FEniCS Course

Lecture 1: Introduction to FEniCS

Contributors
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FENICS PROJECT

What is FEniCS?

FEniCS is an automated programming environment for differential equations

- C++/Python library
- Initiated 2003 in Chicago
- 1000–2000 monthly downloads
- Part of Debian and Ubuntu
- Licensed under the GNU LGPL

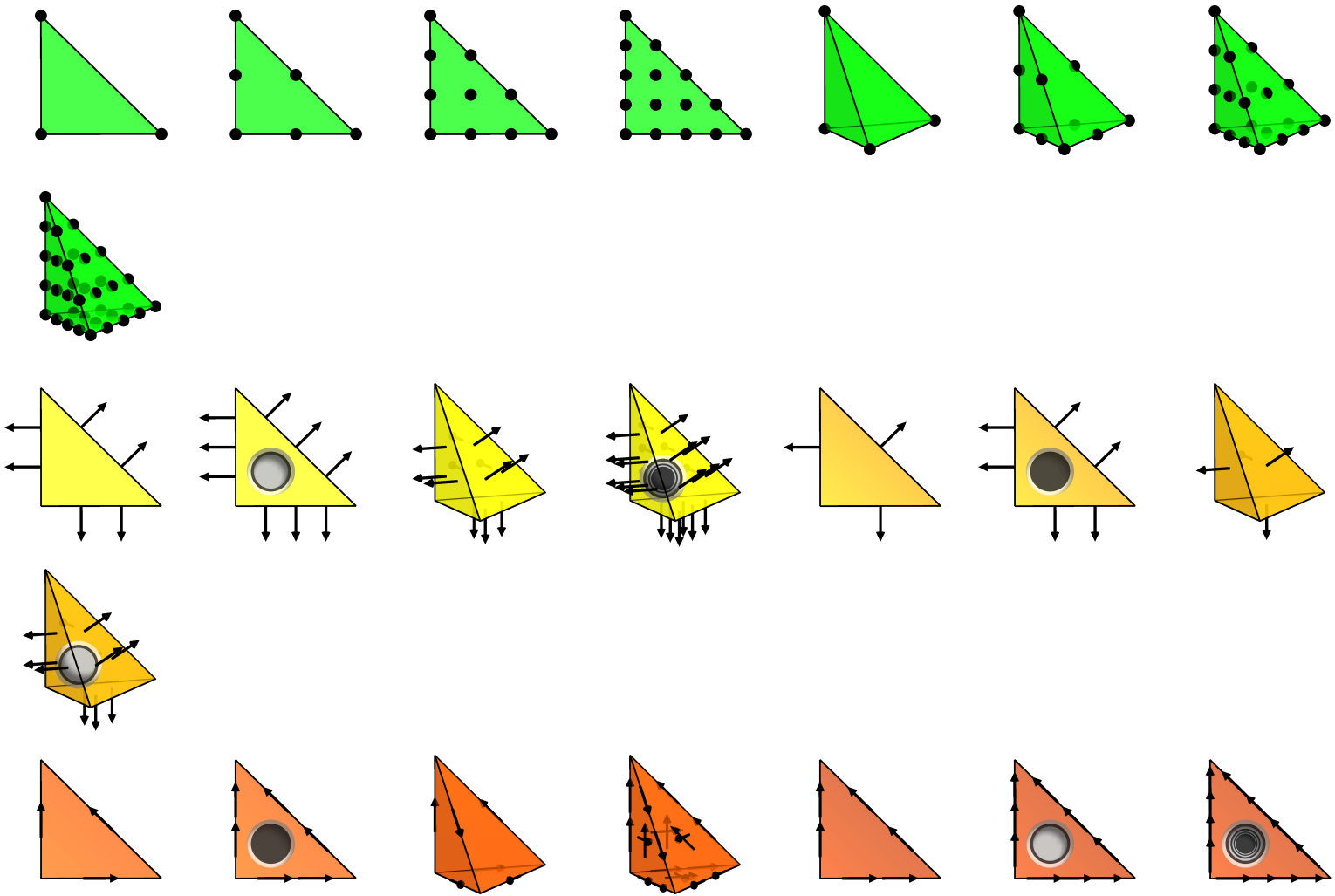
`http://fenicsproject.org/`

Collaborators

Simula Research Laboratory, University of Cambridge, University of Chicago, Texas Tech University, KTH Royal Institute of Technology, Chalmers University of Technology, Imperial College London, University of Oxford, Charles University in Prague, ...

FEniCS is automated FEM

- Automated generation of basis functions
- Automated evaluation of variational forms
- Automated finite element assembly
- Automated adaptive error control



FEniCS code can be readable, scale with mathematical complexity, and provide high-performance

