Testing of Hypothesis

Population.

A population in statistics means a set of objects or mainly the set of numbers which are measurements or observations pertaining to the objects.

The population is finite or infinite. According to the number of element of the set is finite or infinite.

Sampling

A part selected from the population is called a sample. and the process of selection of a sample is called sampling.

Random Sampling

A random sampling is the one in which each number of population has an equal chance of being included in it.

Parameters and statistics.

The statistical constants of the population such as mean (μ), standard deviation (σ) are called parameters.

The mean $\bar{x}$ and the S.D (S) of a sample are known as statistics.

Symbols used for Population & Samples.

Population Parameter	Sample Statistics
1. Population Size : N	Sample Size : n
2. Population Mean : μ	Sample mean : $\bar{x}$
3. Population S.D : σ	Sample S.D : S
4. Population Propotion : P	Sample Propotion : $\bar{P}$

Types of Sampling

1) Purpose in Sampling.
2) Random Sampling.
3) Stratified Sampling.
4) Systemetic Sampling.
5) ~~Sar~~

Sampling Distribution.

From a population, a number of samples are drawn of equal size n. Find out the mean of each sample. The means of samples are not equal. The means with their respective frequencies are grouped. The frequency distribution so formed is known as sampling distribution of the mean. Similarly sampling distribution of S.D can be had.

Test of Significance.

Test of Significance enables us to decide on the basis of the results of the samples, whether

1. The deviation b/w the observed sample statistic and the hypothetical parameter value or

2. The deviation b/w two samples statistics a significant or might be attributed due to the chance of the sampling.

Testing a hypothesis

(i) Null hypothesis (H_0)

(ii) Alternative hypothesis (H_1).

(i) Null hypothesis (H_0) is ~~the~~

Null hypothesis (H_0) is the hypothesis of no difference (i.e) there is no significance difference ~~which~~ b/w the observed value and the expected value. The Null hypothesis is denoted by (H_0).

(ii) Alternative hypothesis (H_1).

Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis and it is denoted by H_1.

Suppose if we said, the null hypothesis as $H_0 : \mu = \mu_0$ then the alternative hypothesis will be

(i) $H_1 : \mu \neq \mu_0$

(ii) $H_1 : \mu > \mu_0$

(iii) $H_1 : \mu < \mu_0$

- H_1 in (i) is called two tailed alternative hypothesis.
- H_1 in (ii) is called a right tailed alternative hypothesis.
- H_1 in (iii) is called a left tailed alternative hypothesis.

Errors

Type I Error

If H_0 is rejected while it should have been accepted.

Type II Error

If H_0 is accepted while it should have been rejected.

Critical Region.

A region corresponding to a statistic in a sample space which amounts to rejection of the null hypothesis H_0 is called critical region or region of rejection.

The region of the sample space which amounts to the acceptance of H_0 is called acceptance region.

Level of Significance.

The probability α that a

Test of significance of small samples

When the size of the sample $n \leq 30$. Then the sample is called small sample.

Test for small sample

Students's 't' test

F - test

χ^2 - test

The students 't' test is defined by $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

where,

Sample Mean, $\bar{x} = \dfrac{\Sigma x_i}{n}$

Population Mean $= \mu$

S.D of Sample,

$$s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}}$$

ii) If the standard deviation of the sample is given directly then the statistic is gn by

$$\cancel{t = \frac{\frac{\bar{x} - \mu}{SD}}{\sqrt{n-1}}}, \quad t = \frac{\bar{x} - \mu}{S.D/\sqrt{n-1}}$$

Confidence limits.

If $t_{0.05}$ is the table value of t for $(n-1)$ decrease of freedom at 5% level of significance then 95% confidence limit for μ is gn by.

$$\bar{x} \pm t_{0.05} \frac{s}{\sqrt{n}}.$$

Ill'ly 99% of confidence limit of μ is gn by

$$\bar{x} \pm t_{0.01} \frac{s}{\sqrt{n}}$$

1) A mechanist is making engine parts with axle diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.0742 inch with the S.D of 0.040 inch. Compute the statistics you would use to test whether the work is meeting the specification.

Soln:-

Sample Size, $n = 10 < 30 \Rightarrow$ Small Sample.

Sample Mean, $\bar{x} = 0.742$

Sample S.D, $s = 0.040$

Population Mean, $\mu = 0.700$

i) Null Hypothesis : $H_0 : \mu = 0.700$

ii) Alternative Hypothesis :

$H_1 : \mu \neq 0.700$ (Two Tailed)

iii) Degree of freedom : $n-1 = 10-1 = 9$

iv) Level of Signification, $\alpha = 5\%$

v) Test of Statistic

$$\cancel{t = \frac{\bar{x} - \mu}{S.D / \sqrt{n}} = \frac{0.742 - 0.700}{0.40 / \sqrt{9}}}$$

$$t = \frac{\bar{x} - \mu}{S.D / \sqrt{n-1}}$$

$$= \frac{0.742 - 0.700}{0.040 / \sqrt{10-1}}$$

$$= \frac{0.042}{0.0133}$$

$$t = 3.15.$$

Calculated value of $t = 3.15$.

vi) Table value of $t = 2.262$;

vii) Conclusion

Cal t > Tab T

$\therefore$ we reject H_0.

$\therefore$ Work is not meeting the specification.

2) A machine is designed to produce insulating washers for electrical devices of ~~an~~ average thickness of 0.025 cm. A random sample of 10 washers was found to have a thickness of 0.024 cm with the S.D of 0.002 cm. Test the significance of the deviation.

3) The mean life time of a sample of 25 ($n=$) flurocent light bulbs produced by a company is computed to be 1570 ($\bar{x}$) hrs with the S.D of 120 (S.D) hrs. The company claims that the avg life of the bulbs produced by the company is 1600 ($\mu=$) hrs. Is the claim acceptable.

4) The avg ... specified to the steal rod is specified to be 18.5 thousand pounds. To test this a sample of 14 rods was tested. The mean and S.D obtained where 17.85 and 1.955 respectively. Is the result of the experiment significant?

Soln:-

Sample Size, $n = 14 < 30 \Rightarrow$ Small Sample.

Sample Mean, $\bar{x} = 17.85$.

Sample SD $= 1.955$.

