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EEE 321

Signals and Systems

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Lab Assignment 4

In this lab task we observed the definition of Fourier Transfer and its application on signals. We examined how to obtain the coefficients of the Fourier transfer and the analysis related to it. We also applied this process on a second order system.

Part 1

Part 1.1: Implementing Fourier Series Analysis

In this part, we did the Matlab implementation of the equation in which we found the coefficients of the Fourier series. We learned that a continious-time periodic signal can be expressed as a summation of harmonically related frequency component and Fourier series coefficients these formulas are as follows:

$$x(t) = \sum_{k=-\infty}^{\infty} \left(a_k e^{jk\left(\frac{2\pi}{To}\right)t} \right) \qquad (synthesis\ formula)$$

$$a_k = \int_{T0} x(t) e^{-jk\left(\frac{2\pi}{T0}\right)t} dt \qquad (analysis\ formula)$$

First, we disritized the coefficient formula with integral. After discriminating, the limits of the addition operation we will perform are between 0 and N-1. The total number of samples in a period that we define as N. Since we know the period, T_0 can be used here instead of N. The desired features of the function we created were as follows:

function [fsCoeffs] = FSAnalysis(x, k)

- **fsCoeffs: An array that contains the Fourier series coefficients of your signal from -k to k**
- **x: One complete period of the sampled continuous-time signal**
- **k: The number of two-sided Fourier series coefficients that you will estimate**

Part 1.2: Testing The Function

First, we found the fundamental period for the given function x1, the period we found is T=1s. Then we calculated the coefficients for x1 and x2 manually. The calculations can be seen below

Part 1.2

$$x_1(t) = 8\cos(\underbrace{10\pi}_{\omega_1} t) + 20\sin(\underbrace{6\pi}_{\omega_2} t) - 11\cos(\underbrace{30\pi}_{\omega_3} t)$$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{10\pi} = \frac{1}{5}, \quad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{6\pi} = \frac{1}{3}, \quad T_3 = \frac{2\pi}{\omega_3} = \frac{2\pi}{30\pi} = \frac{1}{15}$$

$$\text{lcm}\left(\frac{1}{5}, \frac{1}{3}, \frac{1}{15}\right) \Rightarrow T = 1s \text{ (fundamental period)}$$

$$\sin(\omega_0 t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}, \quad \cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

as $\omega_0 \to$ fund freq

$$x_1(t) = 8\cos(5\omega_0 t) + 20\sin(3\omega_0 t) - 11\cos(15\omega_0 t)$$

$$x_1(t) = 8 \cdot \frac{1}{2}\left[e^{j5\omega_0 t} + e^{-j5\omega_0 t}\right] + \frac{20}{2j}\left[e^{j3\omega_0 t} - e^{-j3\omega_0 t}\right]$$

$$- 11 \cdot \frac{1}{2}\left[e^{j15\omega_0 t} + e^{-j15\omega_0 t}\right]$$

figure 1 : Part 1.2 Solution

$$x_1(t) = \underbrace{\left(\frac{-11}{2}\right)}_{a_{-15}} e^{-j15\omega_0 t} + \underbrace{(4)}_{a_{-5}} e^{-j5\omega_0 t} + \underbrace{(10j)}_{a_{-3}} e^{-j3\omega_0 t} \ldots$$

$$\ldots + \underbrace{(-10j)}_{a_3} e^{j3\omega_0 t} + \underbrace{(4)}_{a_5} e^{j5\omega_0 t} + \underbrace{\left(\frac{-11}{2}\right)}_{a_{15}} e^{j15\omega_0 t}$$

$a_{-15} = \frac{-11}{2}$, $a_{-5} = 4$, $a_{-3} = 10j$, $a_3 = -10j$, $a_5 = 4$, $a_{15} = \frac{-11}{2}$

$x_2(t) = e^{-t}$ for $-1 < t < 1 \rightarrow T = 2s$

as $a_k = \frac{1}{T} \int_T x(t)\, e^{-jk\left(\frac{2\pi}{T}\right)t} dt$

$$a_k = \frac{1}{2} \int_{-1}^{1} e^{-t(1+jk\pi)} dt$$

$$a_1 = \frac{1}{2} \int_{-1}^{1} e^{-t(1+j\pi)} = \frac{1}{2} \frac{e^2-1}{e} \frac{1}{1+j\pi}$$

general formula

$$a_k = \frac{1}{2} \frac{e^2-1}{e} \frac{1}{1+jk\pi} (-1)^{k+1}$$

$a_0 = \frac{1}{2} \frac{e^2-1}{e}(-1) = \frac{1-e^2}{2e}$, $a_1 = \frac{e^2-1}{2e(1+j\pi)}$

$a_2 = \frac{1}{2} \frac{1-e^2}{e(1+j2\pi)}$, $a_{-1} = \frac{1}{2} \frac{e^2-1}{e(1-j\pi)}$, $a_{-2} = \frac{1}{2} \frac{1-e^2}{e(1-2j\pi)}$

figure 2 : Part 1.2 Solution Continiued

Then we implemented the given functions in MATLAB and used the FSAnalysis function to estimate the coefficient. Here we used Ts = 0.001s and k = 30. Then we plotted the real and imaginary parts of the coefficients for x1 and x2. Our results for x1 is given in figure 2 and when we calculate for a_{-15} ,a_{-5} ,a_{-3} ,a_{3} ,a_{5} ,a_{15} it gives same result with the result of MATLAB. The plot of results can be seen at figure 3. Also for x2 we have coefficients for a_{-2} to a_{2} and the results are same with the results of MATLAB. Also for Parseval's Relation, it can be seen at the figure 5 that the relation holds computationally for both x1 and x2.