Stokes with nonlinear viscosity

Given temperature T, find velocity u and pressure p such that

$$-\operatorname{div}(2\nu(u,T)\varepsilon(u) + pI) = \operatorname{Ra} T\, g$$
$$\operatorname{div} u = 0$$

in Ω with (for instance)

$$\nu(u,T) = e^{-\alpha T}\,(u \cdot u).$$

Finite element formulation

Given temperature T, find $(u,p) \in W = V \times Q$ such that

$$\int_\Omega 2\nu(u,T)\varepsilon(u) \cdot \varepsilon(v) + \operatorname{div}(v)\, p + \operatorname{div}(u)\, q - \operatorname{Ra} T g \cdot v \,\mathrm{d}x = 0$$

for all $(v,q) \in W$.

```
from fenics import *

# Define viscosity
def nu(u, T):
  return exp(-10.0*T)*dot(u, u)

# Define element spaces
mesh = Mesh(...)
V = VectorFunctionSpace(mesh, "CG", 2)
Q = FunctionSpace(mesh, "CG", 1)
W = V * Q

# Define functions
T = Expression("...")
w = Function(W)
(u, p) = split(w)
(v, q) = TestFunctions(W)

# Define equation
F = (2*nu(u, T)*inner(eps(u), eps(v))
     + div(v)*p + div(u)*q
     + Ra*T*v[1])*dx
bcs = ...

# Solve F = 0 w.r.t w
solve(F == 0, w, bcs)
```

FEniCS provides a wide range of (mixed) finite element spaces

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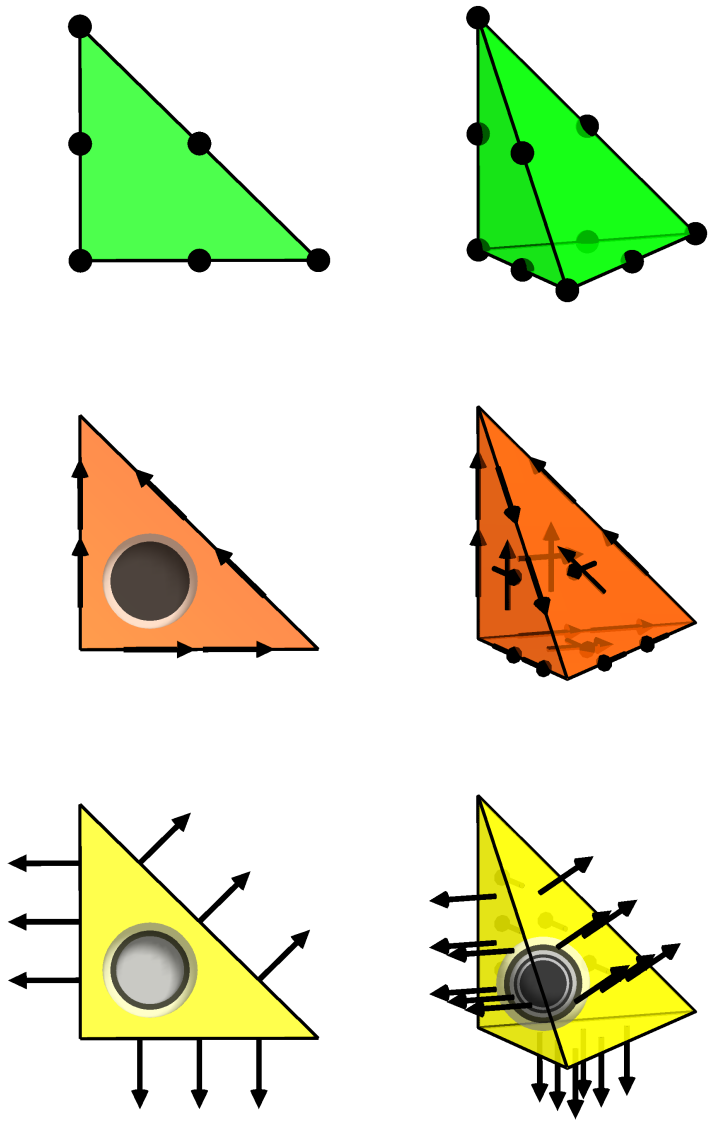

