

DESIGN OF INDUCTOR AND TRANSFORMER

Project Report

EE564 – Project 1

Furkan KARAKAYA
1937051

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INDUCTOR DESIGN

An inductor is a passive basic circuit component that is used in many applications. An inductor is constructed by winding a cable around a magnetic core. The inductor specifications depend on the number of turns of winding and magnetic characteristics of core. As a result, for different applications, different type of cores is preferred. In this study, a powder core, KoolM μ ¹ from the Magnetics, is selected so that it might be used in a power electronics' converter design, which requires higher energy storage capability. Contrary to other core types, powder cores are produced to store energy in the air gaps that exist in the core. Including the air gaps inside the core, the permeability range of the powder cores tend to be small, so the relative permeability of the selected core, 0077620A7, is 125 for the linear region. As shown in Figure 1, the B-H curve is obtained using the curve-fitting parameters given by manufacturer² and it can be seen in (1).

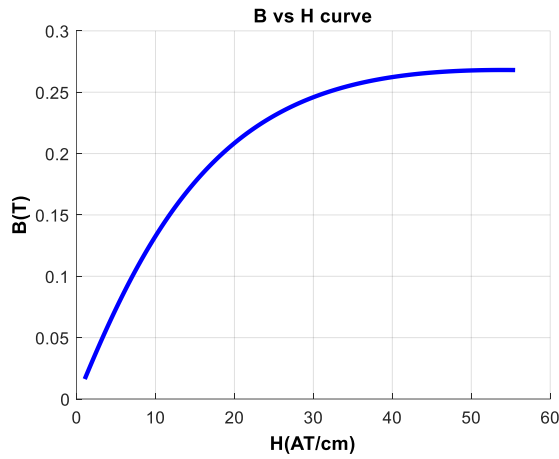


Figure 1: B vs H curve

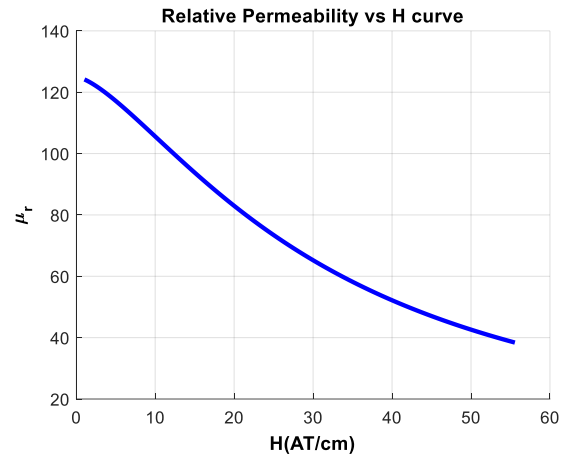


Figure 2: Relative Permeability vs H curve

$$\%Initial\ Permeability = \frac{1}{0.01 + 6.345E-5 * H^{1.462}} \quad (1)$$

Using the Figure 1, the point where the magnetic field strength is equal to 10 AT/cm is assumed as the end of the linear region. Also it is assumed that the rated DC current is 4A. Therefore, the required number of turn to operate in that point is calculated by eqn. (2) and it is 36.

$$N = \frac{H * l_{middle}}{I_{dc}} \quad (2)$$

PART A – Analytical Calculations

In the analytical calculations, the inductance for the cases linear & homogeneous core, linear & nonhomogeneous core, nonlinear & homogenous core and nonlinear & nonhomogeneous core is calculated.

To begin with, in these four different cases, the nonlinear & nonhomogeneous core is the most realistic one. However, it is also the most complex one to solve analytically. The linear core means that the core

¹ <https://www.mag-inc.com/Media/Magnetics/Datasheets/0077620A7.pdf>

² <https://www.mag-inc.com/Media/Magnetics/File-Library/Product%20Literature/Powder%20Core%20Literature/2017-Magnetics-Powder-Core-Catalog.pdf?ext=.pdf>

is never saturated which results from the constant permeability. Therefore, for linear core calculations the relative permeability is taken as 125 in this study. On the other hand, the permeability vs H curve, given in Figure 2, is used as a reference for nonlinear core calculations. Furthermore, the homogeneous core assumption is based on the same amount of flux flow in the core from innermost circle to the outermost circle. Thus, this assumption claims the arbitrary circles in the toroid core have the same reluctance. However, in reality, the innermost circle has the lowest reluctance, so the magnetic field strength is higher in the innermost circle than the others, which results in the possibility of the saturation in the inner circles of the core.

In analytical calculation, the core is discretized, and for each circle, the reluctance is calculated using the eqn. (3). Then, the resulting total reluctance is calculated by paralleling them.

$$R = \frac{l}{\mu * A} \quad (3)$$

For each cases, the basic induction calculation strategy is finding the equivalent reluctance and then the induction is calculated using the eqn. (4) and the results are provided in Table 1.

$$L = \frac{N^2}{R} \quad (4)$$

Linear	Homogenous	Current (A)	Inductance(μH)
✓	✓	4	509
✓	×	4	521
×	✓	4	430
×	×	4	436
✓	✓	6	509
✓	×	6	521
×	✓	6	382
×	×	6	386

Table 1: Analytically calculated inductances when N is 36

In order to observe the differences among the cases, the inductance vs turn number and inductance vs excitation current graphs are plotted as shown in Figure 3 and 4. In both Figure 3 and 4, it is seen that the inductance result deviation is higher for linear and nonlinear cores whereas the homogeneity changes inductance minimally. Therefore, it can be deduced that the inductance homogenous inductance calculation is a useful approximation. On the contrary, the results are significantly different with respect to the linearity. In Figure 3, we observe that the inductance increases as expected for increasing turn number. This relationship was explained in Eqn. (4). In addition, it is seen that for lower turn numbers, the inductances of linear core and nonlinear core calculations overlap as expected but increasing the turn number since the core starts to be saturated slightly, the results become different. However, we observe the inductance increases in each case when the turn number is increased. On the other hand, when the excitation current is increased keeping the turn number constant, the inductance becomes smaller because the core is saturated so the reluctance is high. Increasing the current only saturates the core so it affects the inductance indirectly. This effect is not seen when the core is linear.