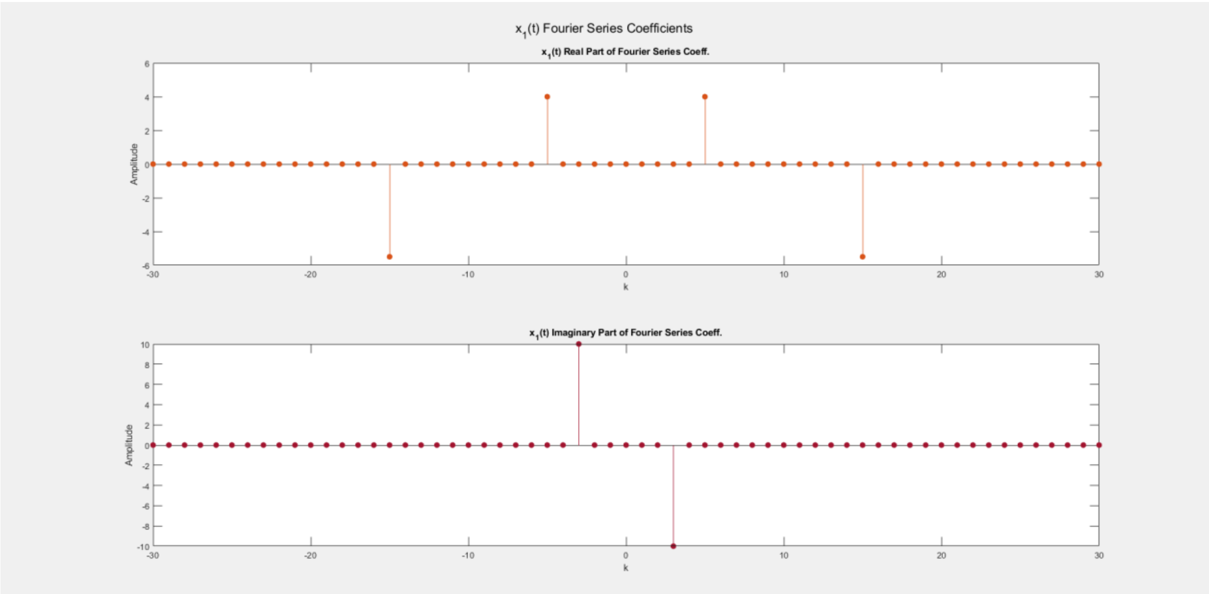

figure 3 : x1(t) Fourier Series Coefficients

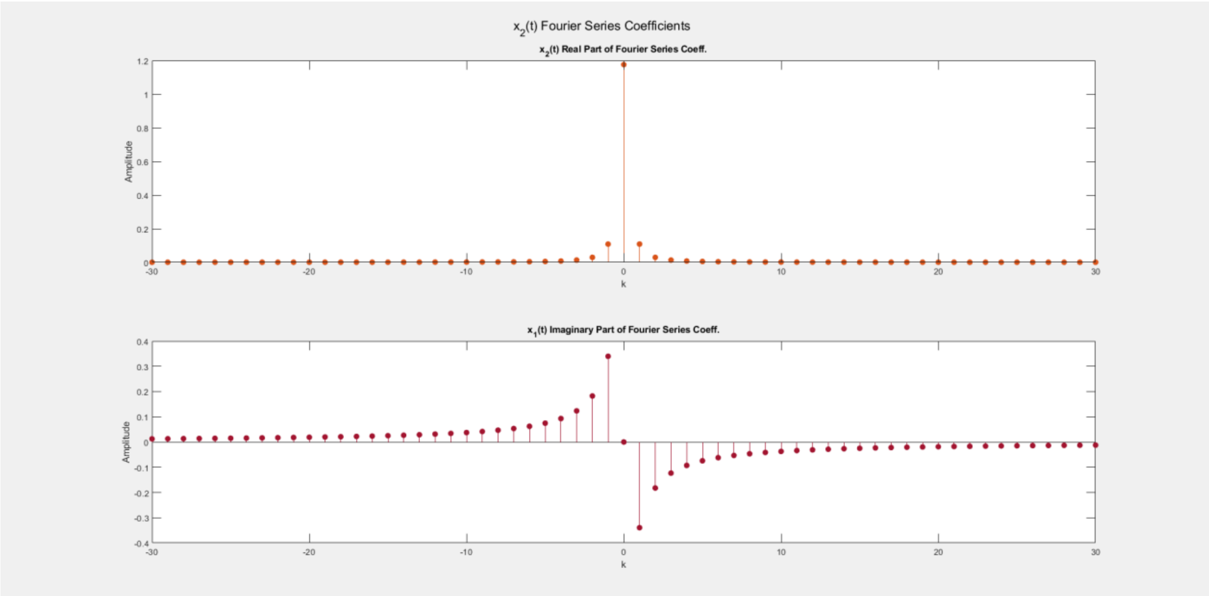

figure 4 : x2(t) Fourier Series Coefficients

```
>> lab4_1_2

x1_par_left =

  292.5000


x1_par_right =

  292.5000


x2_par_left =

    1.8152


x2_par_rigt =

    1.8061
```

figure 5 : Parseval's Relation

Part 2

In this part, we were asked to find the Fourier series coefficients of the given x3. Although it is not wanted, the operations performed manually for the accuracy and practicality of the results can be seen below. We also plotted the real and imaginary components of the coefficients using FSAnalysis on Matlab. Then, we examined the relationship between the coefficients of the given zi(t) equations and the coefficiencies of the x3 equation we have. The equations depending on the coefficient of x3 are as shown in Figure 8. We implemented the zi(t) equations we have separately in MATLAB and plotted the real and imaginary parts of the coefficients in the same way. When we compare the results we obtained with the x3 coefficients corresponding to each value, we can observe that the obtained relation is satisfied. As we stated for z1 coefficient it must be a_{-k} from observing the plot the reel part is symmetry and there is no difference but we can observe this effect on imaginary part as flipped. Using the same logic for z2 ve have mutplication (jk(pi/2) with a_k and we gat this effect from the coeff. Plot of z2. For z3 we have multiplication $(-1)^k$ so it multiplies odd part with -1 and we can clearly see this effect on plot. For z4 we get olnly real part of x3 coeff. So that we get zero at imaginary part. For z5 we have summation with itself for each value so that each value of coefficients doubles and we get this effect from the coeff plot of z5.

