DIFFERENTIATION

Foundational Sessions Housekeeping

- The use of disrespectful language is prohibited in the questions, this is a supportive, learning environment for all - please engage accordingly. **(FBV: Mutual Respect.)**
- No question is daft or silly - **ask them!**
- There are **Q&A sessions** midway and at the end of the session, should you wish to ask any follow-up questions. Moderators are going to be answering questions as the session progresses as well.
- If you have any questions outside of this lecture, or that are not answered during this lecture, please do submit these for upcoming Open Classes. You can submit these questions here: SE Open Class Questions **or** DS Open Class Questions

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Foundational Sessions Housekeeping cont.

- For all **non-academic questions**, please submit a query: www.hyperiondev.com/support

- Report a **safeguarding** incident: www.hyperiondev.com/safeguardreporting

- We would love your **feedback** on lectures: [Feedback on Lectures]()

Reminders!

GLH requirements and lecture materials

Guided Learning Hours

By now, ideally you should have 7 GLHs per week accrued. Remember to attend any and all sessions for support, and to ensure you reach 112 GLHs by the close of your Skills Bootcamp.

Lecture Materials

Lecture materials can be found in the DS repository for Data Science students and SE repository for Software Engineering students.

Progression Criteria

✅ **Criterion 1: Initial Requirements**

- Complete 15 hours of Guided Learning Hours and the first four tasks within two weeks.

✅ **Criterion 2: Mid-Course Progress**

- Software Engineering: Finish 14 tasks by week 8.
- Data Science: Finish 13 tasks by week 8.

✅ **Criterion 3: Post-Course Progress**

- Complete all mandatory tasks by 24th March 2024.
- Record an Invitation to Interview within 4 weeks of course completion, or by 30th March 2024.
- Achieve 112 GLH by 24th March 2024.

✅ **Criterion 4: Employability**

- Record a Final Job Outcome within 12 weeks of graduation, or by 23rd September 2024.

What is a gradient of a linear function?

- **A.** A number which represents the direction of a line
- **B.** A gradual change in steepness of a line
- **C.** A constant value which represents the rate of change of the function
- **D.** A point where the function intercepts the y-axis

How can you calculate the gradient of a linear function?

A. $m = (y + c) / x$

B. $m = (ymax - ymin) / (xmax - xmin)$

C. $m = (x2 - x1) / (y2 - y1)$

D. $m = \Delta y / \Delta x$

What is the derivative of a function?

A. The rate of change of the function’s output with respect to the input variable of the function

B. Another function which looks and behaves similar to the function

C. The average rate of change of the function over the entire domain

D. The gradient of a multivariate function

Recap of Permutations and Combinations

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Permutations

Arrangement of objects where order is important.

To calculate permutations, we use $P(n,r) = \frac{n!}{(n-r)!}$

Where:

- ***n*** is the number of objects available to choose from
- ***r*** is the number of objects that are chosen
- ***!*** is the factorial.

Combinations

Selection where order doesn't matter.

To calculate combinations, we use $C(n, r) = \frac{n!}{r!(n-r)!}$

Where:

- ***n*** is the number of objects available to choose from
- ***r*** is the number of objects that are chosen
- ***!*** is the factorial

Differentiation Topics

1. Gradients in Linear Functions
2. Gradients and Derivatives
3. Rules of Differentiation

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How do sales change with respect to time?

Consider a entertainment company that sells video games. They determine that their sales over time can be represented with the following function:

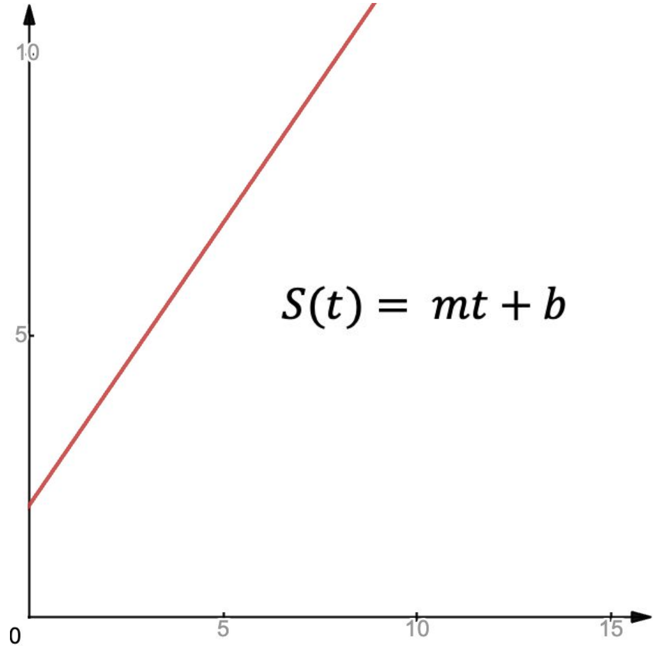
$$S(t) = 5t^2 - 20t + 15$$

where S represents sales in thousands of pounds and t represents time in months

- What can we do to understand more about how sales are changing over time?
- How can we identify periods of growth, decline and stability?

Example: Linear Sales Function

- ***b*** is the initial sales (y-intercept)
- ***m*** is the rate at which the amount of sales changes with respect to time
- The graph shows a function where **m = 1**
- Since the gradient is **positive**, sales **increase** with time



Gradients in Linear Functions

The gradient or slope is a constant value that represents the rate of change of the function.

- The gradient of a linear function tells us how the function changes with respect to the input variable (usually x).
- A **positive** gradient means the function **increases** and a **negative** gradient means the function **decreases**.
- As the magnitude of the gradient **increases**, so does the **steepness** of the slope of the line.
- If the gradient is **0**, the function is a **horizontal line**.