Kirby 2004; Ølgaard, Logg, and Wells 2008; R., Kirby and Logg 2009

FEniCS provides an expressive form language close to mathematical syntax

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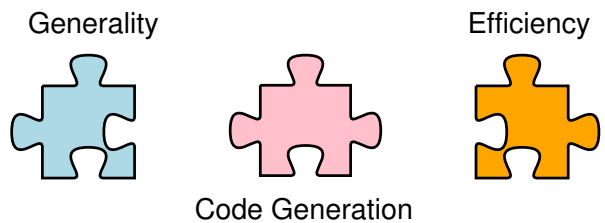
```

Language for variational forms



Kirby and Logg 2006; Alnæs 2012

FEniCS provides automated form assembly over finite element meshes and numerical linear algebra

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from fenics import *

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```
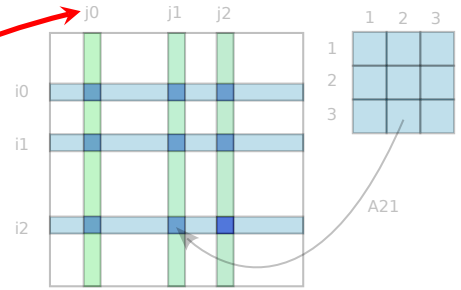
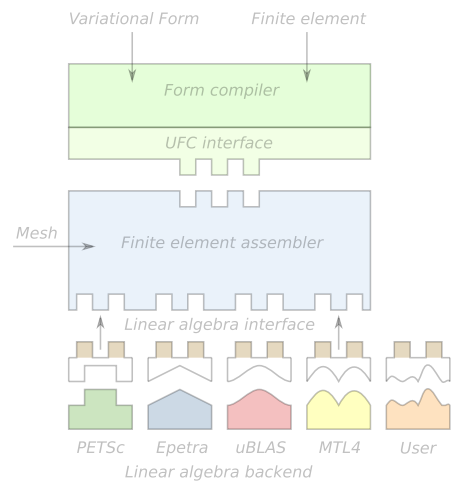

Automated assembly



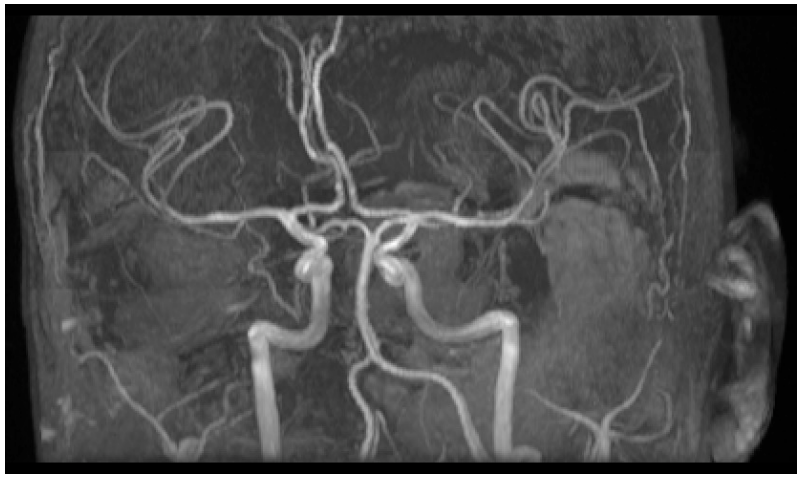
High performance linear algebra

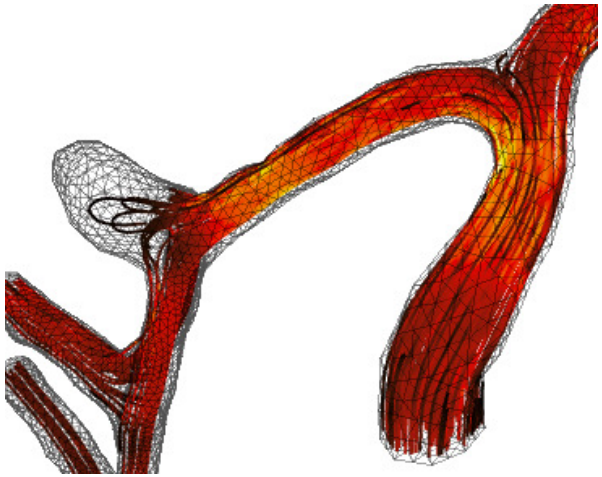


Logg and Wells, 2010; Hake, Logg and Wells 2012

What has FEniCS been used for?

Computational hemodynamics





- Low wall shear stress may trigger aneurysm growth
- Solve the incompressible Navier–Stokes equations on patient-specific geometries

$$\dot{u} + u \cdot \nabla u - \nabla \cdot \sigma(u, p) = f$$
$$\nabla \cdot u = 0$$

Valen-Sendstad, Mardal, Logg, *Computational hemodynamics* (2011)

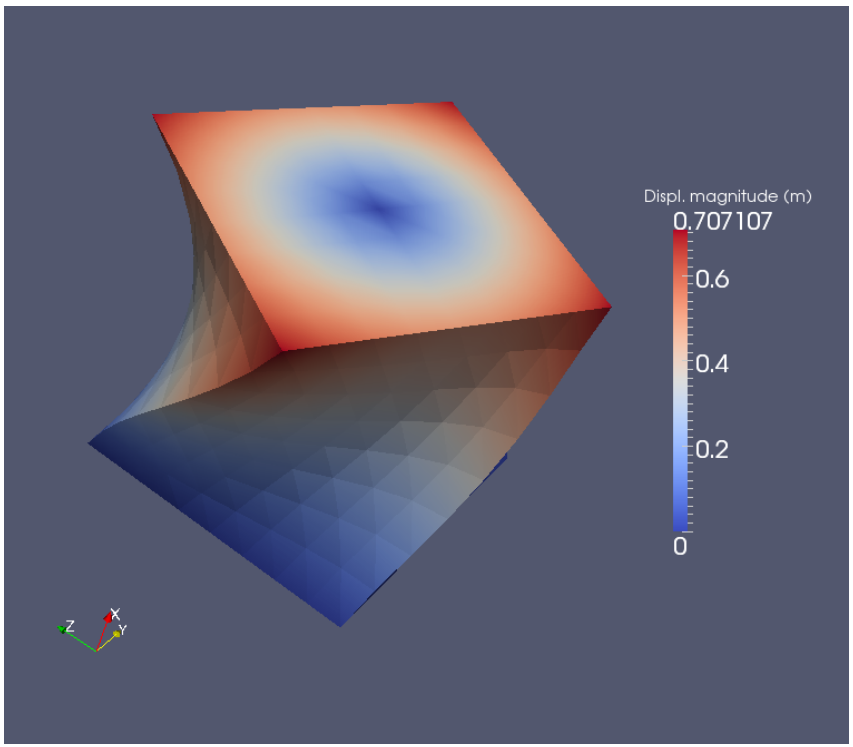
Computational hemodynamics (contd.)

