İşaretler ve Sistemler (Singaller)

27.09.22

İşaret = Veri → Bilgi

$$\xrightarrow[\text{işaret}]{\text{(giriş)}} \boxed{\text{Sistem}} \xrightarrow[\text{Sonuç (işaret)}]{\text{(tepki)}}$$

⇒ $x(t) = 2t + 1$

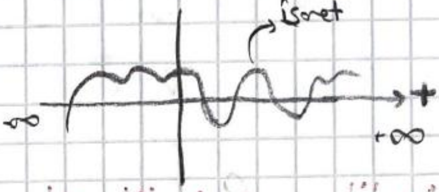
$x(t)$ = işaret : Bağımlı değişken

t = zaman : Bağımsız değişken

İşaretler

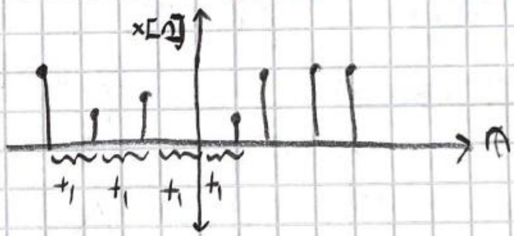
↳ 1. Sürekli zaman işaretleri (Analog) - Continuous Time (CT) $x(t)$

- Bir aralıkta sürekli değişim gösterir
- Zamanın herhangi bir anı için karşılığı var

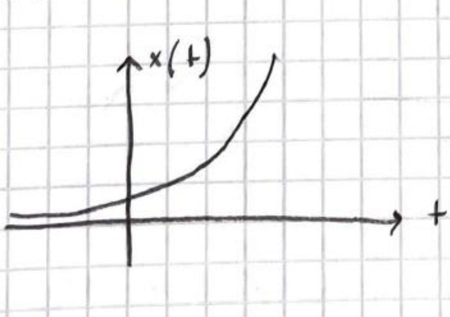


↳ 2. Ayrık zaman İşaretleri - Dirscreete Time (DT) $x[n]$

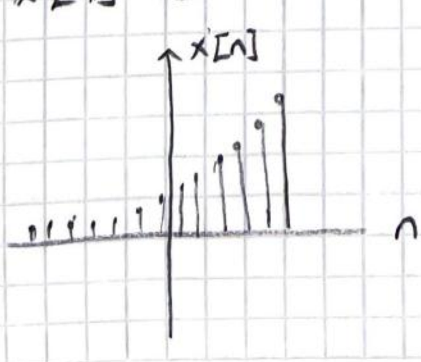
- Zamanın herhangi anı için karşılık olmak zorunda değil. Belirli anlarda değeri olur.
- İşaret eşit zaman aralıklarında incelenir



$x(t) = C \cdot e^{\alpha t}$ $\quad$ C, α sabit, $C > 0$, $\alpha > 1$



$x[n] = C \cdot e^{\alpha n}$ $\quad$ C, α sabit $\quad$ $C > 0$, $\alpha > 1$

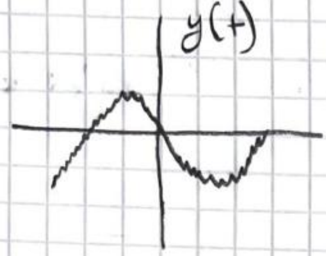


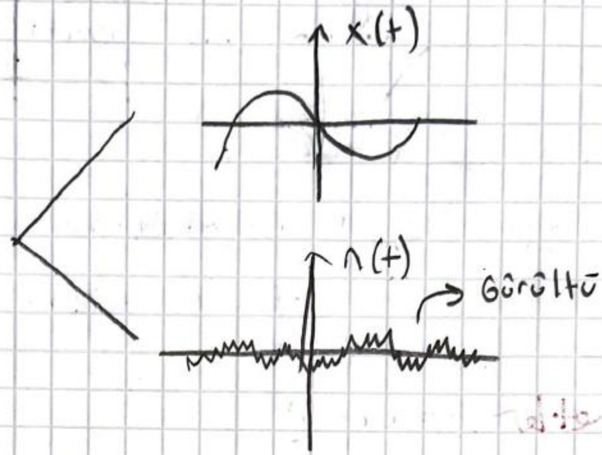
→ n değeri mutlaka tamsayı olmalıdır

1- Deterministik , Stoastik

→ İşaret kesin ve net bir şekilde ifade edilebiliyorsa deterministiktir.

$$x(t) = 2\cos 5t$$





$$y(t) = \underbrace{x(t)}_{\text{orj. Sinyal}} + \underbrace{n(t)}_{\text{Gürültü}}$$

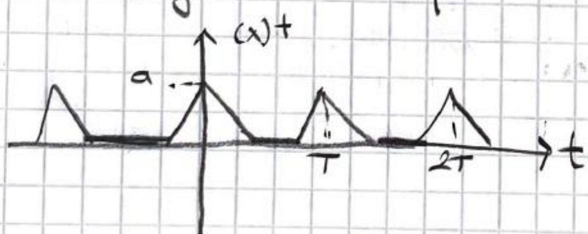
2- Periyodik , Periyodik Olmayan (aperiyodik)

$$x(t) = x(t+T)$$

T : Periyot

(En küçük tekrarlama periyodu temel periyottur)

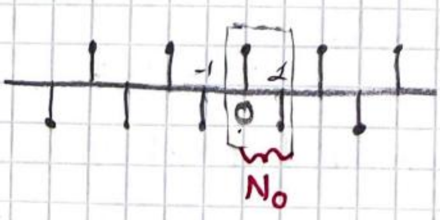
T_0 : Temel Periyot



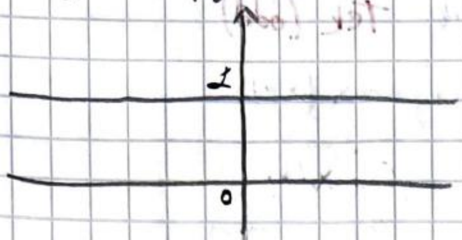
$$x(t) = x(t + k.T) \quad , \quad k = \ldots, -1, 0, 1, \ldots$$

