FT1

Fourier Transformu

Time Domain $\longrightarrow$ Frekans Domain

$e^{-\infty} = 0$

- x(t) verilip X(jw) istendiği durumlarda

$$\Rightarrow F[x(t)] = X(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt \qquad \text{(CTFT)}$$

- X(jw) verilip x(t) istendiği durumda

$$\Rightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) \cdot e^{jwt}\, dw$$

ÖRN: $x(t) = e^{-at} u(t) \quad a>0 \quad X(jw) = ?$

$$X(jw) = \int_{-\infty}^{\infty} e^{-at} \cdot u(t) \cdot e^{-jwt} = \int_{0}^{\infty} e^{-at} \cdot e^{-jwt} = \int_{0}^{\infty} e^{-t(a+jw)}\, dt$$

$$X(jw) = \frac{-1}{a+jw} \cdot e^{-t(a+jw)} \Big|_{0}^{\infty} = 0 + \frac{1}{a+jw} = \frac{1}{a+jw}$$

(önce t yerine ∞ yazıyoruz. $e^{-\infty} = 0$ oldu da ilk ifade komple 0 oluyor. Sonra t yerine 0 yazıyoruz. Sonuç çıkıyor.

ÖRN: $x(t) = e^{-a|t|} \quad , a>0 \quad X(jw) = ?$

$t>0 \; , \; e^{-at}$

$t<0 \; , \; e^{at}$

$$= \int_{-\infty}^{0} e^{at} \cdot e^{-jwt}\, dt + \underbrace{\int_{0}^{\infty} e^{-at} \cdot e^{-jwt}\, dt}_{\frac{1}{a+jw}}$$

$$= \int_{-\infty}^{0} e^{t(a-jw)}\, dt = \frac{e^{t(a-jw)}}{a-jw} \Big|_{-\infty}^{0} = \frac{1}{a-jw} - 0 = \frac{1}{a-jw}$$

$$= \frac{1}{a-jw} + \frac{1}{a+jw} = \frac{2a}{a^2 - j^2 w^2} = \frac{2a}{a^2 + w^2}$$

ÖRN!

$x(t) = \delta(t) \qquad X(jw) = ?$

"birim basamak"

"$\delta(t)$ fonksiyonu sadece $t=0$ da 1 dir."

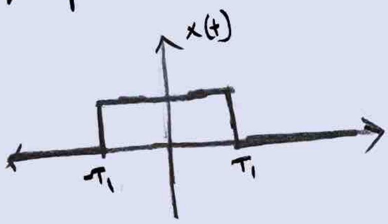
$$X(jw) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-jwt}\, dt$$

$$X(jw) = \delta(0) \cdot e^{-jw0} = 1 \cdot e^{0} = 1$$

ÖRN!

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases} \qquad X(jw) = ?$$

$\rightarrow -T_1 < t < T_1$



$$\boxed{\int e^{at}\, dt = \frac{e^{at}}{a}}$$

$$X(jw) = \int_{-T_1}^{T_1} 1 \cdot e^{-jwt}\, dt = \frac{e^{-jwt}}{-jw}\Bigg|_{-T_1}^{T_1} = \frac{e^{-jwT_1}}{-jw} - \left(-\frac{e^{jwT_1}}{jw}\right)$$

$$= \frac{e^{-jwT_1}}{-jw} + \frac{e^{jwT_1}}{jw} = 2 \cdot \frac{e^{-jwT_1} + e^{jwT_1}}{2 \cdot jw} = \frac{\sin(wT_1)}{w} \cdot 2$$

ÖRN!

$$x(t) = e^{-2(t-1)} \underbrace{u(t-1)}_{t \geq 1 \quad u(t-1)=1}$$

$$\boxed{u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}}$$

$$X(jw) = \int_{1}^{\infty} e^{-2(t-1)} \cdot e^{-jwt}\, dt = \int_{1}^{\infty} e^{-2t} \cdot e^{2} \cdot e^{-jwt}\, dt$$

$$= e^{2} \int_{1}^{\infty} e^{-t(2+jw)}\, dt = e^{2}\left(\frac{e^{-t(2+jw)}}{2+jw}\Bigg|_{1}^{\infty}\right) = e^{2}\left(0 + \frac{e^{-2+jw}}{2+jw}\right)$$

$$X(jw) = \cancel{e^{2}} \cdot \frac{1}{2+jw} \cdot \cancel{e^{-2}} \cdot e^{jw}$$

$$X(jw) = \frac{e^{jw}}{2+jw}$$

Fourier Transform Özellikler

Lineerlik

$$x(t) \xleftrightarrow{FT} X(jw)$$
$$y(t) \xleftrightarrow{FT} Y(jw)$$
$$a x(t) + b y(t) \xleftrightarrow{FT} a.X(jw) + b.Y(jw)$$