Part 2

$$x_3(t) = r(t) - r(t-3) - 3u(t-3) \Rightarrow T = 4s$$

$$a_k = \frac{1}{T}\int_T x_3(t)\, e^{-jk\left(\frac{2\pi}{T}\right)t}\, dt$$

$$= \frac{1}{4}\int_T x_3(t)\, e^{-jk\left(\frac{2\pi}{4}\right)t}\, dt$$

as $0 \leq t < 3 \Rightarrow x_3(t) = t$

$$a_k = \frac{1}{4}\int_0^3 t\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$a_0 = \frac{1}{4}\left(\frac{t^2}{2}\right|_0^3 = \frac{9}{8} \approx 1.125$$

$$a_1 = \frac{1}{4}\int_0^3 t\, e^{-j\frac{\pi}{2}t} = \frac{1}{4}\left(-\frac{e^{-ti\left(\frac{\pi}{2}\right)}\left(1+ti\left(\frac{\pi}{2}\right)\right)}{i\left(\frac{\pi}{2}\right)^2}\right|_0^3$$

from this way we need such unnecessary integral calculations. So that we need to use

$$g(t) = x_3(t+T_1) - x_3(t-T_1)$$

coeff of $g(t)$

$$b_k = e^{jk\omega_0 T_1} a_k - e^{-jk\omega_0 T_1} a_k \quad \text{where } \omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$$

$$b_k = a_k\left(e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}\right) = a_k\, 2j\sin(k\omega_0 T_1)$$

figure 6 : Part2 Solution

let $T_1 = 3$

$$g(t) = x_3(t+3) - x_3(t-3)$$

$$= r(t+3) - r(t) - 3u(t) - r(t-3) + r(t-6) + 3u(t-6)$$

$$\left.\begin{aligned} \text{at } t=1 &\rightarrow g[1] = 0 \\ t=2 &\rightarrow g[2] = 0 \\ t=3 &\rightarrow g[3] = 0 \\ t=4 &\rightarrow g[4] = -1 \\ t=5 &\rightarrow g[5] = -2 \\ t=6 &\rightarrow g[6] = 0 \\ &\vdots \qquad\qquad \vdots \end{aligned}\right\}$$

$$b_k = \frac{1}{N} \sum_{n=\langle N\rangle} g[n]\, e^{-jk\left(\frac{2\pi}{N}\right)n}$$

where $N = 4$

$n = 6$

and $b_0 = \cancel{e^{j0\omega_0 3}}^{\,1} a_0 - \cancel{e^{-j0\omega_0 3}}^{\,1} a_0$

$b_0 = 0$

1.125

$$\Rightarrow b_k = \frac{1}{4} \sum_{n=0}^{6} g[n]\, e^{-jk\left(\frac{\pi}{2}\right)6} = \frac{e^{-jk(3\pi)}}{4}\left[0+0+0+0+(-1)+(-2)+0\right]$$

$$b_k = \frac{-3}{4} e^{-jk(3\pi)}, \text{ for } k>0$$

$b_0 = 0$

$b_1 = \frac{-3}{4} e^{-j(3\pi)}$

$b_2 = \frac{-3}{4} e^{-j(6\pi)}$

$b_3 = \frac{-3}{4} e^{-j(9\pi)}$

then remember $b_k = a_k\, 2j \sin(k\omega_0 3)$ ($\omega_0 = \frac{\pi}{2}$)

we know $a_0 = 1.125$

$a_1 = \left(\frac{-3}{4} e^{-j(3\pi)}\right) \Big/ \left(2j \sin\left(\frac{3\pi}{2}\right)\right) =$

$a_2 = \left(\frac{-3}{4} e^{-j(6\pi)}\right) \Big/ \left(2j \sin\left(\frac{6\pi}{2}\right)\right) =$

$a_3 = \left(\frac{-3}{4} e^{-j(9\pi)}\right) \Big/ \left(2j \sin\left(\frac{9\pi}{2}\right)\right) =$

figure 7 : Part2 Solution

for $z_i(t)$ part   fourier Coeff.

$$z_1(t) = x_3(-t) \rightarrow c_k = a_{-k}$$

$$z_2(t) = \frac{d x_3(t)}{dt} \rightarrow jk\left(\frac{\pi}{2}\right) a_k$$

$$z_3(t) = x_3(t+2) \rightarrow a_k e^{jk\left(\frac{\pi}{2}\right)2} = a_k e^{jk\pi} = (-1)^k a_k$$