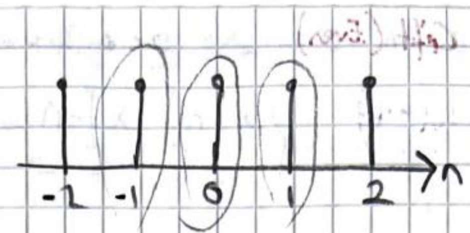
$$x[n] = x[n+N]$$

N : Periyot



$(x)+ = 1$





$x[n] = 1$

$N_0 = 1$ ✓ periyodik

$x(t) = x(k+T)$

$T = 10$ için

$x = 0 \rightarrow x(0) = x(0+10) = 1$

$x = 5 \rightarrow x(5) = x(5+10) = 1$

Aperiyodik

3- Reel ve Kompleks

$x(t) = e^{st}$ s reel ise işaret reel

s kompleks ise işaret kompleks

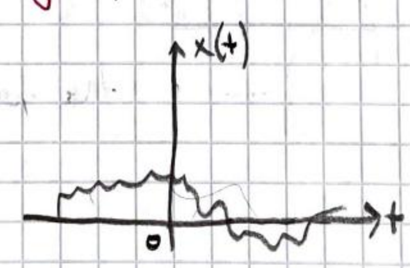
$\left[\; s = r + jw \right.$

$x[n] = z^n$ z reel ise işaret reel

4- Nedersel, Nedersel Olmayan, Anti Nedersel işaretler

$t < 0$ için $x(t) = 0$

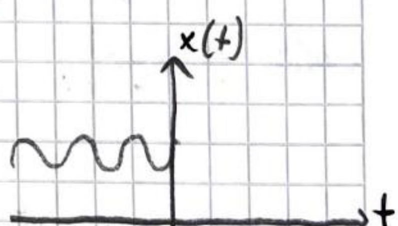
Nedersel



$t < 0$ ve $t > 0$ iken

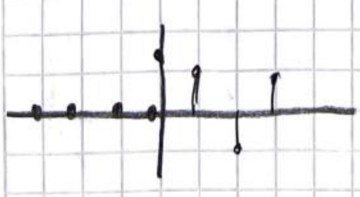
$x(t) \neq 0$

Nedersel Olmayan



$t > 0$ iken $x(t) = 0$

Anti Nedersel

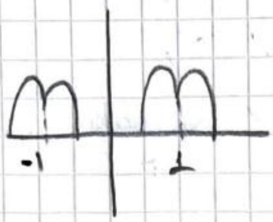


(odd) (even)

5 Tek ve Çift Sinyaller

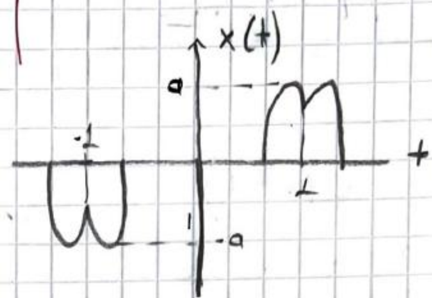
Çift (Even)

$x(t) = x(-t)$ $\quad \{ x[n] = x[-n]$



Tek (odd)

$x(t) = -x(-t)$



Tek fonk : x ve y ekseninden aynalı
Çift fonk : y ekseninden aynalı

⇒ Ne çift ne tek olmayan. Tek ve Çift bileşenlerine ayrılabilir.

$$x(t) = Ev\{x(t)\} + Od\{x(t)\}$$

$$Ev\{x(t)\} = \frac{1}{2}\left(x(t) + x(-t)\right) \qquad Ev\{x[n]\} = \frac{1}{2}\left(x[n] + x[-n]\right)$$

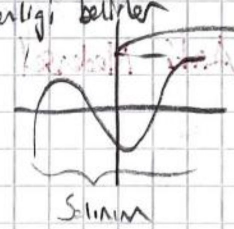
$$Od\{x(t)\} = \frac{1}{2}\left(x(t) - x(-t)\right)$$

ÖRN $x(t) = 2\sin(3t)$ → Salınım sıklığını belirler

2 → Genliği belirler

Sin → tek fonksiyon

Cos → Çift fonksiyon

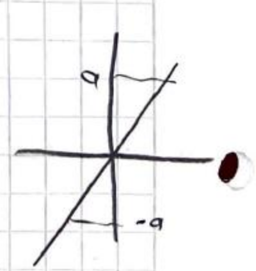


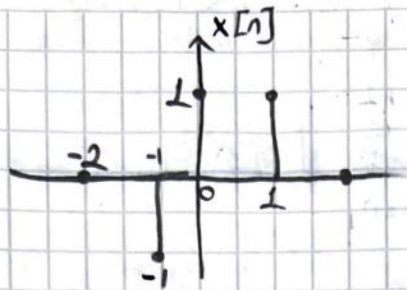
tek fonksiyon

$x(t) = 2t + 1$

$$Ev\{x(t)\} = \frac{1}{2}\left(2t+1 + (-2t+1)\right)$$
$$= 1$$

$$Od\{x(t)\} = \frac{1}{2}\left(2t+1 - (-2t+1)\right)$$
$$= 2t$$

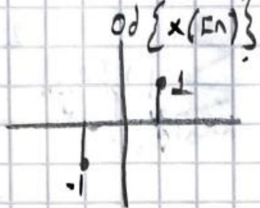


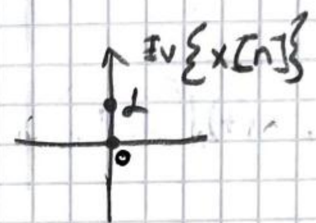


$$Ev\{x(n)\} = \frac{1}{2}\left(x[n] + x[-n]\right)$$

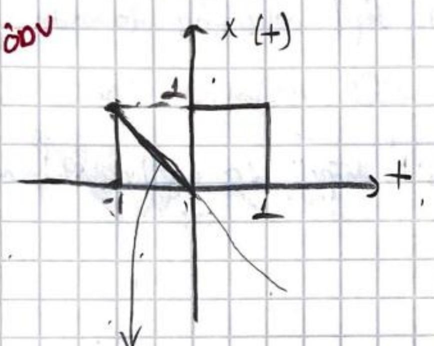
$$n=-1 \Rightarrow Ev\{x(n)\} = \frac{1}{2}(-1+1) = 0$$

$$n=0 \Rightarrow Ev\{x(n)\} = \frac{1}{2}(1+1) = 1$$





ödv



$$\frac{x-1}{x-0} = \frac{t+1}{t-0}$$

$$\cancel{xt} - t = \cancel{xt} + x$$

$$x = -t$$

$$x(t) = -t \cancel{+1}$$

$$x(t) = \begin{cases} -t, & -1 < t < 0 \\ 1, & 0 < t < 1 \end{cases}$$

$$x(t) = -t$$
$$x(-t) = -(-t)$$

$-1 < t < 0$

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} = \frac{1}{2}(\cancel{-t+t}) = 0$$

04.10.2022
2. Hafta

6- Enerji ve Güç İşaretleri

$x(t)$, $t_1 < t < t_2$

$$E = \int_{t_1}^{t_2} |x(t)|^2 dt$$

$x[n]$, $n_1 < n < n_2$

$$E_\infty = \sum_{n=n_1}^{n_2} |x[n]|^2$$

$-\infty < t < \infty$

$$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt$$

$-\infty < n < \infty$

$$E_\infty = \lim_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2$$

Güç = Birim zamanda harcanan enerjidir. Sonsuz aralıkta değil belirli bir zamanda harcanan enerji

Tüm zaman boyunca , $x(t)$, $x[n]$ sabit değerli bir enerjiye sahip mi?

$E_\infty = \infty$ ise enerji sinyali (işareti) değildir.