Zamanda Kaydırma

$$x(t) \xleftrightarrow{FT} X(jw)$$
$$x(t \pm t_0) \longleftrightarrow e^{\pm jwt_0} . X(jw)$$

Zamanda Ölçekleme

$$x(t) \xleftrightarrow{FT} X(jw)$$
$$x(a.t) \xleftrightarrow{FT} \frac{1}{|a|} . X\left(\frac{jw}{a}\right)$$

Zamanda Tersleme

$$x(t) \xleftrightarrow{FT} X(jw)$$
$$x(-t) \xleftrightarrow{FT} X(-jw)$$

Türev ve İntegral

$$x(t) \xleftrightarrow{FT} X(jw)$$
$$\frac{d}{dt} x(t) \xleftrightarrow{FT} jw\, X(jw)$$
$$\int_{-\infty}^{t} x(t).dt \longleftrightarrow \frac{1}{jw} X(jw)$$

Frekans Kaydırma

$$x(t) \xleftrightarrow{FT} X(jw)$$
$$e^{\pm jw_0 t}\, x(t) \longleftrightarrow X(j(w \mp w_0))$$

Convolüsyon

$$x(t) \longleftrightarrow X(jw)$$
$$\boxed{x(t) * y(t) \xleftrightarrow{FT} X(jw) . Y(jw)}$$

Çarpım

$$x(t) . y(t) \xleftrightarrow{FT} \frac{1}{2\pi}\left[X(jw) * Y(jw)\right]$$

Differantiation in Frequency

$$x(t) \xleftrightarrow{FT} X(jw)$$
$$t.x(t) \xleftrightarrow{FT} \frac{j\,d}{dw} . X(jw)$$

Parseval

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw)|^2 d.w$$

FT4

ÖRN!

$x(t) = \sin w_0 t \qquad x(jw) = ?$

$x(t) = \frac{e^{jw_0 t}}{2j} - \frac{e^{-jw_0 t}}{2j}$

$$\boxed{e^{\pm jw_0 t} \xleftrightarrow{FT} X(j(w \mp w_0))}$$

$$\boxed{1 \xleftrightarrow{FT} 2\pi\delta(w)}$$

$\frac{e^{jw_0 t}}{2j} \xleftrightarrow{FS} \frac{1}{\cancel{2}j} \cancel{2}\pi\delta(w - w_0)$

$-\frac{1}{2j} \cdot e^{-jw_0 t} \xleftrightarrow{FS} -\frac{1}{\cancel{2}j} \cancel{2}\pi\delta(w + w_0)$

$= \frac{\pi}{j}\delta(w - w_0) - \frac{\pi}{j}\delta(w + w_0)$

$X(jw) = \frac{\pi}{j}\left(\delta(w - w_0) - \delta(w + w_0)\right)$

ÖRN!

$x(t) = \cos w_0 t$

$x(t) = \frac{1}{2}e^{jw_0 t} + \frac{1}{2}e^{-jw_0 t}$

$$\left.\begin{aligned} \frac{1}{2}e^{jw_0 t} &\longrightarrow \frac{1}{\cancel{2}}\cancel{2}\pi\delta(w - w_0) \\ \frac{1}{2}e^{-jw_0 t} &\longrightarrow \frac{1}{\cancel{2}}\cancel{2}\pi\delta(w + w_0) \end{aligned}\right\} = \pi\delta(w - w_0) + \pi\delta(w + w_0)$$

ÖRN!

$x(t) = e^{-2(t-1)} \cdot u(t-1) \qquad x(jw) = ?$

$$\boxed{x(t \pm t_0) \longrightarrow e^{\pm jwt_0} \cdot X(jw)}$$

FT de time shifting de "u" lu bir fonksiyon var ise "u" içi ile e'nin içi aynı olmak zorundadır

$e^{-at}u(t) \rightarrow \frac{1}{a + jw}$

$e^{-2t}u(t) \rightarrow \frac{1}{2 + jw}$

$$\boxed{e^{jw_0 t} \cdot 1 \xrightarrow{FS} 2\pi\delta(w - w_0)}$$

$e^{-2(t-1)} \cdot u(t-1) \longrightarrow e^{-jw \cdot 1} \cdot \frac{1}{2 + jw}$

ÖRN: $x(t) = e^{-2t} \cdot u(t-1)$ $\quad x(jw) = ?$

$$x(t) = e^{2} \cdot e^{-2(t-1)} \cdot u(t-1)$$

$$e^{-2t} u(t) \xrightarrow{FT} \frac{1}{2+jw}$$

$$e^{-2} \cdot e^{-2(t-1)} \cdot u(t-1) \longrightarrow \frac{1}{2+jw} \cdot e^{-jw \cdot 1} \cdot e^{-2} \qquad x(jw) = \frac{e^{-2} \cdot e^{-jw}}{2+jw}$$