Population Mean, $\mu = 18.5$

i) Null Hypothesis: $H_0 : \mu = 18.5$.

ii) Alternative Hypothesis

$H_1 : \mu \neq 18.5$.

iii) Degree of freedom: $n-1 = 14-1 = 13$

iv) Level of significance: $\alpha = 5\%$.

v) Test of statistics.

$$t = \frac{\bar{x} - \mu}{S.D / \sqrt{n-1}}$$

$$t = \frac{17.85 - 18.5}{1.955 - \sqrt{13}}$$

$$= \frac{-0.65}{1.955 / 3.6055}$$

$$= \frac{-0.65}{\frac{0.5422}{\cancel{1.6505}}}$$

$= \cancel{0.3938}\ -1.1988 \quad |t| = 1.199$

Calculated value of $t = \cancel{0.3938}\ +1.1988$

vi) Table value of $t = 2.160$.

vii) Conclusion

Cal t < Tab T

∴ We accept Ho.

∴ The result of the experiment is significant.

2) Soln:-

Sample Size, $n = 10 < 30$ = Small sample.

Sample Mean, $\bar{x} = 0.024$

Sample S.D, = 0.002.

Population Mean, $\mu = 0.025$.

i) Null Hypothesis, Ho: $\mu = 0.025$.

ii) Alternative Hypothesis

$H_1: \mu \neq 0.025$. (two tail)

iii) Degree of freedom, $n-1 = 10-1 = 9$

iv) Level of Significance $\alpha = 5\%$

v) Test of statistics

$$t = \frac{\bar{x} - \mu}{S.D / \sqrt{n-1}}$$

$$= \frac{0.024 - 0.025}{0.002 / \sqrt{9}}$$

$$= \frac{-1 \times 10^{-03}}{6.666 \times 10^{-04}}$$

$t = -1.5$

$|t| = 1.5$

Calculated value of $t = +1.5$.

vi) Table value of $t = 2.262$

vii) Conclusion

Cal t < Tab t

Calculated value of t is less than table value

∴ we accept Ho.

3) Soln:-

Sample Size, $n = 25 < 30$ = Small Sample.

Sample Mean, $\bar{x} = 1570$.

Sample S.D = 120.

Population Mean, $\mu = 1600$.

i) Null Hypothesis, $H_0 : \mu = 1600$

ii) Alternative hypothesis $H_1 : \mu \neq 1600$. (two tail).

iii) Degree of freedom, $n-1 = 25-1 = 24$.

iv) Level of significance : $\alpha = 5\%$.

v) Test statistic

$$t = \frac{\bar{x} - \mu}{SD / \sqrt{n-1}}$$

$$= \frac{1570 - 1600}{120 / \sqrt{24}}$$

$$= \frac{-30}{120 / 4.898}$$

$t = -1.2245$.

$|t| = 1.2245$.

Calculated $t = 1.2245$.

Table Value $t = 2.064$.

$\therefore$ Cat t < Tab t.

$\therefore$ We ~~reject~~ accept H_0.

$\therefore$ The result of the experiment is significant.

5) A random sample of size 16 volts from a normal population showed a mean of 53 and a sum of squares of deviation from the mean equals to 150. Can this sample be regarded as taken from the population having 56 as mean. Obtain 95% confidence limits of the mean of the population.

95% = 5%.
99% = 1%.

Soln:-

Same size, $n = 16 < 30 \Rightarrow$ small sample.

Sum of the squares of the deviation from the mean.

$\Sigma (x - \bar{x})^2 = 150$.

$\mu = 56$.

$\bar{x} = 53$.

$$t = \frac{\bar{x} - \mu}{SD / \sqrt{n-1}}$$

To find SD

$$S = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{150}{15}} = \sqrt{10} = 3.16.$$

i) Null Hypothesis, $H_0 : \mu = 56$.

ii) Alternative hypothesis : $H_1 : \mu \neq 56$. (two tail) ($\because \mu$ is $\neq$)

iii) Degrees of freedom, $n-1 = 16-1 = 15$.

iv) Level of significance, $\alpha = 5\%$.

v) Test statistic.

$$t = \frac{\bar{x} - \mu}{S.D / \sqrt{n}}$$

$$= \frac{53 - 56}{3.16 / \sqrt{16}}$$

$$t = \frac{-3}{3.16/4}$$

$$t = -3.797$$

$|t|$ 3.797.

vi) Table value of $t = 2.131$.

vii) Conclusion

Cal $t >$ Tab T

$\therefore$ We reject Null hypothesis H_0.

The 95% confidence interval of the population is the mean of the population is given by

$$\bar{x} \pm t_{0.05} \frac{s}{\sqrt{n}}$$

i.e

$$\cancel{53 \pm 2.131_{0.05}}$$

$$\cancel{53 \pm 2.0}$$

$$53 \pm 2.131 \left(\frac{3.16}{\sqrt{16}} \right)$$

i.e

$$53 \pm 1.683.$$

$$(53 - 1.683, 53 + 1.683).$$

$$(51.316, 54.683).$$

$\therefore$ 95% of the confidence limit is $(51.316, 54.683)$.

6) The mean weekly sales of soap box in a departmental stores was 146.3 box per store. After an advertising campaign. The mean weekly sales in 22 stores for a typical weak increased to 153.7 and showed a S.D of 17.2. Was the advertising campaign successful.

Soln:-

Sample Size, $n = 22 < 30 \Rightarrow$ Small sample.

$SD = 17.2$

Sample Mean $\bar{x} = 153.7$

Population Mean, $\mu = 146.3$.

i) Null Hypothesis $H_0 : \mu = 146.3$.

ii) Alternative hypothesis

$H_1 : \mu > 146.3$. (one tail)

[$\because \bar{x}$ is >]

iii) Degree of freedom, $n-1 = 22-1 = 21$.

iv) Level of significance $\alpha = 5\%$.

v) Test of statistic

$$t = \frac{\bar{x} - \mu}{SD / \sqrt{n-1}}$$

$$= \frac{153.7 - 146.3}{17.2 / \sqrt{21}}$$

$$= \frac{7.4}{17.2 / 4.582}$$

$$t = 1.971$$

Calculated $t = 1.971$

Table Value $t = 1.721$.

Cat $t >$ Tab t

$\therefore$ We reject H_0.

T test when S.D of the sample is not given directly

7) A random sample of 10 boys had the following I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do this data support the assumption of a population mean I.Q of 100. (T test) Find a reasonable range in which most of the mean I.Q values of sample of 10 boys lie.

Soln:-

Sample Size, $n = 10 < 30 \Rightarrow$ Small sample.

Sample Mean.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{70 + 120 + 110 + 101 + 88 + 83 + 95 + 98 + 107 + 100}{10}$$

$$= 972 / 10$$

$$\bar{x} = 97.2$$

Population mean, $\mu = 100$.

