Otsu's Method for Image Thresholding

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***Abstract*—This paper aims to describe the methodology used in Otsu's method which is an automated image binary threshold method based on the paper "A threshold selection method from gray-level histograms" published in 1979 by Nobuyuki Otsu. Mathematical background of the algorithm is explained. Finally, a code snippet that implements the algorithm is provided written in python.**

***Index Terms*—Otsu, Thresholding, Image Processing, Computer Vision**

I. Introduction

In image processing, the gray levels of pixels belonging to the object are substantially different from the gray levels of the pixels belonging to the background. Because of that thresholding is a simple way to separate foreground pixels from the background. In this regard, there have been many solutions proposed to find the optimal threshold value.

Ideally, gray level histogram of an image has a sharp valley between foreground and background pixels. This allows to find the threshold value by searching the bottom of the valley. Practically however, many pictures can have multiple peaks int their histograms.Flat peaks also makes finding the bottom of the valley hard. Often picture contains noisy pixels which makes finding optimal threshold, even harder.

Taking the advantage of the separability of the foreground/background pixels and the non-idealities at hand, Otsu proposed a simple but elegant solution by minimizing the within-class weighted variance. This alternatively means to maximizing the inner-class variance. So for Otsu's method, threshold is the value that separates the foreground/background the most by measuring the variance between the two cluster.

II. Formulation

Let the range of pixel values of a gray image differs from 1 to L ([1, 2, 3... L]). The number of pixel at level i is denoted by n_i and the total number of pixels by N = $n_1 + n_2 + ... + n_L$. In order to simplify the discussion, the gray-level histogram is normalized and given as a probability distribution.

$$p_i = n_i/N, \quad p_i \geqslant 0, \quad \sum_{i=1}^{L} p_i = 1. \tag{1}$$

Assume the two class separated at some value into C_0 and C_1. C_0 denotes pixels with levels [1, , k], and C_1 denotes pixels with levels [k + 1, , L]. Then the probabilities of class occurrence and the class mean levels, respectively, are given by

$$w_0 = Pr(C_0) = \sum_{i=1}^{k} p_i = w(k) \tag{2}$$

$$w_1 = Pr(C_1) = \sum_{i=k+1}^{L} p_i = 1 - w(k) \tag{3}$$

and mean values are

$$\mu_0 = \sum_{i=1}^{k} i \operatorname{Pr}(i \mid C_0) = \sum_{i=1}^{k} i p_i/\omega_0 = \mu(k)/\omega(k) \tag{4}$$

$$\mu_1 = \sum_{i=k+1}^{L} i \operatorname{Pr}(i \mid C_1) = \sum_{i=k+1}^{L} i p_i/\omega_1 = \frac{\mu_T - \mu(k)}{1 - \omega(k)}, \tag{5}$$

where

$$w(k) = \sum_{i=1}^{k} p_i \tag{6}$$

and

$$\mu(k) = \sum_{i=1}^{k} i p_i \tag{7}$$

are the zeroth- and the first-order cumulative moments of the histogram up to the kth level, respectively, and

$$\mu_T = \mu(L) = \sum_{1}^{L} i p_i \tag{8}$$

is the total mean level of the original picture. We can easily verify the following relation for any choice of k:

$$\omega_0\mu_0 + \omega_1\mu_1 = \mu_T, \quad \omega_0 + \omega_1 = 1. \tag{9}$$

The class variances are given by

$$\sigma_0^2 = \sum_{i=1}^{k} (i - \mu_0)^2 \operatorname{Pr}(i \mid C_0) = \sum_{i=1}^{k} (i - \mu_0)^2 p_i/\omega_0 \tag{10}$$

$$\sigma_1^2 = \sum_{i=k+1}^{L} (i - \mu_1)^2 \operatorname{Pr}(i \mid C_1) = \sum_{i=k+1}^{L} (i - \mu_1)^2 p_i/\omega_1 \tag{11}$$

Following equations are within-class and between class variance

$$\sigma_W^2 = \omega_0\sigma_0^2 + \omega_1\sigma_1^2 \tag{12}$$

$$\begin{aligned} \sigma_B^2 &= \omega_0 (\mu_0 - \mu_T)^2 + \omega_1 (\mu_1 - \mu_T)^2 \\ &= \omega_0\omega_1 (\mu_1 - \mu_0)^2 \end{aligned} \tag{13}$$

Here at its own work Otsu chooses to maximize between class variance since the formulation for between class variance requires first-order statistic. However, minimizing within class variance can also be used interchangeably as it's the method used in the provided algorithm.

Previous argument can be easily verified using the following formula keeping in mind that total variance is not a function of k (threshold value) but within/between-class variances are.

$$\sigma_W^2 + \sigma_B^2 = \sigma_T^2 \tag{14}$$

III. RESULT

Fig. 1. Stock gray image

Fig. 2. Stock gray image after binary threshold applied according to Otsu's method

IV. CONCLUSION

Otsu's method by utilizing variance between foreground/background clusters automatically selects an optimal threshold value from a gray level histogram. Otsu's method can also be used for multiple segmentation with an adjustment in the algorithm.

V. APPENDIX

Applying the algorithm from scratch using python in jupyter notebook :
https://github.com/furkanatak/Otsu-s-method

REFERENCES

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