ÖRN: $x(t) = e^{-2|t-1|}$

$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+w^2}$$

$$e^{-2|t|} \xleftrightarrow{FT} \frac{4}{4+w^2}$$

$$e^{-2 \cdot |t-1|} \longleftrightarrow e^{-jw \cdot 1} \cdot \frac{4}{4+w^2}$$

Bu kısımları anlamakta zorluk çekiyorum, 2 neden içeriye dağıtılmıyor.

ÖRN: $\delta(t+1) + \delta(t-1)$

$$\boxed{\delta(t) \xleftrightarrow{FT} 1}$$

$$\delta(t+1) \xleftrightarrow{FT} e^{+jw \cdot 1}$$

$$\delta(t-1) \xleftrightarrow{FT} e^{-jw}$$

$$x(jw) = \frac{(e^{jw} + e^{-jw})}{2} \cdot 2 \qquad x(jw) = 2\cos w$$

ÖRN: $x(t) = \frac{d}{dt}\left[u(-t+2) + u(t-2)\right]$

ÖRN! $X(jw) = \underbrace{2\pi\delta(w)}_{1} + \pi\delta(w-4\pi) + \pi\delta(w+4\pi) \qquad x(t) = ?$

$\frac{1}{2} 2.\pi\delta(w-4\pi) \xleftrightarrow{FT} \frac{1}{2} e^{-j4\pi t}$

$\frac{1}{2} . 2.\pi\delta(w+4\pi) \xleftrightarrow{FT} \frac{1}{2} e^{+j4\pi t}$

$1 + \frac{1}{2} . e^{-j4\pi t} + \frac{1}{2} e^{+j4\pi t}$

$x(t) = 1 + \cos(4\pi t)$

ÖRN: $x_2(t) = x(3t-6)$

$x(a.t) \xleftrightarrow{FS} \frac{1}{|a|} . X\left(\frac{jw}{a}\right)$

$x_2(t) = x(3(t-2))$

$x(3t) = \frac{1}{3} . X\left(\frac{jw}{3}\right)$

$x(3(t-2)) = \frac{1}{3} . X\left(\frac{jw}{3}\right) . e^{-2jw}$

ÖRN! $x_1(t) = x(1-t) + x(-1-t)$

" t nin yanının karmaşık old. durumlarda $(3t+6)(-t+1)$ içerideki "t" her zaman "t" şeklinde bulunmalıdır."

$\boxed{x(-t) = X(-jw)}$

$x(-(t-1)) \leftrightarrow e^{-jw} . X(-jw)$

$x(-(t+1)) \leftrightarrow e^{jw} . X(-jw)$

$X(-jw) . (e^{jw} + e^{-jw})$

ÖRN! $x_3(t) = \frac{d^2}{dt^2}\left[(x[t-1])\right]$

$x(t-1) \leftrightarrow e^{-jw} X(jw)$

$\frac{d}{dt} x(t-1) \longleftrightarrow jw . e^{-jw} . X(jw)$

$\frac{d^2}{dt^2} x(t-1) \longleftrightarrow (jw)^2 . e^{-jw} X(jw)$

ÖRN! $g(t) = x(3t) * h(3t)$

$g(t) = A \cdot Y(Bt)$

$A = ?$ $Bt = ?$

$$\boxed{\begin{aligned} x(t) * Y(t) &\longrightarrow X(jw) \cdot Y(jw) \\ Y(jw) &= X(jw) \cdot H(jw) \end{aligned}}$$

$x(3t) = \frac{1}{3} X\left(\frac{jw}{3}\right)$

$h(3t) = \frac{1}{3} H\left(\frac{jw}{3}\right)$

$g(t) = \frac{1}{9} \underbrace{X\left(\frac{jw}{3}\right) \cdot H\left(\frac{jw}{3}\right)}_{Y\left(\frac{jw}{3}\right) \text{ diyebiliriz}}$

Özelliği! her şeyi 3e bölecek şekilde kullanırsak

$Y\left(\frac{jw}{3}\right) = X\left(\frac{jw}{3}\right) \cdot H\left(\frac{jw}{3}\right)$

$g(t) = \frac{1}{9} \cdot Y\left(\frac{jw}{3}\right)$

$g(t) = \frac{1}{3}\left(\frac{1}{3} Y\left(\frac{jw}{3}\right)\right) \qquad = \frac{1}{3}\left(Y(3t)\right)$