$$z_4 = \mathcal{E}v\{x_3(t)\} \rightarrow Re\{a_k\}$$

$$z_5 = x_3^2(t) \rightarrow \sum_{n=-\infty}^{+\infty} a_n a_{k-n}$$

figure 8 : Part 2 Solution

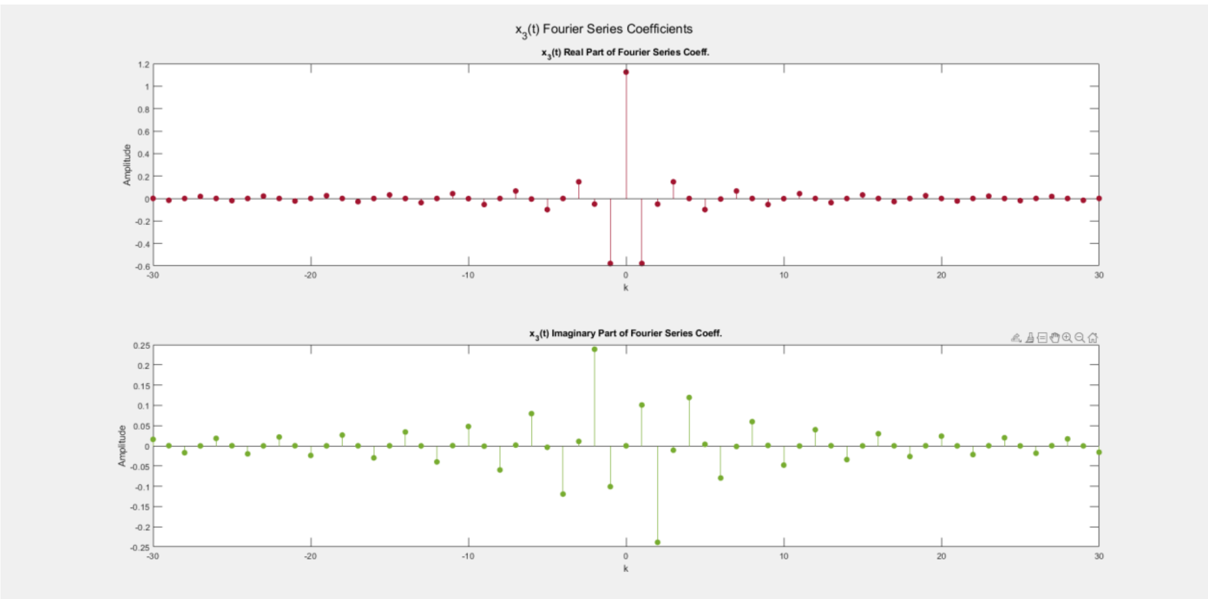

figure 9 : x3(t) Fourier Series Coefficients

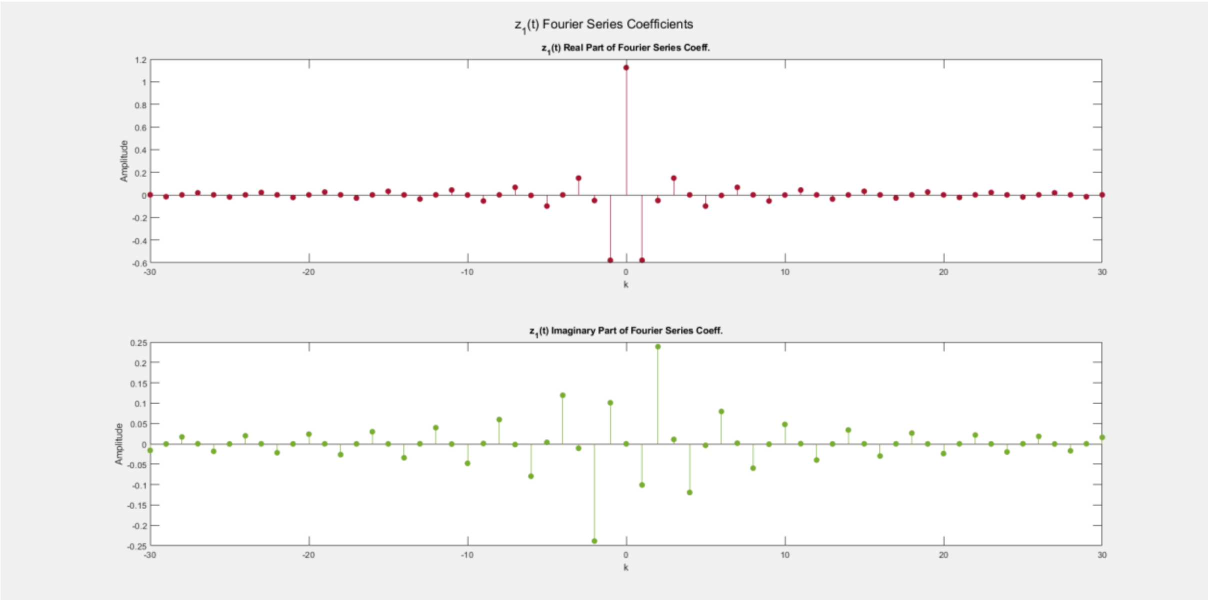

figure 10 : z1(t) Fourier Series Coefficients

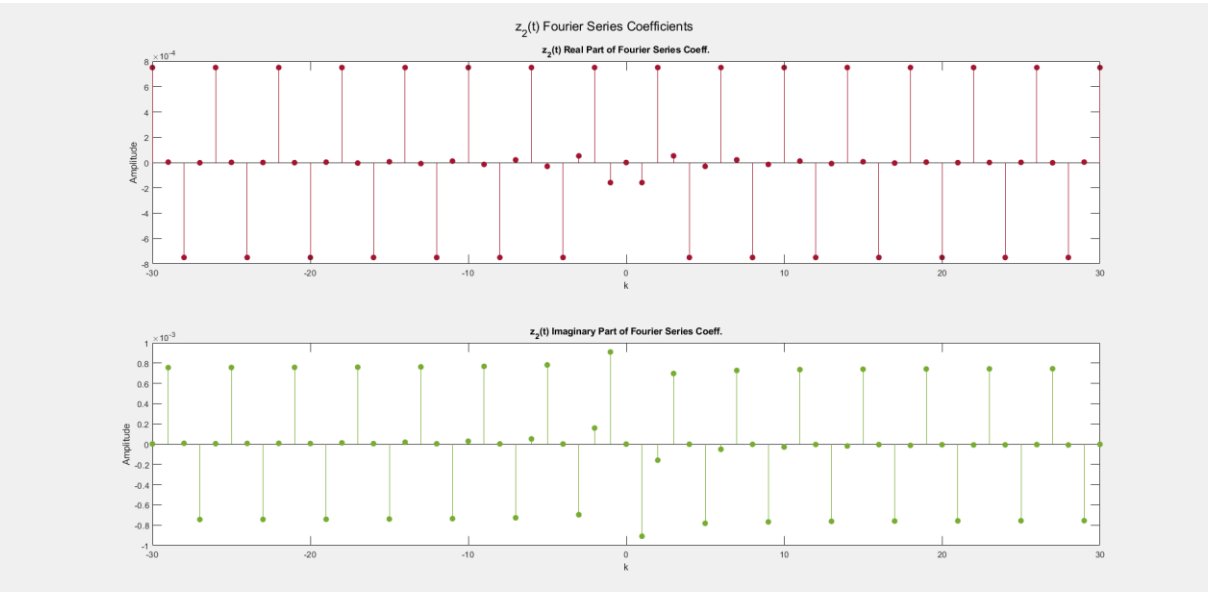

figure 11 : z2(t) Fourier Series Coefficients

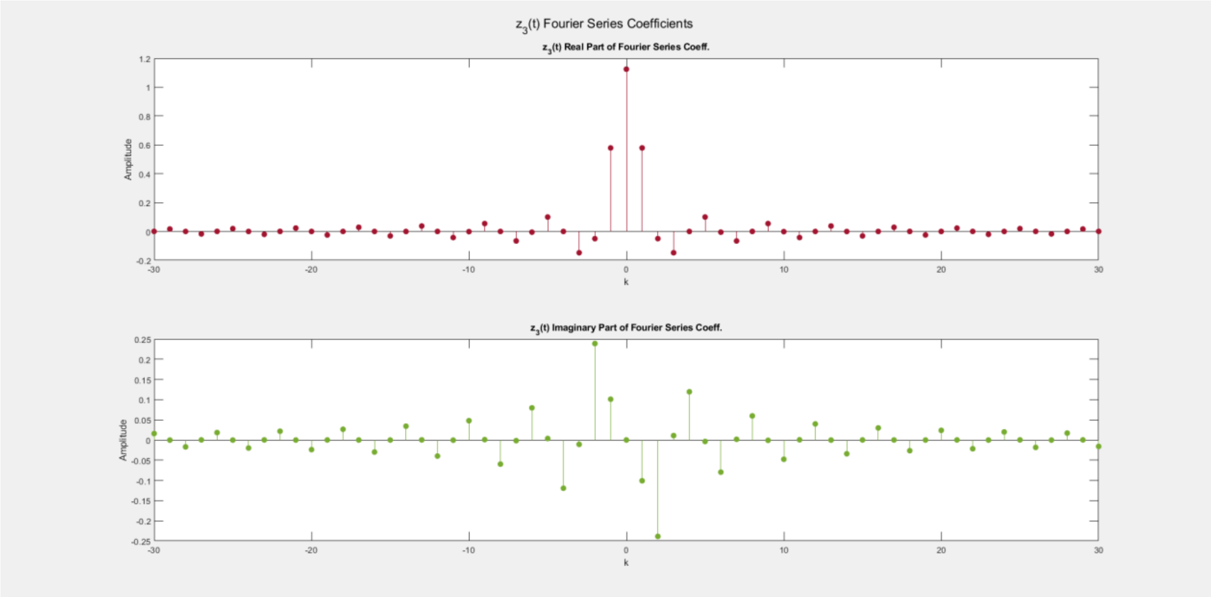

figure 12 : z3(t) Fourier Series Coefficients

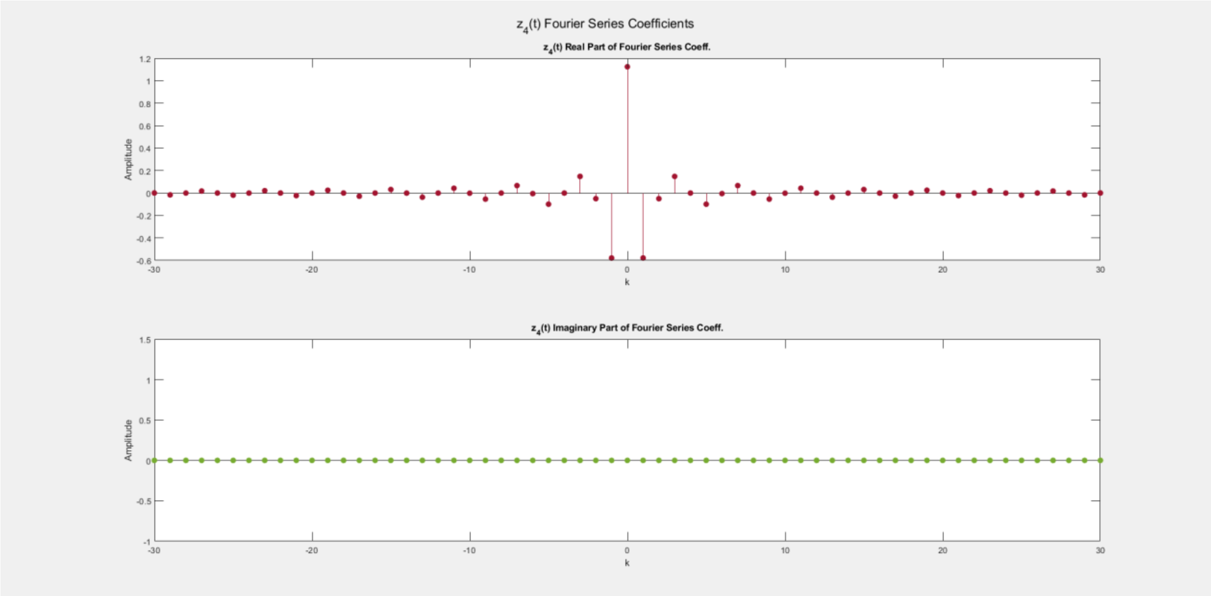

figure 13 : z4(t) Fourier Series Coefficients

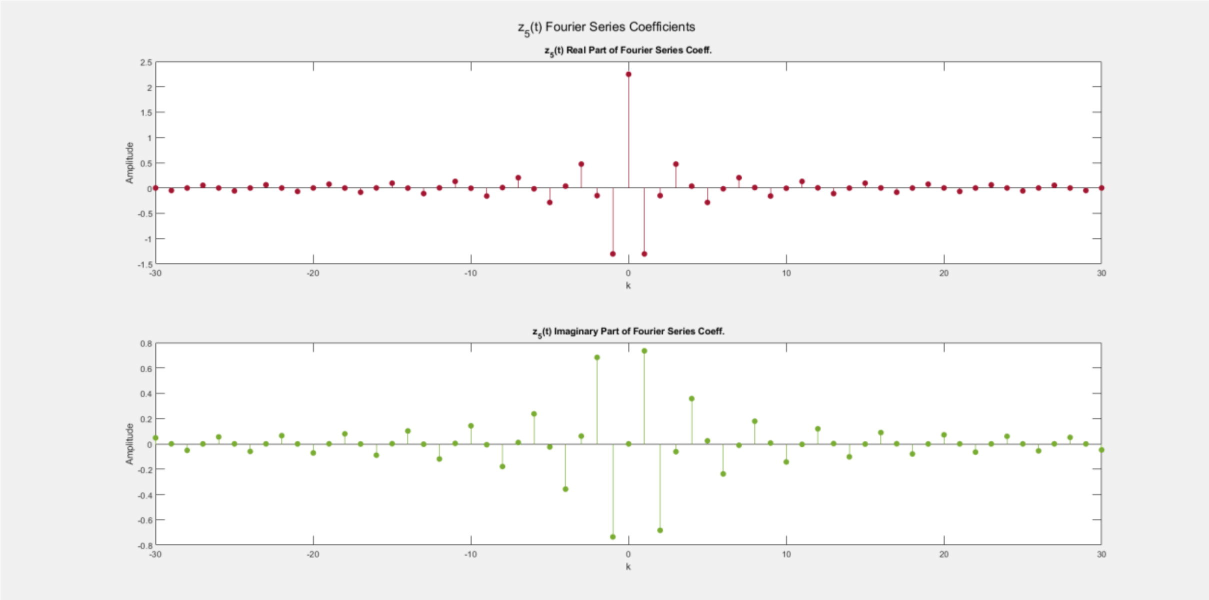

figure 14 : z5(t) Fourier Series Coefficients

<u>Part 3</u>

Part 3.1: A Second-Order System

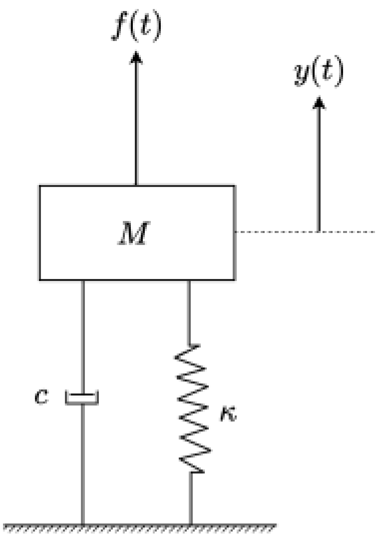

figure 15 : Deception of the second-order physical system

The input-output relationship for figure15 described by second-order differential equation as:

$$\frac{Md^2y(t)}{dt^2} + c\frac{dy(t)}{dt}Ky(t) = f(t)$$

where y(t) is the displacement with respect to the initial position of the center of mass, dy(t)/dt is the velocity, d^2y(t)/dt^2 is the acceleration of the mass in this system, and f(t) is the function that defines the exerted force on the mass with respect to time. The coefficients M, c, and K are the effective mass, damping coefficient, and stiffness of the spring, respectively.