S.D

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

To find $(x-\bar{x})^2$

x	$x-\bar{x}$ $x - 97.2$	$(x-\bar{x})^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84

$\Sigma(x-x)^2 = 1833.6$

$$S = \sqrt{\frac{1833.6}{9}}$$

$S = \sqrt{203.733}$

SD = 14.273

i) Null hypothesis; $H_0: \mu = 100$.

ii) Alternative hypothesis;

$H_1: \mu \neq 100$ (two tail).

iii) Degree of freedom, $n-1 = 10-1 = 9$

iv) Level of significance, $\alpha = 5\%$

v) Test statistic,

$$t = \frac{\bar{x} - \mu}{SD/\sqrt{n}}$$

$$= \frac{97.2 - 100}{14.273/\sqrt{10}} = \frac{-2.8}{14.273/3.162}$$

$$= \frac{-2.8}{4.513} = -0.6204$$

$t = -0.6204 \Rightarrow |t| = 0.6204$ ~~0.588~~

Calculated $t = 0.6204$

Table value $t = 2.262$.

Cal t < Tab t.

$\therefore$ We accept H_0.

8) The 9 items of the sample had the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Thus the mean of the 9 items. Define significantly from the assumed population mean 47.5?

Soln:-

Sample Size, $n = 9 < 30$ = Small sample.

Sample Mean,

$$\bar{x} = \frac{\Sigma x}{n} = \frac{45 + 47 + 50 + 52 + 48 + 47 + 49 + 53 + 51}{9}$$

$$= \frac{442}{9}$$

$$= 49.11$$

Population Mean, $\mu = 47.5$.

S-D

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

To find $(x-\bar{x})^2$

x	$x-\bar{x}$ $x - 49.11$	$(x-\bar{x})^2$
45	-4.11	16.89
47	-2.11	4.452
50	0.89	0.7921
52	2.89	8.3521
48	-1.11	1.2321
47	-2.11	4.4521
49	-0.11	0.0121
53	3.89	15.1321
51	1.89	3.5721

$$\Sigma(x-\bar{x})^2 = 54.8867$$

$$S = \sqrt{\frac{54.8867}{8}}$$

$$S = \sqrt{6.860}$$

$$S = 2.619$$

i) Null hypothesis $H_0: \mu = 47.5$.

ii) Alternative hypothesis: $H_1: \mu \neq 47.5$.

iii) Degree of freedom, $n-1$, $9-1 = 8$.

iv) Level of significance, $\alpha = 5\%$.

v) Test Statistic

$$t = \frac{\bar{x} - \mu}{SD/\sqrt{n}}$$

$$= \frac{49.11 - 47.5}{2.619/\sqrt{9}}$$

$$= \frac{1.61}{2.619/\cancel{2.828}\ 3} = \frac{1.61}{\cancel{0.926}\ 0.873}$$

$t = \cancel{1.7386}$. 1.8442

Calculated $t = \cancel{1.7386}$. 1.8442

vi) Table value $t = 2.306$.

vii) Conclusion :-

Cal t < Tab t

∴ We accept H_0.

9) The ~~heighst~~ of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inchs ~~chers~~. Is it reasonable to believe that average height is greater than 64 inches.

Soln:-

Sample Size, $n = 10 < 30$ = small sample.

sample mean,

$$\bar{x} = \frac{\Sigma x}{n} = \frac{70 + 67 + 62 + 68 + 61 + 68 + 70 + 64 + 64 + 66}{10}$$

$$= \frac{660}{10}$$

$$= 66.$$

Population Mean, $\mu = 64$

S.D

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n-1}}$$

To find $(x - \bar{x})^2$

x	$x - \bar{x}$ $x - 66$	$(x - \bar{x})^2$
70	4	16
67	1	1
62	-4	16
68	2	4
61	-5	25
68	2	4
70	4	16
64	-2	4
64	-2	4
66	0	0
		= 90

$$S = \sqrt{\frac{90}{9}} = \sqrt{10}$$

$= 3.162$

i) Null hypothesis $H_0 : \mu = 64$

ii) Alternative hypothesis $H_1 : \mu > 64$ (one tail)

iii) Degree of freedom, $n-1$, $10-1 = 9$

iv) Level of Significance, $\alpha = 5\%$.

v) Test statistics

$$t = \frac{\bar{x} - \mu}{SD/\sqrt{n}}$$

$$= \frac{66 - 64}{3.162/\sqrt{10}} = \frac{2}{0.999}$$

$= ~~1.891~~ \; 2.002$

Calculated $t = 2.002$

vi) Table value $t = ~~2.262~~ \; 1.833$

vii) Conclusion

Cal $t >$ Tab t

$\therefore$ ~~We accept H_0.~~

$\therefore$ We reject H_0 (Null hypothesis)

(b) The following ... lengths of 12 samples of Egyption cotton taken from a large consiquient 48, 46, 49, 46, 52, 45, 43, 47, 47, 46, 47, 50. Test if the mean length of the consignment can be taken as 46.

Soln:-

Sample size, $n = 12 < 30$ = sample size.

Sample Mean

$$\bar{x} = \frac{\Sigma x}{n} = \frac{48 + 46 + 49 + 46 + 52 + 45 + 43 + 47 + 47 + 46 + 47 + 50}{12}$$

$= 566/12$

$= 47.166.$

Population Mean $= \mu = 46.$

$$S.D = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}}$$

To find $(x - \bar{x})^2$

x	$x - \bar{x}$ $x - 47.166$	$(x - \bar{x})^2$
48	0.834	0.695556
46	-1.166	1.359556
49	1.834	3.363556
46	-1.166	1.359556
52	4.834	23.367556
45	-2.166	4.691556
43	-4.166	17.355556
47	-0.166	0.027556
47	-0.166	0.027556
46	-1.166	1.359556
47	-0.166	0.027556
50	2.834	8.031556
		61.66668.

$$S = \sqrt{\frac{61.66668}{11}}$$

$$S = \sqrt{5.60606}$$

$$S = 2.3677.$$

i) Null hypothesis $H_0: \mu = 46.$

ii) Alternative hypothesis $H_1: \mu \neq 46.$

iii) Degree of freedom, $n-1 = 12-1 = 11$

iv) Level of significance $\alpha = 5\%.$

v) Test statistics

$$t = \frac{\bar{x} - \mu}{SD/\sqrt{n}}$$

$$= \frac{47.166 - 46}{2.3677/\sqrt{12}}$$

$$= \frac{1.166}{2.3677/\cancel{3.3166}\ 3.464} = \frac{1.166}{\cancel{0.7138}\ 0.683}.$$

$$= \cancel{1.6335}\ 1.707$$

Calculated $t = \cancel{1.6335}\ 1.707.$

vi) Table value $t = 2.201.$

vii) Conclusion

Cal t < Tab t

$\therefore$ We accept H_0.

