Fourier Serileri

katsayı) $a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{jk\omega_0 t}$$

$$\cdots a_{-2} \cdot e^{-2j\omega_0 t} + a_{-1} e^{-j\omega_0 t} + a_0 \cdot 1 + a_1 e^{j\omega_0 t} + a_2 e^{2j\omega_0 t}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\omega_0 = \frac{2\pi}{T}$$

T = Temel periyot

ÖRN1 $x(t) = \sin\omega_0 t$ Katsayılar?

$$x(t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t}$$

$a_1 = \frac{1}{2j}$, $a_{-1} = -\frac{1}{2j}$, Diğer durumlarda katsayılar 0 dır.

ÖRN!

$$x(t) = 1 + \sin(\omega_0 t) + 2\cos\omega_0 t + \cos\left(2\omega_0 t + \frac{\pi}{4}\right)$$

$1 \to a_0 = 1$

$\sin(\omega_0 t) \to a_1 = \frac{1}{2j}$ $a_{-1} = \frac{-1}{2j}$

$$2\cos\omega_0 t = \cancel{2}\left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{\cancel{2}}\right) = e^{j\omega_0 t} + e^{-j\omega_0 t}$$

$a_1 = 1$ $a_{-1} = 1$

$$\cos\left(2\omega_0 t + \frac{\pi}{4}\right) = \frac{e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})}}{2}$$

$$= \frac{e^{j\frac{\pi}{4}}}{2} \cdot e^{j2\omega_0 t} + \frac{e^{-j\frac{\pi}{4}}}{2} \cdot e^{-j2\omega_0 t}$$

$a_2 = \frac{e^{j\frac{\pi}{4}}}{2}$ $a_{-2} = \frac{e^{-j\frac{\pi}{4}}}{2}$

$a_0 = 1$

$a_1 = \frac{1}{2}j + 1$

$a_{-1} = -\frac{1}{2}j + 1$

$a_2 = \frac{e^{j\frac{\pi}{4}}}{2}$

$a_{-2} = \frac{e^{-j\frac{\pi}{4}}}{2}$

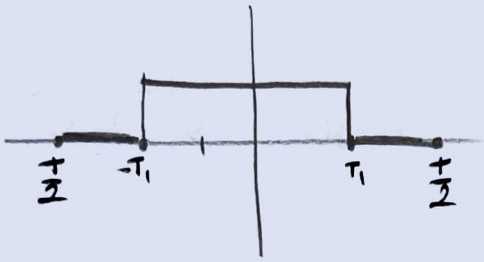
ÖRN!

$$x(t) = \begin{cases} 1 , & |t| < T_1 \\ 0 , & T_1 < |t| < \frac{T}{2} \end{cases}$$

$a_k = ?$

$1 , \quad |t| < T_1 \rightarrow -T_1 < t < T_1$

$0 , \quad T_1 < |t| < \frac{T}{2} \rightarrow -\frac{T}{2} < t < \frac{T}{2}$



$$\boxed{a_k = \frac{1}{T} \int_T x(t)\, e^{-jkW_0 t}\, dt}$$

$$a_k = \frac{1}{T}\left(\cancel{\int_{-\frac{T}{2}}^{-T_1} 0 . e^{-jkW_0 t}\, dt} + \int_{-T_1}^{T_1} 1 . e^{-jkW_0 t}\, dt + \cancel{\int_{-T_1}^{\frac{T}{2}} 0 . e^{-jkW_0 t}\, dt} \right)$$

$$a_k = \frac{1}{T} \cdot \int_{-T_1}^{\frac{T}{2}} e^{-jkW_0 t}\, dt \quad = \frac{1}{T}\left(\frac{-1}{jkW_0} \cdot \left(e^{-jkW_0 T_1} - e^{jkW_0 T_1} \right)\right)$$

$$= \frac{1}{jkW_0 T} \cdot \left(e^{jkW_0 T_1} - e^{-jkW_0 T_1} \right) = \frac{2}{kW_0 T}\left(\frac{e^{jkW_0 T_1} - e^{-jkW_0 T_1}}{2j} \right) = \underline{\frac{1}{kW_0 T} \cdot \sin(kW_0 T_1)}$$

