Optimising tidal turbine farms using the adjoint approach

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in collaboration with
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Imperial College London

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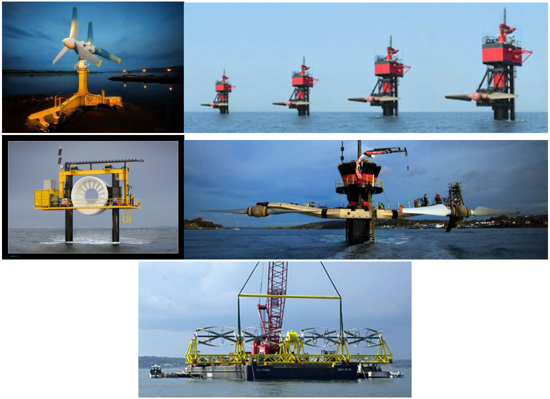

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- Generate between $0.5 - 1.5$ MW.
- Will typically be deployed in farms of $10 - 500$ turbines.
- Very new technology (compared to wind turbines).

Current tidal projects

Inner Sound, Pentland Firth, UK	400×1 MW	starting 2014
Cantick Head, Pentland Firth, UK	200×1 MW	starting 2016
Kaipara Harbour, New Zealand	200 MW	planning stage
Gulf of Kutch, India	50×1 MW	approved 2012

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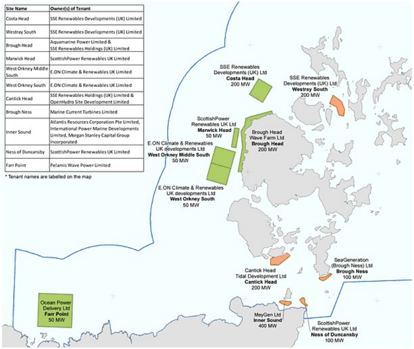

Site Name	Owner(s) of Tenant
Costa Head	SSE Renewables Developments (UK) Limited
Westray South	SSE Renewables Developments (UK) Limited
Brough Head	Aquamarine Power Limited & SSE Renewables Holdings (UK) Limited
Marwick Head	ScottishPower Renewables UK Limited
West Orkney Middle South	E.ON Climate & Renewables UK Limited
West Orkney South	E.ON Climate & Renewables UK Limited
Cantick Head	SSE Renewables Holdings (UK) Limited & OpenHydro Site Development Limited
Brough Ness	Marine Current Turbines Limited
Inner Sound	Atlantis Resources Corporation Pte Limited, International Power Marine Developments Limited, Morgan Stanley Capital Group Incorporated
Ness of Duncansby	ScottishPower Renewables UK Limited
Farr Point	Pelamis Wave Power Limited

* Tenant names are labelled on the map

Figure: Leased tidal projects sites (orange) in Scotland, UK

Tidal farm optimisation

Key question

How should the farm be designed to **maximise the profit**?

Design factors:

- Position and number of turbines.
- Turbine "tuning", e.g. direction, pitch settings.
- Installation and maintenance costs
  (e.g. turbines, cable, electricity transformers).
- Electricity price.

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Literature overview

Tidal farm optimisation

- Analytical approach with 1-D models (*Bryden and Couch (2007), Garrett and Cummins (2008), Vennell (2010, 2011)*).
- Manual comparison of selected farm designs with (more) realistic PDE-based models (*Lee et al. (2010), Divett et al. (2013)*).

Wind farm optimisation

- Related, but has the additional uncertainty of the wind direction.
- Typically apply evolutionary algorithms on simplistic wake models (Bilbao and Alba (2009), *Wan et al. (2010)*).

Goal: Develop a software tool for optimising tidal farms based on a realistic **PDE model** which **scales** up to hundreds of turbines.

Problem formulation of the farm optimisation problem

$$\max_{u,\eta,m} \frac{1}{T}\int_0^T\int_\Omega \rho c_t(m)\|u\|^3 \equiv \text{Power}(u,m)$$

subject to the SWEs:

$$\frac{\partial u}{\partial t} + u\cdot\nabla u - \nu\nabla^2 u + g\nabla\eta + \frac{c_b + c_t(m)}{H}\|u\|u = 0,$$

$$\frac{\partial \eta}{\partial t} + \nabla\cdot(Hu) = 0,$$

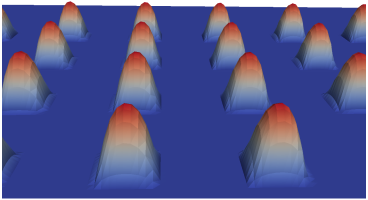
$$g(m) \leq 0,$$

where:

$(0,T)\times\Omega$	Computational domain,
$m : \mathbb{R}^{2N}$	x, y positions for N turbines,
$u : (0,T)\times\Omega \to \mathbb{R}^2$	Velocity,
$\eta : (0,T)\times\Omega \to \mathbb{R}$	Free-surface displacement,
$g : \mathbb{R}^{2N} \to \mathbb{R}^M$	Constraints on m.

