

A wetting and drying algorithm for non-hydrostatic models with combined pressure/free-surface

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1. Introduction

The recent tsunami event in Japan has underscored the importance of understanding free-surface flows and wetting and drying processes. In this work, a wetting and drying algorithm for free-surface problems is proposed, which, in contrast to most other methods, does not assume a hydrostatic pressure. Non-hydrostatic flow features are apparent in coastal regions (thus also in wetting and drying regions), but also play a significant role on a larger scale, see Figure 1.

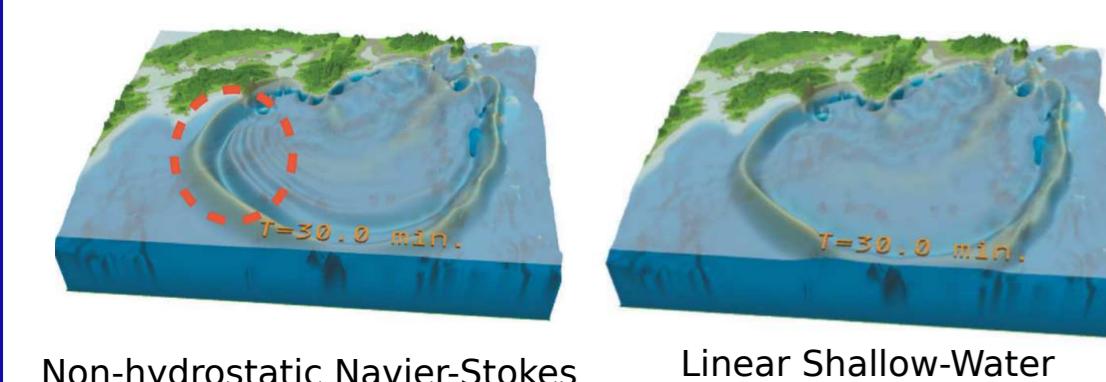


Figure 1: Furumura and Saito (2007) showed the importance of solving the Navier-Stokes equations for tsunami prediction.

2. Underlying equations

- Incompressible Navier Stokes equations with Boussinesq approximation:

$$\rho_0 \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) - \nabla \cdot \mu \nabla \vec{u} + \nabla \tilde{p} = -\rho' g \vec{n}_z,$$

$$\nabla \cdot \vec{u} = 0,$$

where \vec{u} is the velocity, $\tilde{p} = p + \rho_0 g z$ the “piezometric” pressure, ρ_0 and ρ' the background and perturbation density, μ the viscosity and g and \vec{n}_z the gravity magnitude and direction, respectively.

- Boundary conditions in an ocean-like domain:

- Bottom/sides: *No-normal flow boundary condition*: $0 = \vec{n} \cdot \vec{u}$.
- Free-surface: For simplicity, the atmospheric pressure is assumed to be zero. Then the relationship between pressure on the surface and free-surface elevation η is given by:

$$\eta = \max \left(\frac{\tilde{p}}{\rho_0 g}, b + d_0 \right). \quad (1)$$

Here b is the bathymetry function and d_0 enforces a positive water level in the domain. From that, the *combined kinematic free-surface boundary condition with wetting and drying* is derived (Funke et al. (2011)):

$$\frac{\vec{n} \cdot \vec{n}_z}{\rho_0 g} \frac{\partial}{\partial t} \max(\tilde{p}, \rho_0 g(b + d_0)) = \vec{n} \cdot \vec{u}. \quad (2)$$