$E_\infty < \infty$ ise enerji sinyali (işareti) dir.

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

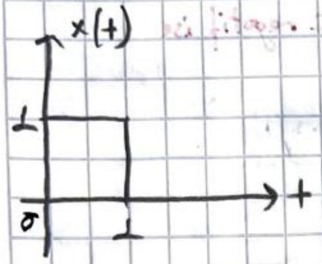
$E_\infty = \infty$ ise bu bir güç işareti değildir

$E_\infty < \infty$ ise bu bir güç işaretidir.

-1- Sonlu Toplam Enerjiye Sahip İşaretler

$E_\infty < \infty$ 'dur — Eğer toplam enerji sonsuzdan küçükse

$P_\infty = 0$ 'dır. — toplam güç sıfırdır



Yansıdı yok

-2- Sonlu Ortalama Güce Sahip İşaretler

$0 < P_\infty < \infty$, $E_\infty = \infty$

-3-

$E_\infty = \infty$

$P_\infty = \infty$

$x(t) = t$

$x[n] = 4$

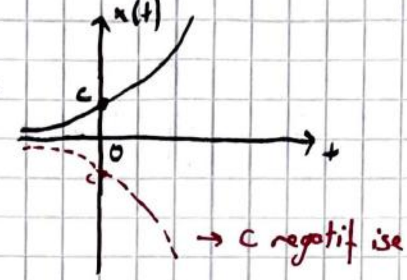
$P_\infty = 16$

Kompleks Üstel İşaretler - CT

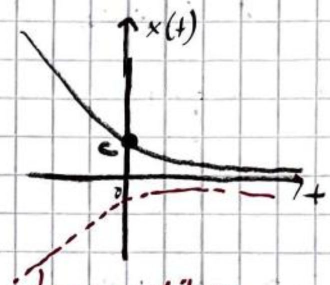
$x(t) = C \cdot e^{at}$, c ve a genelde kompleks

c ve a reel ise

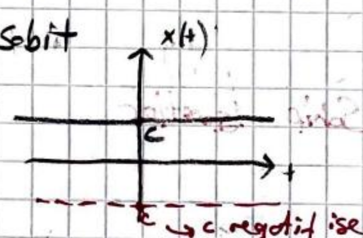
1) $a > 0$ ise , $x(t)$ sürekli artar



2) $a < 0$ ise , $x(t)$ sürekli azalır



3) $a = 0$ ise $x(t)$ sabit



Periyodik Kompleks Üstel ve Sinüsoidal İşaretler

$x(t) = e^{jw_0 t}$, $w_0 = \frac{2\pi}{T}$

$x(t) = e^{jw_0 t}$

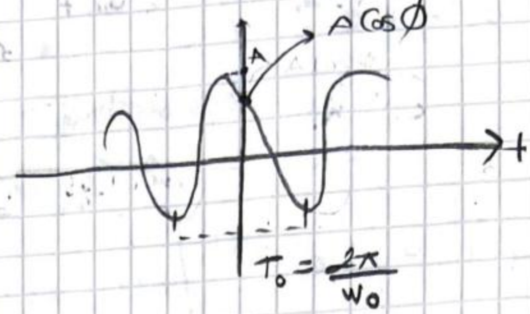
$x(t+T) = e^{jw_0 t} = e^{jw_0 T}$ $\quad e^{jw_0 T} = e^{jw_0 t} \cdot e^{j\frac{2\pi}{T}T}$

$e^{j2\pi} = \cos 2\pi + j \sin 2\pi$

$T_0 = \frac{2\pi}{|w_0|}$

$e^{-jw_0 t}$, $e^{jw_0 t} \rightarrow T_0$

$x(t) = A \cdot \cos(w_0 t + \phi)$



$e^{Jw_0 t} = \cos w_0 t + J \sin w_0 t$

$e^{-Jw_0 t} = \cos w_0 t - J \sin w_0 t$

$A \cdot \cos(w_0 t + \phi) = \frac{A}{2} e^{J\phi} \cdot e^{Jw_0 t} + \frac{A}{2} e^{-J\phi} \cdot e^{-Jw_0 t}$

$A \cdot \sin(w_0 t + \phi) = \frac{A}{2J} e^{J\phi} \cdot e^{Jw_0 t} - \frac{A}{2J} \cdot e^{-J\phi} \cdot e^{-Jw_0 t}$

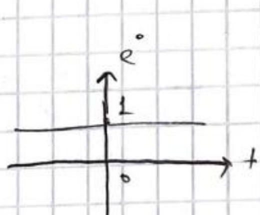
$A \cdot \cos(w_0 t + \phi) = A \cdot Re\{e^{Jw_0 t + \phi}\}$

$A \cdot \sin(w_0 t + \phi) = A \cdot Im\{ - - -$

$T_0 = \frac{2\pi}{|w_0|}$

Temel Periyodu Tanımsız

$e^{Jw_0 t} \xrightarrow[w_0 = 0 \text{ iken}]{}$



Bir peryotluk zaman boyunca enerjisi

$E_{peryot} = \int_0^{T_0} |e^{Jw_0 t}|^2 dt = \int_0^{T_0} \sqrt{\cos^2 w_0 t + \sin^2 w_0 t}^{\,2} \, dt$

$= \int_0^{T_0} 1^2 dt = T_0$ = Sonlu Bir değer

$E_\infty = \infty$ olursa bir güç sinyali olur

ÖRN:

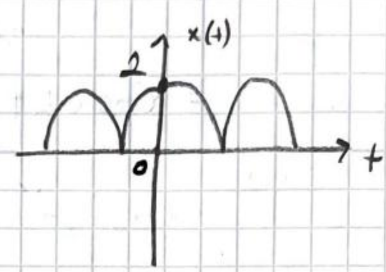
$$x(t) = e^{J2t} + e^{J3t}$$

$$x(t) = e^{J2,5t}\Big(\underbrace{e^{-J0,5t} + e^{J0,5t}}_{2.\,\mathrm{Cos}\,0,5t}\Big)$$

$$\boxed{\mathrm{Cos}x = \frac{e^{x} + e^{-x}}{2}}$$

$$|x(t)| = \underbrace{\left|2e^{J2,5t}\right|}_{2} . \left|\mathrm{Cos}\,0,5t\right|$$

$$\mathrm{Cos}\,0,5t = \sqrt{\mathrm{Cos}^2\,0,5t}$$



Genel Olarak Kompleks Üstel Sinyaller

$$C.\,e^{at}$$

$$C = |C| . e^{J\phi}$$

$$a = r + JW_0$$

$$C.\,e^{at} = |C| . e^{J\phi} . e^{(r + JW_0)t}$$

$$= |C| . e^{rt} . e^{JW_0t + \phi}$$

$$C.\,e^{at} = |C|\,e^{rt}\left[\mathrm{Cos}(W_0t + \phi) + J\sin(W_0t + \phi)\right] = a + Jb$$