Turbine parametrisation

Turbines are modelled via an increased bottom friction over a small area representative of an individual turbine:



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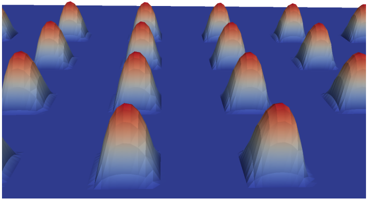
$$c_t(m) = \sum_{(p_x,p_y)\in m} K\psi_{p_x,r}(x)\psi_{p_y,r}(y),$$

where K is a “tuning” parameter of the turbine



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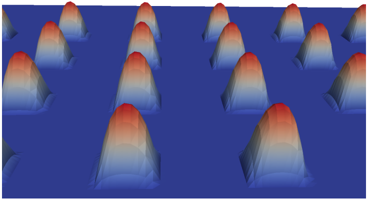
where K is a "tuning" parameter of the turbine and

$$\psi_{p,r}(x) \equiv \begin{cases} e^{1-1/(1-\|\frac{x-p}{r}\|^2)} & \text{for } \|\frac{x-p}{r}\| < 1, \\ 0 & \text{otherwise.} \end{cases}$$



Optimisation algorithms

Optimisation methods

Method	Gradient	Hessian	Typical # iterations
Evolutionary	No	No	$> \mathcal{O}(10^4) - \mathcal{O}(10^6)$
Newton	Yes	Yes	$\mathcal{O}(100)$
Quasi-Newton	Yes	No	$\mathcal{O}(100)$

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Gradient/Hessian computation

Method	# PDE solves
Finite differences	$\mathcal{O}(N)$
Tangent linear/complex step	$\mathcal{O}(N)$
Adjoint	$\mathcal{O}(1)$

Computing gradients with the adjoint approach

- Requires solution of one adjoint PDE (the adjoint shallow water equations), **independently of the number of turbines**.

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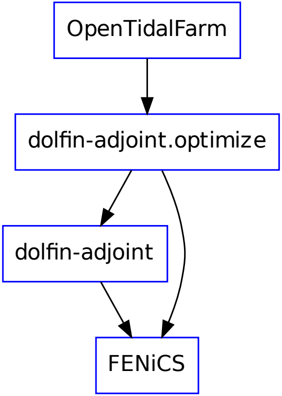
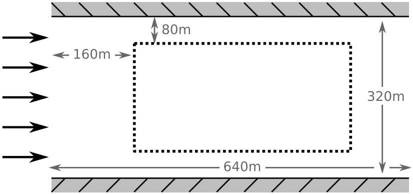
- Requires solution of one adjoint PDE (the adjoint shallow water equations), **independently of the number of turbines**.
- The adjoint PDE (i) depends on the original PDE solution, (ii) is **linear** and (iii) is solved **backwards in time**.
- Computing the gradient is **much faster** than evaluating the farm power:

# turbines	Runtime: $\mathrm{Power}(u, m)$	Runtime: $\mathrm{d}\mathrm{Power}/\mathrm{d}m$
8	58 s	7.3 s
64	56 s	7.3 s
256	70 s	7.8 s

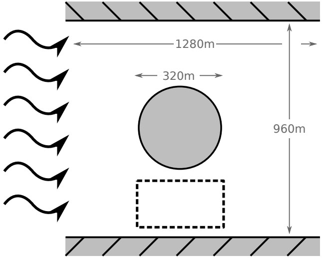
Implementation of *OpenTidalFarm*



Idealised examples



(a) Simple channel



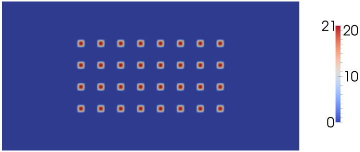
(b) Channel with island and tidal flow

Figure: Geometry of idealised examples.

