$y(t) = x(2t) \quad = ?$

Zamanda değişir?

$y_1(t) = x_1(2t)$

$x_1(t) \rightarrow \boxed{\quad} \rightarrow y_1(t)$

$\boxed{t = t - t_0}$

$x_2(t) \rightarrow \boxed{\quad} \rightarrow y_2(t)$ Test

$x_1(t - t_0)$

$y_1(t_1 - t_0) = \underline{x_1(2t - 2t_0)}$

$x_2(t) = x_1(t - t_0)$

$t = 2t$

$x_2(2t) = x_1(2t - t_0)$

$y_2(t) = \underline{x_1(2t - t_0)}$

$y_2(t) = y_1(t - t_0)$

$x_1(2t - t_0) \neq x_1(2t - 2t_0)$

$=$ zamanda değişir

6) Doğrusallık (Linearity)

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

Doğrusallık için toplamsallık ve homojenlik ilkelerini sağlamalı

1- Toplamsallık İlkesi

$[x_1(t) \mp x_2(t)] = [y_1(t) \mp y_2(t)]$

2- Homojenlik İlkesi

$x_1[n] \rightarrow y_1[n]$

$x_2[n] \rightarrow y_2[n]$

$a . x_1[n] \longrightarrow a . x_2[n]$

$b . x_2[n] \longrightarrow b . x_2[n]$

$y(t) = t \cdot x(t)$

$x_1(t) \rightarrow y_1(t) = t\, x_1(t)$

$x_2(t) \rightarrow y_2(t) = t \cdot x_2(t)$

$x_3(t) = a.x_1(t) + b.x_2(t)$

$x_3(t) \rightarrow y_3(t) = t.\, x_3(t)$

$y_3(t) = t.[a.x_1(t) + b.x_2(t)]$

$y_3(t) = \boxed{a.t.\, x_1(t) + bt.\, x_2(t)}$

$a.x_1(t) + b\,x_2(t) \rightarrow y(t) = ?$

$y_1(t) = t.\, x_1(t)$

$a.y_1(t) = a\,t.\, x_1(t)$

$y_2(t) = t.\, x_2(t)$

$b.y_2(t) = b.t.\, x_2(t)$

$y(t) = a.y_1(t) + b.y_2(t)$

$\boxed{= a\,t\, x_1(t) + bt\, x_2(t)}$

Eşit Old. Lineer Sistem
Doğrusal

?

$y[n] = 2x[n] + 3$

Doğrusallık?

$x_1 \rightarrow y_1[n] = 2\, x_1[n] + 3$

$x_2 \rightarrow y_2[n] = 2.x_2[n] + 3$

$a.y_1[n] = 2a x_1[n] + 3a$

$+\ b y_2[n] = 2.b x_2[n] + 3b$

$a y_1[n] + b.y_2[n] = 2a.x_1[n] + 2b\, x_2[n] + 3a + 3b$

$x_3[n] = a.x_1[n] + b\, x_2[n]$

$y_3[n] = 2x_3[n] + 3$

$= 2a x_1[n] + 2b\, x_2[n] + 3$

Eşit Değil

Doğrusal Değil

y t çıkış

giriş

Doğrusal zamanda değişmeyen

25.10.2022
5. Hafta

LTI SİSTEMLER

1) DT LTI Sistemler

$$x[-1]\cdot\delta[n+1]=\begin{cases}x[-1], & n=-1\\ 0, & n\neq -1\end{cases}$$

$$\downarrow$$

$$x[n]$$

$$x[0]\cdot\delta[n]=\begin{cases}x[0], & n=0\\ 0, & n\neq 0\end{cases}$$

$$x[n]=\ldots+x[-1]\cdot\delta[n+1]+\ldots+x[0]\cdot\delta[n]+x[1]\cdot\delta[n-1]$$