$A = \frac{1}{3}$

$B = 3$

Fourier Trasform ve LTI System

$x(t) \longrightarrow \boxed{\text{LTI Sistem}} \longrightarrow y(t)$

$h(t)$ = Birim dürtü tepkisi

$y(t) = x(t) * h(t)$

$Y(jw) = X(jw) \cdot H(jw)$ $\qquad H(jw) = \frac{Y(jw)}{X(jw)}$

ÖRN! $\left.\begin{aligned} x(t) &= e^{-t} u(t) \\ y(t) &= \delta(t) \end{aligned}\right\} h(t) = ?$

$$\boxed{\delta(t) \xleftrightarrow{FT} 1}$$

$e^{-t} u(t) \Leftrightarrow \frac{1}{jw+1}$

$\delta(t) \longleftrightarrow 1$

$H(jw) = \frac{1}{\frac{1}{jw+1}}$ $\qquad \underbrace{H(jw) = jw + 1 \qquad h(t) = \frac{d}{dt}\delta(t) + \delta(t)}_{\delta(t) \longleftrightarrow 1}$ //

$\frac{d}{dt}\delta(t) \longleftrightarrow 1 \cdot jw$

FT8

ÖRN: $x(t) = e^{-2t}\, u(t)$ $\qquad y(t) = ?$

$h(t) = e^{-3t}\, u(t)$

$$\boxed{e^{-at}\, u(t) = \frac{1}{a + jw}}$$

$$e^{-2t}\, u(t) \leftrightarrow \frac{1}{2 + jw}$$

$$e^{-3t}\, u(t) \leftrightarrow \frac{1}{3 + jw}$$

$$Y(jw) = \frac{1}{jw+2} \cdot \frac{1}{jw+3} = \frac{A}{jw+2} + \frac{B}{jw+3}$$

$(jw+3) \qquad (jw+2)$

$$Ajw + 3A + Bjw + 2B = 1$$

$$jw(A+B) + 3A + 2B = 1$$

$A + B = 0$ $\qquad 3A + 2B = 1$

$A = -B$ $\qquad -3B + 2B = 1$

$\boxed{B = -1}$

$\boxed{A = 1}$

$$Y(jw) = \frac{1}{jw+2} - \frac{1}{jw+3} \qquad y(t) = e^{-2t}\, u(t) - e^{-3t}\, u(t) \; //$$

Sabit Katsayılı Diferansiyel Eşitlikler

$$\frac{d}{dt} x(t) \xleftrightarrow{FT} jw X(jw)$$

ÖRN: $\frac{d}{dt} y(t) + y(t) = x(t)$ $\qquad h(t) = ?$

$$\boxed{\begin{gathered} Y(jw) = X(jw) \cdot H(jw) \\ H(jw) = \frac{Y(jw)}{X(jw)} \end{gathered}}$$

$$jw\, Y(jw) + Y(jw) = X(jw)$$

$$Y(jw)\,(jw+1) = X(jw)$$

$$H(jw) = \frac{Y(jw)}{X(jw)} = \frac{1}{jw+1} \qquad h(t) = e^{-t} \cdot u(t)$$

CTFT FT9

ÖRN!

$$H(jw)=\frac{1}{jw+3} \qquad x(t)=?$$

$$y(t)=e^{-3t}u(t)-e^{-4t}u(t)$$

$$\boxed{\begin{gathered} Y(jw)=H(jw).X(jw) \\ X(jw)=\frac{Y(jw)}{H(jw)} \end{gathered}}$$

$$Y(jw)=\frac{1}{3+jw}-\frac{1}{4+jw}$$

$$X(jw)=\frac{\frac{1}{3+jw}-\frac{1}{4+jw}}{\frac{1}{jw+3}}=jw+3\left(\frac{1}{jw+3}-\frac{1}{jw+4}\right)=1-\frac{jw+3}{jw+4}=\frac{1}{jw+4}$$

$$x(t)=e^{-4t}u(t)$$

ÖRN!

$$\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+3y(t)=\frac{d}{dt}x(t)+2x(t) \qquad h(t)=?$$

$$jw^2Y(jw)+4jwY(jw)+3Y(jw)=jwX(jw)+2X(jw)$$

$$Y(jw)(jw^2+4jw+3)=X(jw)(jw+2)$$

$$H(jw)=\frac{Y(jw)}{X(jw)}=\frac{jw+2}{jw^2+4jw+3}=\frac{jw+2}{(jw+3)(jw+1)}=\frac{A}{\underset{(jw+1)}{jw+3}}+\frac{B}{\underset{(jw+3)}{jw+1}}$$

$$a^2+4a+3$$

$$+a \quad 3$$

$$+a \quad 1$$

$$(a+3)(a+1)$$

$$jw(A+B)+A+3B=jw+2$$

$$-1\ A+B=1$$

$$A+3B=2$$

$$2B=1$$

$$B=\frac{1}{2} \qquad A=\frac{1}{2}$$

$$H(jw)=\frac{1/2}{jw+3}+\frac{1/2}{jw+1}\longleftrightarrow\frac{1}{2}.e^{-3t}.u(t)+\frac{1}{2}e^{-t}u(t)$$

Ayrık Zamanlı F.T.

