CoGrammar

LINEAR ALGEBRA

SKILLS FOR LIFE
SKILLS BOOTCAMPS

Department for Education

Foundational Sessions Housekeeping

- The use of disrespectful language is prohibited in the questions, this is a supportive, learning environment for all - please engage accordingly. **(FBV: Mutual Respect.)**
- No question is daft or silly - **ask them!**
- There are **Q&A sessions** midway and at the end of the session, should you wish to ask any follow-up questions. Moderators are going to be answering questions as the session progresses as well.
- If you have any questions outside of this lecture, or that are not answered during this lecture, please do submit these for upcoming Open Classes. You can submit these questions here: <u>SE Open Class Questions</u> **or** <u>DS Open Class Questions</u>

Foundational Sessions Housekeeping cont.

- For all **non-academic questions**, please submit a query: www.hyperiondev.com/support
- Report a **safeguarding** incident: www.hyperiondev.com/safeguardreporting
- We would love your **feedback** on lectures: Feedback on Lectures

Reminders!

GLH requirements

Guided Learning Hours

By now, ideally you should have 7 GLHs per week accrued. Remember to attend any and all sessions for support, and to ensure you reach 112 GLHs by the close of your Skills Bootcamp.

Progression Criteria

✅ **Criterion 1: Initial Requirements**

- Complete 15 hours of Guided Learning Hours and the first four tasks within two weeks.

✅ **Criterion 2: Mid-Course Progress**

- Software Engineering: Finish 14 tasks by week 8.
- Data Science: Finish 13 tasks by week 8.

✅ **Criterion 3: Post-Course Progress**

- Complete all mandatory tasks by 24th March 2024.
- Record an Invitation to Interview within 4 weeks of course completion, or by 30th March 2024.
- Achieve 112 GLH by 24th March 2024.

✅ **Criterion 4: Employability**

- Record a Final Job Outcome within 12 weeks of graduation, or by 23rd September 2024.

What is a vector in linear algebra?

A. A function that operates on matrices.
B. A series of numbers arranged in a row or column.
C. A mathematical representation of a point in space.
D. A type of matrix with only one row and one column.

What is one of the primary uses of matrices in linear algebra?

A. To solve equations
B. To represent and solve systems of linear equations
C. To calculate probabilities
D. To perform basic arithmetic operations like addition and subtraction

What happens when you multiply a matrix by a scalar?

A. The matrix changes its dimensions
B. Each element of the matrix is multiplied by the scalar
C. The matrix becomes a vector
D. It's not possible to multiply a matrix by a scalar

Recap of Sets, Functions, and Variables

CoGrammar

Sets, Functions, and Variables

Set: a collection of distinct, unordered objects also known as elements or members.

- Set that makes up the input of a function known as **domain**, and set making up the output known as the **codomain**.
- E.g. {1,2,3,4}, {cat,dog,spider}, and {cat,1,spider,4} are all sets.

CoGrammar

Function: a relation between a set of inputs and a set of permissible outputs with the property that each input is related to at most one output.

- **Univariate functions** relate one input to at most one output (i.e. $f(x) = x + 1$)
- **Multivariate functions** relate multiple inputs to at most one output (i.e. $f(x,z) = x - z + 1$)

Variables: Symbols that represent values in mathematical expressions or algorithms.

Linear Algebra Topics

1. Vectors and Matrices
2. Arithmetic Operations on Vectors and Matrices

Using Linear Algebra to Optimise Vehicle Routes

Consider a transportation company wanting to optimise its vehicle routes for efficiency. The company needs to understand the direction and speed of each vehicle to determine the most efficient paths.

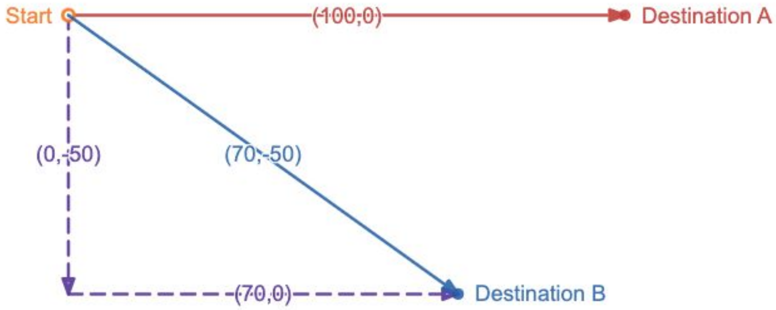
- How do we use vectors to represent the direction and speed of vehicles?
- How do we use matrices to model and analyse complex route networks?