Abstraction of the physical system is provided as a block diagram in Figure 16. Using the properties of continuous-time Fourier series coefficients, express equation terms of their Fourier series coefficients for both side, which are bk and ak. Also, we state this relation, in terms of the system's frequency response H(jω). All these calculations can be seen at figure 17.

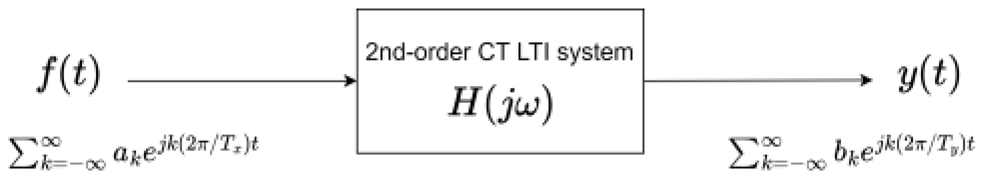

figure 16 : Abstraction of the physical system as a block diagram

Part 3.1

$$b_k = \frac{1}{T_y} \int_{T_y} y(t)\, e^{-jk\omega_0 t}\, dt = \int_{T_y} y(t)\, e^{-jk\left(\frac{2\pi}{T_y}\right)t}\, dt$$

$$a_k = \frac{1}{T_y} \int_{T_y} f(t)\, e^{-jk\left(\frac{2\pi}{T_y}\right)t}\, dt$$

$$= \frac{1}{T_y} \int_{T_y} \left[M \frac{d^2 y(t)}{dt^2} + c\frac{dy(t)}{dt} + K y(t) \right] e^{-jk\left(\frac{2\pi}{T_y}\right)t}\, dt$$

$$= \underbrace{\frac{1}{T_y} M \int_{T_0} \frac{d^2 y(t)}{dt^2} e^{-jk\left(\frac{2\pi}{T_y}\right)t}\, dt}_{\left(jk\frac{2\pi}{T_y}\right)^2 b_k \cdot M} + \underbrace{\frac{1}{T_y} c \int_{T_0} \frac{dy(t)}{dt} e^{-jk\left(\frac{2\pi}{T_y}\right)t}\, dt}_{\left(jk\frac{2\pi}{T_y}\right) b_k c}$$

$$+ \underbrace{\frac{1}{T_y} K \int_{T_0} y(t)\, e^{-jk\left(\frac{2\pi}{T_y}\right)t}\, dt}_{b_k \cdot K}$$

$$a_k = M\left(jk\frac{2\pi}{T_y}\right)^2 b_k + c\left(jk\frac{2\pi}{T_y}\right) b_k + K b_k$$

$$a_k = b_k\left(-\frac{k^2 M 4\pi^2}{T_y^2} + K + c\,jk\frac{2\pi}{T_y}\right)$$

$$= b_k\left(-Mk^2\omega_0^2 + jck\omega_0 + K\right) \Rightarrow H(j\omega) = \frac{Y(j\omega)}{F(j\omega)}$$

$$H(j\omega) = \frac{1}{K - M\omega^2 + jc\omega}$$

Systems frequency response

figure 17 : Part3.1 Solution