$r = 0$ ise

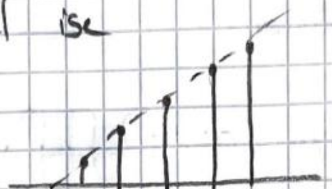
DT Kompleks Üstel ve Sinüsoidal Sinyaller

$x[n] = C \cdot \alpha^n$, C ve α genel kompleks

$\alpha = e^{\phi} \longrightarrow x[n] = C \cdot e^{\phi n}$

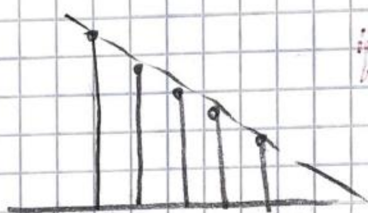
C ve α reel ise

$\alpha > 1$

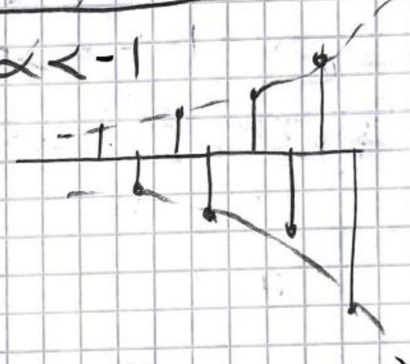


$-1 < \alpha < 0$

$0 < \alpha < 1$



$\alpha < -1$



$x[n] = e^{jw_0 n}$

$x[n] = A \cdot \cos(w_0 n + \phi)$

$e^{jw_0 n} = \cos w_0 n + j \sin w_0 n$ $\qquad \left| e^{jw_0 n} \right| = 1$

$A \cos(w_0 n + \phi) = \frac{A}{2} e^{j\phi} + \frac{A}{2} e^{j\phi} \cdot e^{-jw_0 n}$

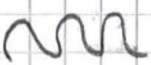
$E_\infty = \infty \quad , \quad P_\infty < \infty$

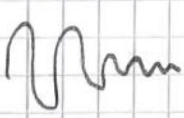
$x[n] = c \cdot \alpha^n$

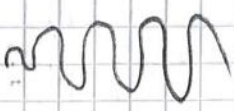
$c = |c| \cdot e^{j\theta}$

$\alpha = |\alpha| e^{jw_0}$

$c \cdot \alpha^n = |c| \cdot |\alpha|^n \left[\cos(w_0 n + \theta) + j \sin(w_0 n + \theta) \right]$

$|\alpha| = 1$ ise sinüs salınımı 

$|\alpha| < 1$ ise azalan sinüs salınımı 

$|\alpha| > 1$ ise artan sinüs salınımı 

DT Kompleksi İşaretin Periyodikliği

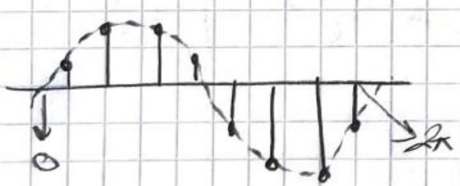
$CT \rightarrow e^{jw_0 t}$

$DT \rightarrow e^{j(w_0 + 2\pi)n} = e^{jw_0 n} \cdot \underbrace{e^{j2\pi n}}_{1} = e^{jw_0 n}$

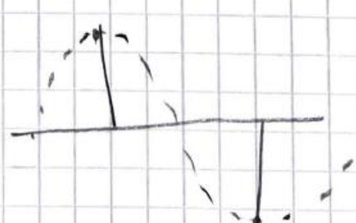
$x[n] = \cos\left(\frac{\pi}{4} n\right) = \cos(w_0 n)$

$$\boxed{w_0 = \frac{2\pi}{N} = \frac{\pi}{4} \qquad N = 8}$$

8 tane örnek arasında geliyor



$$\boxed{\cos \pi n \rightarrow W_0 \ni \pi = \frac{2\pi}{N} \qquad N = 2}$$



$\frac{W_0}{2\pi} = \frac{k}{N}$, k: sayı†

$W_0 N = k \cdot 2\pi$

ÖRN: $x[n] = \cos(2\pi n/12)$ temel Periyotları nedir?

$x(t) = \cos(2\pi t/12)$

$$\boxed{x(t) = \cos wt \qquad w = \frac{2\pi}{T}}$$

$w = \frac{\cancel{2\pi}}{T} = \frac{\cancel{2\pi}}{12}$, $T = 12s$

$x[n] \to \cos w n = \cos\left(\frac{2\pi}{12} n\right)$

$w = \frac{\cancel{2\pi}}{N} = \frac{\cancel{2\pi}}{12}$, $N = 12$

ÖRN:

$x[n] = \cos \frac{8\pi}{31} n$, $x(t) = \cos \frac{8\pi}{31} t$

$w = \frac{2\pi}{N} = \frac{8\pi}{31}$

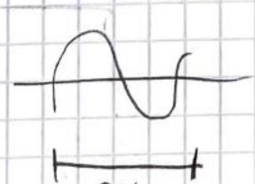
$w = \frac{\cancel{2\pi}}{T} = \frac{\cancel{8\pi}}{31}$

$N = \frac{31}{4} \cdot k$

$T = \frac{31}{4} s$

$k = 4$ için

$N = 31$ Adet



31/4 sn

örn

$x[n] = \cos\frac{n}{6}$, $x(t) = \cos\frac{t}{6}$

$x(t) \rightarrow \frac{2\pi}{T} = \frac{1}{6} \Rightarrow T = 12\pi$

$x[n] \rightarrow \frac{2\pi}{N} = \frac{1}{6}\cdot k \Rightarrow N = \frac{12\pi}{k}$

$x[n]$ periyodik değildir

örnek $x[n] = e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{4}n}$, $N = ?$

$x_1[n] \rightarrow \frac{2\pi}{N_1} = \frac{2\pi}{3} \longrightarrow N_1 = 3.k$

$x_2[n] \rightarrow \frac{2\pi}{N_2} = \frac{3\pi}{4} \longrightarrow N_2 = \frac{8}{3}k$