```
# Define Cauchy stress tensor
def sigma(v,w):
    return 2.0*mu*0.5*(grad(v) + grad(v).T)  -
        w*Identity(v.cell().d)

# Define symmetric gradient
def epsilon(v):
    return  0.5*(grad(v) + grad(v).T)

# Tentative velocity step (sigma formulation)
U = 0.5*(u0 + u)
F1 = rho*(1/k)*inner(v, u - u0)*dx + rho*inner(v,
    grad(u0)*(u0 - w))*dx \
   + inner(epsilon(v), sigma(U, p0))*dx \
   + inner(v, p0*n)*ds - mu*inner(grad(U).T*n,
       v)*ds \
   - inner(v, f)*dx
a1 = lhs(F1)
L1 = rhs(F1)

# Pressure correction
a2 = inner(grad(q), k*grad(p))*dx
L2 = inner(grad(q), k*grad(p0))*dx - q*div(u1)*dx

# Velocity correction
a3 = inner(v, u)*dx
L3 = inner(v, u1)*dx + inner(v, k*grad(p0 -
    p1))*dx
```

- The Navier–Stokes solver is implemented in Python/FEniCS
- FEniCS allows solvers to be implemented in a minimal amount of code

Valen-Sendstad, Mardal, Logg, *Computational hemodynamics* (2011)

Hyperelasticity




```
from fenics import *

mesh = UnitCubeMesh(24, 16, 16)
V = VectorFunctionSpace(mesh, "Lagrange", 1)

left =  CompiledSubDomain("(std::abs(x[0])
    < DOLFIN_EPS) && on_boundary")
right = CompiledSubDomain("(std::abs(x[0] - 1.0)
    < DOLFIN_EPS) && on_boundary")

c = Expression(("0.0", "0.0", "0.0"), degree=0)
r = Expression(("0.0",
"0.5*(y0+(x[1]-y0)*cos(t)-(x[2]-z0)*sin(t)-x[1])",
"0.5*(z0+(x[1]-y0)*sin(t)+(x[2]-z0)*cos(t)-x[2])"),
y0=0.5, z0=0.5, t=pi/3, degree=3)
bcl = DirichletBC(V, c, left)
bcr = DirichletBC(V, r, right)
bcs = [bcl, bcr]
v  = TestFunction(V)
u  = Function(V)
B  = Constant((0.0, -0.5, 0.0))
T  = Constant((0.1,  0.0, 0.0))
I = Identity(V.cell().d)
F = I + grad(u)
Ic = tr(F.T*F)
J  = det(F)
E, nu = 10.0, 0.3
mu, lmbda = Constant(E/(2*(1 + nu))),
    Constant(E*nu/((1 + nu)*(1 - 2*nu)))
psi = (mu/2)*(Ic - 3) - mu*ln(J) +
    (lmbda/2)*(ln(J))**2
Pi = psi*dx - dot(B, u)*dx - dot(T, u)*ds
F = derivative(Pi, u, v)

solve(F == 0, u, bcs)
plot(u, interactive=True, mode="displacement")
```

H. Narayanan, *A computational framework for nonlinear elasticity* (2011)

How to use FEniCS?

Hello World in FEniCS: problem formulation

Poisson's equation

$$\begin{aligned} -\Delta u &= f \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

Finite element formulation

Find $u \in V$ such that

$$\underbrace{\int_\Omega \nabla u \cdot \nabla v \, \mathrm{d}x}_{a(u,v)} = \underbrace{\int_\Omega f \, v \, \mathrm{d}x}_{L(v)} \quad \forall \, v \in V$$

Hello World in FEniCS: implementation

```
from fenics import *

mesh = UnitSquareMesh(32, 32)

V = FunctionSpace(mesh, "Lagrange", 1)
u = TrialFunction(V)
v = TestFunction(V)
f = Expression("x[0]*x[1]", degree=2)

a = dot(grad(u), grad(v))*dx
L = f*v*dx

bc = DirichletBC(V, 0.0, DomainBoundary())