$$x[n]=\sum_{k=-\infty}^{\infty}x[k]\cdot\delta[n-k]$$

Sonsuz Toplam Formülü

$$\sum_{n=0}^{\infty}\alpha^n=\frac{1}{1-\alpha},\quad |\alpha|<1$$

Sonlu Toplam Formülü

$$\sum_{n=0}^{N-1}\alpha^n=\frac{1-\alpha^N}{1-\alpha},\quad \alpha\neq 1$$

$$x[n]=u[n]$$

$$u[n]=\sum_{k=0}^{\infty}\delta[n-k]$$

$$x[n]=\sum_{k=-\infty}^{\infty}x[k]\,\delta[n-k]$$

01.11.2022
6. Hafta

$\delta[n], \delta(t) \rightarrow \boxed{LTI} \rightarrow h[n], h(t)$

Sistemin birim dürtüye verdiği tepkiye, birimdürtü tepkisi denir.

2) ~~DT~~, CT, LTI Sistemler

integral τ ya göre alınıyor

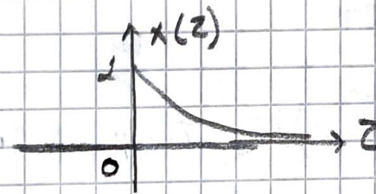
$$x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau$$

$$u(t) = \int_{0}^{\infty} \delta(t-\tau)\, d\tau$$

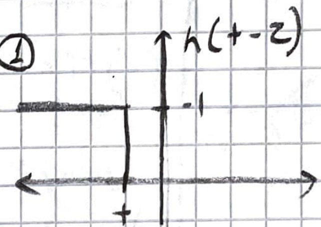
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = x(t) * h(t)$$

Ör $x(t) = e^{-at}\, u(t), \quad a>0$

$h(t) = u(t), \quad y(t) = ?$ → Çıkış konvolüsyon ile bulunabilir $x(t) * h(t) = ?$



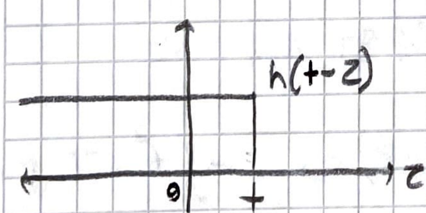
1



$t<0$ için

$y(t) = 0$

2



$t \geq 0$ için

1/ Durumunda grafiklerde çakışma olmadığı için integral aralığı 0'dan başlıyor

2/ durumunda çakışma var

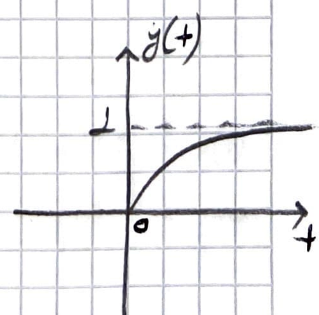
$$y(t) = \int_{0}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

$x(\tau) = e^{-a\tau}, \quad h(t-\tau) = 1$

$$y(t) = \int_{0}^{t} e^{-a\tau} d\tau = -\frac{1}{a} e^{-a\tau}\Big|_{0}^{t} = -\frac{1}{a}\left(e^{-at} - 1\right)$$

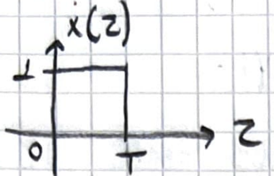
$$y(t) = \frac{1}{a}\left(1 - e^{-at}\right) \cdot u(t)$$

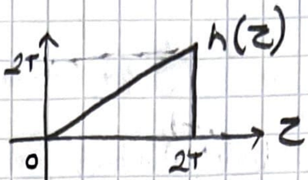
0 dan önce ve sonra ayrı old. için basamak sinyali u(t) ile iki ayrı değer yazmak zorunda kalın

