Homework 4

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Question 1

For $V = \{s, u, v, t\}, E = \{(s, u), (s, v), (u, t), (v, t)\}$, s: source node, t: sink node; the capacity functions are $c(s, u) = 1, c(s, v) = 1, c(u, t) = 1, c(v, t) = 1$. This, flow network can have at least two different max-flow functions.

Max-Flow Function 1: $F(s, u) = 0, F(s, v) = 1, F(u, t) = 0, F(v, t) = 1$
Maximum Flow Value: F = 1,

Max-Flow Function 2: $F(s, u) = 1, F(s, v) = 0, F(u, t) = 1, F(v, t) = 0$
Maximum Flow Value: F = 1

Question 2

Claim A is False. We can use the same flow network which we used in Question 1 for counter example.

$G = (V, E, s, t, c), V = \{s, u, v, t\}, E = \{(s, u), (s, v), (u, t), (v, t)\}$

Capacity of edges: $c(s, u) = 1, c(s, v) = 1, c(u, t) = 1, c(v, t) = 1$

Max-Flow Function 1: $F(s, u) = 0, F(s, v) = 1, F(u, t) = 0, F(v, t) = 1$
Maximum Flow Value: F = 1,

Max-Flow Function 2: $F(s, u) = 1, F(s, v) = 0, F(u, t) = 1, F(v, t) = 0$
Maximum Flow Value: F = 1

As you can see, there are 2 Maximum Flow function has same value. Therefore, Claim A is False.

Question 3

No, $F(u, v) = f_1(u, v) + f_2(u, v)$ is not guaranteed to be a flow on G in general. Consider the flow network $G = (V, E, s, t, c), V = \{s, u, t\}, E = \{(s, u), (u, t)\}$ and capacity functions are $c(s, u) = 5, c(u, t) = 5$.

Let define two flow functions on G called f_1 and f_2.

$f_1(s, u) = f_1(u, t) = 3$, max value is 3.

$f_2(s, u) = f_2(u, t) = 4$, max value is 4.

$F(u, v) = f_1(u, v) + f_2(u, v) => F(s, u) = f_1(s, u) + f_2(s, u) = 7$. This example exceeds the total capacity constraint. That's why it's False.