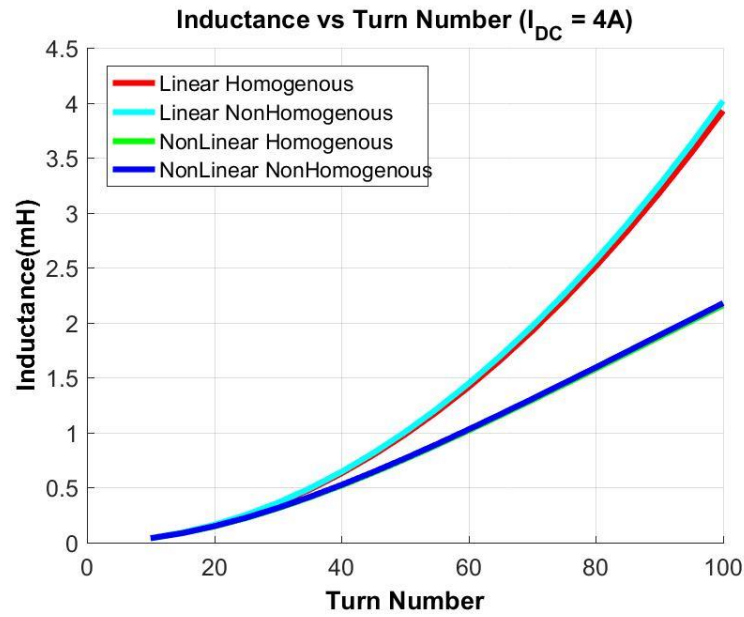


Figure 3: Inductance vs Turn Number

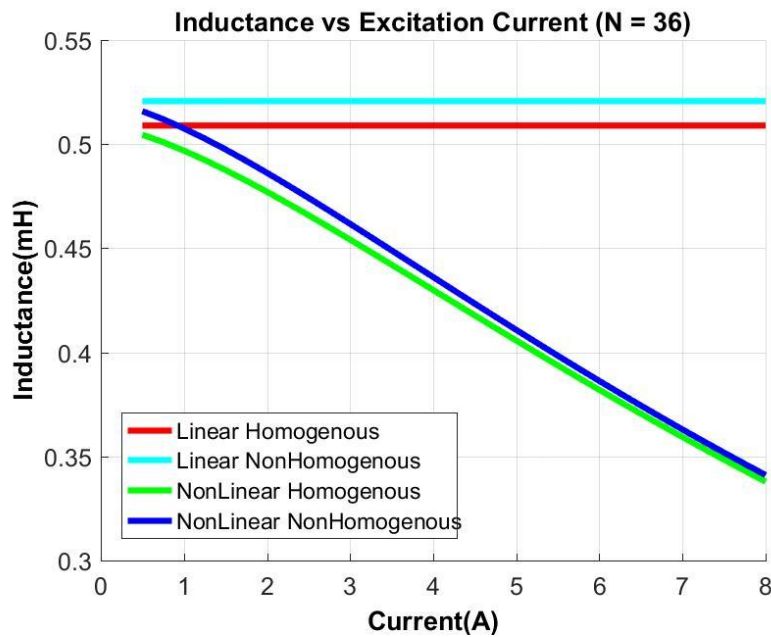


Figure 4: Inductance vs Excitation Current

When the air gap introduced, the equivalent reluctance increases significantly because the air has the lowest permeability, which shows a material magnetic opacity. Therefore, when the air gap exists, the inductance is much lower. Another concept is the leakage inductance which is there are fluxes around the gap with decreasing amplitude as it goes away than the gap. Inevitably, the fringing exists because in air there are actually many paths for flux to flow. The flux follows each path with different amplitudes with respect to the reluctance that depends on the length and area. Therefore, if the fringing fluxes are not taken into consideration the resulting inductance calculation will be slightly lower than the case where the fringing is taken into account. It is because of the fact that the inductance shows how much flux is linked for the given current. Therefore, if the fringing flux is not counted it results in the lower flux linkage so the inductance. The fringing flux is illustrated in Figure 5.

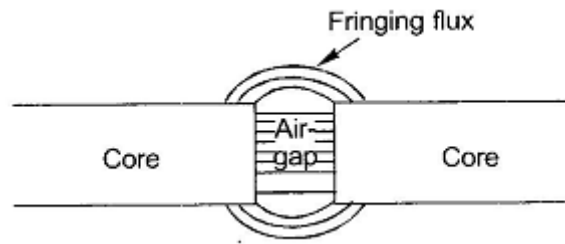


Figure 5: Fringing Flux Illustration

For estimation the effect of fringing, the method proposed in the Mohan's book³ is used. This method suggests to calculation of the air gap reluctance such that the axial length of the air gap is extended as gap length as shown in Figure 6.

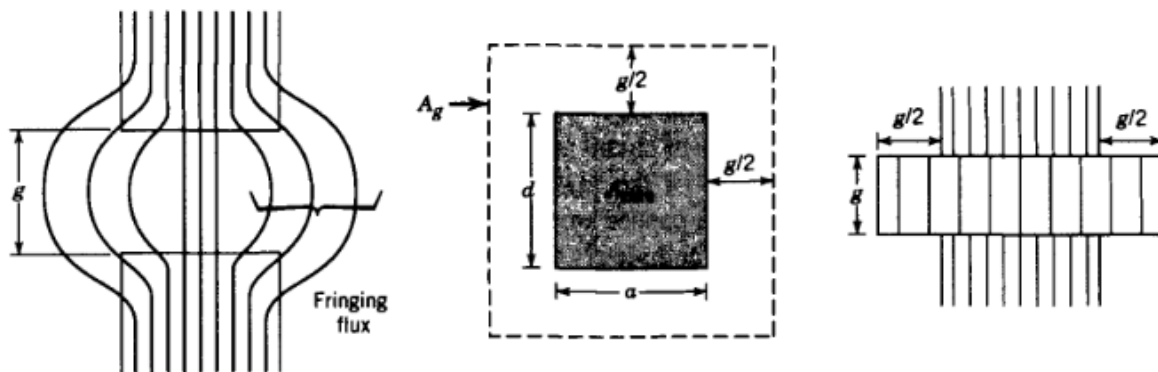


Figure 6: The method to estimate the effect of the fringing

Based on this method, the inductances are calculated as 178 μ H if the fringing does not exist and 194 μ H if the fringing exists.

PART B – Finite Element Analysis

The finite element programs calculate the magneto static solution using the mesh networks. In this project, the toroid core is also analyzed in Maxwell software. The core is plotted in 2D and material B-H parameters are imported into Maxwell as a dataset. The conductors are modeled as circles inside & outside the core. Each conductor is excited with the current whose amplitude is rated current times turn number. The flux density distribution in the core and air is given in Figure 7. Since the core has constant permeability, it is not saturated and we see that the flux density increases up to 0.24 T inside the core. However, when the core is not linear, we observe the inner discs are saturated slightly, so they have higher reluctance, which reduces the flux density. It is also available in Figure 8.