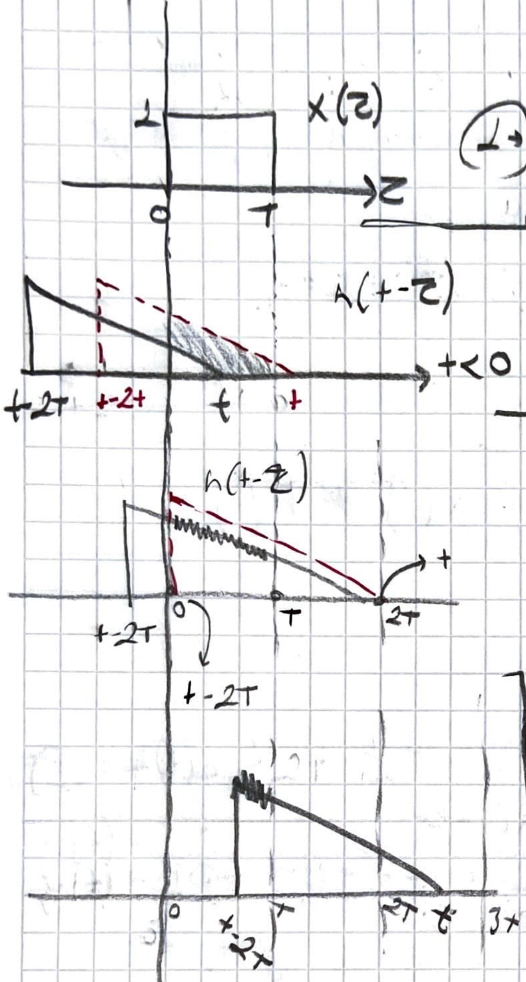


Grafik Çizme ?

ör: $x(t)=\begin{cases} 1, & 0<t<T \\ 0, & \text{diğer} \end{cases}$ $\qquad h(t)=\begin{cases} t, & 0<t<2T \\ 0, & \text{diğer} \end{cases}$







1) $t<0$ için

$y(t)=0$

2) $0<t<T$ için

$$y(t)=\int_0^t x(\tau).h(t-\tau)\,d\tau$$

$$=\int_0^t 1.(t-\tau).d\tau$$

$$=t\tau-\frac{\tau^2}{2}\Big|_0^t=\frac{1}{2}t^2$$

3) $T<t<2T$ için

$$y(t)=\int_0^T (t-\tau)d\tau = T.t-\frac{1}{2}T^2$$

4) $2T<t<3T$ için

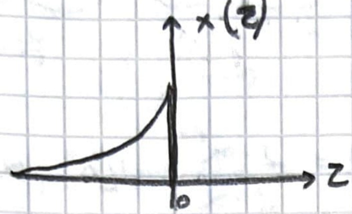
$$y(t)=\int_{t-2T}^{T} t-\tau)\,d\tau = -\frac{1}{2}t^2+T.t+\frac{3}{2}T^2$$

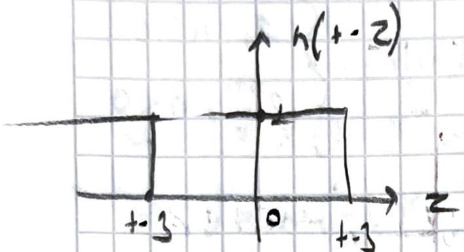
5) $t>3T$ için

$y(t)=0$

$$y(t)=\begin{cases} 0, & t<0 \\ \frac{1}{2}t^2, & 0<t<T \\ \frac{1}{2}t^2, & T<t<2T \\ \frac{1}{2}t^2 & 2T<t<3T \\ 0 & t>3T \end{cases}$$

$$x(t) = e^{2t}\,u(-t) \quad , \quad h(t) = u(t-3) \quad , \quad y(t) = ?$$



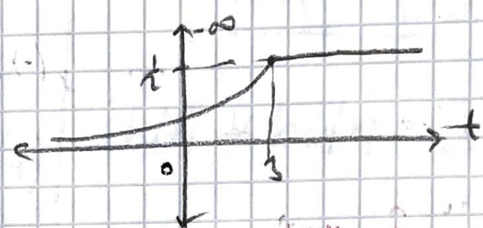


① $t-3<0$ için

$$y(t) = \int_{-\infty}^{t-3} e^{2\tau}\cdot 1 \cdot d\tau = \frac{1}{2} e^{2(t-3)}$$