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jwn}$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) \cdot e^{jwn} \, dw$$

$$\boxed{\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+N_1}}{1-\alpha}}$$

$$\boxed{\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}}$$

ÖRN! $x[n] = a^n \, u[n]$ $|a|<1$ $X(e^{jw}) = ?$

$$X(e^{jw}) = \sum_{-\infty}^{\infty} a^n \overbrace{u[n]}^{} \cdot e^{-jwn}$$

$$X(e^{jw}) = \sum_{n=0}^{\infty} a^n \cdot e^{-jwn} \qquad X(e^{jw}) = \sum_{n=0}^{\infty} (a \cdot e^{-jw})^n$$

$$= \frac{\overbrace{(a \cdot e^{-jw})^0}^{1} - \overbrace{(a \cdot e^{-jw})^{\infty+0}}^{0}}{1 - a \cdot e^{-jw}}$$

$$= \frac{1}{1 - a \cdot e^{-jw}} \quad //$$

ÖRN! $x[n] = a^{|n|}$, $|a|<1$ $X(e^{jw}) = ?$

$$X(e^{jw}) = \sum_{-\infty}^{-1} a^{-n} \cdot e^{-jwn} + \underbrace{\sum_{0}^{+\infty} (a \cdot e^{-jw})^n}_{\frac{1}{1-ae^{-jw}}}$$

$$= \sum_{n=1}^{\infty} a^n \cdot e^{jwn} = \sum_{n=1}^{\infty} (a \cdot e^{jw})^n = \sum_{k=0}^{\infty} (a \cdot e^{jw})^{k+1}$$

$n-1=k$

$$= \sum_{k=0}^{\infty} (a e^{jw})^k \cdot (a \cdot e^{jw}) = a e^{jw} \sum_{k=0}^{\infty} (a e^{jw})^k = \frac{1}{1 - a e^{jw}} \cdot a e^{jw}$$

$$= \frac{1}{\underset{(1-ae^{-jw})}{1-ae^{jw}}} \cdot ae^{jw} + \frac{1}{\underset{(1-ae^{jw})}{1-ae^{-jw}}} = \frac{ae^{jw} - a^2 + 1 - ae^{jw}}{1 - ae^{jw} - ae^{-jw} + a^2}$$

$$= \frac{1-a^2}{a^2 + 1 - a[e^{jw} + e^{-jw}]}$$

$$X(e^{jw}) = \frac{1-a^2}{a^2 + 1 - a(2\cos w)} \quad //$$

FT 11

ÖRN: $x[n] = \delta[n] \qquad X(e^{jw}) = ?$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} \delta[n] \cdot e^{-jwn}$$ Sadece 0 değerinde 1 $= \delta[0] \cdot e^{0}$

$$X(e^{jw}) = 1 \cdot 1 = 1$$

$$\boxed{\delta(n) \xrightarrow{FT} 1}$$

ÖRN:

$x[n] = ?$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) \cdot e^{jwn} \, dw$$

* Grafikte sadece -w +w arası değer old. dan aralıkları ona göre değiştiriyoruz

$$x[n] = \frac{1}{2\pi} \int_{-w}^{w} \underbrace{X(e^{jw})}_{1} \cdot e^{jwn} \, dw = \frac{1}{2\pi} \int_{-w}^{w} 1 \cdot e^{jwn} \, dw = \frac{1}{2\pi} \cdot \frac{e^{jwn}}{jn} \Bigg|_{-w}^{w}$$

$$x[n] = \frac{1}{2\pi} \cdot \frac{e^{jwn} - e^{-jwn}}{jn} = \frac{1}{\pi n} \cdot \frac{e^{jwn} - e^{-jwn}}{2j} = \frac{1}{\pi n} \cdot \sin(wn)$$

DTFT Özellikler

Linearlik

$$x[n] \rightarrow X(e^{jw})$$
$$y[n] \rightarrow Y(e^{jw})$$
$$Ax[n]+By[n] \rightarrow AX(e^{jw})+BY(e^{jw})$$

Zaman ve Frekans Kaydırma

$$x[n \mp n_0] \longrightarrow e^{\mp jwn_0} \cdot X(e^{jw})$$
$$e^{\mp jw_0 n} x[n] \longrightarrow X(e^{j(w \pm w_0)})$$

