x)\, dx = \int_{1/4}^{1/2} 2x\, dx = \left[\frac{2x^2}{2}\right]_{1/4}^{1/2}$

$\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{4 - 1}{16} = +3/16$

iii) $P\left(x > 3/4 \mid x > 1/2\right)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P\left[\left(x > 3/4\right) \cap \left(x > 1/2\right)\right]}{P\left(x > 1/2\right)}$$

$$= \frac{P\left(x > 3/4\right)}{P\left(x > 1/2\right)}$$

Now $P(x \geq 3/4) = \int_{3/4}^{\infty} f(x)\,dx = \int_{3/4}^{1} f(x)\,dx$

$$\int_{3/4}^{1} 2x\,dx = \left[\frac{2x^2}{2}\right]_{3/4}^{1}$$

$$= \left(1 - \frac{9}{16}\right) = 7/16$$

Now, $P(x > 1/2) = \int_{1/2}^{\infty} f(x)\,dx = \int_{1/2}^{1} f(x)\,dx$

$$= \int_{1/2}^{1} 2x\,dx = \left[\frac{2x^2}{2}\right]_{1/2}^{1}$$

$$= \left(1 - \frac{1}{4}\right) = 3/4$$

$$\therefore \frac{P\left(x > 3/4\right)}{P\left(x > 1/2\right)} = \frac{7/16}{3/4}$$

$$\frac{7}{16} \times \frac{4}{3} = \frac{7}{12}$$

Distribution	Probability Mass fn	MGF	Mean	Variance
1) Binomial	$P(X=x) = nc_x\, p^x q^{n-x}$ $x = 0, 1, 2, \dots n.$	$M_X(t) = (pe^t+q)^n$	np	npq
2) Poisson	$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x = 1, 2, 3 \dots \infty$ $p+q=1$	$M_X(t) = e^{\lambda(e^t-1)}$	λ	λ
3) Geometric	$p(X=x) = q^{x-1}p$	$M_X(t) = \frac{pe^t}{1-qe^t}$	$1/p$	q/p^2

Binomial Distribution

5) The Mean and Variance of Binomial distribution are 4 and $4/3$. Find $P(X \geq 1)$.

Soln:-

Gn.

$np = 4$ — (1)

$npq = 4/3$ — (2)

$4q = 4/3$

$\boxed{q = \frac{4}{3}/4}$

$\boxed{q = 1/3}$

But $p + q = 1$

$p = 1 - q$

$p = 1 - 1/3$

$\boxed{p = 2/3}$

$\therefore p = 2/3 \quad q = 1/3$

Subs in (1)

$n(2/3) = 4$

$n = \frac{4 \times 3}{2} = 6 \quad \boxed{n = 6}$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - nC_x \, p^x \, q^{n-x}$$

$$= 1 - 6C_0 (2/3)^0 (1/3)^{6-0}$$

$$= 1 - [1 \cdot 1 \, (1/3)^6]$$

$$= 1 - \frac{1}{3^6}$$

$$= 1 - \frac{1}{729}$$

$$= \frac{728}{729}$$

$$= 0.998.$$

6) Find a probability that tossing a fair coins 5 times.

i) 3 heads.

ii) ~~Hea~~ getting $\boxed{\text{atleast}}$ one head.

iii) getting 3 tails and 2 heads.

iv) getting not more that 1 tail.

Soln:-

$n = 5.$

$p = 1/2$

$q = 1/2$

$\therefore$ Binomial distribution formula.

$$nC_x \, p^x \, q^{n-x}$$

$$x = 0, 1, 2, \ldots n$$

$$5C_x \, (1/2)^x \, (1/2)^{5-x} \quad \text{—(1)}$$

i) $P(X = 3).$

$$(1) \Rightarrow 5C_3 \, (1/2)^3 \, (1/2)^{5-3}$$

$$= 10 \left(\frac{1}{2^3}\right)\left(\frac{1}{4}\right)$$

$$= 10 \left(\frac{1}{8}\right)(1/4).$$

$$= \frac{10}{32}$$

$$= \frac{5}{16}$$

$$= 0.3125.$$

iii) Let X denote the no. of heads

P(getting 3 tails and 2 heads) =

P(2 heads) :

$P(X = 2)$

① $\Rightarrow 5C_2 (1/2)^2 (1/2)^{5-2}$

$= 10 \left(\frac{1}{4}\right)\left(\frac{1}{8}\right)$

$= 10/32$

$= 5/16$.

iv) getting not more than 1 tail.