When the air gap is introduced, we start to observe lower flux densities and fringing fluxes. The figures 9-12 shows the results for both linear core and nonlinear core with air gap.

³ Power Electronics, Ch. 30, P.758, Figure 30-10

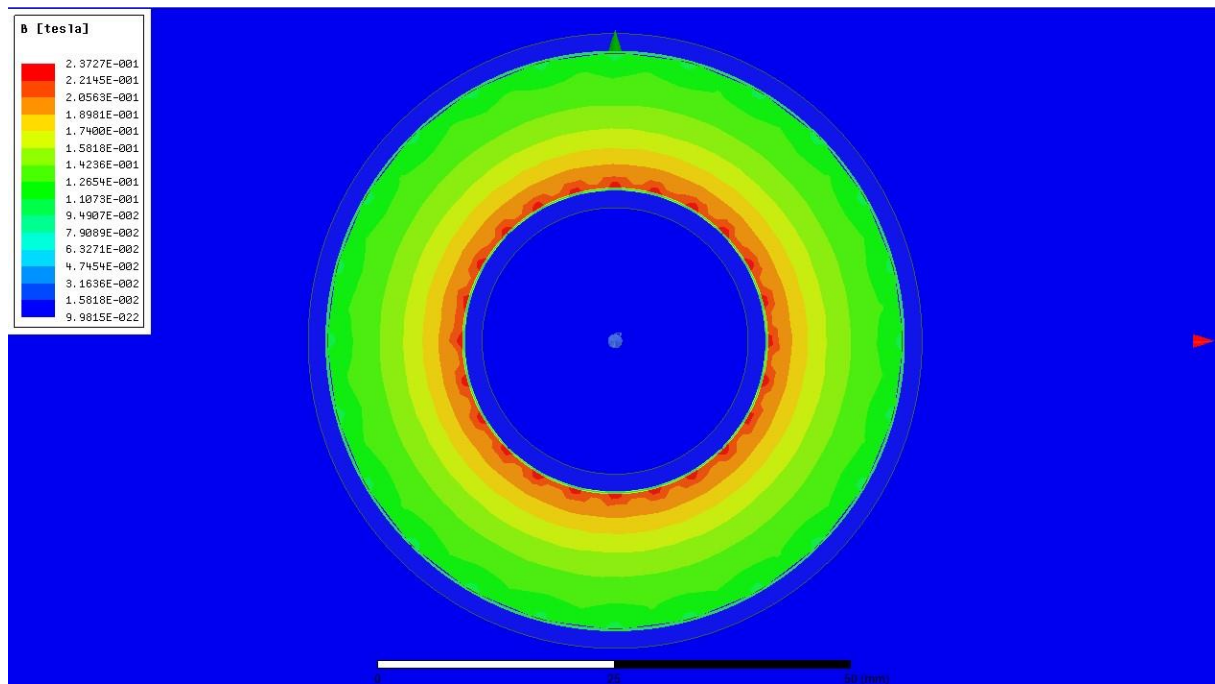


Figure 7: Flux Density for Linear Core

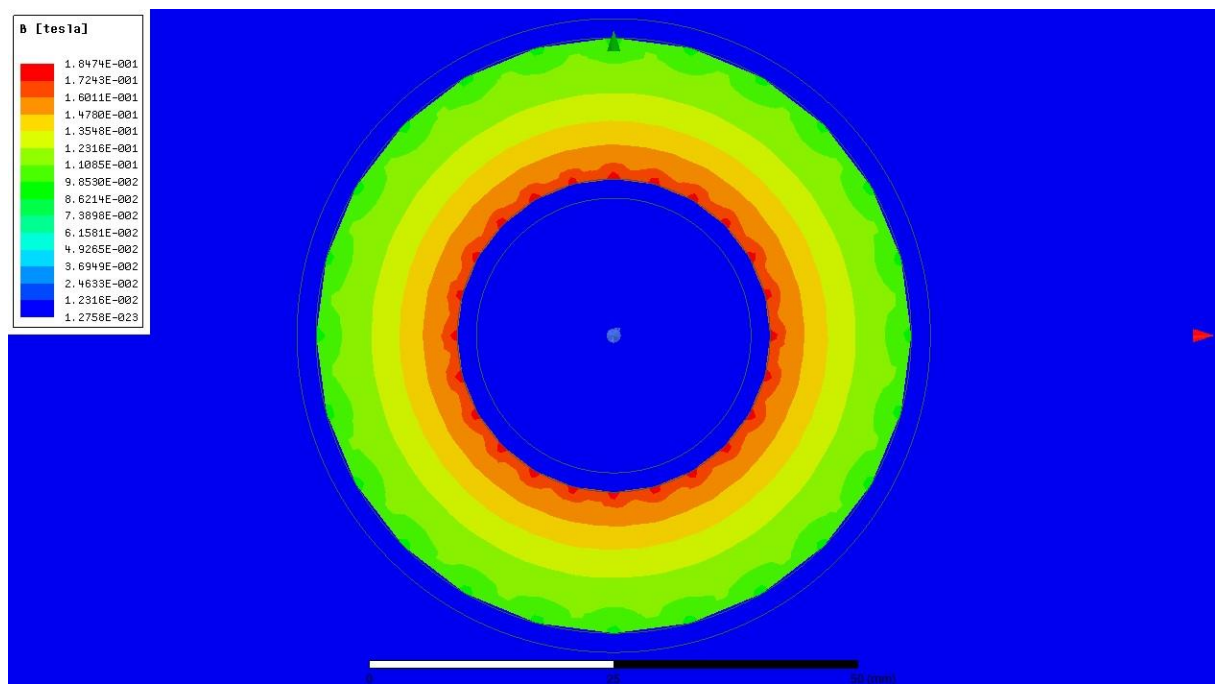


Figure 8: Flux Density for Nonlinear Core

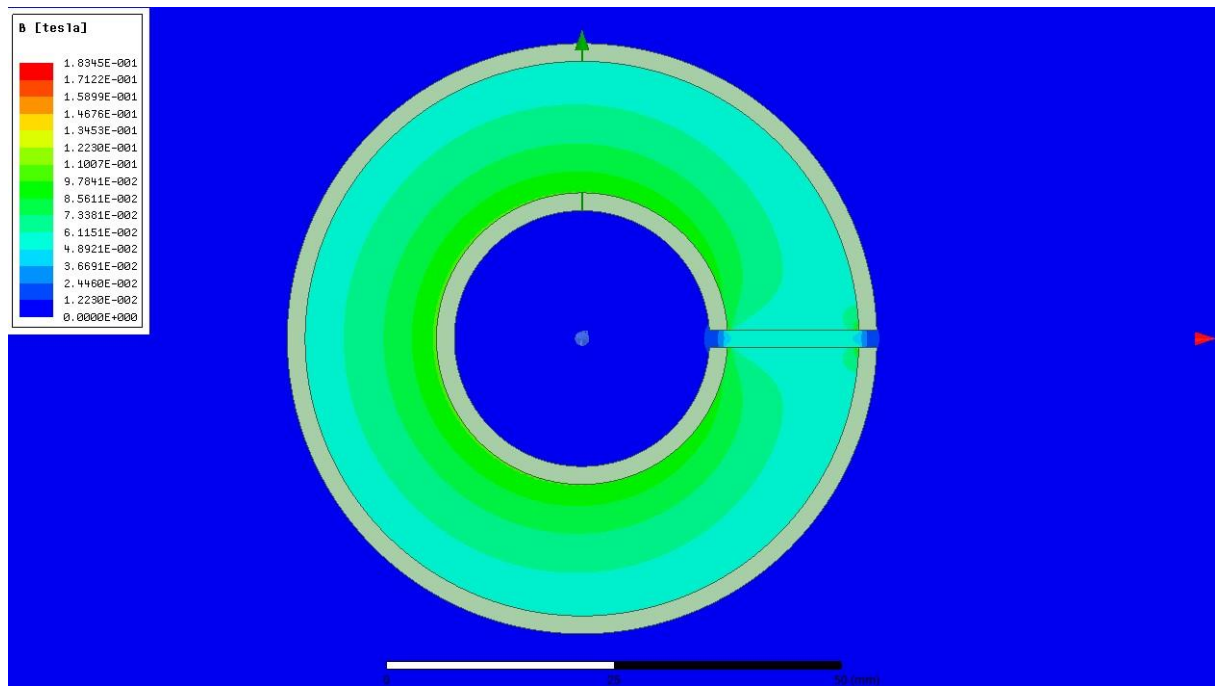


Figure 9: Flux Density for Linear Core with Air Gap

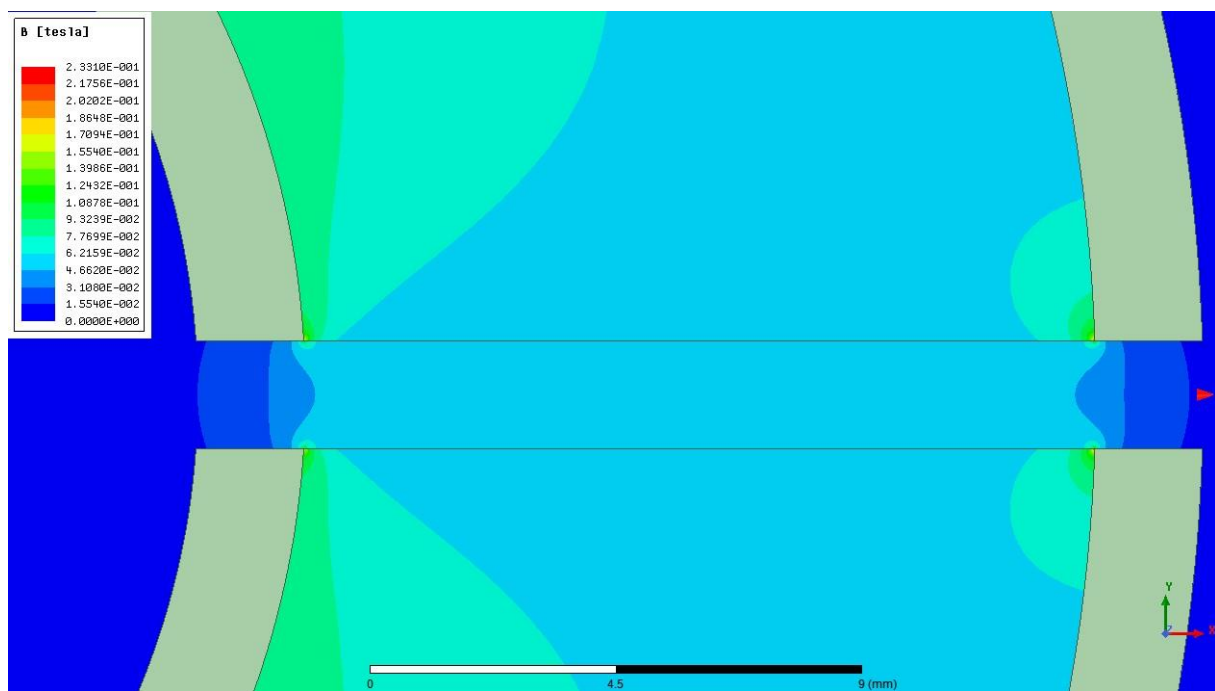


Figure 10: Flux Density around the Gap for Linear Core

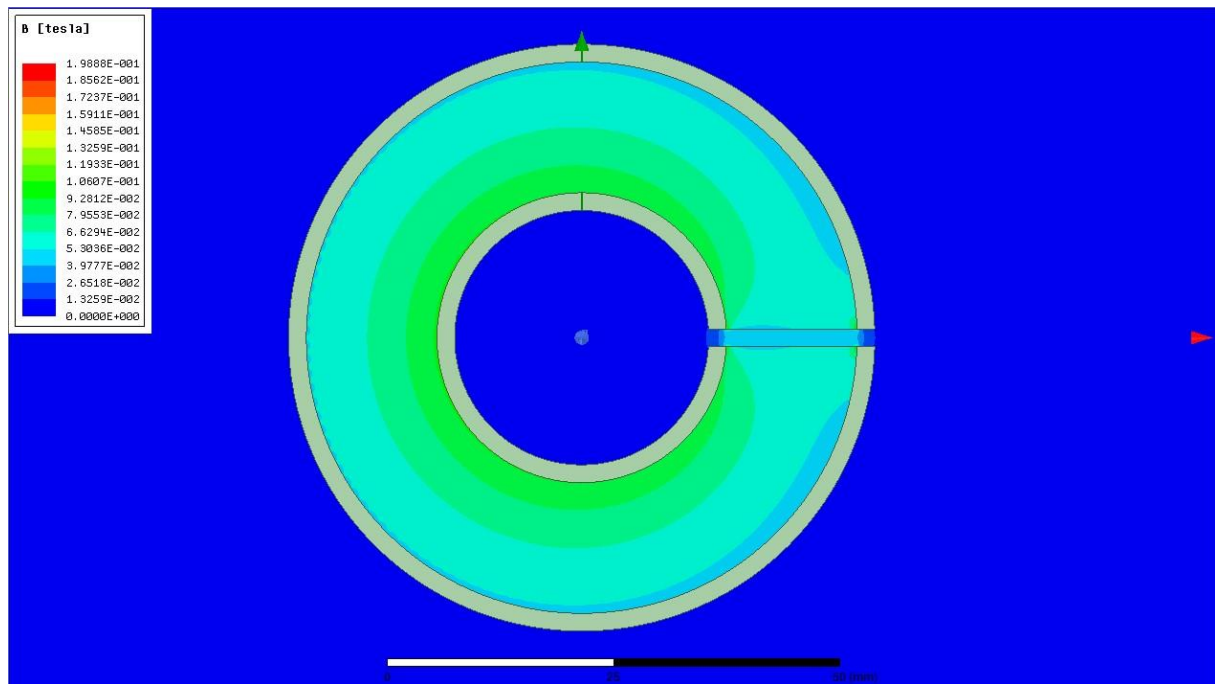


Figure 11: Flux Density for Nonlinear Core with Air Gap

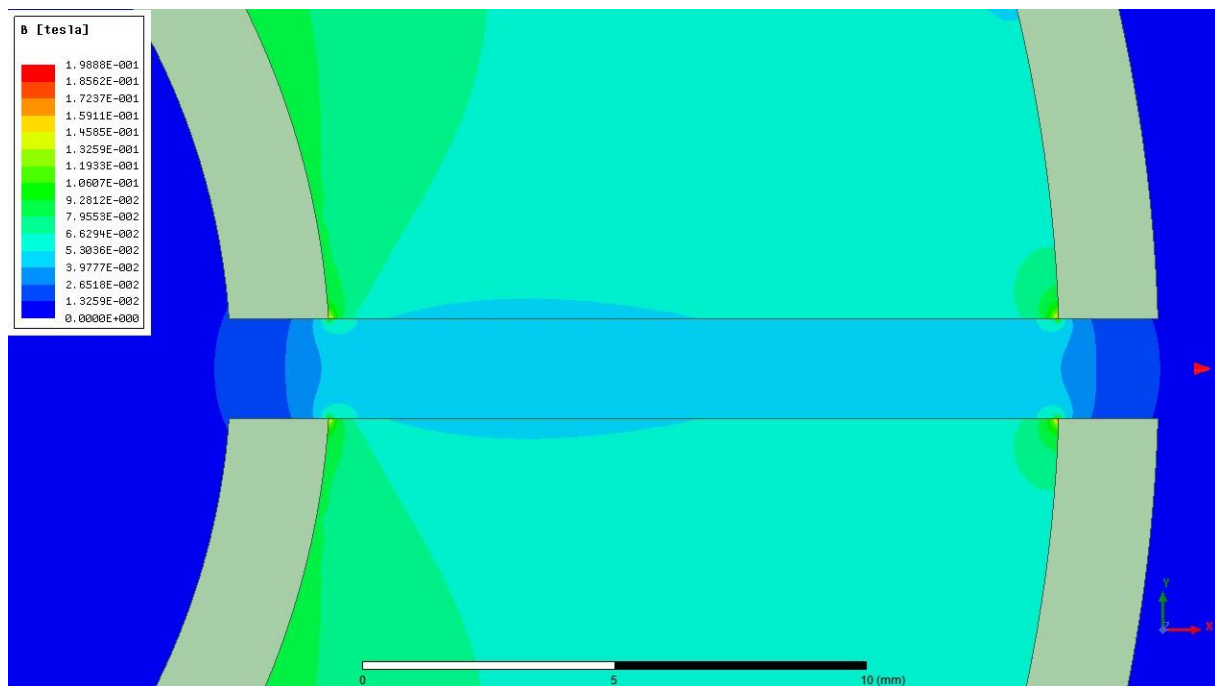


Figure 12: Flux Density around the Gap for Nonlinear Core

Linear	Air Gap	Mutual Ind. (μH)	Leakage Ind. (μH)	Total Ind. (μH)	Analytical Ind. (μH)
×	✓	167	4	171	-
✓	✓	171	4	175	178