Zamanda Tersleme

$$x[n] \rightarrow X(e^{jw})$$
$$x[-n] \rightarrow X(e^{-jw})$$

Türev

$$n x[n] \longrightarrow \frac{jd}{dw} \cdot X(e^{jw})$$

Convolüsyon

$$x[n] * y[n] \longrightarrow X(e^{jw}) \cdot Y(e^{jw})$$

~~Carpım~~ Carpım

$$x[n] \cdot y[n] \longrightarrow \frac{1}{2\pi} \cdot \left[X(e^{jw}) * Y(e^{jw}) \right]$$

Parseval

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{jw})|^2 \cdot dw$$

ÖRN! $x[n] = \cos w_0 n \qquad X(e^{jw}) = ?$

$$= \frac{e^{jw_0 n} + e^{-jw_0 n}}{2}$$

$$\boxed{e^{jw_0 n} \xrightarrow{F} 2\pi \sum_{l=-\infty}^{\infty} \delta(w - w_0 - 2\pi l)}$$

$$\frac{1}{2} \cdot e^{jw_0 n} \longrightarrow \frac{1}{\cancel{2}} \cdot \cancel{2}\pi \sum_{l=-\infty}^{\infty} \delta(w - w_0 - 2\pi l)$$

$$\frac{1}{2} \cdot e^{-jw_0 n} \longrightarrow \frac{1}{\cancel{2}} \cdot \cancel{2}\pi \sum_{l=-\infty}^{\infty} \delta(w + w_0 - 2\pi l)$$

$$X(e^{jw}) = \pi \left[\sum_{-\infty}^{\infty} \delta(w + w_0 - 2\pi l) + \sum_{-\infty}^{\infty} \delta(w - w_0 - 2\pi l) \right] //$$

ÖRN! $x[n] = \sin w_0 n \qquad X(e^{jw}) = ?$

$$= \frac{e^{jw_0 n} - e^{-jw_0 n}}{2j}$$

$$= \frac{1}{2j} \cdot e^{jw_0 n} \longrightarrow \frac{1}{\cancel{2}j} \cancel{2}\pi \sum_{-\infty}^{\infty} \delta(w - w_0 - 2\pi l)$$

$$= \frac{1}{2j} \cdot e^{-jw_0 n} \longrightarrow \frac{1}{\cancel{2}j} \cancel{2}\pi \sum_{-\infty}^{\infty} \delta(w + w_0 - 2\pi l)$$

$$X(e^{jw}) = \frac{\pi}{j} \left[\sum_{-\infty}^{\infty} \delta(w + w_0 - 2\pi l) + \sum_{-\infty}^{\infty} \delta(w - w_0 - 2\pi l) \right] //$$

ÖRN! $x[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1] \qquad X(e^{jw}) = ?$

$$\left(\frac{1}{2}\right)^n u[n] \longrightarrow \frac{1}{1 - \frac{1}{2} e^{-jw}}$$

$$x[n \mp n_0] \longrightarrow X(e^{jw}) \cdot e^{\mp jwn_0}$$

$$\left(\frac{1}{2}\right)^{n-1} u[n-1] \longrightarrow \frac{1}{1 - \frac{1}{2} e^{-jw}} \cdot e^{-jw1}$$

ÖRN! $x[n] = \left(\frac{1}{2}\right)^n \cdot u[n-1] \qquad X(e^{jw}) = ?$

$$= \underbrace{\left(\frac{1}{2}\right)^1}_{} \cdot \underbrace{\left(\frac{1}{2}\right)^{n-1} u[n-1]}_{}$$

$$= \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{2} e^{-jw}} \cdot e^{-jw}$$

$$X(e^{jw}) = \frac{e^{-jw}}{2\left(1 - \frac{1}{2} e^{-jw}\right)}$$

ÖRN! $x[n]=\left(\frac{1}{2}\right)^{|n-1|}$ $\quad X(e^{jw})=?$

$$a^{|n|} \longrightarrow \frac{1-a^2}{1-2a\cos w+a^2}$$

$$\left(\frac{1}{2}\right)^{|n|} \longrightarrow \frac{\frac{3}{4}}{\frac{5}{4}-\cos w}$$

$$\left(\frac{1}{2}\right)^{|n-1|} \longrightarrow e^{-jw}\cdot\frac{3/4}{5/4-\cos w}$$