② $t-3>0$ için

$$y(t) = \int_{-\infty}^{0} e^{2\tau}\, d\tau = \frac{1}{2}$$



LTI Sistemlerin Özellikleri

1- Değişme Özelliği

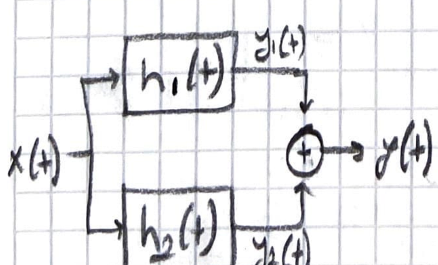
$$x[n] * h[n] = h[n] * x[n]$$

$$\underbrace{\sum_{k=-\infty}^{\infty} x[k].h[n-k]}_{r=n-k} = \sum_{k=-\infty}^{\infty} h[k].\,x[n-k]$$

$$\sum_{r=\infty}^{-\infty} x[n-r]\; h[r]$$

2- Dağılma Özelliği

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$



$$y_1(t) = x(t) * h_1(t)$$

$$y_2(t) = x(t) * h_2(t)$$

$$y(t) = y_1(t) + y_2(t) = x(t) * h_1(t) + x(t) * h_2(t)$$

$$x(t) \longrightarrow \boxed{h_1(t) + h_2(t)} \longrightarrow y(t)$$

$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n]$, $h[n] = u[n]$, $y[n] = ?$

$$y[n] = x[n] * h[n] = (x_1[n] + x_2[n]) * h[n] = \underbrace{x_1[n] * h[n]}_{y_1[n]} + \underbrace{x_2[n] * h[n]}_{y_2[n]}$$

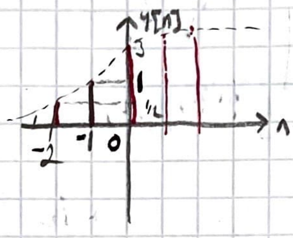
$x[n] = x_1[n] + x_2[n]$

$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$

$x_2[n] = 2^n u[-n]$

$$y_1[n] = \left(\frac{1}{2}\right)^n u[n] * u[n] = \frac{1-\alpha^{n+1}}{1-\alpha} u[n], \quad \alpha = \frac{1}{2}$$

$$y_2[n] = 2^n . u[-n] * u[n] = \begin{cases} 2, & n \geq 0 \\ 2^{n+1}, & n < 0 \end{cases}$$



3- Birleşme Özelliği

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

4- Bellekli ve Belleksiz LTI sistemler

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] . h[n-k]$$

$$h[n] = \begin{cases} 0, & n \neq 0 \\ k, & n = 0 \end{cases} \qquad h(t) = \begin{cases} 0, & t \neq 0 \\ k, & t = 0 \end{cases}$$

$$h[n] = k . \delta[n]$$

$$h(t) = k . \delta(t)$$

5- LTI sistemlerin Tersine Çevrilebilirliği

$$x(t) \rightarrow \boxed{h(t)} \xrightarrow{w(t)} \boxed{h_1(t)} \rightarrow x(t) = y(t)$$

$$x(t) \rightarrow \boxed{\delta(t)} \rightarrow x(t)$$

$$w(t) = x(t) * h(t)$$
$$y(t) = w(t) * h_1(t)$$
$$y(t) = x(t) * h(t) * h_1(t)$$

$$x(t) \rightarrow \boxed{h(t) * h_1(t)} \rightarrow y(t) = x(t)$$

$h(t) * h_1(t) = \delta(t)$ olmalıdır

Soru Gelebilir

6- LTI Sistemlerin Nedenselliği

$h(t) = 0$, $t < 0$ iken
$x(t) = 0$, $t < 0$ iken
sistem nedenseldir

$h[n] = 0$, $n < 0$
$x[n] = 0$, $n < 0$

$$\underbrace{y[n]}_{\text{Şimdiki Zaman}} = \underbrace{2x[n]}_{\text{Şimdi}} - \underbrace{y[n-1]}_{\text{Geçmiş}} + \underbrace{y[n-2]}_{\text{Geçmiş}}$$

$$\underbrace{y[n+1]}_{\text{Gelecek}}$$

7- LTI Sistemlerin Kararlılığı

$|x[n]| < B$ varsayalım

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k] \cdot x[n-k] \right|$$

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]| \cdot \underbrace{|x[n-k]|}_{<B}$$