u = Function(V)
solve(a == L, u, bc)
plot(u)
```

Basic API

- `Mesh`, `Vertex`, `Edge`, `Face`, `Facet`, `Cell`
- `FiniteElement`, `FunctionSpace`
- `TrialFunction`, `TestFunction`, `Function`
- `grad()`, `curl()`, `div()`, ...
- `Matrix`, `Vector`, `KrylovSolver`, `LUSolver`
- `assemble()`, `solve()`, `plot()`

- Python interface generated semi-automatically by SWIG
- C++ and Python interfaces almost identical

Sounds great, but how do I find my way through the jungle?

Three survival advices

Use the right Python tools

Explore the documentation

Ask, report and request

Use the right Python tools!

Python tools

Doc tools

- Standard terminal:
  ```
  > pydoc dolfin
  > pydoc dolfin.Mesh
  ```
- Python console
  ```
  >>> help(dolfin)
  >>> help(dolfin.Mesh)
  ```

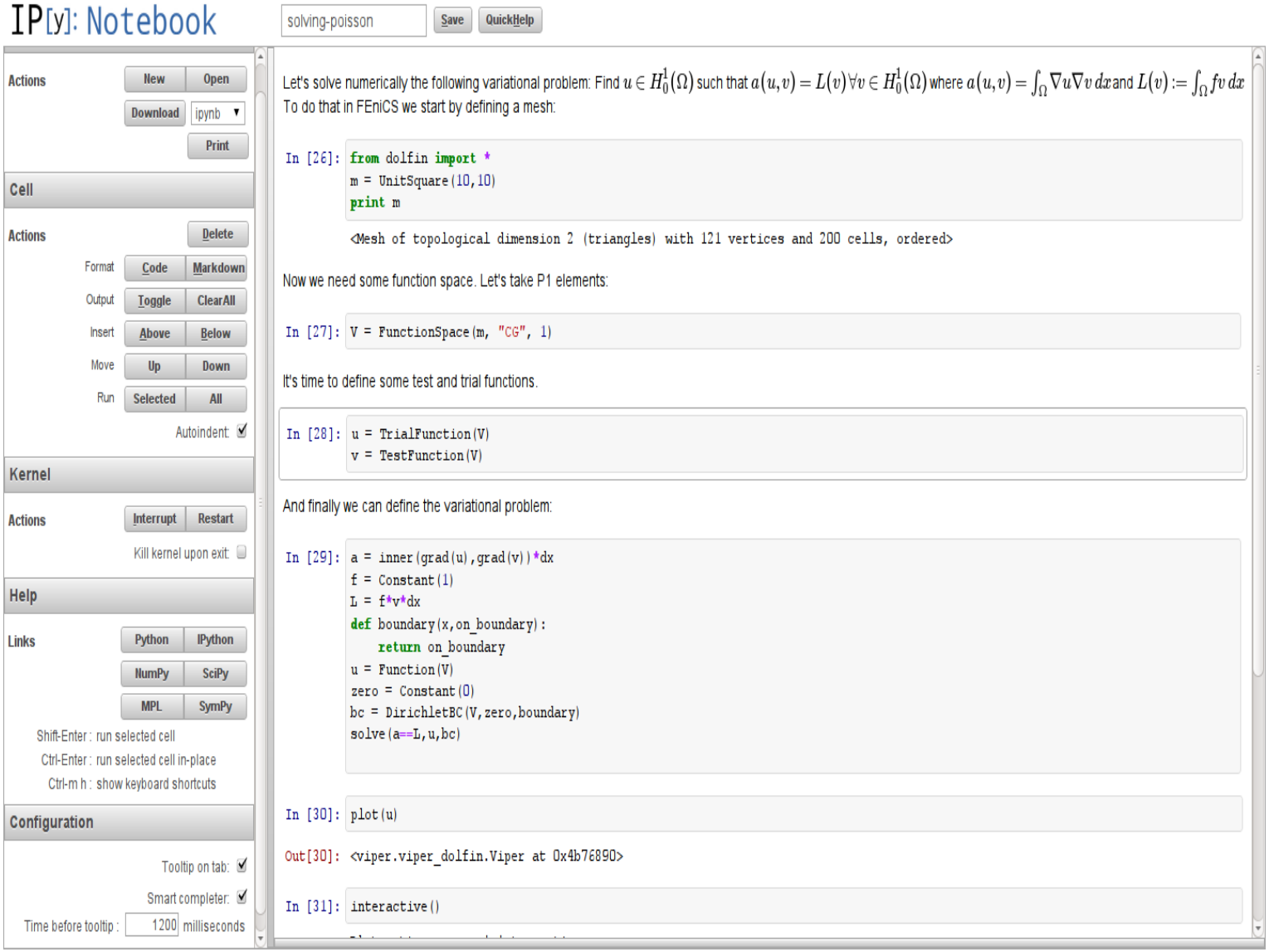