ÖRN!

$$x(t) = \sum_{k=-3}^{3} a_k e^{jk2\pi t} \qquad a_0 = 1 ,\ a_1 = a_{-1} = \frac{1}{4} ,\ a_2 = a_{-2} = \frac{1}{2} ,\ a_3 = a_{-3} = \frac{1}{3}$$

$$= a_{-3}\, e^{-j6\pi t} + a_{-2}\, e^{-j4\pi t} + a_{-1}\, e^{-j\pi t} + a_0\, 1 + a_1\, e^{j2\pi t} + a_2\, e^{j4\pi t} + a_3\, e^{j6\pi t}$$

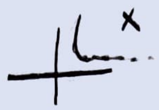
$$x(t) = \frac{1}{3} \cdot \left(e^{-j6\pi t} + e^{j6\pi t} \right) + \frac{1}{2}\left(e^{-j4\pi t} + e^{j4\pi t} \right) + \frac{1}{4}\left(e^{-j2\pi t} + e^{j2\pi t} \right) + 1$$

Fourier Serisi Yakınsaklığı

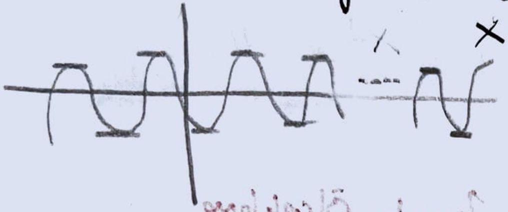
Dirichlet Koşulları

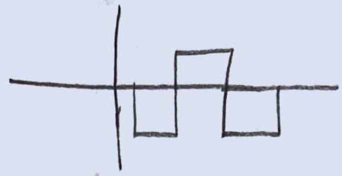
1) Fonksiyonun integrali alınabilmelidir. Yani fonsiyonun sonsuzda küçük değeri olmalıdır.

$$\int |x(t)|\, dt < \infty$$

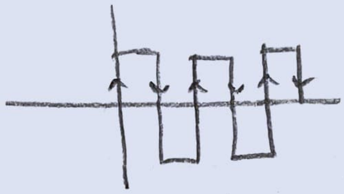


2) Maximum ve Minimum sayısı belli olmalıdır





3) Süreksizlik sınırlı sayıda olmalı



Differentiation (TÜREV)

$$x(t) \xrightarrow{FS} a_k$$

$$\frac{d}{dt} x(t) \longrightarrow j\omega_0 k (a_k)$$

Integration

Arkadaki ok Devamı

$$x(t) \xrightarrow{FS} a_k$$

$$\int_{-\infty}^{t} x(\tau)\, d\tau \xrightarrow{FS} \frac{a_k}{jk\omega_0}$$

Parseval Relation (ortalama Güç)

$$\frac{1}{T}$$

Conjugation

$$x(t) \xrightarrow{FS} a_k$$

$$x^*(t) \xrightarrow{FS} a^*_{-k}$$

$x(t)$ Gerçek bir değer ise

$$x^*(t) = x(t)$$

$$\boxed{a^*_{-k} = a_k} \quad \boxed{a^*_k = a_{-k}}$$

devamı var
yazmadın Video 24

Fourier Seri Özellikleri

Lineerlik

$x(t) \xrightarrow{FS} a_k$

$y(t) \xrightarrow{FS} b_k$

$z(t) = A\,x(t) + B\,y(t)$

$A\,x(t) + B\,y(t) \xrightarrow{FS} A.a_k + B\,b_k$

Time Reversal (Zaman Tersi Almak)

$x(t) \xrightarrow{FS} a_k$

$x(-t) \xrightarrow{FS} a_k$

* Fonksiyon eğer çift ise "x(t) çift"