15.11.2022
8. Hafta

$$|y[n]| \leq B \sum_{k=-\infty}^{\infty} |h[k]|$$

$$\sum |h[k]| < \infty \text{ olmalı}$$

4- Fark Denklemleri ve Dif. Denk. Tanımlanan LTI Sistemler

4.1 = Lineer Sabit Katsayılı Dif. Denklemler

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$$x(t) = K e^{3t} u(t)$$

$$\underbrace{y_h(t)}_{\text{Homojen çözüm}} + \underbrace{y_p(t)}_{\text{özel çözüm}} = y(t)$$

$\underbrace{\text{Homojen çözüm}}$
Doğal Tepki
= Giriş $\varnothing$ olduğunda çıkış

$\underbrace{\text{özel çözüm}}$
zorlanmış tepki

$y_p(t)$ denklemi sağlamalıdır

$$y_p(t) = A \cdot e^{3t} = \frac{K}{5} e^{3t}$$

$$\dot{y}_p + 2y_p = K \cdot e^{3t}, \quad t>0$$

$$3A \cdot e^{3t} + 2 \cdot Ae^{3t} = K \cdot e^{3t}$$

$$5A \cdot \cancel{e^{3t}} = K \cdot \cancel{e^{3t}} \Rightarrow A = \frac{K}{5}$$

$x(t)=0$ iken

$$\dot{y}_h + 2y = x = 0$$

$$y_h(t) = B \cdot e^{st}$$

$$sBe^{st} + 2Be^{st} = 0$$

$$e^{st}(s+2)B = 0$$

$$s = -2$$

$$y_h(t) = B \cdot e^{-2t}$$

$$-2Be^{-2t} + 2Be^{-2t} = 0$$

$$y(t) = y_h(t) + y_p(t)$$

$$y(t) = Be^{-2t} + \frac{K}{5} e^{3t}, \quad t>0$$

$$y(t=0) = 0 = Be^{0} + \frac{K}{5} e^{0} = B + \frac{K}{5}$$

$$B = -\frac{K}{5}$$

$$y(t) = \begin{cases} -\frac{K}{5} e^{-2t} + \frac{K}{5} e^{3t} & t>0 \\ 0 & t<0 \end{cases}$$

Genelleştirilmiş Durum

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

Örnek

$$3\ddot{y}(t) - 2\dot{y}(t) - y(t) = -\dot{x}(t) + x(t)$$

4.2 Sabit Katsayılı Lineer Fark Denklemleri

$$\sum_{k=0}^{N} a_k \cdot y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = y_h[n] + y_p[n]$$

$$y[n] = ? \quad \underbrace{y[n-1], \; y[n-2], \ldots, y[n-N]}_{\text{Bilinmeli}}$$

Nedensel bir sistemde hepsi sıfırdır

$$y[n] = \frac{1}{a_0}\left\{ \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k [n-k] \right\}, \quad N \neq 0 \text{ ise}$$

$$y[n] = \frac{b_k}{a_0} \sum_{k=0}^{M} x[n-k], \quad N=0 \text{ ise}$$

$N=0 \Rightarrow$ Yinelemeli olmayan denklemler

$$x[n] = \delta[n] \longrightarrow h[n]$$

$$h[n] = \begin{cases} \frac{b_n}{a_0}, & 0 \leq n \leq M \\ 0, & \text{diğer} \end{cases}$$

FIR : Finite Impulse Response

Sonlu Birim dürtü yanıtı

$$y[n] = x[n] + \frac{1}{2} y[n-1]$$

$x[n] = K \cdot \delta[n]$ varsayalım $\quad n<0$ için, $x[n]=0$, $y[n]=0$

n'e farklı değerler vererek çözülebilir

$n=0 \rightarrow y[0] = x[0] + \frac{1}{2} y[-1] = k + \frac{1}{2} \cdot 0 = k$

$n=1 \rightarrow y[1] = x[1] + \frac{1}{2} y[0] = 0 + \frac{1}{2} k = \frac{1}{2} k$