×	×	435	0	435	436
✓	×	520	0	520	521

Table 2: Inductances Calculated in Finite Element Analysis

The results provided in Table 2, clearly shows that the inductance calculations for analytical and finite element analysis are very similar. Small differences occur due to the same reason for analytical calculation when the core is homogenous and nonhomogeneous. In analytical calculations, when the core is nonhomogeneous, we observe the inductance is slightly higher. It is because the equivalent reluctance is lower than the actual one when the core is nonhomogeneous due to lower flux flows in outer discs. So, the nonhomogeneous core has higher inductance. The same logic works here, even though analytical calculations are conducted for 40 discretized part of toroidal core, when the finite element analysis is conducted it divides core much smaller pieces, so that it can calculate the reluctance more accurately. Moreover, it is seen that the leakage inductance is zero when the air gap does not exist because the core has very high permeability with respect to the air. Therefore, the flux in the air is nearly zero. Plus, the circular conductor model might be effective on this issue. There might be a leakage inductance if the conductors were modeled more realistic that is each coil to be modeled. In finite element analysis, to calculate the inductance, three lines which are orthogonal to core axial axis are plotted and the total flux is calculated for each line. As seen on the Figure 13 and 14, the blue lines are plotted to see leakage inductance while the orange one to calculate mutual inductance. Then the total flux is multiplied with the turns number so as to find flux linkage. Dividing it by the excitation current the result is obtained for 1m depth as a property of 2D analysis. Thus, it is scaled to core's height.