P(getting not more than 1 tail)

= P(getting 0 tail) + P(getting 1 tail)

= P(getting 5 head) + P(getting 4 head).

$= P(X = 5) + P(X = 4)$.

$= 5C_5 (1/2)^5 (1/2)^0 + 5C_4 (1/2)^4 (1/2)^1$

$= 1\left(\frac{1}{32}\right) 1 + 5\left(\frac{1}{16}\right)\left(\frac{1}{2}\right)$

$= 1/32 + \frac{5}{32}$

$= 6/32 = 3/16$.

ii) P(getting atleast 1 head)

$= P(X \geq 1)$

$= \cancel{P}(1 - P(X < 1))$

$= 1 - P(X = 0)$

$= 1 - 5C_0 (1/2)^0 (1/2)^5$

$= 1 - 1 \cdot 1 \cdot 1/32$

$= 1 - 1/32$

$= 31/32$

7) Out of 800 families with 4 children each, for how many families would be expected to have

i) 2 boys & 2 girls.

ii) atleast 1 boy.

iii) atmost 2 girls.

iv) both are genders.

Soln:-

Let X be the no. of boys in the family.

$p = 1/2, \quad q = 1/2 \quad [\because p + q = 1]$.

$n = 4$

Binomial formula:

$P(X=x) = nC_x \, p^x \, q^{n-x}$

$= 4C_x (1/2)^x (1/2)^{4-x}$

i) $P(X=2) = 4C_2 (1/2)^2 (1/2)^2$

$= 6 (1/4)(1/4)$

$= 3/8$.

iii) Out of 800.

$\therefore 800 \times 3/8$

$= 300$.

ii) atleast 1 boy.

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - 4C_0 (1/2)^0 (1/2)^4$

$= 1 - 1 \times 1 \times 1/16$

$= 1 - 1/16$

$= 15/16$

$\therefore$ Out of 800 families

$800 \times \frac{15}{16}$.

$= 750$.

iii) atmost 2 girls.

$P(X=0) + P(X=1) + P(X=2)$ Girls.

$P(4 \text{ boys}) + P(3 \text{ boys}) + P(2 \text{ boys})$

$P(X=4) + P(X=3) + P(X=2)$.

$4C_4 (1/2)^4 (1/2)^0 + 4C_3 (1/2)^3 (1/2)^1 + \frac{3}{8}$.

$1\left(\frac{1}{16}\right) + 4\left(\frac{1}{8}\right)\left(\frac{1}{2}\right) + \frac{3}{8}$.

$\frac{1}{16} + \frac{4}{16} + \frac{3}{8}$.

$\frac{5}{16} + \frac{3}{8}$.

$\frac{5+6}{16} = 11/16$

$\therefore$ Out of 800 families.

$800 \times 11/16 = 50 \times 11 = 550$.

iv) both are genders.

$P(1 \text{ boy } 3 \text{ girls}) + P(2 \text{ boys } 2 \text{ girls}) + P(3 \text{ boys } 1 \text{ girl})$.

$P(X=1) + P(X=2) + P(X=3)$.

$4C_1 (1/2)^1 (1/2)^3 + \frac{3}{8} + \frac{4}{16}$.

$4(1/2)(1/8) + \frac{3}{8} + \frac{4}{16}$

$= \frac{4}{16} + \frac{3}{8} + \frac{4}{16}$.

$= \frac{8}{16} + \frac{3}{8}$.

$= \frac{1}{2} + \frac{3}{8} = 7/8$

Out of 800 families:

$800 \times \left(\frac{7}{8}\right) = 700$

Problem based on Poisson Distribution

8) ~~If~~ X ~~is~~ ~~a~~

$$P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

Mean $= \lambda$, Variance $= \lambda$.

8) If X is a poisson variate such that $P(X=1) = \frac{3}{10}$ and $P(X=2) = \frac{1}{5}$. Find $P(X=0)$ and $P(X=3)$.