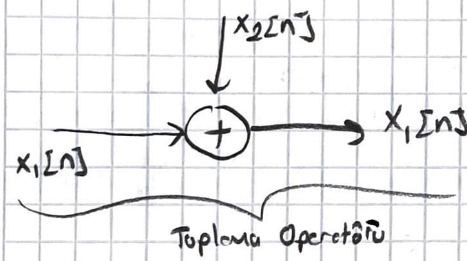
$n=2 \rightarrow y[2] = x[2] + \frac{1}{2} y[1] = 0 + \frac{1}{2} \cdot \frac{1}{2} k = \left(\frac{1}{2}\right)^2 k$

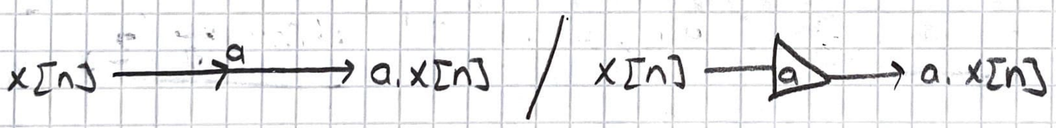
$n=3 \rightarrow y[3] = x[3] + \frac{1}{2} y[2] = 0 + \frac{1}{2} y[2] = \left(\frac{1}{2}\right)^3 k$

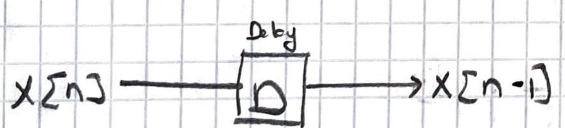
$y[n] = \left(\frac{1}{2}\right)^n k$

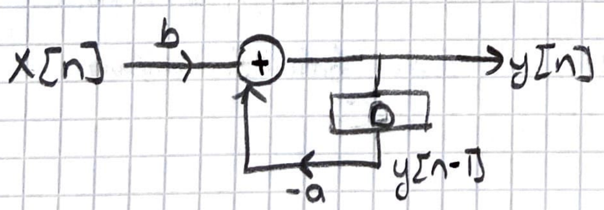
yinelemeli denklemler ile ifade edilen sistemler sonsuz birim dürtü yanıtlı sistemlerdir. (IIR)
infinite impulse response

4.3 Birinci Dereceden Fark / Diferansiyel Denklemler ile tanımlanmış CT/DT LTI sistemlerin Blok Diyagramları







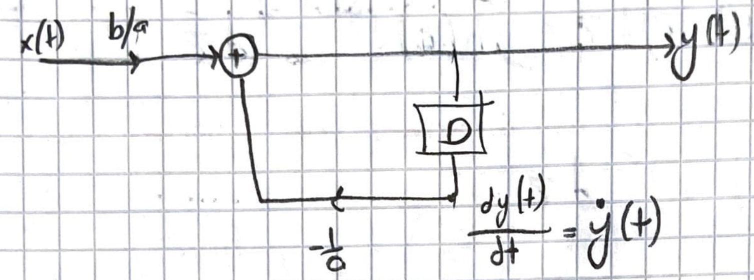


$b.x[n] - a.y[n-1] = y[n]$

$$x(t) \longrightarrow \boxed{D} \longrightarrow \frac{dx(t)}{dt}$$

$$\frac{dy(t)}{dt} + a \cdot y(t) = b \cdot x(t)$$

$$y(t) = \frac{b}{a} x(t) - \frac{1}{a} \frac{dy(t)}{dt}$$



$$\frac{dy(t)}{dt} + ay(t) = b \cdot x(t)$$

$$\cancel{\int} dy(t) = \int [b \cdot x(t) - a \cdot y(t)]\, dt$$

$$y(t) = \int_0^t b x(\tau)\, d\tau - a \int_0^t y(\tau)\, d\tau = \int_0^t (bx(\tau) - ay(\tau))\, d\tau$$