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PERMUTATIONS AND COMBINATIONS

SKILLS FOR LIFE
SKILLS BOOTCAMPS

Department for Education

Foundational Sessions Housekeeping

- The use of disrespectful language is prohibited in the questions, this is a supportive, learning environment for all - please engage accordingly. **(FBV: Mutual Respect.)**
- No question is daft or silly - **ask them!**
- There are **Q&A sessions** midway and at the end of the session, should you wish to ask any follow-up questions. Moderators are going to be answering questions as the session progresses as well.
- If you have any questions outside of this lecture, or that are not answered during this lecture, please do submit these for upcoming Open Classes. You can submit these questions here: SE Open Class Questions **or** DS Open Class Questions

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Foundational Sessions Housekeeping cont.

- For all **non-academic questions**, please submit a query: **www.hyperiondev.com/support**
- Report a **safeguarding** incident: **www.hyperiondev.com/safeguardreporting**
- We would love your **feedback** on lectures: **Feedback on Lectures**

Reminders!

Guided Learning Hours

By now, ideally you should have 7 GLHs per week accrued. Remember to attend any and all sessions for support, and to ensure you reach 112 GLHs by the close of your Skills Bootcamp.

Progression Criteria

✅ **Criterion 1: Initial Requirements**

- Complete 15 hours of Guided Learning Hours and the first four tasks within two weeks.

✅ **Criterion 2: Mid-Course Progress**

- Software Engineering: Finish 14 tasks by week 8.
- Data Science: Finish 13 tasks by week 8.

✅ **Criterion 3: Post-Course Progress**

- Complete all mandatory tasks by 24th March 2024.
- Record an Invitation to Interview within 4 weeks of course completion, or by 30th March 2024.
- Achieve 112 GLH by 24th March 2024.

✅ **Criterion 4: Employability**

- Record a Final Job Outcome within 12 weeks of graduation, or by 23rd September 2024.

What is a permutation?

A. An arrangement of items where order doesn't matter
B. An arrangement of items where order matters
C. The number of ways to choose a subset of items from a set
D. The process of dividing items into different categories

What does a combination represent in mathematics?

A. A sequence of numbers in a specific order
B. The total possible arrangements of a set of numbers
C. The number of ways to choose items from a set where order doesn't matter
D. A method to calculate probabilities

In probability, how are permutations and combinations used?

A. To determine the likelihood of an event occurring.
B. To calculate the average outcome of a random experiment.
C. To organise data into different categories.
D. To measure the spread of data around the mean.

Recap of Probability

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Probability

Foundations of Probability

- **Sample Space:** The set of all possible outcomes.
- **Event:** Any subset of the sample space.

Basic Probability

- Probability of an Event: $P(E) = \frac{Number\ of\ favorable\ outcomes}{Total\ number\ of\ outcomes}$

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Rules of Probability (assuming Independence and mutually exclusive):

- **Addition Rule:** P(A or B) = P(A) + P(B)
- **Multiplication Rule:** P(A and B) = P(A) x P(B)

Conditional Probability:

- Probability of an event A, given event B has occurred [P(A|B)].

Independence and Mutual Exclusion:

- Two events are independent if P(A|B) = P(A) and P(B|A) = P(B), and exclusive if P(A and B) = 0

Probability Distributions

- **Uniform:** Every outcome in the sample space is equally likely.
- **Binomial:** Probability distribution of the number of successes in a sequence of n independent experiments.
- **Normal:** Data tends to be around a central value (mean) with no bias left or right.

Permutations and Combinations Topics

1. Permutations
2. Combinations
3. Permutations and Combinations in Probability
4. Applications with Python

Strategic Team Formation

A coach has 10 players and needs to know how many unique 5-player lineups can be formed.

- How do we use combinations to find the number of unique lineups?
- How do we use permutations if the order of the lineup mattered?

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Example: Coloured Balls

- To find combinations we use the formula $C(n,r) = \binom{n}{r}$, which is read as "n choose r" and equals $\frac{n!}{r!(n-r)!}$

- For example, we have 4 balls (blue, yellow, green, red) and want to choose 3 without regards to order. Then $C(4,3) = \frac{4!}{3!(4-3)!} = 4$ unique combinations.

Example: Coloured Balls

- For the daring amongst you, can you do this example assuming that order matters?

  If not feel free to come back after the lecture 😉

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Detour: What is a factorial?

- A factorial, denoted by an exclamation point (!), is **the product of all positive integers up to a given number**. So
$$\begin{aligned} n! &= n(n-1)! \\ &= n(n-1)(n-2)! \\ &= n(n-1)(n-2)\ldots(n-n)! \end{aligned}$$
Where $(n-n)! = 0! = 1$.

- For example $3! = 3 \times 2 \times 1 \times 0! = 3 \times 2 = 6$. Usually $0!$ is simply omitted.

Permutations

Arrangement of objects where order is important.

- To calculate permutations, we use $P(n, r) = \frac{n!}{(n-r)!}$

- For example, if we have 5 books and want to arrange 3 in a row, we use $P(5,3) = \frac{5!}{(5-3)!} = 60$ arrangements, to calculate the number of permutations.

Combinations

Selection where order doesn't matter.

