Homework 1

Furkan Kerim Yıldırım

28138

Question 1

a) The worst-case scenario for the Insertion Sort algorithm occurs when the input array is in descending order. For example, $[i, i-1, i-2, i-3, i-4, i-5, \dots]$.
b) If the input array is sorted in descending order, resulting in the worst-case time complexity of $\Theta(n^2)$.
c) The Insertion Sort algorithm would need to make the maximum number of shifts and comparisons to sort the array, resulting in the worst-case time complexity.

Question 2

a) The best-case scenario for the Insertion Sort algorithm occurs when the input array is already sorted in ascending order. For example, $[i, i+1, i+2, i+3, i+4, i+5 \ \dots]$.
b) If the input array is and non-descending and already sorted in ascending order, resulting in the best-case time complexity of $\Theta(n)$.
c) The Insertion Sort algorithm would need to make only one comparison for each element to sort the array, resulting in the best-case time complexity.

Question 3

a) $(5n+4)^2 \leq cn^2 \qquad for\ all\ n\ \geq n_0$

$25n^2 + 40n + 16 \leq cn^2 \quad for\ all\ n\ \geq n_0$

$25 + \frac{40}{n} + \frac{16}{n^2} \leq c \qquad for\ all\ n\ \geq n_0$

$25 + \frac{40}{2} + \frac{16}{2^2} \leq c \qquad for\ all\ n\ \geq (n_0 = 2)$

$49\ \leq c \qquad for\ all\ n\ \geq 2$

So, if we choose $c = 49$, then we have $(5n+4)^2 \leq 49n^2$ for all $n\ \geq 2$.

b) $(5n+4)^2 \leq cn^2 \qquad for\ all\ n\ \geq n_0$

$(5n+4)^2 \leq 36n^2 \qquad for\ all\ n\ \geq n_0, \quad c = 36$

$25n^2 + 40n + 16 \leq 36n^2 \quad for\ all\ n\ \geq n_0, \quad c = 36$

$0 \leq 11n^2 - 40n - 16 \qquad for\ all\ n\ \geq n_0, \quad c = 36$

$0 \leq (11n+4)(n-4) \qquad for\ all\ n\ \geq n_0, \quad c = 36$

So if we choose $n_0 = 4$, then we have $(5n+4)^2 \leq 36n^2$ for all $n\ \geq 4$.

Question 4

$$2^{2^n} > n2^n > n! > \lg(n!) > (\lg n)! > lg^2 n$$