11) Certain pesticide is packed into bags by a machine a Random sample of 10 bags is drawn and their contains are found to

weight in kgms as follows
50, 49, 52, 44, 45, 48, 46, 45, 49, 45.
Test if the average packing can be taken to be 50 kg.

Soln:-

Sample size $n = 10 < 30$ = small sample.

Sample Mean

$$\bar{x} = \frac{\Sigma x}{n} = \frac{50+49+52+44+45+48+46+45+49+45}{10}$$

$$= 473/10$$

$$\bar{x} = 47.3$$

$$S.D = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n-1}}$$

x	$x-\bar{x}$ $x-47.3$	$(x-\bar{x})^2$
50	2.7	7.29
49	1.7	2.89
52	4.7	22.09
44	-3.3	10.89
45	-2.3	5.29
48	0.7	0.49
46	-2.3	5.29
45	1.7	2.89
49	-2.3	5.29
45		
	$\Sigma(x-\bar{x})^2$	64.1

$$S = \sqrt{\frac{64.1}{9}} = \sqrt{7.122}$$

$$S = 2.668$$

Population Mean $\mu = 50$

i) Null hypothesis $H_0 : \mu = 50$

ii) Alternative hypothesis $H_1 : \mu \neq 50$.

iii) Degree of freedom, $n-1$, $10-1=9$.

iv) Level of significance, $\alpha = 5\%$.

v) Test statistic

$$t = \frac{\bar{x}-\mu}{SD/\sqrt{n}}$$

$$= \frac{47.3-50}{2.668/\sqrt{10}} = \frac{-2.7}{2.668/3.162}$$

$$t = \cancel{-3.0371}\ -3.2028 \qquad = -2.7/0.843$$

$$|t| = \cancel{3.0371}\ 3.202$$

Calculated t = ~~3.0371~~ 3.202

vi) Table value $t = 2.262$

vii) Conclusion

Cal t > Tab t

∴ We reject H_0 (null hypothesis)

Students T test for difference of Means.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

where

$$s^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

[if the sum of the sequence of the deviation from the mean is given.]

(or)

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$ [∵ if standard deviation is given.]

Degree of freedom = $n_1 + n_2 - 2$.

12) The average number of particles produced by two machines per day are 200 and 250 with standard deviations 20 and 25 respectively on the bases of regards of 25 days production. Can you regard both the machines equally efficient at 1% level of significance.

Soln :-

Gn.

$n_1 = 25$ $\quad$ $n_2 = 25$.

$\bar{x}_1 = 200$ $\quad$ $\bar{x}_2 = 250$

$s_1 = 20$ $\quad$ $s_2 = 25$.

Now $s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$= \frac{25(20)^2 + 25(25)^2}{25 + 25 - 2}$

$= \frac{25(400) + 25(625)}{48}$

$= 25625/48$

$s^2 = 533.85$

i) Null hypothesis : $H_0 : \mu_1 = \mu_2$

ii) Alternative hypothesis: $H_1 : \mu_1 \neq \mu_2$

iii) Degree of freedom = $n_1 + n_2 - 2$

$= 48$.

iv) Level of significance, $\alpha = 1\%$.

v) Test statistic.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$= \frac{200 - 250}{\sqrt{533.85\left[\frac{1}{25} + \frac{1}{25}\right]}}$$

$$= \frac{-50}{\sqrt{533.85(2/25)}} = \frac{-50}{\sqrt{533.85 \times 0.08}}$$

$$= \frac{-50}{\sqrt{42.708}} = \frac{-50}{6.535}$$

$$t = -7.651$$

$$|t| = 7.651$$

vi) Table value $t = 2.58$.

vii) Conclusion.

Cal t > Tab t

$\therefore$ The calculated value t > Table value

$\therefore$ We reject H_0.

3) Samples of 2 types of electric light bulbs where tested for length of life. And the following data where obtained.

Type I

$n_1 = 8$

$\bar{x}_1 = 1234$ hrs

$s_1 = 36$ hrs

Type II

$n_2 = 7$

$\bar{x}_2 = 1036$ hrs.

$s_2 = 40$ hrs.

Is the difference in the mean significant to that Type I is superior (>) to Type II regarding the length of life.

Soln:-

Gn:-

Type I

$n_1 = 8$

$\bar{x}_1 = 1234$

$s_1 = 36$.

Type II

$n_2 = 7$

$\bar{x}_2 = 1036$

$s_2 = 40$.

$$\text{Now } s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{8(36)^2 + 7(40)^2}{8 + 7 - 2}$$

$$= \frac{8(1296) + 7(1600)}{13}$$

$$= 21568/13$$

$$s^2 = 1659.07$$

i) Null hypothesis $H_0: \mu_1 = \mu_2$

ii) Alternative hypothesis $H_1: \mu_1 > \mu_2$.

iii) Degree of freedom, $n_1 + n_2 - 2 = 13$

iv) Level of significan, $\alpha = 5\%$.

v) Test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$= \frac{1234 - 1036}{\sqrt{1659.07\left[\frac{1}{8} + \frac{1}{7}\right]}}$$

$$= \frac{198}{\sqrt{1659.07\left[15/56\right]}}$$

$$= \frac{198}{\sqrt{1659.07 \times 0.267}}$$

$$= 198 / \sqrt{442.97} = \frac{198}{21.046}$$

$t = \cancel{0.446}$ 9.407. $\frac{9.407}{\cancel{0.446}}$

Calculated value of $t =$

vi) Table value of $t = 1.771$

vii) Conclusion

Cal $t >$ Tab t

$\therefore$ We ~~adcept~~ reject H_0. (Null hypothesis)

14) The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively. The sum of the squares of the deviation from the mean of 26.94 and 18.73 respectively. Can the sample be considered to have been drawn from the same normal population.

Soln:-

Gn:-

$n_1 = 9$ $\quad$ $n_2 = 7$.

$\bar{x}_1 = 196.42$ $\quad$ $\bar{x}_2 = 198.82$

$\Sigma(x_1 - \bar{x}_1)^2 = 26.94$ $\quad$ $\Sigma(x_2 - \bar{x}_2)^2 = 18.73$

$$s^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

$$= \frac{26.94 + 18.73}{9 + 7 - 2}$$

$$= \frac{45.67}{14}$$

$s^2 = 3.262$.

i) Null hypothesis $H_0 : \mu_1 = \mu_2$

ii) Alternative hypothesis $H_1 : \mu_1 \neq \mu_2$

iii) Degree of freedom; $n_1 + n_2 - 2 = 14$.

iv) Level of significance; $\alpha = 5\%$.

iv) Test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$= \frac{196.42 - 198.82}{\sqrt{3.262\left[\frac{1}{9} + \frac{1}{7}\right]}}$$

$$= \frac{-2.4}{\sqrt{3.262 \times \left(\frac{16}{63}\right)}}$$

$$= \frac{-2.4}{\sqrt{3.262 \times 0.253}}$$

$$= \frac{-2.4}{\sqrt{0.828}} = \frac{-2.4}{0.910}$$

$t = -2.637$

$|t| = 2.637$.