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Gradients in Linear Functions

- In the equation of a linear graph:

$$y = mx + c$$

m is the gradient of the function

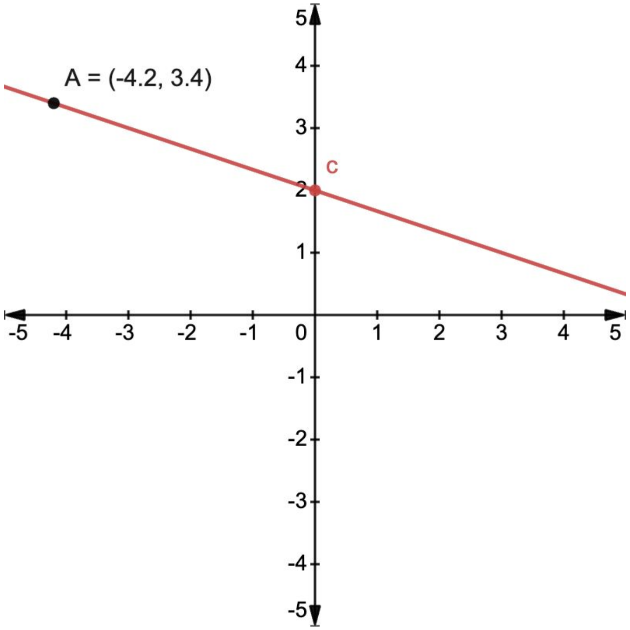
- The gradient is calculated as the **change in *y* over the change in *x*** or "rise over run":

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

using two points on the line ($\mathbf{x_1}$, $\mathbf{y_1}$) and ($\mathbf{x_2}$, $\mathbf{y_2}$)

Example: Gradients

- Given the following graph, calculate the gradient of the function.



$A = (-4.2, 3.4)$ and $c = (0, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 3.4}{0 - (-4.2)}$$

$$= \frac{-1.4}{4.2}$$

$$= -\frac{1}{3}$$

Derivatives

The derivative of a function represents the rate of change of the function with respect to an independent/input variable.

- Linear functions have a **constant rate of change/gradient**, other types of functions do not e.g. quadratic functions
- The derivative of a function is another function which describes the **instantaneous rate of change** of the function at any point.
- The **derivative of a linear function is the gradient**, since the rate of change at any point will be the same, thus a constant.
- Not all functions have a derivative. Functions that have a derivative are called **differentiable functions.**

Rules of Differentiation

- **Notation:** For the function ***f(x) = y***, the derivative of ***f*** with respect to ***x*** is denoted by:

$$\frac{dy}{dx} = f'(x) = \frac{d}{dx}f(x) = D_x y$$

- **Limits:** The derivative is calculated using limits:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This process is long and complicated, instead we use rules that have been derived from this formula.

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Rules of Differentiation

- **Constant rule:** if C is a constant,

$$\frac{d}{dx}C = 0$$

- **Constant multiple rule:** if C is a constant,

$$\frac{d}{dx}Cf(x) = Cf'(x)$$

- **Power rule**

$$\frac{d}{dx}x^n = nx^{n-1}$$

- **Sum and Difference rule**

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$

Rules of Differentiation

- **Product Rule**

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

- **Quotient Rule (derived from product rule)**

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

- **Chain Rule**

$$\frac{d}{dx}f[g(x)] = f'[g(x)]g'(x)$$

Worked Example

Calculate the derivative of the following functions:

1. $f(x) = 1890$
2. $g(x) = 6x$
3. $h(x) = 2x - 7$
4. $m(x) = 16x^6$
5. $S(t) = 5t^2 - 20t + 15$
6. $f(x) = \dfrac{x^3 + 4x - 9}{x + 6}$
7. $g(x) = (x+2)^{16} + 5(x+2)^2 + 6x + 7$

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   $f'(x) = 0$

2. $g(x) = 6x$

   $g'(x) = 6$

3. $h(x) = 2x - 7$

   $h'(x) = 2$

4. $m(x) = 16x^6$

   $m'(x) = 6 \times 16x^5 = 96x^5$

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5. $$S(t) = 5t^2 - 20t + 15$$

S'(t) = 10t - 20

6. $$f(x) = \frac{x^3 + 4x - 9}{x + 6}$$

f'(x) = [(3x^2 + 4) * (x+6) - (x^3 + 4x - 9) * 1]/(x + 6)^2

7. $$g(x) = (x+2)^{16} + 5(x+2)^2 + 6x + 7$$

g'(x) = 16(x+2)^{15} * 1 + 10(x+2) * 1 + 6

Summary

Gradients of Linear Functions

- ★ A constant value representing the rate of change of the function.
- ★ Calculated by dividing the change in y by the change in x.

Derivatives

- ★ A function representing the instantaneous rate of change of a function at any point.
- ★ The gradient of a linear function is equal to the derivative of the function.

Rules of Differentiation

- ★ There are 7 rules of differentiation that we use to determine the derivative.

Further Learning

- Math Centre - Linear Functions
- Khan Academy - Linear equations, functions, & graphs
- LibreTexts - Directional Derivatives and the Gradient
- OpenStax - Calculus 1

Which of the following is a correct differentiation rule?

A. f'(x) = m

B. f'(mx) = m f'(x)

C. f'(a + b) = f'(a) + f'(b)

D. f'(x - a) = -a

What can be said about a linear function with a gradient of -9?

A. The line is curved and increasing.

B. The line is straight and steep, and is decreasing.

C. The line is decreasing, with a gentle slope.

D. The line is straight and decreasing, with a gentle slope.

Questions and Answers

Questions around Sets, Functions and Variables