$x(-t) = x(t)$

$\boxed{a_k = a_k}$

* Fonksiyon eğer tek ise "x(t) tek"

$x(-t) = -x(t)$

$\boxed{a_k = -a_k}$

Periodic Convolusyon

$x(t) \xrightarrow{FS} a_k$

$y(t) \xrightarrow{FS} b_k$

$x(t) * y(t) = z(t)$

Periyot

$z(t) \xrightarrow{FS} a_k . b_k . T$

Zamanda Öteleme

$x(t) \xrightarrow{FS} a_k$

$y(t) \xrightarrow{FS} b_k$

$y(t) = x(t \pm t_0)$

$b_k = e^{\pm jk w_0 t_0} . a_k$

$x(t \pm t_0) \xrightarrow{FS} e^{\pm jk w_0 t_0} . a_k$

Zamanda Ölçekleme

$x(t) \xrightarrow{FS} a_k$

$x(at) \xrightarrow{FS} a_k$

Çarpım (Multiplication)

$x(t) \xrightarrow{FS} a_k$

$y(t) \xrightarrow{FS} b_k$

$x(t).y(t) \longrightarrow \sum_{l=-\infty}^{\infty} a_l . b_{k-l}$

ÖRN: $g(t) = \cos(4\pi t) \cdot \sin(4\pi t)$ $\quad W_0 = 4\pi \quad g(t) = FS$ ...

$$g(t) = x(t) \cdot y(t) \xrightarrow{FS} \sum_{l=-\infty}^{\infty} a_l \cdot b_{k-l}$$

$$\underset{c_k}{g(t)} \quad \underset{a_k}{x(t)} \quad \underset{b_k}{y(t)}$$

$$x(t) = \cos(W_0 t) = \frac{e^{JW_0 t} + e^{-JW_0 t}}{2} = \frac{1}{2} e^{JW_0 t} + \frac{1}{2} e^{-JW_0 t} \quad a_1 = \frac{1}{2} \quad a_{-1} = \frac{1}{2}$$

$$y(t) = \sin(W_0 t) = \frac{e^{JW_0 t} - e^{-JW_0 t}}{2J} = \frac{1}{2J} \cdot e^{JW_0 t} - \frac{1}{2J} \cdot e^{-JW_0 t} \quad b_1 = \frac{1}{2J} \quad b_{-1} = -\frac{1}{2J}$$

$$a_k = \left\{ \frac{1}{2}, 0, \frac{1}{2} \right\} \qquad b_k = \left\{ -\frac{1}{2J}, 0, \frac{1}{2J} \right\}$$

	$\frac{1}{2}$	0	$\frac{1}{2}$
$-\frac{1}{2J}$	$-\frac{1}{4J}$	0	$-\frac{1}{4J}$
0	0	0 c_0	0
$\frac{1}{2J}$	$\frac{1}{4J}$	0	$\frac{1}{4J}$

$$c_k = \left\{ \underset{c_{-2}}{-\frac{1}{4J}}, \underset{c_{-1}}{0}, \underset{c_0}{0}, \underset{c_1}{0}, \underset{c_2}{\frac{1}{4J}} \right\}$$

ÖRN: $x_2(t) = x_1(t-1) + x_1(1-t)$ $\quad x_1$ temel frekansı W_1

$x_2(t)$ yi x_1 cinsinden FS katsayıları olarak bulun?

$$x_1(t) \rightarrow a_k$$

$$x_1(t-1) \rightarrow e^{-JkW_1 \cdot 1} \cdot a_k$$

$$x_1(t+1) \rightarrow e^{JkW_1 \cdot 1} \cdot a_k$$

$$x_1(-t+1) \rightarrow e^{-JkW_1 1} \cdot a_{-k}$$

$$x_2(t) \rightarrow b_k$$

$$b_k = \left(e^{-JkW_1} a_k + e^{-JkW_1} a_{-k} \right)$$

$$b_k = e^{-JkW_1} \left(a_k + a_{-k} \right)$$