Sophisticated Python environments

IDLE the official (but rather limited) Python IDE

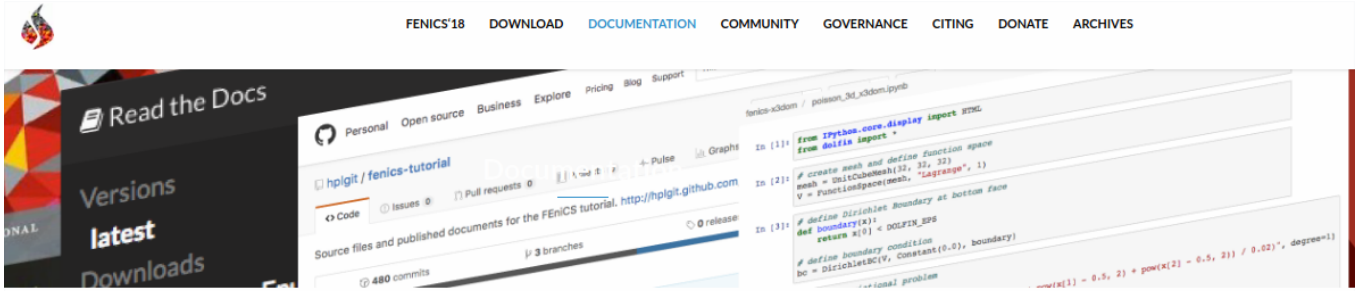
IPython `http://ipython.org/` provides a Python shell and notebook including syntax highlighting, tab-completion, object inspection, debug assisting, history ...

Eclipse plugin `http://pydev.org/` includes syntax highlighting, code completion, unit-testing, refactoring, debugger ...

IPython/Jupyter Notebook



Explore the FEniCS documentation!



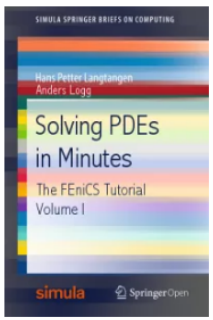
Documentation

Our documentation includes several books, a collection of documented demo programs (notebooks), and a reference manual.

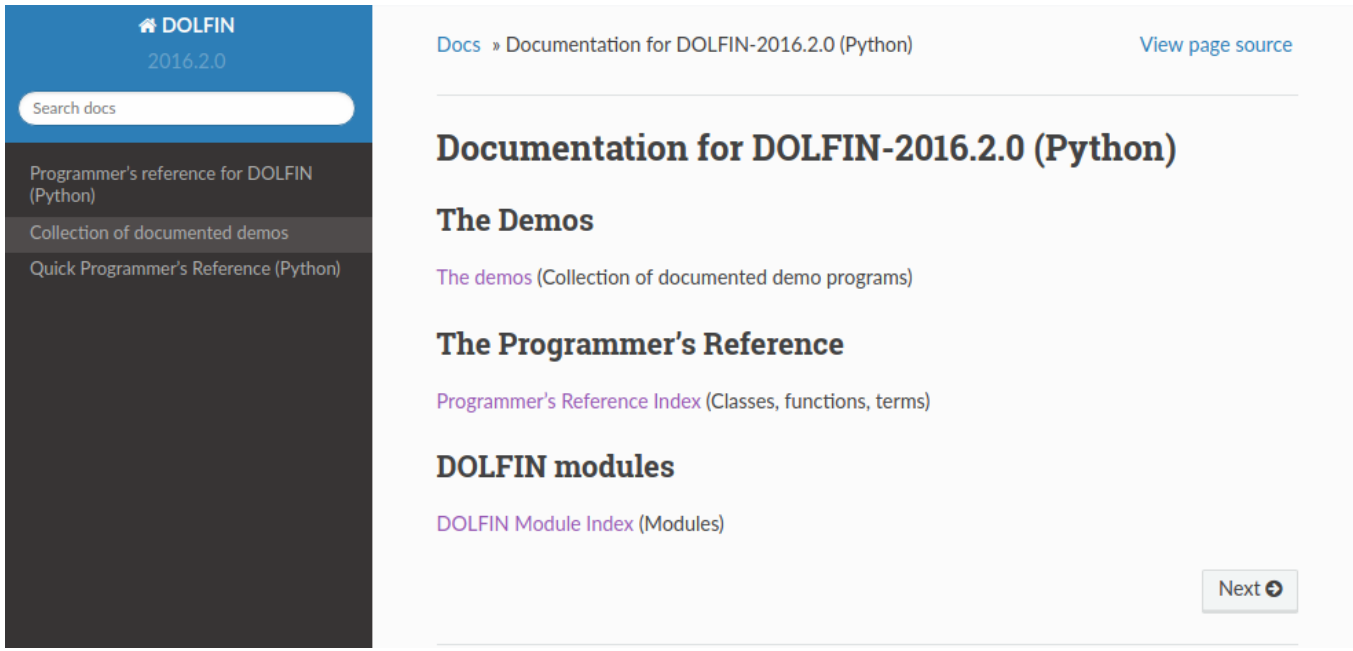
If you are new to FEniCS and want to quickly get started with solving PDEs in Python, the FEniCS Tutorial is the best starting point. For