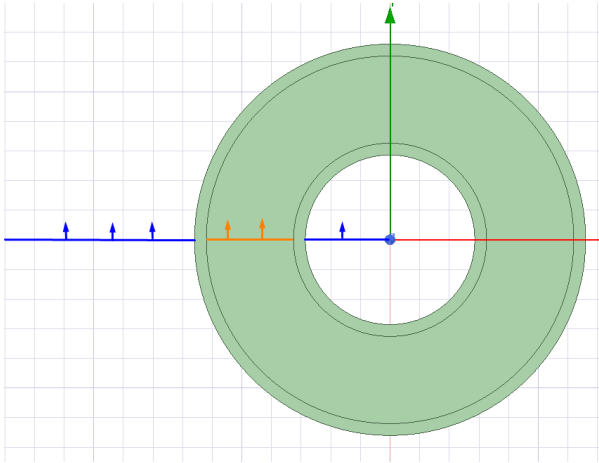


Figure 143: Inductance Calculation Lines

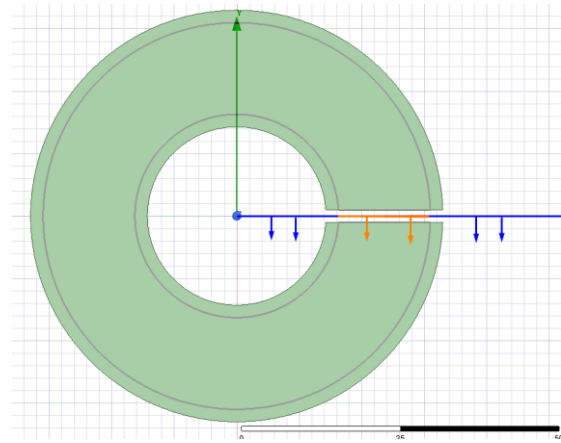


Figure 134: Inductance Calculation Lines for Air Gap

In finite element analysis, 2D or 3D analysis can be conducted. However, for the objects which have uniform cross section, 2D analysis is simpler method. Unlike 2D, if there is no uniform cross section,

3D analysis gives more accurate result. Therefore, for this project, 2D analysis is preferred for its simplicity because the toroidal core has a uniform cross section.

TRANSFORMER DESIGN

Transformers are widely used in power systems and power electronics area. They are preferred to manipulate voltage and current levels with higher efficiency performances for AC excitation. Here, it is acknowledged to design a step down transformer from 34.5 kVrms to 25 kVrms for 50 Hz line frequency. Table 3 gives more illustrative view for design requirements.

S(VA)	500000
Vin(Vrms)	34500
Vo(Vrms)	25000
f(Hz)	50
Temp Min(Celc)	-30
Temp Max(Celc)	50
FF	0.3
J(A/mm2)	3

Table 3: Design Requirements

Here for this inputs, the following results turns number ratio, input and output currents, required cable cross section areas, and window area can be calculated. The turns number ratio is proportional to input and output voltage ratios as seen in eqn. (5). Then for each side the corresponding currents and cable cross section areas are calculated for the given current density, eqn. (6), (7). Lastly, window area is calculated using the areas of each cabling and turns number, eqn. (8). Therefore, a turn number should be selected to find window area.

$$\frac{N_1}{N_2} = \frac{V_{in}}{V_{out}} \quad (5)$$

$$I_{in-out} = \frac{S}{V_{in-out}} \quad (6)$$

$$A_{in-out} = \frac{I_{in-out}}{J} \quad (7)$$

$$Window\ Area = \frac{N_1 * A_{in} + N_2 * A_{out}}{FF} \quad (8)$$

Afterwards, the core should be selected to find the volume and mass of the core. The core should be capable of operating without saturation for higher efficiency. The given core specs, in Table 4, are belong to steel core as an example.

B(T)	1.2
H(A/m)	1000
Core Loss(W/kg)	1.363
Lamination Thickness(m)	3.50E-04
Density(kg/dm3)	7.85
Core Cost (\$/kg)	3

Table 4: Core Specs

Therefore, having this core specs, the cross sectional area can be calculated using eqn. (9). Then, assuming square core and window the middle length for the core and the longest edge of the core can be calculated. Also knowing the core area and middle length the volume of the core can be calculated, so with the given density the mass of the core also to be calculated.

$$A_{core} = \frac{V_{in,rms}}{4.44 * N_1 * f * B} \quad (9)$$

Moreover, the manufacturer provides the core loss (hysteresis and eddy) per kilogram. Then total core loss can be estimated by using eqn. (10). By the way, the lamination thickness is a criterion for efficiency performance of the core. If its lamination is tight, then it means the eddy losses are very small. Although the tightly laminated cores have better performance, their prices are higher due to longer manufacturing process.

$$Core\ Loss(W) = LossConstant \left(\frac{W}{kg} \right) * Mass(kg) \quad (10)$$

Then knowing the input and output number of turns and operation temperature the cable resistances and mass can be calculated. In general, the cables are copper, so the winding specs provided in Table 5 belong to copper.

Density(kg/m3)	8940
Resistivity(ohm-m), ρ @20C	1.68E-08
Temperature Coefficient (α)	0.0038
Winding Cost (\$/kg)	8

Table 5: Winding Specs

The resistance increases with increasing temperature in general for the copper also. Therefore, to find out maximum copper loss the resistance should be calculated for the maximum operation temperature which is 50°C for this case. The temperature dependent resistivity formula is given in eqn. (11). After finding the resistivity the input and output resistances are calculated using eqn. (12), where it is assumed that each side winding has a mean path radius as overflowed core length by a quarter of window length. Additionally, copper mass can be calculated finding the copper total volume.

$$\rho_{Tmax}(ohm - meter) = (1 + \alpha * (T_{max} - 20)) * \rho_{20^\circ C} \quad (11)$$

$$R_{in-out} = N_{in-out} * \rho_{Tmax} * \frac{2\pi * L}{A_{in-out}} \quad (12)$$

Now, it is possible to calculate copper loss and total loss as well. Copper loss is calculated by eqn. (13). Since the core loss and copper loss are calculated, the efficiency can be calculated using eqn. (14). The power factor is assumed to be 1.

$$\text{Copper Loss} = I_{in}^2 * R_{in} + I_{out}^2 * R_{out} \quad (13)$$

$$\eta = \frac{(V_{out} * I_{out} * pf)}{(V_{out} * I_{out} * pf) + \text{Copper Loss} + \text{Core Loss}} \quad (14)$$

Now, it is possible to calculate mutual and leakage inductances of the transformer. Eqn. (15) is the common inductance formula which is used to calculate mutual inductance. However, the leakage inductance is calculated assuming that it has the magnitude of 2% of the base impedance of input side and output side, eqn. (16).

$$L_{mutual}(H) = \frac{N^2}{R}, R = \frac{l_{middle}}{\mu_{core} * A_{core}} \quad (15)$$

$$2\pi * f * L_{leak,in-out}(H) = \frac{V_{base,in-out}}{I_{base,in-out}} * 0.02 \quad (16)$$