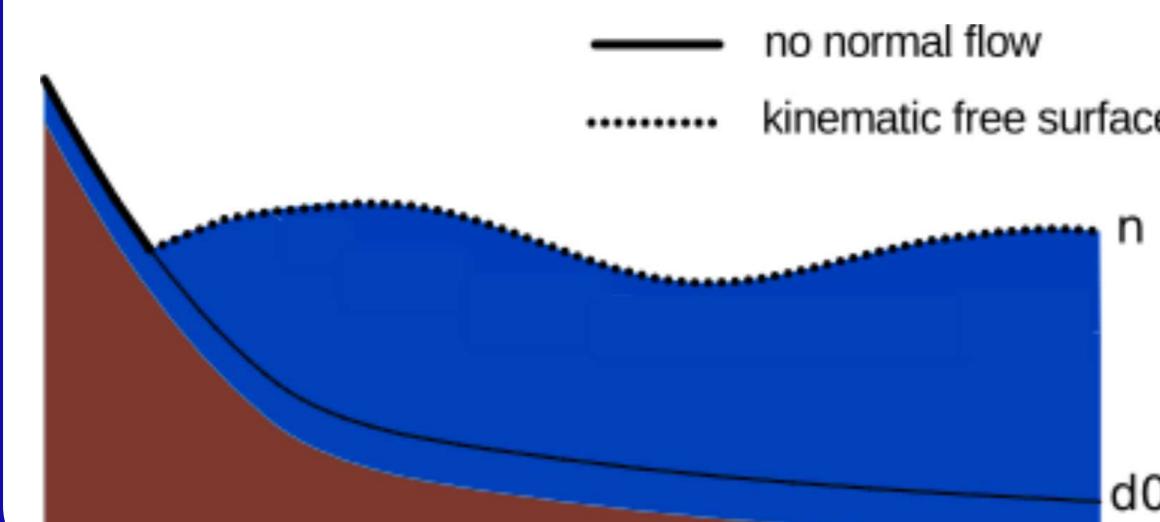


Figure 2: The free-surface boundary condition (2) reduces to the no-normal flow b.c. in dry regions and to the kinematic free-surface b.c. elsewhere.

3. Implementation

- Implemented in the finite-element code Fluidity-ICOM, see Ford et al. (2004).
- The implicit θ -method is used for the time discretisation.
- The resulting non-linear system is solved using an extended pressure-correction scheme (Kramer et al. (2010, in preparation)).
- At the end of each iteration the free-surface elevation is obtained from equation (1) and the mesh is deformed accordingly.
- The mesh deformation is performed only vertically, hence no remeshing is necessary:

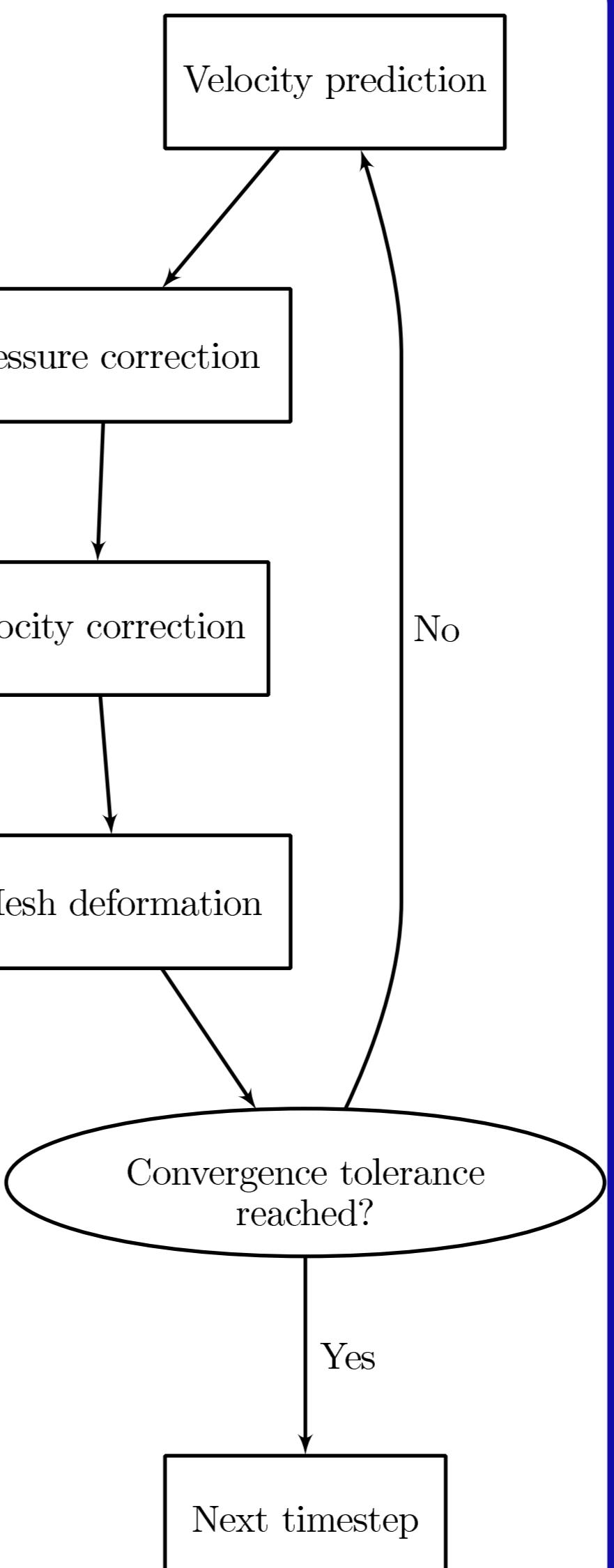
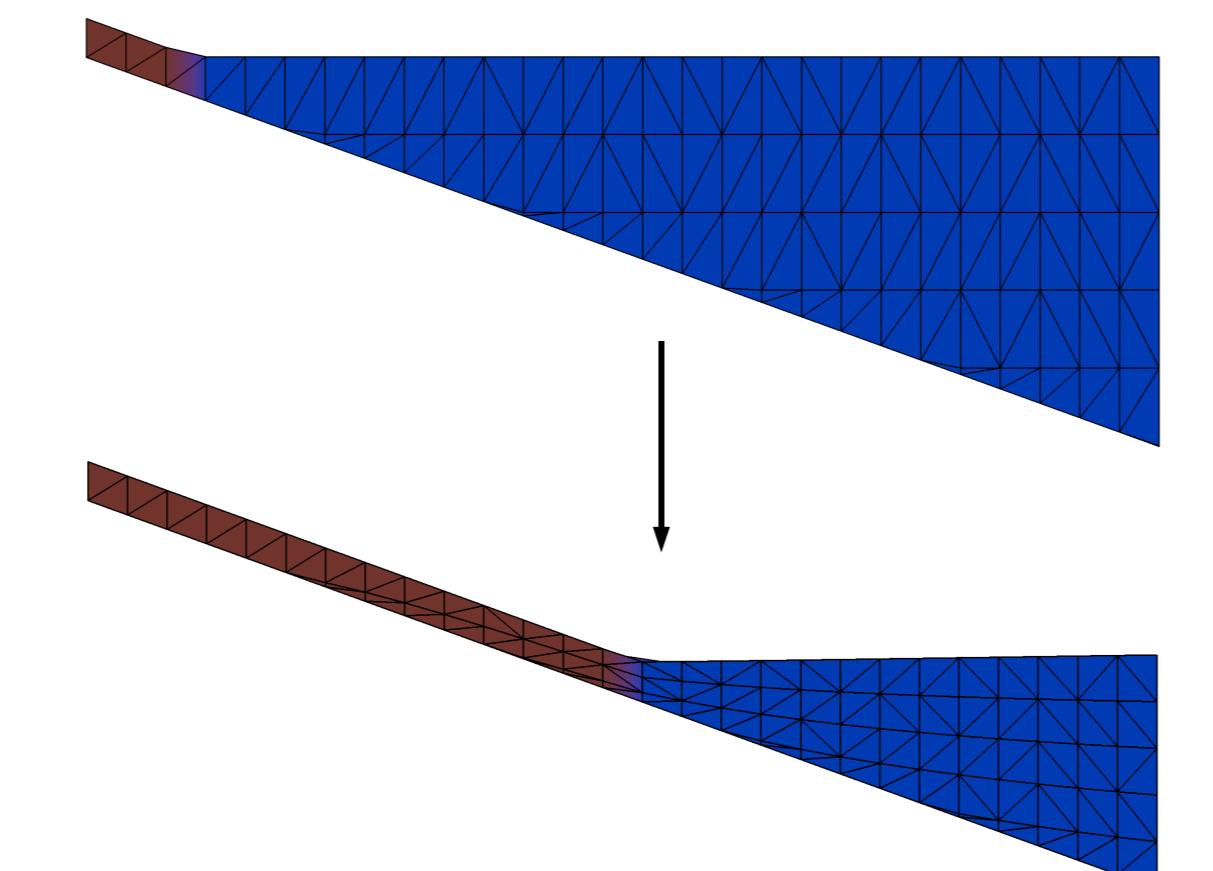


