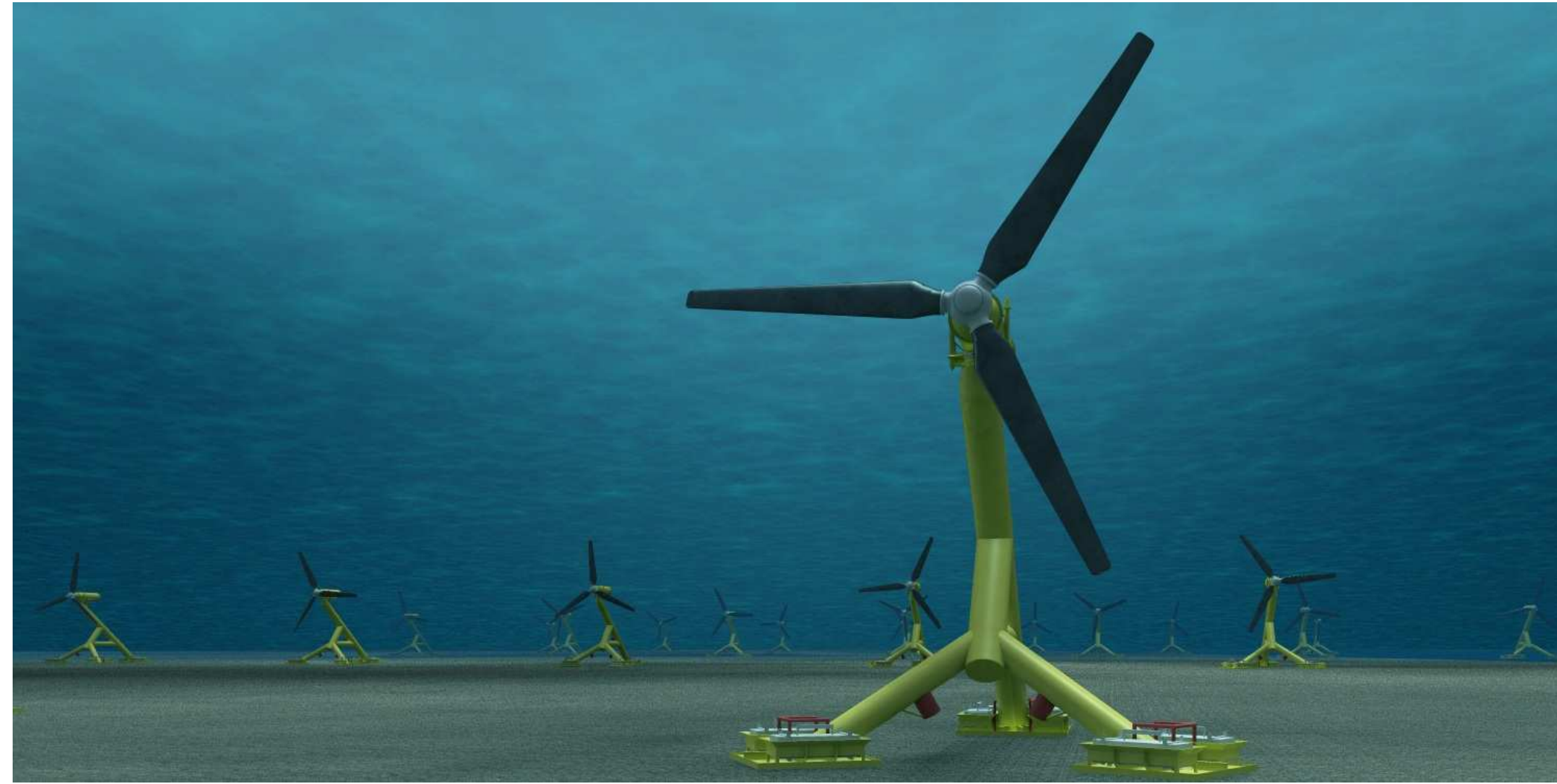


## What is the optimal location of turbines in a tidal stream to maximise the energy output?



Tidal turbines are regarded as a promising technology for renewable electricity generation. One of the main challenges facing the marine industry is to find the optimal location of the tidal turbines within an array to maximise the power output. Answering this question is of huge economic importance as the location of the turbines within the array can substantially change the energy captured.

## Key novelty

The optimal layout problem is formulated here as an optimisation problem and solved numerically. In contrast to previous research, a gradient-based optimisation algorithm is used. The derivative information is obtained by solving the adjoint problem whose computational expense is independent of the number of turbines. This key feature allows for optimising potentially hundreds of turbines in realistic domains.

## Model setup

The flow for a given turbine layout is computed with the non-linear shallow water equations. The tidal turbines are parameterised as increased sea bed friction. The power output is determined from the extracted energy due to the turbine friction. The resulting equations are solved with the finite element method in the FEniCS system (Logg et al. (2011)).

## Optimisation setup and adjoint model

The optimisation problem is formulated and solved in a newly developed optimisation framework (Funke et al. (2012)) using sequential quadratic programming. The required adjoint model is automatically derived as described in Farrell et al. (2013).