ÖRN! $\delta[n-1]+\delta[n+1]$ $\quad X(e^{jw})=?$

$$\boxed{\delta[n] \xrightarrow{FT} 1}$$

$$\delta[n-1] \longrightarrow 1\cdot e^{-jw}$$
$$\delta[n+1] \longrightarrow 1\cdot e^{jw}$$

$$X(e^{jw})=2\frac{e^{-jw}+e^{jw}}{2}=2\cos w$$

ÖRN! $\delta[n+2]-\delta[n-2]$ $\quad X(e^{jw})=?$

$$\delta[n+2] \longrightarrow 1\cdot e^{2jw}$$
$$\delta[n-2] \longrightarrow 1\cdot e^{-2jw}$$

$$X(e^{jw})=2j\frac{e^{2jw}-e^{-2jw}}{2j}=2j\sin 2w$$

ÖRN! $x_1[n]=x[1-n]+x[-1-n]$ $\quad X(e^{jw})=?$

$$\left.\begin{aligned} x[-(n-1)] &\longrightarrow X(e^{-jw})\cdot e^{-jw} \\ x[-(n+1)] &\longrightarrow X(e^{-jw})\cdot e^{jw} \end{aligned}\right\} X(e^{jw})=\frac{X(e^{-jw})(e^{jw}+e^{-jw})2}{2}=X(e^{-jw})2\cos w$$

ÖRN! $x_2[n]=(n-1)^2x[n]$

$$=(n^2-2n+1)x[n]$$

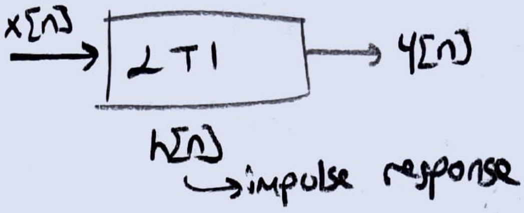
$$=n^2x[n]-2nx[n]+x[n]$$

$$=\frac{j^2d^2}{dw^2}X(e^{jw})-2\frac{jd}{dw}\cdot X(e^{jw})+X(e^{jw})$$

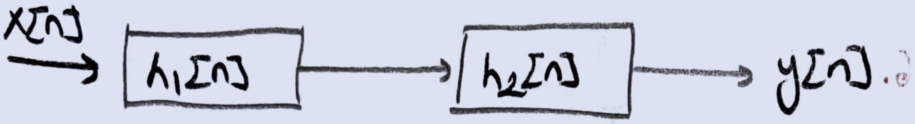
DTFT LTI SYSTEM

$$y[n] = h[n] * x[n]$$

$$Y(e^{jw}) = H(e^{jw}) \cdot X(e^{jw})$$



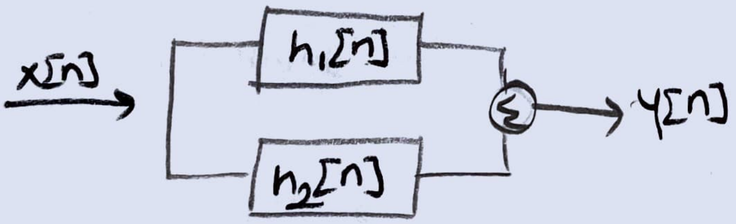
* 2 adet h[n] ve seri bağlılar ise



$$h[n] = h_1[n] * h_2[n]$$

$$y[n] = x[n] * \left[h_1[n] * h_2[n] \right]$$

* 2 adet h[n] ve paralel bağlılar ise



$$h[n] = h_1[n] + h_2[n]$$

$$y[n] = x[n] * \left(h_1[n] + h_2[n] \right)$$

$$y[n] = x[n] * h_1[n] + x[n] * h_2[n]$$

ÖRN: $a\,y[n-1] + y[n] = x[n]$ $\quad H(e^{jw}) = ?$ $\quad h[n] = ?$

$$a \cdot Y(e^{jw}) \cdot e^{-jw} + Y(e^{jw}) = X(e^{jw})$$

$$Y(e^{jw}) \left(a e^{-jw} + 1 \right) = X(e^{jw})$$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{1}{a e^{-jw} + 1}$$

$$h(t) = a^n u[n]$$

FT16

ÖRN: $y[n]-\frac{1}{6}y[n-1]-\frac{1}{6}y[n-2]=x[n]$ $\quad H(e^{jw})=?$ $\quad h(t)=?$

$$Y(e^{jw})-\frac{1}{6}Y(e^{jw}).e^{-jw}-\frac{1}{6}Y(e^{jw}).e^{-2jw}=X(e^{jw})$$

$$Y(e^{jw})\left(-\frac{1}{6}.e^{-jw}-\frac{1}{6}.e^{-2jw}+1\right)=X(e^{jw})$$

$$H(e^{jw})=\frac{Y(e^{jw})}{X(e^{jw})}=\frac{6.\,1}{6.\left(1-\frac{1}{6}e^{-jw}-\frac{1}{6}e^{-2jw}\right)}$$

$$=\frac{6}{6-e^{-jw}-e^{-2jw}}=\frac{6}{-x^2-x+6}=\underbrace{\frac{6}{(2-e^{-jw})(e^{jw}+3)}}$$