Lastly, the cost should be calculated. There are two type of cost, one of them is the material cost which should be paid for only the first time. The second cost is about the losses: the price of the energy lost. These losses are calculated using the eqn. (17) and (18) respectively. The α and β are the prices of the core and copper per kilogram and c is the price of lost energy per kwh.

$$\text{MaterialCost} = \alpha * \text{CoreMass} + \beta * \text{CopperMass} \quad (17)$$

$$\text{LossCost} = c * \text{TotalLoss}(kW) * \text{Year} * 365 * 24 \quad (18)$$

Up to now, eqns. from (5) to (18) explains the design steps. However, to find the optimal solution there are some tricks. Designing a transformer, it is aimed to have the best efficiency with lowest cost and lowest volume. In order to maximize the energy, the copper loss and core loss should be close as possible as. Since the transformers are designed to be used for long years, having low efficiency results in much more money loss than the material cost. Therefore, buying a core with tight laminations will be more profitable. To reduce the material cost, the volume of the core and copper should be minimized because the prices are given per kilogram. Thus, to reduce volume a core material with higher magnetic flux density should be selected; however, actually it means you will be pay more for that material. Thus, the optimum solution should be searched for the material selection also.

In Figure 15, it is seen that just changing the turn numbers, the amount of losses changes significantly. As stated earlier, when the core loss and copper loss are the closest to each other, the total loss has the lowest value which results in higher efficiency ratio. This fact is reflected also the Figure 16 where the costs vs turn number is plotted. Firstly, it is seen that the material cost is much lower than the loss cost for 20 years. Therefore, aiming the efficiency is very important. To sum up, for 1000 turns of output, the transformer obtains the most preferable ratings.