advanced users looking for details on how to build, program and develop with FEniCS, the FEniCS Reference Manual contains all you need to know about FEniCS.

The FEniCS Tutorial

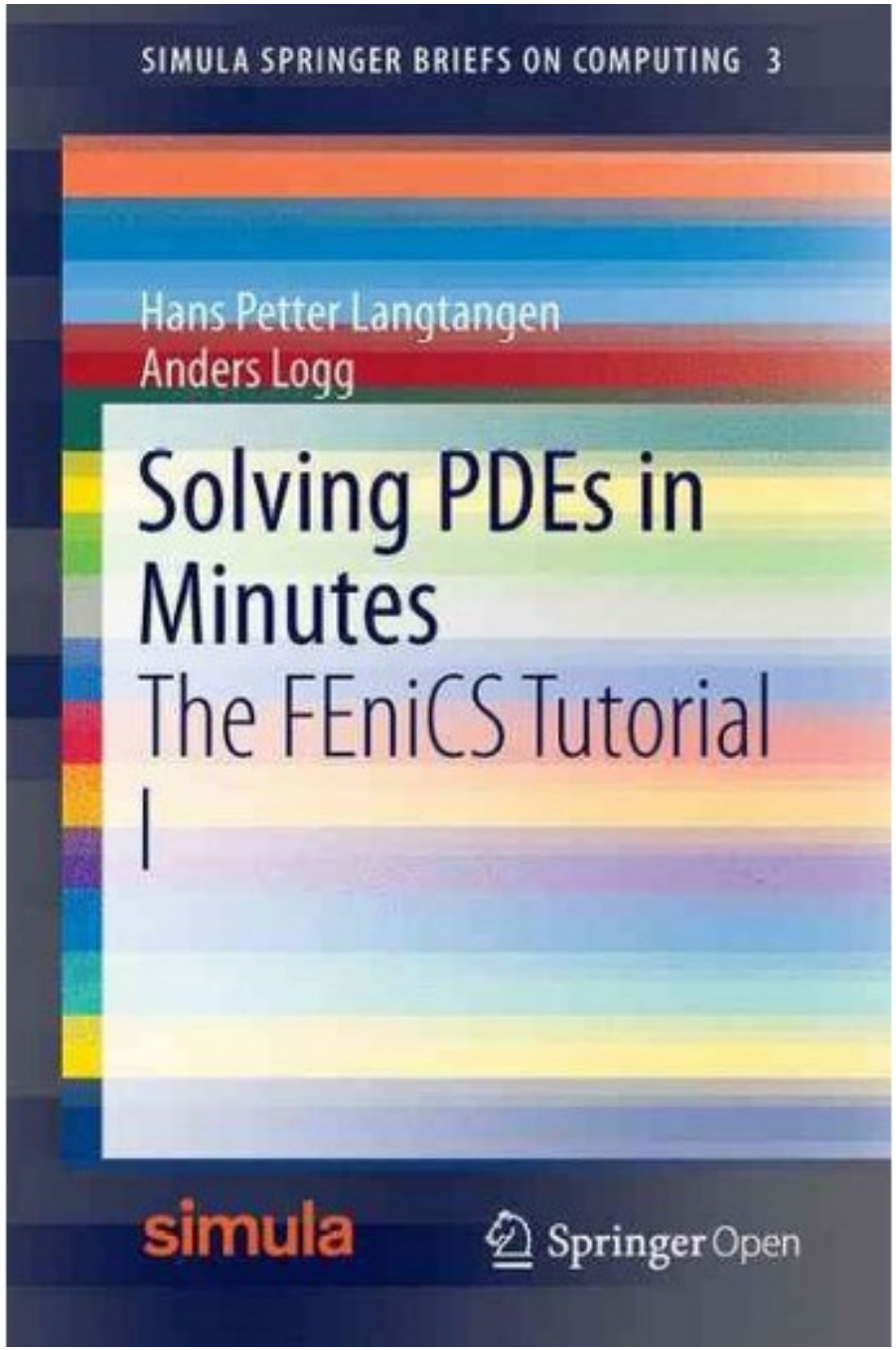
The book Solving PDEs in Python – The FEniCS Tutorial Volume I is the is the perfect guide for new users. The tutorial explains fundamental concepts of the finite element method, FEniCS programming, and demonstrates how to quickly solve a number of PDEs. The tutorial assumes no prior knowledge of the finite element method. The tutorial is an updated and expanded version of the popular first chapter of the FEniCS Book.



http://fenicsproject.org/documentation/



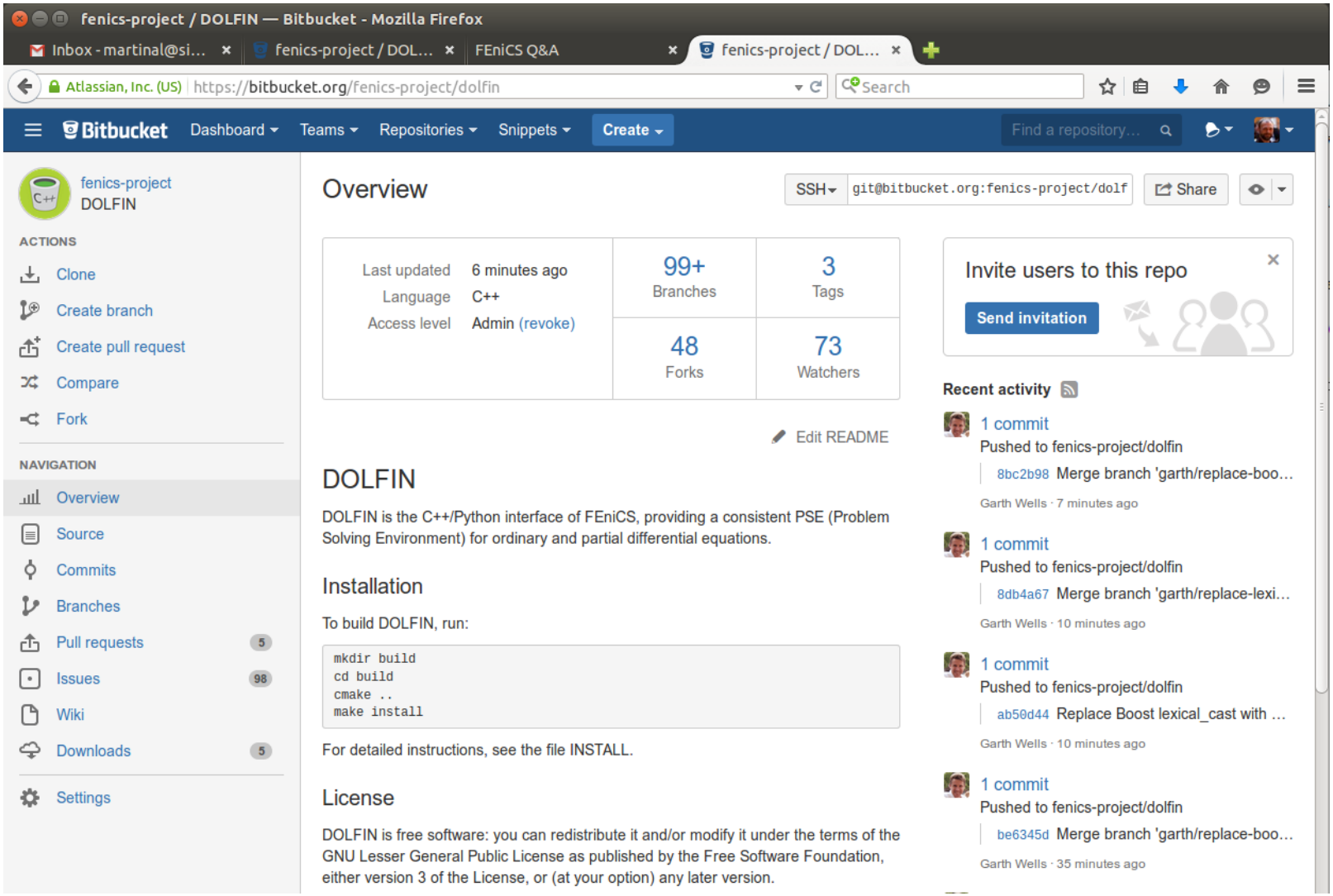
https://fenicsproject.org/olddocs/dolfin/2016.2.0/python/



https://fenicsproject.org/tutorial/