Figure 3: The extended pressure-correction scheme.

4. Conservation properties and validation

- Volume/mass conservation was proved (Funke et al. (2011)) for:
 - the continuous equations from Section 2.
 - the extended pressure-correction scheme from Section 3.
- Three-dimensional, non-viscous testcase in a closed basin, with analytical solution by Thacker (1981).
 - Highly accurate, see Figure 4. No visible numerical damping.
 - Volume is conserved up to machine precision ($< 10^{-14}$).

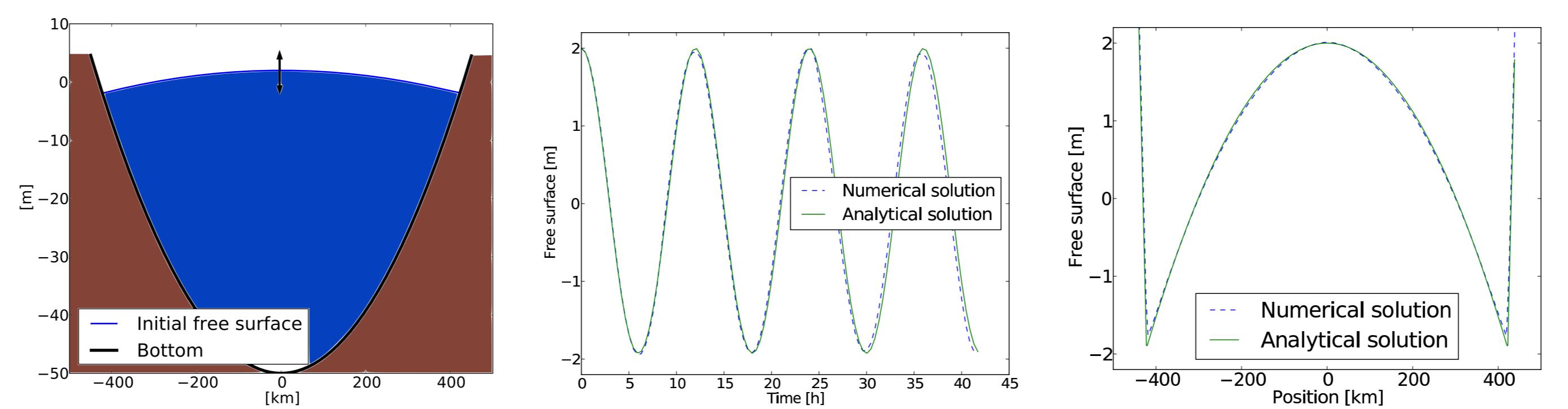
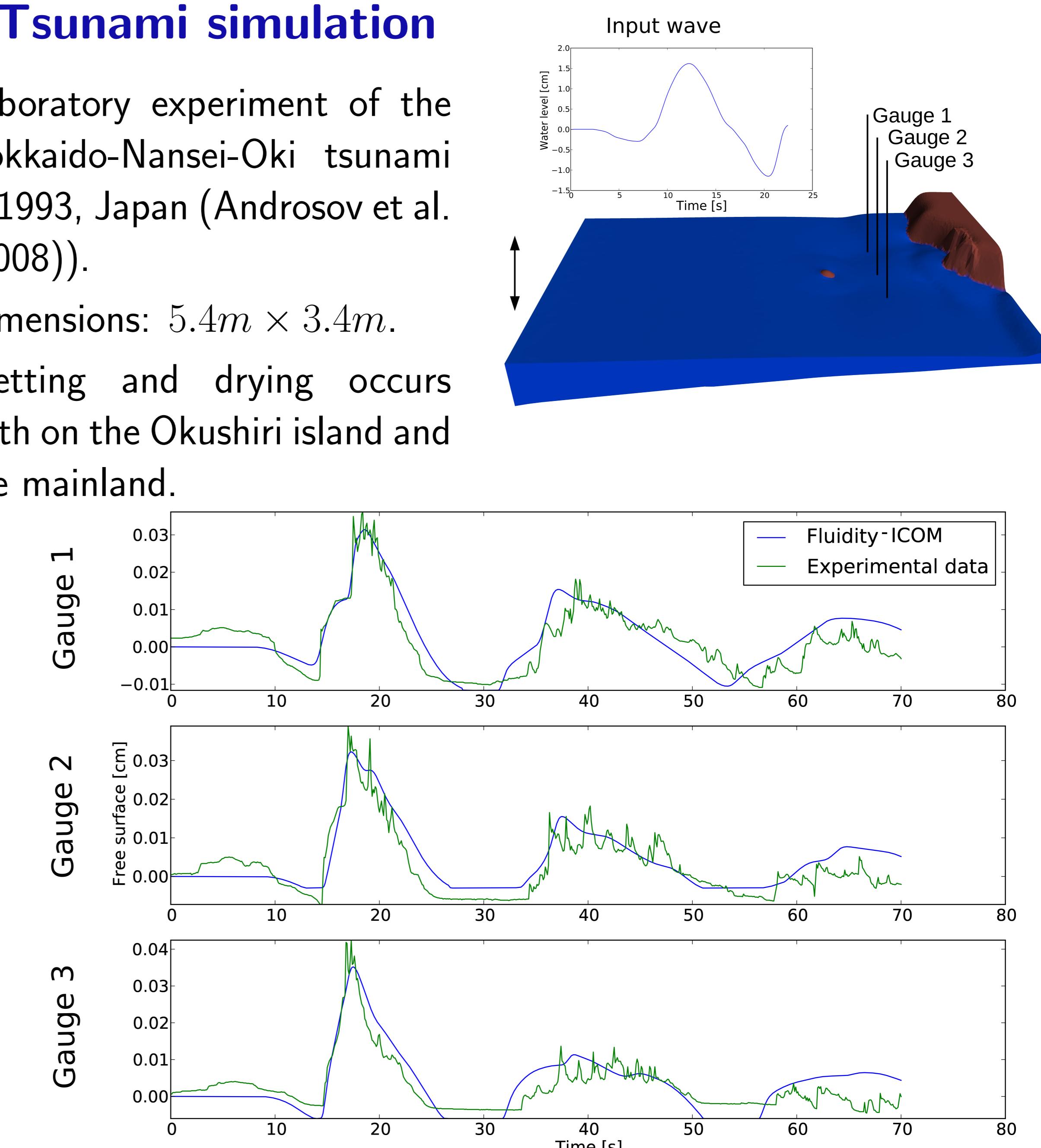


Figure 4: From left to right: Initial setup, solution at center of the domain and after 24h.

5. Tsunami simulation

- Laboratory experiment of the Hokkaido-Nansei-Oki tsunami in 1993, Japan (Androsov et al. (2008)).
- Dimensions: $5.4m \times 3.4m$.
- Wetting and drying occurs both on the Okushiri island and the mainland.



6. Summary

The proposed wetting and drying method

- is stable and accurate.
- conserves volume/mass.
- uses a computationally cheap mesh deformation instead of remeshing.
- does not introduce any restrictions to the discretisation, in particular unstructured meshes and/or implicit timestepping methods can be used (and therefore the timestep size is not restricted by the CFL condition).

References

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