CoGrammar

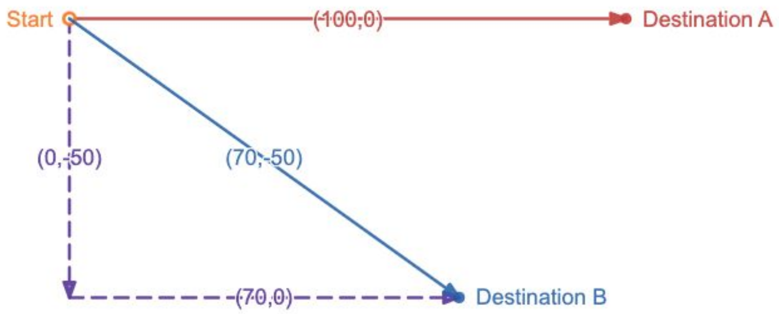
Example: Routes as Vectors

Routes

- **Route A:** 100 km East
- **Route B:** 50 km South, then 70 km East.



- In vector notation **A=(100,0)** and **B=(70,-50)** where the **first number is the x-coordinate and the second number is the y-coordinate**.



- This graph shows a lot about how we represent vectors. Notice how **B = (70,-50) = (0,-50) + (70,0)**. Also notice how each vector is relative to their starting points, such as vector (70,0) which starts from the point [0,-50].
- For clarity in this lecture we will stick to the notation of **(..., ...) for vectors and [..., ...] for points**.

Vectors

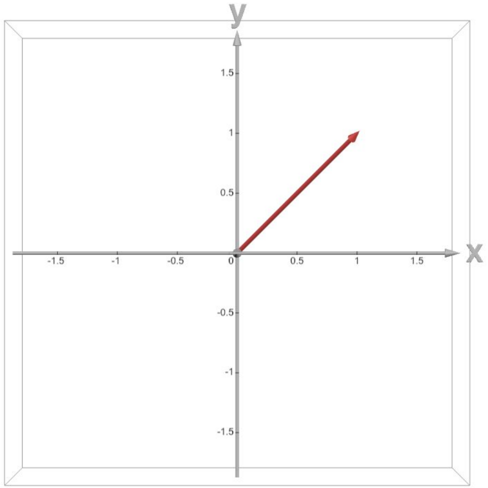
A mathematical object that has magnitude (size or length) and direction

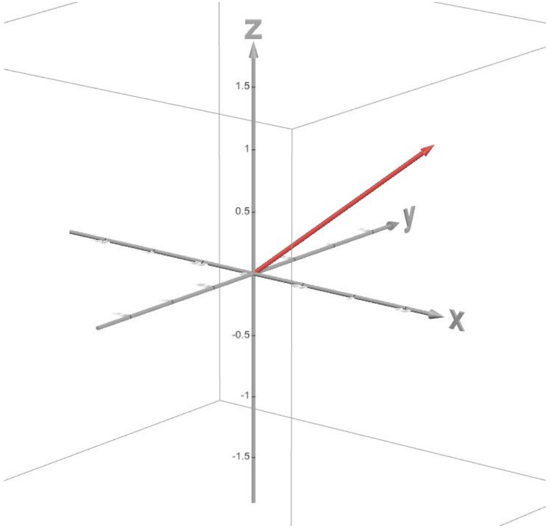
- **Vectors** can be used to represent **physical quantities** that have both **magnitude** and **direction**, such as velocity, force, or displacement.

- In 2D a vector can be represented by **v** = (x, y), where **x** and **y** are the components of the vector along the horizontal and vertical axes, respectively. In a 3D vector we have (x, y, z) where z represents the third dimension.

- For example, if a car moves 3 units to the right and 4 units up, then **v** = (3, 4).

- We can extend this to a 3D example as well.
- The below example may seem like vector (1,1) if we look at the x-y plane, but once we look at it in 3 dimensions we realise it is vector (1,1,1).