## Summary

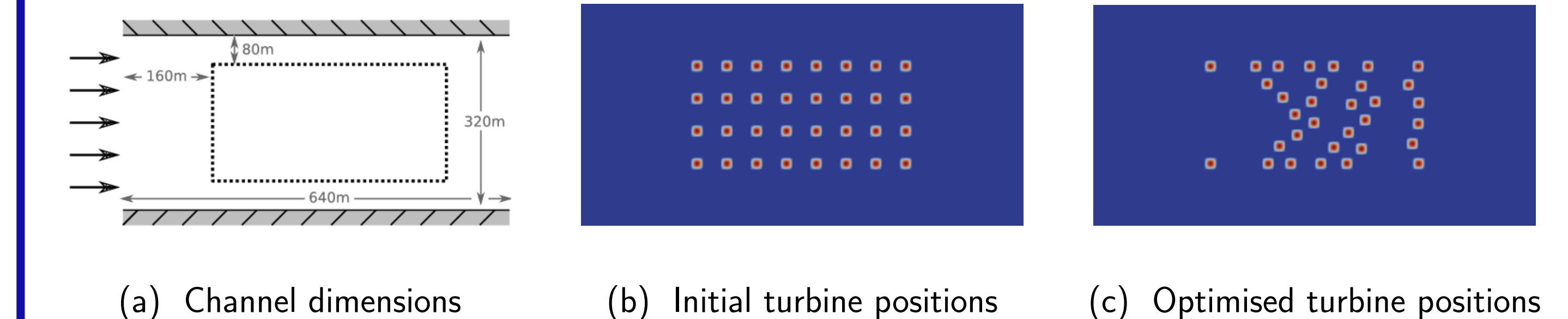
- The optimisation of tidal farm layouts has significant potential for improving the power extraction.
- 22% — 40% power increase compared to reference design.
- Additional results are published in Funke (2012).
- Future work includes application to realistic domains and improved turbine parametrisation.

P. E. Farrell, D. A. Ham, S. W. Funke, and M. E. Rognes. Automated derivation of the adjoint of high-level transient finite element programs. *SIAM Journal on Scientific Computing*, 2013. accepted.  
S. W. Funke. *The automation of PDE-constrained optimisation and its applications*. PhD thesis, Imperial College London, London, UK, 2012.  
S. W. Funke, P. E. Farrell, and D. A. Ham. A framework for automated PDE-constrained optimisation. *ACM Transactions on Mathematical Software*, 2012. in preparation.  
A. Logg, K.-A. Mardal, G. N. Wells, et al. *Automated Solution of Differential Equations By the Finite Element Method*. Springer-Verlag, Berlin, Heidelberg, New-York, 2011. doi: 10.1007/978-3-642-23099-8.

## Optimisation of 32 tidal turbines

### Channel scenario

A 50 m deep channel with a constant 2 m/s inflow from the left.

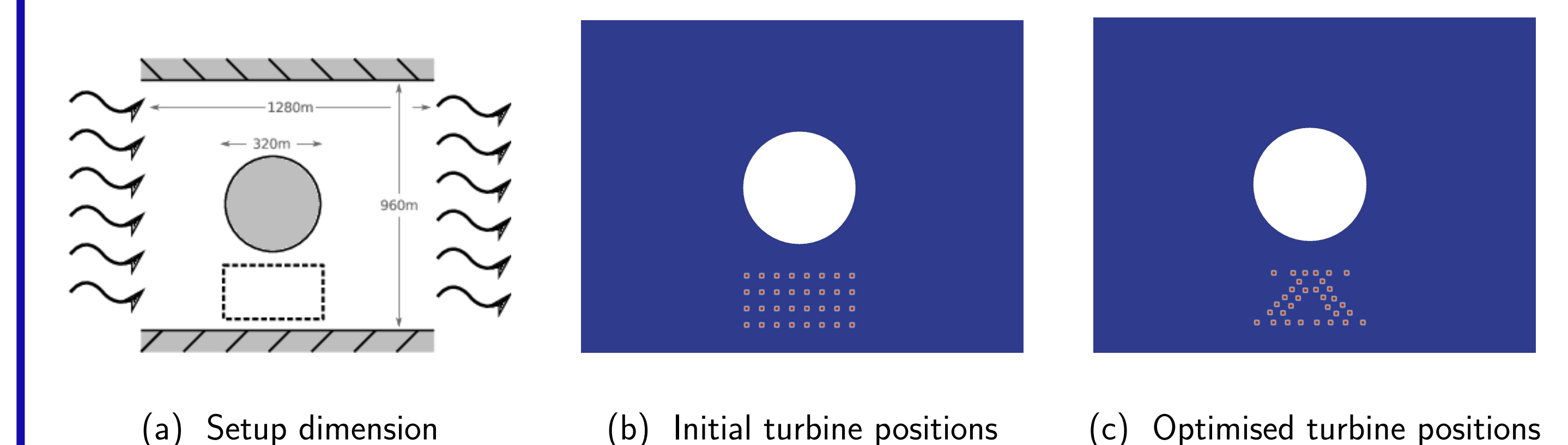


### Power production

- Initial layout: 54.5 MW
- Optimised layout: 76.1 MW

### Island scenario

A wide channel with a sinusoidal inflow. The turbines are to be deployed below the island.



### Power production

- Initial layout: 48.4 MW
- Optimised layout: 59.0 MW