$x=e^{-jw}$

$-x^2-x+6$: $\; -x \;\; 2$, $\; x \;\; 3$

$(-x+2)(x+3)$

$a^n u[n] \Longrightarrow \frac{1}{1-ae^{-jw}}$

Benzetilmelidir !

$$=\frac{\cancel{6}}{\cancel{2}\left(1-\frac{1}{2}e^{-jw}\right)\cancel{3}\left(1+\frac{1}{3}e^{jw}\right)}$$

$$H(e^{jw})=\frac{1}{\left(1-\frac{1}{2}e^{jw}\right)\left(1+\frac{1}{3}e^{jw}\right)}=\frac{A}{1-\frac{1}{2}e^{jw}}+\frac{B}{1+\frac{1}{3}e^{jw}}$$

$$=A+B+e^{-jw}\left(\frac{1}{3}A-\frac{1}{2}B\right)=1$$

$6/\ \frac{1}{3}A-\frac{1}{2}B=0 \qquad A+B=1$

$$=\frac{3/5}{1-\frac{1}{2}e^{-jw}}+\frac{2/5}{1+\frac{1}{3}e^{-jw}}$$

$2A-3B=0$

$-2/\ A+B=1$

$-5B=-2$

$B=\frac{2}{5} \qquad A=\frac{3}{5}$

$$h[n]=\frac{3}{5}\cdot\left(\frac{1}{2}\right)^n u[n]+\frac{2}{5}\left(-\frac{1}{3}\right)^n u[n]$$

37. Vid 18 dk

FTA.

ÖRN! $\underbrace{\left(\frac{4}{5}\right)^n u[n]}_{\text{Sistem Girişi } x[n]} \longrightarrow \underbrace{n\left(\frac{4}{5}\right)^n u[n]}_{\text{Sistem çıkışı } y[n]}$ a) $H(e^{jw})$ b) Denklem olarak yazın

$$\boxed{a^n u[n] \rightarrow \frac{1}{1-ae^{-jw}}}$$

$$\frac{Y(e^{jw})}{X(e^{jw})} = \frac{\frac{1}{1-\frac{4}{5}e^{-jw}}\,\frac{jd}{dw}}{\frac{1}{1-\frac{4}{5}e^{-jw}}} =$$

ÖRN: $X(e^{jw}) = \frac{1-\frac{1}{3}e^{-jw}}{\left(1-\frac{1}{4}e^{-jw}-\frac{1}{8}e^{-2jw}\right)}$ $x[n]=?$

$-x^2-2x+8$
$-x$ $\quad -4$
$+x$ $\quad -2$
$(-x-4)(x-2) = -x^2+2x-4x+8$

$$= \frac{8-2e^{-jw}-e^{-2jw}}{8} \qquad e^{-jw}=x \qquad \frac{-x^2-2x+8}{8} = \frac{(-x-4)(x-2)}{8}$$

$$\boxed{a^n u[n] \rightarrow \frac{1}{1-ae^{-jw}}}$$

$$\frac{(-e^{-jw}-4)(e^{-jw}-2)}{8} = \frac{\cancel{-4}\left(1+\frac{1}{4}e^{-jw}\right)\cancel{-2}\left(1+\frac{1}{2}e^{-jw}\right)}{\cancel{8}}$$

$$X(e^{jw}) = \frac{1-\frac{1}{3}e^{-jw}}{\left(1+\frac{1}{4}e^{-jw}\right)\left(1+\frac{1}{2}e^{-jw}\right)} = \frac{A}{1+\frac{1}{4}e^{-jw}} + \frac{B}{1-\frac{1}{2}e^{-jw}}$$

$\left(1-\frac{1}{2}e^{-jw}\right) \qquad \left(1+\frac{1}{4}e^{-jw}\right)$

? A ve B yeri yanlış gibi

$$A+B+e^{-jw}\left(-\frac{1}{2}A+\frac{1}{4}B\right) = 1-\frac{1}{3}e^{-jw}$$

6/ $A+B=1$

$-6A+3B=-4$

$9B=2$

$B=2/9 \quad A=7/9$

$$X(e^{jw}) = \frac{7/9}{1+\frac{1}{4}e^{-jw}} + \frac{2/9}{1-\frac{1}{2}e^{-jw}}$$

38. Video

$$x[n] = \frac{7}{9}\left(\frac{-1}{4}\right)^n u[n] + \frac{2}{9}\left(\frac{1}{2}\right)^n u[n]$$