Goal

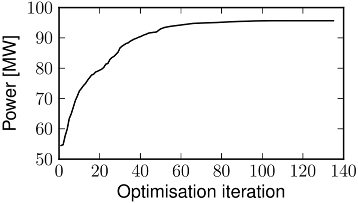
Optimally deploy 32 turbines in the area marked by dashed lines.

Simple channel

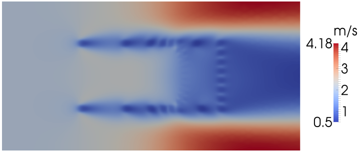


(a) Initial turbine positions

(b) Optimised turbine positions

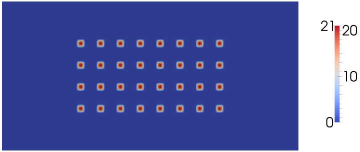


(c) Optimisation convergence



(d) Velocity magnitude

Simple channel



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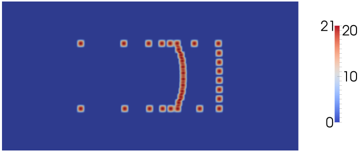

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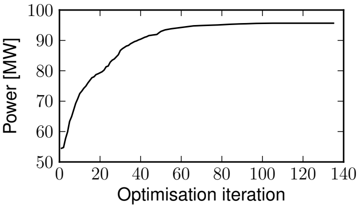

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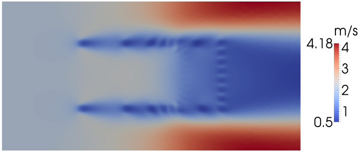

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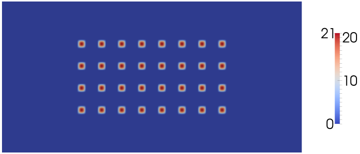
Figure: Results of the simple channel optimisation.

Simple channel with minimum distance constraint



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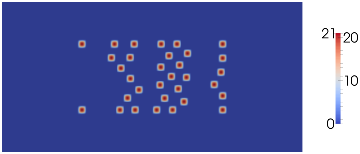
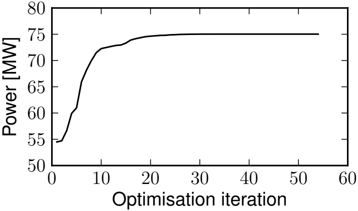


(b) Optimised turbine positions



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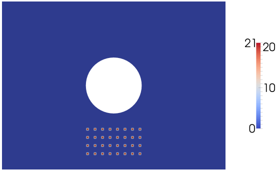
Channel with island and tidal flow



(a) Initial turbine positions

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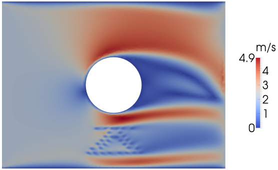
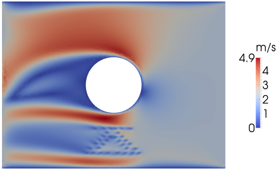


(c) Velocity magnitude reaching its maximum from the left



(d) Velocity magnitude when reaching its maximum from the right

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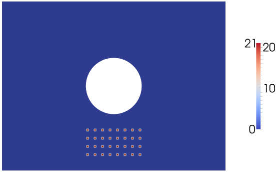
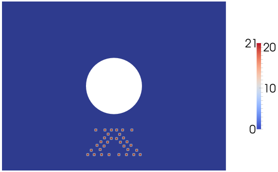
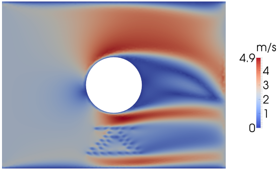


(e) Initial turbine positions



(f) Optimised turbine positions



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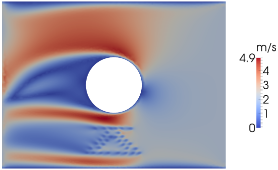

(h) Velocity magnitude when reaching its maximum from the right

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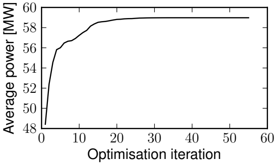
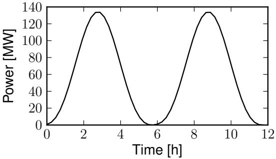


(i) Optimisation convergence

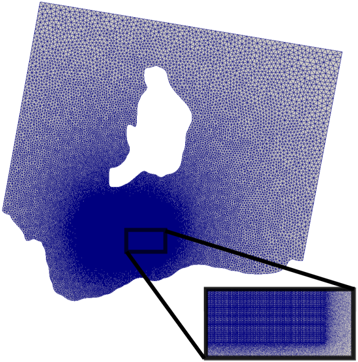


(j) Power extraction over time of the optimised configuration

Figure: Results of the channel with island and tidal flow optimisation.