Calculated $t = 2.637$.

vi) Table value of $t = 2.145$.

vii) Conclusion

Cal t > Tab t

$\therefore$ We reject Ho (Null hypothesis)

b) The average no. of articles produced by 2 machines per day are 200 and

15) Below are given the gain in weights of pigs fed on two diets A and B.

Diet A	25, 32, 30, 34, 24, 14, 32, 24, 30, 31, 35, 25.
Diet B	44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18, 21, 35, 29, 22

Test if the two diets differ significantly as regards their effect on increase in weight.

Soln:-

Gn:-

$n_1 = 12$; $n_2 = 15$.

x_1	x_2	$x_1 - \bar{x}_1$ $x_1 - 28$	$x_2 - \bar{x}_2$ $x_2 - 30$	$(x_1 - \bar{x}_1)^2$	$(x_2 - \bar{x}_2)^2$
25	44	-3	14	9	196
32	34	4	4	16	16
30	22	2	-8	4	64
34	10	6	-20	36	400
24	47	-4	17	16	289
14	31	-14	1	196	1
32	40	4	10	16	100
24	30	-4	0	16	0
30	32	2	2	4	4
31	35	3	5	9	25
35	18	7	-12	49	144
25	21	-3	-9	9	81

-	35	5		25
-	29	-1		
-	22	-8		64
336	450		380	1410

$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{336}{12} = 28$

$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{450}{15} = 30$

$\Sigma(x_1 - \bar{x}_1)^2 = 380$

$\Sigma(x_2 - \bar{x}_2)^2 = 1410$

$$s^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

$$= \frac{380 + 1410}{25} = 1790/25$$

$s^2 = 71.6$

i) Null hypothesis $H_0 : \mu_1 = \mu_2$

ii) Alternative hypothesis $H_1 : \mu_1 \neq \mu_2$

iii) Degree of freedom $n_1 + n_2 - 2 = 25$

iv) Level of significance, $\alpha = 5\%$

v) Test statistic.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$= \frac{28 - 30}{\sqrt{71.6\left[\frac{1}{12} + \frac{1}{15}\right]}}$$

$$= \frac{-2}{\sqrt{71.6\left[\frac{27}{180}\right]}}$$

$$= \frac{-2}{\sqrt{71.6 \times 0.15}} = \frac{-2}{\sqrt{10.74}}$$

$$= \frac{-2}{3.277}$$

$t = -0.6103$

$|t| = 0.6103$

Calculated $t = 0.6103$

vi) Table value of t is 2.060.

vii) Conclusion.

Cal t < Tab t

$\therefore$ We reject H_0 (Null hypothesis).

16) The heights of six randomly choosen sailors are in inches 63, 65, 68, 69, 71, and 72 and. Those of 10 randomly choosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72, and 73. Discuss the light that these data throw on the suggestion that sailors are on the average taller than soldiers.

Soln:-

Gn:- $n_1 = 6$; $n_2 = 10$.

x_1	x_2	$(x_1 - \bar{x}_1)^2$ $x_1 - 68$	$(x_2 - \bar{x}_2)^2$ $x_2 - 67.8$
63	61	25	46.24
65	62	9	33.64
68	65	0	7.84
69	66	1	3.24
71	69	9	1.44
72	69	16	1.44
	70		4.84
	71		10.24
	72		17.64
	73		27.04
408	678	60	153.6

$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = 68$

$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = 67.8.$

$\Sigma(x_1 - \bar{x}_1)^2 = 60$

$\Sigma(x_2 - \bar{x}_2) = 153.6.$

$S^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$

$= \frac{60 + 153.6}{6 + 10 - 2}$

$= \frac{213.6}{14}$

$S^2 = 15.257$

i) Null hypothesis $H_0 : \mu_1 = \mu_2$

ii) Alternative hypothesis $H_1 : \mu_1 \geq \mu_2$

iii) Degree of freedom, $n_1 + n_2 - 2 = 14$.

iv) Level of significance, $\alpha = 5\%$.

v) Test statistic.

$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$

$$= \frac{68 - 67.8}{\sqrt{15.257\left[\frac{1}{6} + \frac{1}{10}\right]}}$$

$$= \frac{0.2}{\sqrt{15.257\left[16/60\right]}}$$

$$= \frac{0.2}{\sqrt{15.257 \times 0.266}}$$

$$= \frac{0.2}{\sqrt{4.068}} = \frac{0.2}{2.017}$$

$t = 0.099$

Calculated $t = 0.099$

vi) Table value of $t = 1.761$

vii) Conclusion

Cal $t <$ Tab t

∴ We accept Ho. (Null hypothesis).

i.e, the sailors are not on the average taller than the soldiers.

17) The horses A

tested according to the time (in seconds) to run a particular track with the following results.

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	27	-

Test whether you can discriminate b/w two horses. You can use the fact that 5% value of t for 11 degrees of freedom is 2.2.

Soln:-

$n_1 = 7$; $n_2 = 6$.

x_1	x_2	$(x_1 - \bar{x}_1)^2$ $\bar{x}_1 = 31.28$	$(x_2 - \bar{x}_2)^2$ $\bar{x}_2 = 27.8$
28	29	10.75	1.44
30	30	1.44	4.84
32	30	0.64	4.84
33	24	3.24	14.44
33	27	3.24	0.64
29	27	4.84	0.64
34	-	7.84	
219	167	31.48	26.84

$$\bar{x}_1 = \frac{\Sigma x_1}{n} = \frac{219}{7} = 31.28$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n} = \frac{167}{6} = 27.83$$

$$\Sigma(x_1 - \bar{x}_1)^2 = 31.48$$

$$\Sigma(x_2 - \bar{x}_2)^2 = 26.84$$

$$S^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

$$= \frac{31.48 + 26.84}{11}$$

$$= 58.32/11$$

$$S^2 = 5.301$$

$$S = 2.3$$

i) Null hypothesis, $H_0 : \mu_1 = \mu_2$

ii) Alternative hypothesis, $H_1 : \mu_1 \neq \mu_2$

iii) Degree of freedom, $n_1 + n_2 - 2 = 11$

iv) Level of significance, $\alpha = 5\%$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$= \frac{31.28 - 27.83}{\sqrt{5.301\left[\frac{1}{7} + \frac{1}{6}\right]}}$$

$$= \frac{\cancel{59.11}\ 3.45}{\sqrt{5.301\left[\frac{13}{42}\right]}}$$

$$= \frac{\cancel{59.11}\ 3.45}{\sqrt{5.301 \times 0.309}} = \frac{59.11}{\sqrt{1.640}}$$

$$= \frac{\cancel{59.11}\ 3.45}{1.280} \quad 0.843$$

$$t = \cancel{46.17}\ 2.69$$

vi) Table value of $t = 2.201$

vii) Conclusion

Cal t > Tab t

∴ We reject H_0 (Null hypothesis).