Part 3.2: Implementation of the Second-Order System

In this part, we implemented the given system in MATLAB, for this the values we used were determined as M=100, c=0.1 and K= 0.1. The Ts, k values we used were set to be the same as in Part 1.2. First, we discretized the given function as follows.

Part 3.2

$$M \frac{d^2y(t)}{dt^2} + c\frac{dy(t)}{dt} + K\, y(t) = f(t) = x_3(t)$$

$$\downarrow$$

$$M\left[\frac{(y[n]-2y[n-1]+y[n-2])}{\Delta T^2}\right] + c\left[\frac{(y[n]-y[n-1])}{\Delta T}\right]$$

$$+ K\, y[n] = x_3[n]$$

$$\Rightarrow y[n]\left(\frac{M}{\Delta T^2}+\frac{c}{\Delta T}+K\right) - \left[y[n-1]\left(\frac{2M}{\Delta T^2}-\frac{c}{\Delta T}\right)\right] + y[n-2]\frac{M}{\Delta T^2} = x_3[n]$$

$$y[n] = \left(\frac{M}{\Delta T^2}+\frac{c}{\Delta T}+K\right)^{-1}\left[y[n-1]\left(\frac{2M}{\Delta T^2}-\frac{c}{\Delta T}\right) - y[n-2]\frac{M}{\Delta T^2} + x_3[n]\right]$$

figure 18 : Part 3.2 Solution

Then we implemented the function we had discitized into Matlab. And we used x3 given in the previous part as input. When we plotted the input and output discrete functions we obtained with the plot command, we obtained the images in Figure 19. Then, as we did in the previous parts, we plotted the real and imaginary parts with the FSAnalsis function. The effect we observed here increased the high coefficient values of the input in both the real and imaginary parts and dampened the low values, and thus we were able to capture the coefficient values that were important to us.