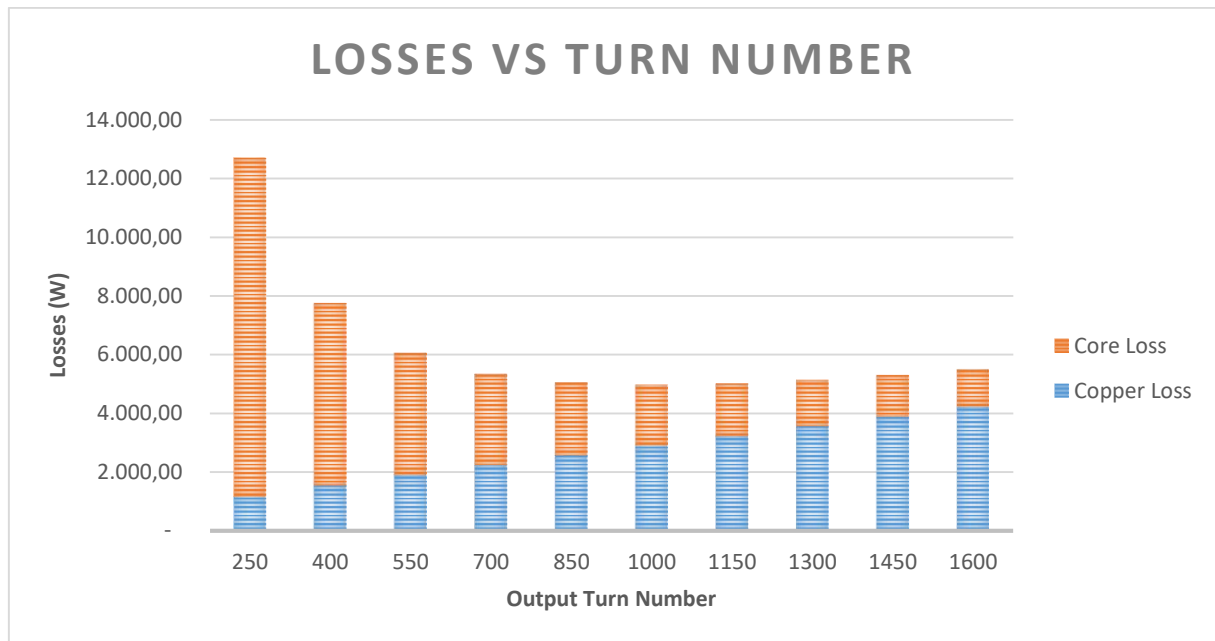


Figure 15: Core and Copper Losses vs Output Turn Numbers

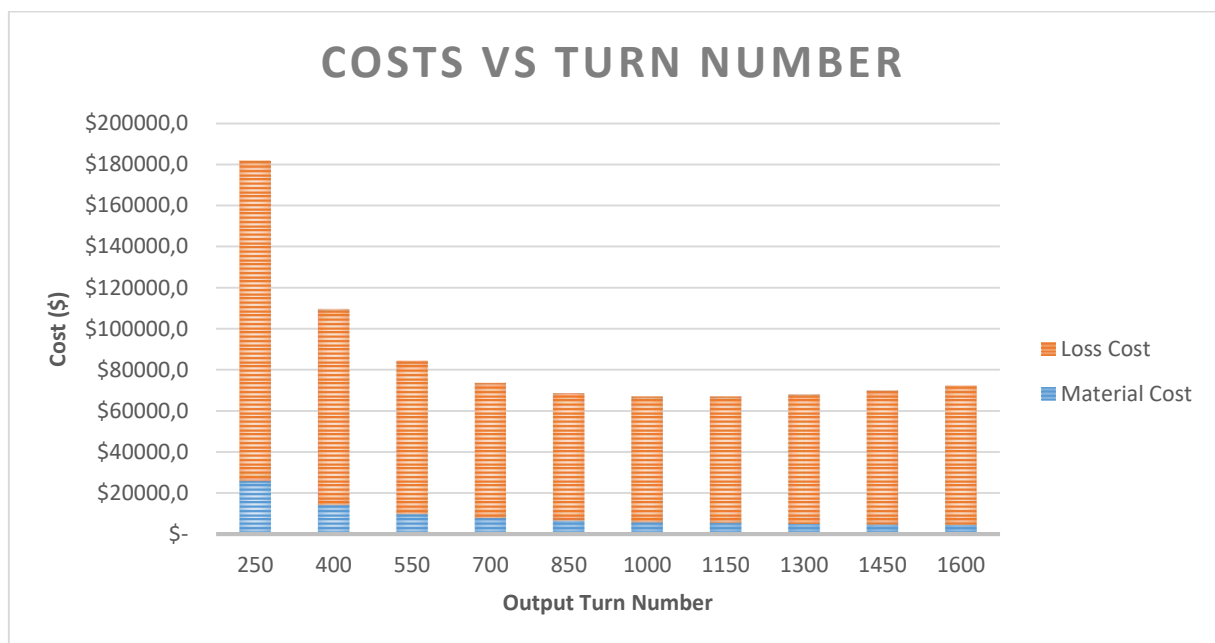


Figure 16: Material and Loss Costs vs Turn Numbers

For 1000 of turns at the output side, different core types are compared with each other in Figure 17 and 18. Figure 17 shows that, the copper loss does not vary if the turns number is kept constant. On the other hand, the core losses vary significantly. More interestingly, even though the steel has the lowest loss, since its lamination is taken as 0.35mm, its price is higher, the minimum cost is obtained for M250-50A because its lamination thickness is 0.5 mm so it is cheaper. Also, nearly all of the cores' material cost is negligible comparing to energy loss cost in 20 years. Therefore, it clearly shows that for cost saving, the long term calculation is definitely required. It is important to say that, since the price for each core is not available on the internet, the prices are set as inversely proportional to their lamination thickness.

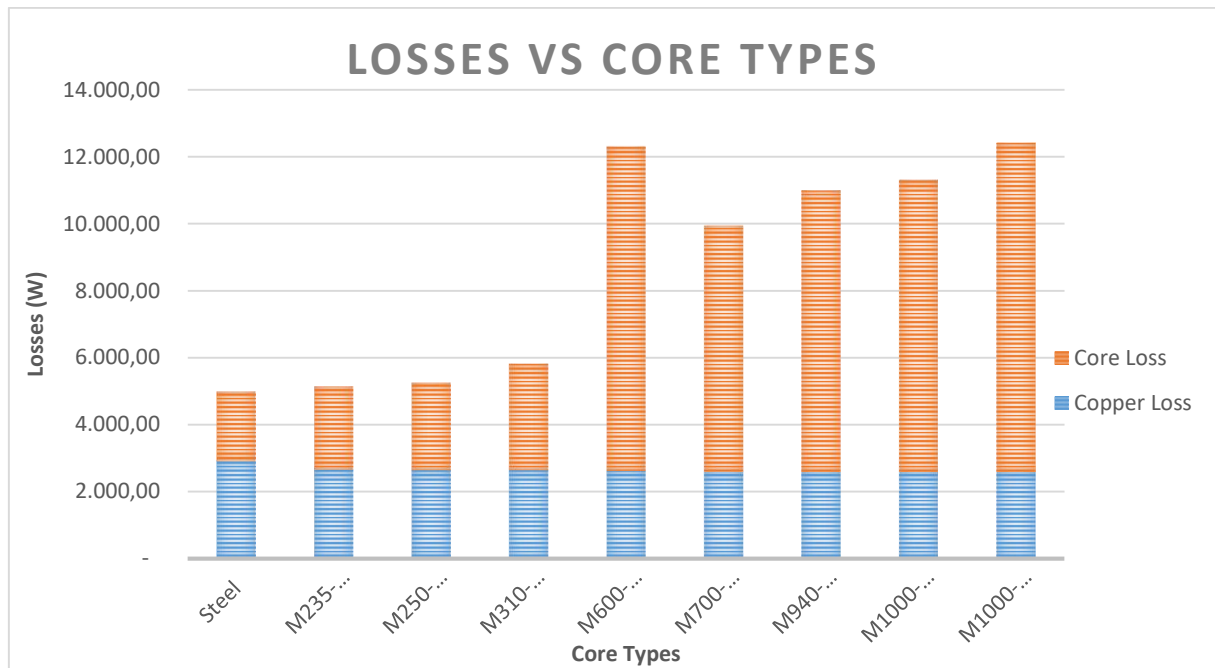


Figure 17: Losses for Different Core Types

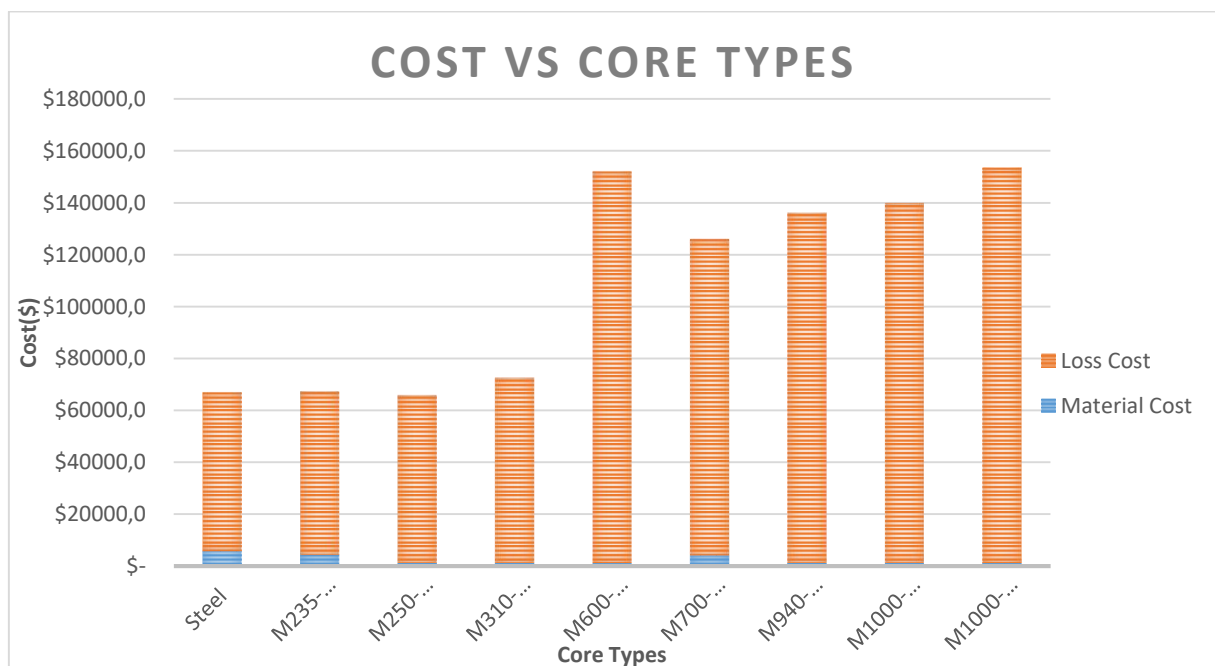


Figure 18: Costs for Different Core Types

Core types in order: Steel, M235-35A, M250-50A, M310-65A, M600-100A, M700-35A, M940-50A, M1000-65A, M1000-100A.