18) The nicotine content in milligrams of two samples of tobacco were found to be as follows.

Sample A	24	27	26	21	25	
Sample B	27	30	28	31	22	36

Can it be said that two samples come from normal populations having the same mean.

Soln:-

$n_1 = 5$; $n_2 = 6$.

x_1	x_2	$(x_1-\bar{x}_1)^2$ $x_1 - 24.6$	$(x_2-\bar{x}_2)^2$ $x_2 - 29$
24	27	0.36	4
27	30	5.76	1
26	28	1.96	1
21	31	12.96	4
25	22	0.16	49
-	36		49
123	174	21.2	108.

$\bar{x}_1 = \frac{\Sigma x_1}{n} = 24.6$

$\bar{x}_2 = \frac{\Sigma x_2}{n} = 29$

$\Sigma(x_1-\bar{x}_1)^2 = 21.2$

$\Sigma(x_2-\bar{x}_2)^2 = 108$

$S^2 = \frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}$

$= \frac{21.2 + 108}{5+6-2} = \frac{129.2}{9}$

$S^2 = 14.35$

i) Null hypothesis $H_0: \mu_1 = \mu_2$

ii) Alternative hypothesis $\mu_1 \neq \mu_2$

iii) Degree of freedom, $n_1+n_2-2 = 9$.

iv) Level of significance, $\alpha = 5\%$.

v) Test statistics.

$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}}$

$= \frac{24.6 - 29}{\sqrt{14.35\left[\frac{1}{5}+\frac{1}{6}\right]}}$

$= \frac{-4.4}{\sqrt{14.35\left(\frac{11}{30}\right)}}$

$= \frac{-4.4}{\sqrt{14.35(0.366)}} = \frac{-4.4}{\sqrt{5.261}}$

$t = \frac{-4.4}{2.293} = -1.9188$

$|t| = 1.9188.$

vi) Table value of $t = 2.262$

vii) Conclusion.

Calt < Tabt

∴ We accept Ho (Null hypothesis)

19) A group of 5 patients treated with medicine A weigh 42, 39, 48, 60 and 41 kg. Second group of 7 patient from the same hospital treated with medicine of weigh 38, 42, 56, 64, 68, 69 and 62 kg. Do you agree with the claim that medicine B increased the weight significantly.

Soln:-

$n_1 = 5$, $n_2 = 7$

x_1	x_2	$(x_1-\bar{x}_1)^2$	$(x_2-\bar{x}_2)^2$
42	38	16	361
39	42	49	225
48	56	4	1
60	64	196	49
41	68	25	121
	69		144
	62		25
230	399	290	926

$x_1 = 46$, x_2

$\Sigma(x_1-\bar{x}_1)^2 = 290$

$\Sigma(x_2-\bar{x}_2)^2 = 926$

$$S^2 = \frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}$$

$$= \frac{290+926}{10}$$

$S^2 = 121.6$

i) Null hypothesis Ho $\mu_1 = \mu_2$

ii) Alternative hypothesis. H_1 $\mu_1 < \mu_2$

iii) Degree of freedom. $n_1+n_2-2 = 10$.

iv) Level of significance, $\alpha = 5\%$

v) Test Statistic.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}}$$

$$= \frac{46-57}{\sqrt{121.6\left[\frac{1}{5}+\frac{1}{7}\right]}}$$

$$= \frac{-11}{\sqrt{121.6\left[\frac{12}{35}\right]}}$$

$$= \frac{-11}{\sqrt{12.1 \cdot 6(0.342)}}$$

$$= \frac{-11}{\sqrt{41.58}} = \frac{-11}{6.448} = -1.7$$

$|t| = 1.7.$

vi) Table value of $t = 1.81$

vii) Conclusion

Cal t < Tab t

$\therefore$ We accept H_0 (Null hypothesis).

F Test

To test whether if there is any significant difference b/w two estimates of population variance or to test if the two samples have come from the same population, we use F - Test.

Null hypothesis: $H_0: \sigma_1^2 = \sigma_2^2$

Alternative Hypothesis: $H_1: \sigma_1^2 \neq \sigma_2^2$

Degree of Freedom: $(n_1 - 1, n_2 - 1)$

Test statistic:

$$F = \frac{S_1^2}{S_2^2} \text{ when } S_1^2 > S_2^2$$

(or)

$$F = \frac{S_2^2}{S_1^2} \text{ when } S_2^2 > S_1^2$$

where

$$S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1}$$

$$S_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}$$

Note :-

If the sample variance s_1^2 is given we can obtain population variance S_1^2 by using the relation

$$n_1 s_1^2 = (n_1 - 1) S_1^2$$

40) If one sample of 10 observation from a normal population, the sum of the squares of the deviation of the sample values from the sample mean is 102.4 and in another sample of 12 observations from another population,

the sum of squares of the deviations of the sample values from the sample mean is 120·5. Examine whether the two normal populations have the same variances.

Soln:-

Gn.

$n_1 = 10, \quad n_2 = 12$

$\Sigma(x_1-\bar{x}_1)^2 = 102.4 \; ; \; \Sigma(x_2-\bar{x}_2)^2 = 120.5.$

$$S_1^2 = \frac{\Sigma(x_1-\bar{x}_1)^2}{n_1-1}$$

$$= \frac{102.4}{9}$$

$$= 11.37.$$

$$S_2^2 = \frac{\Sigma(x_2-\bar{x}_2)^2}{n_2-1} = \frac{120.5}{11}$$

$$= 10.95.$$

i) Null hypothesis : $H_0: \sigma_1^2 = \sigma_2^2$

ii) Alternative hypothesis : $H_1: \sigma_1^2 \neq \sigma_2^2$

iii) Degree of freedom, (n_1-1, n_2-1)
$= (9, 11)$

iv) Level of significance, $\alpha = 5\%$

v) Test statistic

$$F = \frac{S_1^2}{S_2^2}$$

$$= \frac{11.37}{10.95}$$

$$= 1.038.$$

Calculated value of F = 1·038.

vi) Table value = 2·90.

∴ Tabulated value of F = 2·90.

vii) Conclusion

Cal F < Tab F

∴ We accept the Null hypothesis(H_0). i.e the two normal populations have the same variance.