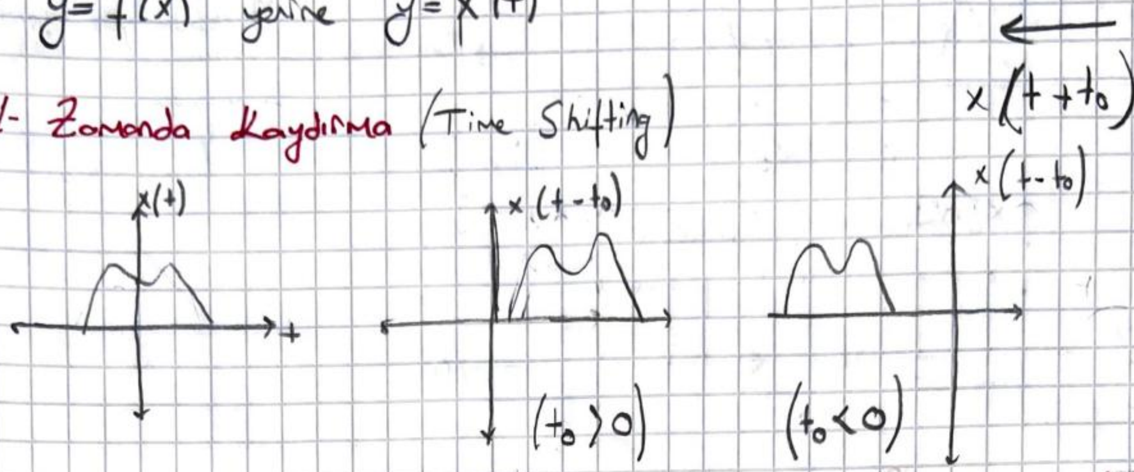
$N = N_1 \cdot N_2 = 3 \cdot 8 = 24$ $k = 3$, $N_2 = 8$

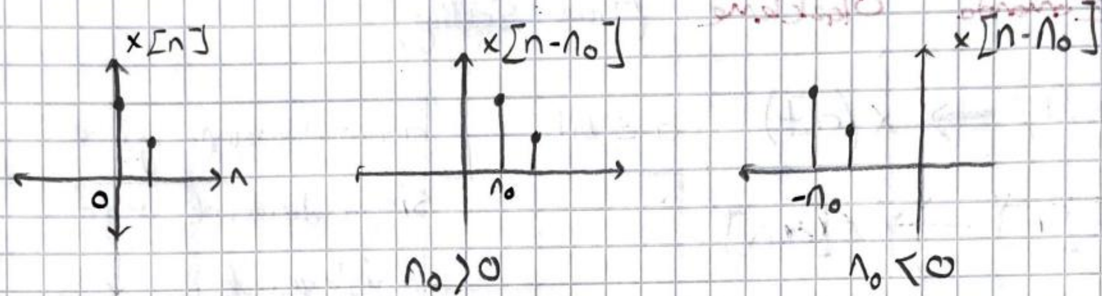
21.10.2022
3. Hafta

Sinyal Dönüşümleri

$y = f(x)$ yerine $y = x(t)$

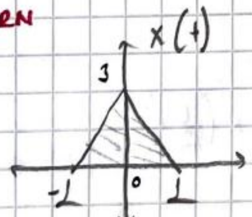
1- Zamanda Kaydırma (Time Shifting)



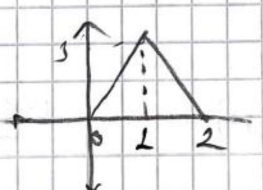


Gerçek hayatta ise $\Rightarrow$ $x(t)$, $x(t-1)$, $x(t+2)$
$x[n]$, $x[n-3]$, $x[n+5]$

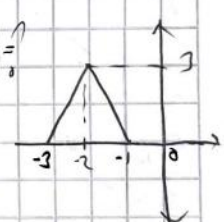
ÖRN



$x(t-1) = ?$

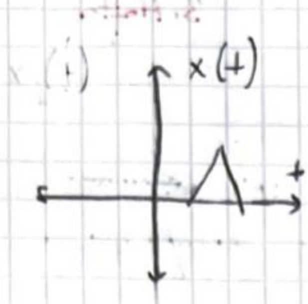


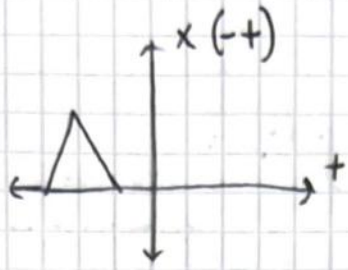
$x(t+2) = ?$



$$x(t) = \begin{cases} 3t+3, & -1 < t < 0 \\ -3t+3, & 0 < t < 1 \\ 0, & \text{diğer} \end{cases}$$

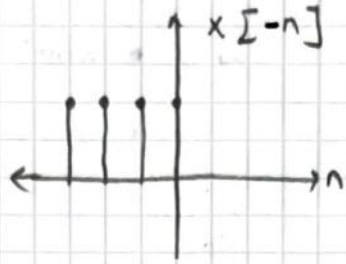
2. Zamanda Ters Çevirme (Time Reversal)





Zamanda ters çevirdik

y eksenine göre simetriğini al



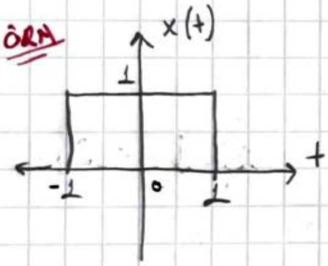
3- Zamanda Ölçekleme (Time Scaling)

$x(t) \longrightarrow x(a.t)$ a sabit

$x[n] \longrightarrow x[a.n]$

a nın durumuna göne sinyol daralacak veya genişleyecek

ÖRN

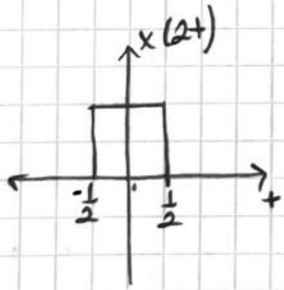


– Zaman sıkıştırması veya zaman genişletmesi

$\underbrace{\text{Zaman sıkıştırması}}_{a>1}$ veya $\underbrace{\text{zaman genişletmesi}}_{a<1}$