- To calculate combinations, we use $C(n,r) = \frac{n!}{r!(n-r)!}$

- For example, if we have 10 flowers and want to make bouquets using 3 of them (order does not matter) we can calculate the number of possible bouquets using this formula: $C(10,3) = \frac{10!}{3!(10-3)!} = \boxed{120}$ bouquets.

Probability Example with Permutations

Scenario: Calculate the probability of arranging 3 specific books in order from a set of 5.

Permutations: Total ways to arrange 3 out of 5 books = $P(5, 3)$.

Favorable Outcome: Only 1 specific arrangement is favorable.

Calculation: Probability = 1 / P(5, 3).

- Since we know P(5, 3) is 60, **Probability = 1/60 = 0.017**

Probability Example with Combinations

Scenario: Calculate the probability of selecting 3 specific books out of 5.

Combinations: Total ways to select 3 out of 5 books = $C(5, 3)$.

Favorable Outcome: Only 1 specific selection is favorable.

Calculation: Probability = 1 / C(5, 3).

- Since C(5, 3) = 5!/(3! x 2!) = 10, **Probability = 1/10 = 0.1**

- Thus we can observe that arranging 3 specific books **in order** from a set of 5 is much less likely than picking 3 specific books from a set of 5.

- This is what makes permutations and combinations important in software engineering and data science, **they help us estimate the likelihood of events so we could optimise our programs or models for the most likely future outcomes**.

Application in Python

- Once you are comfortable with the concepts of permutations and combinations applying it in python becomes a breeze as **it abstracts the complexity**:

```python
import math

# Calculate permutations of 3 out of 5 items
total_permutations = math.perm(5, 3)
print("Total permutations:", total_permutations)

# Calculate combinations of 3 out of 5 items
total_combinations = math.comb(5, 3)
print("Total combinations:", total_combinations)
```

```python
# Probability of a specific permutation
prob_permutation = 1 / math.perm(5, 3)
print("Probability of a specific permutation:", prob_permutation)

# Probability of a specific combination
prob_combination = 1 / math.comb(5, 3)
print("Probability of a specific combination:", prob_combination)
```

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Back to Strategic Team Formation

A coach has 10 players and needs to know how many unique 5-player lineups can be formed.

- Combinations are used when the order of the selection **is not important**.
- Permutations are used when the order of the selection **is important**.

- Here, **n = 10** (total players) and **r = 5** (players in a lineup)
- **C(10, 5) = 252** and **P(10, 5) = 30 240** (Can you calculate these given what we learned in the lecture? 😃)
- Thus if order mattered there would be **29 988 more possible lineups**

Probability Example with Permutations

Scenario: Calculate the probability of arranging 3 specific books in order from a set of 5.

Permutations: Total ways to arrange 3 out of 5 books = $P(5, 3)$.

Favorable Outcome: Only 1 specific arrangement is favorable.

Calculation: Probability = 1 / P(5, 3).

- Since we know P(5, 3) is 60, **Probability = 1/60 = 0.017**

Worked Example

Five friends are forming a study group and have five different subjects to study. They plan to study one subject each day from Monday to Friday, with each subject being studied on a different day.

1. How many unique study schedules can the friends create if they plan to study one subject each day?

2. If the friends decide to study only three different subjects during the week and the order in which they study these subjects does not matter, how many different subject combinations can they choose?

Worked Example

Five friends are forming a study group and have five different subjects to study. They plan to study one subject each day from Monday to Friday, with each subject being studied on a different day.

- How many unique study schedules can the friends create if they plan to study one subject each day?

The order in which the subjects are studied matters, as studying Math on Monday and Chemistry on Tuesday is a different schedule from studying Chemistry on Monday and Math on Tuesday. Since there are 5 subjects and 5 days, we want to arrange these subjects in a specific order, which is a permutation problem. The number of different schedules is P(5, 5), which equals 5 factorial (5!) and gives us 120 unique schedules.

Worked Example

Five friends are forming a study group and have five different subjects to study. They plan to study one subject each day from Monday to Friday, with each subject being studied on a different day.

- If the friends decide to study only three different subjects during the week and the order in which they study these subjects does not matter, how many different subject combinations can they choose?

Since the order of subjects does not matter in this case, we are dealing with combinations. The friends are choosing 3 out of the 5 subjects, and the combination formula C(n, r) is used to determine the number of ways to choose these subjects. The number of ways to choose 3 subjects out of 5 is C(5, 3), which equals 10 combinations.

Summary

Permutations

★ Arrangements **where order matters**

Combinations

★ Selections **where order doesn't matter**

Probability Applications

★ Calculating likelihood of arrangements/selections

Further Learning

- Better Explained - Easy Permutations and Combinations
- Math is Fun - Combinations and Permutations
- Khan Academy - Counting, permutations, and combinations

Which scenario is an example of a permutation?

A. Selecting 3 members to form a committee from a group of 10
B. Arranging 4 books on a shelf from a collection of 6
C. Choosing toppings for a pizza from a list of ingredients
D. Picking a team captain and goalkeeper from a football team

Where would you apply combinations in real life?

A. Decoding a secret message

B. Determining the outcome of a dice roll

C. Selecting a team of 5 from 10 candidates

D. Arranging songs in a playlist

Questions and Answers

Questions around Combinations and Permutations