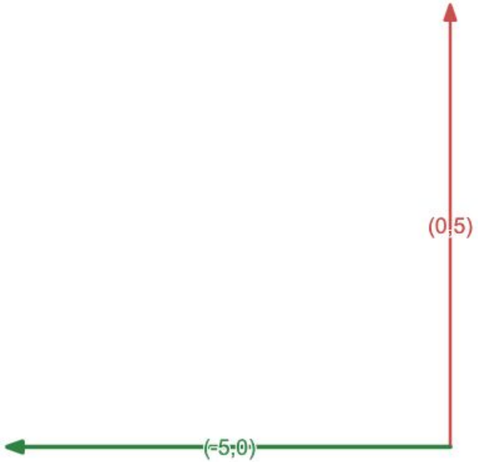


Matrices

A rectangular array of numbers, symbols, or expressions arranged in rows and columns

- Matrices are used to represent and solve systems of linear equations, perform linear transformations, and handle multiple data sets in various fields.
- A 2x2 matrix can be written as $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where each element correspond to a number or expression in a matrix.
- For example, a matrix could be used to transform vectors in a plane. To rotate vectors by 90 degrees clockwise, we could use $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

- Here is the matrix transformation in action, rotating the red vector 90 degrees counterclockwise to become the green vector.



$$\begin{pmatrix} 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Solution

$$\begin{pmatrix} -5 & 0 \end{pmatrix}$$

- One of the most common uses of matrices is in representing linear equations. For example, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ represents 2 linear equations, the first being **f(x) = ax + b**, and the second **f(x) = cx + d**.
- Once you've mastered the basics you could dive into **solving systems of linear equations** using matrices. This will come in very handy for those pursuing a career in data science, although out of scope for our current lecture.

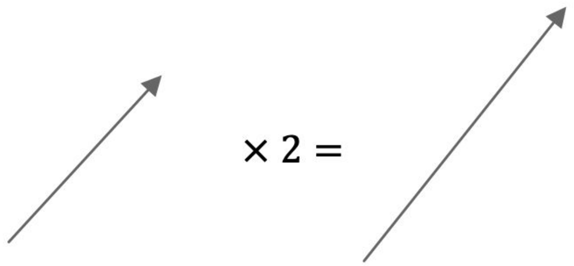
Scalar Multiplication of Vectors

- **Scalar Multiplication** involves multiplying each component of a vector by a scalar (a single number), which changes the magnitude of the vector but not its direction.

Let $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ be a vector.

Let $k = 2$ be a scalar.

Then $k \times \vec{v} = \begin{bmatrix} k \times 3 \\ k \times 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$



Dot Product and Cosine Rule

- **The Dot Product** is a mathematical operation that multiplies two vectors to **yield a scalar**, reflecting the product of the vectors' magnitudes and the cosine of the angle between them.

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

The dot product is computed as $\vec{u} * \vec{v} = u_1 v_1 + u_2 v_2$.

- In simpler words: the dot product is a way to **combine two arrows (vectors)** to get **a single number** that **measures how much one arrow points in the same direction as the other**.

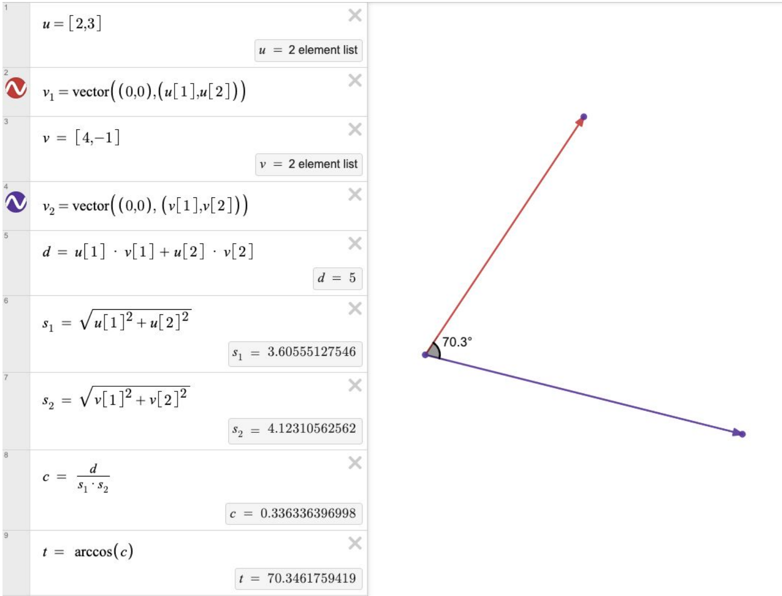
- **The cosine rule** states that the dot product of two vectors is equal to the product of their magnitudes and the cosine of the angle between them:

$$\vec{u} * \vec{v} = |\vec{u}||\vec{v}|\cos(\theta), \text{ so } \cos(\theta) = \frac{\vec{u} * \vec{v}}{|\vec{u}||\vec{v}|}.$$

The magnitude $|\vec{u}| = \sqrt{u_1^2 + u_2^2}$.

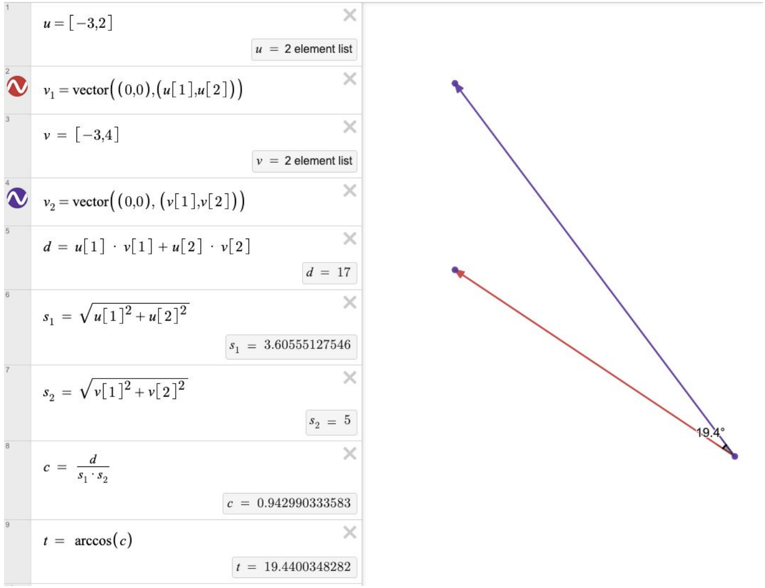
The angle, θ, is found by taking the inverse of $\cos(\theta)$.

- To exemplify the dot product and the cosine rule we could use **Desmos' geometry tool**. For example let us take the vectors **u=(2,3)** and **v=(4,-1)** then do the calculations:



- d is the dot product.
- s_1 is the magnitude of u.
- s_2 is the magnitude of v.
- c is the cosine of the angle.
- t is the angle between the vectors.

- We can do this for any two vectors, feel free to play around with it yourself. It might come in useful for the worked example.



CoGrammar

Matrix Operations

- **Matrix operations** include **addition**, **subtraction**, and **multiplication**. Scalar multiplication involves multiplying every element of the matrix by the scalar, similar to vectors.
- Addition and subtraction are straight forward, adding or subtracting all the corresponding elements of both matrices.
- Multiplication of matrices are more complex, so let’s try the basics

- Matrix multiplication is only valid if the dimensions line up. The columns of the first matrix need to equal the number of rows of the seconds matrix.
- We can use one simple 2x2 example that can be extrapolated to all other matrix multiplications:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

Solution

$$\begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

CoGrammar

- Here are a few examples using **Symbolab** to solve the operations.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

Multiply the rows of the first matrix by the columns of the second matrix

$$= \begin{pmatrix} 1\cdot 5+2\cdot 7 & 1\cdot 6+2\cdot 8 \\ 3\cdot 5+4\cdot 7 & 3\cdot 6+4\cdot 8 \end{pmatrix}$$

Simplify each element

$$= \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

$$\begin{pmatrix} 11 & 3 \\ 7 & 11 \end{pmatrix}\begin{pmatrix} 8 & 0 & 1 \\ 0 & 3 & 5 \end{pmatrix}$$