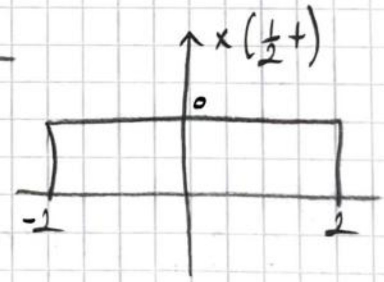
$a>1$ ise

$a=2$

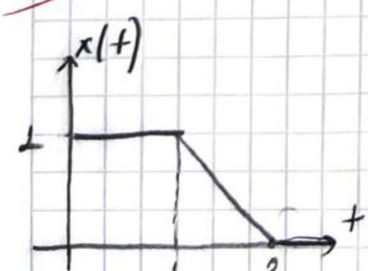


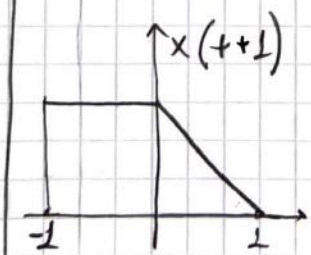
$0<a<1$

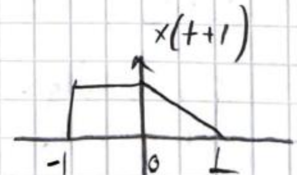
$a=\frac{1}{2}$

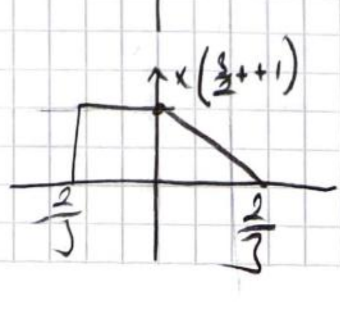


ÖRN









$\frac{3}{2} > 1$

Zaman Daralma

$x(t+1)=$

$x(-t-1)=$

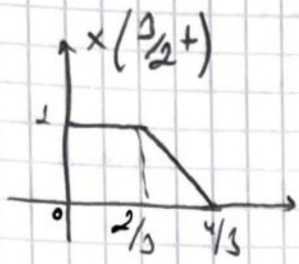
$x(\frac{3}{2}t)=$

$x(\frac{3}{2}t+1)=$

Hem ölçekleme hem kaydırma
veya
hem ters çevirme hem kaydırma

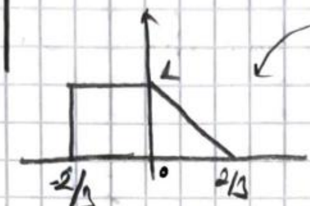
1) Önce kaydırma, sonra ölçekleme (ters) yap

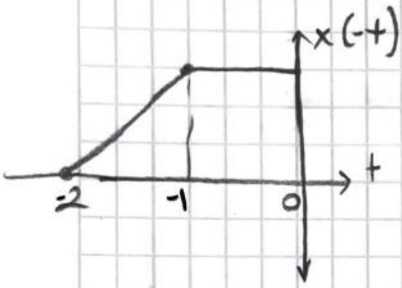
2) Önce ölçekleme(ters) sonra kaydırma → İste tersiyen

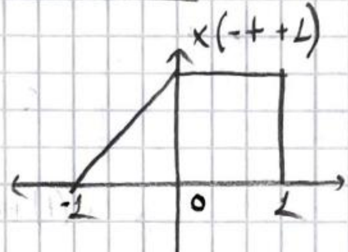


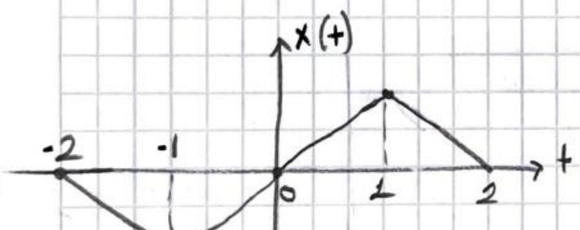
1) $x(t) \rightarrow x(t+1)$: bul

$a = \frac{3}{2}$ ile ölçekle $\longrightarrow x\left(\frac{3}{2}t+1\right)$



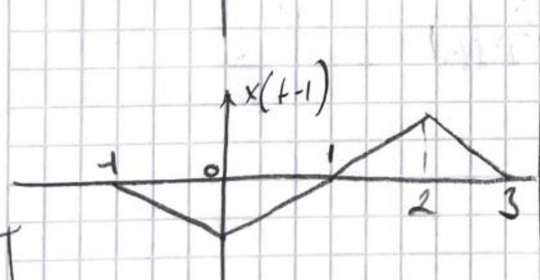


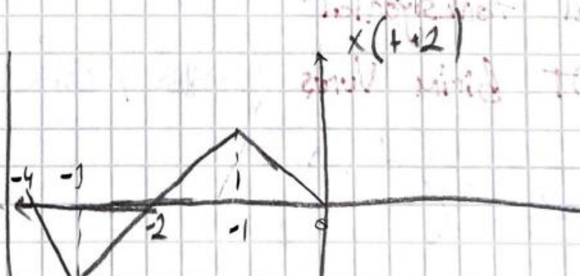




$x(2t-1) = ?$ $\quad x(-3t+2) = ?$

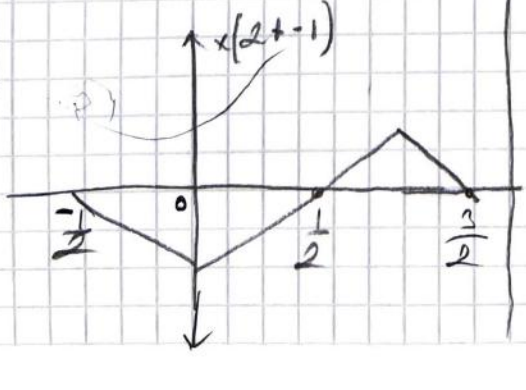
--- Sol kaydır

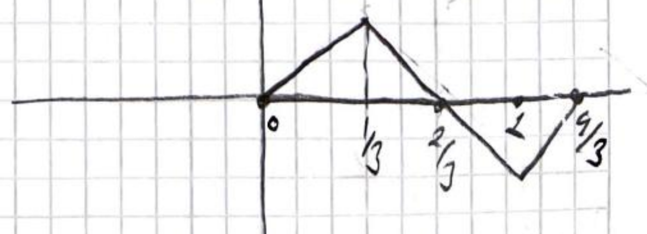




2>1
Zaman
Daraltma

3>1
Daraltma





4. Genlik Ölçekleme

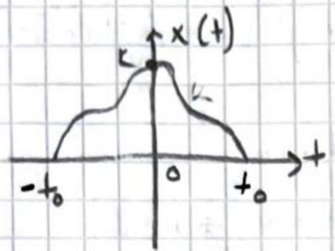
* Sinyalin şekli eskisiyle aynı kalır

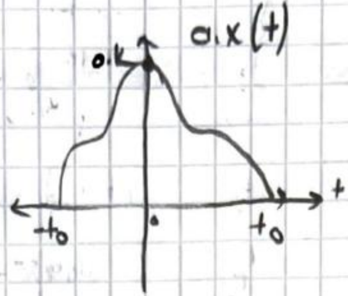
$x(t) \rightarrow a.x(t)$

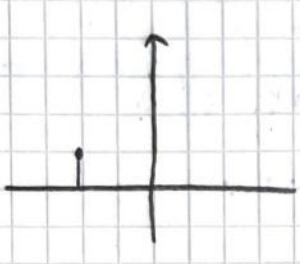
$x[n] \rightarrow a.x[n]$

$a>1$ Genlik Artar

$a<1$ Genlik Azalır



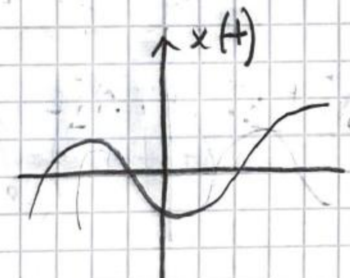


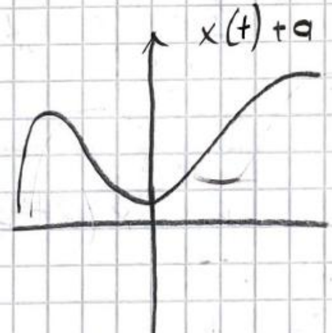


5. Genlik Kaydırma

$x(t) \rightarrow x(t)+a$

$x[n] \rightarrow x[n]+a$

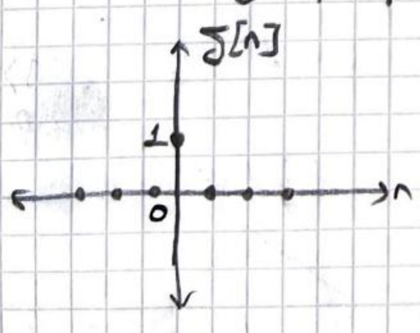




Temel Fonksiyonlar

1) DT Birim Vuruş (Impulse / Dirac Delta)