Optimisation of 256 turbines in the Pentland Firth

(a) Satellite image of Stroma Island and Caithness (Bing Maps, Microsoft). Turbine site marked in pink.



(b) Mesh with $1.5 - 200$ m element size. 5.6×10^6 DOFs.

Figure: Geometry and mesh.

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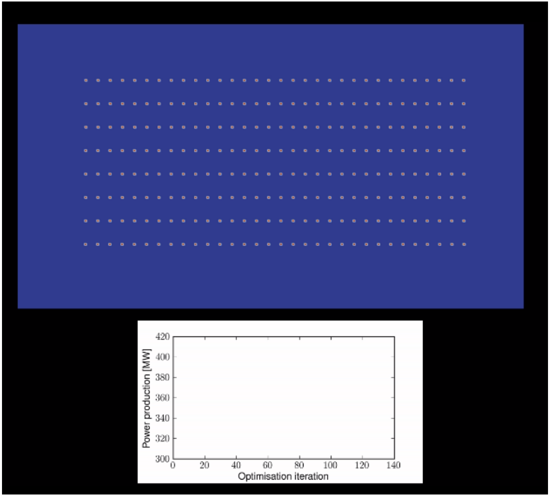

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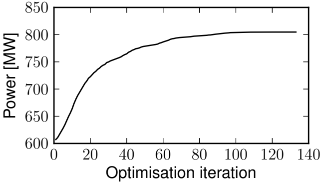

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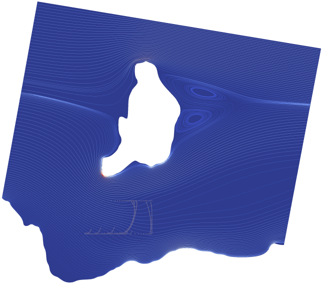
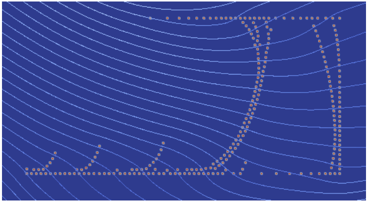


(b) Streamline flow



(c) Streamline flow (zoom)

(d) Turbine power map

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- maximum/minimum installation depth,
- maximum bathymetry gradient,
- complicated (non-convex) domain sites and/or
- optimisation of multiple farms.

Problem formulation for complex constraints

$$\max_{u,m} \mathrm{Power}(u, m)$$

subject to

$$\frac{\partial u}{\partial t} + u \cdot \nabla u - \nu \nabla^2 u + g \nabla \eta + \frac{c_b + c_t(m)}{H} \|u\| u = 0,$$

$$\frac{\partial \eta}{\partial t} + \nabla \cdot (H u) = 0,$$

$$g(m) \leq 0,$$

Problem: The inequality constraint function g becomes highly non-convex.

Problem formulation for complex constraints

$$\max_{u,\tilde{m}} \mathrm{Power}(u, \tilde{m})$$

subject to

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$$\frac{\partial \eta}{\partial t} + \nabla \cdot (Hu) = 0,$$

$$0 \leq \tilde{m},$$

Problem: The inequality constraint function g becomes highly non-convex.
Solution: Optimise the "concentration" of turbines $\tilde{m} : \Omega \to \mathbb{R}$ instead of parametrising turbines individually.

Multifarm optimisation in the Pentland Firth

Figure: Computational domain in the wider Pentland Firth with four tidal turbine sites. The valid deployment locations (based in the bathymetry gradient) of the sites are marked in pink.

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- Combined with the adjoint approach, the optimum is obtained in typically $100 - 200$ **flow solves**.
- The farm optimisation problem is non-convex (different initial guesses yield different optimal solutions), but **relatively well behaved** (good optimal solutions are found from any initial guess).

Future work

- Combine "smooth" and individual turbine optimisation.
- Validate wakes produced by the turbines.
- Incorporate financial (e.g. cable routing costs) and ecological model in the optimisation.

`OpenTidalFarm`

http://opentidalfarm.org