Multiply the rows of the first matrix by the columns of the second matrix

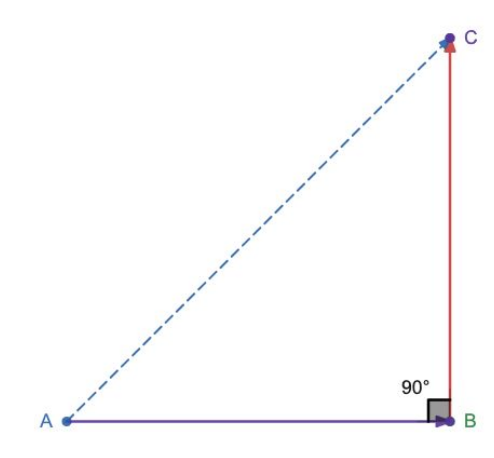
$$= \begin{pmatrix} 11\cdot 8+3\cdot 0 & 11\cdot 0+3\cdot 3 & 11\cdot 1+3\cdot 5 \\ 7\cdot 8+11\cdot 0 & 7\cdot 0+11\cdot 3 & 7\cdot 1+11\cdot 5 \end{pmatrix}$$

Simplify each element

$$= \begin{pmatrix} 88 & 9 & 26 \\ 56 & 33 & 62 \end{pmatrix}$$

Worked Example

Imagine a vehicle is traveling from point A to point B and then to point C. The vehicle starts at point A, moves 80 miles east to point B, then makes a turn and moves to point C, which is 80 miles from point B in a direction making a 90-degree angle with the AB path.



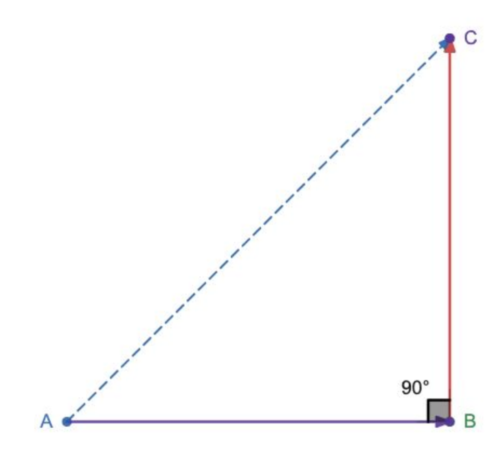
1. Calculate the vector that represents AC, assuming the turn from B to C is northwards.

2. Calculate the dot product between vectors AC and BC.

3. Calculate the angle between vectors AC and BC.

Worked Example

Imagine a vehicle is traveling from point A to point B and then to point C. The vehicle starts at point A, moves 80 miles east to point B, then makes a turn and moves to point C, which is 80 miles from point B in a direction making a 90-degree angle with the AB path.



1. Calculate the vector that represents AC, assuming the turn from B to C is northwards.

   AC = (80,80)

2. Calculate the dot product between vectors AC and BC.

   AC = (80,80) | BC = (0, 80) | AC * BC = 80*80 = 6400

3. Calculate the angle between vectors AC and BC.

   Angle = arccos(6500/(80 * 80 * sqrt(2))) = arccos(6500/(6500 8 sqrt(2))) = arccos(1/sqrt(2)) = 45 deg

CoGrammar

Summary

Vectors

★ A **mathematical object** that has a **magnitude** and **direction**.

Matrices

★ A **rectangular array** of numbers, symbols, or expressions **arranged in rows and columns**.

Dot Product and Cosine Rule

★ **Scalar** that represents the **magnitude** of **one vector projected onto the other**.

★ The **cosine rule** states that the dot product of two vectors is **equal** to the product of their magnitudes and the cosine of the angle between them.

Further Learning

- Khan Academy - Basic Linear Algebra vectors and matrix transformations

- LibreTexts - Linear Algebra Textbook which covers basics and advanced topics

- Machine Learning Mastery - Introduction to Linear Algebra specifically for coders

What does the dot product of two vectors represent?

A. The angle between the vectors
B. A vector perpendicular to both vectors
C. The multiplication of their magnitudes
D. A scalar representing the product of their magnitudes and the cosine of the angle between them

How can vectors be used to model real-world scenarios?

A. To represent quantities like temperature and pressure

B. To represent directions and magnitudes, such as wind speed and direction

C. To calculate the area of geometric shapes

D. To represent colors in digital images

Questions and Answers

Questions around Linear Algebra