$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n\neq 0 \end{cases}$$

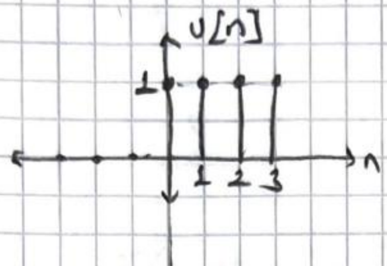


Sıfırdayken değeri 1

Diğer heryerde 0

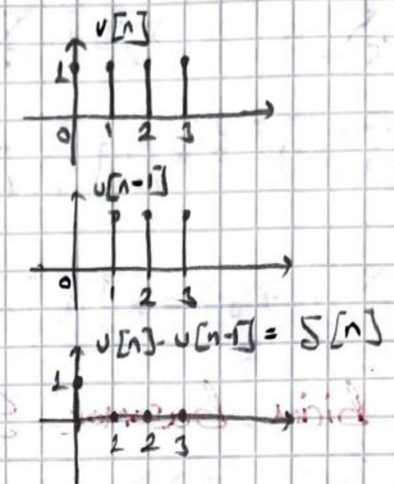
2) DT birim Basamak (Unit Step)

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



ÖZELLİK 1

$$\delta[n] = u[n] - u[n-1]$$



$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

$$u[3] = \sum_{m=-\infty}^{3} \delta[m]$$

$$= \cancel{\delta[-\infty]} + \ldots + \cancel{\delta[-1]} + \overbrace{\delta[0]}^{1} + \cancel{\delta[1]} + \cancel{\delta[2]} + \cancel{\delta[3]}$$

$$= 1$$

$$m = n-k \quad , \quad \overbrace{k = n-m}^{k = n-(-\infty) = \infty}$$

$$u[n] = \sum_{k=\infty}^{0} \delta[n-k] \Rightarrow \boxed{u[n] = \sum_{k=0}^{\infty} \delta[n-k]} \; ?$$

$n=1$

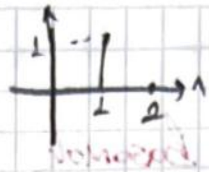
$$u[1] = \sum_{k=0}^{\infty} \delta[1-k] = \cancel{\delta[1]} + \overbrace{\delta[0]}^{1} + \ldots + \cancel{\delta[-1]} + \ldots$$

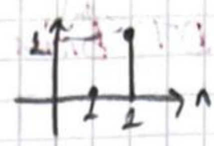
$$= 1$$

→ içeriyi topluca kaydırma yap

$\delta[n-1]$, $n=1$ için 1'dir.

$\delta[n-2]$, $n=2$ için 1 dir.





$$x[n]\cdot\delta[n] = x[0]\cdot\delta[n]$$

$$x[n]\cdot\delta[n-1] = x[1]\cdot\delta[n-1]$$

$$x[n]\cdot\delta[n-n_0] = x[n_0]\cdot\delta[n-n_0]$$

3. CT Birim Basamak Sinyali

$$u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$$

*Sıfır anında süreksizdir

4. CT birim Vuruş Sinyali

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

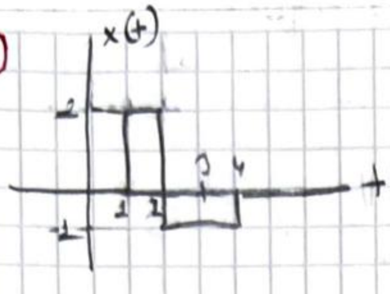
$$\underbrace{u(t) = \int_{\tau=-\infty}^{t} \delta(\tau)\, d\tau}$$

(Her iki tarafın türevini alıp integrali yok ederiz)

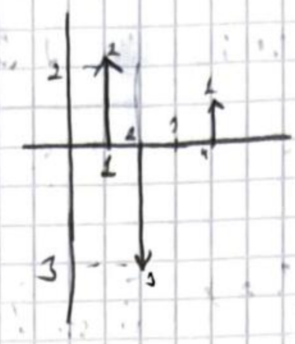
$$\delta(t) = \frac{du(t)}{dt}$$

Yaz

ÖRN



$\frac{dx(t)}{dt}$



$2\delta(t-1)$

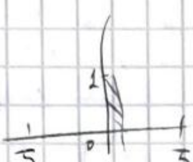
$-3\delta(t-2)$

$+\delta(t-4)$

ÖRN

$\int_{-5}^{5} \delta(t)\, dt = ?\ \underline{1}$

$\int_{-5}^{-1} \delta(t)\, dt = ?\ 0$



$\int_{0}^{5} \delta(t-3)\, dt = ?\ 0$

ÖRN

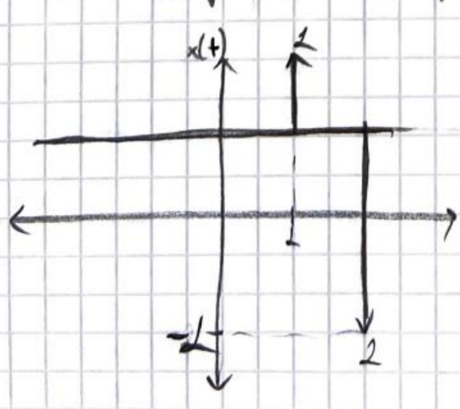
$$I = \int_{-2}^{3} f(t)\left[2 + \delta(t+1) - 3\delta(t-1) + 2\delta(t-3)\right] dt$$

$$I = 2\int_{-2}^{3} f(t)\,dt + \int_{-2}^{3} f(t)\cdot\delta(t+1)\,dt - 3\int_{-2}^{3} f(t)\,\delta(t-1)\,dt$$

$$+2\int_{2}^{3} f(t)\,\delta(t+3)\,dt$$

$$I = 2\int_{2}^{3} f(t)\,dt + f(-1) - 3f(1)$$

$x(t) = 1 + \delta(t-1) - 2\delta(t-2)$ Grafiksel olarak çizin......



Sistemler

18.10.2022
4. Hafta

* Girişe göre çıkışı yorumlicaz veya oluşturacağız.