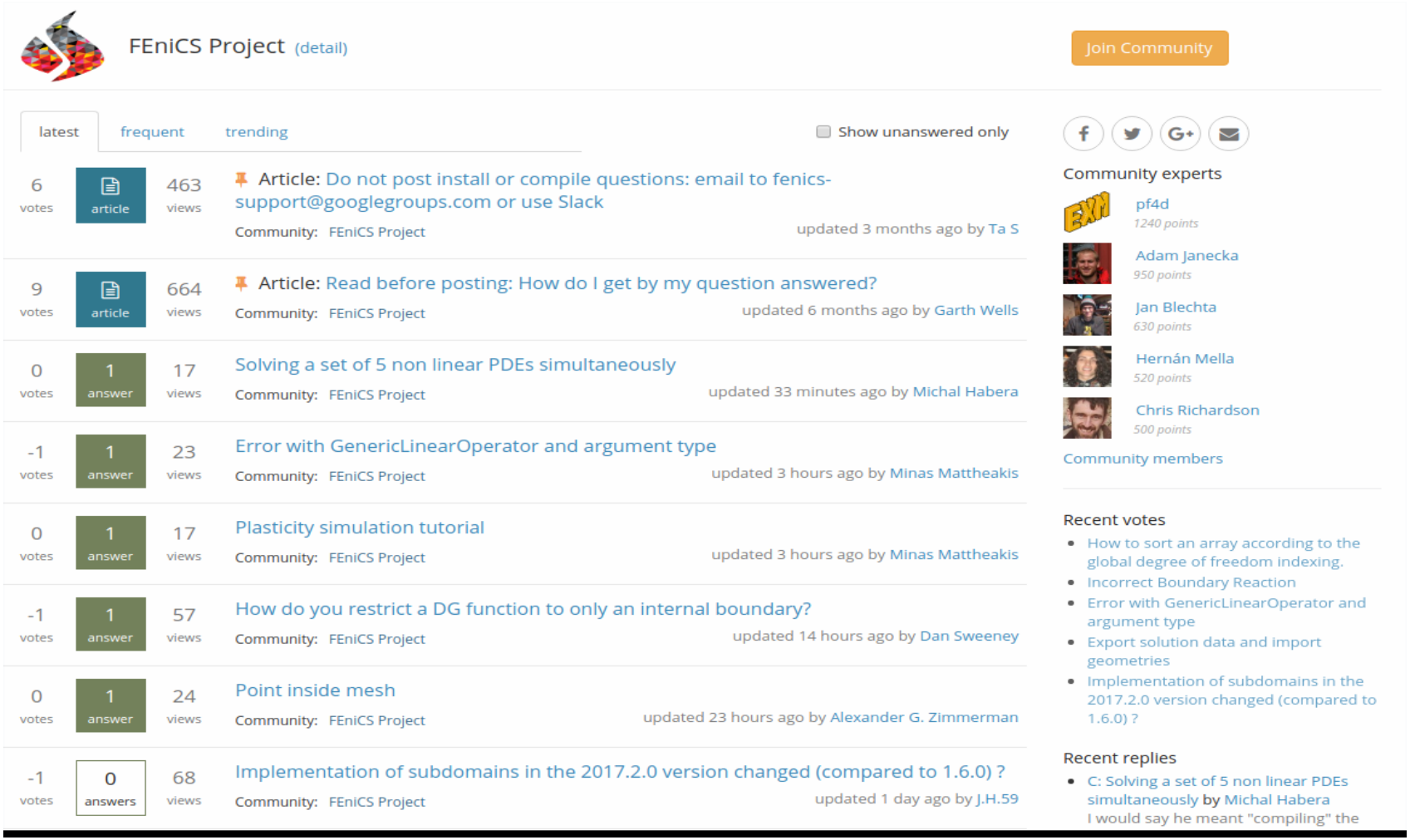
Ask questions, report bugs and request new features!

Development community is organized via bitbucket.org



http://bitbucket.org/fenics-project/

Community help is available via QA forum



`allanswered.com/community/s/fenics-project` (new)

`fenicsproject.org/qa` (old)

Installation alternatives

☞ Docker images on Linux, Mac, Windows

☞ PPA with apt packages for Debian and Ubuntu

☞ Anaconda packages

☞ Build from source

http://fenicsproject.org/download/

The FEniCS challenge!

1. Install FEniCS on your laptop!

   `http://fenicsproject.org/download/`

2. Download and execute demo_cahn-hilliard.py, try to visualize the results with Paraview.
3. Which elements are supported in `dolfin`
   Hint: Check documentation of *dolfin.FunctionSpace*?

See also `https://femtable.org`