Homework 2

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Question 1

a) To select the first m students to send n students to the "Center for Advanced Algorithms" in Westeros, we can use the Counting Sort with assuming students' grades are integer from 0 to 100. First the occurrences of each note are counted and recorded by calculating the cumulative sum of the numbers. Then, using the cumulative sum of each grade, the m students with the highest grades are taken in descending order of grades.

b) Time complexity of counting sort is $O(n + k)$, where n is the number of students and k is the range of the grades. students grades between 0 and 100, therefore, the time complexity of counting sort is $O(n + 101)$. Since 101 is a constant, the time complexity can be simplified to $O(n)$. After sorting the grades of students, we need to select the top m students from the sorted list, which can be done in $O(m)$ time complexity by simply iterating over the first m grades of the sorted list. Therefore, the overall time complexity of the algorithm is $O(n + m)$, which is a tight upper bound for the runtime complexity of the algorithm.

Question 2

Given a set of n friends $F = \{f_1, f_2, \ldots, f_n\}$ and a set of pairs of friends who know each other, represented as $K = \{(f_i, f_j)\}$, determine whether it is possible to send a unique message to each friend in F such that no two friends who know each other get the same message.

For $F = \{f_1, f_2, \ldots, f_n\}$ and $K = \{(f_i, f_j)\}$, is there a function $M: F \to S$ such that $|S| = n$, and for every $(f_i, f_j)) \in K$, M$(f_i) \neq M(f_j)$?

Question 3

a) False. Red-black tree insertion requires $O(\log n)$ in the worst case. This is because the height of a red-black tree is limited to $2 \times \log(n + 1)$, so insertion may require balancing operations to maintain the red-black tree properties.

b) False. Red-black tree insertion requires $O(\log n)$ in the worst case. This is because the insertion operation may violate one or more of the red-black tree properties, and fixing those violations often requires recoloring nodes in tree.

c) False. Walking a red-black tree in pre-order takes O(n) time complexity. This is because pre-order traversal visits each node in the tree exactly once. Also, time spend is constant for each node.

Question 4

a) It stands for "Nondeterministic Polynomial Time."

b) When the solution should be verified in polynomial time, we say problem is NP problem. This can be if Q is in NP and there exist NP-complete problem P such that $P \to Q$.