21) Two random samples gave the following results

Sample	Size	Sample Mean	Sum of the squares of the deviation from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same population.

Soln:-

Gn.

$n_1 = 10$, $n_2 = 12$

$\bar{x}_1 = 15$; $\bar{x}_2 = 14$

$\Sigma(x_1 - \bar{x}_1)^2 = 90$; $\Sigma(x_2 - \bar{x}_2)^2 = 108$.

i) Null hypothesis : $H_0 : \mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$

~~ii) Alternative hypoth~~

Here we have to use two test

i) T test

ii) F test

(i) T test (to test the equality of Means).

(i) Null hypothesis : $H_0 : \mu_1 = \mu_2$

(ii) Alternative hypothesis : $H_1 : \mu_1 \neq \mu_2$

iii) Degrees of freedom, $n_1 + n_2 - 2$
$= 20$.

iv) Level of Significance, $\alpha = 5\%$

v) Test Statistic

~~$t = \Sigma(x_1 - x_2)$~~

$$S^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}.$$

$$= \frac{90 + 108}{20} = 9.9.$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$= \frac{15 - 14}{\sqrt{9.9\left[\frac{1}{10} + \frac{1}{12}\right]}} = \frac{1}{\sqrt{9.9\left[22/12\right]}}$$

$$= \frac{1}{\sqrt{9.9\,[0.183]}} = \frac{1}{\sqrt{1.8117}}$$

$$= \frac{1}{1.345}$$

$t = 0.743$.

vi) Table value $t = 2.086$.

vii) Conclusion

Cal t < Tab t

$\therefore$ We accept the Null hypothesis H_0.

ii) F test. (to test the equality of variance).

$$s_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{90}{9} = 10.$$

$$s_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{108}{11} = 9.82$$

i) Null hypothesis: $H_0: \sigma_1^2 = \sigma_2^2$

ii) Alternative hypothesis: $H_1: \sigma_1^2 \neq \sigma_2^2$

iii) Degree of freedom $(n_1 - 1, n_2 - 1)$

$(9, 11)$.

iv) Level of significance, $\alpha = 5\%$.

v) Test statistic

$$F = \frac{s_1^2}{s_2^2}$$

$$= \frac{10}{9.81}$$

$$F = 1.018$$

vi) Table value of $F = 2.90$

vii) Conclusion

Cal F < Tab F

$\therefore$ we accept the Null hypothesis H_0.

From the conclusion of F test we can conclude that the given samples have been drawn from the same normal population.

Q.2) It is known that the mean diameters of rivets produced by two firms A and B are practically the same but the standard deviations may differ for 22 rivets produced by firm A, the standard deviation is 2.9 mm while for 16 rivets manufactured by firm B, the S.D is 3.8 mm. Compute the statistic you would use to test whether the products of firm A have the same variability as those of firm B and test its significance.

Soln:-

Gn:-

$n_1 = 22$; $s_1 = 2.9$.

$n_2 = 16$; $s_2 = 3.8$.

W.K.T

$$n_1 s_1^2 = (n_1 - 1) S_1^2$$

i.e

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$$

$S_1^2 = \frac{22 \times (2.9)^2}{21} = \frac{\cancel{63.18}}{21}$

$S_1^2 = \cancel{3.038}\ 8.8$

Similarly.

$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$

$= \frac{16 \times (3.8)^2}{15} = \frac{16 \times 14.44}{15}$

$S_2^2 = 15.40. \quad \therefore S_2^2 > S_1^2$

i) Null hypothesis $H_0: \sigma_1^2 = \sigma_2^2$

ii) Alternative hypothesis, $H_1: \sigma_1^2 \neq \sigma_2^2$

iii) Degrees of freedom $= (n_2 - 1, n_1 - 1)$

$= (15, 21) \; [\because S_2^2 > S_1^2$

iv) Level of significance, $\alpha = 5\%$.

v) Test statistic.

$$F = \frac{S_2^2}{S_1^2}$$

$$= \frac{15.40}{8.8}$$

$$F = 1.75.$$

Calculated $F = 1.75$

vi) Table value of $F = 2.18$.

$\therefore$ Cal $F <$ Tab F

vii) Conclusion.

$\therefore$ We accept the Null hypothesis H_0.

i.e, the products of both the firms A and B have the same variability.

23) The time taken by the workers in performing a job by method I and method II is given below.

Method I	20	16	26	27	23	22	
Method II	27	33	42	35	32	34	38.

To the data show that the variances of time distribution from population from which these samples are drawn do not differ significantly.

Soln:-

$n_1 = 6$; $n_2 = 7$

$\bar{x}_1 = \frac{\Sigma x_1}{n} = 22.3$; $\bar{x}_2 = \frac{\Sigma x_2}{n_2} = 34.4$.

$S_1^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1}$

$S_2^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1}$

x_1	x_2	$(x_1-\bar{x}_1)^2$ $x_1-22.3$	$(x_2-\bar{x}_2)^2$ $x_2-34.4$
20	27	5.29	54.76
16	33	39.69	1.96
26	42	13.69	57.76
27	35	22.09	0.36
23	32	0.49	5.76
22	34	0.09	0.16
	38		12.96
134		81.34	133.72

$\bar{x}_1 = 22.3$, $\bar{x}_2 = 34.4$.

$S_1^2 = \frac{81.34}{5} = 16.268$

$S_2^2 = \frac{133.72}{6} = 22.286$.

i) Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

ii) Alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$

iii) Degree of freedom (n_2-1, n_1-1)

$= (6, 5)$.

iv) Level of significance, $\alpha = 5\%$.

v) Test statistic

$$F = \frac{S_2^2}{S_1^2}$$

$$= \frac{22.286}{16.268}$$

$F = 1.369$.

vi) Table value of $F = 4.95$.

$\therefore$ Cal F < Tab F.

vii) Conclusion.

$\therefore$ We accept Null hypothesis H_0.
i.e there is no significance difference b/w method I & method II.

24) Pumpkins were grown under two experimental conditions. Two random samples of 11 and 9 pumkins show the sample standards deviation of their weights as 0.8 and 0.5 respectively. Assuming that the weight distributions are normal test hypothesis that the true variances are equal.

Soln :-

Gn: $n_1 = 11$. , $S_1 = 0.8$.
$n_2 = 9$, $S_2 = 0.5$

W.K.T

$n_1 s_1^2 = (n_1 - 1) S_1^2$

i.e

$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$

$S_1^2 = \frac{11 \times (0.8)^2}{10} = \frac{11 \times 0.64}{10}$

$S_1^2 = 0.704$

III^ly

$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$

$= \frac{9 \times (0.5)^2}{8} = \frac{2.25}{8}$

$S_2^2 = 0.281$

$\therefore S_1^2 > S_2^2$

i) Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

ii) Alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$

iii) Degrees of freedom $(n_1 - 1, n_2 - 1)$

$= (10, 8)$

iv) Level of significance, $\alpha = 5\%$.

v) Test statistic.