Soln:-

$$P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$P(X = 1) = \frac{e^{-\lambda} \lambda^{1}}{1!} = \frac{3}{10} \quad \text{—— (1)}.$$

$$P(X = 2) = \frac{e^{-\lambda} \lambda^{2}}{2!} = \frac{1}{5}$$

$$= e^{-\lambda} \lambda^{2} = \frac{2}{5} \quad \text{—— (2)}.$$

Solving eqn (1) & (2).

$$\frac{(2)}{(1)} \quad \frac{e^{-\lambda} \lambda^{2} = 2/5}{e^{-\lambda} \cdot \lambda = 3/10}$$

$$\lambda = \frac{2}{5} \times \frac{10}{3}$$

$$\lambda = 4/3$$

$$P(X = 0) = \frac{e^{-4/3} (4/3)^{0}}{0!}$$

$$= e^{-4/3} \quad [\because 0! = 1].$$

$$P(X = 3) = \frac{e^{-4/3} (4/3)^{3}}{3!}$$

$$= \frac{e^{-4/3}\ 64/27}{6}$$

$$= \frac{e^{-4/3} \times 64}{162}$$

9) A manufacture of pins knows that 2% of his product are defective, If he sells pins in boxes of 100 he guarantees that not more than 4 pins be defective.

ii) What is the probability that a box will fail to meet the guarantee quality? $[e^{-2} = 0.13534]$.

Soln

$n = 100$

$P = 2\% = 2/100 = 0.025$.

$\lambda = np$.

$= 100 \times 0.025$.

$= 2$.

The Poison distribution is gn by.

$$\frac{e^{-\lambda}\lambda^{x}}{x!}$$

P(not more than 4 pins will be defective).

$P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$.

$$= \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}.2^2}{2!} + \frac{e^{-2}.2^3}{3!} + \frac{e^{-2}.2^4}{4!}$$

$$= \frac{0.13534 \times 1}{1} + \frac{0.13534 \times 2}{1} + \frac{0.13534 \times 4}{2} + \frac{0.13534 \times 8}{6} + \frac{0.13534 \times 16}{24}.$$

$= 0.13534 + 0.27068 + 0.27068 + 0.18045 + 0.09022$

$= 0.94737$.

(ii) P(a box will fail to meet the guarantee qualities) = P(more than 4 pin will be defective).

$P(X > 4) = 1 - P(X \leq 4)$.

$= 1 - 0.94737$.

$= 0.0526$.

10) If X is a poisson variable.

$P(x=2) = 9P(X=4) + 90P(x=6)$.

Find i) Mean of X

ii) Variance of X.

Soln:-

$P(X=2) = 9P(X=4) + 90P(x=6)$

$$\frac{e^{-\lambda}\lambda^2}{2!} = 9\left(\frac{e^{-\lambda}\lambda^4}{4!}\right) + 90\left(\frac{e^{-\lambda}\lambda^6}{6!}\right)$$

$$\frac{e^{-\lambda}\lambda^2}{2!} = e^{-\lambda}\lambda^2\left[\frac{9\lambda^2}{4!} + \frac{90\lambda^4}{6!}\right]$$

$$\frac{1}{2} = \left[\frac{\cancel{9}^{3}\lambda^2}{\cancel{24}_{8}} + \frac{\cancel{90}^{\cancel{9}\,1}\lambda^4}{\cancel{720}_{\cancel{72}\,8}}\right]$$

$$\frac{1}{2} = \frac{3\lambda^2 + \lambda^4}{8}$$

$$\lambda^4 + 3\lambda^2 - 4 = 0.$$

$$\lambda = \frac{-b \pm \sqrt{\cancel{4ac}\, b^2 - 4ac}}{2a}.$$

$$\frac{-3 \pm \sqrt{9 - 4(1)(-4)}}{2(1)}$$

$$\lambda^2 = \frac{-3 \pm \sqrt{9+16}}{2}$$

$$\lambda^2 = \frac{-3 \pm \sqrt{25}}{2}.$$

$$\lambda^2 = \frac{-3 \pm 5}{2}$$

$$\lambda^2 = \frac{-3+5}{2}, \frac{-3-5}{2}$$

$$\lambda^2 = \frac{2}{2}, \frac{-8}{2} \cdot \lambda^2 = (1, -4).$$

$$\lambda = \sqrt{1}, \sqrt{-4}$$

$$\lambda = \pm 1, \pm 2i$$ [Neglect the negative term]

$\therefore \lambda = 1$

Mean, $\lambda = 1$

Variance, $\lambda = 1$

S.D. $\lambda = \sqrt{1}$.