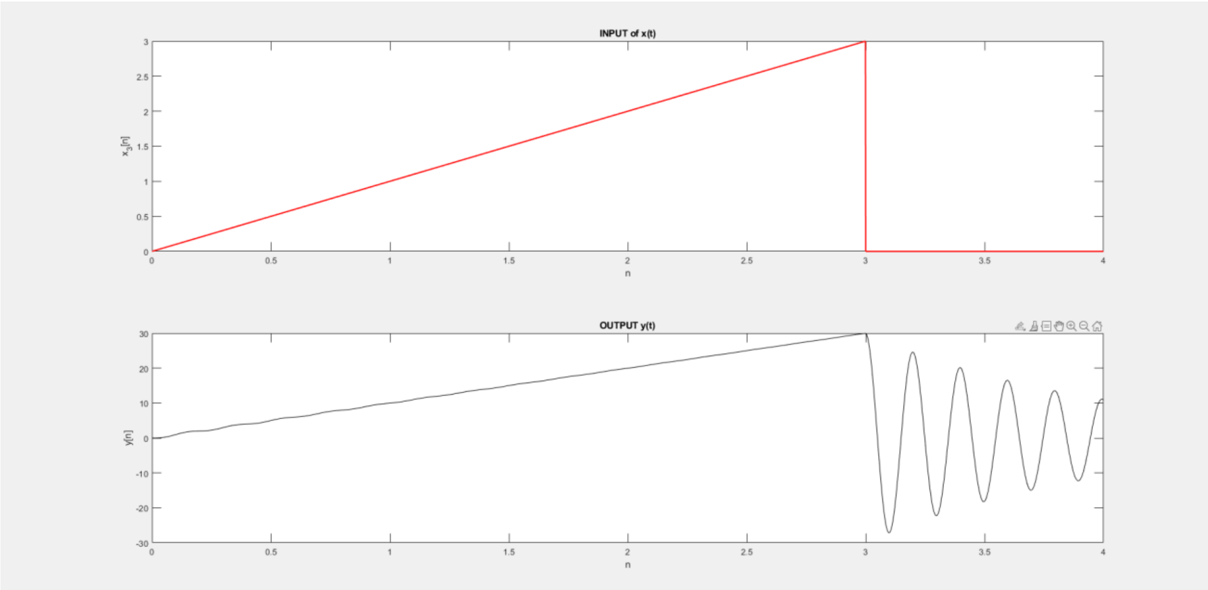

figure 19 : Input and Output Signals

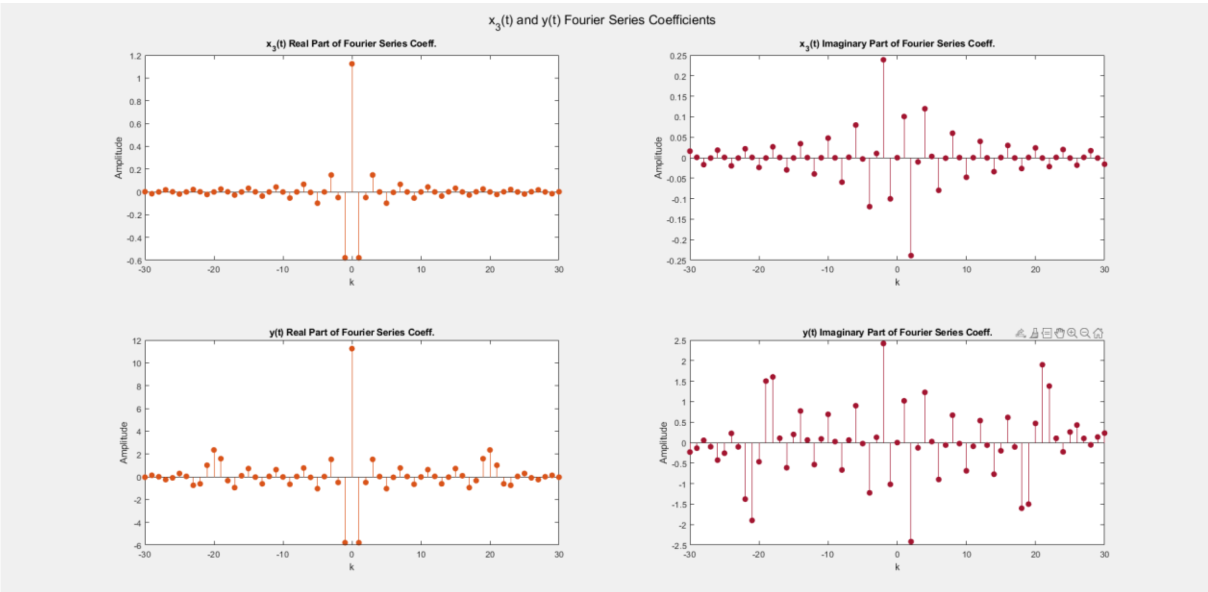

figure 20 : x3(t) and y(t) Fourier Series Coefficients

Conclusion

In this lab, we have done a Matlab implementation to find the Fourier series and coefficients and to give the desired values according to the function we gave input to. Also, the second order behavior of forrier series was examined over a system. It was observed how the Fourier series coefficients changed according to the operation performed.