$F = \frac{S_1^2}{S_2^2}$

$F = \frac{0.704}{0.281}$

$F = 2.505$

vi) Table value of $F = 3.35$.

viii) Conclusion:

Cal F < Tab F

$\therefore$ We accept Null hypothesis H_0.

i.e the true variances are equal.

25) For a random sample of 10 pigs fed on diet A the increases in weight in a certain period were 10, 6, 16, 17, 13, 12, 8, 14, 15, 9. For another random sample of 12 pigs, fed on diet B, the increases in the same period were 7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17. Show that the estimates of the population variance from the samples are not significantly different.

Soln:-

Gn.

$n_1 = 10 \quad , \quad n_2 = 12$

$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{120}{10} = 12$

$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{180}{12} = 15.$

$S_1^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{120}{9} = 13.33$

$S_2^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{314}{11} = 28.54.$

x_1	x_2	$(x_1 - \bar{x}_1)^2$ $x_1 - 12$	$(x_2 - \bar{x}_2)^2$ $x_2 - 15$
10	7	4	64
6	13	36	4
16	22	16	49
17	15	25	0
13	12	1	9
12	14	0	
8	18	16	9
14	8	4	49
15	21	9	36
9	23	9	64
	10		25
	17		4
120	180	120	~~305~~ 314

i) Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

ii) Alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$

iii) Degree of freedom $(n_2 - 1, n_1 - 1)$.
$(11, 9)$

iv) Level of significance, $\alpha = 5\%$.

v) Test statistic.

$$F = \frac{S_2^2}{S_1^2}$$

$$F = \frac{28.54}{13.33} = 2.14$$

vii) Conclusion

Cal F < Tab F

$\therefore$ We accept the null hypothesis H_0.

i.e there is no significant difference b/w the two samples.

26) The nicotine contents in milligrams in two samples of tobacco were found to be as follows.

Sample A	24	27	26	21	25	
Sample B	27	30	28	31	22	36.

Can it be said that the two samples come from same normal population.

Soln:-

Gn:-

$n_1 = 5, \quad n_2 = 6$

x_1	x_2	$(x_1 - \bar{x}_1)^2$ $x_1 - 24.6$	$(x_2 - \bar{x}_2)^2$ $x_2 = 29$
24	27	0.36	4
27	30	5.76	1
26	28	1.96	1
21	31	12.96	4
25	22	0.16.	49
	36		49
123	174	~~Q.12~~ 21.2	108.

$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{123}{5} = 24.6$

$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{174}{6} = 29$

$S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{21.2}{4} = 5.3$

$S_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{108}{5} = 21.6$

i) Null hypothesis : $H_0 : \mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$

Here we have to use two test.

i) T test

ii) F test.

(i) T test (to test the equality of Mean)

i) Null hypothesis $H_0 : \mu_1 = \mu_2$.

ii) Alternative hypothesis H_1 $\mu_1 \neq \mu_2$.

iii) Degree of freedom $n_1 + n_2 - 2 = 9$

iv) Level of significance, $\alpha = 5\%$.

v) Test statistic.

$$S^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

$$= \frac{21.2 + 108}{9} = \cancel{12.235} \; 14.355.$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$= \frac{24.6 - 29}{\sqrt{14.35\left[\frac{1}{5} + \frac{1}{6}\right]}} = \frac{-4.4}{\sqrt{14.35\left[11/30\right]}}$$

$$= \frac{-4.4}{\sqrt{14.35 \times 0.366}} = \frac{-4.4}{2.293}$$

$t = -1.918$.

$|t| = 1.918$.

Calculated $t = 1.918$.

vi) Table value of $t = \cancel{1.833}$ 2.262.

vii) Conclusion.

Cal t < Tab t

∴ We accept the Null hypothesis H_0.

ii) F test (to test the equality of Variance.

i) Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

ii) Alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$.

iii) Degree of freedom, $(n_2 - 1, n_1 - 1) = (5, 4)$.

iv) Level of significance, $\alpha = 5\%$.

$$F = \frac{s_2^2}{s_1^2}$$

$$= \frac{21.6}{5.3} = 4.075.$$

vi) Table value of $F = 6.26$.

vii) Conclusion.

Cal t < Tab t.

∴ We accept the Null hypothesis Ho.

∴ The two samples come from the same normal population.

27) Two horses A and B were tested according to the time (in seconds) to run a particular track with the following results.

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	29	

Test whether the two horses have the same running capacity.

Soln:-

Gn:-

$n_1 = 7, \quad n_2 = 6$

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{219}{7} = 31.285$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{169}{6} = 28.166$$

28	29	10.7584	0.7056
30	30	1.63	3.385
32	30	0.518	3.385
33	24	2.95	17.30
33	27	2.95	1.345
29	29	5.19	0.705
34		7.39	
219	169	31.37	26.8

$$s_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{31.37}{6} = 5.22.$$

$$s_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{26.8}{5} = 5.36.$$

i) Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

ii) Alternative hypothesis $H_1 = \sigma_1^2 \neq \sigma_2^2$

iii) Degree of freedom $= \cancel{n_1 + n_2 - 2 = 11}$ $[(n_2 - 1), (n_1 - 1)] = (5, 6)$

iv) Level of Significance, $\alpha = 5\%$.

v) Test statistic

$$F = \frac{s_2^2}{s_1^2}$$

$$= \frac{5.36}{5.22}$$

$$= 1.026.$$

vii) Conclusion

Cal F < Tab F

∴ We accept the Null hypothesis. Ho.

i.e the two horses have the same running capacity.

2mrk

1) Type I and Type II error.

Type I error: Reject Ho when it is true.

Type II error: Accept Ho when it is wrong.

i.e accept Ho when H_1 is true.

For eg:-

P(Reject Ho when it is true) = P(Type I error) = α.

& P (Accept Ho when it is wrong) = P (Type II error) = β.

where

α & β are called sizes of Type I and Type II errors. It is also called as producer's risk and Consumer's risk.

2) Level of Significance.

The probability α that a random value of the statistic 't' belongs to the critical region is known as the level of significance. Significance is the size of the Type I error. The level of significance usually employed in testing of hypothesis are 5% and 1%.

3) Student t test.

The student 't' is defined by the statistic,

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where

$\bar{x} = \frac{1}{n} \Sigma x_1$ = sample mean.

μ = population mean.

s^2 n = Sample size.