$x(t) \longrightarrow CT \longrightarrow y(t)$

$x[n] \longrightarrow DT \longrightarrow y[n]$

$$\frac{d\,v_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t)$$

$$\frac{dy(t)}{dt} + a.y(t) = b.x(t)$$

- Diff denklem

Sürekli zaman için sistem örneği

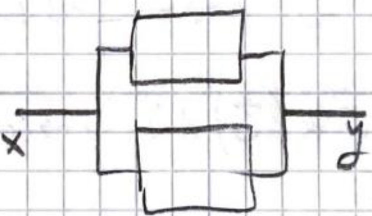
$$y[n] - 1{,}01\, y[n-1] = x[n]$$

$$y[n] + a.y[n-1] = b.x[n]$$

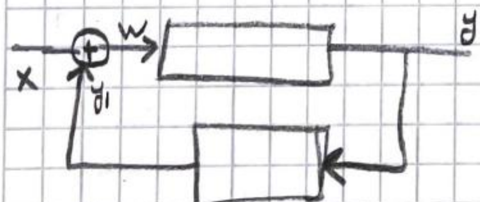
- Fark Denklem

Ayrık zamanda sist. örneği

Seri bağlı sistem



Paralel bağlı Sistem

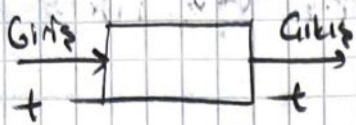


Geri beslemeli sistem

$w = x + y_1$

Sistemlerin Özellikleri

1- Bellekli ve Belleksiz Sistemler



Giriş ve çıkış ikisi de t ye bağlıysa...
Belleksiz sistemler

$t \rightarrow (t \mp t_0)$ Çıkışta $(t \mp t_0)$ olursa bellekli sistemlerdir

$y[n] = \left(2x[n] - x^2[n]\right)^2 \rightarrow$ Belleksiz bir sistem

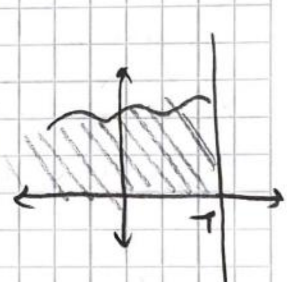
$y(t) = \alpha . x(t) \longrightarrow$ Belleksiz

$y[n] = \sum_{k=-\infty}^{n} x[k] \rightarrow$ Bellekli

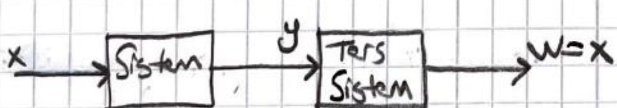
$y[n] = \dots, x[-2] + x[-1] + \dots + x[n] \rightarrow$ Akümülatör

$y[n] = x[n-1]$ n = zaman $\rightarrow$ bellekli

$y(t) = \frac{1}{c} \int_{-\infty}^{t} x(\tau)\, d\tau \rightarrow$ bellekli



2- Tersine Çevrilebilir Sistemler



$\underbrace{y(t) = 2x(t)} \Longrightarrow x(t) = \frac{1}{2} y(t) = w(t)$

x yalnız bırakılarak tersi alındı. Fakat her sistemin tersi alınmaz

$y[n] = y[n-1] + x[n]$ (sistem)

$x[n] = y[n] - y[n-1] = w(n)$ (ters sistem)

Giriş = $x(t)$
Çıkış = $y(t)$

Bütün belleksiz sistemler Nedenseldir

3) Nedensellik

Çıkış sadece geçmişteki veya o andaki değere bağlı ise nedenseldir.

Çıkış gelecekteki bir girişe bağlı ise nedensel değildir

$y[n] = x[n] - x[n+1]$ Nedensel değil

$y(t) = x(t+1)$ Nedensel değil

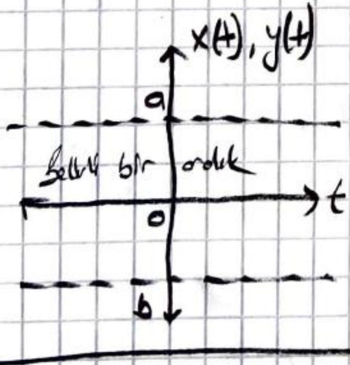
$y[n] = x[-n]$

$$\left.\begin{aligned} n=1 &\rightarrow y[1] = x[-1] \\ n=0 &\rightarrow y[0] = x[0] \end{aligned}\right\} \; n \geq 0 \text{ için nedenseldir}$$

$$\left.n=-1 \rightarrow y[-1] = x[1] \right\} \; n < 0 \text{ için nedensel değildir}$$

$y(t) = x(t) . \underbrace{\cos(t+1)}_{\text{bir fonksiyondur } f(t) \text{ gibi düşünülebilir}}$ Nedenseldir. Cos bir fonksiyondur

4) Kararlılık



Sistemin kararlı olabilmesi için giriş veya çıkışının sınırlı aralık içinde olması gerekiyor

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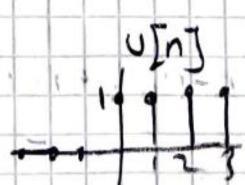
$y[n] = \sum_{k=-\infty}^{n} u[n] = (n+1).u[n]$



$n \geq 0 \rightarrow 1$
$n < 0 \rightarrow 0$

$y[0] = 1$
$y[1] = 2$
$y[2] = 3$

$y[n] \rightarrow$ Sonsuza gidebildiği için sistem kararsız

5) Zamanda Değişmezlik (Time Invariant)

Bir işaretin girişi zamanda belli bir aralıkta kaydırılırsa ve çıkışta o aralıkta kayıyorsa zamanda değişmezdir

$$x[n] \rightarrow \boxed{} \rightarrow y[n] \qquad x[n \mp n_0] \rightarrow \boxed{} \rightarrow y[n \mp n_0]$$

Zamanda Nedir?

$$y(t) = \sin[x(t)]$$

$$y_1(t) = \sin[x_1(t)] \rightarrow \boxed{t = t - t_0}$$

$$y_1[t - t_0] = \sin[x_1(t - t_0)]$$

$$x_2(t) = x_1(t - t_0)$$

$$y_2(t) = \sin[x_2(t)] = \sin[x_1(t - t_0)]$$

$$y_2(t) \overset{?}{=} y_1(t - t_0) \quad (\text{Eşit Mi Sorgularız } ?)$$

$$\sin[x_1(t - t_0)] = \sin[x_1(t - t_0)]$$

Zamanda Nedir?

$$y[n] = n . x[n]$$

$$y_1[n] = n . x_1[n]$$

$$n = n - n_0 ;$$

$$y_1[n - n_0] = (n - n_0) . x[n - n_0]$$

$$x_2[n] = x_1[n - n_0]$$

Zamanda Değişir

$$y_2[n] = n . x_2[n]$$

$$y_2[n] = n . x_1[n - n_0]$$

$$\underbrace{n . x_1[n - n_0]}_{y_2[n]} \neq \underbrace{(n - n_0) . x_1(n - n_0)}_{y_2[n - n_0]}$$