Codes

FSAnalysis

```
function [fsCoeffs] = FSAnalysis(x,k)
% fsCoeffs: An array that contains the Fourier series coefficients of your signal
from -k to k
% x :  One complete period of the sampled continuous-time signal
% k: The number of two-sided Fourier series coefficients that you will estimate

N = length(x); %total number of sample in single period
n = 0:1:N-1; %sample index
fsCoeffs = zeros(1,2*k+1);
for i = -k:1:k
fsCoeffs(1,i+k+1) = (1/N)*sum(x.*exp(-1j*i*(2*pi/N)*n));
end
```

Part 1.2 : Testing the Function

```
clear
%parameters
Ts = 0.001;
k = 30;
x_k = -k:1:k;

x1 = @(t) 8*cos(10*pi*t) + 20*sin(6*pi*t) - 11 * cos(30*pi*t);
x2 = @(t) exp(-t);

x1_1 = x1(0:Ts:1-Ts);
x1_coeff = FSAnalysis(x1_1,k);

figure
sgtitle("x_{1}(t) Fourier Series Coefficients")
subplot(2,1,1)
stem(x_k,real(x1_coeff),"filled",Color="#D95319")
title("x_{1}(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,1,2)
stem(x_k,imag(x1_coeff),"filled",Color="#A2142F")
title("x_{1}(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")

x2_1 = x2(-1:Ts:(1-Ts));
x2_coeff = FSAnalysis(x2_1,k);

figure
sgtitle("x_{2}(t) Fourier Series Coefficients")
subplot(2,1,1)
stem(x_k,real(x2_coeff),"filled",Color="#D95319")
title("x_{2}(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,1,2)
```

```
stem(x_k,imag(x2_coeff),"filled",Color="#A2142F")
title("x_{1}(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")


x1_par_left = Ts*(1/1) * sum(abs(x1_1).^2)
x1_par_right = sum(abs(x1_coeff).^2)

x2_par_left = Ts*(1/2) * sum(abs(x2_1).^2)
x2_par_rigt = sum(abs(x2_coeff).^2)
```

Part 2

```
clear
%parameters
Ts = 0.001;
k = 30;
x_k = -k:1:k;


u = @(t) double(t>=0);
r = @(t) t .* u(t);

x3 = @(t) r(mod(t,4)) - r(mod(t,4)-3) - 3*u(mod(t,4)-3);
x3_1 = x3(0:Ts:4-Ts);
x3_coeff = FSAnalysis(x3_1,k);

figure
sgtitle("x_{3}(t) Fourier Series Coefficients")
subplot(2,1,1)
stem(x_k,real(x3_coeff),"filled",Color="#A2142F")
title("x_{3}(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,1,2)
stem(x_k,imag(x3_coeff),"filled",Color="#77AC30")
title("x_{3}(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")


z1 = @(t) x3(-t);
z2 = @(t) diff(x3(t));
z3 = @(t) (x3(t+2));
z4 = @(t) ((x3(t)+x3(-t))/2);
z5 = @(t) (x3(t).^2);

z1_1 = z1(0:Ts:4-Ts);
z2_1 = z2(0:Ts:4-Ts);
z3_1 = z3(0:Ts:4-Ts);
z4_1 = z4(0:Ts:4-Ts);
z5_1 = z5(0:Ts:4-Ts);

z1_coeff = FSAnalysis(z1_1,k);
z2_coeff = FSAnalysis(z2_1,k);
z3_coeff = FSAnalysis(z3_1,k);
```

```
z4_coeff = FSAnalysis(z4_1,k);
z5_coeff = FSAnalysis(z5_1,k);

%z1
figure
sgtitle("z_{1}(t) Fourier Series Coefficients")
subplot(2,1,1)
stem(x_k,real(z1_coeff),"filled",Color="#A2142F")
title("z_{1}(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,1,2)
stem(x_k,imag(z1_coeff),"filled",Color="#77AC30")
title("z_{1}(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")

%z2
figure
sgtitle("z_{2}(t) Fourier Series Coefficients")
subplot(2,1,1)
stem(x_k,real(z2_coeff),"filled",Color="#A2142F")
title("z_{2}(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,1,2)
stem(x_k,imag(z2_coeff),"filled",Color="#77AC30")
title("z_{2}(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")

%z3
figure
sgtitle("z_{3}(t) Fourier Series Coefficients")
subplot(2,1,1)
stem(x_k,real(z3_coeff),"filled",Color="#A2142F")
title("z_{3}(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,1,2)
stem(x_k,imag(z3_coeff),"filled",Color="#77AC30")
title("z_{3}(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")

%z4
figure
sgtitle("z_{4}(t) Fourier Series Coefficients")
subplot(2,1,1)
stem(x_k,real(z4_coeff),"filled",Color="#A2142F")
title("z_{4}(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,1,2)
stem(x_k,imag(z4_coeff),"filled",Color="#77AC30")
title("z_{4}(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
ylim([-1 1.5])
```

```
figure
sgtitle("z_{5}(t) Fourier Series Coefficients")
subplot(2,1,1)
stem(x_k,real(z5_coeff),"filled",Color="#A2142F")
title("z_{5}(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,1,2)
stem(x_k,imag(z5_coeff),"filled",Color="#77AC30")
title("z_{5}(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
```

Part 3

```
clear;
%parameters
M = 100;
c = 0.1;
kk = 0.1;

Ts = 0.001;
k = 30;
x_k = -k:1:k;

u = @(t) double(t>=0);
r = @(t) t .* u(t);


x3 = @(t) r(mod(t,4)) - r(mod(t,4)-3) - 3*u(mod(t,4)-3);
x3_1 = x3(0:Ts:4-Ts);

y = zeros(1,length(0:Ts:(4.5-Ts)));


for n= 3:length(0:Ts:4.5-Ts)
y(n) = (1/(M/(1^2)+c/1+kk))*(y(n-1)*(2*M/(1^2)-c/1)-y(n-2)*(M/1^2) +x3(n));
end

figure;

subplot(2,1,1)
plot(0:Ts:4-Ts,x3_1,Color="r")
xlabel("n")
ylabel("x_{3}[n]")
title("Input Signal x_{3}[n]")
subplot(2,1,2)
plot(0:Ts:4.5-Ts,y,Color="k")
xlabel("n")
ylabel("y[n]")
title("Output Signal y[n]")

x3_coeff = FSAnalysis(x3_1,k);
y_coeff = FSAnalysis(y,k);
```

```
figure
sgtitle("x_{3}(t) and y(t) Fourier Series Coefficients")
subplot(2,2,1)
stem(x_k,real(x3_coeff),"filled",Color="#D95319")
title("x_{3}(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,2,2)
stem(x_k,imag(x3_coeff),"filled",Color="#A2142F")
title("x_{3}(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")

subplot(2,2,3)
stem(x_k,real(y_coeff),"filled",Color="#D95319")
title("y(t) Real Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
subplot(2,2,4)
stem(x_k,imag(y_coeff),"filled",Color="#A2142F")
title("y(t) Imaginary Part of Fourier Series Coeff.")
xlabel("k")
ylabel("Amplitude")
```